Computer algebra independent integration tests

1-Algebraic-functions/1.1-Binomial-products/1.1.3-General/1.1.3.3-a+b-x^n-^p-c+d-x^n-^q

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [385]. This is test number [26].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (385)	% 0. (0)
Mathematica	% 99.48 (383)	% 0.52 (2)
Maple	% 51.17 (197)	% 48.83 (188)
Maxima	% 16.88 (65)	% 83.12 (320)
Fricas	% 55.32 (213)	% 44.68 (172)
Sympy	% 38.18 (147)	% 61.82 (238)
Giac	% 34.03 (131)	% 65.97 (254)

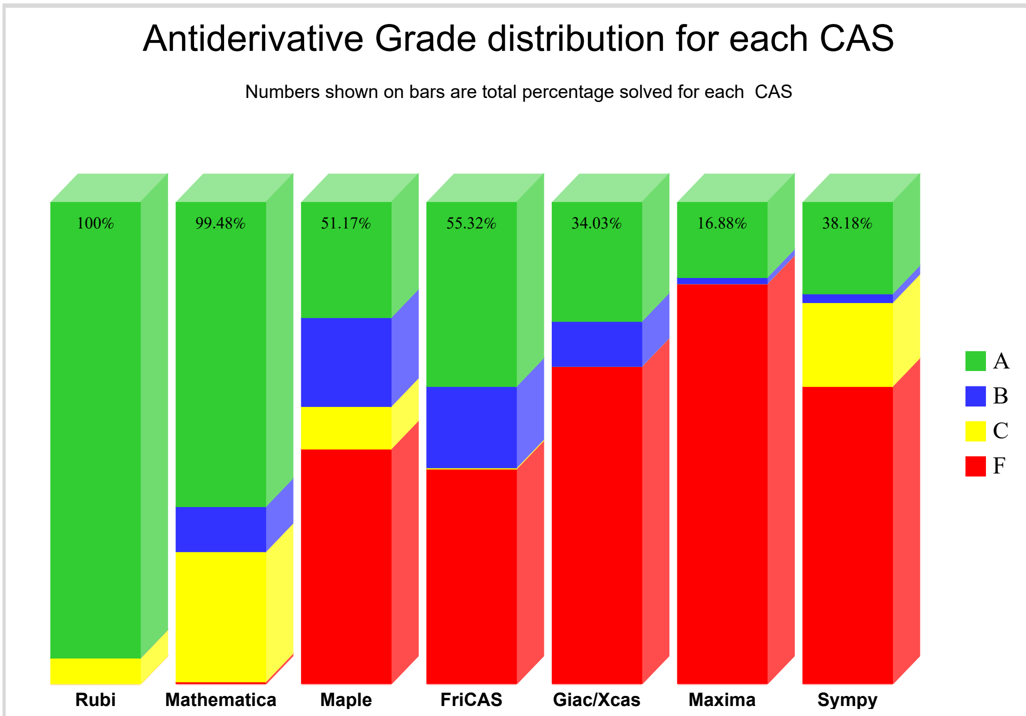
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

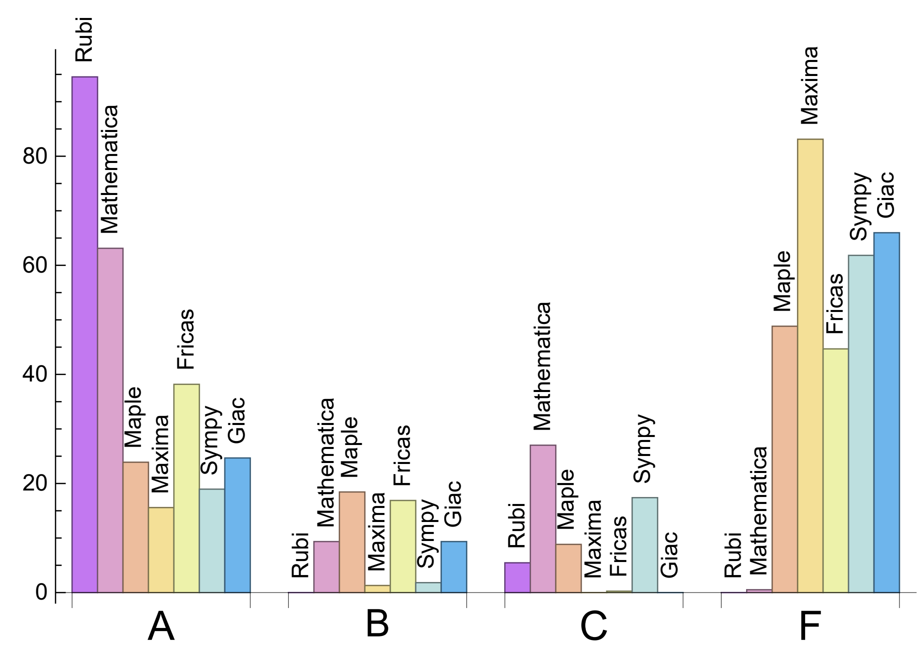
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	94.55	0.	5.45	0.
Mathematica	63.12	9.35	27.01	0.52
Maple	23.9	18.44	8.83	48.83
Maxima	15.58	1.3	0.	83.12
Fricas	38.18	16.88	0.26	44.68
Sympy	18.96	1.82	17.4	61.82
Giac	24.68	9.35	0.	65.97

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.21	169.1	1.01	118.	1.
Mathematica	0.87	197.78	1.38	131.	0.97
Maple	0.01	410.13	2.22	198.	1.34
Maxima	1.04	145.86	1.55	128.	1.38
Fricas	6.3	1215.	6.35	527.	4.83
Sympy	18.6	280.98	2.54	163.	1.31
Giac	1.22	332.5	2.15	234.	1.78

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {34, 35, 36, 37, 38, 39, 86, 87, 88, 91, 92, 98, 99, 102, 103, 109, 110, 111, 115, 116}

Mathematica {34, 35, 36, 37, 38, 39, 70, 71, 75, 79, 80, 81, 82, 83, 84, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 136, 137, 141, 142, 143, 144, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 209, 210, 211, 212, 213, 214, 215, 218, 221, 222, 269, 274, 306, 307, 312, 317, 318, 319, 320, 328, 334, 335, 352, 384}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima abs_integrate was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error Exception raised: NotImplementedError

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function LeafSize is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-s

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

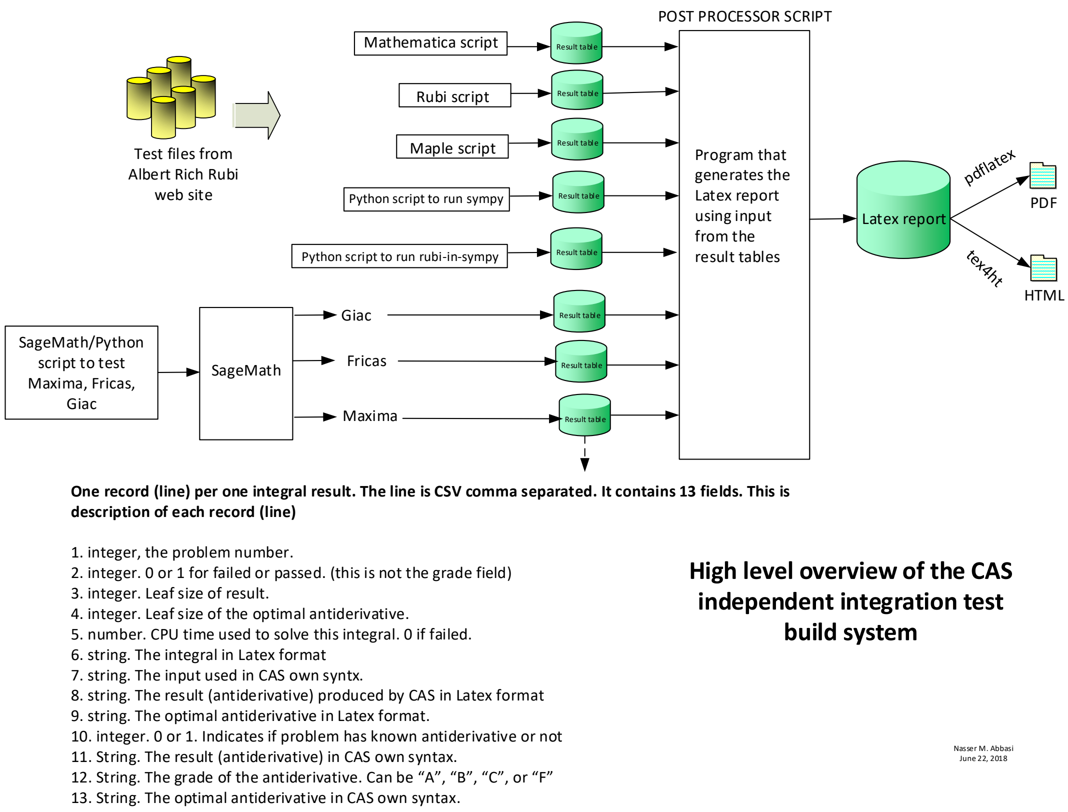
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 89, 90, 93, 94, 95, 96, 97, 100, 101, 104, 105, 106, 107, 108, 112, 113, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385 }

B grade: { }

C grade: { 34, 35, 36, 37, 38, 39, 86, 87, 88, 91, 92, 98, 99, 102, 103, 109, 110, 111, 114, 115, 116 }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 30, 31, 32, 33, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 58, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 89, 116, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 134, 135, 138, 139, 140, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 195, 216, 217, 219, 220, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 253, 259, 266, 267, 268, 270, 272, 274, 275, 276, 277, 278, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 301, 302, 303, 304, 305, 308, 309, 310, 311, 313, 314, 315, 316, 320, 321, 322, 324, 325, 326, 327, 329, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 353, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 376, 378, 380, 381, 383, 385 }

B grade: { 82, 93, 94, 95, 96, 97, 104, 105, 106, 107, 108, 117, 118, 119, 120, 121, 133, 136, 137, 142, 143, 144, 218, 221, 222, 269, 312, 317, 318, 319, 328, 334, 335, 352, 354, 384 }

C grade: { 27, 29, 34, 35, 36, 37, 38, 39, 56, 57, 59, 70, 71, 86, 87, 88, 90, 91, 92, 98, 99, 100, 101, 102, 103, 109, 110, 111, 112, 113, 114, 115, 141, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 223, 252, 254, 255, 256, 257, 258, 260, 261, 262, 263, 264, 265, 271, 273, 279, 298, 299, 300, 306, 307, 323, 330, 331, 332, 333, 375, 377, 379 }

F grade: { 336, 382}

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 13, 17, 18, 19, 24, 25, 26, 30, 31, 32, 33, 44, 45, 46, 47, 60, 61, 62, 63, 75, 76, 77, 78, 85, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 159, 163, 164, 165, 170, 171, 172, 191, 192, 224, 248, 270, 271, 272, 273, 275, 276, 277, 278, 279, 280, 284, 285, 286, 287, 292, 293, 294, 337, 338, 339, 349, 351, 353, 355, 356, 357, 359, 361, 366, 369, 371, 374, 376, 378, 379 }

B grade: { 11, 12, 14, 15, 16, 20, 21, 22, 23, 157, 158, 160, 161, 162, 166, 167, 168, 169, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 324, 340, 341, 342, 363, 365, 367, 373, 375, 377 }

C grade: { 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 343, 344, 345, 346, 347, 348, 350, 352, 354, 358, 360, 362, 364, 368, 370, 372 }

F grade: { 27, 28, 29, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 269, 274, 281, 282, 283, 288, 289, 290, 291, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 380, 381, 382, 383, 384, 385 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 8, 9, 10, 30, 31, 44, 45, 60, 61, 62, 63, 75, 76, 77, 78, 85, 146, 147, 148, 149, 153, 154, 155, 156, 276, 277, 278, 279, 280, 337, 338, 339, 343, 344, 345, 348, 349, 350, 351, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 368, 369, 370, 371, 372, 378, 379

B grade: { 32, 33, 46, 47, 352 }

C grade: { }

F grade: { 5, 6, 7, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 150, 151, 152, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 340, 341, 342, 346, 347, 363, 364, 365, 366, 367, 373, 374, 375, 376, 377, 380, 381, 382, 383, 384, 385 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 14, 15, 16, 17, 18, 19, 24, 25, 30, 31, 32, 33, 40, 41, 44, 45, 46, 47, 56, 57, 58, 60, 61, 62, 63, 70, 71, 72, 75, 76, 77, 78, 85, 145, 146, 147, 148, 149, 153, 154, 155, 156, 224, 225, 226, 227, 228, 229, 231, 232, 233, 234, 235, 236, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 252, 253, 254, 255, 259, 260, 261, 262, 266, 267, 268, 270, 275, 276, 277, 278, 279, 280, 287, 322, 323, 324, 329, 330, 331, 332, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 373, 374, 375, 376, 377, 378, 379, 381, 385 }

B grade: { 7, 12, 13, 20, 21, 22, 23, 27, 28, 29, 36, 42, 43, 59, 73, 74, 86, 87, 88, 98, 99, 110, 111, 150, 151, 152, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 191, 192, 193, 194, 205, 206, 216, 230, 237, 251, 256, 257, 258, 263, 264, 265, 284, 285, 286, 292, 293, 294, 321, 372 }

C grade: { 380 }

F grade: { 26, 34, 35, 37, 38, 39, 48, 49, 50, 51, 52, 53, 54, 55, 64, 65, 66, 67, 68, 69, 79, 80, 81, 82, 83, 84, 89, 90, 91, 92, 93, 94, 95, 96, 97, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 207, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220, 221, 222, 223, 269, 271, 272, 273, 274, 281, 282, 283, 288, 289, 290, 291, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 325, 326, 327, 328, 333, 334, 335, 336, 382, 383, 384 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 167, 168, 169, 170, 224, 225, 226, 227, 231, 232, 233, 238, 239, 240, 241, 245, 246, 247, 248, 255, 275, 276, 277, 278, 279, 280, 284, 285, 286, 287, 294, 324 }

B grade: { 30, 60, 234, 254, 261, 262, 270 }

C grade: { 27, 28, 29, 40, 41, 48, 49, 50, 56, 57, 58, 59, 64, 65, 66, 67, 68, 70, 71, 72, 79, 80, 81, 82, 141, 220, 288, 289, 295, 298, 299, 300, 301, 308, 314, 315, 316, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 378 }

F grade: { 19, 25, 26, 31, 32, 33, 34, 35, 36, 37, 38, 39, 42, 43, 44, 45, 46, 47, 51, 52, 53, 54, 55, 61, 62, 63, 69, 73, 74, 75, 76, 77, 78, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 142, 143, 144, 145, 164, 165, 166, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 221, 222, 223, 228, 229, 230, 235, 236, 237, 242, 243, 244, 249, 250, 251, 252, 253, 256, 257, 258, 259, 260, 263, 264, 265, 266, 267, 268, 269, 271, 272, 273, 274, 281, 282, 283, 290, 291, 292, 293, 296, 297, 302, 303, 304, 305, 306, 307, 309, 310, 311, 312, 313, 317, 318, 319, 320, 321, 322, 323, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 377, 379, 380, 381, 382, 383, 384, 385 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 16, 17, 18, 19, 22, 23, 24, 25, 26, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 162, 163, 165, 168, 169, 170, 171, 245, 246, 247, 249, 250, 251, 252, 253, 254, 255, 256, 258, 259, 260, 261, 262, 263, 265, 270, 275, 276, 277, 278, 280, 337, 338, 339, 340, 341, 343, 344, 345, 346, 348, 349, 350, 351, 352, 353, 354, 358, 359, 360, 361, 362, 363, 364, 368, 369, 370, 371, 378 }

B grade: { 14, 15, 20, 21, 160, 161, 166, 167, 227, 230, 237, 244, 248, 257, 264, 284, 285, 286, 287, 292, 293, 294, 342, 347, 355, 356, 357, 365, 366, 367, 372, 373, 374, 375, 376, 377 }

C grade: { }

F grade: { 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 164, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 228, 229, 231, 232, 233, 234, 235, 236, 238, 239, 240, 241, 242, 243, 266, 267, 268, 269, 271, 272, 273, 274, 279, 281, 282, 283, 288, 289, 290, 291, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 379, 380, 381, 382, 383, 384, 385 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	94	97	130	234	104	131
normalized size	1	1.	1.	1.03	1.38	2.49	1.11	1.39
time (sec)	N/A	0.072	0.02	0.	0.979	1.268	0.101	1.11

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	70	73	95	182	80	100
normalized size	1	1.	1.	1.04	1.36	2.6	1.14	1.43
time (sec)	N/A	0.043	0.013	0.001	0.955	1.343	0.077	1.104

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	65	123	51	68
normalized size	1	1.	1.	0.98	1.3	2.46	1.02	1.36
time (sec)	N/A	0.028	0.008	0.001	0.941	1.287	0.101	1.09

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	32	66	26	35
normalized size	1	1.	1.	0.89	1.14	2.36	0.93	1.25
time (sec)	N/A	0.013	0.005	0.	0.959	1.393	0.075	1.114

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	128	195	0	900	71	217
normalized size	1	1.	0.89	1.35	0.	6.25	0.49	1.51
time (sec)	N/A	0.094	0.078	0.005	0.	1.704	0.831	1.13

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	145	221	0	1233	97	246
normalized size	1	1.	0.86	1.31	0.	7.3	0.57	1.46
time (sec)	N/A	0.082	0.092	0.009	0.	1.706	0.947	1.11

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	175	249	0	1648	133	273
normalized size	1	1.	0.89	1.26	0.	8.37	0.68	1.39
time (sec)	N/A	0.106	0.132	0.01	0.	1.723	1.363	1.148

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	122	125	167	313	139	178
normalized size	1	1.	1.	1.02	1.37	2.57	1.14	1.46
time (sec)	N/A	0.071	0.016	0.	0.957	1.317	0.087	1.072

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	82	87	111	215	90	123
normalized size	1	1.	1.	1.06	1.35	2.62	1.1	1.5
time (sec)	N/A	0.046	0.011	0.001	0.968	1.304	0.112	1.103

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	65	123	51	68
normalized size	1	1.	1.	0.98	1.3	2.46	1.02	1.36
time (sec)	N/A	0.029	0.007	0.	0.956	1.298	0.101	1.119

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	167	334	0	1183	156	336
normalized size	1	1.	0.97	1.93	0.	6.84	0.9	1.94
time (sec)	N/A	0.128	0.091	0.003	0.	1.685	1.309	1.244

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	210	367	0	1635	189	358
normalized size	1	1.	1.03	1.81	0.	8.05	0.93	1.76
time (sec)	N/A	0.244	0.194	0.01	0.	1.721	1.758	1.12

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	234	388	0	2296	233	400
normalized size	1	1.	0.91	1.5	0.	8.9	0.9	1.55
time (sec)	N/A	0.233	0.271	0.01	0.	1.748	2.874	1.58

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	253	661	0	1916	369	622
normalized size	1	1.	1.	2.62	0.	7.6	1.46	2.47
time (sec)	N/A	0.191	0.116	0.005	0.	1.726	2.14	1.126

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	203	486	0	1544	255	473
normalized size	1	1.	0.98	2.34	0.	7.42	1.23	2.27
time (sec)	N/A	0.148	0.091	0.003	0.	1.789	2.034	1.101

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	167	334	0	1183	156	336
normalized size	1	1.	0.97	1.93	0.	6.84	0.9	1.94
time (sec)	N/A	0.124	0.113	0.003	0.	1.705	1.347	1.139

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	129	195	0	926	71	217
normalized size	1	1.	0.89	1.34	0.	6.39	0.49	1.5
time (sec)	N/A	0.078	0.065	0.002	0.	1.716	0.781	1.098

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	224	222	0	606	447	375
normalized size	1	1.	0.78	0.77	0.	2.1	1.55	1.3
time (sec)	N/A	0.147	0.11	0.006	0.	2.123	43.129	1.126

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	346	336	406	0	994	0	598
normalized size	1	1.	0.97	1.17	0.	2.87	0.	1.73
time (sec)	N/A	0.27	0.204	0.012	0.	44.628	0.	1.121

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	313	905	0	3490	536	822
normalized size	1	1.	0.98	2.83	0.	10.91	1.68	2.57
time (sec)	N/A	0.298	0.253	0.013	0.	1.811	15.539	1.111

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	260	708	0	2808	403	643
normalized size	1	1.	0.97	2.65	0.	10.52	1.51	2.41
time (sec)	N/A	0.226	0.217	0.011	0.	1.892	7.678	1.143

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	227	529	0	2225	289	495
normalized size	1	1.	0.97	2.26	0.	9.51	1.24	2.12
time (sec)	N/A	0.22	0.159	0.009	0.	1.713	4.005	1.115

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	205	367	0	1661	189	350
normalized size	1	1.	1.01	1.81	0.	8.18	0.93	1.72
time (sec)	N/A	0.231	0.213	0.008	0.	1.711	2.377	1.129

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	145	221	0	1233	97	246
normalized size	1	1.	0.86	1.31	0.	7.3	0.57	1.46
time (sec)	N/A	0.084	0.09	0.007	0.	1.706	1.076	1.124

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	346	337	406	0	1004	0	598
normalized size	1	1.	0.97	1.17	0.	2.9	0.	1.73
time (sec)	N/A	0.255	0.213	0.01	0.	43.049	0.	1.171

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	419	419	381	606	0	0	0	896
normalized size	1	1.	0.91	1.45	0.	0.	0.	2.14
time (sec)	N/A	0.493	0.64	0.014	0.	0.	0.	1.653

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	B	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	62	0	0	1022	80	0
normalized size	1	1.	0.55	0.	0.	9.12	0.71	0.
time (sec)	N/A	0.032	0.066	0.295	0.	1.665	5.08	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	134	0	0	950	76	0
normalized size	1	1.	1.47	0.	0.	10.44	0.84	0.
time (sec)	N/A	0.02	0.075	0.229	0.	1.715	3.38	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	B	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	62	0	0	976	70	0
normalized size	1	1.	0.73	0.	0.	11.48	0.82	0.
time (sec)	N/A	0.013	0.036	0.226	0.	1.827	12.763	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	28	25	68	96	190	0
normalized size	1	1.	0.6	0.53	1.45	2.04	4.04	0.
time (sec)	N/A	0.009	0.017	0.004	1.095	1.617	100.391	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	40	37	115	150	0	0
normalized size	1	1.	0.73	0.67	2.09	2.73	0.	0.
time (sec)	N/A	0.013	0.019	0.003	0.973	1.744	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	51	48	161	197	0	0
normalized size	1	1.	0.69	0.65	2.18	2.66	0.	0.
time (sec)	N/A	0.019	0.021	0.003	1.191	1.53	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	62	59	207	262	0	0
normalized size	1	1.	0.67	0.63	2.23	2.82	0.	0.
time (sec)	N/A	0.026	0.023	0.004	1.384	1.482	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F(-1)	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	483	56	232	0	0	0	0	0
normalized size	1	0.12	0.48	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.273	0.592	0.	0.	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	464	55	217	0	0	0	0	0
normalized size	1	0.12	0.47	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.115	0.472	0.	0.	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	B	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	398	58	151	0	0	1494	0	0
normalized size	1	0.15	0.38	0.	0.	3.75	0.	0.
time (sec)	N/A	0.026	0.143	0.42	0.	170.566	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	452	58	153	0	0	0	0	0
normalized size	1	0.13	0.34	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.051	0.428	0.	0.	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F(-1)	F(-2)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	473	58	213	0	0	0	0	0
normalized size	1	0.12	0.45	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	0.116	0.453	0.	0.	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F(-1)	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	492	58	240	0	0	0	0	0
normalized size	1	0.12	0.49	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.16	0.423	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	151	0	0	1076	126	0
normalized size	1	1.	1.09	0.	0.	7.74	0.91	0.
time (sec)	N/A	0.057	0.137	0.233	0.	2.63	4.777	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	141	0	0	1029	121	0
normalized size	1	1.	1.18	0.	0.	8.57	1.01	0.
time (sec)	N/A	0.042	0.06	0.216	0.	2.601	3.867	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	137	0	0	1062	0	0
normalized size	1	1.	1.21	0.	0.	9.4	0.	0.
time (sec)	N/A	0.042	0.083	0.41	0.	2.397	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	131	0	0	1253	0	0
normalized size	1	1.	1.19	0.	0.	11.39	0.	0.
time (sec)	N/A	0.04	0.067	0.368	0.	1.992	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	40	37	142	142	0	0
normalized size	1	1.	0.53	0.49	1.87	1.87	0.	0.
time (sec)	N/A	0.021	0.015	0.005	1.002	1.988	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	51	48	209	203	0	0
normalized size	1	1.	0.49	0.46	1.99	1.93	0.	0.
time (sec)	N/A	0.035	0.032	0.007	0.977	1.947	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	62	59	278	259	0	0
normalized size	1	1.	0.63	0.6	2.84	2.64	0.	0.
time (sec)	N/A	0.035	0.035	0.007	0.964	2.042	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	73	70	347	311	0	0
normalized size	1	1.	0.62	0.6	2.97	2.66	0.	0.
time (sec)	N/A	0.044	0.037	0.007	0.981	2.19	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	97	0	0	0	168	0
normalized size	1	1.	1.03	0.	0.	0.	1.79	0.
time (sec)	N/A	0.034	0.048	0.215	0.	0.	5.115	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	85	0	0	0	126	0
normalized size	1	1.	0.9	0.	0.	0.	1.34	0.
time (sec)	N/A	0.034	0.038	0.217	0.	0.	2.983	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	75	0	0	0	121	0
normalized size	1	1.	0.8	0.	0.	0.	1.29	0.
time (sec)	N/A	0.034	0.036	0.233	0.	0.	3.742	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	62	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	0.038	0.421	0.	0.	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	70	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.057	0.376	0.	0.	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	85	0	0	0	0	0
normalized size	1	1.	1.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.07	0.358	0.	0.	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	95	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.033	0.084	0.37	0.	0.	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	106	0	0	0	0	0
normalized size	1	1.	1.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	0.103	0.388	0.	0.	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	75	0	0	1222	170	0
normalized size	1	1.	0.43	0.	0.	7.02	0.98	0.
time (sec)	N/A	0.059	0.066	0.214	0.	2.064	6.709	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	72	0	0	1087	82	0
normalized size	1	1.	0.51	0.	0.	7.71	0.58	0.
time (sec)	N/A	0.045	0.069	0.222	0.	2.022	3.23	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	141	0	0	965	78	0
normalized size	1	1.	1.27	0.	0.	8.69	0.7	0.
time (sec)	N/A	0.03	0.13	0.217	0.	1.975	2.317	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	B	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	61	0	0	1187	71	0
normalized size	1	1.	0.62	0.	0.	11.99	0.72	0.
time (sec)	N/A	0.024	0.035	0.221	0.	1.99	10.404	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	37	34	69	117	190	0
normalized size	1	1.	0.79	0.72	1.47	2.49	4.04	0.
time (sec)	N/A	0.01	0.02	0.005	0.958	1.661	96.653	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	59	57	116	188	0	0
normalized size	1	1.	0.65	0.63	1.27	2.07	0.	0.
time (sec)	N/A	0.028	0.028	0.003	0.965	1.712	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	80	81	162	263	0	0
normalized size	1	1.	0.66	0.67	1.34	2.17	0.	0.
time (sec)	N/A	0.035	0.031	0.006	0.944	1.657	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	100	105	208	352	0	0
normalized size	1	1.	0.66	0.7	1.38	2.33	0.	0.
time (sec)	N/A	0.047	0.038	0.004	0.974	1.789	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	77	0	0	0	265	0
normalized size	1	1.	0.91	0.	0.	0.	3.12	0.
time (sec)	N/A	0.023	0.037	0.22	0.	0.	7.031	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	75	0	0	0	170	0
normalized size	1	1.	0.9	0.	0.	0.	2.05	0.
time (sec)	N/A	0.022	0.062	0.224	0.	0.	3.79	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	72	0	0	0	82	0
normalized size	1	1.	0.88	0.	0.	0.	1.	0.
time (sec)	N/A	0.021	0.067	0.208	0.	0.	2.037	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	73	0	0	0	78	0
normalized size	1	1.	0.89	0.	0.	0.	0.95	0.
time (sec)	N/A	0.022	0.038	0.212	0.	0.	1.737	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	66	0	0	0	78	0
normalized size	1	1.	0.71	0.	0.	0.	0.84	0.
time (sec)	N/A	0.025	0.034	0.224	0.	0.	14.131	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	75	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.039	0.211	0.	0.	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	A	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	262	262	176	0	0	1713	270	0
normalized size	1	1.	0.67	0.	0.	6.54	1.03	0.
time (sec)	N/A	0.165	4.871	0.22	0.	1.773	14.929	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	A	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	219	219	179	0	0	1519	131	0
normalized size	1	1.	0.82	0.	0.	6.94	0.6	0.
time (sec)	N/A	0.166	4.149	0.219	0.	1.809	4.884	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	172	0	0	1341	126	0
normalized size	1	1.	0.98	0.	0.	7.66	0.72	0.
time (sec)	N/A	0.096	5.138	0.215	0.	1.755	3.903	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	168	0	0	1538	0	0
normalized size	1	1.	1.06	0.	0.	9.67	0.	0.
time (sec)	N/A	0.102	5.149	0.416	0.	1.736	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	180	0	0	1617	0	0
normalized size	1	1.	1.18	0.	0.	10.64	0.	0.
time (sec)	N/A	0.068	5.211	0.376	0.	1.761	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	78	78	126	76	147	217	0	0
normalized size	1	1.	1.62	0.97	1.88	2.78	0.	0.
time (sec)	N/A	0.021	0.08	0.006	0.96	1.669	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	106	115	215	328	0	0
normalized size	1	1.	0.61	0.66	1.24	1.89	0.	0.
time (sec)	N/A	0.073	5.061	0.007	0.972	1.737	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	138	156	284	433	0	0
normalized size	1	1.	0.65	0.74	1.35	2.05	0.	0.
time (sec)	N/A	0.127	5.074	0.007	0.978	1.847	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	169	197	352	544	0	0
normalized size	1	1.	0.67	0.78	1.39	2.15	0.	0.
time (sec)	N/A	0.207	5.086	0.008	0.97	2.066	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	135	135	177	0	0	0	418	0
normalized size	1	1.	1.31	0.	0.	0.	3.1	0.
time (sec)	N/A	0.067	2.214	0.216	0.	0.	13.769	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	133	133	176	0	0	0	270	0
normalized size	1	1.	1.32	0.	0.	0.	2.03	0.
time (sec)	N/A	0.077	3.316	0.23	0.	0.	6.099	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	131	131	179	0	0	0	131	0
normalized size	1	1.	1.37	0.	0.	0.	1.	0.
time (sec)	N/A	0.062	2.282	0.222	0.	0.	2.903	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	132	132	304	0	0	0	126	0
normalized size	1	1.	2.3	0.	0.	0.	0.95	0.
time (sec)	N/A	0.075	4.109	0.217	0.	0.	2.889	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	146	146	171	0	0	0	0	0
normalized size	1	1.	1.17	0.	0.	0.	0.	0.
time (sec)	N/A	0.095	2.525	0.427	0.	0.	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	147	147	171	0	0	0	0	0
normalized size	1	1.	1.16	0.	0.	0.	0.	0.
time (sec)	N/A	0.088	4.471	0.369	0.	0.	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	120	134	246	356	0	0
normalized size	1	1.	1.1	1.23	2.26	3.27	0.	0.
time (sec)	N/A	0.036	0.037	0.005	0.976	1.657	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	B	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	331	62	655	0	0	1486	0	0
normalized size	1	0.19	1.98	0.	0.	4.49	0.	0.
time (sec)	N/A	0.028	0.789	0.426	0.	61.647	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	B	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	273	60	443	0	0	1258	0	0
normalized size	1	0.22	1.62	0.	0.	4.61	0.	0.
time (sec)	N/A	0.027	0.477	0.427	0.	6.457	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	B	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	233	59	161	0	0	1102	0	0
normalized size	1	0.25	0.69	0.	0.	4.73	0.	0.
time (sec)	N/A	0.027	0.154	0.428	0.	1.887	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	207	168	0	0	0	0	0
normalized size	1	1.4	1.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.192	0.149	0.414	0.	0.	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	179	238	256	0	0	0	0	0
normalized size	1	1.33	1.43	0.	0.	0.	0.	0.
time (sec)	N/A	0.193	0.43	0.396	0.	0.	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	226	621	621	0	0	0	0	0
normalized size	1	2.75	2.75	0.	0.	0.	0.	0.
time (sec)	N/A	2.58	1.927	0.403	0.	0.	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F(-1)	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	280	1172	1172	0	0	0	0	0
normalized size	1	4.19	4.19	0.	0.	0.	0.	0.
time (sec)	N/A	6.642	4.355	0.408	0.	0.	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	60	60	346	0	0	0	0	0
normalized size	1	1.	5.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.461	0.415	0.	0.	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	59	59	160	0	0	0	0	0
normalized size	1	1.	2.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.151	0.426	0.	0.	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	59	59	161	0	0	0	0	0
normalized size	1	1.	2.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.045	0.418	0.	0.	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	62	62	332	0	0	0	0	0
normalized size	1	1.	5.35	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.257	0.418	0.	0.	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	62	62	429	0	0	0	0	0
normalized size	1	1.	6.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	0.785	0.408	0.	0.	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	B	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	351	62	698	0	0	1810	0	0
normalized size	1	0.18	1.99	0.	0.	5.16	0.	0.
time (sec)	N/A	0.029	0.891	0.248	0.	40.081	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	B	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	301	60	450	0	0	1454	0	0
normalized size	1	0.2	1.5	0.	0.	4.83	0.	0.
time (sec)	N/A	0.028	0.607	0.405	0.	3.599	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	182	241	78	0	0	0	0	0
normalized size	1	1.32	0.43	0.	0.	0.	0.	0.
time (sec)	N/A	0.21	0.03	0.401	0.	0.	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	276	99	0	0	0	0	0
normalized size	1	1.27	0.46	0.	0.	0.	0.	0.
time (sec)	N/A	0.204	0.106	0.408	0.	0.	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F(-1)	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	261	625	625	0	0	0	0	0
normalized size	1	2.39	2.39	0.	0.	0.	0.	0.
time (sec)	N/A	1.934	1.451	0.403	0.	0.	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F(-1)	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	324	1214	1216	0	0	0	0	0
normalized size	1	3.75	3.75	0.	0.	0.	0.	0.
time (sec)	N/A	5.692	4.103	0.423	0.	0.	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	60	60	341	0	0	0	0	0
normalized size	1	1.	5.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.334	0.441	0.	0.	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	59	59	232	0	0	0	0	0
normalized size	1	1.	3.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.202	0.417	0.	0.	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	59	59	393	0	0	0	0	0
normalized size	1	1.	6.66	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.272	0.412	0.	0.	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	62	62	386	0	0	0	0	0
normalized size	1	1.	6.23	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.505	0.411	0.	0.	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	62	62	550	0	0	0	0	0
normalized size	1	1.	8.87	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.983	0.405	0.	0.	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F(-1)	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	541	62	1171	0	0	0	0	0
normalized size	1	0.11	2.16	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	2.193	0.263	0.	0.	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	B	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	458	62	908	0	0	2693	0	0
normalized size	1	0.14	1.98	0.	0.	5.88	0.	0.
time (sec)	N/A	0.028	1.742	0.258	0.	174.293	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	B	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	391	62	651	0	0	2080	0	0
normalized size	1	0.16	1.66	0.	0.	5.32	0.	0.
time (sec)	N/A	0.028	0.902	0.423	0.	16.033	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	217	276	79	0	0	0	0	0
normalized size	1	1.27	0.36	0.	0.	0.	0.	0.
time (sec)	N/A	0.239	0.026	0.438	0.	0.	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	326	153	0	0	0	0	0
normalized size	1	1.22	0.57	0.	0.	0.	0.	0.
time (sec)	N/A	0.243	0.161	0.432	0.	0.	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	167	168	0	0	0	0	0
normalized size	1	0.54	0.55	0.	0.	0.	0.	0.
time (sec)	N/A	0.31	0.226	0.441	0.	0.	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	F	F(-1)	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	377	428	428	0	0	0	0	0
normalized size	1	1.14	1.14	0.	0.	0.	0.	0.
time (sec)	N/A	2.742	2.266	0.435	0.	0.	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	F	F	F(-1)	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	463	1990	337	0	0	0	0	0
normalized size	1	4.3	0.73	0.	0.	0.	0.	0.
time (sec)	N/A	8.662	5.848	0.428	0.	0.	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	60	60	316	0	0	0	0	0
normalized size	1	1.	5.27	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	0.5	0.436	0.	0.	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	59	59	431	0	0	0	0	0
normalized size	1	1.	7.31	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.541	0.43	0.	0.	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	59	59	442	0	0	0	0	0
normalized size	1	1.	7.49	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.684	0.422	0.	0.	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	62	62	531	0	0	0	0	0
normalized size	1	1.	8.56	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	0.917	0.429	0.	0.	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	62	62	515	0	0	0	0	0
normalized size	1	1.	8.31	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	1.633	0.45	0.	0.	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	155	155	90	0	0	0	0	0
normalized size	1	1.	0.58	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	0.033	0.466	0.	0.	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	155	155	90	0	0	0	0	0
normalized size	1	1.	0.58	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	0.032	0.487	0.	0.	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	122	122	89	0	0	0	0	0
normalized size	1	1.	0.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	0.02	0.444	0.	0.	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	122	122	89	0	0	0	0	0
normalized size	1	1.	0.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	0.024	0.453	0.	0.	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	87	87	86	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.029	0.446	0.	0.	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	87	87	86	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	0.021	0.433	0.	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	87	87	89	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.016	0.022	0.449	0.	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	121	121	89	0	0	0	0	0
normalized size	1	1.	0.74	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	0.025	0.433	0.	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	121	121	89	0	0	0	0	0
normalized size	1	1.	0.74	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	0.026	0.433	0.	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	153	153	90	0	0	0	0	0
normalized size	1	1.	0.59	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	0.03	0.5	0.	0.	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	153	153	90	0	0	0	0	0
normalized size	1	1.	0.59	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	0.026	0.439	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	79	79	172	0	0	0	0	0
normalized size	1	1.	2.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	0.22	0.503	0.	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	176	106	0	0	0	0	0
normalized size	1	1.05	0.63	0.	0.	0.	0.	0.
time (sec)	N/A	0.126	0.051	0.431	0.	0.	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	85	90	0	0	0	0	0
normalized size	1	1.01	1.07	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.029	0.239	0.	0.	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0
normalized size	1	1.	2.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.172	0.44	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0
normalized size	1	1.	2.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.194	0.443	0.	0.	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	137	0	0	0	0	0
normalized size	1	1.	0.46	0.	0.	0.	0.	0.
time (sec)	N/A	0.304	5.064	0.41	0.	0.	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	106	0	0	0	0	0
normalized size	1	1.	0.6	0.	0.	0.	0.	0.
time (sec)	N/A	0.126	5.042	0.398	0.	0.	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	85	90	0	0	0	0	0
normalized size	1	0.91	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.042	0.031	0.225	0.	0.	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	44	44	196	0	0	0	34	0
normalized size	1	1.	4.45	0.	0.	0.	0.77	0.
time (sec)	N/A	0.01	0.155	0.25	0.	0.	15.311	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0
normalized size	1	1.	2.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	0.185	0.431	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0
normalized size	1	1.	2.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	0.191	0.461	0.	0.	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0
normalized size	1	1.	2.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.238	0.442	0.	0.	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	52	71	0	182	0	0
normalized size	1	1.	0.98	1.34	0.	3.43	0.	0.
time (sec)	N/A	0.019	0.033	0.003	0.	1.472	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	94	97	130	242	107	132
normalized size	1	1.	1.	1.03	1.38	2.57	1.14	1.4
time (sec)	N/A	0.068	0.021	0.001	0.942	1.099	0.08	1.106

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	70	73	95	182	76	100
normalized size	1	1.	1.	1.04	1.36	2.6	1.09	1.43
time (sec)	N/A	0.049	0.015	0.	0.94	1.025	0.073	1.106

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	65	123	53	68
normalized size	1	1.	1.	0.98	1.3	2.46	1.06	1.36
time (sec)	N/A	0.029	0.012	0.	0.948	1.05	0.068	1.092

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	32	66	26	35
normalized size	1	1.	1.	0.89	1.14	2.36	0.93	1.25
time (sec)	N/A	0.014	0.005	0.001	0.948	0.946	0.058	1.103

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	196	266	0	1315	87	331
normalized size	1	1.	0.88	1.19	0.	5.9	0.39	1.48
time (sec)	N/A	0.151	0.129	0.007	0.	1.368	0.589	1.1

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	212	295	0	1526	112	359
normalized size	1	1.	0.87	1.2	0.	6.23	0.46	1.47
time (sec)	N/A	0.147	0.172	0.008	0.	1.419	0.839	1.127

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	273	273	243	314	0	1752	151	386
normalized size	1	1.	0.89	1.15	0.	6.42	0.55	1.41
time (sec)	N/A	0.174	0.207	0.01	0.	1.398	1.65	1.135

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	154	163	213	413	185	234
normalized size	1	1.	1.	1.06	1.38	2.68	1.2	1.52
time (sec)	N/A	0.114	0.031	0.	0.95	1.111	0.091	1.112

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	122	125	167	315	139	178
normalized size	1	1.	1.	1.02	1.37	2.58	1.14	1.46
time (sec)	N/A	0.077	0.022	0.001	0.956	1.023	0.085	1.109

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	82	87	111	217	97	123
normalized size	1	1.	1.	1.06	1.35	2.65	1.18	1.5
time (sec)	N/A	0.049	0.017	0.001	0.945	1.088	0.076	1.096

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	65	123	53	68
normalized size	1	1.	1.	0.98	1.3	2.46	1.06	1.36
time (sec)	N/A	0.03	0.008	0.001	0.971	1.073	0.13	1.091

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	231	436	0	2547	187	477
normalized size	1	1.	0.91	1.72	0.	10.07	0.74	1.89
time (sec)	N/A	0.194	0.109	0.003	0.	1.507	1.418	1.097

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	298	475	0	2909	219	508
normalized size	1	1.	1.02	1.63	0.	10.	0.75	1.75
time (sec)	N/A	0.366	0.173	0.01	0.	1.573	2.745	1.135

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	349	319	499	0	3302	264	549
normalized size	1	1.	0.91	1.43	0.	9.46	0.76	1.57
time (sec)	N/A	0.266	0.186	0.008	0.	1.575	8.058	1.141

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	332	332	322	837	0	5701	430	833
normalized size	1	1.	0.97	2.52	0.	17.17	1.3	2.51
time (sec)	N/A	0.267	0.192	0.007	0.	1.728	2.906	1.11

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	271	627	0	4034	301	649
normalized size	1	1.	0.94	2.18	0.	14.01	1.05	2.25
time (sec)	N/A	0.223	0.143	0.001	0.	1.687	1.736	1.109

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	253	231	436	0	2547	187	477
normalized size	1	1.	0.91	1.72	0.	10.07	0.74	1.89
time (sec)	N/A	0.19	0.106	0.003	0.	1.44	1.077	1.437

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	196	266	0	1316	87	331
normalized size	1	1.	0.88	1.19	0.	5.9	0.39	1.48
time (sec)	N/A	0.138	0.127	0.003	0.	1.497	0.634	1.139

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	449	449	340	320	0	2668	0	0
normalized size	1	1.	0.76	0.71	0.	5.94	0.	0.
time (sec)	N/A	0.268	0.128	0.006	0.	2.717	0.	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	513	513	498	550	0	6808	0	900
normalized size	1	1.	0.97	1.07	0.	13.27	0.	1.75
time (sec)	N/A	0.42	0.317	0.01	0.	117.466	0.	3.02

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	407	407	391	1118	0	8400	0	1077
normalized size	1	1.	0.96	2.75	0.	20.64	0.	2.65
time (sec)	N/A	0.396	0.365	0.012	0.	2.347	0.	1.115

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	357	357	341	885	0	6400	466	867
normalized size	1	1.	0.96	2.48	0.	17.93	1.31	2.43
time (sec)	N/A	0.367	0.288	0.01	0.	2.214	37.26	1.114

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	317	301	669	0	4251	335	670
normalized size	1	1.	0.95	2.11	0.	13.41	1.06	2.11
time (sec)	N/A	0.317	0.222	0.009	0.	2.089	6.684	1.107

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	297	475	0	2909	219	508
normalized size	1	1.	1.02	1.63	0.	10.	0.75	1.75
time (sec)	N/A	0.377	0.177	0.009	0.	1.926	2.065	1.127

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	212	295	0	1526	112	359
normalized size	1	1.	0.87	1.2	0.	6.23	0.46	1.47
time (sec)	N/A	0.153	0.166	0.007	0.	1.646	0.886	1.109

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	513	513	499	550	0	6809	0	900
normalized size	1	1.	0.97	1.07	0.	13.27	0.	1.75
time (sec)	N/A	0.428	0.324	0.01	0.	120.692	0.	1.137

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	596	596	561	784	0	0	0	0
normalized size	1	1.	0.94	1.32	0.	0.	0.	0.
time (sec)	N/A	0.739	1.412	0.015	0.	0.	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	321	321	290	408	0	0	0	0
normalized size	1	1.	0.9	1.27	0.	0.	0.	0.
time (sec)	N/A	0.383	0.716	0.051	0.	0.	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	277	277	341	311	0	0	0	0
normalized size	1	1.	1.23	1.12	0.	0.	0.	0.
time (sec)	N/A	0.262	0.351	0.019	0.	0.	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	240	240	155	259	0	0	0	0
normalized size	1	1.	0.65	1.08	0.	0.	0.	0.
time (sec)	N/A	0.163	0.164	0.017	0.	0.	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	162	162	156	183	0	0	0	0
normalized size	1	1.	0.96	1.13	0.	0.	0.	0.
time (sec)	N/A	0.115	0.13	0.016	0.	0.	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	281	281	381	301	0	0	0	0
normalized size	1	1.	1.36	1.07	0.	0.	0.	0.
time (sec)	N/A	0.222	0.231	0.052	0.	0.	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	334	334	422	361	0	0	0	0
normalized size	1	1.	1.26	1.08	0.	0.	0.	0.
time (sec)	N/A	0.4	0.725	0.032	0.	0.	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	926	926	346	322	0	0	0	0
normalized size	1	1.	0.37	0.35	0.	0.	0.	0.
time (sec)	N/A	1.655	0.462	0.026	0.	0.	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	881	881	161	273	0	0	0	0
normalized size	1	1.	0.18	0.31	0.	0.	0.	0.
time (sec)	N/A	0.88	0.152	0.017	0.	0.	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	742	742	161	191	0	0	0	0
normalized size	1	1.	0.22	0.26	0.	0.	0.	0.
time (sec)	N/A	0.633	0.051	0.016	0.	0.	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	913	913	331	313	0	0	0	0
normalized size	1	1.	0.36	0.34	0.	0.	0.	0.
time (sec)	N/A	1.132	0.246	0.025	0.	0.	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	976	976	429	371	0	0	0	0
normalized size	1	1.	0.44	0.38	0.	0.	0.	0.
time (sec)	N/A	1.683	0.779	0.03	0.	0.	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	426	426	477	540	0	0	0	0
normalized size	1	1.	1.12	1.27	0.	0.	0.	0.
time (sec)	N/A	0.536	0.821	0.029	0.	0.	0.	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	365	365	396	412	0	0	0	0
normalized size	1	1.	1.08	1.13	0.	0.	0.	0.
time (sec)	N/A	0.405	0.579	0.028	0.	0.	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	309	309	342	329	0	0	0	0
normalized size	1	1.	1.11	1.06	0.	0.	0.	0.
time (sec)	N/A	0.277	0.3	0.026	0.	0.	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	276	276	233	294	0	0	0	0
normalized size	1	1.	0.84	1.07	0.	0.	0.	0.
time (sec)	N/A	0.215	0.132	0.025	0.	0.	0.	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	310	310	386	322	0	0	0	0
normalized size	1	1.	1.25	1.04	0.	0.	0.	0.
time (sec)	N/A	0.247	0.259	0.024	0.	0.	0.	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	362	362	374	375	0	0	0	0
normalized size	1	1.	1.03	1.04	0.	0.	0.	0.
time (sec)	N/A	0.404	0.524	0.038	0.	0.	0.	0.

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	439	439	382	484	0	0	0	0
normalized size	1	1.	0.87	1.1	0.	0.	0.	0.
time (sec)	N/A	0.536	0.822	0.038	0.	0.	0.	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	103	103	155	103	0	787	0	0
normalized size	1	1.	1.5	1.	0.	7.64	0.	0.
time (sec)	N/A	0.056	0.147	0.106	0.	5.719	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	116	116	155	158	0	822	0	0
normalized size	1	1.	1.34	1.36	0.	7.09	0.	0.
time (sec)	N/A	0.023	0.154	0.018	0.	5.	0.	0.

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	211	211	364	0	0	5146	0	0
normalized size	1	1.	1.73	0.	0.	24.39	0.	0.
time (sec)	N/A	0.225	0.445	0.423	0.	16.548	0.	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	173	173	161	0	0	1794	0	0
normalized size	1	1.	0.93	0.	0.	10.37	0.	0.
time (sec)	N/A	0.103	0.158	0.415	0.	2.196	0.	0.

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	84	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	0.041	0.405	0.	0.	0.	0.

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	134	134	256	0	0	0	0	0
normalized size	1	1.	1.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.096	0.346	0.411	0.	0.	0.	0.

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	180	180	621	0	0	0	0	0
normalized size	1	1.	3.45	0.	0.	0.	0.	0.
time (sec)	N/A	0.203	1.662	0.41	0.	0.	0.	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	233	233	1172	0	0	0	0	0
normalized size	1	1.	5.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.295	4.363	0.425	0.	0.	0.	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	316	316	294	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.338	0.66	0.431	0.	0.	0.	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	274	274	346	0	0	0	0	0
normalized size	1	1.	1.26	0.	0.	0.	0.	0.
time (sec)	N/A	0.156	0.322	0.416	0.	0.	0.	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	166	166	160	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.089	0.153	0.407	0.	0.	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	259	259	161	0	0	0	0	0
normalized size	1	1.	0.62	0.	0.	0.	0.	0.
time (sec)	N/A	0.154	0.032	0.409	0.	0.	0.	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	304	304	332	0	0	0	0	0
normalized size	1	1.	1.09	0.	0.	0.	0.	0.
time (sec)	N/A	0.238	0.256	0.403	0.	0.	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	357	357	430	0	0	0	0	0
normalized size	1	1.	1.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.401	0.725	0.402	0.	0.	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	280	280	560	0	0	7871	0	0
normalized size	1	1.	2.	0.	0.	28.11	0.	0.
time (sec)	N/A	0.359	0.794	0.245	0.	87.739	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	230	230	358	0	0	3656	0	0
normalized size	1	1.	1.56	0.	0.	15.9	0.	0.
time (sec)	N/A	0.175	0.554	0.434	0.	6.943	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	135	135	78	0	0	0	0	0
normalized size	1	1.	0.58	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	0.033	0.41	0.	0.	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	99	0	0	0	0	0
normalized size	1	1.	0.61	0.	0.	0.	0.	0.
time (sec)	N/A	0.109	0.106	0.41	0.	0.	0.	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	205	205	625	0	0	0	0	0
normalized size	1	1.	3.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.184	1.373	0.409	0.	0.	0.	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	266	266	1216	0	0	0	0	0
normalized size	1	1.	4.57	0.	0.	0.	0.	0.
time (sec)	N/A	0.293	3.814	0.425	0.	0.	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	353	353	392	0	0	0	0	0
normalized size	1	1.	1.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.348	0.503	0.253	0.	0.	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	298	298	341	0	0	0	0	0
normalized size	1	1.	1.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.24	0.325	0.403	0.	0.	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	308	308	233	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.203	0.212	0.435	0.	0.	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	330	330	337	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.25	0.341	0.441	0.	0.	0.	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	390	390	387	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.406	0.535	0.431	0.	0.	0.	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	44	0	0	583	0	0
normalized size	1	1.	0.83	0.	0.	11.	0.	0.
time (sec)	N/A	0.018	0.015	0.388	0.	44.14	0.	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	48	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	0.02	0.457	0.	0.	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	79	79	172	0	0	0	0	0
normalized size	1	1.	2.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	0.227	0.477	0.	0.	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	106	0	0	0	0	0
normalized size	1	1.	0.6	0.	0.	0.	0.	0.
time (sec)	N/A	0.133	0.053	0.361	0.	0.	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	85	90	0	0	0	75	0
normalized size	1	0.91	0.97	0.	0.	0.	0.81	0.
time (sec)	N/A	0.041	0.03	0.212	0.	0.	92.166	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0
normalized size	1	1.	2.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	0.173	0.414	0.	0.	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0
normalized size	1	1.	2.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.188	0.413	0.	0.	0.	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	545	545	49	0	0	0	0	0
normalized size	1	1.	0.09	0.	0.	0.	0.	0.
time (sec)	N/A	1.085	0.02	0.427	0.	0.	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	118	248	0	672	454	0
normalized size	1	1.	0.83	1.73	0.	4.7	3.17	0.
time (sec)	N/A	0.123	0.146	0.016	0.	1.271	26.27	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	84	191	0	462	121	0
normalized size	1	1.	0.85	1.93	0.	4.67	1.22	0.
time (sec)	N/A	0.067	0.088	0.01	0.	1.334	17.749	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	52	163	0	300	87	0
normalized size	1	1.	0.7	2.2	0.	4.05	1.18	0.
time (sec)	N/A	0.048	0.041	0.009	0.	1.313	20.045	0.

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	74	0	234	42	86
normalized size	1	1.	1.	1.9	0.	6.	1.08	2.21
time (sec)	N/A	0.019	0.016	0.003	0.	1.328	1.824	1.145

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	100	287	0	1110	0	0
normalized size	1	1.	0.96	2.76	0.	10.67	0.	0.
time (sec)	N/A	0.111	0.192	0.062	0.	1.497	0.	0.

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	122	943	0	1760	0	0
normalized size	1	1.	0.83	6.41	0.	11.97	0.	0.
time (sec)	N/A	0.209	0.312	0.013	0.	1.502	0.	0.

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	330	1972	0	3664	0	1107
normalized size	1	1.	1.55	9.26	0.	17.2	0.	5.2
time (sec)	N/A	0.34	0.61	0.015	0.	2.028	0.	1.331

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	159	353	0	845	1817	0
normalized size	1	1.	0.97	2.15	0.	5.15	11.08	0.
time (sec)	N/A	0.142	0.169	0.012	0.	1.377	68.451	0.

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	106	260	0	610	534	0
normalized size	1	1.	0.84	2.06	0.	4.84	4.24	0.
time (sec)	N/A	0.08	0.177	0.011	0.	1.248	44.128	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	73	205	0	392	163	0
normalized size	1	1.	0.73	2.05	0.	3.92	1.63	0.
time (sec)	N/A	0.063	0.069	0.009	0.	1.294	27.871	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	46	100	0	246	92	0
normalized size	1	1.	0.85	1.85	0.	4.56	1.7	0.
time (sec)	N/A	0.025	0.019	0.007	0.	1.523	2.376	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	102	528	0	1156	0	0
normalized size	1	1.	0.96	4.98	0.	10.91	0.	0.
time (sec)	N/A	0.128	0.191	0.011	0.	1.768	0.	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	143	834	0	1682	0	0
normalized size	1	1.	0.92	5.35	0.	10.78	0.	0.
time (sec)	N/A	0.222	0.285	0.01	0.	1.808	0.	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	168	1817	0	3646	0	981
normalized size	1	1.	0.8	8.69	0.	17.44	0.	4.69
time (sec)	N/A	0.345	0.448	0.011	0.	1.816	0.	1.317

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	201	457	0	1115	5513	0
normalized size	1	1.	1.02	2.31	0.	5.63	27.84	0.
time (sec)	N/A	0.158	0.212	0.013	0.	1.371	91.14	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	121	336	0	807	1841	0
normalized size	1	1.	0.8	2.21	0.	5.31	12.11	0.
time (sec)	N/A	0.102	0.136	0.013	0.	1.273	57.478	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	94	253	0	527	520	0
normalized size	1	1.	0.75	2.02	0.	4.22	4.16	0.
time (sec)	N/A	0.077	0.094	0.01	0.	1.238	34.91	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	64	120	0	329	99	0
normalized size	1	1.	0.9	1.69	0.	4.63	1.39	0.
time (sec)	N/A	0.034	0.044	0.007	0.	1.423	3.371	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	116	859	0	1431	0	0
normalized size	1	1.	0.87	6.41	0.	10.68	0.	0.
time (sec)	N/A	0.221	0.183	0.013	0.	2.267	0.	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	145	1323	0	2125	0	0
normalized size	1	1.	0.87	7.97	0.	12.8	0.	0.
time (sec)	N/A	0.234	0.34	0.034	0.	1.951	0.	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	191	1638	0	3040	0	1276
normalized size	1	1.	0.81	6.91	0.	12.83	0.	5.38
time (sec)	N/A	0.373	0.546	0.011	0.	1.896	0.	1.378

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	95	535	0	510	386	207
normalized size	1	1.	0.75	4.25	0.	4.05	3.06	1.64
time (sec)	N/A	0.09	0.109	0.014	0.	1.376	40.841	1.201

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	66	348	0	359	114	131
normalized size	1	1.	0.9	4.77	0.	4.92	1.56	1.79
time (sec)	N/A	0.054	0.071	0.013	0.	1.364	25.404	1.202

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	53	173	0	277	82	99
normalized size	1	1.	1.04	3.39	0.	5.43	1.61	1.94
time (sec)	N/A	0.033	0.034	0.01	0.	1.295	24.435	1.189

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	70	0	239	44	96
normalized size	1	1.	1.	1.63	0.	5.56	1.02	2.23
time (sec)	N/A	0.02	0.018	0.006	0.	1.237	2.143	1.155

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	104	228	0	1215	0	174
normalized size	1	1.	0.96	2.11	0.	11.25	0.	1.61
time (sec)	N/A	0.096	0.177	0.011	0.	1.511	0.	1.193

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	150	1135	0	2408	0	393
normalized size	1	1.	0.87	6.6	0.	14.	0.	2.28
time (sec)	N/A	0.218	0.61	0.017	0.	1.894	0.	1.211

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	216	2269	0	4749	0	459
normalized size	1	1.	0.86	9.08	0.	19.	0.	1.84
time (sec)	N/A	0.4	1.404	0.014	0.	3.794	0.	1.224

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	92	969	0	699	0	297
normalized size	1	1.	0.7	7.34	0.	5.3	0.	2.25
time (sec)	N/A	0.101	0.055	0.016	0.	1.324	0.	1.205

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	90	81	789	0	586	0	217
normalized size	1	0.96	0.86	8.39	0.	6.23	0.	2.31
time (sec)	N/A	0.079	0.086	0.012	0.	1.304	0.	1.713

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	48	387	0	473	224	165
normalized size	1	1.	0.63	5.09	0.	6.22	2.95	2.17
time (sec)	N/A	0.05	0.021	0.01	0.	1.354	21.589	1.199

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	61	36	198	0	352	71	116
normalized size	1	1.02	0.6	3.3	0.	5.87	1.18	1.93
time (sec)	N/A	0.031	0.012	0.009	0.	1.311	3.066	1.191

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	106	962	0	2195	0	261
normalized size	1	1.	0.72	6.54	0.	14.93	0.	1.78
time (sec)	N/A	0.194	0.055	0.016	0.	2.032	0.	1.19

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	164	3119	0	4641	0	560
normalized size	1	1.	0.73	13.92	0.	20.72	0.	2.5
time (sec)	N/A	0.323	0.116	0.014	0.	4.018	0.	1.186

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	239	5158	0	8142	0	678
normalized size	1	1.	0.75	16.12	0.	25.44	0.	2.12
time (sec)	N/A	0.525	0.212	0.016	0.	10.481	0.	1.207

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	145	1150	0	1002	0	267
normalized size	1	1.	1.01	8.04	0.	7.01	0.	1.87
time (sec)	N/A	0.151	0.294	0.014	0.	1.327	0.	1.195

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	118	97	588	0	864	0	217
normalized size	1	0.97	0.8	4.82	0.	7.08	0.	1.78
time (sec)	N/A	0.095	0.049	0.011	0.	1.368	0.	1.202

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	60	541	0	726	1479	185
normalized size	1	1.	0.58	5.25	0.	7.05	14.36	1.8
time (sec)	N/A	0.065	0.023	0.011	0.	1.396	57.651	1.194

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	82	38	271	0	501	774	132
normalized size	1	1.04	0.48	3.43	0.	6.34	9.8	1.67
time (sec)	N/A	0.038	0.016	0.008	0.	1.274	4.937	1.225

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	118	1767	0	4049	0	332
normalized size	1	1.	0.59	8.79	0.	20.14	0.	1.65
time (sec)	N/A	0.315	0.059	0.013	0.	4.558	0.	1.182

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	178	4644	0	7776	0	779
normalized size	1	1.	0.62	16.18	0.	27.09	0.	2.71
time (sec)	N/A	0.448	0.132	0.017	0.	11.729	0.	1.231

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	409	409	239	7300	0	12556	0	703
normalized size	1	1.	0.58	17.85	0.	30.7	0.	1.72
time (sec)	N/A	0.702	0.214	0.019	0.	31.435	0.	1.23

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	167	253	0	1995	0	0
normalized size	1	1.	1.36	2.06	0.	16.22	0.	0.
time (sec)	N/A	0.094	1.01	0.046	0.	5.403	0.	0.

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	81	155	0	551	0	0
normalized size	1	1.	1.	1.91	0.	6.8	0.	0.
time (sec)	N/A	0.052	0.061	0.024	0.	2.211	0.	0.

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	87	280	0	710	0	0
normalized size	1	1.	0.71	2.3	0.	5.82	0.	0.
time (sec)	N/A	0.08	0.095	0.032	0.	2.082	0.	0.

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	96	96	206	0	0	0	0	0
normalized size	1	1.	2.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.061	0.303	0.121	0.	0.	0.	0.

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	40	45	0	223	82	45
normalized size	1	1.	1.03	1.15	0.	5.72	2.1	1.15
time (sec)	N/A	0.02	0.026	0.004	0.	1.233	0.404	1.135

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	205	277	0	0	0	0
normalized size	1	1.	0.88	1.19	0.	0.	0.	0.
time (sec)	N/A	0.218	0.296	0.038	0.	0.	0.	0.

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	86	94	0	0	0	0
normalized size	1	1.	0.37	0.41	0.	0.	0.	0.
time (sec)	N/A	0.205	0.061	0.018	0.	0.	0.	0.

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	191	187	0	0	0	0
normalized size	1	1.	0.73	0.71	0.	0.	0.	0.
time (sec)	N/A	0.28	0.224	0.071	0.	0.	0.	0.

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	79	79	104	0	0	0	0	0
normalized size	1	1.	1.32	0.	0.	0.	0.	0.
time (sec)	N/A	0.088	0.11	0.093	0.	0.	0.	0.

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	129	195	0	926	71	217
normalized size	1	1.	0.89	1.34	0.	6.39	0.49	1.5
time (sec)	N/A	0.106	0.084	0.004	0.	1.343	0.518	1.132

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	41	59	63	111	82	66
normalized size	1	1.	0.84	1.2	1.29	2.27	1.67	1.35
time (sec)	N/A	0.049	0.041	0.004	0.967	1.308	0.271	1.098

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	21	27	65	24	27
normalized size	1	1.	1.	0.81	1.04	2.5	0.92	1.04
time (sec)	N/A	0.016	0.01	0.003	0.941	1.33	0.167	1.104

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	24	31	77	27	32
normalized size	1	1.	1.	1.41	1.82	4.53	1.59	1.88
time (sec)	N/A	0.015	0.004	0.004	0.973	1.422	0.224	1.361

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	22	66	96	261	102	0
normalized size	1	1.	0.21	0.63	0.92	2.51	0.98	0.
time (sec)	N/A	0.086	0.003	0.006	1.456	1.577	8.583	0.

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	30	66	26	31
normalized size	1	1.	1.	0.77	1.	2.2	0.87	1.03
time (sec)	N/A	0.019	0.013	0.003	0.932	1.442	0.16	1.135

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	79	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.033	0.034	0.641	0.	0.	0.	0.

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	76	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.016	0.376	0.	0.	0.	0.

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	72	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	0.019	0.856	0.	0.	0.	0.

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	110	138	0	1166	2744	999
normalized size	1	1.	0.83	1.05	0.	8.83	20.79	7.57
time (sec)	N/A	0.111	0.148	0.059	0.	1.645	3.279	1.15

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	90	104	0	695	1540	608
normalized size	1	1.	0.91	1.05	0.	7.02	15.56	6.14
time (sec)	N/A	0.073	0.106	0.009	0.	1.599	1.761	1.112

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	70	74	0	382	726	313
normalized size	1	1.	1.	1.06	0.	5.46	10.37	4.47
time (sec)	N/A	0.045	0.081	0.009	0.	1.656	0.928	1.129

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	37	43	0	155	236	112
normalized size	1	1.	0.92	1.08	0.	3.88	5.9	2.8
time (sec)	N/A	0.021	0.062	0.006	0.	1.604	0.44	1.094

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	40	0	0	0	73	0
normalized size	1	1.	0.93	0.	0.	0.	1.7	0.
time (sec)	N/A	0.017	0.012	0.407	0.	0.	1.744	0.

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	56	0	0	0	592	0
normalized size	1	1.	0.77	0.	0.	0.	8.11	0.
time (sec)	N/A	0.031	0.037	0.36	0.	0.	4.608	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	58	0	0	0	0	0
normalized size	1	1.	0.74	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.042	0.373	0.	0.	0.	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	58	0	0	0	0	0
normalized size	1	1.	0.74	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.049	0.373	0.	0.	0.	0.

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	149	164	0	1434	0	1278
normalized size	1	1.	0.94	1.04	0.	9.08	0.	8.09
time (sec)	N/A	0.13	0.159	0.055	0.	1.701	0.	1.648

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	105	117	0	787	0	728
normalized size	1	1.	0.94	1.04	0.	7.03	0.	6.5
time (sec)	N/A	0.082	0.158	0.01	0.	1.645	0.	1.134

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	70	74	0	382	726	313
normalized size	1	1.	1.	1.06	0.	5.46	10.37	4.47
time (sec)	N/A	0.047	0.086	0.008	0.	1.635	0.971	1.114

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	82	0	0	0	170	0
normalized size	1	1.	0.98	0.	0.	0.	2.02	0.
time (sec)	N/A	0.094	0.044	0.353	0.	0.	3.319	0.

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	95	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.092	0.113	0.385	0.	0.	0.	0.

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	133	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.16	0.107	0.353	0.	0.	0.	0.

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	133	0	0	0	369	0
normalized size	1	1.	0.43	0.	0.	0.	1.19	0.
time (sec)	N/A	0.501	2.471	0.364	0.	0.	6.646	0.

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	104	0	0	0	269	0
normalized size	1	1.	0.6	0.	0.	0.	1.55	0.
time (sec)	N/A	0.267	0.983	0.382	0.	0.	4.735	0.

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	75	0	0	0	170	0
normalized size	1	1.	0.89	0.	0.	0.	2.02	0.
time (sec)	N/A	0.097	0.295	0.429	0.	0.	3.182	0.

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	40	0	0	0	73	0
normalized size	1	1.	0.95	0.	0.	0.	1.74	0.
time (sec)	N/A	0.016	0.01	0.416	0.	0.	1.697	0.

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	64	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	0.034	0.711	0.	0.	0.	0.

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	121	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.146	0.133	0.721	0.	0.	0.	0.

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	210	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.329	0.198	0.757	0.	0.	0.	0.

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	341	341	217	0	0	0	0	0
normalized size	1	1.	0.64	0.	0.	0.	0.	0.
time (sec)	N/A	0.548	5.19	0.379	0.	0.	0.	0.

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	200	200	2050	0	0	0	0	0
normalized size	1	1.	10.25	0.	0.	0.	0.	0.
time (sec)	N/A	0.26	4.21	0.368	0.	0.	0.	0.

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	115	115	666	0	0	0	0	0
normalized size	1	1.	5.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.094	1.575	0.369	0.	0.	0.	0.

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	56	0	0	0	592	0
normalized size	1	1.	0.78	0.	0.	0.	8.22	0.
time (sec)	N/A	0.031	0.039	0.356	0.	0.	4.515	0.

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	108	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.142	0.151	0.719	0.	0.	0.	0.

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	147	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.271	0.209	0.734	0.	0.	0.	0.

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	233	0	0	0	0	0
normalized size	1	1.	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.547	0.324	0.734	0.	0.	0.	0.

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	81	81	190	0	0	0	0	0
normalized size	1	1.	2.35	0.	0.	0.	0.	0.
time (sec)	N/A	0.053	0.393	0.816	0.	0.	0.	0.

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	402	401	168	0	0	0	0	0
normalized size	1	1.	0.42	0.	0.	0.	0.	0.
time (sec)	N/A	0.578	5.254	0.623	0.	0.	0.	0.

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	197	140	0	0	0	143	0
normalized size	1	0.98	0.69	0.	0.	0.	0.71	0.
time (sec)	N/A	0.269	5.158	0.55	0.	0.	23.894	0.

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	89	94	0	0	0	87	0
normalized size	1	0.91	0.96	0.	0.	0.	0.89	0.
time (sec)	N/A	0.047	0.043	0.401	0.	0.	4.298	0.

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	37	0
normalized size	1	1.	1.	0.	0.	0.	0.8	0.
time (sec)	N/A	0.011	0.005	0.	0.	0.	1.434	0.

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	59	59	180	0	0	0	0	0
normalized size	1	1.	3.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	0.256	0.711	0.	0.	0.	0.

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	59	59	180	0	0	0	0	0
normalized size	1	1.	3.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.03	0.278	0.676	0.	0.	0.	0.

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	59	59	180	0	0	0	0	0
normalized size	1	1.	3.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.029	0.406	0.708	0.	0.	0.	0.

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	93	93	94	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	0.067	0.773	0.	0.	0.	0.

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	218	0	0	971	0	0
normalized size	1	1.	1.22	0.	0.	5.46	0.	0.
time (sec)	N/A	0.086	0.129	0.594	0.	1.663	0.	0.

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	113	0	0	468	0	0
normalized size	1	1.	0.97	0.	0.	4.03	0.	0.
time (sec)	N/A	0.036	0.073	0.577	0.	1.585	0.	0.

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	82	0	0	177	0	0
normalized size	1	1.	1.41	0.	0.	3.05	0.	0.
time (sec)	N/A	0.014	0.107	0.421	0.	1.61	0.	0.

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	53	0	61	211	0
normalized size	1	1.	1.	2.94	0.	3.39	11.72	0.
time (sec)	N/A	0.003	0.027	0.061	0.	1.581	35.648	0.

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	52	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	0.013	0.714	0.	0.	0.	0.

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	53	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	0.013	0.7	0.	0.	0.	0.

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	55	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.011	0.724	0.	0.	0.	0.

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	193	179	1414	0	0	0	0	0
normalized size	1	0.93	7.33	0.	0.	0.	0.	0.
time (sec)	N/A	0.081	45.82	0.793	0.	0.	0.	0.

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	55	0	0	209	0	0
normalized size	1	1.	0.96	0.	0.	3.67	0.	0.
time (sec)	N/A	0.027	0.049	0.817	0.	1.719	0.	0.

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	327	327	153	0	0	819	0	0
normalized size	1	1.	0.47	0.	0.	2.5	0.	0.
time (sec)	N/A	0.184	0.198	0.586	0.	1.656	0.	0.

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	96	0	0	360	0	0
normalized size	1	1.	0.76	0.	0.	2.83	0.	0.
time (sec)	N/A	0.064	0.097	0.449	0.	1.611	0.	0.

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	55	0	0	139	0	0
normalized size	1	1.	1.1	0.	0.	2.78	0.	0.
time (sec)	N/A	0.012	0.026	0.398	0.	1.586	0.	0.

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	153	0	0	0	0	0
normalized size	1	1.	1.61	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	6.441	0.696	0.	0.	0.	0.

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	127	127	1070	0	0	0	0	0
normalized size	1	1.	8.43	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	49.839	0.682	0.	0.	0.	0.

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	131	131	1241	0	0	0	0	0
normalized size	1	1.	9.47	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	42.82	0.702	0.	0.	0.	0.

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	133	133	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	180.006	0.717	0.	0.	0.	0.

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	164	110	92	240	246	0	309
normalized size	1	1.08	0.72	0.61	1.58	1.62	0.	2.03
time (sec)	N/A	0.118	0.073	0.006	0.974	1.937	0.	1.298

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	118	88	68	167	193	0	227
normalized size	1	1.08	0.81	0.62	1.53	1.77	0.	2.08
time (sec)	N/A	0.087	0.053	0.006	0.984	1.679	0.	1.191

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	72	62	44	95	140	0	126
normalized size	1	1.07	0.93	0.66	1.42	2.09	0.	1.88
time (sec)	N/A	0.044	0.031	0.002	0.961	1.549	0.	1.518

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	85	174	0	174	0	109
normalized size	1	1.	1.06	2.17	0.	2.17	0.	1.36
time (sec)	N/A	0.078	0.161	0.036	0.	1.555	0.	1.695

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	114	114	182	0	182	0	212
normalized size	1	1.19	1.19	1.9	0.	1.9	0.	2.21
time (sec)	N/A	0.084	0.049	0.014	0.	1.547	0.	1.182

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	164	137	226	0	213	0	437
normalized size	1	1.36	1.13	1.87	0.	1.76	0.	3.61
time (sec)	N/A	0.104	0.084	0.014	0.	1.58	0.	1.214

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	161	298	356	300	0	394
normalized size	1	1.	0.77	1.43	1.71	1.44	0.	1.89
time (sec)	N/A	0.149	0.214	0.022	0.949	1.829	0.	1.35

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	135	240	284	243	0	311
normalized size	1	1.	0.85	1.51	1.79	1.53	0.	1.96
time (sec)	N/A	0.123	0.163	0.013	0.97	1.65	0.	1.312

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	129	180	205	190	0	204
normalized size	1	1.	1.13	1.58	1.8	1.67	0.	1.79
time (sec)	N/A	0.047	0.235	0.01	0.942	1.587	0.	1.261

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	101	153	0	181	0	149
normalized size	1	1.	0.97	1.47	0.	1.74	0.	1.43
time (sec)	N/A	0.087	0.08	0.014	0.	1.579	0.	1.243

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	105	153	0	216	0	231
normalized size	1	1.	1.25	1.82	0.	2.57	0.	2.75
time (sec)	N/A	0.08	0.076	0.016	0.	1.534	0.	1.321

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	117	191	231	224	216	205
normalized size	1	1.	0.94	1.53	1.85	1.79	1.73	1.64
time (sec)	N/A	0.085	0.103	0.077	0.961	1.547	59.95	1.239

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	70	57	128	128	216	130
normalized size	1	1.	0.68	0.55	1.24	1.24	2.1	1.26
time (sec)	N/A	0.075	0.042	0.003	0.946	1.542	45.711	1.188

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	98	147	177	180	212	151
normalized size	1	1.	1.13	1.69	2.03	2.07	2.44	1.74
time (sec)	N/A	0.069	0.066	0.018	0.967	1.533	40.417	1.228

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	52	38	73	88	202	74
normalized size	1	1.	0.8	0.58	1.12	1.35	3.11	1.14
time (sec)	N/A	0.04	0.027	0.004	0.987	1.476	27.926	1.164

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	B	A	C	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	47	47	101	103	120	136	182	96
normalized size	1	1.	2.15	2.19	2.55	2.89	3.87	2.04
time (sec)	N/A	0.02	0.152	0.013	0.978	1.452	14.664	1.228

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	66	62	42	122	162	61
normalized size	1	1.	1.43	1.35	0.91	2.65	3.52	1.33
time (sec)	N/A	0.061	0.025	0.018	1.459	1.497	18.002	1.138

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	73	77	68	131	148	78
normalized size	1	1.	2.21	2.33	2.06	3.97	4.48	2.36
time (sec)	N/A	0.057	0.023	0.018	1.477	1.56	18.89	1.657

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	77	84	66	143	141	154
normalized size	1	1.	1.28	1.4	1.1	2.38	2.35	2.57
time (sec)	N/A	0.063	0.045	0.019	1.429	1.519	26.115	1.19

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	51	37	73	120	146	157
normalized size	1	1.	0.82	0.6	1.18	1.94	2.35	2.53
time (sec)	N/A	0.062	0.017	0.004	1.463	1.522	37.407	1.189

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	102	125	120	186	148	362
normalized size	1	1.	1.03	1.26	1.21	1.88	1.49	3.66
time (sec)	N/A	0.078	0.091	0.02	1.412	1.521	52.027	1.169

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	148	240	289	251	240	246
normalized size	1	1.	0.9	1.46	1.76	1.53	1.46	1.5
time (sec)	N/A	0.12	0.116	0.028	0.961	1.586	87.195	1.27

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	87	68	167	144	240	151
normalized size	1	1.	0.74	0.58	1.42	1.22	2.03	1.28
time (sec)	N/A	0.086	0.054	0.006	0.967	1.572	60.904	1.212

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	121	182	216	196	236	176
normalized size	1	1.	1.03	1.54	1.83	1.66	2.	1.49
time (sec)	N/A	0.097	0.082	0.02	0.993	1.493	49.787	1.219

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	61	43	93	93	223	82
normalized size	1	1.	0.85	0.6	1.29	1.29	3.1	1.14
time (sec)	N/A	0.046	0.033	0.004	0.981	1.475	33.112	1.144

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	119	124	140	142	199	107
normalized size	1	1.	1.75	1.82	2.06	2.09	2.93	1.57
time (sec)	N/A	0.032	0.261	0.016	0.977	1.522	16.483	1.202

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	87	108	0	135	178	74
normalized size	1	1.	1.55	1.93	0.	2.41	3.18	1.32
time (sec)	N/A	0.069	0.032	0.019	0.	1.519	21.157	1.163

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	90	97	0	143	165	89
normalized size	1	1.	1.58	1.7	0.	2.51	2.89	1.56
time (sec)	N/A	0.075	0.039	0.02	0.	1.524	21.491	1.189

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	102	158	0	165	162	190
normalized size	1	1.	1.34	2.08	0.	2.17	2.13	2.5
time (sec)	N/A	0.073	0.057	0.022	0.	1.517	28.438	1.244

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	66	49	0	144	170	185
normalized size	1	1.	0.88	0.65	0.	1.92	2.27	2.47
time (sec)	N/A	0.069	0.026	0.005	0.	1.486	44.32	1.197

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	144	227	0	219	172	439
normalized size	1	1.	1.17	1.85	0.	1.78	1.4	3.57
time (sec)	N/A	0.095	0.094	0.02	0.	1.58	61.592	1.209

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	119	316	289	390	233	290
normalized size	1	1.	0.74	1.96	1.8	2.42	1.45	1.8
time (sec)	N/A	0.123	0.142	0.03	0.952	1.512	99.723	1.326

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	72	68	166	161	226	194
normalized size	1	1.	0.63	0.59	1.44	1.4	1.97	1.69
time (sec)	N/A	0.094	0.05	0.005	0.951	1.519	79.907	1.23

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	90	254	211	325	212	208
normalized size	1	1.	0.59	1.67	1.39	2.14	1.39	1.37
time (sec)	N/A	0.115	0.108	0.022	0.956	1.548	66.747	1.29

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	45	43	93	107	201	127
normalized size	1	1.	0.59	0.57	1.22	1.41	2.64	1.67
time (sec)	N/A	0.054	0.028	0.004	0.967	1.507	52.863	1.25

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	86	160	115	252	182	153
normalized size	1	1.	1.37	2.54	1.83	4.	2.89	2.43
time (sec)	N/A	0.032	0.167	0.018	0.99	1.512	39.629	1.259

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	84	188	0	201	172	155
normalized size	1	1.	1.29	2.89	0.	3.09	2.65	2.38
time (sec)	N/A	0.081	0.039	0.021	0.	1.538	51.56	1.318

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	51	48	0	197	165	296
normalized size	1	1.	0.76	0.72	0.	2.94	2.46	4.42
time (sec)	N/A	0.077	0.024	0.004	0.	1.497	58.032	1.368

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	75	315	0	278	165	285
normalized size	1	1.	0.64	2.69	0.	2.38	1.41	2.44
time (sec)	N/A	0.098	0.028	0.024	0.	1.599	79.398	1.388

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	77	73	0	265	165	327
normalized size	1	1.	0.65	0.61	0.	2.23	1.39	2.75
time (sec)	N/A	0.096	0.028	0.006	0.	1.537	129.492	1.504

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	78	387	0	335	0	543
normalized size	1	1.	0.47	2.33	0.	2.02	0.	3.27
time (sec)	N/A	0.12	0.03	0.026	0.	1.622	0.	1.617

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	56	53	34	103	148	54
normalized size	1	1.	1.4	1.32	0.85	2.58	3.7	1.35
time (sec)	N/A	0.054	0.024	0.071	1.655	1.551	16.137	1.13

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	244	66	107	128	0	0
normalized size	1	1.	4.6	1.25	2.02	2.42	0.	0.
time (sec)	N/A	0.091	0.258	0.008	1.614	1.792	0.	0.

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	0	0	194	0	0
normalized size	1	1.	1.	0.	0.	5.39	0.	0.
time (sec)	N/A	0.02	0.016	0.601	0.	1.703	0.	0.

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	75	0	0	124	0	0
normalized size	1	1.	1.	0.	0.	1.65	0.	0.
time (sec)	N/A	0.05	0.033	0.663	0.	1.812	0.	0.

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	113	113	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.09	0.239	1.492	0.	0.	0.	0.

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	87	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	0.042	1.464	0.	0.	0.	0.

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	76	76	258	0	0	0	0	0
normalized size	1	1.	3.39	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	0.29	1.048	0.	0.	0.	0.

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	104	103	0	0	369	0	0
normalized size	1	1.08	1.07	0.	0.	3.84	0.	0.
time (sec)	N/A	0.122	0.276	2.152	0.	1.901	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [279] had the largest ratio of [0.5294]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.	17	0.059
2	A	2	1	1.	17	0.059
3	A	2	1	1.	17	0.059
4	A	2	1	1.	15	0.067
5	A	7	7	1.	17	0.412
6	A	7	7	1.	17	0.412
7	A	8	8	1.	17	0.471
8	A	2	1	1.	19	0.053
9	A	2	1	1.	19	0.053
10	A	2	1	1.	17	0.059
11	A	8	7	1.	19	0.368
12	A	9	8	1.	19	0.421
13	A	8	8	1.	19	0.421
14	A	8	7	1.	19	0.368

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
15	A	8	7	1.	19	0.368
16	A	8	7	1.	19	0.368
17	A	7	7	1.	17	0.412
18	A	13	7	1.	19	0.368
19	A	14	8	1.	19	0.421
20	A	9	8	1.	19	0.421
21	A	9	8	1.	19	0.421
22	A	9	8	1.	19	0.421
23	A	9	8	1.	19	0.421
24	A	7	7	1.	17	0.412
25	A	14	8	1.	19	0.421
26	A	15	9	1.	19	0.474
27	A	3	3	1.	20	0.15
28	A	2	2	1.	20	0.1
29	A	2	2	1.	20	0.1
30	A	2	2	1.	20	0.1
31	A	3	3	1.	20	0.15
32	A	4	3	1.	20	0.15
33	A	5	3	1.	20	0.15
34	C	2	2	0.12	22	0.091
35	C	2	2	0.12	22	0.091
36	C	2	2	0.15	22	0.091
37	C	2	2	0.13	22	0.091
38	C	2	2	0.12	22	0.091
39	C	2	2	0.12	22	0.091
40	A	4	4	1.	22	0.182
41	A	3	3	1.	22	0.136
42	A	3	3	1.	22	0.136
43	A	3	3	1.	22	0.136
44	A	3	2	1.	22	0.091
45	A	4	3	1.	22	0.136
46	A	5	4	1.	22	0.182
47	A	6	4	1.	22	0.182
48	A	4	4	1.	22	0.182
49	A	4	4	1.	22	0.182
50	A	4	4	1.	22	0.182
51	A	4	4	1.	22	0.182
52	A	4	4	1.	22	0.182
53	A	4	4	1.	22	0.182
54	A	4	4	1.	22	0.182
55	A	4	4	1.	22	0.182
56	A	4	3	1.	19	0.158
57	A	3	3	1.	19	0.158
58	A	2	2	1.	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
59	A	2	2	1.	19	0.105
60	A	2	2	1.	19	0.105
61	A	3	3	1.	19	0.158
62	A	4	3	1.	19	0.158
63	A	5	3	1.	19	0.158
64	A	3	3	1.	19	0.158
65	A	3	3	1.	19	0.158
66	A	3	3	1.	19	0.158
67	A	3	3	1.	19	0.158
68	A	3	3	1.	19	0.158
69	A	3	3	1.	19	0.158
70	A	5	4	1.	21	0.19
71	A	4	4	1.	21	0.19
72	A	3	3	1.	21	0.143
73	A	3	3	1.	21	0.143
74	A	3	3	1.	21	0.143
75	A	3	2	1.	21	0.095
76	A	4	3	1.	21	0.143
77	A	5	4	1.	21	0.19
78	A	6	4	1.	21	0.19
79	A	4	4	1.	21	0.19
80	A	4	4	1.	21	0.19
81	A	4	4	1.	21	0.19
82	A	4	4	1.	21	0.19
83	A	4	4	1.	21	0.19
84	A	4	4	1.	21	0.19
85	A	4	2	1.	21	0.095
86	C	2	2	0.19	21	0.095
87	C	2	2	0.22	21	0.095
88	C	2	2	0.25	21	0.095
89	A	7	7	1.4	21	0.333
90	A	8	8	1.33	21	0.381
91	C	2	2	2.75	21	0.095
92	C	2	2	4.19	21	0.095
93	A	2	2	1.	21	0.095
94	A	2	2	1.	21	0.095
95	A	2	2	1.	21	0.095
96	A	2	2	1.	21	0.095
97	A	2	2	1.	21	0.095
98	C	2	2	0.18	21	0.095
99	C	2	2	0.2	21	0.095
100	A	8	8	1.32	21	0.381
101	A	8	8	1.27	21	0.381
102	C	2	2	2.39	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
103	C	2	2	3.75	21	0.095
104	A	2	2	1.	21	0.095
105	A	2	2	1.	21	0.095
106	A	2	2	1.	21	0.095
107	A	2	2	1.	21	0.095
108	A	2	2	1.	21	0.095
109	C	2	2	0.11	21	0.095
110	C	2	2	0.14	21	0.095
111	C	2	2	0.16	21	0.095
112	A	9	8	1.27	21	0.381
113	A	9	9	1.22	21	0.429
114	C	2	2	0.54	21	0.095
115	C	2	2	1.14	21	0.095
116	C	2	2	4.3	21	0.095
117	A	2	2	1.	21	0.095
118	A	2	2	1.	21	0.095
119	A	2	2	1.	21	0.095
120	A	2	2	1.	21	0.095
121	A	2	2	1.	21	0.095
122	A	3	2	1.	23	0.087
123	A	3	2	1.	23	0.087
124	A	2	2	1.	23	0.087
125	A	2	2	1.	23	0.087
126	A	1	1	1.	23	0.043
127	A	1	1	1.	23	0.043
128	A	1	1	1.	23	0.043
129	A	2	2	1.	23	0.087
130	A	2	2	1.	23	0.087
131	A	3	2	1.	23	0.087
132	A	3	2	1.	23	0.087
133	A	3	2	1.	19	0.105
134	A	4	4	1.05	19	0.21
135	A	3	3	1.01	17	0.176
136	A	2	2	1.	19	0.105
137	A	2	2	1.	19	0.105
138	A	5	5	1.	19	0.263
139	A	4	4	1.	19	0.21
140	A	3	3	0.91	17	0.176
141	A	2	2	1.	9	0.222
142	A	2	2	1.	19	0.105
143	A	2	2	1.	19	0.105
144	A	2	2	1.	19	0.105
145	A	1	1	1.	50	0.02
146	A	2	1	1.	17	0.059

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
147	A	2	1	1.	17	0.059
148	A	2	1	1.	17	0.059
149	A	2	1	1.	15	0.067
150	A	10	7	1.	17	0.412
151	A	10	7	1.	17	0.412
152	A	11	8	1.	17	0.471
153	A	2	1	1.	19	0.053
154	A	2	1	1.	19	0.053
155	A	2	1	1.	19	0.053
156	A	2	1	1.	17	0.059
157	A	11	7	1.	19	0.368
158	A	12	8	1.	19	0.421
159	A	11	8	1.	19	0.421
160	A	11	7	1.	19	0.368
161	A	11	7	1.	19	0.368
162	A	11	7	1.	19	0.368
163	A	10	7	1.	17	0.412
164	A	19	7	1.	19	0.368
165	A	20	8	1.	19	0.421
166	A	12	8	1.	19	0.421
167	A	12	8	1.	19	0.421
168	A	12	8	1.	19	0.421
169	A	12	8	1.	19	0.421
170	A	10	7	1.	17	0.412
171	A	20	8	1.	19	0.421
172	A	21	9	1.	19	0.474
173	A	10	8	1.	23	0.348
174	A	9	7	1.	23	0.304
175	A	8	6	1.	23	0.261
176	A	5	3	1.	23	0.13
177	A	9	7	1.	23	0.304
178	A	10	8	1.	23	0.348
179	A	10	6	1.	21	0.286
180	A	9	5	1.	21	0.238
181	A	7	4	1.	21	0.19
182	A	10	6	1.	21	0.286
183	A	11	7	1.	21	0.333
184	A	11	8	1.	23	0.348
185	A	10	8	1.	23	0.348
186	A	9	7	1.	23	0.304
187	A	9	7	1.	23	0.304
188	A	9	7	1.	23	0.304
189	A	10	8	1.	23	0.348
190	A	11	8	1.	23	0.348

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
191	A	4	4	1.	25	0.16
192	A	1	1	1.	25	0.04
193	A	10	9	1.	21	0.429
194	A	9	8	1.	21	0.381
195	A	4	4	1.	21	0.19
196	A	5	5	1.	21	0.238
197	A	7	7	1.	21	0.333
198	A	8	7	1.	21	0.333
199	A	11	10	1.	21	0.476
200	A	10	9	1.	21	0.429
201	A	4	3	1.	21	0.143
202	A	9	8	1.	21	0.381
203	A	10	9	1.	21	0.429
204	A	11	10	1.	21	0.476
205	A	11	10	1.	21	0.476
206	A	10	9	1.	21	0.429
207	A	5	5	1.	21	0.238
208	A	5	5	1.	21	0.238
209	A	7	7	1.	21	0.333
210	A	8	7	1.	21	0.333
211	A	11	10	1.	21	0.476
212	A	10	9	1.	21	0.429
213	A	10	9	1.	21	0.429
214	A	10	9	1.	21	0.429
215	A	11	10	1.	21	0.476
216	A	4	4	1.	17	0.235
217	A	4	4	1.	26	0.154
218	A	3	2	1.	19	0.105
219	A	4	4	1.	19	0.21
220	A	3	3	0.91	17	0.176
221	A	2	2	1.	19	0.105
222	A	2	2	1.	19	0.105
223	A	7	7	1.	21	0.333
224	A	6	6	1.	21	0.286
225	A	6	6	1.	21	0.286
226	A	5	5	1.	19	0.263
227	A	4	4	1.	11	0.364
228	A	7	6	1.	21	0.286
229	A	8	7	1.	21	0.333
230	A	9	7	1.	21	0.333
231	A	7	7	1.	21	0.333
232	A	7	6	1.	21	0.286
233	A	6	5	1.	19	0.263
234	A	5	5	1.	11	0.454

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
235	A	7	6	1.	21	0.286
236	A	8	7	1.	21	0.333
237	A	9	7	1.	21	0.333
238	A	8	7	1.	21	0.333
239	A	8	6	1.	21	0.286
240	A	7	5	1.	19	0.263
241	A	6	5	1.	11	0.454
242	A	8	7	1.	21	0.333
243	A	8	7	1.	21	0.333
244	A	9	8	1.	21	0.381
245	A	5	5	1.	21	0.238
246	A	5	5	1.	21	0.238
247	A	4	4	1.	19	0.21
248	A	4	4	1.	11	0.364
249	A	7	6	1.	21	0.286
250	A	8	7	1.	21	0.333
251	A	9	7	1.	21	0.333
252	A	5	5	1.	21	0.238
253	A	5	5	0.96	21	0.238
254	A	5	5	1.	19	0.263
255	A	5	4	1.02	11	0.364
256	A	8	7	1.	21	0.333
257	A	9	8	1.	21	0.381
258	A	10	8	1.	21	0.381
259	A	5	5	1.	21	0.238
260	A	6	6	0.97	21	0.286
261	A	6	5	1.	19	0.263
262	A	6	4	1.04	11	0.364
263	A	9	7	1.	21	0.333
264	A	10	8	1.	21	0.381
265	A	11	8	1.	21	0.381
266	A	8	8	1.	23	0.348
267	A	4	4	1.	23	0.174
268	A	5	5	1.	23	0.217
269	A	3	3	1.	19	0.158
270	A	3	3	1.	17	0.176
271	A	6	6	1.	23	0.261
272	A	7	7	1.	23	0.304
273	A	7	7	1.	23	0.304
274	A	4	3	1.	19	0.158
275	A	8	8	1.	17	0.471
276	A	3	2	1.	21	0.095
277	A	3	2	1.	17	0.118
278	A	4	4	1.	17	0.235

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
279	A	9	9	1.	17	0.529
280	A	4	3	1.	17	0.176
281	A	3	3	1.	24	0.125
282	A	3	3	1.	24	0.125
283	A	3	3	1.	20	0.15
284	A	2	1	1.	17	0.059
285	A	2	1	1.	17	0.059
286	A	2	1	1.	17	0.059
287	A	2	1	1.	15	0.067
288	A	2	2	1.	17	0.118
289	A	2	2	1.	17	0.118
290	A	2	2	1.	17	0.118
291	A	2	2	1.	17	0.118
292	A	2	1	1.	19	0.053
293	A	2	1	1.	19	0.053
294	A	2	1	1.	17	0.059
295	A	3	3	1.	19	0.158
296	A	3	3	1.	19	0.158
297	A	3	3	1.	19	0.158
298	A	5	4	1.	19	0.21
299	A	4	4	1.	19	0.21
300	A	3	3	1.	19	0.158
301	A	2	2	1.	17	0.118
302	A	3	2	1.	19	0.105
303	A	4	3	1.	19	0.158
304	A	5	4	1.	19	0.21
305	A	5	4	1.	19	0.21
306	A	4	4	1.	19	0.21
307	A	3	3	1.	19	0.158
308	A	2	2	1.	17	0.118
309	A	4	3	1.	19	0.158
310	A	5	4	1.	19	0.21
311	A	6	4	1.	19	0.21
312	A	3	2	1.	19	0.105
313	A	5	5	1.	19	0.263
314	A	4	4	0.98	19	0.21
315	A	3	3	0.91	17	0.176
316	A	2	2	1.	9	0.222
317	A	2	2	1.	19	0.105
318	A	2	2	1.	19	0.105
319	A	2	2	1.	19	0.105
320	A	1	1	1.	28	0.036
321	A	4	2	1.	25	0.08
322	A	3	2	1.	25	0.08

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
323	A	2	2	1.	23	0.087
324	A	1	1	1.	15	0.067
325	A	1	1	1.	23	0.043
326	A	1	1	1.	25	0.04
327	A	1	1	1.	25	0.04
328	A	2	2	0.93	28	0.071
329	A	1	1	1.	69	0.014
330	A	5	3	1.	25	0.12
331	A	3	3	1.	23	0.13
332	A	2	2	1.	15	0.133
333	A	2	2	1.	25	0.08
334	A	2	2	1.	23	0.087
335	A	2	2	1.	25	0.08
336	A	2	2	1.	25	0.08
337	A	6	4	1.08	31	0.129
338	A	4	4	1.08	31	0.129
339	A	2	2	1.07	29	0.069
340	A	5	5	1.	31	0.161
341	A	5	5	1.19	31	0.161
342	A	5	4	1.36	31	0.129
343	A	9	8	1.	31	0.258
344	A	7	7	1.	31	0.226
345	A	5	5	1.	28	0.179
346	A	5	5	1.	31	0.161
347	A	6	6	1.	31	0.194
348	A	5	5	1.	29	0.172
349	A	4	4	1.	29	0.138
350	A	3	3	1.	29	0.103
351	A	2	2	1.	27	0.074
352	A	2	2	1.	26	0.077
353	A	3	3	1.	29	0.103
354	A	2	2	1.	29	0.069
355	A	3	3	1.	29	0.103
356	A	2	2	1.	29	0.069
357	A	5	5	1.	29	0.172
358	A	8	7	1.	31	0.226
359	A	4	4	1.	31	0.129
360	A	6	6	1.	31	0.194
361	A	2	2	1.	29	0.069
362	A	4	4	1.	28	0.143
363	A	3	3	1.	31	0.097
364	A	4	4	1.	31	0.129
365	A	3	3	1.	31	0.097
366	A	2	2	1.	31	0.065

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
367	A	5	5	1.	31	0.161
368	A	8	8	1.	31	0.258
369	A	4	4	1.	31	0.129
370	A	7	7	1.	31	0.226
371	A	2	2	1.	29	0.069
372	A	4	4	1.	28	0.143
373	A	3	3	1.	31	0.097
374	A	2	2	1.	31	0.065
375	A	5	5	1.	31	0.161
376	A	4	4	1.	31	0.129
377	A	7	7	1.	31	0.226
378	A	3	3	1.	31	0.097
379	A	1	1	1.	57	0.018
380	A	3	3	1.	32	0.094
381	A	4	4	1.	41	0.098
382	A	4	3	1.	31	0.097
383	A	4	4	1.	35	0.114
384	A	3	3	1.	31	0.097
385	A	2	2	1.08	76	0.026

Chapter 3

Listing of integrals

3.1 $\int \left(a+bx^3\right)\left(c+dx^3\right)^4 \, dx$

Optimal. Leaf size=94

$$\frac{2}{7}c^2dx^7(3ad+2bc)+\frac{1}{4}c^3x^4(4ad+bc)+\frac{1}{13}d^3x^{13}(ad+4bc)+\frac{1}{5}cd^2x^{10}(2ad+3bc)+ac^4x+\frac{1}{16}bd^4x^{16}$$

```
[Out] a*c^4*x + (c^3*(b*c + 4*a*d)*x^4)/4 + (2*c^2*d*(2*b*c + 3*a*d)*x^7)/7 + (c*
d^2*(3*b*c + 2*a*d)*x^10)/5 + (d^3*(4*b*c + a*d)*x^13)/13 + (b*d^4*x^16)/16
```

Rubi [A] time = 0.0724306, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$\frac{2}{7}c^2dx^7(3ad+2bc)+\frac{1}{4}c^3x^4(4ad+bc)+\frac{1}{13}d^3x^{13}(ad+4bc)+\frac{1}{5}cd^2x^{10}(2ad+3bc)+ac^4x+\frac{1}{16}bd^4x^{16}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)*(c + d*x^3)^4,x]
```

```
[Out] a*c^4*x + (c^3*(b*c + 4*a*d)*x^4)/4 + (2*c^2*d*(2*b*c + 3*a*d)*x^7)/7 + (c*
d^2*(3*b*c + 2*a*d)*x^10)/5 + (d^3*(4*b*c + a*d)*x^13)/13 + (b*d^4*x^16)/16
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^3\right)\left(c+dx^3\right)^4 \, dx &= \int \left(ac^4+c^3(bc+4ad)x^3+2c^2d(2bc+3ad)x^6+2cd^2(3bc+2ad)x^9+d^3(4bc+ad)x^{12}+b\right.\\ &= ac^4x+\frac{1}{4}c^3(bc+4ad)x^4+\frac{2}{7}c^2d(2bc+3ad)x^7+\frac{1}{5}cd^2(3bc+2ad)x^{10}+\frac{1}{13}d^3(4bc+ad)x^{1}\end{aligned}$$

Mathematica [A] time = 0.0204303, size = 94, normalized size = 1.

$$\frac{2}{7}c^2dx^7(3ad+2bc)+\frac{1}{4}c^3x^4(4ad+bc)+\frac{1}{13}d^3x^{13}(ad+4bc)+\frac{1}{5}cd^2x^{10}(2ad+3bc)+ac^4x+\frac{1}{16}bd^4x^{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)*(c + d*x^3)^4,x]
```

```
[Out] a*c^4*x + (c^3*(b*c + 4*a*d)*x^4)/4 + (2*c^2*d*(2*b*c + 3*a*d)*x^7)/7 + (c*
d^2*(3*b*c + 2*a*d)*x^10)/5 + (d^3*(4*b*c + a*d)*x^13)/13 + (b*d^4*x^16)/16
```

Maple [A] time = 0., size = 97, normalized size = 1.

$$\frac{bd^4x^{16}}{16}+\frac{\left(ad^4+4\,bcd^3\right)x^{13}}{13}+\frac{\left(4\,acd^3+6\,c^2d^2b\right)x^{10}}{10}+\frac{\left(6\,ac^2d^2+4\,c^3db\right)x^7}{7}+\frac{\left(4\,ac^3d+bc^4\right)x^4}{4}+ac^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(d*x^3+c)^4,x)
```

```
[Out] 1/16*b*d^4*x^16+1/13*(a*d^4+4*b*c*d^3)*x^13+1/10*(4*a*c*d^3+6*b*c^2*d^2)*x^
10+1/7*(6*a*c^2*d^2+4*b*c^3*d)*x^7+1/4*(4*a*c^3*d+b*c^4)*x^4+a*c^4*x
```

Maxima [A] time = 0.979166, size = 130, normalized size = 1.38

$$\frac{1}{16}\,bd^4x^{16}+\frac{1}{13}\left(4\,bcd^3+ad^4\right)x^{13}+\frac{1}{5}\left(3\,bc^2d^2+2\,acd^3\right)x^{10}+\frac{2}{7}\left(2\,bc^3d+3\,ac^2d^2\right)x^7+ac^4x+\frac{1}{4}\left(bc^4+4\,ac^3d\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c)^4,x, algorithm="maxima")
```

```
[Out] 1/16*b*d^4*x^16 + 1/13*(4*b*c*d^3 + a*d^4)*x^13 + 1/5*(3*b*c^2*d^2 + 2*a*c*
d^3)*x^10 + 2/7*(2*b*c^3*d + 3*a*c^2*d^2)*x^7 + a*c^4*x + 1/4*(b*c^4 + 4*a*
c^3*d)*x^4
```

Fricas [A] time = 1.26752, size = 234, normalized size = 2.49

$$\frac{1}{16}x^{16}d^4b+\frac{4}{13}x^{13}d^3cb+\frac{1}{13}x^{13}d^4a+\frac{3}{5}x^{10}d^2c^2b+\frac{2}{5}x^{10}d^3ca+\frac{4}{7}x^7dc^3b+\frac{6}{7}x^7d^2c^2a+\frac{1}{4}x^4c^4b+x^4dc^3a+xc^4a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c)^4,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*d^4*b + 4/13*x^13*d^3*c*b + 1/13*x^13*d^4*a + 3/5*x^10*d^2*c^2*b
 + 2/5*x^10*d^3*c*a + 4/7*x^7*d*c^3*b + 6/7*x^7*d^2*c^2*a + 1/4*x^4*c^4*b +
x^4*d*c^3*a + x*c^4*a
```

Sympy [A] time = 0.101292, size = 104, normalized size = 1.11

$$ac^4x+\frac{bd^4x^{16}}{16}+x^{13}\left(\frac{ad^4}{13}+\frac{4bcd^3}{13}\right)+x^{10}\left(\frac{2acd^3}{5}+\frac{3bc^2d^2}{5}\right)+x^7\left(\frac{6ac^2d^2}{7}+\frac{4bc^3d}{7}\right)+x^4\left(ac^3d+\frac{bc^4}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(d*x**3+c)**4,x)
```

```
[Out] a*c**4*x + b*d**4*x**16/16 + x**13*(a*d**4/13 + 4*b*c*d**3/13) + x**10*(2*a
*c*d**3/5 + 3*b*c**2*d**2/5) + x**7*(6*a*c**2*d**2/7 + 4*b*c**3*d/7) + x**4
*(a*c**3*d + b*c**4/4)
```

Giac [A] time = 1.10963, size = 131, normalized size = 1.39

$$\frac{1}{16}bd^4x^{16} + \frac{4}{13}bcd^3x^{13} + \frac{1}{13}ad^4x^{13} + \frac{3}{5}bc^2d^2x^{10} + \frac{2}{5}acd^3x^{10} + \frac{4}{7}bc^3dx^7 + \frac{6}{7}ac^2d^2x^7 + \frac{1}{4}bc^4x^4 + ac^3dx^4 + ac^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c)^4,x, algorithm="giac")
```

```
[Out] 1/16*b*d^4*x^16 + 4/13*b*c*d^3*x^13 + 1/13*a*d^4*x^13 + 3/5*b*c^2*d^2*x^10
+ 2/5*a*c*d^3*x^10 + 4/7*b*c^3*d*x^7 + 6/7*a*c^2*d^2*x^7 + 1/4*b*c^4*x^4 +
a*c^3*d*x^4 + a*c^4*x
```

3.2 $\int \left(a+bx^3\right) \left(c+dx^3\right)^3 \, dx$

Optimal. Leaf size=70

$$\frac{1}{4}c^2x^4(3ad+bc)+\frac{1}{10}d^2x^{10}(ad+3bc)+\frac{3}{7}cdx^7(ad+bc)+ac^3x+\frac{1}{13}bd^3x^{13}$$

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^4)/4 + (3*c*d*(b*c + a*d)*x^7)/7 + (d^2*(3*b
*c + a*d)*x^10)/10 + (b*d^3*x^13)/13
```

Rubi [A] time = 0.0426525, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$\frac{1}{4}c^2x^4(3ad+bc)+\frac{1}{10}d^2x^{10}(ad+3bc)+\frac{3}{7}cdx^7(ad+bc)+ac^3x+\frac{1}{13}bd^3x^{13}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)*(c + d*x^3)^3,x]
```

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^4)/4 + (3*c*d*(b*c + a*d)*x^7)/7 + (d^2*(3*b
*c + a*d)*x^10)/10 + (b*d^3*x^13)/13
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^3\right)\left(c+dx^3\right)^3 \, dx &= \int \left(ac^3+c^2(bc+3ad)x^3+3cd(bc+ad)x^6+d^2(3bc+ad)x^9+bd^3x^{12}\right) dx\\ &= ac^3x+\frac{1}{4}c^2(bc+3ad)x^4+\frac{3}{7}cd(bc+ad)x^7+\frac{1}{10}d^2(3bc+ad)x^{10}+\frac{1}{13}bd^3x^{13}\end{aligned}$$

Mathematica [A] time = 0.0125842, size = 70, normalized size = 1.

$$\frac{1}{4}c^2x^4(3ad+bc)+\frac{1}{10}d^2x^{10}(ad+3bc)+\frac{3}{7}cdx^7(ad+bc)+ac^3x+\frac{1}{13}bd^3x^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)*(c + d*x^3)^3,x]
```

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^4)/4 + (3*c*d*(b*c + a*d)*x^7)/7 + (d^2*(3*b
*c + a*d)*x^10)/10 + (b*d^3*x^13)/13
```

Maple [A] time = 0.001, size = 73, normalized size = 1.

$$\frac{bd^3x^{13}}{13}+\frac{\left(ad^3+3\,bcd^2\right)x^{10}}{10}+\frac{\left(3\,acd^2+3\,bc^2d\right)x^7}{7}+\frac{\left(3\,ac^2d+bc^3\right)x^4}{4}+ac^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(d*x^3+c)^3,x)
```

```
[Out] 1/13*b*d^3*x^13+1/10*(a*d^3+3*b*c*d^2)*x^10+1/7*(3*a*c*d^2+3*b*c^2*d)*x^7+1
/4*(3*a*c^2*d+b*c^3)*x^4+a*c^3*x
```

Maxima [A] time = 0.955029, size = 95, normalized size = 1.36

$$\frac{1}{13}\,bd^3x^{13} + \frac{1}{10}\left(3\,bcd^2 + ad^3\right)x^{10} + \frac{3}{7}\left(bc^2d + acd^2\right)x^7 + ac^3x + \frac{1}{4}\left(bc^3 + 3\,ac^2d\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] 1/13*b*d^3*x^13 + 1/10*(3*b*c*d^2 + a*d^3)*x^10 + 3/7*(b*c^2*d + a*c*d^2)*x
^7 + a*c^3*x + 1/4*(b*c^3 + 3*a*c^2*d)*x^4
```

Fricas [A] time = 1.34321, size = 182, normalized size = 2.6

$$\frac{1}{13}x^{13}d^3b + \frac{3}{10}x^{10}d^2cb + \frac{1}{10}x^{10}d^3a + \frac{3}{7}x^7dc^2b + \frac{3}{7}x^7d^2ca + \frac{1}{4}x^4c^3b + \frac{3}{4}x^4dc^2a + xc^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*d^3*b + 3/10*x^10*d^2*c*b + 1/10*x^10*d^3*a + 3/7*x^7*d*c^2*b + 3
/7*x^7*d^2*c*a + 1/4*x^4*c^3*b + 3/4*x^4*d*c^2*a + x*c^3*a
```

Sympy [A] time = 0.077301, size = 80, normalized size = 1.14

$$ac^3x + \frac{bd^3x^{13}}{13} + x^{10}\left(\frac{ad^3}{10} + \frac{3bcd^2}{10}\right) + x^7\left(\frac{3acd^2}{7} + \frac{3bc^2d}{7}\right) + x^4\left(\frac{3ac^2d}{4} + \frac{bc^3}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(d*x**3+c)**3,x)
```

```
[Out] a*c**3*x + b*d**3*x**13/13 + x**10*(a*d**3/10 + 3*b*c*d**2/10) + x**7*(3*a*
c*d**2/7 + 3*b*c**2*d/7) + x**4*(3*a*c**2*d/4 + b*c**3/4)
```

Giac [A] time = 1.10375, size = 100, normalized size = 1.43

$$\frac{1}{13}\,bd^3x^{13} + \frac{3}{10}\,bcd^2x^{10} + \frac{1}{10}\,ad^3x^{10} + \frac{3}{7}\,bc^2dx^7 + \frac{3}{7}\,acd^2x^7 + \frac{1}{4}\,bc^3x^4 + \frac{3}{4}\,ac^2dx^4 + ac^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] 1/13*b*d^3*x^13 + 3/10*b*c*d^2*x^10 + 1/10*a*d^3*x^10 + 3/7*b*c^2*d*x^7 + 3
/7*a*c*d^2*x^7 + 1/4*b*c^3*x^4 + 3/4*a*c^2*d*x^4 + a*c^3*x
```

3.3 $\int \left(a+bx^3\right)\left(c+dx^3\right)^2\,dx$

Optimal. Leaf size=50

$$\frac{1}{7}dx^7(ad+2bc)+\frac{1}{4}cx^4(2ad+bc)+ac^2x+\frac{1}{10}bd^2x^{10}$$

```
[Out] a*c^2*x + (c*(b*c + 2*a*d)*x^4)/4 + (d*(2*b*c + a*d)*x^7)/7 + (b*d^2*x^10)/
10
```

Rubi [A] time = 0.0284518, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$\frac{1}{7}dx^7(ad+2bc)+\frac{1}{4}cx^4(2ad+bc)+ac^2x+\frac{1}{10}bd^2x^{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)*(c + d*x^3)^2,x]
```

```
[Out] a*c^2*x + (c*(b*c + 2*a*d)*x^4)/4 + (d*(2*b*c + a*d)*x^7)/7 + (b*d^2*x^10)/
10
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^3\right)\left(c+dx^3\right)^2\,dx &= \int \left(ac^2+c(bc+2ad)x^3+d(2bc+ad)x^6+bd^2x^9\right)dx\\ &= ac^2x+\frac{1}{4}c(bc+2ad)x^4+\frac{1}{7}d(2bc+ad)x^7+\frac{1}{10}bd^2x^{10}\end{aligned}$$

Mathematica [A] time = 0.0081618, size = 50, normalized size = 1.

$$\frac{1}{7}dx^7(ad+2bc)+\frac{1}{4}cx^4(2ad+bc)+ac^2x+\frac{1}{10}bd^2x^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)*(c + d*x^3)^2,x]
```

```
[Out] a*c^2*x + (c*(b*c + 2*a*d)*x^4)/4 + (d*(2*b*c + a*d)*x^7)/7 + (b*d^2*x^10)/
10
```

Maple [A] time = 0.001, size = 49, normalized size = 1.

$$\frac{bd^2x^{10}}{10}+\frac{\left(ad^2+2\,bcd\right)x^7}{7}+\frac{\left(2\,acd+bc^2\right)x^4}{4}+ac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(d*x^3+c)^2,x)
```

```
[Out] 1/10*b*d^2*x^10+1/7*(a*d^2+2*b*c*d)*x^7+1/4*(2*a*c*d+b*c^2)*x^4+a*c^2*x
```

Maxima [A] time = 0.940585, size = 65, normalized size = 1.3

$$\frac{1}{10}\,bd^2x^{10} + \frac{1}{7}\left(2\,bcd + ad^2\right)x^7 + \frac{1}{4}\left(bc^2 + 2\,acd\right)x^4 + ac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] 1/10*b*d^2*x^10 + 1/7*(2*b*c*d + a*d^2)*x^7 + 1/4*(b*c^2 + 2*a*c*d)*x^4 + a
*c^2*x
```

Fricas [A] time = 1.28691, size = 123, normalized size = 2.46

$$\frac{1}{10}x^{10}d^2b + \frac{2}{7}x^7dcb + \frac{1}{7}x^7d^2a + \frac{1}{4}x^4c^2b + \frac{1}{2}x^4dca + xc^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*d^2*b + 2/7*x^7*d*c*b + 1/7*x^7*d^2*a + 1/4*x^4*c^2*b + 1/2*x^4*d
*c*a + x*c^2*a
```

Sympy [A] time = 0.101341, size = 51, normalized size = 1.02

$$ac^2x + \frac{bd^2x^{10}}{10} + x^7\left(\frac{ad^2}{7} + \frac{2bcd}{7}\right) + x^4\left(\frac{acd}{2} + \frac{bc^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(d*x**3+c)**2,x)
```

```
[Out] a*c**2*x + b*d**2*x**10/10 + x**7*(a*d**2/7 + 2*b*c*d/7) + x**4*(a*c*d/2 +
b*c**2/4)
```

Giac [A] time = 1.08964, size = 68, normalized size = 1.36

$$\frac{1}{10}\,bd^2x^{10} + \frac{2}{7}\,bcdx^7 + \frac{1}{7}\,ad^2x^7 + \frac{1}{4}\,bc^2x^4 + \frac{1}{2}\,acdx^4 + ac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] 1/10*b*d^2*x^10 + 2/7*b*c*d*x^7 + 1/7*a*d^2*x^7 + 1/4*b*c^2*x^4 + 1/2*a*c*d
*x^4 + a*c^2*x
```

3.4 $\int \left(a+bx^3\right)\left(c+dx^3\right)\,dx$

Optimal. Leaf size=28

$$\frac{1}{4}x^4(ad+bc)+acx+\frac{1}{7}bdx^7$$

```
[Out] a*c*x + ((b*c + a*d)*x^4)/4 + (b*d*x^7)/7
```

Rubi [A] time = 0.0126816, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {373}

$$\frac{1}{4}x^4(ad+bc)+acx+\frac{1}{7}bdx^7$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)*(c + d*x^3),x]
```

```
[Out] a*c*x + ((b*c + a*d)*x^4)/4 + (b*d*x^7)/7
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^3\right)\left(c+dx^3\right)\,dx &= \int \left(ac+(bc+ad)x^3+bdx^6\right)\,dx\\ &= acx+\frac{1}{4}(bc+ad)x^4+\frac{1}{7}bdx^7\end{aligned}$$

Mathematica [A] time = 0.0050431, size = 28, normalized size = 1.

$$\frac{1}{4}x^4(ad+bc)+acx+\frac{1}{7}bdx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)*(c + d*x^3),x]
```

```
[Out] a*c*x + ((b*c + a*d)*x^4)/4 + (b*d*x^7)/7
```

Maple [A] time = 0., size = 25, normalized size = 0.9

$$acx+\frac{(ad+bc)\,x^4}{4}+\frac{bdx^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(d*x^3+c),x)
```

```
[Out] a*c*x+1/4*(a*d+b*c)*x^4+1/7*b*d*x^7
```

Maxima [A] time = 0.959284, size = 32, normalized size = 1.14

$$\frac{1}{7}bdx^7 + \frac{1}{4}(bc + ad)x^4 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/7*b*d*x^7 + 1/4*(b*c + a*d)*x^4 + a*c*x
```

Fricas [A] time = 1.39328, size = 66, normalized size = 2.36

$$\frac{1}{7}x^7db + \frac{1}{4}x^4cb + \frac{1}{4}x^4da + xca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*d*b + 1/4*x^4*c*b + 1/4*x^4*d*a + x*c*a
```

Sympy [A] time = 0.075002, size = 26, normalized size = 0.93

$$acx + \frac{bdx^7}{7} + x^4\left(\frac{ad}{4} + \frac{bc}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(d*x**3+c),x)
```

```
[Out] a*c*x + b*d*x**7/7 + x**4*(a*d/4 + b*c/4)
```

Giac [A] time = 1.11432, size = 35, normalized size = 1.25

$$\frac{1}{7}bdx^7 + \frac{1}{4}bcx^4 + \frac{1}{4}adx^4 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/7*b*d*x^7 + 1/4*b*c*x^4 + 1/4*a*d*x^4 + a*c*x
```

3.5 $\int \frac{a+bx^3}{c+dx^3}\,dx$

Optimal. Leaf size=144

$$\frac{(bc-ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}}-\frac{(bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{4/3}}+\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{4/3}}+\frac{bx}{d}$$

```
[Out] (b*x)/d + ((b*c - a*d)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(
Sqrt[3]*c^(2/3)*d^(4/3)) - ((b*c - a*d)*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3
)*d^(4/3)) + ((b*c - a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(
6*c^(2/3)*d^(4/3))
```

Rubi [A] time = 0.0937481, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {388, 200, 31, 634, 617, 204, 628}

$$\frac{(bc-ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}}-\frac{(bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{4/3}}+\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{4/3}}+\frac{bx}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)/(c + d*x^3),x]
```

```
[Out] (b*x)/d + ((b*c - a*d)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(
Sqrt[3]*c^(2/3)*d^(4/3)) - ((b*c - a*d)*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3
)*d^(4/3)) + ((b*c - a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(
6*c^(2/3)*d^(4/3))
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^3}{c+dx^3}\,dx &= \frac{bx}{d}-\frac{(bc-ad)\int \frac{1}{c+dx^3}\,dx}{d}\\
&= \frac{bx}{d}-\frac{(bc-ad)\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{3c^{2/3}d}-\frac{(bc-ad)\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{3c^{2/3}d}\\
&= \frac{bx}{d}-\frac{(bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{4/3}}+\frac{(bc-ad)\int \frac{-\sqrt[3]{c}\sqrt[3]{d}+2d^{2/3}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{6c^{2/3}d^{4/3}}-\frac{(bc-ad)\int \frac{1}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{2\sqrt[3]{cd}}\\
&= \frac{bx}{d}-\frac{(bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{4/3}}+\frac{(bc-ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}}-\frac{(bc-ad)\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,d\right.}{c^{2/3}d^{4/3}}\\
&= \frac{bx}{d}+\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{4/3}}-\frac{(bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{4/3}}+\frac{(bc-ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}}\end{aligned}$$

Mathematica [A] time = 0.0778663, size = 128, normalized size = 0.89

$$\frac{(bc-ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)-2(bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)+2\sqrt{3}(bc-ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)+6bc^{2/3}\sqrt[3]{d}x}{6c^{2/3}d^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)/(c + d*x^3),x]
```

```
[Out] (6*b*c^(2/3)*d^(1/3)*x + 2*Sqrt[3]*(b*c - a*d)*ArcTan[(1 - (2*d^(1/3)*x)/c^
(1/3))/Sqrt[3]] - 2*(b*c - a*d)*Log[c^(1/3) + d^(1/3)*x] + (b*c - a*d)*Log[
c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c^(2/3)*d^(4/3))
```

Maple [A] time = 0.005, size = 195, normalized size = 1.4

$$\frac{bx}{d}+\frac{a}{3d}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}-\frac{bc}{3d^2}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}-\frac{a}{6d}\ln\left(x^2-\sqrt[3]{\frac{c}{d}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}+\frac{bc}{6d^2}\ln\left(x^2-\sqrt[3]{\frac{c}{d}}x+\left(\frac{c}{d}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)/(d*x^3+c),x)
```

```
[Out] b*x/d+1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*a-1/3/d^2/(c/d)^(2/3)*ln(x+(c/d)^
(1/3))*b*c-1/6/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*a+1/6/d^2/(c
/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*b*c+1/3/d/(c/d)^(2/3)*3^(1/2)*a
rctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*a-1/3/d^2/(c/d)^(2/3)*3^(1/2)*arctan
(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*b*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.7042, size = 900, normalized size = 6.25

$$\left[\frac{6\,bc^2dx-3\sqrt{\frac{1}{3}}\left(bc^2d-acd^2\right)\sqrt{-\frac{\left(c^2d\right)^{\frac{1}{3}}}{d}}\log\left(\frac{2\,cdx^3-3\left(c^2d\right)^{\frac{1}{3}}cx-c^2+3\sqrt{\frac{1}{3}}\left(2\,cdx^2+\left(c^2d\right)^{\frac{2}{3}}x-\left(c^2d\right)^{\frac{1}{3}}c\right)\sqrt{-\frac{\left(c^2d\right)^{\frac{1}{3}}}{d}}}{dx^3+c}\right)+\left(c^2d\right)^{\frac{2}{3}}(bc-ad)\log\left(\right.}{6\,c^2d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/6*(6*b*c^2*d*x - 3*sqrt(1/3)*(b*c^2*d - a*c*d^2)*sqrt(-(c^2*d)^(1/3)/d)*
log((2*c*d*x^3 - 3*(c^2*d)^(1/3)*c*x - c^2 + 3*sqrt(1/3)*(2*c*d*x^2 + (c^2*
d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt(-(c^2*d)^(1/3)/d))/(d*x^3 + c)) + (c^2*d
)^(2/3)*(b*c - a*d)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) - 2*(c
^2*d)^(2/3)*(b*c - a*d)*log(c*d*x + (c^2*d)^(2/3)))/(c^2*d^2), 1/6*(6*b*c^2
*d*x - 6*sqrt(1/3)*(b*c^2*d - a*c*d^2)*sqrt((c^2*d)^(1/3)/d)*arctan(sqrt(1/
3)*(2*(c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt((c^2*d)^(1/3)/d)/c^2) + (c^2*
d)^(2/3)*(b*c - a*d)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) - 2*(
c^2*d)^(2/3)*(b*c - a*d)*log(c*d*x + (c^2*d)^(2/3)))/(c^2*d^2)]
```

Sympy [A] time = 0.831132, size = 71, normalized size = 0.49

$$\frac{bx}{d} + \operatorname{RootSum}\left(27t^3c^2d^4 - a^3d^3 + 3a^2bcd^2 - 3ab^2c^2d + b^3c^3, \left(t \mapsto t\log\left(\frac{3tcd}{ad - bc} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)/(d*x**3+c),x)
```

```
[Out] b*x/d + RootSum(27*_t**3*c**2*d**4 - a**3*d**3 + 3*a**2*b*c*d**2 - 3*a*b**2
*c**2*d + b**3*c**3, Lambda(_t, _t*log(3*_t*c*d/(a*d - b*c) + x)))
```

Giac [A] time = 1.12975, size = 217, normalized size = 1.51

$$\frac{bx}{d} + \frac{(bc - ad)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\,cd} - \frac{\sqrt{3}\left(\left(-cd^2\right)^{\frac{1}{3}}bc - \left(-cd^2\right)^{\frac{1}{3}}ad\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\,cd^2} - \frac{\left(\left(-cd^2\right)^{\frac{1}{3}}bc - \left(-cd^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] b*x/d + 1/3*(b*c - a*d)*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(c*d) - 1/3
*sqrt(3)*((-c*d^2)^(1/3)*b*c - (-c*d^2)^(1/3)*a*d)*arctan(1/3*sqrt(3)*(2*x
+ (-c/d)^(1/3))/(-c/d)^(1/3))/(c*d^2) - 1/6*((-c*d^2)^(1/3)*b*c - (-c*d^2)^
(1/3)*a*d)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(c*d^2)
```

3.6 $\int \frac{a+bx^3}{\left(c+dx^3\right)^2} dx$

Optimal. Leaf size=169

$$-\frac{(2ad+bc)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{18c^{5/3}d^{4/3}}+\frac{(2ad+bc)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}d^{4/3}}-\frac{(2ad+bc)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}d^{4/3}}-\frac{x(bc-ad)}{3cd\left(c+dx^3\right)}$$

```
[Out] -((b*c - a*d)*x)/(3*c*d*(c + d*x^3)) - ((b*c + 2*a*d)*ArcTan[(c^(1/3) - 2*d
^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(3*Sqrt[3]*c^(5/3)*d^(4/3)) + ((b*c + 2*a*d)*
Log[c^(1/3) + d^(1/3)*x])/(9*c^(5/3)*d^(4/3)) - ((b*c + 2*a*d)*Log[c^(2/3)
- c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(18*c^(5/3)*d^(4/3))
```

Rubi [A] time = 0.082311, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {385, 200, 31, 634, 617, 204, 628}

$$-\frac{(2ad+bc)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{18c^{5/3}d^{4/3}}+\frac{(2ad+bc)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}d^{4/3}}-\frac{(2ad+bc)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}d^{4/3}}-\frac{x(bc-ad)}{3cd\left(c+dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)/(c + d*x^3)^2,x]
```

```
[Out] -((b*c - a*d)*x)/(3*c*d*(c + d*x^3)) - ((b*c + 2*a*d)*ArcTan[(c^(1/3) - 2*d
^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(3*Sqrt[3]*c^(5/3)*d^(4/3)) + ((b*c + 2*a*d)*
Log[c^(1/3) + d^(1/3)*x])/(9*c^(5/3)*d^(4/3)) - ((b*c + 2*a*d)*Log[c^(2/3)
- c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(18*c^(5/3)*d^(4/3))
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{a+bx^3}{\left(c+dx^3\right)^2}\,dx = -\frac{(bc-ad)x}{3cd\left(c+dx^3\right)} + \frac{(bc+2ad)\int \frac{1}{c+dx^3}\,dx}{3cd}$$

$$= -\frac{(bc-ad)x}{3cd\left(c+dx^3\right)} + \frac{(bc+2ad)\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{9c^{5/3}d} + \frac{(bc+2ad)\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{9c^{5/3}d}$$

$$= -\frac{(bc-ad)x}{3cd\left(c+dx^3\right)} + \frac{(bc+2ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}d^{4/3}} - \frac{(bc+2ad)\int \frac{-\sqrt[3]{c}\sqrt[3]{d}+2d^{2/3}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{18c^{5/3}d^{4/3}} + \frac{(bc+2ad)\int -}{6c}$$

$$= -\frac{(bc-ad)x}{3cd\left(c+dx^3\right)} + \frac{(bc+2ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}d^{4/3}} - \frac{(bc+2ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{18c^{5/3}d^{4/3}} + \frac{(bc+2a}{}$$

$$= -\frac{(bc-ad)x}{3cd\left(c+dx^3\right)} - \frac{(bc+2ad)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}d^{4/3}} + \frac{(bc+2ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}d^{4/3}} - \frac{(bc+2ad)\log\left(c^{2/3}\right.}{18c^{5}}$$

Mathematica [A] time = 0.0924508, size = 145, normalized size = 0.86

$$\frac{-(2ad+bc)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right) - \frac{6c^{2/3}\sqrt[3]{d}x(bc-ad)}{c+dx^3} + 2(2ad+bc)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right) - 2\sqrt{3}(2ad+bc)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}}{\sqrt[3]{c}}}{\sqrt{3}}\right.}{18c^{5/3}d^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)/(c + d*x^3)^2,x]
```

```
[Out] ((-6*c^(2/3)*d^(1/3)*(b*c - a*d)*x)/(c + d*x^3) - 2*Sqrt[3]*(b*c + 2*a*d)*A
rcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]] + 2*(b*c + 2*a*d)*Log[c^(1/3) +
d^(1/3)*x] - (b*c + 2*a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/
```

```
(18*c^(5/3)*d^(4/3))
```

Maple [A] time = 0.009, size = 221, normalized size = 1.3

$$\frac{(ad-bc)\,x}{3\,cd\left(dx^{3}+c\right)}+\frac{2\,a}{9\,cd}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}+\frac{b}{9\,d^{2}}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}-\frac{a}{9\,cd}\ln\left(x^{2}-\sqrt[3]{\frac{c}{d}}x+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}-\frac{b}{18\,d^{2}}\ln\left(x^{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)/(d*x^3+c)^2,x)
```

```
[Out] 1/3*(a*d-b*c)/c/d*x/(d*x^3+c)+2/9/c/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*a+1/9/d
^2/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*b-1/9/c/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x
+(c/d)^(2/3))*a-1/18/d^2/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*b+2/
9/c/d/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*a+1/9/d^2
/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.70572, size = 1233, normalized size = 7.3

$$\left[\frac{3\sqrt{\frac{1}{3}}\left(bc^{3}d+2\,ac^{2}d^{2}+\left(bc^{2}d^{2}+2\,acd^{3}\right)x^{3}\right)\sqrt{-\frac{\left(c^{2}d\right)^{\frac{1}{3}}}{d}}\log\left(\frac{2\,cdx^{3}-3\left(c^{2}d\right)^{\frac{1}{3}}cx-c^{2}+3\sqrt{\frac{1}{3}}\left(2\,cdx^{2}+\left(c^{2}d\right)^{\frac{2}{3}}x-\left(c^{2}d\right)^{\frac{1}{3}}c\right)\sqrt{-\frac{\left(c^{2}d\right)^{\frac{1}{3}}}{d}}}{dx^{3}+c}\right)-\left(\left(bcd+\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*(b*c^3*d + 2*a*c^2*d^2 + (b*c^2*d^2 + 2*a*c*d^3)*x^3)*sq
rt(-(c^2*d)^(1/3)/d)*log((2*c*d*x^3 - 3*(c^2*d)^(1/3)*c*x - c^2 + 3*sqrt(1/
3)*(2*c*d*x^2 + (c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt(-(c^2*d)^(1/3)/d))/
(d*x^3 + c)) - ((b*c*d + 2*a*d^2)*x^3 + b*c^2 + 2*a*c*d)*(c^2*d)^(2/3)*log(
c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) + 2*((b*c*d + 2*a*d^2)*x^3 + b
*c^2 + 2*a*c*d)*(c^2*d)^(2/3)*log(c*d*x + (c^2*d)^(2/3)) - 6*(b*c^3*d - a*c
^2*d^2)*x)/(c^3*d^3*x^3 + c^4*d^2), 1/18*(6*sqrt(1/3)*(b*c^3*d + 2*a*c^2*d^
```

```
2 + (b*c^2*d^2 + 2*a*c*d^3)*x^3)*sqrt((c^2*d)^(1/3)/d)*arctan(sqrt(1/3)*(2*
(c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt((c^2*d)^(1/3)/d)/c^2) - ((b*c*d + 2
*a*d^2)*x^3 + b*c^2 + 2*a*c*d)*(c^2*d)^(2/3)*log(c*d*x^2 - (c^2*d)^(2/3)*x
+ (c^2*d)^(1/3)*c) + 2*((b*c*d + 2*a*d^2)*x^3 + b*c^2 + 2*a*c*d)*(c^2*d)^(2
/3)*log(c*d*x + (c^2*d)^(2/3)) - 6*(b*c^3*d - a*c^2*d^2)*x)/(c^3*d^3*x^3 +
c^4*d^2)]
```

Sympy [A] time = 0.946584, size = 97, normalized size = 0.57

$$\frac{x\,(ad-bc)}{3c^2d+3cd^2x^3}+\text{RootSum}\left(729t^3c^5d^4-8a^3d^3-12a^2bcd^2-6ab^2c^2d-b^3c^3,\left(t\mapsto t\log\left(\frac{9tc^2d}{2ad+bc}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)/(d*x**3+c)**2,x)
```

```
[Out] x*(a*d - b*c)/(3*c**2*d + 3*c*d**2*x**3) + RootSum(729*_t**3*c**5*d**4 - 8*
a**3*d**3 - 12*a**2*b*c*d**2 - 6*a*b**2*c**2*d - b**3*c**3, Lambda(_t, _t*l
og(9*_t*c**2*d/(2*a*d + b*c) + x)))
```

Giac [A] time = 1.11013, size = 246, normalized size = 1.46

$$-\frac{(bc+2\,ad)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{9\,c^2d}+\frac{\sqrt{3}\left(\left(-cd^2\right)^{\frac{1}{3}}bc+2\left(-cd^2\right)^{\frac{1}{3}}ad\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9\,c^2d^2}-\frac{bcx-adx}{3\left(dx^3+c\right)cd}+\frac{\left(\left(\cdots\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] -1/9*(b*c + 2*a*d)*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(c^2*d) + 1/9*sq
rt(3)*((-c*d^2)^(1/3)*b*c + 2*(-c*d^2)^(1/3)*a*d)*arctan(1/3*sqrt(3)*(2*x +
 (-c/d)^(1/3))/(-c/d)^(1/3))/(c^2*d^2) - 1/3*(b*c*x - a*d*x)/((d*x^3 + c)*c
*d) + 1/18*((-c*d^2)^(1/3)*b*c + 2*(-c*d^2)^(1/3)*a*d)*log(x^2 + x*(-c/d)^(
1/3) + (-c/d)^(2/3))/(c^2*d^2)
```

3.7 $\int \frac{a+bx^3}{\left(c+dx^3\right)^3} dx$

Optimal. Leaf size=197

$$-\frac{(5ad+bc)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{54c^{8/3}d^{4/3}}+\frac{(5ad+bc)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{27c^{8/3}d^{4/3}}-\frac{(5ad+bc)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{9\sqrt{3}c^{8/3}d^{4/3}}+\frac{x(5ad+bc)}{18c^2d\left(c+dx^3\right)}$$

```
[Out] -((b*c - a*d)*x)/(6*c*d*(c + d*x^3)^2) + ((b*c + 5*a*d)*x)/(18*c^2*d*(c + d
*x^3)) - ((b*c + 5*a*d)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/
(9*Sqrt[3]*c^(8/3)*d^(4/3)) + ((b*c + 5*a*d)*Log[c^(1/3) + d^(1/3)*x])/(27*
c^(8/3)*d^(4/3)) - ((b*c + 5*a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)
*x^2])/(54*c^(8/3)*d^(4/3))
```

Rubi [A] time = 0.106395, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {385, 199, 200, 31, 634, 617, 204, 628}

$$-\frac{(5ad+bc)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{54c^{8/3}d^{4/3}}+\frac{(5ad+bc)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{27c^{8/3}d^{4/3}}-\frac{(5ad+bc)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{9\sqrt{3}c^{8/3}d^{4/3}}+\frac{x(5ad+bc)}{18c^2d\left(c+dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)/(c + d*x^3)^3,x]
```


```
[Out] -((b*c - a*d)*x)/(6*c*d*(c + d*x^3)^2) + ((b*c + 5*a*d)*x)/(18*c^2*d*(c + d
*x^3)) - ((b*c + 5*a*d)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/
(9*Sqrt[3]*c^(8/3)*d^(4/3)) + ((b*c + 5*a*d)*Log[c^(1/3) + d^(1/3)*x])/(27*
c^(8/3)*d^(4/3)) - ((b*c + 5*a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)
*x^2])/(54*c^(8/3)*d^(4/3))
```


Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{a+bx^3}{\left(c+dx^3\right)^3}\,dx = -\frac{(bc-ad)x}{6cd\left(c+dx^3\right)^2}+\frac{(bc+5ad)\int \frac{1}{\left(c+dx^3\right)^2}\,dx}{6cd}$$

$$= -\frac{(bc-ad)x}{6cd\left(c+dx^3\right)^2}+\frac{(bc+5ad)x}{18c^2d\left(c+dx^3\right)}+\frac{(bc+5ad)\int \frac{1}{c+dx^3}\,dx}{9c^2d}$$

$$= -\frac{(bc-ad)x}{6cd\left(c+dx^3\right)^2}+\frac{(bc+5ad)x}{18c^2d\left(c+dx^3\right)}+\frac{(bc+5ad)\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{27c^{8/3}d}+\frac{(bc+5ad)\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{27c^{8/3}d}$$

$$= -\frac{(bc-ad)x}{6cd\left(c+dx^3\right)^2}+\frac{(bc+5ad)x}{18c^2d\left(c+dx^3\right)}+\frac{(bc+5ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{27c^{8/3}d^{4/3}}-\frac{(bc+5ad)\int \frac{-\sqrt[3]{c}\sqrt[3]{d}+2d^{2/3}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,d}{54c^{8/3}d^{4/3}}$$

$$= -\frac{(bc-ad)x}{6cd\left(c+dx^3\right)^2}+\frac{(bc+5ad)x}{18c^2d\left(c+dx^3\right)}+\frac{(bc+5ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{27c^{8/3}d^{4/3}}-\frac{(bc+5ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x-\right.}{54c^{8/3}d^{4/3}}$$

$$= -\frac{(bc-ad)x}{6cd\left(c+dx^3\right)^2}+\frac{(bc+5ad)x}{18c^2d\left(c+dx^3\right)}-\frac{(bc+5ad)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{9\sqrt{3}c^{8/3}d^{4/3}}+\frac{(bc+5ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{27c^{8/3}d^{4/3}}-$$

Mathematica [A] time = 0.131517, size = 175, normalized size = 0.89

$$\frac{-(5ad+bc)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)-\frac{9c^{5/3}\sqrt[3]{d}x(bc-ad)}{\left(c+dx^3\right)^2}+\frac{3c^{2/3}\sqrt[3]{d}x(5ad+bc)}{c+dx^3}+2(5ad+bc)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)-2\sqrt{3}(5ad+bc}{54c^{8/3}d^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)/(c + d*x^3)^3,x]
```

```
[Out] ((-9*c^(5/3)*d^(1/3)*(b*c - a*d)*x)/(c + d*x^3)^2 + (3*c^(2/3)*d^(1/3)*(b*c
 + 5*a*d)*x)/(c + d*x^3) - 2*Sqrt[3]*(b*c + 5*a*d)*ArcTan[(1 - (2*d^(1/3)*x
)/c^(1/3))/Sqrt[3]] + 2*(b*c + 5*a*d)*Log[c^(1/3) + d^(1/3)*x] - (b*c + 5*a
*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(54*c^(8/3)*d^(4/3))
```

Maple [A] time = 0.01, size = 249, normalized size = 1.3

$$\frac{1}{\left(dx^3+c\right)^2}\left(\frac{(5\,ad+bc)\,x^4}{18\,c^2}+\frac{(4\,ad-bc)\,x}{9\,cd}\right)+\frac{5\,a}{27\,c^2d}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}+\frac{b}{27\,cd^2}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}-\frac{5\,a}{54\,c^2d}\ln\left(x^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)/(d*x^3+c)^3,x)
```

```
[Out] (1/18*(5*a*d+b*c)/c^2*x^4+1/9*(4*a*d-b*c)/c/d*x)/(d*x^3+c)^2+5/27/c^2/d/(c/
d)^(2/3)*ln(x+(c/d)^(1/3))*a+1/27/c/d^2/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*b-5/5
4/c^2/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*a-1/54/c/d^2/(c/d)^(2
/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*b+5/27/c^2/d/(c/d)^(2/3)*3^(1/2)*arct
an(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*a+1/27/c/d^2/(c/d)^(2/3)*3^(1/2)*arctan
(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.72292, size = 1648, normalized size = 8.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(3*(b*c^3*d^2 + 5*a*c^2*d^3)*x^4 + 3*sqrt(1/3)*((b*c^2*d^3 + 5*a*c*d^
4)*x^6 + b*c^4*d + 5*a*c^3*d^2 + 2*(b*c^3*d^2 + 5*a*c^2*d^3)*x^3)*sqrt(-(c^
2*d)^(1/3)/d)*log((2*c*d*x^3 - 3*(c^2*d)^(1/3)*c*x - c^2 + 3*sqrt(1/3)*(2*c
*d*x^2 + (c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt(-(c^2*d)^(1/3)/d))/(d*x^3
+ c)) - ((b*c*d^2 + 5*a*d^3)*x^6 + b*c^3 + 5*a*c^2*d + 2*(b*c^2*d + 5*a*c*d
^2)*x^3)*(c^2*d)^(2/3)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) + 2
*((b*c*d^2 + 5*a*d^3)*x^6 + b*c^3 + 5*a*c^2*d + 2*(b*c^2*d + 5*a*c*d^2)*x^3
)*(c^2*d)^(2/3)*log(c*d*x + (c^2*d)^(2/3)) - 6*(b*c^4*d - 4*a*c^3*d^2)*x)/(
c^4*d^4*x^6 + 2*c^5*d^3*x^3 + c^6*d^2), 1/54*(3*(b*c^3*d^2 + 5*a*c^2*d^3)*x
^4 + 6*sqrt(1/3)*((b*c^2*d^3 + 5*a*c*d^4)*x^6 + b*c^4*d + 5*a*c^3*d^2 + 2*(
b*c^3*d^2 + 5*a*c^2*d^3)*x^3)*sqrt((c^2*d)^(1/3)/d)*arctan(sqrt(1/3)*(2*(c^
2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt((c^2*d)^(1/3)/d)/c^2) - ((b*c*d^2 + 5*
a*d^3)*x^6 + b*c^3 + 5*a*c^2*d + 2*(b*c^2*d + 5*a*c*d^2)*x^3)*(c^2*d)^(2/3)
*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) + 2*((b*c*d^2 + 5*a*d^3)*
x^6 + b*c^3 + 5*a*c^2*d + 2*(b*c^2*d + 5*a*c*d^2)*x^3)*(c^2*d)^(2/3)*log(c*
d*x + (c^2*d)^(2/3)) - 6*(b*c^4*d - 4*a*c^3*d^2)*x)/(c^4*d^4*x^6 + 2*c^5*d^
3*x^3 + c^6*d^2)]
```

Sympy [A] time = 1.36349, size = 133, normalized size = 0.68

$$\frac{x^4\left(5ad^2+bcd\right)+x\left(8acd-2bc^2\right)}{18c^4d+36c^3d^2x^3+18c^2d^3x^6}+\operatorname{RootSum}\left(19683t^3c^8d^4-125a^3d^3-75a^2bcd^2-15ab^2c^2d-b^3c^3,\left(t\mapsto t\log\left(\frac{2}{5a}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)/(d*x**3+c)**3,x)
```

```
[Out] (x**4*(5*a*d**2 + b*c*d) + x*(8*a*c*d - 2*b*c**2))/(18*c**4*d + 36*c**3*d**
2*x**3 + 18*c**2*d**3*x**6) + RootSum(19683*_t**3*c**8*d**4 - 125*a**3*d**3
 - 75*a**2*b*c*d**2 - 15*a*b**2*c**2*d - b**3*c**3, Lambda(_t, _t*log(27*_t
*c**3*d/(5*a*d + b*c) + x)))
```

Giac [A] time = 1.14816, size = 273, normalized size = 1.39

$$-\frac{(bc+5\,ad)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{27\,c^3d}+\frac{\sqrt{3}\left(\left(-cd^2\right)^{\frac{1}{3}}bc+5\left(-cd^2\right)^{\frac{1}{3}}ad\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{27\,c^3d^2}+\frac{\left(\left(-cd^2\right)^{\frac{1}{3}}bc+5\left(-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] -1/27*(b*c + 5*a*d)*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(c^3*d) + 1/27*
sqrt(3)*((-c*d^2)^(1/3)*b*c + 5*(-c*d^2)^(1/3)*a*d)*arctan(1/3*sqrt(3)*(2*x
 + (-c/d)^(1/3))/(-c/d)^(1/3))/(c^3*d^2) + 1/54*((-c*d^2)^(1/3)*b*c + 5*(-c
*d^2)^(1/3)*a*d)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(c^3*d^2) + 1/18*
(b*c*d*x^4 + 5*a*d^2*x^4 - 2*b*c^2*x + 8*a*c*d*x)/((d*x^3 + c)^2*c^2*d)
```

3.8 $\int \left(a+bx^3\right)^2 \left(c+dx^3\right)^3 \, dx$

Optimal. Leaf size=122

$$\frac{1}{10}dx^{10}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{7}cx^7\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{4}ac^2x^4(3ad+2bc)+\frac{1}{13}bd^2x^{13}(2ad+3bc)+\frac{1}{16}$$

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^4)/4 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2
*d^2)*x^7)/7 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^10)/10 + (b*d^2*(3*b*
c + 2*a*d)*x^13)/13 + (b^2*d^3*x^16)/16
```

Rubi [A] time = 0.0710383, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {373}

$$\frac{1}{10}dx^{10}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{7}cx^7\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{4}ac^2x^4(3ad+2bc)+\frac{1}{13}bd^2x^{13}(2ad+3bc)+\frac{1}{16}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2*(c + d*x^3)^3,x]
```

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^4)/4 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2
*d^2)*x^7)/7 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^10)/10 + (b*d^2*(3*b*
c + 2*a*d)*x^13)/13 + (b^2*d^3*x^16)/16
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \left(a+bx^3\right)^2 \left(c+dx^3\right)^3 \, dx = \int \left(a^2c^3+ac^2(2bc+3ad)x^3+c\left(b^2c^2+6abcd+3a^2d^2\right)x^6+d\left(3b^2c^2+6abcd+a^2d^2\right)x^9+\right.$$
$$= a^2c^3x+\frac{1}{4}ac^2(2bc+3ad)x^4+\frac{1}{7}c\left(b^2c^2+6abcd+3a^2d^2\right)x^7+\frac{1}{10}d\left(3b^2c^2+6abcd+a^2d^2\right)$$

Mathematica [A] time = 0.0161661, size = 122, normalized size = 1.

$$\frac{1}{10}dx^{10}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{7}cx^7\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{4}ac^2x^4(3ad+2bc)+\frac{1}{13}bd^2x^{13}(2ad+3bc)+\frac{1}{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2*(c + d*x^3)^3,x]
```

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^4)/4 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2
*d^2)*x^7)/7 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^10)/10 + (b*d^2*(3*b*
c + 2*a*d)*x^13)/13 + (b^2*d^3*x^16)/16
```

Maple [A] time = 0., size = 125, normalized size = 1.

$$\frac{b^2d^3x^{16}}{16}+\frac{\left(2\,abd^3+3\,b^2cd^2\right)x^{13}}{13}+\frac{\left(a^2d^3+6\,abcd^2+3\,b^2c^2d\right)x^{10}}{10}+\frac{\left(3\,a^2cd^2+6\,abc^2d+b^2c^3\right)x^7}{7}+\frac{\left(3\,a^2c^2d+2\,abc\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(d*x^3+c)^3,x)
```

```
[Out] 1/16*b^2*d^3*x^16+1/13*(2*a*b*d^3+3*b^2*c*d^2)*x^13+1/10*(a^2*d^3+6*a*b*c*d
^2+3*b^2*c^2*d)*x^10+1/7*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3)*x^7+1/4*(3*a^2*c
^2*d+2*a*b*c^3)*x^4+a^2*c^3*x
```

Maxima [A] time = 0.956539, size = 167, normalized size = 1.37

$$\frac{1}{16}\,b^2d^3x^{16}+\frac{1}{13}\left(3\,b^2cd^2+2\,abd^3\right)x^{13}+\frac{1}{10}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^{10}+\frac{1}{7}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^7+a^2c^3x+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] 1/16*b^2*d^3*x^16 + 1/13*(3*b^2*c*d^2 + 2*a*b*d^3)*x^13 + 1/10*(3*b^2*c^2*d
 + 6*a*b*c*d^2 + a^2*d^3)*x^10 + 1/7*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*
x^7 + a^2*c^3*x + 1/4*(2*a*b*c^3 + 3*a^2*c^2*d)*x^4
```

Fricas [A] time = 1.31728, size = 313, normalized size = 2.57

$$\frac{1}{16}x^{16}d^3b^2+\frac{3}{13}x^{13}d^2cb^2+\frac{2}{13}x^{13}d^3ba+\frac{3}{10}x^{10}dc^2b^2+\frac{3}{5}x^{10}d^2cba+\frac{1}{10}x^{10}d^3a^2+\frac{1}{7}x^7c^3b^2+\frac{6}{7}x^7dc^2ba+\frac{3}{7}x^7d^2ca^2+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] 1/16*x^16*d^3*b^2 + 3/13*x^13*d^2*c*b^2 + 2/13*x^13*d^3*b*a + 3/10*x^10*d*c
^2*b^2 + 3/5*x^10*d^2*c*b*a + 1/10*x^10*d^3*a^2 + 1/7*x^7*c^3*b^2 + 6/7*x^7
*d*c^2*b*a + 3/7*x^7*d^2*c*a^2 + 1/2*x^4*c^3*b*a + 3/4*x^4*d*c^2*a^2 + x*c^
3*a^2
```

Sympy [A] time = 0.086971, size = 139, normalized size = 1.14

$$a^2c^3x+\frac{b^2d^3x^{16}}{16}+x^{13}\left(\frac{2abd^3}{13}+\frac{3b^2cd^2}{13}\right)+x^{10}\left(\frac{a^2d^3}{10}+\frac{3abcd^2}{5}+\frac{3b^2c^2d}{10}\right)+x^7\left(\frac{3a^2cd^2}{7}+\frac{6abc^2d}{7}+\frac{b^2c^3}{7}\right)+x^4\left(\frac{3a^2}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(d*x**3+c)**3,x)
```

```
[Out] a**2*c**3*x + b**2*d**3*x**16/16 + x**13*(2*a*b*d**3/13 + 3*b**2*c*d**2/13)
 + x**10*(a**2*d**3/10 + 3*a*b*c*d**2/5 + 3*b**2*c**2*d/10) + x**7*(3*a**2*
c*d**2/7 + 6*a*b*c**2*d/7 + b**2*c**3/7) + x**4*(3*a**2*c**2*d/4 + a*b*c**3
```

/2)

Giac [A] time = 1.07199, size = 178, normalized size = 1.46

$$\frac{1}{16}b^2d^3x^{16} + \frac{3}{13}b^2cd^2x^{13} + \frac{2}{13}abd^3x^{13} + \frac{3}{10}b^2c^2dx^{10} + \frac{3}{5}abcd^2x^{10} + \frac{1}{10}a^2d^3x^{10} + \frac{1}{7}b^2c^3x^7 + \frac{6}{7}abc^2dx^7 + \frac{3}{7}a^2cd^2x^7 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] 1/16*b^2*d^3*x^16 + 3/13*b^2*c*d^2*x^13 + 2/13*a*b*d^3*x^13 + 3/10*b^2*c^2*
d*x^10 + 3/5*a*b*c*d^2*x^10 + 1/10*a^2*d^3*x^10 + 1/7*b^2*c^3*x^7 + 6/7*a*b
*c^2*d*x^7 + 3/7*a^2*c*d^2*x^7 + 1/2*a*b*c^3*x^4 + 3/4*a^2*c^2*d*x^4 + a^2*
c^3*x
```

3.9 $\int \left(a+bx^3\right)^2 \left(c+dx^3\right)^2 \, dx$

Optimal. Leaf size=82

$$\frac{1}{7}x^7\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{1}{5}bdx^{10}(ad+bc)+\frac{1}{2}acx^4(ad+bc)+\frac{1}{13}b^2d^2x^{13}$$

```
[Out] a^2*c^2*x + (a*c*(b*c + a*d)*x^4)/2 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^7)
/7 + (b*d*(b*c + a*d)*x^10)/5 + (b^2*d^2*x^13)/13
```

Rubi [A] time = 0.0460077, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {373}

$$\frac{1}{7}x^7\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{1}{5}bdx^{10}(ad+bc)+\frac{1}{2}acx^4(ad+bc)+\frac{1}{13}b^2d^2x^{13}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2*(c + d*x^3)^2,x]
```

```
[Out] a^2*c^2*x + (a*c*(b*c + a*d)*x^4)/2 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^7)
/7 + (b*d*(b*c + a*d)*x^10)/5 + (b^2*d^2*x^13)/13
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^3\right)^2\left(c+dx^3\right)^2 dx &= \int \left(a^2c^2+2ac(bc+ad)x^3+\left(b^2c^2+4abcd+a^2d^2\right)x^6+2bd(bc+ad)x^9+b^2d^2x^{12}\right) dx\\ &= a^2c^2x+\frac{1}{2}ac(bc+ad)x^4+\frac{1}{7}\left(b^2c^2+4abcd+a^2d^2\right)x^7+\frac{1}{5}bd(bc+ad)x^{10}+\frac{1}{13}b^2d^2x^{13}\end{aligned}$$

Mathematica [A] time = 0.0110951, size = 82, normalized size = 1.

$$\frac{1}{7}x^7\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{1}{5}bdx^{10}(ad+bc)+\frac{1}{2}acx^4(ad+bc)+\frac{1}{13}b^2d^2x^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2*(c + d*x^3)^2,x]
```

```
[Out] a^2*c^2*x + (a*c*(b*c + a*d)*x^4)/2 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^7)
/7 + (b*d*(b*c + a*d)*x^10)/5 + (b^2*d^2*x^13)/13
```

Maple [A] time = 0.001, size = 87, normalized size = 1.1

$$\frac{b^2d^2x^{13}}{13}+\frac{\left(2\,abd^2+2\,b^2cd\right)x^{10}}{10}+\frac{\left(a^2d^2+4\,cabd+b^2c^2\right)x^7}{7}+\frac{\left(2\,a^2cd+2\,abc^2\right)x^4}{4}+a^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(d*x^3+c)^2,x)
```

```
[Out] 1/13*b^2*d^2*x^13+1/10*(2*a*b*d^2+2*b^2*c*d)*x^10+1/7*(a^2*d^2+4*a*b*c*d+b^
2*c^2)*x^7+1/4*(2*a^2*c*d+2*a*b*c^2)*x^4+a^2*c^2*x
```

Maxima [A] time = 0.967997, size = 111, normalized size = 1.35

$$\frac{1}{13}\,b^2d^2x^{13} + \frac{1}{5}\left(b^2cd + abd^2\right)x^{10} + \frac{1}{7}\left(b^2c^2 + 4\,abcd + a^2d^2\right)x^7 + a^2c^2x + \frac{1}{2}\left(abc^2 + a^2cd\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] 1/13*b^2*d^2*x^13 + 1/5*(b^2*c*d + a*b*d^2)*x^10 + 1/7*(b^2*c^2 + 4*a*b*c*d
 + a^2*d^2)*x^7 + a^2*c^2*x + 1/2*(a*b*c^2 + a^2*c*d)*x^4
```

Fricas [A] time = 1.30395, size = 215, normalized size = 2.62

$$\frac{1}{13}x^{13}d^2b^2 + \frac{1}{5}x^{10}dcb^2 + \frac{1}{5}x^{10}d^2ba + \frac{1}{7}x^7c^2b^2 + \frac{4}{7}x^7dcba + \frac{1}{7}x^7d^2a^2 + \frac{1}{2}x^4c^2ba + \frac{1}{2}x^4dca^2 + xc^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*d^2*b^2 + 1/5*x^10*d*c*b^2 + 1/5*x^10*d^2*b*a + 1/7*x^7*c^2*b^2 +
 4/7*x^7*d*c*b*a + 1/7*x^7*d^2*a^2 + 1/2*x^4*c^2*b*a + 1/2*x^4*d*c*a^2 + x*
c^2*a^2
```

Sympy [A] time = 0.112381, size = 90, normalized size = 1.1

$$a^2c^2x + \frac{b^2d^2x^{13}}{13} + x^{10}\left(\frac{abd^2}{5} + \frac{b^2cd}{5}\right) + x^7\left(\frac{a^2d^2}{7} + \frac{4abcd}{7} + \frac{b^2c^2}{7}\right) + x^4\left(\frac{a^2cd}{2} + \frac{abc^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(d*x**3+c)**2,x)
```

```
[Out] a**2*c**2*x + b**2*d**2*x**13/13 + x**10*(a*b*d**2/5 + b**2*c*d/5) + x**7*(
a**2*d**2/7 + 4*a*b*c*d/7 + b**2*c**2/7) + x**4*(a**2*c*d/2 + a*b*c**2/2)
```

Giac [A] time = 1.10344, size = 123, normalized size = 1.5

$$\frac{1}{13}\,b^2d^2x^{13} + \frac{1}{5}\,b^2cdx^{10} + \frac{1}{5}\,abd^2x^{10} + \frac{1}{7}\,b^2c^2x^7 + \frac{4}{7}\,abcdx^7 + \frac{1}{7}\,a^2d^2x^7 + \frac{1}{2}\,abc^2x^4 + \frac{1}{2}\,a^2cdx^4 + a^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] 1/13*b^2*d^2*x^13 + 1/5*b^2*c*d*x^10 + 1/5*a*b*d^2*x^10 + 1/7*b^2*c^2*x^7 +
 4/7*a*b*c*d*x^7 + 1/7*a^2*d^2*x^7 + 1/2*a*b*c^2*x^4 + 1/2*a^2*c*d*x^4 + a^
2*c^2*x
```

3.10 $\int \left(a+bx^3\right)^2 \left(c+dx^3\right)\,dx$

Optimal. Leaf size=50

$$a^2cx+\frac{1}{7}bx^7(2ad+bc)+\frac{1}{4}ax^4(ad+2bc)+\frac{1}{10}b^2dx^{10}$$

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^4)/4 + (b*(b*c + 2*a*d)*x^7)/7 + (b^2*d*x^10)/
10
```

Rubi [A] time = 0.0292321, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$a^2cx+\frac{1}{7}bx^7(2ad+bc)+\frac{1}{4}ax^4(ad+2bc)+\frac{1}{10}b^2dx^{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2*(c + d*x^3),x]
```

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^4)/4 + (b*(b*c + 2*a*d)*x^7)/7 + (b^2*d*x^10)/
10
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^3\right)^2\left(c+dx^3\right)\,dx &= \int \left(a^2c+a(2bc+ad)x^3+b(bc+2ad)x^6+b^2dx^9\right)\,dx\\ &= a^2cx+\frac{1}{4}a(2bc+ad)x^4+\frac{1}{7}b(bc+2ad)x^7+\frac{1}{10}b^2dx^{10}\end{aligned}$$

Mathematica [A] time = 0.0070966, size = 50, normalized size = 1.

$$a^2cx+\frac{1}{7}bx^7(2ad+bc)+\frac{1}{4}ax^4(ad+2bc)+\frac{1}{10}b^2dx^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2*(c + d*x^3),x]
```

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^4)/4 + (b*(b*c + 2*a*d)*x^7)/7 + (b^2*d*x^10)/
10
```

Maple [A] time = 0., size = 49, normalized size = 1.

$$\frac{b^2dx^{10}}{10}+\frac{\left(2\,abd+b^2c\right)x^7}{7}+\frac{\left(a^2d+2\,abc\right)x^4}{4}+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(d*x^3+c),x)
```

```
[Out] 1/10*b^2*d*x^10+1/7*(2*a*b*d+b^2*c)*x^7+1/4*(a^2*d+2*a*b*c)*x^4+a^2*c*x
```

Maxima [A] time = 0.955875, size = 65, normalized size = 1.3

$$\frac{1}{10}\,b^2dx^{10}+\frac{1}{7}\left(b^2c+2\,abd\right)x^7+\frac{1}{4}\left(2\,abc+a^2d\right)x^4+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(d*x^3+c),x, algorithm="maxima")
```

```
[Out] 1/10*b^2*d*x^10 + 1/7*(b^2*c + 2*a*b*d)*x^7 + 1/4*(2*a*b*c + a^2*d)*x^4 + a
^2*c*x
```

Fricas [A] time = 1.29771, size = 123, normalized size = 2.46

$$\frac{1}{10}x^{10}db^2+\frac{1}{7}x^7cb^2+\frac{2}{7}x^7dba+\frac{1}{2}x^4cba+\frac{1}{4}x^4da^2+xca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/10*x^10*d*b^2 + 1/7*x^7*c*b^2 + 2/7*x^7*d*b*a + 1/2*x^4*c*b*a + 1/4*x^4*d
*a^2 + x*c*a^2
```

Sympy [A] time = 0.100682, size = 51, normalized size = 1.02

$$a^2cx+\frac{b^2dx^{10}}{10}+x^7\left(\frac{2abd}{7}+\frac{b^2c}{7}\right)+x^4\left(\frac{a^2d}{4}+\frac{abc}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(d*x**3+c),x)
```

```
[Out] a**2*c*x + b**2*d*x**10/10 + x**7*(2*a*b*d/7 + b**2*c/7) + x**4*(a**2*d/4 +
 a*b*c/2)
```

Giac [A] time = 1.11853, size = 68, normalized size = 1.36

$$\frac{1}{10}\,b^2dx^{10}+\frac{1}{7}\,b^2cx^7+\frac{2}{7}\,abdx^7+\frac{1}{2}\,abcx^4+\frac{1}{4}\,a^2dx^4+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/10*b^2*d*x^10 + 1/7*b^2*c*x^7 + 2/7*a*b*d*x^7 + 1/2*a*b*c*x^4 + 1/4*a^2*d
*x^4 + a^2*c*x
```

3.11 $\int \frac{(a+bx^3)^2}{c+dx^3} \, dx$

Optimal. Leaf size=173

$$-\frac{(bc-ad)^2 \log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{7/3}}+\frac{(bc-ad)^2 \log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{7/3}}-\frac{(bc-ad)^2 \tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{7/3}}-\frac{bx(bc-2ad)}{d^2}+$$

```
[Out] -((b*(b*c - 2*a*d)*x)/d^2) + (b^2*x^4)/(4*d) - ((b*c - a*d)^2*ArcTan[(c^(1/
3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(2/3)*d^(7/3)) + ((b*c - a
*d)^2*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3)*d^(7/3)) - ((b*c - a*d)^2*Log[c^
(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c^(2/3)*d^(7/3))
```

Rubi [A] time = 0.128459, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {390, 200, 31, 634, 617, 204, 628}

$$-\frac{(bc-ad)^2 \log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{7/3}}+\frac{(bc-ad)^2 \log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{7/3}}-\frac{(bc-ad)^2 \tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{7/3}}-\frac{bx(bc-2ad)}{d^2}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2/(c + d*x^3),x]
```

```
[Out] -((b*(b*c - 2*a*d)*x)/d^2) + (b^2*x^4)/(4*d) - ((b*c - a*d)^2*ArcTan[(c^(1/
3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(2/3)*d^(7/3)) + ((b*c - a
*d)^2*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3)*d^(7/3)) - ((b*c - a*d)^2*Log[c^
(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(6*c^(2/3)*d^(7/3))
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^2}{c+dx^3}\,dx = \int \left(-\frac{b(bc-2ad)}{d^2}+\frac{b^2x^3}{d}+\frac{b^2c^2-2abcd+a^2d^2}{d^2\left(c+dx^3\right)}\right)dx$$

$$= -\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^4}{4d}+\frac{(bc-ad)^2\int \frac{1}{c+dx^3}\,dx}{d^2}$$

$$= -\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^4}{4d}+\frac{(bc-ad)^2\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{3c^{2/3}d^2}+\frac{(bc-ad)^2\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{3c^{2/3}d^2}$$

$$= -\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^4}{4d}+\frac{(bc-ad)^2\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{7/3}}-\frac{(bc-ad)^2\int \frac{-\sqrt[3]{c}\sqrt[3]{d}+2d^{2/3}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{6c^{2/3}d^{7/3}}+\frac{(bc-ad)^2\int}{}$$

$$= -\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^4}{4d}+\frac{(bc-ad)^2\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{7/3}}-\frac{(bc-ad)^2\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{7/3}}+\frac{(bc-}{}$$

$$= -\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^4}{4d}-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{7/3}}+\frac{(bc-ad)^2\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{7/3}}-\frac{(bc-ad)^2\log\left(c\right.}{6}$$

Mathematica [A] time = 0.0908675, size = 167, normalized size = 0.97

$$\frac{-2(bc-ad)^2\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)-12bc^{2/3}\sqrt[3]{d}x(bc-2ad)+4(bc-ad)^2\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)+4\sqrt{3}(bc-ad)^2\tan^{-1}\left(\right.}{12c^{2/3}d^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2/(c + d*x^3),x]
```

```
[Out] (-12*b*c^(2/3)*d^(1/3)*(b*c - 2*a*d)*x + 3*b^2*c^(2/3)*d^(4/3)*x^4 + 4*Sqrt
[3]*(b*c - a*d)^2*ArcTan[(-c^(1/3) + 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))] + 4*(b
*c - a*d)^2*Log[c^(1/3) + d^(1/3)*x] - 2*(b*c - a*d)^2*Log[c^(2/3) - c^(1/3
)*d^(1/3)*x + d^(2/3)*x^2])/(12*c^(2/3)*d^(7/3))
```

Maple [B] time = 0.003, size = 334, normalized size = 1.9

$$\frac{b^2x^4}{4d} + 2\frac{xab}{d} - \frac{b^2xc}{d^2} + \frac{a^2}{3d}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}} - \frac{2\,abc}{3\,d^2}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}} + \frac{b^2c^2}{3\,d^3}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}} - \frac{a^2}{6\,d}\ln\left(x^2 -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2/(d*x^3+c),x)
```

```
[Out] 1/4*b^2*x^4/d+2*b/d*a*x-b^2/d^2*x*c+1/3/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*a^2
-2/3/d^2/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*c*a*b+1/3/d^3/(c/d)^(2/3)*ln(x+(c/d)
^(1/3))*b^2*c^2-1/6/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*a^2+1/3
/d^2/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*c*a*b-1/6/d^3/(c/d)^(2/3
)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*b^2*c^2+1/3/d/(c/d)^(2/3)*3^(1/2)*arcta
n(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*a^2-2/3/d^2/(c/d)^(2/3)*3^(1/2)*arctan(1
/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*c*a*b+1/3/d^3/(c/d)^(2/3)*3^(1/2)*arctan(1/
3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*b^2*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.68486, size = 1183, normalized size = 6.84

$$\left[\frac{3\,b^2c^2d^2x^4 + 6\sqrt{\frac{1}{3}}\left(b^2c^3d - 2\,abc^2d^2 + a^2cd^3\right)\sqrt{-\frac{\left(c^2d\right)^{\frac{1}{3}}}{d}}\log\left(\frac{2\,cdx^3-3\left(c^2d\right)^{\frac{1}{3}}cx-c^2+3\sqrt{\frac{1}{3}}\left(2\,cdx^2+\left(c^2d\right)^{\frac{2}{3}}x-\left(c^2d\right)^{\frac{1}{3}}c\right)\sqrt{-\frac{\left(c^2d\right)^{\frac{1}{3}}}{d}}}{dx^3+c}\right) - 2}{\quad}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [1/12*(3*b^2*c^2*d^2*x^4 + 6*sqrt(1/3)*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d
^3)*sqrt(-(c^2*d)^(1/3)/d)*log((2*c*d*x^3 - 3*(c^2*d)^(1/3)*c*x - c^2 + 3*s
qrt(1/3)*(2*c*d*x^2 + (c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt(-(c^2*d)^(1/3
)/d))/(d*x^3 + c)) - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(c^2*d)^(2/3)*log(c*
d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) + 4*(b^2*c^2 - 2*a*b*c*d + a^2*d
^2)*(c^2*d)^(2/3)*log(c*d*x + (c^2*d)^(2/3)) - 12*(b^2*c^3*d - 2*a*b*c^2*d^
2)*x)/(c^2*d^3), 1/12*(3*b^2*c^2*d^2*x^4 + 12*sqrt(1/3)*(b^2*c^3*d - 2*a*b*
c^2*d^2 + a^2*c*d^3)*sqrt((c^2*d)^(1/3)/d)*arctan(sqrt(1/3)*(2*(c^2*d)^(2/3
```

```
)*x - (c^2*d)^(1/3)*c)*sqrt((c^2*d)^(1/3)/d)/c^2) - 2*(b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)*(c^2*d)^(2/3)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) +
 4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(c^2*d)^(2/3)*log(c*d*x + (c^2*d)^(2/3))
 - 12*(b^2*c^3*d - 2*a*b*c^2*d^2)*x)/(c^2*d^3)]
```

Sympy [A] time = 1.30891, size = 156, normalized size = 0.9

$$\frac{b^2x^4}{4d} + \text{RootSum}\left(27t^3c^2d^7 - a^6d^6 + 6a^5bcd^5 - 15a^4b^2c^2d^4 + 20a^3b^3c^3d^3 - 15a^2b^4c^4d^2 + 6ab^5c^5d - b^6c^6, \left(t \mapsto t\log\left(\frac{}{a^2 \cdots}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2/(d*x**3+c),x)
```

```
[Out] b**2*x**4/(4*d) + RootSum(27*_t**3*c**2*d**7 - a**6*d**6 + 6*a**5*b*c*d**5
- 15*a**4*b**2*c**2*d**4 + 20*a**3*b**3*c**3*d**3 - 15*a**2*b**4*c**4*d**2
+ 6*a*b**5*c**5*d - b**6*c**6, Lambda(_t, _t*log(3*_t*c*d**2/(a**2*d**2 - 2
*a*b*c*d + b**2*c**2) + x))) + x*(2*a*b*d - b**2*c)/d**2
```

Giac [A] time = 1.24444, size = 336, normalized size = 1.94

$$\frac{\sqrt{3}\left(\left(-cd^2\right)^{\frac{1}{3}}b^2c^2 - 2\left(-cd^2\right)^{\frac{1}{3}}abcd + \left(-cd^2\right)^{\frac{1}{3}}a^2d^2\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3cd^3} + \frac{\left(\left(-cd^2\right)^{\frac{1}{3}}b^2c^2 - 2\left(-cd^2\right)^{\frac{1}{3}}abcd + \left(-cd^2\right)^{\frac{1}{3}} \cdots\right.}{6cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/(d*x^3+c),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*((-c*d^2)^(1/3)*b^2*c^2 - 2*(-c*d^2)^(1/3)*a*b*c*d + (-c*d^2)^(
1/3)*a^2*d^2)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))/(c*d^3)
 + 1/6*((-c*d^2)^(1/3)*b^2*c^2 - 2*(-c*d^2)^(1/3)*a*b*c*d + (-c*d^2)^(1/3)*
a^2*d^2)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(c*d^3) - 1/3*(b^2*c^2*d^
2 - 2*a*b*c*d^3 + a^2*d^4)*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(c*d^4)
+ 1/4*(b^2*d^3*x^4 - 4*b^2*c*d^2*x + 8*a*b*d^3*x)/d^4
```

3.12 $\int \frac{(a+bx^3)^2}{(c+dx^3)^2}\,dx$

Optimal. Leaf size=203

$$\frac{(bc-ad)(ad+2bc)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{9c^{5/3}d^{7/3}}-\frac{2(bc-ad)(ad+2bc)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}d^{7/3}}+\frac{2(bc-ad)(ad+2bc)\tan^{-1}}{3\sqrt{3}c^{5/3}d^{7/3}}$$

```
[Out] (b^2*x)/d^2 + ((b*c - a*d)^2*x)/(3*c*d^2*(c + d*x^3)) + (2*(b*c - a*d)*(2*b
*c + a*d)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(3*Sqrt[3]*c^(
5/3)*d^(7/3)) - (2*(b*c - a*d)*(2*b*c + a*d)*Log[c^(1/3) + d^(1/3)*x])/(9*c
^(5/3)*d^(7/3)) + ((b*c - a*d)*(2*b*c + a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*
x + d^(2/3)*x^2])/(9*c^(5/3)*d^(7/3))
```

Rubi [A] time = 0.243648, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {390, 385, 200, 31, 634, 617, 204, 628}

$$\frac{(bc-ad)(ad+2bc)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{9c^{5/3}d^{7/3}}-\frac{2(bc-ad)(ad+2bc)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}d^{7/3}}+\frac{2(bc-ad)(ad+2bc)\tan^{-1}}{3\sqrt{3}c^{5/3}d^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2/(c + d*x^3)^2,x]
```

```
[Out] (b^2*x)/d^2 + ((b*c - a*d)^2*x)/(3*c*d^2*(c + d*x^3)) + (2*(b*c - a*d)*(2*b
*c + a*d)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(3*Sqrt[3]*c^(
5/3)*d^(7/3)) - (2*(b*c - a*d)*(2*b*c + a*d)*Log[c^(1/3) + d^(1/3)*x])/(9*c
^(5/3)*d^(7/3)) + ((b*c - a*d)*(2*b*c + a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*
x + d^(2/3)*x^2])/(9*c^(5/3)*d^(7/3))
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^2}{\left(c+dx^3\right)^2}\,dx = \int \left(\frac{b^2}{d^2}-\frac{b^2c^2-a^2d^2+2bd(bc-ad)x^3}{d^2\left(c+dx^3\right)^2}\right)dx$$

$$= \frac{b^2x}{d^2}-\frac{\int \frac{b^2c^2-a^2d^2+2bd(bc-ad)x^3}{\left(c+dx^3\right)^2}\,dx}{d^2}$$

$$= \frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{3cd^2\left(c+dx^3\right)}-\frac{(2(bc-ad)(2bc+ad))\int \frac{1}{c+dx^3}\,dx}{3cd^2}$$

$$= \frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{3cd^2\left(c+dx^3\right)}-\frac{(2(bc-ad)(2bc+ad))\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{9c^{5/3}d^2}-\frac{(2(bc-ad)(2bc+ad))\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}\cdots}\cdots}{9c^{5/3}d^2}$$

$$= \frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{3cd^2\left(c+dx^3\right)}-\frac{2(bc-ad)(2bc+ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}d^{7/3}}+\frac{((bc-ad)(2bc+ad))\int \frac{-\sqrt[3]{c}\sqrt[3]{d}+2d^{2/3}\cdots}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/\cdots}}\cdots}{9c^{5/3}d^{7/3}}$$

$$= \frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{3cd^2\left(c+dx^3\right)}-\frac{2(bc-ad)(2bc+ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}d^{7/3}}+\frac{(bc-ad)(2bc+ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\cdots\right)}{9c^{5/3}d^{7/3}}$$

$$= \frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{3cd^2\left(c+dx^3\right)}+\frac{2(bc-ad)(2bc+ad)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}d^{7/3}}-\frac{2(bc-ad)(2bc+ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\cdots\right)}{9c^{5/3}d^{7/3}}$$

Mathematica [A] time = 0.194235, size = 210, normalized size = 1.03

$$\frac{\frac{\left(-a^2d^2-abcd+2b^2c^2\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{c^{5/3}}-\frac{2\left(-a^2d^2-abcd+2b^2c^2\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{c^{5/3}}+\frac{2\sqrt{3}\left(-a^2d^2-abcd+2b^2c^2\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right)}{c^{5/3}}+\frac{3\sqrt[3]{d}x(bc-ad)^2}{c\left(c+dx^3\right)}}{9d^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2/(c + d*x^3)^2,x]
```

```
[Out] (9*b^2*d^(1/3)*x + (3*d^(1/3)*(b*c - a*d)^2*x)/(c*(c + d*x^3)) + (2*Sqrt[3]
*(2*b^2*c^2 - a*b*c*d - a^2*d^2)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]
])/c^(5/3) - (2*(2*b^2*c^2 - a*b*c*d - a^2*d^2)*Log[c^(1/3) + d^(1/3)*x])/c
^(5/3) + ((2*b^2*c^2 - a*b*c*d - a^2*d^2)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x +
 d^(2/3)*x^2])/c^(5/3))/(9*d^(7/3))
```

Maple [B] time = 0.01, size = 367, normalized size = 1.8

$$\frac{b^2x}{d^2}+\frac{a^2x}{3c\left(dx^3+c\right)}-\frac{2xab}{3d\left(dx^3+c\right)}+\frac{cxb^2}{3d^2\left(dx^3+c\right)}+\frac{2a^2}{9cd}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}+\frac{2ab}{9d^2}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}-\frac{4b^2c}{9d^3}\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2/(d*x^3+c)^2,x)
```

```
[Out] b^2*x/d^2+1/3/c*x/(d*x^3+c)*a^2-2/3/d*x/(d*x^3+c)*a*b+1/3/d^2*c*x/(d*x^3+c)
*b^2+2/9/d/c/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*a^2+2/9/d^2/(c/d)^(2/3)*ln(x+(c/
d)^(1/3))*a*b-4/9/d^3*c/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*b^2-1/9/d/c/(c/d)^(2/
3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*a^2-1/9/d^2/(c/d)^(2/3)*ln(x^2-(c/d)^(
1/3)*x+(c/d)^(2/3))*a*b+2/9/d^3*c/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2
/3))*b^2+2/9/d/c/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1)
)*a^2+2/9/d^2/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*a
*b-4/9/d^3*c/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*b^
2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.72141, size = 1635, normalized size = 8.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] [1/9*(9*b^2*c^3*d^2*x^4 - 3*sqrt(1/3)*(2*b^2*c^4*d - a*b*c^3*d^2 - a^2*c^2*
d^3 + (2*b^2*c^3*d^2 - a*b*c^2*d^3 - a^2*c*d^4)*x^3)*sqrt(-(c^2*d)^(1/3)/d)
*log((2*c*d*x^3 - 3*(c^2*d)^(1/3)*c*x - c^2 + 3*sqrt(1/3)*(2*c*d*x^2 + (c^2
*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt(-(c^2*d)^(1/3)/d))/(d*x^3 + c)) + (2*b^
2*c^3 - a*b*c^2*d - a^2*c*d^2 + (2*b^2*c^2*d - a*b*c*d^2 - a^2*d^3)*x^3)*(c
^2*d)^(2/3)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) - 2*(2*b^2*c^3
 - a*b*c^2*d - a^2*c*d^2 + (2*b^2*c^2*d - a*b*c*d^2 - a^2*d^3)*x^3)*(c^2*d)
^(2/3)*log(c*d*x + (c^2*d)^(2/3)) + 3*(4*b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^
2*d^3)*x)/(c^3*d^4*x^3 + c^4*d^3), 1/9*(9*b^2*c^3*d^2*x^4 - 6*sqrt(1/3)*(2*
b^2*c^4*d - a*b*c^3*d^2 - a^2*c^2*d^3 + (2*b^2*c^3*d^2 - a*b*c^2*d^3 - a^2*
c*d^4)*x^3)*sqrt((c^2*d)^(1/3)/d)*arctan(sqrt(1/3)*(2*(c^2*d)^(2/3)*x - (c^
2*d)^(1/3)*c)*sqrt((c^2*d)^(1/3)/d)/c^2) + (2*b^2*c^3 - a*b*c^2*d - a^2*c*d
^2 + (2*b^2*c^2*d - a*b*c*d^2 - a^2*d^3)*x^3)*(c^2*d)^(2/3)*log(c*d*x^2 - (
c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) - 2*(2*b^2*c^3 - a*b*c^2*d - a^2*c*d^2 +
(2*b^2*c^2*d - a*b*c*d^2 - a^2*d^3)*x^3)*(c^2*d)^(2/3)*log(c*d*x + (c^2*d)^
(2/3)) + 3*(4*b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*x)/(c^3*d^4*x^3 + c^
4*d^3)]
```

Sympy [A] time = 1.75829, size = 189, normalized size = 0.93

$$\frac{b^2x}{d^2} + \frac{x\left(a^2d^2 - 2abcd + b^2c^2\right)}{3c^2d^2 + 3cd^3x^3} + \text{RootSum}\left(729t^3c^5d^7 - 8a^6d^6 - 24a^5bcd^5 + 24a^4b^2c^2d^4 + 88a^3b^3c^3d^3 - 48a^2b^4c^4d^2 - 9\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2/(d*x**3+c)**2,x)
```

```
[Out] b**2*x/d**2 + x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(3*c**2*d**2 + 3*c*d**3
*x**3) + RootSum(729*_t**3*c**5*d**7 - 8*a**6*d**6 - 24*a**5*b*c*d**5 + 24*
a**4*b**2*c**2*d**4 + 88*a**3*b**3*c**3*d**3 - 48*a**2*b**4*c**4*d**2 - 96*
a*b**5*c**5*d + 64*b**6*c**6, Lambda(_t, _t*log(9*_t*c**2*d**2/(2*a**2*d**2
 + 2*a*b*c*d - 4*b**2*c**2) + x)))
```

Giac [A] time = 1.11978, size = 358, normalized size = 1.76

$$\frac{b^2x}{d^2} + \frac{2\left(2b^2c^2 - abcd - a^2d^2\right)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{9c^2d^2} - \frac{2\sqrt{3}\left(2\left(-cd^2\right)^{\frac{1}{3}}b^2c^2 - \left(-cd^2\right)^{\frac{1}{3}}abcd - \left(-cd^2\right)^{\frac{1}{3}}a^2d^2\right)\arctan\left(\right.}{9c^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] b^2*x/d^2 + 2/9*(2*b^2*c^2 - a*b*c*d - a^2*d^2)*(-c/d)^(1/3)*log(abs(x - (-
c/d)^(1/3)))/(c^2*d^2) - 2/9*sqrt(3)*(2*(-c*d^2)^(1/3)*b^2*c^2 - (-c*d^2)^(
1/3)*a*b*c*d - (-c*d^2)^(1/3)*a^2*d^2)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/
3))/(-c/d)^(1/3))/(c^2*d^3) + 1/3*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((d
*x^3 + c)*c*d^2) - 1/9*(2*(-c*d^2)^(1/3)*b^2*c^2 - (-c*d^2)^(1/3)*a*b*c*d -
```

```
(-c*d^2)^(1/3)*a^2*d^2)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(c^2*d^3)
```

3.13 $\int \frac{(a+bx^3)^2}{(c+dx^3)^3} dx$

Optimal. Leaf size=258

$$-\frac{(5a^2d^2+2abcd+2b^2c^2)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{54c^{8/3}d^{7/3}}+\frac{(5a^2d^2+2abcd+2b^2c^2)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{27c^{8/3}d^{7/3}}-\frac{(5a^2d^2+2abcd+2}{9\sqrt{3}}$$

```
[Out] -((b*c - a*d)*x*(a + b*x^3))/(6*c*d*(c + d*x^3)^2) - ((b*c - a*d)*(4*b*c +
5*a*d)*x)/(18*c^2*d^2*(c + d*x^3)) - ((2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*A
rcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(9*Sqrt[3]*c^(8/3)*d^(7/3
)) + ((2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*Log[c^(1/3) + d^(1/3)*x])/(27*c^(
8/3)*d^(7/3)) - ((2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*Log[c^(2/3) - c^(1/3)*
d^(1/3)*x + d^(2/3)*x^2])/(54*c^(8/3)*d^(7/3))
```

Rubi [A] time = 0.232737, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {413, 385, 200, 31, 634, 617, 204, 628}

$$-\frac{(5a^2d^2+2abcd+2b^2c^2)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{54c^{8/3}d^{7/3}}+\frac{(5a^2d^2+2abcd+2b^2c^2)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{27c^{8/3}d^{7/3}}-\frac{(5a^2d^2+2abcd+2}{9\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2/(c + d*x^3)^3,x]
```


```
[Out] -((b*c - a*d)*x*(a + b*x^3))/(6*c*d*(c + d*x^3)^2) - ((b*c - a*d)*(4*b*c +
5*a*d)*x)/(18*c^2*d^2*(c + d*x^3)) - ((2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*A
rcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(9*Sqrt[3]*c^(8/3)*d^(7/3
)) + ((2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*Log[c^(1/3) + d^(1/3)*x])/(27*c^(
8/3)*d^(7/3)) - ((2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*Log[c^(2/3) - c^(1/3)*
d^(1/3)*x + d^(2/3)*x^2])/(54*c^(8/3)*d^(7/3))
```


Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^2}{(c+dx^3)^3}\,dx = -\frac{(bc-ad)x\left(a+bx^3\right)}{6cd\left(c+dx^3\right)^2} + \frac{\int \frac{a(bc+5ad)+2b(2bc+ad)x^3}{(c+dx^3)^2}\,dx}{6cd}$$

$$= -\frac{(bc-ad)x\left(a+bx^3\right)}{6cd\left(c+dx^3\right)^2} - \frac{(bc-ad)(4bc+5ad)x}{18c^2d^2\left(c+dx^3\right)} + \frac{\left(2b^2c^2+2abcd+5a^2d^2\right)\int \frac{1}{c+dx^3}\,dx}{9c^2d^2}$$

$$= -\frac{(bc-ad)x\left(a+bx^3\right)}{6cd\left(c+dx^3\right)^2} - \frac{(bc-ad)(4bc+5ad)x}{18c^2d^2\left(c+dx^3\right)} + \frac{\left(2b^2c^2+2abcd+5a^2d^2\right)\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{27c^{8/3}d^2} + \frac{(2b^2c^2+2}{}$$

$$= -\frac{(bc-ad)x\left(a+bx^3\right)}{6cd\left(c+dx^3\right)^2} - \frac{(bc-ad)(4bc+5ad)x}{18c^2d^2\left(c+dx^3\right)} + \frac{\left(2b^2c^2+2abcd+5a^2d^2\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{27c^{8/3}d^{7/3}} - \frac{(2b^2c^2}{}$$

$$= -\frac{(bc-ad)x\left(a+bx^3\right)}{6cd\left(c+dx^3\right)^2} - \frac{(bc-ad)(4bc+5ad)x}{18c^2d^2\left(c+dx^3\right)} + \frac{\left(2b^2c^2+2abcd+5a^2d^2\right)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{27c^{8/3}d^{7/3}} - \frac{(2b^2c^2}{}$$

$$= -\frac{(bc-ad)x\left(a+bx^3\right)}{6cd\left(c+dx^3\right)^2} - \frac{(bc-ad)(4bc+5ad)x}{18c^2d^2\left(c+dx^3\right)} - \frac{\left(2b^2c^2+2abcd+5a^2d^2\right)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{9\sqrt{3}c^{8/3}d^{7/3}} + \frac{(2b^2c^2}{}$$

Mathematica [A] time = 0.271457, size = 234, normalized size = 0.91

$$\frac{-\frac{3c^{2/3}\sqrt[3]{d}x\left(-a^2d^2\left(8c+5dx^3\right)+2abcd\left(2c-dx^3\right)+b^2c^2\left(4c+7dx^3\right)\right)}{\left(c+dx^3\right)^2} - \left(5a^2d^2+2abcd+2b^2c^2\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)+2\left(5a^2d^2+2ab\right.}{54c^{8/3}d^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2/(c + d*x^3)^3,x]
```

```
[Out] ((-3*c^(2/3)*d^(1/3)*x*(2*a*b*c*d*(2*c - d*x^3) - a^2*d^2*(8*c + 5*d*x^3) +
 b^2*c^2*(4*c + 7*d*x^3)))/(c + d*x^3)^2 - 2*Sqrt[3]*(2*b^2*c^2 + 2*a*b*c*d
 + 5*a^2*d^2)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]] + 2*(2*b^2*c^2 +
2*a*b*c*d + 5*a^2*d^2)*Log[c^(1/3) + d^(1/3)*x] - (2*b^2*c^2 + 2*a*b*c*d +
5*a^2*d^2)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(54*c^(8/3)*d^(7
/3))
```

Maple [A] time = 0.01, size = 388, normalized size = 1.5

$$\frac{1}{\left(dx^3+c\right)^2}\left(\frac{\left(5\,a^2d^2+2\,cabd-7\,b^2c^2\right)x^4}{18\,c^2d}+\frac{\left(4\,a^2d^2-2\,cabd-2\,b^2c^2\right)x}{9\,d^2c}\right)+\frac{5\,a^2}{27\,c^2d}\ln\left(x+\sqrt[3]{\frac{c}{d}}\right)\left(\frac{c}{d}\right)^{-\frac{2}{3}}+\frac{2\,ab}{27\,d^2c}\ln\left(x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2/(d*x^3+c)^3,x)
```

```
[Out] (1/18*(5*a^2*d^2+2*a*b*c*d-7*b^2*c^2)/c^2/d*x^4+2/9*(2*a^2*d^2-a*b*c*d-b^2*
c^2)/d^2/c*x)/(d*x^3+c)^2+5/27/c^2/d/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*a^2+2/27
/c/d^2/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*a*b+2/27/d^3/(c/d)^(2/3)*ln(x+(c/d)^(1
```

```
/3))*b^2-5/54/c^2/d/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*a^2-1/27/
c/d^2/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*a*b-1/27/d^3/(c/d)^(2/3
)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*b^2+5/27/c^2/d/(c/d)^(2/3)*3^(1/2)*arct
an(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*a^2+2/27/c/d^2/(c/d)^(2/3)*3^(1/2)*arct
an(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*a*b+2/27/d^3/(c/d)^(2/3)*3^(1/2)*arctan
(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.74767, size = 2296, normalized size = 8.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] [-1/54*(3*(7*b^2*c^4*d^2 - 2*a*b*c^3*d^3 - 5*a^2*c^2*d^4)*x^4 - 3*sqrt(1/3)
*(2*b^2*c^5*d + 2*a*b*c^4*d^2 + 5*a^2*c^3*d^3 + (2*b^2*c^3*d^3 + 2*a*b*c^2*
d^4 + 5*a^2*c*d^5)*x^6 + 2*(2*b^2*c^4*d^2 + 2*a*b*c^3*d^3 + 5*a^2*c^2*d^4)*
x^3)*sqrt(-(c^2*d)^(1/3)/d)*log((2*c*d*x^3 - 3*(c^2*d)^(1/3)*c*x - c^2 + 3*
sqrt(1/3)*(2*c*d*x^2 + (c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt(-(c^2*d)^(1/
3)/d))/(d*x^3 + c)) + ((2*b^2*c^2*d^2 + 2*a*b*c*d^3 + 5*a^2*d^4)*x^6 + 2*b^
2*c^4 + 2*a*b*c^3*d + 5*a^2*c^2*d^2 + 2*(2*b^2*c^3*d + 2*a*b*c^2*d^2 + 5*a^
2*c*d^3)*x^3)*(c^2*d)^(2/3)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c
) - 2*((2*b^2*c^2*d^2 + 2*a*b*c*d^3 + 5*a^2*d^4)*x^6 + 2*b^2*c^4 + 2*a*b*c^
3*d + 5*a^2*c^2*d^2 + 2*(2*b^2*c^3*d + 2*a*b*c^2*d^2 + 5*a^2*c*d^3)*x^3)*(c
^2*d)^(2/3)*log(c*d*x + (c^2*d)^(2/3)) + 12*(b^2*c^5*d + a*b*c^4*d^2 - 2*a^
2*c^3*d^3)*x)/(c^4*d^5*x^6 + 2*c^5*d^4*x^3 + c^6*d^3), -1/54*(3*(7*b^2*c^4*
d^2 - 2*a*b*c^3*d^3 - 5*a^2*c^2*d^4)*x^4 - 6*sqrt(1/3)*(2*b^2*c^5*d + 2*a*b
*c^4*d^2 + 5*a^2*c^3*d^3 + (2*b^2*c^3*d^3 + 2*a*b*c^2*d^4 + 5*a^2*c*d^5)*x^
6 + 2*(2*b^2*c^4*d^2 + 2*a*b*c^3*d^3 + 5*a^2*c^2*d^4)*x^3)*sqrt((c^2*d)^(1/
3)/d)*arctan(sqrt(1/3)*(2*(c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt((c^2*d)^(
1/3)/d)/c^2) + ((2*b^2*c^2*d^2 + 2*a*b*c*d^3 + 5*a^2*d^4)*x^6 + 2*b^2*c^4 +
 2*a*b*c^3*d + 5*a^2*c^2*d^2 + 2*(2*b^2*c^3*d + 2*a*b*c^2*d^2 + 5*a^2*c*d^3
)*x^3)*(c^2*d)^(2/3)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) - 2*(
(2*b^2*c^2*d^2 + 2*a*b*c*d^3 + 5*a^2*d^4)*x^6 + 2*b^2*c^4 + 2*a*b*c^3*d + 5
*a^2*c^2*d^2 + 2*(2*b^2*c^3*d + 2*a*b*c^2*d^2 + 5*a^2*c*d^3)*x^3)*(c^2*d)^(
2/3)*log(c*d*x + (c^2*d)^(2/3)) + 12*(b^2*c^5*d + a*b*c^4*d^2 - 2*a^2*c^3*d
^3)*x)/(c^4*d^5*x^6 + 2*c^5*d^4*x^3 + c^6*d^3)]
```

Sympy [A] time = 2.87419, size = 233, normalized size = 0.9

$$\frac{x^4\left(5a^2d^3+2abcd^2-7b^2c^2d\right)+x\left(8a^2cd^2-4abc^2d-4b^2c^3\right)}{18c^4d^2+36c^3d^3x^3+18c^2d^4x^6}+\mathrm{RootSum}\left(19683t^3c^8d^7-125a^6d^6-150a^5bcd^5-210\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2/(d*x**3+c)**3,x)
```

```
[Out] (x**4*(5*a**2*d**3 + 2*a*b*c*d**2 - 7*b**2*c**2*d) + x*(8*a**2*c*d**2 - 4*a
*b*c**2*d - 4*b**2*c**3))/(18*c**4*d**2 + 36*c**3*d**3*x**3 + 18*c**2*d**4*
x**6) + RootSum(19683*_t**3*c**8*d**7 - 125*a**6*d**6 - 150*a**5*b*c*d**5 -
 210*a**4*b**2*c**2*d**4 - 128*a**3*b**3*c**3*d**3 - 84*a**2*b**4*c**4*d**2
 - 24*a*b**5*c**5*d - 8*b**6*c**6, Lambda(_t, _t*log(27*_t*c**3*d**2/(5*a**
2*d**2 + 2*a*b*c*d + 2*b**2*c**2) + x)))
```

Giac [A] time = 1.57974, size = 400, normalized size = 1.55

$$-\frac{\left(2\,b^2c^2+2\,abcd+5\,a^2d^2\right)\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{27\,c^3d^2}+\frac{\sqrt{3}\left(2\,\left(-cd^2\right)^{\frac{1}{3}}b^2c^2+2\,\left(-cd^2\right)^{\frac{1}{3}}abcd+5\,\left(-cd^2\right)^{\frac{1}{3}}a^2d^2\right)\arctan\left(\right.}{27\,c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2/(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] -1/27*(2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*(-c/d)^(1/3)*log(abs(x - (-c/d)^(
1/3)))/(c^3*d^2) + 1/27*sqrt(3)*(2*(-c*d^2)^(1/3)*b^2*c^2 + 2*(-c*d^2)^(1/3
)*a*b*c*d + 5*(-c*d^2)^(1/3)*a^2*d^2)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3
))/(-c/d)^(1/3))/(c^3*d^3) + 1/54*(2*(-c*d^2)^(1/3)*b^2*c^2 + 2*(-c*d^2)^(1
/3)*a*b*c*d + 5*(-c*d^2)^(1/3)*a^2*d^2)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(
2/3))/(c^3*d^3) - 1/18*(7*b^2*c^2*d*x^4 - 2*a*b*c*d^2*x^4 - 5*a^2*d^3*x^4 +
 4*b^2*c^3*x + 4*a*b*c^2*d*x - 8*a^2*c*d^2*x)/((d*x^3 + c)^2*c^2*d^2)
```

3.14 $\int \frac{(c+dx^3)^4}{a+bx^3} dx$

Optimal. Leaf size=252

$$\frac{d^2x^4\left(a^2d^2-4abcd+6b^2c^2\right)}{4b^3}+\frac{dx(2bc-ad)\left(a^2d^2-2abcd+2b^2c^2\right)}{b^4}-\frac{(bc-ad)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{13/3}}+\frac{(bc-$$

```
[Out] (d*(2*b*c - a*d)*(2*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/b^4 + (d^2*(6*b^2*c^2
 - 4*a*b*c*d + a^2*d^2)*x^4)/(4*b^3) + (d^3*(4*b*c - a*d)*x^7)/(7*b^2) + (d
^4*x^10)/(10*b) - ((b*c - a*d)^4*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^
(1/3))])/(Sqrt[3]*a^(2/3)*b^(13/3)) + ((b*c - a*d)^4*Log[a^(1/3) + b^(1/3)*
x])/(3*a^(2/3)*b^(13/3)) - ((b*c - a*d)^4*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2])/(6*a^(2/3)*b^(13/3))
```

Rubi [A] time = 0.191462, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {390, 200, 31, 634, 617, 204, 628}

$$\frac{d^2x^4\left(a^2d^2-4abcd+6b^2c^2\right)}{4b^3}+\frac{dx(2bc-ad)\left(a^2d^2-2abcd+2b^2c^2\right)}{b^4}-\frac{(bc-ad)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{13/3}}+\frac{(bc-$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^4/(a + b*x^3),x]
```

```
[Out] (d*(2*b*c - a*d)*(2*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/b^4 + (d^2*(6*b^2*c^2
 - 4*a*b*c*d + a^2*d^2)*x^4)/(4*b^3) + (d^3*(4*b*c - a*d)*x^7)/(7*b^2) + (d
^4*x^10)/(10*b) - ((b*c - a*d)^4*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^
(1/3))])/(Sqrt[3]*a^(2/3)*b^(13/3)) + ((b*c - a*d)^4*Log[a^(1/3) + b^(1/3)*
x])/(3*a^(2/3)*b^(13/3)) - ((b*c - a*d)^4*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2])/(6*a^(2/3)*b^(13/3))
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^4}{a+bx^3}\,dx = \int \left(\frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)}{b^4}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^3}{b^3}+\frac{d^3(4bc-ad)x^6}{b^2}+\frac{d^4x^9}{b}+\frac{b\ldots}{\ldots}\right.$$

$$=\frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^4}{4b^3}+\frac{d^3(4bc-ad)x^7}{7b^2}+\frac{d^4x^{10}}{10b}+\frac{(bc\ldots}{\ldots}$$

$$=\frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^4}{4b^3}+\frac{d^3(4bc-ad)x^7}{7b^2}+\frac{d^4x^{10}}{10b}+\frac{(bc\ldots}{\ldots}$$

$$=\frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^4}{4b^3}+\frac{d^3(4bc-ad)x^7}{7b^2}+\frac{d^4x^{10}}{10b}+\frac{(bc\ldots}{\ldots}$$

$$=\frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^4}{4b^3}+\frac{d^3(4bc-ad)x^7}{7b^2}+\frac{d^4x^{10}}{10b}+\frac{(bc\ldots}{\ldots}$$

$$=\frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^4}{4b^3}+\frac{d^3(4bc-ad)x^7}{7b^2}+\frac{d^4x^{10}}{10b}-\frac{(bc\ldots}{\ldots}$$

Mathematica [A] time = 0.115872, size = 253, normalized size = 1.

$$\frac{105b^{4/3}d^2x^4\left(a^2d^2-4abcd+6b^2c^2\right)+420\sqrt[3]{b}dx\left(4a^2bcd^2-a^3d^3-6ab^2c^2d+4b^3c^3\right)-\frac{70(bc-ad)^4\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{2/3}}+\frac{140\ldots}{\ldots}}{420b^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^4/(a + b*x^3),x]
```

```
[Out] (420*b^(1/3)*d*(4*b^3*c^3 - 6*a*b^2*c^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*x + 10
5*b^(4/3)*d^2*(6*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x^4 + 60*b^(7/3)*d^3*(4*b*c
```

```
 - a*d)*x^7 + 42*b^(10/3)*d^4*x^10 + (140*Sqrt[3]*(b*c - a*d)^4*ArcTan[(-a^
(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/a^(2/3) + (140*(b*c - a*d)^4*Log[a
^(1/3) + b^(1/3)*x])/a^(2/3) - (70*(b*c - a*d)^4*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2])/a^(2/3))/(420*b^(13/3))
```

Maple [B] time = 0.005, size = 661, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^4/(b*x^3+a),x)
```

```
[Out] -4/3/b^4/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*a^3*c*d^3+2/b^3/(1/b*a)^(2/3)*ln
(x+(1/b*a)^(1/3))*a^2*c^2*d^2-4/3/b^2/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*a*c
^3*d+2/3/b^4/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*a^3*c*d^3-
1/b^3/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*a^2*c^2*d^2+2/3/b
^2/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*a*c^3*d+1/3/b^5/(1/b
*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*a^4*d^4+1/3/b/(
1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*c^4-1/6/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/
3)*x+(1/b*a)^(2/3))*c^4+1/4*d^4/b^3*x^4*a^2+3/2*d^2/b*x^4*c^2-d^4/b^4*a^3*x
+4*d/b*c^3*x-1/6/b^5/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*a^
4*d^4+1/3/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))
*c^4+4*d^3/b^3*a^2*c*x-d^3/b^2*x^4*a*c-6*d^2/b^2*a*c^2*x+1/3/b^5/(1/b*a)^(2
/3)*ln(x+(1/b*a)^(1/3))*a^4*d^4-4/3/b^4/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^
(1/2)*(2/(1/b*a)^(1/3)*x-1))*a^3*c*d^3+2/b^3/(1/b*a)^(2/3)*3^(1/2)*arctan(1
/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*a^2*c^2*d^2-4/3/b^2/(1/b*a)^(2/3)*3^(1/2)
*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*a*c^3*d+1/10*d^4*x^10/b-1/7*d^4/
b^2*x^7*a+4/7*d^3/b*x^7*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^4/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.72613, size = 1916, normalized size = 7.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^4/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/420*(42*a^2*b^4*d^4*x^10 + 60*(4*a^2*b^4*c*d^3 - a^3*b^3*d^4)*x^7 + 105*
(6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*x^4 + 210*sqrt(1/3)*(a*
b^5*c^4 - 4*a^2*b^4*c^3*d + 6*a^3*b^3*c^2*d^2 - 4*a^4*b^2*c*d^3 + a^5*b*d^4
)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqr
```

```
t(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/
b))/(b*x^3 + a)) - 70*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*
b*c*d^3 + a^4*d^4)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1
/3)*a) + 140*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 +
 a^4*d^4)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 420*(4*a^2*b^4*c^3*d -
 6*a^3*b^3*c^2*d^2 + 4*a^4*b^2*c*d^3 - a^5*b*d^4)*x)/(a^2*b^5), 1/420*(42*a
^2*b^4*d^4*x^10 + 60*(4*a^2*b^4*c*d^3 - a^3*b^3*d^4)*x^7 + 105*(6*a^2*b^4*c
^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*x^4 + 420*sqrt(1/3)*(a*b^5*c^4 - 4*
a^2*b^4*c^3*d + 6*a^3*b^3*c^2*d^2 - 4*a^4*b^2*c*d^3 + a^5*b*d^4)*sqrt((a^2*
b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^
2*b)^(1/3)/b)/a^2) - 70*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^
3*b*c*d^3 + a^4*d^4)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^
(1/3)*a) + 140*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
 + a^4*d^4)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 420*(4*a^2*b^4*c^3*d
 - 6*a^3*b^3*c^2*d^2 + 4*a^4*b^2*c*d^3 - a^5*b*d^4)*x)/(a^2*b^5)]
```

Sympy [A] time = 2.14005, size = 369, normalized size = 1.46

$$\text{RootSum}\left(27t^3a^2b^{13} - a^{12}d^{12} + 12a^{11}bcd^{11} - 66a^{10}b^2c^2d^{10} + 220a^9b^3c^3d^9 - 495a^8b^4c^4d^8 + 792a^7b^5c^5d^7 - 924a^6b^6c^6d^6\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**4/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**13 - a**12*d**12 + 12*a**11*b*c*d**11 - 66*a**10*b
**2*c**2*d**10 + 220*a**9*b**3*c**3*d**9 - 495*a**8*b**4*c**4*d**8 + 792*a*
*7*b**5*c**5*d**7 - 924*a**6*b**6*c**6*d**6 + 792*a**5*b**7*c**7*d**5 - 495
*a**4*b**8*c**8*d**4 + 220*a**3*b**9*c**9*d**3 - 66*a**2*b**10*c**10*d**2 +
 12*a*b**11*c**11*d - b**12*c**12, Lambda(_t, _t*log(3*_t*a*b**4/(a**4*d**4
 - 4*a**3*b*c*d**3 + 6*a**2*b**2*c**2*d**2 - 4*a*b**3*c**3*d + b**4*c**4) +
 x))) + d**4*x**10/(10*b) - x**7*(a*d**4 - 4*b*c*d**3)/(7*b**2) + x**4*(a**
2*d**4 - 4*a*b*c*d**3 + 6*b**2*c**2*d**2)/(4*b**3) - x*(a**3*d**4 - 4*a**2*
b*c*d**3 + 6*a*b**2*c**2*d**2 - 4*b**3*c**3*d)/b**4
```

Giac [B] time = 1.12613, size = 622, normalized size = 2.47

$$\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^4c^4 - 4\left(-ab^2\right)^{\frac{1}{3}}ab^3c^3d + 6\left(-ab^2\right)^{\frac{1}{3}}a^2b^2c^2d^2 - 4\left(-ab^2\right)^{\frac{1}{3}}a^3bcd^3 + \left(-ab^2\right)^{\frac{1}{3}}a^4d^4\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^4/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*((-a*b^2)^(1/3)*b^4*c^4 - 4*(-a*b^2)^(1/3)*a*b^3*c^3*d + 6*(-a*
b^2)^(1/3)*a^2*b^2*c^2*d^2 - 4*(-a*b^2)^(1/3)*a^3*b*c*d^3 + (-a*b^2)^(1/3)*
a^4*d^4)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^5) + 1/
6*((-a*b^2)^(1/3)*b^4*c^4 - 4*(-a*b^2)^(1/3)*a*b^3*c^3*d + 6*(-a*b^2)^(1/3)
*a^2*b^2*c^2*d^2 - 4*(-a*b^2)^(1/3)*a^3*b*c*d^3 + (-a*b^2)^(1/3)*a^4*d^4)*l
og(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^5) - 1/3*(b^10*c^4 - 4*a*b^9*c
^3*d + 6*a^2*b^8*c^2*d^2 - 4*a^3*b^7*c*d^3 + a^4*b^6*d^4)*(-a/b)^(1/3)*log(
abs(x - (-a/b)^(1/3)))/(a*b^10) + 1/140*(14*b^9*d^4*x^10 + 80*b^9*c*d^3*x^7
```

```
 - 20*a*b^8*d^4*x^7 + 210*b^9*c^2*d^2*x^4 - 140*a*b^8*c*d^3*x^4 + 35*a^2*b^
7*d^4*x^4 + 560*b^9*c^3*d*x - 840*a*b^8*c^2*d^2*x + 560*a^2*b^7*c*d^3*x - 1
40*a^3*b^6*d^4*x)/b^10
```

3.15 $\int \frac{(c+dx^3)^3}{a+bx^3}\, dx$

Optimal. Leaf size=208

$$\frac{dx\left(a^2d^2-3abcd+3b^2c^2\right)}{b^3}-\frac{(bc-ad)^3\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{10/3}}+\frac{(bc-ad)^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{10/3}}-\frac{(bc-ad)^3\tan^{-1}\left(\cdots\right)}{\sqrt{3}a^{2/3}b^{10/3}}$$

```
[Out] (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x)/b^3 + (d^2*(3*b*c - a*d)*x^4)/(4*b^
2) + (d^3*x^7)/(7*b) - ((b*c - a*d)^3*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[
3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(10/3)) + ((b*c - a*d)^3*Log[a^(1/3) + b^(
1/3)*x])/(3*a^(2/3)*b^(10/3)) - ((b*c - a*d)^3*Log[a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(10/3))
```

Rubi [A] time = 0.147699, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {390, 200, 31, 634, 617, 204, 628}

$$\frac{dx\left(a^2d^2-3abcd+3b^2c^2\right)}{b^3}-\frac{(bc-ad)^3\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{10/3}}+\frac{(bc-ad)^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{10/3}}-\frac{(bc-ad)^3\tan^{-1}\left(\cdots\right)}{\sqrt{3}a^{2/3}b^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^3/(a + b*x^3),x]
```

```
[Out] (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x)/b^3 + (d^2*(3*b*c - a*d)*x^4)/(4*b^
2) + (d^3*x^7)/(7*b) - ((b*c - a*d)^3*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[
3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(10/3)) + ((b*c - a*d)^3*Log[a^(1/3) + b^(
1/3)*x])/(3*a^(2/3)*b^(10/3)) - ((b*c - a*d)^3*Log[a^(2/3) - a^(1/3)*b^(1/3
)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(10/3))
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^3}{a+bx^3}\,dx = \int \left(\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)}{b^3}+\frac{d^2(3bc-ad)x^3}{b^2}+\frac{d^3x^6}{b}+\frac{b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3}{b^3\left(a+bx^3\right)}\right)dx$$

$$=\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x}{b^3}+\frac{d^2(3bc-ad)x^4}{4b^2}+\frac{d^3x^7}{7b}+\frac{(bc-ad)^3\int\frac{1}{a+bx^3}\,dx}{b^3}$$

$$=\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x}{b^3}+\frac{d^2(3bc-ad)x^4}{4b^2}+\frac{d^3x^7}{7b}+\frac{(bc-ad)^3\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{2/3}b^3}+\frac{(bc-ad)^3\int\ldots}{3a^{2/}\ldots}$$

$$=\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x}{b^3}+\frac{d^2(3bc-ad)x^4}{4b^2}+\frac{d^3x^7}{7b}+\frac{(bc-ad)^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{10/3}}-\frac{(bc-ad)^3\int\ldots}{6\ldots}$$

$$=\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x}{b^3}+\frac{d^2(3bc-ad)x^4}{4b^2}+\frac{d^3x^7}{7b}+\frac{(bc-ad)^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{10/3}}-\frac{(bc-ad)^3\log\ldots}{\ldots}$$

$$=\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x}{b^3}+\frac{d^2(3bc-ad)x^4}{4b^2}+\frac{d^3x^7}{7b}-\frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{10/3}}+\frac{(bc-ad)^3\log\ldots}{3a^2\ldots}$$

Mathematica [A] time = 0.0911959, size = 203, normalized size = 0.98

$$\frac{84\sqrt[3]{b}dx\left(a^2d^2-3abcd+3b^2c^2\right)+\frac{14(ad-bc)^3\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{2/3}}+\frac{28(bc-ad)^3\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{2/3}}+\frac{28\sqrt{3}(bc-ad)^3\tan^{-1}\left(\frac{2\sqrt[3]{b}x-\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}}+\ldots}{84b^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^3/(a + b*x^3),x]
```

```
[Out] (84*b^(1/3)*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x + 21*b^(4/3)*d^2*(3*b*c -
 a*d)*x^4 + 12*b^(7/3)*d^3*x^7 + (28*Sqrt[3]*(b*c - a*d)^3*ArcTan[(-a^(1/3)
 + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/a^(2/3) + (28*(b*c - a*d)^3*Log[a^(1/3)
```

```
 + b^(1/3)*x])/a^(2/3) + (14*(-(b*c) + a*d)^3*Log[a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2])/a^(2/3))/(84*b^(10/3))
```

Maple [B] time = 0.003, size = 486, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^3/(b*x^3+a),x)
```

```
[Out] 1/7*d^3*x^7/b-1/4*d^3/b^2*x^4*a+3/4*d^2/b*x^4*c+d^3/b^3*a^2*x-3*d^2/b^2*c*a
*x+3*d/b*c^2*x-1/3/b^4/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*a^3*d^3+1/b^3/(1/b
*a)^(2/3)*ln(x+(1/b*a)^(1/3))*a^2*c*d^2-1/b^2/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1
/3))*a*c^2*d+1/3/b/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*c^3+1/6/b^4/(1/b*a)^(2
/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*a^3*d^3-1/2/b^3/(1/b*a)^(2/3)*ln(
x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*a^2*c*d^2+1/2/b^2/(1/b*a)^(2/3)*ln(x^2-(
1/b*a)^(1/3)*x+(1/b*a)^(2/3))*a*c^2*d-1/6/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1
/3)*x+(1/b*a)^(2/3))*c^3-1/3/b^4/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(
2/(1/b*a)^(1/3)*x-1))*a^3*d^3+1/b^3/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2
)*(2/(1/b*a)^(1/3)*x-1))*a^2*c*d^2-1/b^2/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3
^(1/2)*(2/(1/b*a)^(1/3)*x-1))*a*c^2*d+1/3/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/
3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^3/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.78899, size = 1544, normalized size = 7.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^3/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/84*(12*a^2*b^3*d^3*x^7 + 21*(3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*x^4 - 42*sqr
t(1/3)*(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2 - a^4*b*d^3)*sqrt((-a
^2*b)^(1/3)/b)*log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2
*a*b*x^2 + (-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*
x^3 + a)) - 14*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(-a^2*b)
^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 28*(b^3*c^3 - 3
*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)
^(2/3)) + 84*(3*a^2*b^3*c^2*d - 3*a^3*b^2*c*d^2 + a^4*b*d^3)*x)/(a^2*b^4),
1/84*(12*a^2*b^3*d^3*x^7 + 21*(3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*x^4 + 84*sqrt
(1/3)*(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 3*a^3*b^2*c*d^2 - a^4*b*d^3)*sqrt(-(-a
```

```
^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqr
t(-(-a^2*b)^(1/3)/b)/a^2) - 14*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a
^3*d^3)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) +
 28*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*(-a^2*b)^(2/3)*log(
a*b*x + (-a^2*b)^(2/3)) + 84*(3*a^2*b^3*c^2*d - 3*a^3*b^2*c*d^2 + a^4*b*d^3
)*x)/(a^2*b^4)]
```

Sympy [A] time = 2.03412, size = 255, normalized size = 1.23

$$\mathrm{RootSum}\Big(27t^3a^2b^{10} + a^9d^9 - 9a^8bcd^8 + 36a^7b^2c^2d^7 - 84a^6b^3c^3d^6 + 126a^5b^4c^4d^5 - 126a^4b^5c^5d^4 + 84a^3b^6c^6d^3 - 36a^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**3/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**10 + a**9*d**9 - 9*a**8*b*c*d**8 + 36*a**7*b**2*c*
*2*d**7 - 84*a**6*b**3*c**3*d**6 + 126*a**5*b**4*c**4*d**5 - 126*a**4*b**5*
c**5*d**4 + 84*a**3*b**6*c**6*d**3 - 36*a**2*b**7*c**7*d**2 + 9*a*b**8*c**8
*d - b**9*c**9, Lambda(_t, _t*log(-3*_t*a*b**3/(a**3*d**3 - 3*a**2*b*c*d**2
 + 3*a*b**2*c**2*d - b**3*c**3) + x))) + d**3*x**7/(7*b) - x**4*(a*d**3 - 3
*b*c*d**2)/(4*b**2) + x*(a**2*d**3 - 3*a*b*c*d**2 + 3*b**2*c**2*d)/b**3
```

Giac [B] time = 1.10148, size = 473, normalized size = 2.27

$$\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^3c^3 - 3\left(-ab^2\right)^{\frac{1}{3}}ab^2c^2d + 3\left(-ab^2\right)^{\frac{1}{3}}a^2bcd^2 - \left(-ab^2\right)^{\frac{1}{3}}a^3d^3\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab^4} + \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}b^3c^3 - \cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^3/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*((-a*b^2)^(1/3)*b^3*c^3 - 3*(-a*b^2)^(1/3)*a*b^2*c^2*d + 3*(-a*
b^2)^(1/3)*a^2*b*c*d^2 - (-a*b^2)^(1/3)*a^3*d^3)*arctan(1/3*sqrt(3)*(2*x +
(-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^4) + 1/6*((-a*b^2)^(1/3)*b^3*c^3 - 3*(-a*b
^2)^(1/3)*a*b^2*c^2*d + 3*(-a*b^2)^(1/3)*a^2*b*c*d^2 - (-a*b^2)^(1/3)*a^3*d
^3)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^4) - 1/3*(b^7*c^3 - 3*a*b
^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(
1/3)))/(a*b^7) + 1/28*(4*b^6*d^3*x^7 + 21*b^6*c*d^2*x^4 - 7*a*b^5*d^3*x^4 +
 84*b^6*c^2*d*x - 84*a*b^5*c*d^2*x + 28*a^2*b^4*d^3*x)/b^7
```

3.16 $\int \frac{(c+dx^3)^2}{a+bx^3}\,dx$

Optimal. Leaf size=173

$$-\frac{(bc-ad)^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{7/3}}+\frac{(bc-ad)^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{7/3}}-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{7/3}}+\frac{dx(2bc-ad)}{b^2}+$$

```
[Out] (d*(2*b*c - a*d)*x)/b^2 + (d^2*x^4)/(4*b) - ((b*c - a*d)^2*ArcTan[(a^(1/3)
- 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(7/3)) + ((b*c - a*d)
^2*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(7/3)) - ((b*c - a*d)^2*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(7/3))
```

Rubi [A] time = 0.124228, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {390, 200, 31, 634, 617, 204, 628}

$$-\frac{(bc-ad)^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{7/3}}+\frac{(bc-ad)^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{7/3}}-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{7/3}}+\frac{dx(2bc-ad)}{b^2}+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^2/(a + b*x^3),x]
```

```
[Out] (d*(2*b*c - a*d)*x)/b^2 + (d^2*x^4)/(4*b) - ((b*c - a*d)^2*ArcTan[(a^(1/3)
- 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*b^(7/3)) + ((b*c - a*d)
^2*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*b^(7/3)) - ((b*c - a*d)^2*Log[a^(2/
3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(7/3))
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(c+dx^3\right)^2}{a+bx^3}\,dx &= \int \left(\frac{d(2bc-ad)}{b^2}+\frac{d^2x^3}{b}+\frac{b^2c^2-2abcd+a^2d^2}{b^2\left(a+bx^3\right)}\right)dx\\
&= \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^4}{4b}+\frac{(bc-ad)^2\int \frac{1}{a+bx^3}\,dx}{b^2}\\
&= \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^4}{4b}+\frac{(bc-ad)^2\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{2/3}b^2}+\frac{(bc-ad)^2\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}b^2}\\
&= \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^4}{4b}+\frac{(bc-ad)^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{7/3}}-\frac{(bc-ad)^2\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{2/3}b^{7/3}}+\frac{(bc-ad)^2}{\cdots}\\
&= \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^4}{4b}+\frac{(bc-ad)^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{7/3}}-\frac{(bc-ad)^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{7/3}}+\frac{(bc\cdots}{\cdots}\\
&= \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^4}{4b}-\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{7/3}}+\frac{(bc-ad)^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{7/3}}-\frac{(bc-ad)^2\log\cdots}{\cdots}
\end{aligned}$$

Mathematica [A] time = 0.112812, size = 167, normalized size = 0.97

$$\frac{-2(bc-ad)^2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+3a^{2/3}b^{4/3}d^2x^4-12a^{2/3}\sqrt[3]{b}dx(ad-2bc)+4(bc-ad)^2\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)+4\sqrt{\cdots}}{12a^{2/3}b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3),x]
```

```
[Out] (-12*a^(2/3)*b^(1/3)*d*(-2*b*c + a*d)*x + 3*a^(2/3)*b^(4/3)*d^2*x^4 + 4*Sqr
t[3]*(b*c - a*d)^2*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))] + 4*(
b*c - a*d)^2*Log[a^(1/3) + b^(1/3)*x] - 2*(b*c - a*d)^2*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/(12*a^(2/3)*b^(7/3))
```

Maple [B] time = 0.003, size = 334, normalized size = 1.9

$$\frac{d^2x^4}{4b} - \frac{ad^2x}{b^2} + 2\frac{dxc}{b} + \frac{a^2d^2}{3b^3}\ln\left(x + \sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{2acd}{3b^2}\ln\left(x + \sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{c^2}{3b}\ln\left(x + \sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} - \frac{a^2d^2}{6b^3}\ln\left(x^2 -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^2/(b*x^3+a),x)
```

```
[Out] 1/4*d^2*x^4/b-d^2/b^2*a*x+2*d/b*x*c+1/3/b^3/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3
))*a^2*d^2-2/3/b^2/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*c*a*d+1/3/b/(1/b*a)^(2
/3)*ln(x+(1/b*a)^(1/3))*c^2-1/6/b^3/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1
/b*a)^(2/3))*a^2*d^2+1/3/b^2/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(
2/3))*c*a*d-1/6/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c^2+1
/3/b^3/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*a^2*
d^2-2/3/b^2/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))
*c*a*d+1/3/b/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1)
)*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.70513, size = 1183, normalized size = 6.84

$$\left[\frac{3a^2b^2d^2x^4 + 6\sqrt{\frac{1}{3}}\left(ab^3c^2 - 2a^2b^2cd + a^3bd^2\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2abx^3 - 3\left(a^2b\right)^{\frac{1}{3}}ax - a^2 + 3\sqrt{\frac{1}{3}}\left(2abx^2 + \left(a^2b\right)^{\frac{2}{3}}x - \left(a^2b\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}}{bx^3 + a}\right) - 2\left(b\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/12*(3*a^2*b^2*d^2*x^4 + 6*sqrt(1/3)*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d
^2)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*s
qrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3
)/b))/(b*x^3 + a)) - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(a^2*b)^(2/3)*log(a*
b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 4*(b^2*c^2 - 2*a*b*c*d + a^2*d
^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 12*(2*a^2*b^2*c*d - a^3*b*d^
2)*x)/(a^2*b^3), 1/12*(3*a^2*b^2*d^2*x^4 + 12*sqrt(1/3)*(a*b^3*c^2 - 2*a^2*
```

```
b^2*c*d + a^3*b*d^2)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3
)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 2*(b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) +
 4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3))
 + 12*(2*a^2*b^2*c*d - a^3*b*d^2)*x)/(a^2*b^3)]
```

Sympy [A] time = 1.34681, size = 156, normalized size = 0.9

$$\operatorname{RootSum}\left(27t^3a^2b^7 - a^6d^6 + 6a^5bcd^5 - 15a^4b^2c^2d^4 + 20a^3b^3c^3d^3 - 15a^2b^4c^4d^2 + 6ab^5c^5d - b^6c^6, \left(t \mapsto t\log\left(\frac{}{a^2d^2 - \ldots}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**2/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**7 - a**6*d**6 + 6*a**5*b*c*d**5 - 15*a**4*b**2*c**
2*d**4 + 20*a**3*b**3*c**3*d**3 - 15*a**2*b**4*c**4*d**2 + 6*a*b**5*c**5*d
- b**6*c**6, Lambda(_t, _t*log(3*_t*a*b**2/(a**2*d**2 - 2*a*b*c*d + b**2*c*
*2) + x))) + d**2*x**4/(4*b) - x*(a*d**2 - 2*b*c*d)/b**2
```

Giac [A] time = 1.13895, size = 336, normalized size = 1.94

$$\frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^2c^2 - 2\left(-ab^2\right)^{\frac{1}{3}}abcd + \left(-ab^2\right)^{\frac{1}{3}}a^2d^2\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3ab^3} + \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}b^2c^2 - 2\left(-ab^2\right)^{\frac{1}{3}}abcd + \left(-a\right.\right.}{6a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*((-a*b^2)^(1/3)*b^2*c^2 - 2*(-a*b^2)^(1/3)*a*b*c*d + (-a*b^2)^(
1/3)*a^2*d^2)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^3)
 + 1/6*((-a*b^2)^(1/3)*b^2*c^2 - 2*(-a*b^2)^(1/3)*a*b*c*d + (-a*b^2)^(1/3)*
a^2*d^2)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^3) - 1/3*(b^4*c^2 -
2*a*b^3*c*d + a^2*b^2*d^2)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^4)
+ 1/4*(b^3*d^2*x^4 + 8*b^3*c*d*x - 4*a*b^2*d^2*x)/b^4
```

3.17 $\int \frac{c+dx^3}{a+bx^3}\,dx$

Optimal. Leaf size=145

$$-\frac{(bc-ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}}+\frac{(bc-ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{4/3}}+\frac{dx}{b}$$

```
[Out] (d*x)/b - ((b*c - a*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(
Sqrt[3]*a^(2/3)*b^(4/3)) + ((b*c - a*d)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3
)*b^(4/3)) - ((b*c - a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(
6*a^(2/3)*b^(4/3))
```

Rubi [A] time = 0.0780682, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {388, 200, 31, 634, 617, 204, 628}

$$-\frac{(bc-ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}}+\frac{(bc-ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{4/3}}+\frac{dx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)/(a + b*x^3),x]
```

```
[Out] (d*x)/b - ((b*c - a*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(
Sqrt[3]*a^(2/3)*b^(4/3)) + ((b*c - a*d)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3
)*b^(4/3)) - ((b*c - a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(
6*a^(2/3)*b^(4/3))
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx^3}{a+bx^3}\,dx = \frac{dx}{b} - \frac{(-bc+ad)\int \frac{1}{a+bx^3}\,dx}{b}$$

$$= \frac{dx}{b} + \frac{(bc-ad)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{2/3}b} + \frac{(bc-ad)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}b}$$

$$= \frac{dx}{b} + \frac{(bc-ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}} - \frac{(bc-ad)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{2/3}b^{4/3}} + \frac{(bc-ad)\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{2\sqrt[3]{ab}}$$

$$= \frac{dx}{b} + \frac{(bc-ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}} - \frac{(bc-ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}} + \frac{(bc-ad)\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\right.}{a^{2/3}b^{4/3}}$$

$$= \frac{dx}{b} - \frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{4/3}} + \frac{(bc-ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}} - \frac{(bc-ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}}$$

Mathematica [A] time = 0.0647341, size = 129, normalized size = 0.89

$$\frac{-(bc-ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+6a^{2/3}\sqrt[3]{b}dx+2(bc-ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)-2\sqrt{3}(bc-ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{6a^{2/3}b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)/(a + b*x^3),x]
```

```
[Out] (6*a^(2/3)*b^(1/3)*d*x - 2*Sqrt[3]*(b*c - a*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^
(1/3))/Sqrt[3]] + 2*(b*c - a*d)*Log[a^(1/3) + b^(1/3)*x] - (b*c - a*d)*Log[
a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3)*b^(4/3))
```

Maple [A] time = 0.002, size = 195, normalized size = 1.3

$$\frac{dx}{b}-\frac{ad}{3b^{2}}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{c}{3b}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{ad}{6b^{2}}\ln\left(x^{2}-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{c}{6b}\ln\left(x^{2}-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)/(b*x^3+a),x)
```

```
[Out] d*x/b-1/3/b^2/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*a*d+1/3/b/(1/b*a)^(2/3)*ln(
x+(1/b*a)^(1/3))*c+1/6/b^2/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/
3))*a*d-1/6/b/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c-1/3/b^2
/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*a*d+1/3/b/
(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.71623, size = 926, normalized size = 6.39

$$\left[\frac{6a^{2}bdx-3\sqrt{\frac{1}{3}}\left(ab^{2}c-a^{2}bd\right)\sqrt{\frac{\left(-a^{2}b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2abx^{3}+3\left(-a^{2}b\right)^{\frac{1}{3}}ax-a^{2}-3\sqrt{\frac{1}{3}}\left(2abx^{2}+\left(-a^{2}b\right)^{\frac{2}{3}}x+\left(-a^{2}b\right)^{\frac{1}{3}}a\right)\sqrt{\frac{\left(-a^{2}b\right)^{\frac{1}{3}}}{b}}}{bx^{3}+a}\right)-\left(-a^{2}b\right)^{\frac{2}{3}}(bc-ad)}{6a^{2}b^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(6*a^2*b*d*x - 3*sqrt(1/3)*(a*b^2*c - a^2*b*d)*sqrt((-a^2*b)^(1/3)/b)*
log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^
2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) - (-a
^2*b)^(2/3)*(b*c - a*d)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a)
+ 2*(-a^2*b)^(2/3)*(b*c - a*d)*log(a*b*x + (-a^2*b)^(2/3)))/(a^2*b^2), 1/6*
(6*a^2*b*d*x + 6*sqrt(1/3)*(a*b^2*c - a^2*b*d)*sqrt(-(-a^2*b)^(1/3)/b)*arct
an(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3)/b
)/a^2) - (-a^2*b)^(2/3)*(b*c - a*d)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*
b)^(1/3)*a) + 2*(-a^2*b)^(2/3)*(b*c - a*d)*log(a*b*x + (-a^2*b)^(2/3)))/(a^
2*b^2)]
```

Sympy [A] time = 0.780621, size = 71, normalized size = 0.49

$$\operatorname{RootSum}\left(27t^3a^2b^4 + a^3d^3 - 3a^2bcd^2 + 3ab^2c^2d - b^3c^3, \left(t \mapsto t\log\left(-\frac{3tab}{ad - bc} + x\right)\right)\right) + \frac{dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)/(b*x**3+a),x)
```

```
[Out] RootSum(27*_t**3*a**2*b**4 + a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d
- b**3*c**3, Lambda(_t, _t*log(-3*_t*a*b/(a*d - b*c) + x))) + d*x/b
```

Giac [A] time = 1.09841, size = 217, normalized size = 1.5

$$\frac{dx}{b} - \frac{(bc - ad)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\,ab} + \frac{\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}bc - \left(-ab^2\right)^{\frac{1}{3}}ad\right)\arctan\left(\frac{\sqrt{3}\left(2\,x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,ab^2} + \frac{\left(\left(-ab^2\right)^{\frac{1}{3}}bc - \left(-a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a),x, algorithm="giac")
```

```
[Out] d*x/b - 1/3*(b*c - a*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b) + 1/3
*sqrt(3)*((-a*b^2)^(1/3)*b*c - (-a*b^2)^(1/3)*a*d)*arctan(1/3*sqrt(3)*(2*x
+ (-a/b)^(1/3))/(-a/b)^(1/3))/(a*b^2) + 1/6*((-a*b^2)^(1/3)*b*c - (-a*b^2)^
(1/3)*a*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^2)
```

3.18 $\int \frac{1}{(a+bx^3)(c+dx^3)}\,dx$

Optimal. Leaf size=288

$$-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)}-\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(bc-ad)}+\frac{d^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}(bc-ad)}-$$

```
[Out] -((b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2
/3)*(b*c - a*d))) + (d^(2/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3
))])/(Sqrt[3]*c^(2/3)*(b*c - a*d)) + (b^(2/3)*Log[a^(1/3) + b^(1/3)*x])/(3*
a^(2/3)*(b*c - a*d)) - (d^(2/3)*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3)*(b*c -
 a*d)) - (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3
)*(b*c - a*d)) + (d^(2/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(
6*c^(2/3)*(b*c - a*d))
```

Rubi [A] time = 0.146722, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {391, 200, 31, 634, 617, 204, 628}

$$-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)}+\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)}-\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(bc-ad)}+\frac{d^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}(bc-ad)}-$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] -((b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2
/3)*(b*c - a*d))) + (d^(2/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3
))])/(Sqrt[3]*c^(2/3)*(b*c - a*d)) + (b^(2/3)*Log[a^(1/3) + b^(1/3)*x])/(3*
a^(2/3)*(b*c - a*d)) - (d^(2/3)*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3)*(b*c -
 a*d)) - (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(2/3
)*(b*c - a*d)) + (d^(2/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(
6*c^(2/3)*(b*c - a*d))
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx = \frac{b\int \frac{1}{a+bx^3}\,dx}{bc-ad} - \frac{d\int \frac{1}{c+dx^3}\,dx}{bc-ad}$$

$$= \frac{b\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{2/3}(bc-ad)} + \frac{b\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}(bc-ad)} - \frac{d\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}\,dx}{3c^{2/3}(bc-ad)} - \frac{d\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}\,dx}{3c^{2/3}(bc-ad)}$$

$$= \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)} - \frac{d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}(bc-ad)} - \frac{b^{2/3}\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{6a^{2/3}(bc-ad)} + \frac{b\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x}}{2\sqrt[3]{a}(bc-}$$

$$= \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)} - \frac{d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}(bc-ad)} - \frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)} + \frac{d^{2/3}\log}{}$$

$$= -\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(bc-ad)} + \frac{d^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}(bc-ad)} + \frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)} - \frac{d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{}\right.}{3c^{2/3}(bc-ad}$$

Mathematica [A] time = 0.109617, size = 224, normalized size = 0.78

$$\frac{\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{2/3}} - \frac{2b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{2/3}} + \frac{2\sqrt{3}b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} - \frac{d^{2/3}\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{c^{2/3}} + \frac{2d^{2/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{c^{2/3}} - \frac{2\sqrt{3}d^{2/3}}{}}{6ad-6bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^3)*(c + d*x^3)),x]
```

```
[Out] ((2*Sqrt[3]*b^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) -
(2*Sqrt[3]*d^(2/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/c^(2/3) - (
```

```
2*b^(2/3)*Log[a^(1/3) + b^(1/3)*x]]/a^(2/3) + (2*d^(2/3)*Log[c^(1/3) + d^(1
/3)*x]])/c^(2/3) + (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]])/
a^(2/3) - (d^(2/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2]])/c^(2/3))
/(-6*b*c + 6*a*d)
```

Maple [A] time = 0.006, size = 222, normalized size = 0.8

$$-\frac{1}{3\,ad-3\,bc}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{1}{6\,ad-6\,bc}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{\sqrt{3}}{3\,ad-3\,bc}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)\left(\frac{a}{b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)/(d*x^3+c),x)
```

```
[Out] -1/3/(a*d-b*c)/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))+1/6/(a*d-b*c)/(1/b*a)^(2/3
)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))-1/3/(a*d-b*c)/(1/b*a)^(2/3)*3^(1/2)
*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+1/3/(a*d-b*c)/(c/d)^(2/3)*ln(x+(
c/d)^(1/3))-1/6/(a*d-b*c)/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))+1/3
/(a*d-b*c)/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.12349, size = 606, normalized size = 2.1

$$-\frac{2\,\sqrt{3}\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3\,b}\right)+2\,\sqrt{3}\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\,\sqrt{3}cx\left(\frac{d^2}{c^2}\right)^{\frac{2}{3}}-\sqrt{3}d}{3\,d}\right)-\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2+abx\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}+a^2\left(-\right.\right.}{6\,(bc-ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*(-b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2/3)
 - sqrt(3)*b)/b) + 2*sqrt(3)*(d^2/c^2)^(1/3)*arctan(1/3*(2*sqrt(3)*c*x*(d^2
/c^2)^(2/3) - sqrt(3)*d)/d) - (-b^2/a^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^
2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) - (d^2/c^2)^(1/3)*log(d^2*x^2 - c*d*x*(d^2
/c^2)^(1/3) + c^2*(d^2/c^2)^(2/3)) + 2*(-b^2/a^2)^(1/3)*log(b*x - a*(-b^2/a
^2)^(1/3)) + 2*(d^2/c^2)^(1/3)*log(d*x + c*(d^2/c^2)^(1/3)))/(b*c - a*d)
```

Sympy [A] time = 43.1292, size = 447, normalized size = 1.55

$$\mathrm{RootSum}\left(t^3\left(27a^5d^3-81a^4bcd^2+81a^3b^2c^2d-27a^2b^3c^3\right)+b^2,\left(t\mapsto t\log\left(x+\frac{81t^4a^7c^2d^5-243t^4a^6bc^3d^4+162t^4a^5\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)/(d*x**3+c),x)
```

```
[Out] RootSum(_t**3*(27*a**5*d**3 - 81*a**4*b*c*d**2 + 81*a**3*b**2*c**2*d - 27*a
**2*b**3*c**3) + b**2, Lambda(_t, _t*log(x + (81*_t**4*a**7*c**2*d**5 - 243
*_t**4*a**6*b*c**3*d**4 + 162*_t**4*a**5*b**2*c**4*d**3 + 162*_t**4*a**4*b*
*3*c**5*d**2 - 243*_t**4*a**3*b**4*c**6*d + 81*_t**4*a**2*b**5*c**7 - 3*_t*
a**4*d**4 + 3*_t*a**3*b*c*d**3 + 3*_t*a*b**3*c**3*d - 3*_t*b**4*c**4)/(a**2
*b*d**3 + b**3*c**2*d)))) + RootSum(_t**3*(27*a**3*c**2*d**3 - 81*a**2*b*c*
*3*d**2 + 81*a*b**2*c**4*d - 27*b**3*c**5) - d**2, Lambda(_t, _t*log(x + (8
1*_t**4*a**7*c**2*d**5 - 243*_t**4*a**6*b*c**3*d**4 + 162*_t**4*a**5*b**2*c
**4*d**3 + 162*_t**4*a**4*b**3*c**5*d**2 - 243*_t**4*a**3*b**4*c**6*d + 81*
_t**4*a**2*b**5*c**7 - 3*_t*a**4*d**4 + 3*_t*a**3*b*c*d**3 + 3*_t*a*b**3*c*
*3*d - 3*_t*b**4*c**4)/(a**2*b*d**3 + b**3*c**2*d))))
```

Giac [A] time = 1.12593, size = 375, normalized size = 1.3

$$-\frac{b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(abc-a^2d\right)}+\frac{d\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(bc^2-acd\right)}+\frac{\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}abc-\sqrt{3}a^2d}-\frac{\left(-cd^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x\right.}{3\left(\right.}\right.}{\sqrt{3}bc^2-\sqrt{3}acd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*b*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b*c - a^2*d) + 1/3*d*(-c/
d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b*c^2 - a*c*d) + (-a*b^2)^(1/3)*arctan
(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a*b*c - sqrt(3)*a^
2*d) - (-c*d^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3))
/(sqrt(3)*b*c^2 - sqrt(3)*a*c*d) + 1/6*(-a*b^2)^(1/3)*log(x^2 + x*(-a/b)^(1
/3) + (-a/b)^(2/3))/(a*b*c - a^2*d) - 1/6*(-c*d^2)^(1/3)*log(x^2 + x*(-c/d)
^(1/3) + (-c/d)^(2/3))/(b*c^2 - a*c*d)
```

3.19 $\int \frac{1}{\left(a+bx^3\right)\left(c+dx^3\right)^2}\,dx$

Optimal. Leaf size=346

$$-\frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)^2}+\frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)^2}-\frac{b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(bc-ad)^2}+\frac{d^{2/3}(5bc-2ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+\right.}{18c^{5/3}(bc-ad)^2}$$

```
[Out] -(d*x)/(3*c*(b*c - a*d)*(c + d*x^3)) - (b^(5/3)*ArcTan[(a^(1/3) - 2*b^(1/3)
*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*(b*c - a*d)^2) + (d^(2/3)*(5*b*c -
 2*a*d)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(3*Sqrt[3]*c^(5/
3)*(b*c - a*d)^2) + (b^(5/3)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*(b*c - a*
d)^2) - (d^(2/3)*(5*b*c - 2*a*d)*Log[c^(1/3) + d^(1/3)*x])/(9*c^(5/3)*(b*c
- a*d)^2) - (b^(5/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(
2/3)*(b*c - a*d)^2) + (d^(2/3)*(5*b*c - 2*a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3
)*x + d^(2/3)*x^2])/(18*c^(5/3)*(b*c - a*d)^2)
```

Rubi [A] time = 0.270003, antiderivative size = 346, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {414, 522, 200, 31, 634, 617, 204, 628}

$$-\frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}(bc-ad)^2}+\frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)^2}-\frac{b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(bc-ad)^2}+\frac{d^{2/3}(5bc-2ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+\right.}{18c^{5/3}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)*(c + d*x^3)^2),x]
```


```
[Out] -(d*x)/(3*c*(b*c - a*d)*(c + d*x^3)) - (b^(5/3)*ArcTan[(a^(1/3) - 2*b^(1/3)
*x)/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*(b*c - a*d)^2) + (d^(2/3)*(5*b*c -
 2*a*d)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(3*Sqrt[3]*c^(5/
3)*(b*c - a*d)^2) + (b^(5/3)*Log[a^(1/3) + b^(1/3)*x])/(3*a^(2/3)*(b*c - a*
d)^2) - (d^(2/3)*(5*b*c - 2*a*d)*Log[c^(1/3) + d^(1/3)*x])/(9*c^(5/3)*(b*c
- a*d)^2) - (b^(5/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(6*a^(
2/3)*(b*c - a*d)^2) + (d^(2/3)*(5*b*c - 2*a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3
)*x + d^(2/3)*x^2])/(18*c^(5/3)*(b*c - a*d)^2)
```


Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)\left(c+dx^3\right)^2}\,dx = -\frac{dx}{3c(bc-ad)\left(c+dx^3\right)} + \frac{\int \frac{3bc-2ad-2bdx^3}{\left(a+bx^3\right)\left(c+dx^3\right)}\,dx}{3c(bc-ad)}$$

$$= -\frac{dx}{3c(bc-ad)\left(c+dx^3\right)} + \frac{b^2\int \frac{1}{a+bx^3}\,dx}{(bc-ad)^2} - \frac{(d(5bc-2ad))\int \frac{1}{c+dx^3}\,dx}{3c(bc-ad)^2}$$

$$= -\frac{dx}{3c(bc-ad)\left(c+dx^3\right)} + \frac{b^2\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{3a^{2/3}(bc-ad)^2} + \frac{b^2\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{3a^{2/3}(bc-ad)^2} - \frac{(d(5bc-2ad))\int \frac{}{\sqrt[3]{c}-}}{9c^{5/3}(bc-ad)}$$

$$= -\frac{dx}{3c(bc-ad)\left(c+dx^3\right)} + \frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)^2} - \frac{d^{2/3}(5bc-2ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}(bc-ad)^2} - \frac{b^{5/3}\int}{6a}$$

$$= -\frac{dx}{3c(bc-ad)\left(c+dx^3\right)} + \frac{b^{5/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}(bc-ad)^2} - \frac{d^{2/3}(5bc-2ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{9c^{5/3}(bc-ad)^2} - \frac{b^{5/3}\log}{}$$

$$= -\frac{dx}{3c(bc-ad)\left(c+dx^3\right)} - \frac{b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(bc-ad)^2} + \frac{d^{2/3}(5bc-2ad)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}(bc-ad)^2} + \frac{b^{5/3}\log}{3a^2}$$

Mathematica [A] time = 0.204087, size = 336, normalized size = 0.97

$$-3b^{5/3}c^{5/3}\left(c+dx^3\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+a^{2/3}d^{2/3}\left(c+dx^3\right)(5bc-2ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)+6a^{2/3}c^{2/3}d$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^3)*(c + d*x^3)^2),x]
```

```
[Out] (6*a^(2/3)*c^(2/3)*d*(-(b*c) + a*d)*x - 6*Sqrt[3]*b^(5/3)*c^(5/3)*(c + d*x^
3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 2*Sqrt[3]*a^(2/3)*d^(2/3)*
(-5*b*c + 2*a*d)*(c + d*x^3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]] +
6*b^(5/3)*c^(5/3)*(c + d*x^3)*Log[a^(1/3) + b^(1/3)*x] + 2*a^(2/3)*d^(2/3)*
(-5*b*c + 2*a*d)*(c + d*x^3)*Log[c^(1/3) + d^(1/3)*x] - 3*b^(5/3)*c^(5/3)*(
c + d*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] + a^(2/3)*d^(2/3)
*(5*b*c - 2*a*d)*(c + d*x^3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2]
)/(18*a^(2/3)*c^(5/3)*(b*c - a*d)^2*(c + d*x^3))
```

Maple [A] time = 0.012, size = 406, normalized size = 1.2

$$\frac{b}{3\,(ad-bc)^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{b}{6\,(ad-bc)^2}\ln\left(x^2-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{b\sqrt{3}}{3\,(ad-bc)^2}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,x\frac{1}{\sqrt[3]{\frac{a}{b}}}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)/(d*x^3+c)^2,x)
```

```
[Out] 1/3*b/(a*d-b*c)^2/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))-1/6*b/(a*d-b*c)^2/(1/b*
a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))+1/3*b/(a*d-b*c)^2/(1/b*a)^(2
```

```
/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))+1/3*d^2/(a*d-b*c)^2/c
*x/(d*x^3+c)*a-1/3*d/(a*d-b*c)^2*x/(d*x^3+c)*b+2/9*d/(a*d-b*c)^2/c/(c/d)^(2
/3)*ln(x+(c/d)^(1/3))*a-5/9/(a*d-b*c)^2/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*b-1/9
*d/(a*d-b*c)^2/c/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*a+5/18/(a*d-
b*c)^2/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*b+2/9*d/(a*d-b*c)^2/c/
(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*a-5/9/(a*d-b*c)
^2/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 44.628, size = 994, normalized size = 2.87

$$\frac{6\sqrt{3}\left(bcdx^3+bc^2\right)\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)-2\sqrt{3}\left(\left(5bcd-2ad^2\right)x^3+5bc^2-2acd\right)\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}cx\left(\frac{d^2}{c^2}\right)^{\frac{2}{3}}}{3d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] 1/18*(6*sqrt(3)*(b*c*d*x^3 + b*c^2)*(b^2/a^2)^(1/3)*arctan(1/3*(2*sqrt(3)*a
*x*(b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 2*sqrt(3)*((5*b*c*d - 2*a*d^2)*x^3 + 5
*b*c^2 - 2*a*c*d)*(d^2/c^2)^(1/3)*arctan(1/3*(2*sqrt(3)*c*x*(d^2/c^2)^(2/3)
 - sqrt(3)*d)/d) - 3*(b*c*d*x^3 + b*c^2)*(b^2/a^2)^(1/3)*log(b^2*x^2 - a*b*
x*(b^2/a^2)^(1/3) + a^2*(b^2/a^2)^(2/3)) + ((5*b*c*d - 2*a*d^2)*x^3 + 5*b*c
^2 - 2*a*c*d)*(d^2/c^2)^(1/3)*log(d^2*x^2 - c*d*x*(d^2/c^2)^(1/3) + c^2*(d^
2/c^2)^(2/3)) + 6*(b*c*d*x^3 + b*c^2)*(b^2/a^2)^(1/3)*log(b*x + a*(b^2/a^2)
^(1/3)) - 2*((5*b*c*d - 2*a*d^2)*x^3 + 5*b*c^2 - 2*a*c*d)*(d^2/c^2)^(1/3)*l
og(d*x + c*(d^2/c^2)^(1/3)) - 6*(b*c*d - a*d^2)*x)/(b^2*c^4 - 2*a*b*c^3*d +
 a^2*c^2*d^2 + (b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)/(d*x**3+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12082, size = 598, normalized size = 1.73

$$-\frac{b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3\left(ab^2c^2-2a^2bcd+a^3d^2\right)}+\frac{\left(-ab^2\right)^{\frac{1}{3}}b\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}ab^2c^2-2\sqrt{3}a^2bcd+\sqrt{3}a^3d^2}+\frac{\left(-ab^2\right)^{\frac{1}{3}}b\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(ab^2c^2-2a^2bcd+a^3d^2\right)}+\frac{\left(5bcd-2a\right.}{9\left(b^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] -1/3*b^2*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a*b^2*c^2 - 2*a^2*b*c*d +
 a^3*d^2) + (-a*b^2)^(1/3)*b*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)
^(1/3))/(sqrt(3)*a*b^2*c^2 - 2*sqrt(3)*a^2*b*c*d + sqrt(3)*a^3*d^2) + 1/6*(
-a*b^2)^(1/3)*b*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a*b^2*c^2 - 2*a^2
*b*c*d + a^3*d^2) + 1/9*(5*b*c*d - 2*a*d^2)*(-c/d)^(1/3)*log(abs(x - (-c/d)
^(1/3)))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2) - 1/3*(5*(-c*d^2)^(1/3)*b*c
- 2*(-c*d^2)^(1/3)*a*d)*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)^(1/3
))/(sqrt(3)*b^2*c^4 - 2*sqrt(3)*a*b*c^3*d + sqrt(3)*a^2*c^2*d^2) - 1/18*(5*
(-c*d^2)^(1/3)*b*c - 2*(-c*d^2)^(1/3)*a*d)*log(x^2 + x*(-c/d)^(1/3) + (-c/d
)^(2/3))/(b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2) - 1/3*d*x/((d*x^3 + c)*(b*c^
2 - a*c*d))
```

3.20 $\int \frac{(c+dx^3)^5}{(a+bx^3)^2} \, dx$

Optimal. Leaf size=320

$$\frac{d^3x^4\left(3a^2d^2-10abcd+10b^2c^2\right)}{4b^4}+\frac{d^2x\left(15a^2bcd^2-4a^3d^3-20ab^2c^2d+10b^3c^3\right)}{b^5}-\frac{(bc-ad)^4(13ad+2bc)\log\left(a^{2/3}-\sqrt[3]{}\right.}{18a^{5/3}b^{16/3}}$$

```
[Out] (d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3)*x)/b^5 + (d
^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^4)/(4*b^4) + (d^4*(5*b*c - 2*a*d
)*x^7)/(7*b^3) + (d^5*x^10)/(10*b^2) + ((b*c - a*d)^5*x)/(3*a*b^5*(a + b*x^
3)) - ((b*c - a*d)^4*(2*b*c + 13*a*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[
3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(16/3)) + ((b*c - a*d)^4*(2*b*c + 13*a*d
)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(16/3)) - ((b*c - a*d)^4*(2*b*c +
13*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(16/3
))
```

Rubi [A] time = 0.298205, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {390, 385, 200, 31, 634, 617, 204, 628}

$$\frac{d^3x^4\left(3a^2d^2-10abcd+10b^2c^2\right)}{4b^4}+\frac{d^2x\left(15a^2bcd^2-4a^3d^3-20ab^2c^2d+10b^3c^3\right)}{b^5}-\frac{(bc-ad)^4(13ad+2bc)\log\left(a^{2/3}-\sqrt[3]{}\right.}{18a^{5/3}b^{16/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^5/(a + b*x^3)^2,x]
```


```
[Out] (d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3)*x)/b^5 + (d
^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^4)/(4*b^4) + (d^4*(5*b*c - 2*a*d
)*x^7)/(7*b^3) + (d^5*x^10)/(10*b^2) + ((b*c - a*d)^5*x)/(3*a*b^5*(a + b*x^
3)) - ((b*c - a*d)^4*(2*b*c + 13*a*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[
3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(16/3)) + ((b*c - a*d)^4*(2*b*c + 13*a*d
)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(16/3)) - ((b*c - a*d)^4*(2*b*c +
13*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(16/3
))
```


Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^5}{\left(a+bx^3\right)^2}\,dx = \int \left(\frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)}{b^5} + \frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^3}{b^4} + \frac{d^4(5bc-2ad}{b^3}\right.$$

$$= \frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)x}{b^5} + \frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^4}{4b^4} + \frac{d^4(5bc-2ad)x}{7b^3}$$

$$= \frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)x}{b^5} + \frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^4}{4b^4} + \frac{d^4(5bc-2ad)x}{7b^3}$$

$$= \frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)x}{b^5} + \frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^4}{4b^4} + \frac{d^4(5bc-2ad)x}{7b^3}$$

$$= \frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)x}{b^5} + \frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^4}{4b^4} + \frac{d^4(5bc-2ad)x}{7b^3}$$

$$= \frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)x}{b^5} + \frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^4}{4b^4} + \frac{d^4(5bc-2ad)x}{7b^3}$$

$$= \frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)x}{b^5} + \frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^4}{4b^4} + \frac{d^4(5bc-2ad)x}{7b^3}$$

Mathematica [A] time = 0.252526, size = 313, normalized size = 0.98

$$315b^{4/3}d^3x^4\left(3a^2d^2-10abcd+10b^2c^2\right)+1260\sqrt[3]{b}d^2x\left(15a^2bcd^2-4a^3d^3-20ab^2c^2d+10b^3c^3\right)-\frac{70(bc-ad)^4(13ad+2bc)\log(}{a^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^5/(a + b*x^3)^2,x]
```

```
[Out] (1260*b^(1/3)*d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3
)*x + 315*b^(4/3)*d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^4 + 180*b^(7/
3)*d^4*(5*b*c - 2*a*d)*x^7 + 126*b^(10/3)*d^5*x^10 + (420*b^(1/3)*(b*c - a*
d)^5*x)/(a*(a + b*x^3)) + (140*Sqrt[3]*(b*c - a*d)^4*(2*b*c + 13*a*d)*ArcTa
n[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/a^(5/3) + (140*(b*c - a*d)^4
*(2*b*c + 13*a*d)*Log[a^(1/3) + b^(1/3)*x])/a^(5/3) - (70*(b*c - a*d)^4*(2*
b*c + 13*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3))/(126
0*b^(16/3))
```

Maple [B] time = 0.013, size = 905, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^5/(b*x^3+a)^2,x)
```

```
[Out] 5/9/b^2/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*c^4*d-13/18/b^6*a^4/(1/b*a)^(2/3)
*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d^5-20*d^3/b^3*a*c^2*x+13/9/b^6*a^4/
```

```
(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*d^5-5/3/b*x/(b*x^3+a)*c^4*d-50/9/b^5*a^3/
(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c*d^4+70/9/
b^4*a^2/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c^2
*d^3-40/9/b^3*a/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x
-1))*c^3*d^2-50/9/b^5*a^3/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*c*d^4+70/9/b^4*
a^2/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*c^2*d^3-40/9/b^3*a/(1/b*a)^(2/3)*ln(x
+(1/b*a)^(1/3))*c^3*d^2+25/9/b^5*a^3/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(
1/b*a)^(2/3))*c*d^4-35/9/b^4*a^2/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*
a)^(2/3))*c^2*d^3+20/9/b^3*a/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(
2/3))*c^3*d^2+13/9/b^6*a^4/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b
*a)^(1/3)*x-1))*d^5+5/9/b^2/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/
b*a)^(1/3)*x-1))*c^4*d+2/9/b/a/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/
(1/b*a)^(1/3)*x-1))*c^5+5/3/b^4*a^3*x/(b*x^3+a)*c*d^4-10/3/b^3*a^2*x/(b*x^3
+a)*c^2*d^3+10/3/b^2*a*x/(b*x^3+a)*c^3*d^2+1/10*d^5*x^10/b^2-5/18/b^2/(1/b*
a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c^4*d-1/9/b/a/(1/b*a)^(2/3)*
ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c^5+1/3/a*x/(b*x^3+a)*c^5+5/2*d^3/b^2
*x^4*c^2-4*d^5/b^5*a^3*x+10*d^2/b^2*c^3*x-2/7*d^5/b^3*x^7*a+5/7*d^4/b^2*x^7
*c+3/4*d^5/b^4*x^4*a^2-5/2*d^4/b^3*x^4*a*c+15*d^4/b^4*a^2*c*x-1/3/b^5*a^4*x
/(b*x^3+a)*d^5+2/9/b/a/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*c^5
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^5/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.81118, size = 3490, normalized size = 10.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^5/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/1260*(126*a^3*b^5*d^5*x^13 + 18*(50*a^3*b^5*c*d^4 - 13*a^4*b^4*d^5)*x^10
 + 45*(70*a^3*b^5*c^2*d^3 - 50*a^4*b^4*c*d^4 + 13*a^5*b^3*d^5)*x^7 + 315*(4
0*a^3*b^5*c^3*d^2 - 70*a^4*b^4*c^2*d^3 + 50*a^5*b^3*c*d^4 - 13*a^6*b^2*d^5)
*x^4 + 210*sqrt(1/3)*(2*a^2*b^6*c^5 + 5*a^3*b^5*c^4*d - 40*a^4*b^4*c^3*d^2
+ 70*a^5*b^3*c^2*d^3 - 50*a^6*b^2*c*d^4 + 13*a^7*b*d^5 + (2*a*b^7*c^5 + 5*a
^2*b^6*c^4*d - 40*a^3*b^5*c^3*d^2 + 70*a^4*b^4*c^2*d^3 - 50*a^5*b^3*c*d^4 +
 13*a^6*b^2*d^5)*x^3)*sqrt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/
3)*a*x - a^2 + 3*sqrt(1/3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*
sqrt(-(a^2*b)^(1/3)/b))/(b*x^3 + a)) - 70*(2*a*b^5*c^5 + 5*a^2*b^4*c^4*d -
40*a^3*b^3*c^3*d^2 + 70*a^4*b^2*c^2*d^3 - 50*a^5*b*c*d^4 + 13*a^6*d^5 + (2*
b^6*c^5 + 5*a*b^5*c^4*d - 40*a^2*b^4*c^3*d^2 + 70*a^3*b^3*c^2*d^3 - 50*a^4*
b^2*c*d^4 + 13*a^5*b*d^5)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x
+ (a^2*b)^(1/3)*a) + 140*(2*a*b^5*c^5 + 5*a^2*b^4*c^4*d - 40*a^3*b^3*c^3*d^
2 + 70*a^4*b^2*c^2*d^3 - 50*a^5*b*c*d^4 + 13*a^6*d^5 + (2*b^6*c^5 + 5*a*b^5
*c^4*d - 40*a^2*b^4*c^3*d^2 + 70*a^3*b^3*c^2*d^3 - 50*a^4*b^2*c*d^4 + 13*a^
5*b*d^5)*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 420*(a^2*b^6*c^5 -
 5*a^3*b^5*c^4*d + 40*a^4*b^4*c^3*d^2 - 70*a^5*b^3*c^2*d^3 + 50*a^6*b^2*c*d
```

```
^4 - 13*a^7*b*d^5)*x)/(a^3*b^7*x^3 + a^4*b^6), 1/1260*(126*a^3*b^5*d^5*x^13
 + 18*(50*a^3*b^5*c*d^4 - 13*a^4*b^4*d^5)*x^10 + 45*(70*a^3*b^5*c^2*d^3 - 5
0*a^4*b^4*c*d^4 + 13*a^5*b^3*d^5)*x^7 + 315*(40*a^3*b^5*c^3*d^2 - 70*a^4*b^
4*c^2*d^3 + 50*a^5*b^3*c*d^4 - 13*a^6*b^2*d^5)*x^4 + 420*sqrt(1/3)*(2*a^2*b
^6*c^5 + 5*a^3*b^5*c^4*d - 40*a^4*b^4*c^3*d^2 + 70*a^5*b^3*c^2*d^3 - 50*a^6
*b^2*c*d^4 + 13*a^7*b*d^5 + (2*a*b^7*c^5 + 5*a^2*b^6*c^4*d - 40*a^3*b^5*c^3
*d^2 + 70*a^4*b^4*c^2*d^3 - 50*a^5*b^3*c*d^4 + 13*a^6*b^2*d^5)*x^3)*sqrt((a
^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(
(a^2*b)^(1/3)/b)/a^2) - 70*(2*a*b^5*c^5 + 5*a^2*b^4*c^4*d - 40*a^3*b^3*c^3*
d^2 + 70*a^4*b^2*c^2*d^3 - 50*a^5*b*c*d^4 + 13*a^6*d^5 + (2*b^6*c^5 + 5*a*b
^5*c^4*d - 40*a^2*b^4*c^3*d^2 + 70*a^3*b^3*c^2*d^3 - 50*a^4*b^2*c*d^4 + 13*
a^5*b*d^5)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)
*a) + 140*(2*a*b^5*c^5 + 5*a^2*b^4*c^4*d - 40*a^3*b^3*c^3*d^2 + 70*a^4*b^2*
c^2*d^3 - 50*a^5*b*c*d^4 + 13*a^6*d^5 + (2*b^6*c^5 + 5*a*b^5*c^4*d - 40*a^2
*b^4*c^3*d^2 + 70*a^3*b^3*c^2*d^3 - 50*a^4*b^2*c*d^4 + 13*a^5*b*d^5)*x^3)*(
a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 420*(a^2*b^6*c^5 - 5*a^3*b^5*c^4*
d + 40*a^4*b^4*c^3*d^2 - 70*a^5*b^3*c^2*d^3 + 50*a^6*b^2*c*d^4 - 13*a^7*b*d
^5)*x)/(a^3*b^7*x^3 + a^4*b^6)]
```

Sympy [A] time = 15.5391, size = 536, normalized size = 1.68

$$-\frac{x\left(a^5d^5-5a^4bcd^4+10a^3b^2c^2d^3-10a^2b^3c^3d^2+5ab^4c^4d-b^5c^5\right)}{3a^2b^5+3ab^6x^3}+\operatorname{RootSum}\left(729t^3a^5b^{16}-2197a^{15}d^{15}+25350a^{14}b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**5/(b*x**3+a)**2,x)
```

```
[Out] -x*(a**5*d**5 - 5*a**4*b*c*d**4 + 10*a**3*b**2*c**2*d**3 - 10*a**2*b**3*c**
3*d**2 + 5*a*b**4*c**4*d - b**5*c**5)/(3*a**2*b**5 + 3*a*b**6*x**3) + RootS
um(729*_t**3*a**5*b**16 - 2197*a**15*d**15 + 25350*a**14*b*c*d**14 - 132990
*a**13*b**2*c**2*d**13 + 418280*a**12*b**3*c**3*d**12 - 874635*a**11*b**4*c
**4*d**11 + 1271886*a**10*b**5*c**5*d**10 - 1302400*a**9*b**6*c**6*d**9 + 9
22680*a**8*b**7*c**7*d**8 - 422235*a**7*b**8*c**8*d**7 + 97570*a**6*b**9*c*
*9*d**6 + 7194*a**5*b**10*c**10*d**5 - 10200*a**4*b**11*c**11*d**4 + 1435*a
**3*b**12*c**12*d**3 + 330*a**2*b**13*c**13*d**2 - 60*a*b**14*c**14*d - 8*b
**15*c**15, Lambda(_t, _t*log(9*_t*a**2*b**5/(13*a**5*d**5 - 50*a**4*b*c*d*
*4 + 70*a**3*b**2*c**2*d**3 - 40*a**2*b**3*c**3*d**2 + 5*a*b**4*c**4*d + 2*
b**5*c**5) + x))) + d**5*x**10/(10*b**2) - x**7*(2*a*d**5 - 5*b*c*d**4)/(7*
b**3) + x**4*(3*a**2*d**5 - 10*a*b*c*d**4 + 10*b**2*c**2*d**3)/(4*b**4) - x
*(4*a**3*d**5 - 15*a**2*b*c*d**4 + 20*a*b**2*c**2*d**3 - 10*b**3*c**3*d**2)
/b**5
```

Giac [B] time = 1.11113, size = 822, normalized size = 2.57

$$-\frac{\left(2\,b^5c^5+5\,ab^4c^4d-40\,a^2b^3c^3d^2+70\,a^3b^2c^2d^3-50\,a^4bcd^4+13\,a^5d^5\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b^5}+\frac{\sqrt{3}\left(2\,\left(-ab^2\right)^{\frac{1}{3}}b^5c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^5/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*(2*b^5*c^5 + 5*a*b^4*c^4*d - 40*a^2*b^3*c^3*d^2 + 70*a^3*b^2*c^2*d^3 -
 50*a^4*b*c*d^4 + 13*a^5*d^5)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*
b^5) + 1/9*sqrt(3)*(2*(-a*b^2)^(1/3)*b^5*c^5 + 5*(-a*b^2)^(1/3)*a*b^4*c^4*d
 - 40*(-a*b^2)^(1/3)*a^2*b^3*c^3*d^2 + 70*(-a*b^2)^(1/3)*a^3*b^2*c^2*d^3 -
50*(-a*b^2)^(1/3)*a^4*b*c*d^4 + 13*(-a*b^2)^(1/3)*a^5*d^5)*arctan(1/3*sqrt(
3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^6) + 1/3*(b^5*c^5*x - 5*a*b^4*
c^4*d*x + 10*a^2*b^3*c^3*d^2*x - 10*a^3*b^2*c^2*d^3*x + 5*a^4*b*c*d^4*x - a
^5*d^5*x)/((b*x^3 + a)*a*b^5) + 1/18*(2*(-a*b^2)^(1/3)*b^5*c^5 + 5*(-a*b^2)
^(1/3)*a*b^4*c^4*d - 40*(-a*b^2)^(1/3)*a^2*b^3*c^3*d^2 + 70*(-a*b^2)^(1/3)*
a^3*b^2*c^2*d^3 - 50*(-a*b^2)^(1/3)*a^4*b*c*d^4 + 13*(-a*b^2)^(1/3)*a^5*d^5
)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^6) + 1/140*(14*b^18*d^5*x
^10 + 100*b^18*c*d^4*x^7 - 40*a*b^17*d^5*x^7 + 350*b^18*c^2*d^3*x^4 - 350*a
*b^17*c*d^4*x^4 + 105*a^2*b^16*d^5*x^4 + 1400*b^18*c^3*d^2*x - 2800*a*b^17*
c^2*d^3*x + 2100*a^2*b^16*c*d^4*x - 560*a^3*b^15*d^5*x)/b^20
```

3.21 $\int \frac{\left(c+dx^3\right)^4}{\left(a+bx^3\right)^2}\,dx$

Optimal. Leaf size=267

$$\frac{d^2x\left(3a^2d^2-8abcd+6b^2c^2\right)}{b^4}-\frac{(bc-ad)^3(5ad+bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{5/3}b^{13/3}}+\frac{2(bc-ad)^3(5ad+bc)\log\left(\sqrt[3]{a}+\sqrt[3]{\right.}}{9a^{5/3}b^{13/3}}$$

```
[Out] (d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x)/b^4 + (d^3*(2*b*c - a*d)*x^4)/(
2*b^3) + (d^4*x^7)/(7*b^2) + ((b*c - a*d)^4*x)/(3*a*b^4*(a + b*x^3)) - (2*(
b*c - a*d)^3*(b*c + 5*a*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))
])/(3*Sqrt[3]*a^(5/3)*b^(13/3)) + (2*(b*c - a*d)^3*(b*c + 5*a*d)*Log[a^(1/3
) + b^(1/3)*x])/(9*a^(5/3)*b^(13/3)) - ((b*c - a*d)^3*(b*c + 5*a*d)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(9*a^(5/3)*b^(13/3))
```

Rubi [A] time = 0.226458, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {390, 385, 200, 31, 634, 617, 204, 628}

$$\frac{d^2x\left(3a^2d^2-8abcd+6b^2c^2\right)}{b^4}-\frac{(bc-ad)^3(5ad+bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{5/3}b^{13/3}}+\frac{2(bc-ad)^3(5ad+bc)\log\left(\sqrt[3]{a}+\sqrt[3]{\right.}}{9a^{5/3}b^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^4/(a + b*x^3)^2,x]
```

```
[Out] (d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x)/b^4 + (d^3*(2*b*c - a*d)*x^4)/(
2*b^3) + (d^4*x^7)/(7*b^2) + ((b*c - a*d)^4*x)/(3*a*b^4*(a + b*x^3)) - (2*(
b*c - a*d)^3*(b*c + 5*a*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))
])/(3*Sqrt[3]*a^(5/3)*b^(13/3)) + (2*(b*c - a*d)^3*(b*c + 5*a*d)*Log[a^(1/3
) + b^(1/3)*x])/(9*a^(5/3)*b^(13/3)) - ((b*c - a*d)^3*(b*c + 5*a*d)*Log[a^(
2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(9*a^(5/3)*b^(13/3))
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

```
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(c+dx^3)^4}{(a+bx^3)^2}\,dx = \int \left(\frac{d^2(6b^2c^2-8abcd+3a^2d^2)}{b^4} + \frac{2d^3(2bc-ad)x^3}{b^3} + \frac{d^4x^6}{b^2} + \frac{(bc-ad)^3(bc+3ad)+4bd(bc-ad)^3}{b^4(a+bx^3)^2}\right.$$

$$= \frac{d^2(6b^2c^2-8abcd+3a^2d^2)x}{b^4} + \frac{d^3(2bc-ad)x^4}{2b^3} + \frac{d^4x^7}{7b^2} + \frac{\int \frac{(bc-ad)^3(bc+3ad)+4bd(bc-ad)^3x^3}{(a+bx^3)^2}\,dx}{b^4}$$

$$= \frac{d^2(6b^2c^2-8abcd+3a^2d^2)x}{b^4} + \frac{d^3(2bc-ad)x^4}{2b^3} + \frac{d^4x^7}{7b^2} + \frac{(bc-ad)^4x}{3ab^4(a+bx^3)} + \frac{(2(bc-ad)^3(bc+5ad)}{3ab^4}$$

$$= \frac{d^2(6b^2c^2-8abcd+3a^2d^2)x}{b^4} + \frac{d^3(2bc-ad)x^4}{2b^3} + \frac{d^4x^7}{7b^2} + \frac{(bc-ad)^4x}{3ab^4(a+bx^3)} + \frac{(2(bc-ad)^3(bc+5ad)}{9a^{5/3}b^4}$$

$$= \frac{d^2(6b^2c^2-8abcd+3a^2d^2)x}{b^4} + \frac{d^3(2bc-ad)x^4}{2b^3} + \frac{d^4x^7}{7b^2} + \frac{(bc-ad)^4x}{3ab^4(a+bx^3)} + \frac{2(bc-ad)^3(bc+5ad)}{9a^{5/3}b^{13}}$$

$$= \frac{d^2(6b^2c^2-8abcd+3a^2d^2)x}{b^4} + \frac{d^3(2bc-ad)x^4}{2b^3} + \frac{d^4x^7}{7b^2} + \frac{(bc-ad)^4x}{3ab^4(a+bx^3)} + \frac{2(bc-ad)^3(bc+5ad)}{9a^{5/3}b^{13}}$$

$$= \frac{d^2(6b^2c^2-8abcd+3a^2d^2)x}{b^4} + \frac{d^3(2bc-ad)x^4}{2b^3} + \frac{d^4x^7}{7b^2} + \frac{(bc-ad)^4x}{3ab^4(a+bx^3)} - \frac{2(bc-ad)^3(bc+5ad)}{3\sqrt{3}a^{5/3}b}$$

Mathematica [A] time = 0.216735, size = 260, normalized size = 0.97

$$\frac{126\sqrt[3]{b}d^2x(3a^2d^2-8abcd+6b^2c^2) + \frac{14(ad-bc)^3(5ad+bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{5/3}} + \frac{28(bc-ad)^3(5ad+bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{5/3}} + \frac{28\sqrt{3}(bc-ad)^3(}{}}{126b^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^4/(a + b*x^3)^2,x]
```

```
[Out] (126*b^(1/3)*d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x + 63*b^(4/3)*d^3*(2*
b*c - a*d)*x^4 + 18*b^(7/3)*d^4*x^7 + (42*b^(1/3)*(b*c - a*d)^4*x)/(a*(a +
b*x^3)) + (28*Sqrt[3]*(b*c - a*d)^3*(b*c + 5*a*d)*ArcTan[(-a^(1/3) + 2*b^(1
/3)*x)/(Sqrt[3]*a^(1/3))])/a^(5/3) + (28*(b*c - a*d)^3*(b*c + 5*a*d)*Log[a^
(1/3) + b^(1/3)*x])/a^(5/3) + (14*(-(b*c) + a*d)^3*(b*c + 5*a*d)*Log[a^(2/3
) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3))/(126*b^(13/3))
```

Maple [B] time = 0.011, size = 708, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^4/(b*x^3+a)^2,x)
```

```
[Out] 2/b^2*a*x/(b*x^3+a)*c^2*d^2-14/9/b^4*a^2/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)
*x+(1/b*a)^(2/3))*c*d^3+4/3/b^3*a/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b
*a)^(2/3))*c^2*d^2-10/9/b^5*a^3/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2
/(1/b*a)^(1/3)*x-1))*d^4+4/9/b^2/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(
```

```
2/(1/b*a)^(1/3)*x-1))*c^3*d+2/9/b/a/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2
)*(2/(1/b*a)^(1/3)*x-1))*c^4+28/9/b^4*a^2/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))
*c*d^3-8/3/b^3*a/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*c^2*d^2-4/3/b^3*a^2*x/(b
*x^3+a)*c*d^3+28/9/b^4*a^2/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b
*a)^(1/3)*x-1))*c*d^3-8/3/b^3*a/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2
/(1/b*a)^(1/3)*x-1))*c^2*d^2-8*d^3/b^3*c*a*x-2/9/b^2/(1/b*a)^(2/3)*ln(x^2-(
1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c^3*d-1/9/b/a/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1
/3)*x+(1/b*a)^(2/3))*c^4-10/9/b^5*a^3/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*d^4
+4/9/b^2/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*c^3*d+2/9/b/a/(1/b*a)^(2/3)*ln(x
+(1/b*a)^(1/3))*c^4+5/9/b^5*a^3/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a
)^(2/3))*d^4+1/7*d^4*x^7/b^2+1/3/b^4*a^3*x/(b*x^3+a)*d^4-4/3/b*x/(b*x^3+a)*
c^3*d+1/3/a*x/(b*x^3+a)*c^4-1/2*d^4/b^3*x^4*a+d^3/b^2*x^4*c+3*d^4/b^4*a^2*x
+6*d^2/b^2*c^2*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^4/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.89248, size = 2808, normalized size = 10.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^4/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/126*(18*a^3*b^4*d^4*x^10 + 9*(14*a^3*b^4*c*d^3 - 5*a^4*b^3*d^4)*x^7 + 63
*(12*a^3*b^4*c^2*d^2 - 14*a^4*b^3*c*d^3 + 5*a^5*b^2*d^4)*x^4 - 42*sqrt(1/3)
*(a^2*b^5*c^4 + 2*a^3*b^4*c^3*d - 12*a^4*b^3*c^2*d^2 + 14*a^5*b^2*c*d^3 - 5
*a^6*b*d^4 + (a*b^6*c^4 + 2*a^2*b^5*c^3*d - 12*a^3*b^4*c^2*d^2 + 14*a^4*b^3
*c*d^3 - 5*a^5*b^2*d^4)*x^3)*sqrt((-a^2*b)^(1/3)/b)*log((2*a*b*x^3 + 3*(-a^
2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a^2*b)^(2/3)*x + (-a^2*b)
^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) - 14*(a*b^4*c^4 + 2*a^2*b^3*
c^3*d - 12*a^3*b^2*c^2*d^2 + 14*a^4*b*c*d^3 - 5*a^5*d^4 + (b^5*c^4 + 2*a*b^
4*c^3*d - 12*a^2*b^3*c^2*d^2 + 14*a^3*b^2*c*d^3 - 5*a^4*b*d^4)*x^3)*(-a^2*b
)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 28*(a*b^4*c^4
+ 2*a^2*b^3*c^3*d - 12*a^3*b^2*c^2*d^2 + 14*a^4*b*c*d^3 - 5*a^5*d^4 + (b^5*
c^4 + 2*a*b^4*c^3*d - 12*a^2*b^3*c^2*d^2 + 14*a^3*b^2*c*d^3 - 5*a^4*b*d^4)*
x^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)) + 42*(a^2*b^5*c^4 - 4*a^3*b
^4*c^3*d + 24*a^4*b^3*c^2*d^2 - 28*a^5*b^2*c*d^3 + 10*a^6*b*d^4)*x)/(a^3*b^
6*x^3 + a^4*b^5), 1/126*(18*a^3*b^4*d^4*x^10 + 9*(14*a^3*b^4*c*d^3 - 5*a^4*
b^3*d^4)*x^7 + 63*(12*a^3*b^4*c^2*d^2 - 14*a^4*b^3*c*d^3 + 5*a^5*b^2*d^4)*x
^4 + 84*sqrt(1/3)*(a^2*b^5*c^4 + 2*a^3*b^4*c^3*d - 12*a^4*b^3*c^2*d^2 + 14*
a^5*b^2*c*d^3 - 5*a^6*b*d^4 + (a*b^6*c^4 + 2*a^2*b^5*c^3*d - 12*a^3*b^4*c^2
*d^2 + 14*a^4*b^3*c*d^3 - 5*a^5*b^2*d^4)*x^3)*sqrt(-(-a^2*b)^(1/3)/b)*arcta
n(sqrt(1/3)*(2*(-a^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3)/b)
/a^2) - 14*(a*b^4*c^4 + 2*a^2*b^3*c^3*d - 12*a^3*b^2*c^2*d^2 + 14*a^4*b*c*d
^3 - 5*a^5*d^4 + (b^5*c^4 + 2*a*b^4*c^3*d - 12*a^2*b^3*c^2*d^2 + 14*a^3*b^2
*c*d^3 - 5*a^4*b*d^4)*x^3)*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x -
```

```
(-a^2*b)^(1/3)*a) + 28*(a*b^4*c^4 + 2*a^2*b^3*c^3*d - 12*a^3*b^2*c^2*d^2 +
14*a^4*b*c*d^3 - 5*a^5*d^4 + (b^5*c^4 + 2*a*b^4*c^3*d - 12*a^2*b^3*c^2*d^2
+ 14*a^3*b^2*c*d^3 - 5*a^4*b*d^4)*x^3)*(-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^
(2/3)) + 42*(a^2*b^5*c^4 - 4*a^3*b^4*c^3*d + 24*a^4*b^3*c^2*d^2 - 28*a^5*b^
2*c*d^3 + 10*a^6*b*d^4)*x)/(a^3*b^6*x^3 + a^4*b^5)]
```

Sympy [A] time = 7.67771, size = 403, normalized size = 1.51

$$\frac{x\left(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4\right)}{3a^2b^4+3ab^5x^3}+\operatorname{RootSum}\left(729t^3a^5b^{13}+1000a^{12}d^{12}-8400a^{11}bcd^{11}+30720a^{10}b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**4/(b*x**3+a)**2,x)
```

```
[Out] x*(a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*b**2*c**2*d**2 - 4*a*b**3*c**3*d +
b**4*c**4)/(3*a**2*b**4 + 3*a*b**5*x**3) + RootSum(729*_t**3*a**5*b**13 + 1
000*a**12*d**12 - 8400*a**11*b*c*d**11 + 30720*a**10*b**2*c**2*d**10 - 6347
2*a**9*b**3*c**3*d**9 + 79848*a**8*b**4*c**4*d**8 - 60192*a**7*b**5*c**5*d*
*7 + 22848*a**6*b**6*c**6*d**6 + 288*a**5*b**7*c**7*d**5 - 3528*a**4*b**8*c
**8*d**4 + 752*a**3*b**9*c**9*d**3 + 192*a**2*b**10*c**10*d**2 - 48*a*b**11
*c**11*d - 8*b**12*c**12, Lambda(_t, _t*log(-9*_t*a**2*b**4/(10*a**4*d**4 -
 28*a**3*b*c*d**3 + 24*a**2*b**2*c**2*d**2 - 4*a*b**3*c**3*d - 2*b**4*c**4)
 + x))) + d**4*x**7/(7*b**2) - x**4*(a*d**4 - 2*b*c*d**3)/(2*b**3) + x*(3*a
**2*d**4 - 8*a*b*c*d**3 + 6*b**2*c**2*d**2)/b**4
```

Giac [B] time = 1.14312, size = 643, normalized size = 2.41

$$-\frac{2\left(b^4c^4+2\,ab^3c^3d-12\,a^2b^2c^2d^2+14\,a^3bcd^3-5\,a^4d^4\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b^4}+\frac{2\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^4c^4+2\left(-ab^2\right)^{\frac{1}{3}}ab\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^4/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -2/9*(b^4*c^4 + 2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 14*a^3*b*c*d^3 - 5*a^4
*d^4)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b^4) + 2/9*sqrt(3)*((-a*
b^2)^(1/3)*b^4*c^4 + 2*(-a*b^2)^(1/3)*a*b^3*c^3*d - 12*(-a*b^2)^(1/3)*a^2*b
^2*c^2*d^2 + 14*(-a*b^2)^(1/3)*a^3*b*c*d^3 - 5*(-a*b^2)^(1/3)*a^4*d^4)*arct
an(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^5) + 1/3*(b^4*c^4*
x - 4*a*b^3*c^3*d*x + 6*a^2*b^2*c^2*d^2*x - 4*a^3*b*c*d^3*x + a^4*d^4*x)/((
b*x^3 + a)*a*b^4) + 1/9*((-a*b^2)^(1/3)*b^4*c^4 + 2*(-a*b^2)^(1/3)*a*b^3*c^
3*d - 12*(-a*b^2)^(1/3)*a^2*b^2*c^2*d^2 + 14*(-a*b^2)^(1/3)*a^3*b*c*d^3 - 5
*(-a*b^2)^(1/3)*a^4*d^4)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^5)
 + 1/14*(2*b^12*d^4*x^7 + 14*b^12*c*d^3*x^4 - 7*a*b^11*d^4*x^4 + 84*b^12*c^
2*d^2*x - 112*a*b^11*c*d^3*x + 42*a^2*b^10*d^4*x)/b^14
```

3.22 $\int \frac{(c+dx^3)^3}{(a+bx^3)^2} \, dx$

Optimal. Leaf size=234

$$-\frac{(bc-ad)^2(7ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{10/3}}+\frac{(bc-ad)^2(7ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{10/3}}-\frac{(bc-ad)^2(7ad+2bc)\tan}{3\sqrt{3}a^{5/3}b^{10/}}$$

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^4)/(4*b^2) + ((b*c - a*d)^3*x)/(3*a*b^
3*(a + b*x^3)) - ((b*c - a*d)^2*(2*b*c + 7*a*d)*ArcTan[(a^(1/3) - 2*b^(1/3)
*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(10/3)) + ((b*c - a*d)^2*(2*b*
c + 7*a*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(10/3)) - ((b*c - a*d)^2*
(2*b*c + 7*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)
*b^(10/3))
```

Rubi [A] time = 0.219666, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {390, 385, 200, 31, 634, 617, 204, 628}

$$-\frac{(bc-ad)^2(7ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{10/3}}+\frac{(bc-ad)^2(7ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{10/3}}-\frac{(bc-ad)^2(7ad+2bc)\tan}{3\sqrt{3}a^{5/3}b^{10/}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^3/(a + b*x^3)^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^4)/(4*b^2) + ((b*c - a*d)^3*x)/(3*a*b^
3*(a + b*x^3)) - ((b*c - a*d)^2*(2*b*c + 7*a*d)*ArcTan[(a^(1/3) - 2*b^(1/3)
*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(10/3)) + ((b*c - a*d)^2*(2*b*
c + 7*a*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(10/3)) - ((b*c - a*d)^2*
(2*b*c + 7*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)
*b^(10/3))
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

```
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(c+dx^3)^3}{(a+bx^3)^2}\,dx = \int \left(\frac{d^2(3bc-2ad)}{b^3}+\frac{d^3x^3}{b^2}+\frac{(bc-ad)^2(bc+2ad)+3bd(bc-ad)^2x^3}{b^3(a+bx^3)^2}\right)dx$$

$$= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^4}{4b^2}+\frac{\int \frac{(bc-ad)^2(bc+2ad)+3bd(bc-ad)^2x^3}{(a+bx^3)^2}\,dx}{b^3}$$

$$= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^4}{4b^2}+\frac{(bc-ad)^3x}{3ab^3(a+bx^3)}+\frac{\left((bc-ad)^2(2bc+7ad)\right)\int \frac{1}{a+bx^3}\,dx}{3ab^3}$$

$$= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^4}{4b^2}+\frac{(bc-ad)^3x}{3ab^3(a+bx^3)}+\frac{\left((bc-ad)^2(2bc+7ad)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{5/3}b^3}+\frac{\left((bc-ad)^2(2bc+\right.}{}$$

$$= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^4}{4b^2}+\frac{(bc-ad)^3x}{3ab^3(a+bx^3)}+\frac{(bc-ad)^2(2bc+7ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{10/3}}-\frac{\left((bc-ad)^2(2bc\right.}{}$$

$$= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^4}{4b^2}+\frac{(bc-ad)^3x}{3ab^3(a+bx^3)}+\frac{(bc-ad)^2(2bc+7ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{10/3}}-\frac{(bc-ad)^2(2bc+}{}$$

$$= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^4}{4b^2}+\frac{(bc-ad)^3x}{3ab^3(a+bx^3)}-\frac{(bc-ad)^2(2bc+7ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{10/3}}+\frac{(bc-ad)^2(2bc}{9}$$

Mathematica [A] time = 0.15917, size = 227, normalized size = 0.97

$$\frac{-\frac{2(bc-ad)^2(7ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{5/3}}+\frac{4(bc-ad)^2(7ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{5/3}}+\frac{4\sqrt{3}(bc-ad)^2(7ad+2bc)\tan^{-1}\left(\frac{2\sqrt[3]{b}x-\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{5/3}}+36\sqrt[3]{b}d^2x(3bc-2}{36b^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^3/(a + b*x^3)^2,x]
```

```
[Out] (36*b^(1/3)*d^2*(3*b*c - 2*a*d)*x + 9*b^(4/3)*d^3*x^4 + (12*b^(1/3)*(b*c -
a*d)^3*x)/(a*(a + b*x^3)) + (4*Sqrt[3]*(b*c - a*d)^2*(2*b*c + 7*a*d)*ArcTan
[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/a^(5/3) + (4*(b*c - a*d)^2*(2
*b*c + 7*a*d)*Log[a^(1/3) + b^(1/3)*x])/a^(5/3) - (2*(b*c - a*d)^2*(2*b*c +
 7*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/a^(5/3))/(36*b^(10/
3))
```

Maple [B] time = 0.009, size = 529, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^3/(b*x^3+a)^2,x)
```

```
[Out] 1/4*d^3*x^4/b^2-2*d^3/b^3*a*x+3*d^2/b^2*x*c-1/3/b^3*a^2*x/(b*x^3+a)*d^3+1/b
^2*a*x/(b*x^3+a)*c*d^2-1/b*x/(b*x^3+a)*c^2*d+1/3/a*x/(b*x^3+a)*c^3+7/9/b^4*
a^2/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*d^3-4/3/b^3*a/(1/b*a)^(2/3)*ln(x+(1/b
*a)^(1/3))*c*d^2+1/3/b^2/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*c^2*d+2/9/b/a/(1
```

```
/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*c^3-7/18/b^4*a^2/(1/b*a)^(2/3)*ln(x^2-(1/b*
a)^(1/3)*x+(1/b*a)^(2/3))*d^3+2/3/b^3*a/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*
x+(1/b*a)^(2/3))*c*d^2-1/6/b^2/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)
^(2/3))*c^2*d-1/9/b/a/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c
^3+7/9/b^4*a^2/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-
1))*d^3-4/3/b^3*a/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)
*x-1))*c*d^2+1/3/b^2/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1
/3)*x-1))*c^2*d+2/9/b/a/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)
^(1/3)*x-1))*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^3/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.71334, size = 2225, normalized size = 9.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^3/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/36*(9*a^3*b^3*d^3*x^7 + 9*(12*a^3*b^3*c*d^2 - 7*a^4*b^2*d^3)*x^4 + 6*sqr
t(1/3)*(2*a^2*b^4*c^3 + 3*a^3*b^3*c^2*d - 12*a^4*b^2*c*d^2 + 7*a^5*b*d^3 +
(2*a*b^5*c^3 + 3*a^2*b^4*c^2*d - 12*a^3*b^3*c*d^2 + 7*a^4*b^2*d^3)*x^3)*sqr
t(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/3
)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/(
b*x^3 + a)) - 2*(2*a*b^3*c^3 + 3*a^2*b^2*c^2*d - 12*a^3*b*c*d^2 + 7*a^4*d^3
 + (2*b^4*c^3 + 3*a*b^3*c^2*d - 12*a^2*b^2*c*d^2 + 7*a^3*b*d^3)*x^3)*(a^2*b
)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 4*(2*a*b^3*c^3 +
 3*a^2*b^2*c^2*d - 12*a^3*b*c*d^2 + 7*a^4*d^3 + (2*b^4*c^3 + 3*a*b^3*c^2*d
- 12*a^2*b^2*c*d^2 + 7*a^3*b*d^3)*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2
/3)) + 12*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 12*a^4*b^2*c*d^2 - 7*a^5*b*d^3)*
x)/(a^3*b^5*x^3 + a^4*b^4), 1/36*(9*a^3*b^3*d^3*x^7 + 9*(12*a^3*b^3*c*d^2 -
 7*a^4*b^2*d^3)*x^4 + 12*sqrt(1/3)*(2*a^2*b^4*c^3 + 3*a^3*b^3*c^2*d - 12*a^
4*b^2*c*d^2 + 7*a^5*b*d^3 + (2*a*b^5*c^3 + 3*a^2*b^4*c^2*d - 12*a^3*b^3*c*d
^2 + 7*a^4*b^2*d^3)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^
(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - 2*(2*a*b^3*c^3 + 3*
a^2*b^2*c^2*d - 12*a^3*b*c*d^2 + 7*a^4*d^3 + (2*b^4*c^3 + 3*a*b^3*c^2*d - 1
2*a^2*b^2*c*d^2 + 7*a^3*b*d^3)*x^3)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/
3)*x + (a^2*b)^(1/3)*a) + 4*(2*a*b^3*c^3 + 3*a^2*b^2*c^2*d - 12*a^3*b*c*d^2
 + 7*a^4*d^3 + (2*b^4*c^3 + 3*a*b^3*c^2*d - 12*a^2*b^2*c*d^2 + 7*a^3*b*d^3)
*x^3)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 12*(a^2*b^4*c^3 - 3*a^3*b^
3*c^2*d + 12*a^4*b^2*c*d^2 - 7*a^5*b*d^3)*x)/(a^3*b^5*x^3 + a^4*b^4)]
```

Sympy [A] time = 4.0055, size = 289, normalized size = 1.24

$$-\frac{x\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}{3a^2b^3+3ab^4x^3}+\operatorname{RootSum}\left(729t^3a^5b^{10}-343a^9d^9+1764a^8bcd^8-3465a^7b^2c^2d^7+2946a^6b^3c^3d^6\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**3/(b*x**3+a)**2,x)
```

```
[Out] -x*(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3)/(3*a**2*b**3
 + 3*a*b**4*x**3) + RootSum(729*_t**3*a**5*b**10 - 343*a**9*d**9 + 1764*a**
8*b*c*d**8 - 3465*a**7*b**2*c**2*d**7 + 2946*a**6*b**3*c**3*d**6 - 477*a**5
*b**4*c**4*d**5 - 792*a**4*b**5*c**5*d**4 + 321*a**3*b**6*c**6*d**3 + 90*a*
*2*b**7*c**7*d**2 - 36*a*b**8*c**8*d - 8*b**9*c**9, Lambda(_t, _t*log(9*_t*
a**2*b**3/(7*a**3*d**3 - 12*a**2*b*c*d**2 + 3*a*b**2*c**2*d + 2*b**3*c**3)
+ x))) + d**3*x**4/(4*b**2) - x*(2*a*d**3 - 3*b*c*d**2)/b**3
```

Giac [A] time = 1.11544, size = 495, normalized size = 2.12

$$-\frac{\left(2\,b^3c^3+3\,ab^2c^2d-12\,a^2bcd^2+7\,a^3d^3\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b^3}+\frac{\sqrt{3}\left(2\,\left(-ab^2\right)^{\frac{1}{3}}b^3c^3+3\,\left(-ab^2\right)^{\frac{1}{3}}ab^2c^2d-12\,\left(-ab^2\right)\right.}{9\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^3/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*(2*b^3*c^3 + 3*a*b^2*c^2*d - 12*a^2*b*c*d^2 + 7*a^3*d^3)*(-a/b)^(1/3)*
log(abs(x - (-a/b)^(1/3)))/(a^2*b^3) + 1/9*sqrt(3)*(2*(-a*b^2)^(1/3)*b^3*c^
3 + 3*(-a*b^2)^(1/3)*a*b^2*c^2*d - 12*(-a*b^2)^(1/3)*a^2*b*c*d^2 + 7*(-a*b^
2)^(1/3)*a^3*d^3)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(a^
2*b^4) + 1/3*(b^3*c^3*x - 3*a*b^2*c^2*d*x + 3*a^2*b*c*d^2*x - a^3*d^3*x)/((
b*x^3 + a)*a*b^3) + 1/18*(2*(-a*b^2)^(1/3)*b^3*c^3 + 3*(-a*b^2)^(1/3)*a*b^2
*c^2*d - 12*(-a*b^2)^(1/3)*a^2*b*c*d^2 + 7*(-a*b^2)^(1/3)*a^3*d^3)*log(x^2
+ x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^4) + 1/4*(b^6*d^3*x^4 + 12*b^6*c*d^
2*x - 8*a*b^5*d^3*x)/b^8
```

3.23 $\int \frac{(c+dx^3)^2}{(a+bx^3)^2} dx$

Optimal. Leaf size=203

$$-\frac{(bc-ad)(2ad+bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{5/3}b^{7/3}}+\frac{2(bc-ad)(2ad+bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{7/3}}-\frac{2(bc-ad)(2ad+bc)\tan}{3\sqrt{3}a^{5/3}b^{7/3}}$$

```
[Out] (d^2*x)/b^2 + ((b*c - a*d)^2*x)/(3*a*b^2*(a + b*x^3)) - (2*(b*c - a*d)*(b*c
 + 2*a*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(
5/3)*b^(7/3)) + (2*(b*c - a*d)*(b*c + 2*a*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a
^(5/3)*b^(7/3)) - ((b*c - a*d)*(b*c + 2*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2])/(9*a^(5/3)*b^(7/3))
```

Rubi [A] time = 0.231096, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {390, 385, 200, 31, 634, 617, 204, 628}

$$-\frac{(bc-ad)(2ad+bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{5/3}b^{7/3}}+\frac{2(bc-ad)(2ad+bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{7/3}}-\frac{2(bc-ad)(2ad+bc)\tan}{3\sqrt{3}a^{5/3}b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^2,x]
```

```
[Out] (d^2*x)/b^2 + ((b*c - a*d)^2*x)/(3*a*b^2*(a + b*x^3)) - (2*(b*c - a*d)*(b*c
 + 2*a*d)*ArcTan[(a^(1/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(
5/3)*b^(7/3)) + (2*(b*c - a*d)*(b*c + 2*a*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a
^(5/3)*b^(7/3)) - ((b*c - a*d)*(b*c + 2*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*
x + b^(2/3)*x^2])/(9*a^(5/3)*b^(7/3))
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^2}\,dx = \int \left(\frac{d^2}{b^2}+\frac{b^2c^2-a^2d^2+2bd(bc-ad)x^3}{b^2\left(a+bx^3\right)^2}\right)dx$$

$$= \frac{d^2x}{b^2}+\frac{\int \frac{b^2c^2-a^2d^2+2bd(bc-ad)x^3}{\left(a+bx^3\right)^2}\,dx}{b^2}$$

$$= \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{3ab^2\left(a+bx^3\right)}+\frac{(2(bc-ad)(bc+2ad))\int \frac{1}{a+bx^3}\,dx}{3ab^2}$$

$$= \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{3ab^2\left(a+bx^3\right)}+\frac{(2(bc-ad)(bc+2ad))\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{5/3}b^2}+\frac{(2(bc-ad)(bc+2ad))\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/}}}{9a^{5/3}b^2}$$

$$= \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{3ab^2\left(a+bx^3\right)}+\frac{2(bc-ad)(bc+2ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{7/3}}-\frac{((bc-ad)(bc+2ad))\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2}}}{9a^{5/3}b^{7/3}}$$

$$= \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{3ab^2\left(a+bx^3\right)}+\frac{2(bc-ad)(bc+2ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{7/3}}-\frac{(bc-ad)(bc+2ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\right.}{9a^{5/3}b^{7/3}}$$

$$= \frac{d^2x}{b^2}+\frac{(bc-ad)^2x}{3ab^2\left(a+bx^3\right)}-\frac{2(bc-ad)(bc+2ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{7/3}}+\frac{2(bc-ad)(bc+2ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right.}{9a^{5/3}b^{7/3}}$$

Mathematica [A] time = 0.212557, size = 205, normalized size = 1.01

$$\frac{-\frac{\left(-2a^2d^2+abcd+b^2c^2\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{5/3}}+\frac{2\left(-2a^2d^2+abcd+b^2c^2\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{5/3}}-\frac{2\sqrt{3}\left(-2a^2d^2+abcd+b^2c^2\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}}+\frac{3\sqrt[3]{b}x(bc-ad}{a\left(a+bx^3\right)}}{9b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^2,x]
```

```
[Out] (9*b^(1/3)*d^2*x + (3*b^(1/3)*(b*c - a*d)^2*x)/(a*(a + b*x^3)) - (2*Sqrt[3]
*(b^2*c^2 + a*b*c*d - 2*a^2*d^2)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]
])/a^(5/3) + (2*(b^2*c^2 + a*b*c*d - 2*a^2*d^2)*Log[a^(1/3) + b^(1/3)*x])/a
^(5/3) - ((b^2*c^2 + a*b*c*d - 2*a^2*d^2)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x +
 b^(2/3)*x^2])/a^(5/3))/(9*b^(7/3))
```

Maple [B] time = 0.008, size = 367, normalized size = 1.8

$$\frac{d^2x}{b^2}+\frac{axd^2}{3b^2\left(bx^3+a\right)}-\frac{2cxd}{3b\left(bx^3+a\right)}+\frac{xc^2}{3a\left(bx^3+a\right)}-\frac{4ad^2}{9b^3}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{2cd}{9b^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{2c^2}{9ab}\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^2/(b*x^3+a)^2,x)
```

```
[Out] d^2*x/b^2+1/3/b^2*a*x/(b*x^3+a)*d^2-2/3/b*x/(b*x^3+a)*c*d+1/3/a*x/(b*x^3+a)
*c^2-4/9/b^3*a/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*d^2+2/9/b^2/(1/b*a)^(2/3)*
ln(x+(1/b*a)^(1/3))*c*d+2/9/b/a/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*c^2+2/9/b
^3*a/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d^2-1/9/b^2/(1/b*a
)^(2/3)*ln(x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c*d-1/9/b/a/(1/b*a)^(2/3)*ln(
x^2-(1/b*a)^(1/3)*x+(1/b*a)^(2/3))*c^2-4/9/b^3*a/(1/b*a)^(2/3)*3^(1/2)*arct
an(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d^2+2/9/b^2/(1/b*a)^(2/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c*d+2/9/b/a/(1/b*a)^(2/3)*3^(1/2)*ar
ctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.71062, size = 1661, normalized size = 8.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/9*(9*a^3*b^2*d^2*x^4 - 3*sqrt(1/3)*(a^2*b^3*c^2 + a^3*b^2*c*d - 2*a^4*b*
d^2 + (a*b^4*c^2 + a^2*b^3*c*d - 2*a^3*b^2*d^2)*x^3)*sqrt((-a^2*b)^(1/3)/b)
*log((2*a*b*x^3 + 3*(-a^2*b)^(1/3)*a*x - a^2 - 3*sqrt(1/3)*(2*a*b*x^2 + (-a
^2*b)^(2/3)*x + (-a^2*b)^(1/3)*a)*sqrt((-a^2*b)^(1/3)/b))/(b*x^3 + a)) - (a
*b^2*c^2 + a^2*b*c*d - 2*a^3*d^2 + (b^3*c^2 + a*b^2*c*d - 2*a^2*b*d^2)*x^3)
*(-a^2*b)^(2/3)*log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 2*(a*b
^2*c^2 + a^2*b*c*d - 2*a^3*d^2 + (b^3*c^2 + a*b^2*c*d - 2*a^2*b*d^2)*x^3)*(
-a^2*b)^(2/3)*log(a*b*x + (-a^2*b)^(2/3)) + 3*(a^2*b^3*c^2 - 2*a^3*b^2*c*d
+ 4*a^4*b*d^2)*x)/(a^3*b^4*x^3 + a^4*b^3), 1/9*(9*a^3*b^2*d^2*x^4 + 6*sqrt(
1/3)*(a^2*b^3*c^2 + a^3*b^2*c*d - 2*a^4*b*d^2 + (a*b^4*c^2 + a^2*b^3*c*d -
2*a^3*b^2*d^2)*x^3)*sqrt(-(-a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(-a^2*b)^(2
/3)*x + (-a^2*b)^(1/3)*a)*sqrt(-(-a^2*b)^(1/3)/b)/a^2) - (a*b^2*c^2 + a^2*b
*c*d - 2*a^3*d^2 + (b^3*c^2 + a*b^2*c*d - 2*a^2*b*d^2)*x^3)*(-a^2*b)^(2/3)*
log(a*b*x^2 - (-a^2*b)^(2/3)*x - (-a^2*b)^(1/3)*a) + 2*(a*b^2*c^2 + a^2*b*c
*d - 2*a^3*d^2 + (b^3*c^2 + a*b^2*c*d - 2*a^2*b*d^2)*x^3)*(-a^2*b)^(2/3)*lo
g(a*b*x + (-a^2*b)^(2/3)) + 3*(a^2*b^3*c^2 - 2*a^3*b^2*c*d + 4*a^4*b*d^2)*x
)/(a^3*b^4*x^3 + a^4*b^3)]
```

Sympy [A] time = 2.37695, size = 189, normalized size = 0.93

$$\frac{x\left(a^2d^2-2abcd+b^2c^2\right)}{3a^2b^2+3ab^3x^3}+\operatorname{RootSum}\left(729t^3a^5b^7+64a^6d^6-96a^5bcd^5-48a^4b^2c^2d^4+88a^3b^3c^3d^3+24a^2b^4c^4d^2-24ab^5c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**2,x)
```

```
[Out] x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(3*a**2*b**2 + 3*a*b**3*x**3) + RootS
um(729*_t**3*a**5*b**7 + 64*a**6*d**6 - 96*a**5*b*c*d**5 - 48*a**4*b**2*c**
2*d**4 + 88*a**3*b**3*c**3*d**3 + 24*a**2*b**4*c**4*d**2 - 24*a*b**5*c**5*d
 - 8*b**6*c**6, Lambda(_t, _t*log(-9*_t*a**2*b**2/(4*a**2*d**2 - 2*a*b*c*d
- 2*b**2*c**2) + x))) + d**2*x/b**2
```

Giac [A] time = 1.12866, size = 350, normalized size = 1.72

$$\frac{d^2x}{b^2}-\frac{2\left(b^2c^2+abcd-2a^2d^2\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9a^2b^2}+\frac{2\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^2c^2+\left(-ab^2\right)^{\frac{1}{3}}abcd-2\left(-ab^2\right)^{\frac{1}{3}}a^2d^2\right)\arctan}{9a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] d^2*x/b^2 - 2/9*(b^2*c^2 + a*b*c*d - 2*a^2*d^2)*(-a/b)^(1/3)*log(abs(x - (-
a/b)^(1/3)))/(a^2*b^2) + 2/9*sqrt(3)*((-a*b^2)^(1/3)*b^2*c^2 + (-a*b^2)^(1/
3)*a*b*c*d - 2*(-a*b^2)^(1/3)*a^2*d^2)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/
3))/(-a/b)^(1/3))/(a^2*b^3) + 1/3*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((b
*x^3 + a)*a*b^2) + 1/9*((-a*b^2)^(1/3)*b^2*c^2 + (-a*b^2)^(1/3)*a*b*c*d - 2
```

```
*(-a*b^2)^(1/3)*a^2*d^2)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^3)
```

3.24 $\int \frac{c+dx^3}{\left(a+bx^3\right)^2} dx$

Optimal. Leaf size=169

$$-\frac{(ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{4/3}}+\frac{(ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{4/3}}-\frac{(ad+2bc)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{4/3}}+\frac{x(bc-ad)}{3ab\left(a+bx^3\right)}$$

```
[Out] ((b*c - a*d)*x)/(3*a*b*(a + b*x^3)) - ((2*b*c + a*d)*ArcTan[(a^(1/3) - 2*b^
(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(4/3)) + ((2*b*c + a*d)*L
og[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(4/3)) - ((2*b*c + a*d)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(4/3))
```

Rubi [A] time = 0.0842995, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {385, 200, 31, 634, 617, 204, 628}

$$-\frac{(ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{4/3}}+\frac{(ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{4/3}}-\frac{(ad+2bc)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{4/3}}+\frac{x(bc-ad)}{3ab\left(a+bx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)/(a + b*x^3)^2,x]
```

```
[Out] ((b*c - a*d)*x)/(3*a*b*(a + b*x^3)) - ((2*b*c + a*d)*ArcTan[(a^(1/3) - 2*b^
(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(4/3)) + ((2*b*c + a*d)*L
og[a^(1/3) + b^(1/3)*x])/(9*a^(5/3)*b^(4/3)) - ((2*b*c + a*d)*Log[a^(2/3) -
 a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(18*a^(5/3)*b^(4/3))
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{c+dx^3}{\left(a+bx^3\right)^2}\,dx = \frac{(bc-ad)x}{3ab\left(a+bx^3\right)} + \frac{(2bc+ad)\int \frac{1}{a+bx^3}\,dx}{3ab}$$

$$= \frac{(bc-ad)x}{3ab\left(a+bx^3\right)} + \frac{(2bc+ad)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx}{9a^{5/3}b} + \frac{(2bc+ad)\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{9a^{5/3}b}$$

$$= \frac{(bc-ad)x}{3ab\left(a+bx^3\right)} + \frac{(2bc+ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{4/3}} - \frac{(2bc+ad)\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2b^{2/3}x}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2}\,dx}{18a^{5/3}b^{4/3}} + \frac{(2bc+ad)\int \frac{\ldots}{a^{2/3}\ldots}}{6a\ldots}$$

$$= \frac{(bc-ad)x}{3ab\left(a+bx^3\right)} + \frac{(2bc+ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{4/3}} - \frac{(2bc+ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{4/3}} + \frac{(2bc+ad\ldots}{\ldots}$$

$$= \frac{(bc-ad)x}{3ab\left(a+bx^3\right)} - \frac{(2bc+ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{4/3}} + \frac{(2bc+ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{4/3}} - \frac{(2bc+ad)\log\left(a^{2/3}\ldots\right.}{18a^{5/3}\ldots}$$

Mathematica [A] time = 0.0899517, size = 145, normalized size = 0.86

$$\frac{-(ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right) - \frac{6a^{2/3}\sqrt[3]{b}x(ad-bc)}{a+bx^3} + 2(ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right) - 2\sqrt{3}(ad+2bc)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\ldots}{\sqrt[3]{a}\ldots}}{\sqrt{3}}\ldots\right.}{18a^{5/3}b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)/(a + b*x^3)^2,x]
```

```
[Out] ((-6*a^(2/3)*b^(1/3)*(-(b*c) + a*d)*x)/(a + b*x^3) - 2*Sqrt[3]*(2*b*c + a*d
)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(2*b*c + a*d)*Log[a^(1/3)
 + b^(1/3)*x] - (2*b*c + a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
```

```
])/(18*a^(5/3)*b^(4/3))
```

Maple [A] time = 0.007, size = 221, normalized size = 1.3

$$-\frac{(ad-bc)\,x}{3\,ab\left(bx^{3}+a\right)}+\frac{d}{9\,b^{2}}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}+\frac{2\,c}{9\,ab}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{d}{18\,b^{2}}\ln\left(x^{2}-\sqrt[3]{\frac{a}{b}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}}-\frac{c}{9\,ab}\ln\left(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)/(b*x^3+a)^2,x)
```

```
[Out] -1/3*(a*d-b*c)/a/b*x/(b*x^3+a)+1/9/b^2/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*d+
2/9/b/a/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*c-1/18/b^2/(1/b*a)^(2/3)*ln(x^2-(
1/b*a)^(1/3)*x+(1/b*a)^(2/3))*d-1/9/b/a/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/3)*
x+(1/b*a)^(2/3))*c+1/9/b^2/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b
*a)^(1/3)*x-1))*d+2/9/b/a/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*
a)^(1/3)*x-1))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.70564, size = 1233, normalized size = 7.3

$$\left[\frac{3\sqrt{\frac{1}{3}}\left(2\,a^{2}b^{2}c+a^{3}bd+\left(2\,ab^{3}c+a^{2}b^{2}d\right)x^{3}\right)\sqrt{-\frac{\left(a^{2}b\right)^{\frac{1}{3}}}{b}}\log\left(\frac{2\,abx^{3}-3\left(a^{2}b\right)^{\frac{1}{3}}ax-a^{2}+3\sqrt{\frac{1}{3}}\left(2\,abx^{2}+\left(a^{2}b\right)^{\frac{2}{3}}x-\left(a^{2}b\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(a^{2}b\right)^{\frac{1}{3}}}{b}}}{bx^{3}+a}\right)-\left(\left(2\,b^{2}\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] [1/18*(3*sqrt(1/3)*(2*a^2*b^2*c + a^3*b*d + (2*a*b^3*c + a^2*b^2*d)*x^3)*sq
rt(-(a^2*b)^(1/3)/b)*log((2*a*b*x^3 - 3*(a^2*b)^(1/3)*a*x - a^2 + 3*sqrt(1/
3)*(2*a*b*x^2 + (a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b))/
(b*x^3 + a)) - ((2*b^2*c + a*b*d)*x^3 + 2*a*b*c + a^2*d)*(a^2*b)^(2/3)*log(
a*b*x^2 - (a^2*b)^(2/3)*x + (a^2*b)^(1/3)*a) + 2*((2*b^2*c + a*b*d)*x^3 + 2
*a*b*c + a^2*d)*(a^2*b)^(2/3)*log(a*b*x + (a^2*b)^(2/3)) + 6*(a^2*b^2*c - a
```

```
^3*b*d)*x)/(a^3*b^3*x^3 + a^4*b^2), 1/18*(6*sqrt(1/3)*(2*a^2*b^2*c + a^3*b*
d + (2*a*b^3*c + a^2*b^2*d)*x^3)*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*
(a^2*b)^(2/3)*x - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - ((2*b^2*c +
 a*b*d)*x^3 + 2*a*b*c + a^2*d)*(a^2*b)^(2/3)*log(a*b*x^2 - (a^2*b)^(2/3)*x
+ (a^2*b)^(1/3)*a) + 2*((2*b^2*c + a*b*d)*x^3 + 2*a*b*c + a^2*d)*(a^2*b)^(2
/3)*log(a*b*x + (a^2*b)^(2/3)) + 6*(a^2*b^2*c - a^3*b*d)*x)/(a^3*b^3*x^3 +
a^4*b^2)]
```

Sympy [A] time = 1.07586, size = 97, normalized size = 0.57

$$-\frac{x(ad-bc)}{3a^2b+3ab^2x^3}+\operatorname{RootSum}\left(729t^3a^5b^4-a^3d^3-6a^2bcd^2-12ab^2c^2d-8b^3c^3,\left(t\mapsto t\log\left(\frac{9ta^2b}{ad+2bc}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)/(b*x**3+a)**2,x)
```

```
[Out] -x*(a*d - b*c)/(3*a**2*b + 3*a*b**2*x**3) + RootSum(729*_t**3*a**5*b**4 - a
**3*d**3 - 6*a**2*b*c*d**2 - 12*a*b**2*c**2*d - 8*b**3*c**3, Lambda(_t, _t*
log(9*_t*a**2*b/(a*d + 2*b*c) + x)))
```

Giac [A] time = 1.12387, size = 246, normalized size = 1.46

$$-\frac{(2\,bc+ad)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9\,a^2b}+\frac{\sqrt{3}\left(2\,\left(-ab^2\right)^{\frac{1}{3}}bc+\left(-ab^2\right)^{\frac{1}{3}}ad\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\,a^2b^2}+\frac{bcx-adx}{3\left(bx^3+a\right)ab}+\frac{(2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -1/9*(2*b*c + a*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b) + 1/9*sq
rt(3)*(2*(-a*b^2)^(1/3)*b*c + (-a*b^2)^(1/3)*a*d)*arctan(1/3*sqrt(3)*(2*x +
 (-a/b)^(1/3))/(-a/b)^(1/3))/(a^2*b^2) + 1/3*(b*c*x - a*d*x)/((b*x^3 + a)*a
*b) + 1/18*(2*(-a*b^2)^(1/3)*b*c + (-a*b^2)^(1/3)*a*d)*log(x^2 + x*(-a/b)^(
1/3) + (-a/b)^(2/3))/(a^2*b^2)
```

3.25 $\int \frac{1}{\left(a+bx^3\right)^2\left(c+dx^3\right)} dx$

Optimal. Leaf size=346

$$-\frac{b^{2/3}(2bc-5ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}(bc-ad)^2}+\frac{b^{2/3}(2bc-5ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}(bc-ad)^2}-\frac{b^{2/3}(2bc-5ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}(bc-ad)^2}-\frac{d^{5}}{}$$

```
[Out] (b*x)/(3*a*(b*c - a*d)*(a + b*x^3)) - (b^(2/3)*(2*b*c - 5*a*d)*ArcTan[(a^(1
/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*(b*c - a*d)^2) -
(d^(5/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(2/3
)*(b*c - a*d)^2) + (b^(2/3)*(2*b*c - 5*a*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a^
(5/3)*(b*c - a*d)^2) + (d^(5/3)*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3)*(b*c -
 a*d)^2) - (b^(2/3)*(2*b*c - 5*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/(18*a^(5/3)*(b*c - a*d)^2) - (d^(5/3)*Log[c^(2/3) - c^(1/3)*d^(1/3
)*x + d^(2/3)*x^2])/(6*c^(2/3)*(b*c - a*d)^2)
```

Rubi [A] time = 0.254639, antiderivative size = 346, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {414, 522, 200, 31, 634, 617, 204, 628}

$$-\frac{b^{2/3}(2bc-5ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}(bc-ad)^2}+\frac{b^{2/3}(2bc-5ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}(bc-ad)^2}-\frac{b^{2/3}(2bc-5ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}(bc-ad)^2}-\frac{d^{5}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^2*(c + d*x^3)),x]
```

```
[Out] (b*x)/(3*a*(b*c - a*d)*(a + b*x^3)) - (b^(2/3)*(2*b*c - 5*a*d)*ArcTan[(a^(1
/3) - 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*(b*c - a*d)^2) -
(d^(5/3)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(2/3
)*(b*c - a*d)^2) + (b^(2/3)*(2*b*c - 5*a*d)*Log[a^(1/3) + b^(1/3)*x])/(9*a^
(5/3)*(b*c - a*d)^2) + (d^(5/3)*Log[c^(1/3) + d^(1/3)*x])/(3*c^(2/3)*(b*c -
 a*d)^2) - (b^(2/3)*(2*b*c - 5*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/
3)*x^2])/(18*a^(5/3)*(b*c - a*d)^2) - (d^(5/3)*Log[c^(2/3) - c^(1/3)*d^(1/3
)*x + d^(2/3)*x^2])/(6*c^(2/3)*(b*c - a*d)^2)
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^2\left(c+dx^3\right)}dx = \frac{bx}{3a(bc-ad)\left(a+bx^3\right)} - \frac{\int \frac{-2bc+3ad-2bdx^3}{\left(a+bx^3\right)\left(c+dx^3\right)}dx}{3a(bc-ad)}$$

$$= \frac{bx}{3a(bc-ad)\left(a+bx^3\right)} + \frac{d^2\int \frac{1}{c+dx^3}dx}{(bc-ad)^2} + \frac{(b(2bc-5ad))\int \frac{1}{a+bx^3}dx}{3a(bc-ad)^2}$$

$$= \frac{bx}{3a(bc-ad)\left(a+bx^3\right)} + \frac{d^2\int \frac{1}{\sqrt[3]{c}+\sqrt[3]{d}x}dx}{3c^{2/3}(bc-ad)^2} + \frac{d^2\int \frac{2\sqrt[3]{c}-\sqrt[3]{d}x}{c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2}dx}{3c^{2/3}(bc-ad)^2} + \frac{(b(2bc-5ad))\int \frac{1}{\sqrt[3]{a}+}}{9a^{5/3}(bc-ad)^2}$$

$$= \frac{bx}{3a(bc-ad)\left(a+bx^3\right)} + \frac{b^{2/3}(2bc-5ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}(bc-ad)^2} + \frac{d^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}(bc-ad)^2} - \frac{d^{5/3}\int}{6c^{2/}}$$

$$= \frac{bx}{3a(bc-ad)\left(a+bx^3\right)} + \frac{b^{2/3}(2bc-5ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}(bc-ad)^2} + \frac{d^{5/3}\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}(bc-ad)^2} - b^{2/3}(2bc$$

$$= \frac{bx}{3a(bc-ad)\left(a+bx^3\right)} - \frac{b^{2/3}(2bc-5ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}(bc-ad)^2} - \frac{d^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}(bc-ad)^2} + b^{2/3}(2b$$

Mathematica [A] time = 0.213159, size = 337, normalized size = 0.97

$$-b^{2/3}c^{2/3}\left(a+bx^3\right)(2bc-5ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right) - 3a^{5/3}d^{5/3}\left(a+bx^3\right)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right) + 6a^{2/3}bc^{2/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^3)^2*(c + d*x^3)),x]
```

```
[Out] (6*a^(2/3)*b*c^(2/3)*(b*c - a*d)*x - 2*Sqrt[3]*b^(2/3)*c^(2/3)*(2*b*c - 5*a
*d)*(a + b*x^3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] - 6*Sqrt[3]*a^(
5/3)*d^(5/3)*(a + b*x^3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]] + 2*b^
(2/3)*c^(2/3)*(2*b*c - 5*a*d)*(a + b*x^3)*Log[a^(1/3) + b^(1/3)*x] + 6*a^(5
/3)*d^(5/3)*(a + b*x^3)*Log[c^(1/3) + d^(1/3)*x] - b^(2/3)*c^(2/3)*(2*b*c -
 5*a*d)*(a + b*x^3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2] - 3*a^(5
/3)*d^(5/3)*(a + b*x^3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(18
*a^(5/3)*c^(2/3)*(b*c - a*d)^2*(a + b*x^3))
```

Maple [A] time = 0.01, size = 406, normalized size = 1.2

$$-\frac{bdx}{3\,(ad-bc)^2\left(bx^3+a\right)} + \frac{b^2xc}{3\,(ad-bc)^2\,a\left(bx^3+a\right)} - \frac{5\,d}{9\,(ad-bc)^2}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)^{-\frac{2}{3}} + \frac{2\,bc}{9\,(ad-bc)^2\,a}\ln\left(x+\sqrt[3]{\frac{a}{b}}\right)\left(\frac{a}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^2/(d*x^3+c),x)
```

```
[Out] -1/3*b/(a*d-b*c)^2*x/(b*x^3+a)*d+1/3*b^2/(a*d-b*c)^2/a*x/(b*x^3+a)*c-5/9/(a
*d-b*c)^2/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*d+2/9*b/(a*d-b*c)^2/a/(1/b*a)^(
```

```
2/3)*ln(x+(1/b*a)^(1/3))*c+5/18/(a*d-b*c)^2/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1
/3)*x+(1/b*a)^(2/3))*d-1/9*b/(a*d-b*c)^2/a/(1/b*a)^(2/3)*ln(x^2-(1/b*a)^(1/
3)*x+(1/b*a)^(2/3))*c-5/9/(a*d-b*c)^2/(1/b*a)^(2/3)*3^(1/2)*arctan(1/3*3^(1
/2)*(2/(1/b*a)^(1/3)*x-1))*d+2/9*b/(a*d-b*c)^2/a/(1/b*a)^(2/3)*3^(1/2)*arct
an(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c+1/3*d/(a*d-b*c)^2/(c/d)^(2/3)*ln(x+
(c/d)^(1/3))-1/6*d/(a*d-b*c)^2/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3)
)+1/3*d/(a*d-b*c)^2/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x
-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 43.0494, size = 1004, normalized size = 2.9

$$-\frac{2\sqrt{3}\left(\left(2b^2c-5abd\right)x^3+2abc-5a^2d\right)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)-6\sqrt{3}\left(abdx^3+a^2d\right)\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}cx}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/18*(2*sqrt(3)*((2*b^2*c - 5*a*b*d)*x^3 + 2*a*b*c - 5*a^2*d)*(-b^2/a^2)^(
1/3)*arctan(1/3*(2*sqrt(3)*a*x*(-b^2/a^2)^(2/3) - sqrt(3)*b)/b) - 6*sqrt(3)
*(a*b*d*x^3 + a^2*d)*(d^2/c^2)^(1/3)*arctan(1/3*(2*sqrt(3)*c*x*(d^2/c^2)^(2
/3) - sqrt(3)*d)/d) - ((2*b^2*c - 5*a*b*d)*x^3 + 2*a*b*c - 5*a^2*d)*(-b^2/a
^2)^(1/3)*log(b^2*x^2 + a*b*x*(-b^2/a^2)^(1/3) + a^2*(-b^2/a^2)^(2/3)) + 3*
(a*b*d*x^3 + a^2*d)*(d^2/c^2)^(1/3)*log(d^2*x^2 - c*d*x*(d^2/c^2)^(1/3) + c
^2*(d^2/c^2)^(2/3)) + 2*((2*b^2*c - 5*a*b*d)*x^3 + 2*a*b*c - 5*a^2*d)*(-b^2
/a^2)^(1/3)*log(b*x - a*(-b^2/a^2)^(1/3)) - 6*(a*b*d*x^3 + a^2*d)*(d^2/c^2)
^(1/3)*log(d*x + c*(d^2/c^2)^(1/3)) - 6*(b^2*c - a*b*d)*x)/(a^2*b^2*c^2 - 2
*a^3*b*c*d + a^4*d^2 + (a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*x^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**2/(d*x**3+c),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17086, size = 598, normalized size = 1.73

$$-\frac{d^2\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^2c^3-2\,abc^2d+a^2cd^2\right)}+\frac{\left(-cd^2\right)^{\frac{1}{3}}d\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c^3-2\,\sqrt{3}abc^2d+\sqrt{3}a^2cd^2}+\frac{\left(-cd^2\right)^{\frac{1}{3}}d\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(b^2c^3-2\,abc^2d+a^2cd^2\right)}-\frac{\left(2\,b^2c-5\,a\right.}{9\left(a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c),x, algorithm="giac")
```

```
[Out] -1/3*d^2*(-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b^2*c^3 - 2*a*b*c^2*d + a
^2*c*d^2) + (-c*d^2)^(1/3)*d*arctan(1/3*sqrt(3)*(2*x + (-c/d)^(1/3))/(-c/d)
^(1/3))/(sqrt(3)*b^2*c^3 - 2*sqrt(3)*a*b*c^2*d + sqrt(3)*a^2*c*d^2) + 1/6*(
-c*d^2)^(1/3)*d*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b^2*c^3 - 2*a*b*c
^2*d + a^2*c*d^2) - 1/9*(2*b^2*c - 5*a*b*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)
^(1/3)))/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2) + 1/3*(2*(-a*b^2)^(1/3)*b*c
- 5*(-a*b^2)^(1/3)*a*d)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3
))/(sqrt(3)*a^2*b^2*c^2 - 2*sqrt(3)*a^3*b*c*d + sqrt(3)*a^4*d^2) + 1/18*(2*
(-a*b^2)^(1/3)*b*c - 5*(-a*b^2)^(1/3)*a*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b
)^(2/3))/(a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2) + 1/3*b*x/((b*x^3 + a)*(a*b*
c - a^2*d))
```

3.26 $\int \frac{1}{\left(a+bx^3\right)^2\left(c+dx^3\right)^2}\,dx$

Optimal. Leaf size=419

$$-\frac{b^{5/3}(bc-4ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{5/3}(bc-ad)^3}+\frac{2b^{5/3}(bc-4ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}(bc-ad)^3}-\frac{2b^{5/3}(bc-4ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}(bc-ad)^3}-$$

```
[Out] (d*(b*c + a*d)*x)/(3*a*c*(b*c - a*d)^2*(c + d*x^3)) + (b*x)/(3*a*(b*c - a*d
)*(a + b*x^3)*(c + d*x^3)) - (2*b^(5/3)*(b*c - 4*a*d)*ArcTan[(a^(1/3) - 2*b
^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*(b*c - a*d)^3) - (2*d^(5/3
)*(4*b*c - a*d)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(3*Sqrt[
3]*c^(5/3)*(b*c - a*d)^3) + (2*b^(5/3)*(b*c - 4*a*d)*Log[a^(1/3) + b^(1/3)*
x])/(9*a^(5/3)*(b*c - a*d)^3) + (2*d^(5/3)*(4*b*c - a*d)*Log[c^(1/3) + d^(1
/3)*x])/(9*c^(5/3)*(b*c - a*d)^3) - (b^(5/3)*(b*c - 4*a*d)*Log[a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(9*a^(5/3)*(b*c - a*d)^3) - (d^(5/3)*(4*b*c
 - a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(9*c^(5/3)*(b*c - a
*d)^3)
```

Rubi [A] time = 0.493458, antiderivative size = 419, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {414, 527, 522, 200, 31, 634, 617, 204, 628}

$$-\frac{b^{5/3}(bc-4ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{5/3}(bc-ad)^3}+\frac{2b^{5/3}(bc-4ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}(bc-ad)^3}-\frac{2b^{5/3}(bc-4ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}(bc-ad)^3}-$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^2*(c + d*x^3)^2),x]
```


```
[Out] (d*(b*c + a*d)*x)/(3*a*c*(b*c - a*d)^2*(c + d*x^3)) + (b*x)/(3*a*(b*c - a*d
)*(a + b*x^3)*(c + d*x^3)) - (2*b^(5/3)*(b*c - 4*a*d)*ArcTan[(a^(1/3) - 2*b
^(1/3)*x)/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*(b*c - a*d)^3) - (2*d^(5/3
)*(4*b*c - a*d)*ArcTan[(c^(1/3) - 2*d^(1/3)*x)/(Sqrt[3]*c^(1/3))])/(3*Sqrt[
3]*c^(5/3)*(b*c - a*d)^3) + (2*b^(5/3)*(b*c - 4*a*d)*Log[a^(1/3) + b^(1/3)*
x])/(9*a^(5/3)*(b*c - a*d)^3) + (2*d^(5/3)*(4*b*c - a*d)*Log[c^(1/3) + d^(1
/3)*x])/(9*c^(5/3)*(b*c - a*d)^3) - (b^(5/3)*(b*c - 4*a*d)*Log[a^(2/3) - a^
(1/3)*b^(1/3)*x + b^(2/3)*x^2])/(9*a^(5/3)*(b*c - a*d)^3) - (d^(5/3)*(4*b*c
 - a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/(9*c^(5/3)*(b*c - a
*d)^3)
```


Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
```

```
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^2\left(c+dx^3\right)^2}\,dx = \frac{bx}{3a(bc-ad)\left(a+bx^3\right)\left(c+dx^3\right)} - \frac{\int \frac{-2bc+3ad-5bdx^3}{\left(a+bx^3\right)\left(c+dx^3\right)^2}\,dx}{3a(bc-ad)}$$

$$= \frac{d(bc+ad)x}{3ac(bc-ad)^2\left(c+dx^3\right)} + \frac{bx}{3a(bc-ad)\left(a+bx^3\right)\left(c+dx^3\right)} - \frac{\int \frac{-6\left(b^2c^2-3abcd+a^2d^2\right)-6bd(bc+\dots}{\left(a+bx^3\right)\left(c+dx^3\right)}}{9ac(bc-ad)^2}$$

$$= \frac{d(bc+ad)x}{3ac(bc-ad)^2\left(c+dx^3\right)} + \frac{bx}{3a(bc-ad)\left(a+bx^3\right)\left(c+dx^3\right)} + \frac{\left(2b^2(bc-4ad)\right)\int \frac{1}{a+bx^3}\,dx}{3a(bc-ad)^3}$$

$$= \frac{d(bc+ad)x}{3ac(bc-ad)^2\left(c+dx^3\right)} + \frac{bx}{3a(bc-ad)\left(a+bx^3\right)\left(c+dx^3\right)} + \frac{\left(2b^2(bc-4ad)\right)\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\dots}{9a^{5/3}(bc-ad)^3}$$

$$= \frac{d(bc+ad)x}{3ac(bc-ad)^2\left(c+dx^3\right)} + \frac{bx}{3a(bc-ad)\left(a+bx^3\right)\left(c+dx^3\right)} + \frac{2b^{5/3}(bc-4ad)\log\left(\sqrt[3]{a}+\dots\right.}{9a^{5/3}(bc-ad)^3}$$

$$= \frac{d(bc+ad)x}{3ac(bc-ad)^2\left(c+dx^3\right)} + \frac{bx}{3a(bc-ad)\left(a+bx^3\right)\left(c+dx^3\right)} + \frac{2b^{5/3}(bc-4ad)\log\left(\sqrt[3]{a}+\dots\right.}{9a^{5/3}(bc-ad)^3}$$

$$= \frac{d(bc+ad)x}{3ac(bc-ad)^2\left(c+dx^3\right)} + \frac{bx}{3a(bc-ad)\left(a+bx^3\right)\left(c+dx^3\right)} - \frac{2b^{5/3}(bc-4ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-\dots}{\sqrt{3}\dots}\right.}{3\sqrt{3}a^{5/3}(bc-ad)^3}$$

Mathematica [A] time = 0.639885, size = 381, normalized size = 0.91

$$\frac{1}{9}\left(\frac{b^{5/3}(bc-4ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{5/3}(ad-bc)^3} + \frac{2b^{5/3}(4ad-bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{5/3}(ad-bc)^3} + \frac{2\sqrt{3}b^{5/3}(bc-4ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{5/3}(ad-bc)^3}\dots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^3)^2*(c + d*x^3)^2),x]
```

```
[Out] ((3*b^2*x)/(a*(b*c - a*d)^2*(a + b*x^3)) + (3*d^2*x)/(c*(b*c - a*d)^2*(c +
d*x^3)) + (2*Sqrt[3]*b^(5/3)*(b*c - 4*a*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3
))/Sqrt[3]])/(a^(5/3)*(-(b*c) + a*d)^3) + (2*Sqrt[3]*d^(5/3)*(-4*b*c + a*d)
*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/(c^(5/3)*(b*c - a*d)^3) + (2*
b^(5/3)*(-(b*c) + 4*a*d)*Log[a^(1/3) + b^(1/3)*x])/(a^(5/3)*(-(b*c) + a*d)^
3) + (2*d^(5/3)*(4*b*c - a*d)*Log[c^(1/3) + d^(1/3)*x])/(c^(5/3)*(b*c - a*d
)^3) + (b^(5/3)*(b*c - 4*a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2
])/(a^(5/3)*(-(b*c) + a*d)^3) + (d^(5/3)*(-4*b*c + a*d)*Log[c^(2/3) - c^(1/
3)*d^(1/3)*x + d^(2/3)*x^2])/(c^(5/3)*(b*c - a*d)^3))/9
```

Maple [A] time = 0.014, size = 606, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^2/(d*x^3+c)^2,x)
```

```
[Out] 1/3*b^2/(a*d-b*c)^3*x/(b*x^3+a)*d-1/3*b^3/(a*d-b*c)^3/a*x/(b*x^3+a)*c+8/9*b
/(a*d-b*c)^3/(1/b*a)^(2/3)*ln(x+(1/b*a)^(1/3))*d-2/9*b^2/(a*d-b*c)^3/a/(1/b
*a)^(2/3)*ln(x+(1/b*a)^(1/3))*c-4/9*b/(a*d-b*c)^3/(1/b*a)^(2/3)*ln(x^2-(1/b
*a)^(1/3)*x+(1/b*a)^(2/3))*d+1/9*b^2/(a*d-b*c)^3/a/(1/b*a)^(2/3)*ln(x^2-(1/
b*a)^(1/3)*x+(1/b*a)^(2/3))*c+8/9*b/(a*d-b*c)^3/(1/b*a)^(2/3)*3^(1/2)*arcta
n(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*d-2/9*b^2/(a*d-b*c)^3/a/(1/b*a)^(2/3)*
3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/b*a)^(1/3)*x-1))*c+1/3*d^3/(a*d-b*c)^3/c*x
/(d*x^3+c)*a-1/3*d^2/(a*d-b*c)^3*x/(d*x^3+c)*b+2/9*d^2/(a*d-b*c)^3/c/(c/d)^
(2/3)*ln(x+(c/d)^(1/3))*a-8/9*d/(a*d-b*c)^3/(c/d)^(2/3)*ln(x+(c/d)^(1/3))*b
-1/9*d^2/(a*d-b*c)^3/c/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*a+4/9*
d/(a*d-b*c)^3/(c/d)^(2/3)*ln(x^2-(c/d)^(1/3)*x+(c/d)^(2/3))*b+2/9*d^2/(a*d-
b*c)^3/c/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*a-8/9*
d/(a*d-b*c)^3/(c/d)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(c/d)^(1/3)*x-1))*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**2/(d*x**3+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.65298, size = 896, normalized size = 2.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^2/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] -2/9*(b^3*c - 4*a*b^2*d)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b^3*c
^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3) - 2/9*(4*b*c*d^2 - a*d^3)*(
-c/d)^(1/3)*log(abs(x - (-c/d)^(1/3)))/(b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c
^3*d^2 - a^3*c^2*d^3) + 2/3*((-a*b^2)^(1/3)*b^2*c - 4*(-a*b^2)^(1/3)*a*b*d)
*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)*a^2*b^3*c^3
 - 3*sqrt(3)*a^3*b^2*c^2*d + 3*sqrt(3)*a^4*b*c*d^2 - sqrt(3)*a^5*d^3) + 2/3
*(4*(-c*d^2)^(1/3)*b*c*d - (-c*d^2)^(1/3)*a*d^2)*arctan(1/3*sqrt(3)*(2*x +
(-c/d)^(1/3))/(-c/d)^(1/3))/(sqrt(3)*b^3*c^5 - 3*sqrt(3)*a*b^2*c^4*d + 3*sq
rt(3)*a^2*b*c^3*d^2 - sqrt(3)*a^3*c^2*d^3) + 1/9*((-a*b^2)^(1/3)*b^2*c - 4*
(-a*b^2)^(1/3)*a*b*d)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/(a^2*b^3*c^3
 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3) + 1/9*(4*(-c*d^2)^(1/3)*b*c*d
 - (-c*d^2)^(1/3)*a*d^2)*log(x^2 + x*(-c/d)^(1/3) + (-c/d)^(2/3))/(b^3*c^5
- 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3) + 1/3*(b^2*c*d*x^4 + a*b*d
^2*x^4 + b^2*c^2*x + a^2*d^2*x)/((b*d*x^6 + b*c*x^3 + a*d*x^3 + a*c)*(a*b^2
*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2))
```

3.27 $\int \left(a-bx^3\right)\left(a+bx^3\right)^{2/3}\,dx$

Optimal. Leaf size=112

$$-\frac{7a^2\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{18\sqrt[3]{b}}+\frac{7a^2\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\sqrt[3]{b}}+\frac{7}{18}ax\left(a+bx^3\right)^{2/3}-\frac{1}{6}x\left(a+bx^3\right)^{5/3}$$

```
[Out] (7*a*x*(a + b*x^3)^(2/3))/18 - (x*(a + b*x^3)^(5/3))/6 + (7*a^2*ArcTan[(1 +
 (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(1/3)) - (7*a^2*Lo
g[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(18*b^(1/3))
```

Rubi [A] time = 0.0322832, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {388, 195, 239}

$$-\frac{7a^2\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{18\sqrt[3]{b}}+\frac{7a^2\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\sqrt[3]{b}}+\frac{7}{18}ax\left(a+bx^3\right)^{2/3}-\frac{1}{6}x\left(a+bx^3\right)^{5/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)*(a + b*x^3)^(2/3),x]
```

```
[Out] (7*a*x*(a + b*x^3)^(2/3))/18 - (x*(a + b*x^3)^(5/3))/6 + (7*a^2*ArcTan[(1 +
 (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(1/3)) - (7*a^2*Lo
g[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(18*b^(1/3))
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int (a-bx^3)(a+bx^3)^{2/3}\,dx = -\frac{1}{6}x\left(a+bx^3\right)^{5/3}+\frac{1}{6}(7a)\int \left(a+bx^3\right)^{2/3}\,dx$$
$$=\frac{7}{18}ax\left(a+bx^3\right)^{2/3}-\frac{1}{6}x\left(a+bx^3\right)^{5/3}+\frac{1}{9}\left(7a^2\right)\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx$$
$$=\frac{7}{18}ax\left(a+bx^3\right)^{2/3}-\frac{1}{6}x\left(a+bx^3\right)^{5/3}+\frac{7a^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}\sqrt[3]{b}}-\frac{7a^2\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+b\right.}{18\sqrt[3]{b}}$$

Mathematica [C] time = 0.066334, size = 62, normalized size = 0.55

$$\frac{1}{6}x\left(a+bx^3\right)^{2/3}\left(\frac{7a\,{}_2F_1\left(-\frac{2}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\left(\frac{bx^3}{a}+1\right)^{2/3}}-a-bx^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)*(a + b*x^3)^(2/3),x]
```

```
[Out] (x*(a + b*x^3)^(2/3)*(-a - b*x^3 + (7*a*Hypergeometric2F1[-2/3, 1/3, 4/3, -
((b*x^3)/a)])/(1 + (b*x^3)/a)^(2/3)))/6
```

Maple [F] time = 0.295, size = 0, normalized size = 0.

$$\int \left(-bx^3+a\right)\left(bx^3+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)*(b*x^3+a)^(2/3),x)
```

```
[Out] int((-b*x^3+a)*(b*x^3+a)^(2/3),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)*(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.66543, size = 1022, normalized size = 9.12

$$\left[\frac{21\,\sqrt{\frac{1}{3}}a^2b\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3\,bx^3-3\left(bx^3+a\right)^{\frac{1}{3}}(-b)^{\frac{2}{3}}x^2-3\,\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}bx^3-\left(bx^3+a\right)^{\frac{1}{3}}bx^2+2\left(bx^3+a\right)^{\frac{2}{3}}(-b)^{\frac{2}{3}}x\right)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}+\cdots}{54\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)*(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] [1/54*(21*sqrt(1/3)*a^2*b*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1
/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^
2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) - 14*a^2*(-
b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + 7*a^2*(-b)^(2/3)*log((
(-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) -
 3*(3*b^2*x^4 - 4*a*b*x)*(b*x^3 + a)^(2/3))/b, -1/54*(42*sqrt(1/3)*a^2*b*sq
rt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sq
rt(-(-b)^(1/3)/b)/x) + 14*a^2*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1
/3))/x) - 7*a^2*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/
3)*x + (b*x^3 + a)^(2/3))/x^2) + 3*(3*b^2*x^4 - 4*a*b*x)*(b*x^3 + a)^(2/3))
/b]
```

Sympy [C] time = 5.08027, size = 80, normalized size = 0.71

$$\frac{a^{\frac{5}{3}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}-\frac{a^{\frac{2}{3}}bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)*(b*x**3+a)**(2/3),x)
```

```
[Out] a**(5/3)*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(4/3)) - a**(2/3)*b*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/3,), b*x*
*3*exp_polar(I*pi)/a)/(3*gamma(7/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\left(bx^3+a\right)^{\frac{2}{3}}\left(bx^3-a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)*(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(2/3)*(b*x^3 - a), x)
```

3.28 $\int \frac{a-bx^3}{\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=91

$$-\frac{1}{3}x\left(a+bx^3\right)^{2/3}-\frac{2a\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{3\sqrt[3]{b}}+\frac{4a\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{b}}$$

```
[Out] -(x*(a + b*x^3)^(2/3))/3 + (4*a*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3)
)/Sqrt[3]])/(3*Sqrt[3]*b^(1/3)) - (2*a*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)
])/(3*b^(1/3))
```

Rubi [A] time = 0.0195956, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {388, 239}

$$-\frac{1}{3}x\left(a+bx^3\right)^{2/3}-\frac{2a\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{3\sqrt[3]{b}}+\frac{4a\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)/(a + b*x^3)^(1/3),x]
```


```
[Out] -(x*(a + b*x^3)^(2/3))/3 + (4*a*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3)
)/Sqrt[3]])/(3*Sqrt[3]*b^(1/3)) - (2*a*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)
])/(3*b^(1/3))
```


Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a-bx^3}{\sqrt[3]{a+bx^3}}\,dx &= -\frac{1}{3}x\left(a+bx^3\right)^{2/3}+\frac{1}{3}(4a)\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx\\ &= -\frac{1}{3}x\left(a+bx^3\right)^{2/3}+\frac{4a\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{b}}-\frac{2a\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{3\sqrt[3]{b}}\end{aligned}$$

Mathematica [A] time = 0.0746889, size = 134, normalized size = 1.47

$$\frac{2a\log\left(\frac{b^{2/3}x^2}{(a+bx^3)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right)-3\sqrt[3]{b}x\left(a+bx^3\right)^{2/3}-4a\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)+4\sqrt{3}a\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)/(a + b*x^3)^(1/3),x]
```

```
[Out] (-3*b^(1/3)*x*(a + b*x^3)^(2/3) + 4*Sqrt[3]*a*ArcTan[(1 + (2*b^(1/3)*x)/(a
+ b*x^3)^(1/3))/Sqrt[3]] - 4*a*Log[1 - (b^(1/3)*x)/(a + b*x^3)^(1/3)] + 2*a
*Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)])/
(9*b^(1/3))
```

Maple [F] time = 0.229, size = 0, normalized size = 0.

$$\int (-bx^3+a)\frac{1}{\sqrt[3]{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)/(b*x^3+a)^(1/3),x)
```

```
[Out] int((-b*x^3+a)/(b*x^3+a)^(1/3),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.71535, size = 950, normalized size = 10.44

$$\left[\frac{6\sqrt{\frac{1}{3}}ab\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3bx^3-3\left(bx^3+a\right)^{\frac{1}{3}}(-b)^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}bx^3-\left(bx^3+a\right)^{\frac{1}{3}}bx^2+2\left(bx^3+a\right)^{\frac{2}{3}}(-b)^{\frac{2}{3}}x\right)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}+2\right.}{9b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/9*(6*sqrt(1/3)*a*b*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*
(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 +
2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) - 3*(b*x^3 + a)
^(2/3)*b*x - 4*a*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + 2*a
*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 +
 a)^(2/3))/x^2))/b, -1/9*(12*sqrt(1/3)*a*b*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt
(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sqrt(-(-b)^(1/3)/b)/x) + 3*(b*x^
3 + a)^(2/3)*b*x + 4*a*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x)
 - 2*a*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b
*x^3 + a)^(2/3))/x^2))/b]
```

Sympy [C] time = 3.37955, size = 76, normalized size = 0.84

$$\frac{a^{\frac{2}{3}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}-\frac{bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)/(b*x**3+a)**(1/3),x)
```

```
[Out] a**(2/3)*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(
3*gamma(4/3)) - b*x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), b*x**3*exp_pola
r(I*pi)/a)/(3*a**(1/3)*gamma(7/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{bx^3 - a}{\left(bx^3 + a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 - a)/(b*x^3 + a)^(1/3), x)
```

3.29 $\int \frac{a-bx^3}{\left(a+bx^3\right)^{4/3}}\,dx$

Optimal. Leaf size=85

$$\frac{2x}{\sqrt[3]{a+bx^3}}+\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}-\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}$$

```
[Out] (2*x)/(a + b*x^3)^(1/3) - ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt
[3]]/(Sqrt[3]*b^(1/3)) + Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))
```

Rubi [A] time = 0.0125869, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {385, 239}

$$\frac{2x}{\sqrt[3]{a+bx^3}}+\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}-\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)/(a + b*x^3)^(4/3),x]
```

```
[Out] (2*x)/(a + b*x^3)^(1/3) - ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt
[3]]/(Sqrt[3]*b^(1/3)) + Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{a-bx^3}{\left(a+bx^3\right)^{4/3}}\,dx &= \frac{2x}{\sqrt[3]{a+bx^3}}-\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx\\ &= \frac{2x}{\sqrt[3]{a+bx^3}}-\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}+\frac{\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}}\end{aligned}$$

Mathematica [C] time = 0.0364964, size = 62, normalized size = 0.73

$$\frac{4ax - bx^4\sqrt[3]{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{4}{3},\frac{4}{3};\frac{7}{3};-\frac{bx^3}{a}\right)}{4a\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)/(a + b*x^3)^(4/3),x]
```

```
[Out] (4*a*x - b*x^4*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[4/3, 4/3, 7/3, -((b*
x^3)/a)])/(4*a*(a + b*x^3)^(1/3))
```

Maple [F] time = 0.226, size = 0, normalized size = 0.

$$\int \left(-bx^3+a\right)\left(bx^3+a\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)/(b*x^3+a)^(4/3),x)
```

```
[Out] int((-b*x^3+a)/(b*x^3+a)^(4/3),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.82697, size = 976, normalized size = 11.48

$$\left[\frac{3\sqrt{\frac{1}{3}}\left(b^2x^3+ab\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3\,bx^3-3\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^3+\left(bx^3+a\right)^{\frac{1}{3}}bx^2-2\left(bx^3+a\right)^{\frac{2}{3}}b^{\frac{2}{3}}x\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}+2\,a\right.}{6\left(b^2x^3+ab\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*(b^2*x^3 + a*b)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 +
 a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2
```

```
- 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 12*(b*x^3 + a)^(
2/3)*b*x + 2*(b*x^3 + a)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) -
(b*x^3 + a)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3
 + a)^(2/3))/x^2))/(b^2*x^3 + a*b), 1/6*(12*(b*x^3 + a)^(2/3)*b*x + 2*(b*x^
3 + a)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - (b*x^3 + a)*b^(2/3
)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2)
+ 6*sqrt(1/3)*(b^2*x^3 + a*b)*arctan(sqrt(1/3)*(b^(1/3)*x + 2*(b*x^3 + a)^(
1/3))/(b^(1/3)*x))/b^(1/3))/(b^2*x^3 + a*b)]
```

Sympy [C] time = 12.763, size = 70, normalized size = 0.82

$$\frac{x\Gamma\left(\frac{1}{3}\right)}{3\sqrt[3]{a}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{4}{3}\right)} - \frac{bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)/(b*x**3+a)**(4/3),x)
```

```
[Out] x*gamma(1/3)/(3*a**(1/3)*(1 + b*x**3/a)**(1/3)*gamma(4/3)) - b*x**4*gamma(4
/3)*hyper((4/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(4/3)*gamma(7
/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{bx^3-a}{\left(bx^3+a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 - a)/(b*x^3 + a)^(4/3), x)
```

3.30 $\int \frac{a-bx^3}{\left(a+bx^3\right)^{7/3}}\,dx$

Optimal. Leaf size=47

$$\frac{3x}{4a\sqrt[3]{a+bx^3}}+\frac{x\left(a-bx^3\right)}{4a\left(a+bx^3\right)^{4/3}}$$

```
[Out] (x*(a - b*x^3))/(4*a*(a + b*x^3)^(4/3)) + (3*x)/(4*a*(a + b*x^3)^(1/3))
```

Rubi [A] time = 0.0093022, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {378, 191}

$$\frac{3x}{4a\sqrt[3]{a+bx^3}}+\frac{x\left(a-bx^3\right)}{4a\left(a+bx^3\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)/(a + b*x^3)^(7/3),x]
```

```
[Out] (x*(a - b*x^3))/(4*a*(a + b*x^3)^(4/3)) + (3*x)/(4*a*(a + b*x^3)^(1/3))
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a-bx^3}{\left(a+bx^3\right)^{7/3}}\,dx &= \frac{x\left(a-bx^3\right)}{4a\left(a+bx^3\right)^{4/3}}+\frac{3}{4}\int \frac{1}{\left(a+bx^3\right)^{4/3}}\,dx\\ &= \frac{x\left(a-bx^3\right)}{4a\left(a+bx^3\right)^{4/3}}+\frac{3x}{4a\sqrt[3]{a+bx^3}}\end{aligned}$$

Mathematica [A] time = 0.0168929, size = 28, normalized size = 0.6

$$\frac{x\left(2a+bx^3\right)}{2a\left(a+bx^3\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)/(a + b*x^3)^(7/3),x]

[Out] (x*(2*a + b*x^3))/(2*a*(a + b*x^3)^(4/3))
```

Maple [A] time = 0.004, size = 25, normalized size = 0.5

$$\frac{x\left(bx^3+2\,a\right)}{2\,a}\left(bx^3+a\right)^{-\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)/(b*x^3+a)^(7/3),x)

[Out] 1/2*x*(b*x^3+2*a)/(b*x^3+a)^(4/3)/a
```

Maxima [A] time = 1.09459, size = 68, normalized size = 1.45

$$-\frac{\left(b-\frac{4\left(bx^3+a\right)}{x^3}\right)x^4}{4\left(bx^3+a\right)^{\frac{4}{3}}a}-\frac{bx^4}{4\left(bx^3+a\right)^{\frac{4}{3}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(7/3),x, algorithm="maxima")

[Out] -1/4*(b - 4*(b*x^3 + a)/x^3)*x^4/((b*x^3 + a)^(4/3)*a) - 1/4*b*x^4/((b*x^3
+ a)^(4/3)*a)
```

Fricas [A] time = 1.61742, size = 96, normalized size = 2.04

$$\frac{\left(bx^4+2\,ax\right)\left(bx^3+a\right)^{\frac{2}{3}}}{2\left(ab^2x^6+2\,a^2bx^3+a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(7/3),x, algorithm="fricas")

[Out] 1/2*(b*x^4 + 2*a*x)*(b*x^3 + a)^(2/3)/(a*b^2*x^6 + 2*a^2*b*x^3 + a^3)
```

Sympy [B] time = 100.391, size = 190, normalized size = 4.04

$$a\left(\frac{4ax\Gamma\left(\frac{1}{3}\right)}{9a^{\frac{10}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)+9a^{\frac{7}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)}+\frac{3bx^4\Gamma\left(\frac{1}{3}\right)}{9a^{\frac{10}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)+9a^{\frac{7}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)}\right)-\frac{bx^4\Gamma(}{3a^{\frac{7}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)+3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)/(b*x**3+a)**(7/3),x)
```

```
[Out] a*(4*a*x*gamma(1/3)/(9*a**(10/3)*(1 + b*x**3/a)**(1/3)*gamma(7/3) + 9*a**(7
/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(7/3)) + 3*b*x**4*gamma(1/3)/(9*a**(1
0/3)*(1 + b*x**3/a)**(1/3)*gamma(7/3) + 9*a**(7/3)*b*x**3*(1 + b*x**3/a)**(
1/3)*gamma(7/3))) - b*x**4*gamma(4/3)/(3*a**(7/3)*(1 + b*x**3/a)**(1/3)*gam
ma(7/3) + 3*a**(4/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(7/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{bx^3 - a}{\left(bx^3 + a\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(7/3),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 - a)/(b*x^3 + a)^(7/3), x)
```

3.31 $\int \frac{a-bx^3}{\left(a+bx^3\right)^{10/3}}\,dx$

Optimal. Leaf size=55

$$\frac{15x}{28a^2\sqrt[3]{a+bx^3}}+\frac{5x}{28a\left(a+bx^3\right)^{4/3}}+\frac{2x}{7\left(a+bx^3\right)^{7/3}}$$

```
[Out] (2*x)/(7*(a + b*x^3)^(7/3)) + (5*x)/(28*a*(a + b*x^3)^(4/3)) + (15*x)/(28*a
^2*(a + b*x^3)^(1/3))
```

Rubi [A] time = 0.0131894, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {385, 192, 191}

$$\frac{15x}{28a^2\sqrt[3]{a+bx^3}}+\frac{5x}{28a\left(a+bx^3\right)^{4/3}}+\frac{2x}{7\left(a+bx^3\right)^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)/(a + b*x^3)^(10/3),x]
```

```
[Out] (2*x)/(7*(a + b*x^3)^(7/3)) + (5*x)/(28*a*(a + b*x^3)^(4/3)) + (15*x)/(28*a
^2*(a + b*x^3)^(1/3))
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{a-bx^3}{\left(a+bx^3\right)^{10/3}}\,dx = \frac{2x}{7\left(a+bx^3\right)^{7/3}} + \frac{5}{7}\int \frac{1}{\left(a+bx^3\right)^{7/3}}\,dx$$

$$= \frac{2x}{7\left(a+bx^3\right)^{7/3}} + \frac{5x}{28a\left(a+bx^3\right)^{4/3}} + \frac{15\int \frac{1}{\left(a+bx^3\right)^{4/3}}\,dx}{28a}$$

$$= \frac{2x}{7\left(a+bx^3\right)^{7/3}} + \frac{5x}{28a\left(a+bx^3\right)^{4/3}} + \frac{15x}{28a^2\sqrt[3]{a+bx^3}}$$

Mathematica [A] time = 0.0190547, size = 40, normalized size = 0.73

$$\frac{x\left(28a^2+35abx^3+15b^2x^6\right)}{28a^2\left(a+bx^3\right)^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)/(a + b*x^3)^(10/3),x]
```

```
[Out] (x*(28*a^2 + 35*a*b*x^3 + 15*b^2*x^6))/(28*a^2*(a + b*x^3)^(7/3))
```

Maple [A] time = 0.003, size = 37, normalized size = 0.7

$$\frac{x\left(15\,b^2x^6+35\,bx^3a+28\,a^2\right)}{28\,a^2}\left(bx^3+a\right)^{-\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)/(b*x^3+a)^(10/3),x)
```

```
[Out] 1/28*x*(15*b^2*x^6+35*a*b*x^3+28*a^2)/(b*x^3+a)^(7/3)/a^2
```

Maxima [A] time = 0.972712, size = 115, normalized size = 2.09

$$\frac{\left(4\,b-\frac{7\,\left(bx^3+a\right)}{x^3}\right)bx^7}{28\,\left(bx^3+a\right)^{\frac{7}{3}}a^2} + \frac{\left(2\,b^2-\frac{7\,\left(bx^3+a\right)b}{x^3}+\frac{14\,\left(bx^3+a\right)^2}{x^6}\right)x^7}{14\,\left(bx^3+a\right)^{\frac{7}{3}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(10/3),x, algorithm="maxima")
```

```
[Out] 1/28*(4*b - 7*(b*x^3 + a)/x^3)*b*x^7/((b*x^3 + a)^(7/3)*a^2) + 1/14*(2*b^2
- 7*(b*x^3 + a)*b/x^3 + 14*(b*x^3 + a)^2/x^6)*x^7/((b*x^3 + a)^(7/3)*a^2)
```

Fricas [A] time = 1.74418, size = 150, normalized size = 2.73

$$\frac{\left(15\,b^2x^7+35\,abx^4+28\,a^2x\right)\left(bx^3+a\right)^{\frac{2}{3}}}{28\,\left(a^2b^3x^9+3\,a^3b^2x^6+3\,a^4bx^3+a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(10/3),x, algorithm="fricas")
```

```
[Out] 1/28*(15*b^2*x^7 + 35*a*b*x^4 + 28*a^2*x)*(b*x^3 + a)^(2/3)/(a^2*b^3*x^9 +
3*a^3*b^2*x^6 + 3*a^4*b*x^3 + a^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)/(b*x**3+a)**(10/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{bx^3 - a}{\left(bx^3 + a\right)^{\frac{10}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(10/3),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 - a)/(b*x^3 + a)^(10/3), x)
```

3.32 $\int \frac{a-bx^3}{\left(a+bx^3\right)^{13/3}}\,dx$

Optimal. Leaf size=74

$$\frac{18x}{35a^3\sqrt[3]{a+bx^3}}+\frac{6x}{35a^2\left(a+bx^3\right)^{4/3}}+\frac{4x}{35a\left(a+bx^3\right)^{7/3}}+\frac{x}{5\left(a+bx^3\right)^{10/3}}$$

```
[Out] x/(5*(a + b*x^3)^(10/3)) + (4*x)/(35*a*(a + b*x^3)^(7/3)) + (6*x)/(35*a^2*(
a + b*x^3)^(4/3)) + (18*x)/(35*a^3*(a + b*x^3)^(1/3))
```

Rubi [A] time = 0.0189451, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {385, 192, 191}

$$\frac{18x}{35a^3\sqrt[3]{a+bx^3}}+\frac{6x}{35a^2\left(a+bx^3\right)^{4/3}}+\frac{4x}{35a\left(a+bx^3\right)^{7/3}}+\frac{x}{5\left(a+bx^3\right)^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)/(a + b*x^3)^(13/3),x]
```

```
[Out] x/(5*(a + b*x^3)^(10/3)) + (4*x)/(35*a*(a + b*x^3)^(7/3)) + (6*x)/(35*a^2*(
a + b*x^3)^(4/3)) + (18*x)/(35*a^3*(a + b*x^3)^(1/3))
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{a-bx^3}{(a+bx^3)^{13/3}}\,dx = \frac{x}{5(a+bx^3)^{10/3}}+\frac{4}{5}\int \frac{1}{(a+bx^3)^{10/3}}\,dx$$
$$= \frac{x}{5(a+bx^3)^{10/3}}+\frac{4x}{35a(a+bx^3)^{7/3}}+\frac{24\int \frac{1}{(a+bx^3)^{7/3}}\,dx}{35a}$$
$$= \frac{x}{5(a+bx^3)^{10/3}}+\frac{4x}{35a(a+bx^3)^{7/3}}+\frac{6x}{35a^2(a+bx^3)^{4/3}}+\frac{18\int \frac{1}{(a+bx^3)^{4/3}}\,dx}{35a^2}$$
$$= \frac{x}{5(a+bx^3)^{10/3}}+\frac{4x}{35a(a+bx^3)^{7/3}}+\frac{6x}{35a^2(a+bx^3)^{4/3}}+\frac{18x}{35a^3\sqrt[3]{a+bx^3}}$$

Mathematica [A] time = 0.0210671, size = 51, normalized size = 0.69

$$\frac{x\left(70a^2bx^3+35a^3+60ab^2x^6+18b^3x^9\right)}{35a^3(a+bx^3)^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)/(a + b*x^3)^(13/3),x]
```

```
[Out] (x*(35*a^3 + 70*a^2*b*x^3 + 60*a*b^2*x^6 + 18*b^3*x^9))/(35*a^3*(a + b*x^3)
^(10/3))
```

Maple [A] time = 0.003, size = 48, normalized size = 0.7

$$\frac{x\left(18\,b^3x^9+60\,b^2x^6a+70\,bx^3a^2+35\,a^3\right)}{35\,a^3}\left(bx^3+a\right)^{-\frac{10}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)/(b*x^3+a)^(13/3),x)
```

```
[Out] 1/35*x*(18*b^3*x^9+60*a*b^2*x^6+70*a^2*b*x^3+35*a^3)/(b*x^3+a)^(10/3)/a^3
```

Maxima [B] time = 1.1912, size = 161, normalized size = 2.18

$$-\frac{\left(14\,b^2-\frac{40\,(bx^3+a)b}{x^3}+\frac{35\,(bx^3+a)^2}{x^6}\right)bx^{10}}{140\,(bx^3+a)^{\frac{10}{3}}a^3}-\frac{\left(14\,b^3-\frac{60\,(bx^3+a)b^2}{x^3}+\frac{105\,(bx^3+a)^2b}{x^6}-\frac{140\,(bx^3+a)^3}{x^9}\right)x^{10}}{140\,(bx^3+a)^{\frac{10}{3}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(13/3),x, algorithm="maxima")
```

```
[Out] -1/140*(14*b^2 - 40*(b*x^3 + a)*b/x^3 + 35*(b*x^3 + a)^2/x^6)*b*x^10/((b*x^
3 + a)^(10/3)*a^3) - 1/140*(14*b^3 - 60*(b*x^3 + a)*b^2/x^3 + 105*(b*x^3 +
```

```
a)^2*b/x^6 - 140*(b*x^3 + a)^3/x^9)*x^10/((b*x^3 + a)^(10/3)*a^3)
```

Fricas [A] time = 1.53024, size = 197, normalized size = 2.66

$$\frac{\left(18\,b^{3}x^{10}+60\,ab^{2}x^{7}+70\,a^{2}bx^{4}+35\,a^{3}x\right)\left(bx^{3}+a\right)^{\frac{2}{3}}}{35\left(a^{3}b^{4}x^{12}+4\,a^{4}b^{3}x^{9}+6\,a^{5}b^{2}x^{6}+4\,a^{6}bx^{3}+a^{7}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(13/3),x, algorithm="fricas")
```

```
[Out] 1/35*(18*b^3*x^10 + 60*a*b^2*x^7 + 70*a^2*b*x^4 + 35*a^3*x)*(b*x^3 + a)^(2/
3)/(a^3*b^4*x^12 + 4*a^4*b^3*x^9 + 6*a^5*b^2*x^6 + 4*a^6*b*x^3 + a^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)/(b*x**3+a)**(13/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{bx^{3}-a}{\left(bx^{3}+a\right)^{\frac{13}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(13/3),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 - a)/(b*x^3 + a)^(13/3), x)
```

3.33 $\int \frac{a-bx^3}{\left(a+bx^3\right)^{16/3}}\,dx$

Optimal. Leaf size=93

$$\frac{891x}{1820a^4\sqrt[3]{a+bx^3}}+\frac{297x}{1820a^3\left(a+bx^3\right)^{4/3}}+\frac{99x}{910a^2\left(a+bx^3\right)^{7/3}}+\frac{11x}{130a\left(a+bx^3\right)^{10/3}}+\frac{2x}{13\left(a+bx^3\right)^{13/3}}$$

```
[Out] (2*x)/(13*(a + b*x^3)^(13/3)) + (11*x)/(130*a*(a + b*x^3)^(10/3)) + (99*x)/
(910*a^2*(a + b*x^3)^(7/3)) + (297*x)/(1820*a^3*(a + b*x^3)^(4/3)) + (891*x
)/(1820*a^4*(a + b*x^3)^(1/3))
```

Rubi [A] time = 0.0263103, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {385, 192, 191}

$$\frac{891x}{1820a^4\sqrt[3]{a+bx^3}}+\frac{297x}{1820a^3\left(a+bx^3\right)^{4/3}}+\frac{99x}{910a^2\left(a+bx^3\right)^{7/3}}+\frac{11x}{130a\left(a+bx^3\right)^{10/3}}+\frac{2x}{13\left(a+bx^3\right)^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)/(a + b*x^3)^(16/3),x]
```


```
[Out] (2*x)/(13*(a + b*x^3)^(13/3)) + (11*x)/(130*a*(a + b*x^3)^(10/3)) + (99*x)/
(910*a^2*(a + b*x^3)^(7/3)) + (297*x)/(1820*a^3*(a + b*x^3)^(4/3)) + (891*x
)/(1820*a^4*(a + b*x^3)^(1/3))
```


Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{a-bx^3}{\left(a+bx^3\right)^{16/3}}\,dx = \frac{2x}{13\left(a+bx^3\right)^{13/3}} + \frac{11}{13}\int \frac{1}{\left(a+bx^3\right)^{13/3}}\,dx$$

$$= \frac{2x}{13\left(a+bx^3\right)^{13/3}} + \frac{11x}{130a\left(a+bx^3\right)^{10/3}} + \frac{99\int \frac{1}{\left(a+bx^3\right)^{10/3}}\,dx}{130a}$$

$$= \frac{2x}{13\left(a+bx^3\right)^{13/3}} + \frac{11x}{130a\left(a+bx^3\right)^{10/3}} + \frac{99x}{910a^2\left(a+bx^3\right)^{7/3}} + \frac{297\int \frac{1}{\left(a+bx^3\right)^{7/3}}\,dx}{455a^2}$$

$$= \frac{2x}{13\left(a+bx^3\right)^{13/3}} + \frac{11x}{130a\left(a+bx^3\right)^{10/3}} + \frac{99x}{910a^2\left(a+bx^3\right)^{7/3}} + \frac{297x}{1820a^3\left(a+bx^3\right)^{4/3}} + \frac{891\int \frac{1}{\left(a+bx^{3}\right.}}{1820a}$$

$$= \frac{2x}{13\left(a+bx^3\right)^{13/3}} + \frac{11x}{130a\left(a+bx^3\right)^{10/3}} + \frac{99x}{910a^2\left(a+bx^3\right)^{7/3}} + \frac{297x}{1820a^3\left(a+bx^3\right)^{4/3}} + \frac{891x}{1820a^4\sqrt[3]{a}}$$

Mathematica [A] time = 0.0231695, size = 62, normalized size = 0.67

$$\frac{x\left(6435a^2b^2x^6+5005a^3bx^3+1820a^4+3861ab^3x^9+891b^4x^{12}\right)}{1820a^4\left(a+bx^3\right)^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)/(a + b*x^3)^(16/3),x]
```

```
[Out] (x*(1820*a^4 + 5005*a^3*b*x^3 + 6435*a^2*b^2*x^6 + 3861*a*b^3*x^9 + 891*b^4
*x^12))/(1820*a^4*(a + b*x^3)^(13/3))
```

Maple [A] time = 0.004, size = 59, normalized size = 0.6

$$\frac{x\left(891\,b^4x^{12}+3861\,b^3x^9a+6435\,b^2x^6a^2+5005\,bx^3a^3+1820\,a^4\right)}{1820\,a^4}\left(bx^3+a\right)^{-\frac{13}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)/(b*x^3+a)^(16/3),x)
```

```
[Out] 1/1820*x*(891*b^4*x^12+3861*a*b^3*x^9+6435*a^2*b^2*x^6+5005*a^3*b*x^3+1820*
a^4)/(b*x^3+a)^(13/3)/a^4
```

Maxima [B] time = 1.38391, size = 207, normalized size = 2.23

$$\frac{\left(140\,b^3-\frac{546\left(bx^3+a\right)b^2}{x^3}+\frac{780\left(bx^3+a\right)^2b}{x^6}-\frac{455\left(bx^3+a\right)^3}{x^9}\right)bx^{13}}{1820\left(bx^3+a\right)^{\frac{13}{3}}a^4}+\frac{\left(35\,b^4-\frac{182\left(bx^3+a\right)b^3}{x^3}+\frac{390\left(bx^3+a\right)^2b^2}{x^6}-\frac{455\left(bx^3+a\right)^3b}{x^9}+\frac{455\left(bx^3+a\right.}{x^{12}}\right.}{455\left(bx^3+a\right)^{\frac{13}{3}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(16/3),x, algorithm="maxima")
```

```
[Out] 1/1820*(140*b^3 - 546*(b*x^3 + a)*b^2/x^3 + 780*(b*x^3 + a)^2*b/x^6 - 455*(
b*x^3 + a)^3/x^9)*b*x^13/((b*x^3 + a)^(13/3)*a^4) + 1/455*(35*b^4 - 182*(b*
x^3 + a)*b^3/x^3 + 390*(b*x^3 + a)^2*b^2/x^6 - 455*(b*x^3 + a)^3*b/x^9 + 45
5*(b*x^3 + a)^4/x^12)*x^13/((b*x^3 + a)^(13/3)*a^4)
```

Fricas [A] time = 1.48221, size = 262, normalized size = 2.82

$$\frac{\left(891\,b^{4}x^{13} + 3861\,ab^{3}x^{10} + 6435\,a^{2}b^{2}x^{7} + 5005\,a^{3}bx^{4} + 1820\,a^{4}x\right)\left(bx^{3} + a\right)^{\frac{2}{3}}}{1820\,\left(a^{4}b^{5}x^{15} + 5\,a^{5}b^{4}x^{12} + 10\,a^{6}b^{3}x^{9} + 10\,a^{7}b^{2}x^{6} + 5\,a^{8}bx^{3} + a^{9}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(16/3),x, algorithm="fricas")
```

```
[Out] 1/1820*(891*b^4*x^13 + 3861*a*b^3*x^10 + 6435*a^2*b^2*x^7 + 5005*a^3*b*x^4
+ 1820*a^4*x)*(b*x^3 + a)^(2/3)/(a^4*b^5*x^15 + 5*a^5*b^4*x^12 + 10*a^6*b^3
*x^9 + 10*a^7*b^2*x^6 + 5*a^8*b*x^3 + a^9)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)/(b*x**3+a)**(16/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{bx^{3} - a}{\left(bx^{3} + a\right)^{\frac{16}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)/(b*x^3+a)^(16/3),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 - a)/(b*x^3 + a)^(16/3), x)
```

3.34 $\int \frac{(a+bx^3)^{7/3}}{a-bx^3}\,dx$

Optimal. Leaf size=483

$$-\frac{7a^2x\left(\frac{bx^3}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5\left(a+bx^3\right)^{2/3}}-\frac{2\sqrt[3]{2}a^{5/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}}+\frac{2\sqrt[3]{2}a^{5/3}\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\sqrt[3]{b}}$$

```
[Out] (-7*a*x*(a + b*x^3)^(1/3))/5 - (x*(a + b*x^3)^(4/3))/5 - (4*2^(1/3)*a^(5/3)
*ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])
/(Sqrt[3]*b^(1/3)) - (2*2^(1/3)*a^(5/3)*ArcTan[(1 + (2^(1/3)*(a^(1/3) + b^(
1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3)) - (7*a^2*x*(1 + (b*
x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(5*(a + b*x^3
)^(2/3)) - (2*2^(1/3)*a^(5/3)*Log[2^(2/3) - (a^(1/3) + b^(1/3)*x)/(a + b*x^
3)^(1/3)])/(3*b^(1/3)) + (2*2^(1/3)*a^(5/3)*Log[1 + (2^(2/3)*(a^(1/3) + b^(
1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^
(1/3)])/(3*b^(1/3)) - (4*2^(1/3)*a^(5/3)*Log[1 + (2^(1/3)*(a^(1/3) + b^(1/3
)*x))/(a + b*x^3)^(1/3)])/(3*b^(1/3)) + (2^(1/3)*a^(5/3)*Log[2*2^(1/3) + (a
^(1/3) + b^(1/3)*x)^2/(a + b*x^3)^(2/3) + (2^(2/3)*(a^(1/3) + b^(1/3)*x))/(
a + b*x^3)^(1/3)])/(3*b^(1/3))
```

Rubi [C] time = 0.0278397, antiderivative size = 56, normalized size of antiderivative = 0.12, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {430, 429}

$$\frac{ax\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};1,-\frac{7}{3};\frac{4}{3};\frac{bx^3}{a},-\frac{bx^3}{a}\right)}{\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(7/3)/(a - b*x^3),x]
```

```
[Out] (a*x*(a + b*x^3)^(1/3)*AppellF1[1/3, 1, -7/3, 4/3, (b*x^3)/a, -((b*x^3)/a)]
)/(1 + (b*x^3)/a)^(1/3)
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{7/3}}{a-bx^3}\,dx = \frac{\left(a^2\sqrt[3]{a+bx^3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{7/3}}{a-bx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= \frac{ax\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};1,-\frac{7}{3};\frac{4}{3};\frac{bx^3}{a},-\frac{bx^3}{a}\right)}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [C] time = 0.272522, size = 232, normalized size = 0.48

$$\frac{4x\left(\frac{52a^4F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{\left(a-bx^3\right)\left(bx^3\left(3F_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)-2F_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)+4aF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)}-8a^2-9abx^3-b^2x^6\right)+27abx^4\left(\frac{bx^3}{a}+1\right)^{2/3}F_1}{20\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(7/3)/(a - b*x^3),x]
```

```
[Out] (27*a*b*x^4*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a),
(b*x^3)/a] + 4*x*(-8*a^2 - 9*a*b*x^3 - b^2*x^6 + (52*a^4*AppellF1[1/3, 2/3,
 1, 4/3, -((b*x^3)/a), (b*x^3)/a])/((a - b*x^3)*(4*a*AppellF1[1/3, 2/3, 1,
4/3, -((b*x^3)/a), (b*x^3)/a] + b*x^3*(3*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^
3)/a), (b*x^3)/a] - 2*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a]))
))))/(20*(a + b*x^3)^(2/3))
```

Maple [F] time = 0.592, size = 0, normalized size = 0.

$$\int \frac{1}{-bx^3+a}\left(bx^3+a\right)^{\frac{7}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(7/3)/(-b*x^3+a),x)
```

```
[Out] int((b*x^3+a)^(7/3)/(-b*x^3+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(bx^3+a\right)^{\frac{7}{3}}}{bx^3-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(7/3)/(-b*x^3+a),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(7/3)/(b*x^3 - a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(7/3)/(-b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(7/3)/(-b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(bx^3+a\right)^{\frac{7}{3}}}{bx^3-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(7/3)/(-b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(7/3)/(b*x^3 - a), x)
```

3.35 $\int \frac{(a+bx^3)^{4/3}}{a-bx^3} \, dx$

Optimal. Leaf size=464

$$-\frac{\sqrt[3]{2}a^{2/3}\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{bx}}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}}+\frac{\sqrt[3]{2}a^{2/3}\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\sqrt[3]{b}}-\frac{2\sqrt[3]{2}a^{2/3}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\sqrt[3]{b}}+\frac{a^{2/3}\log}{}$$

```
[Out] -(x*(a + b*x^3)^(1/3))/2 - (2*2^(1/3)*a^(2/3)*ArcTan[(1 - (2*2^(1/3)*(a^(1/
3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(1/3)) - (2^(1/3)*
a^(2/3)*ArcTan[(1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt
[3]])/(Sqrt[3]*b^(1/3)) - (a*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3,
 2/3, 4/3, -((b*x^3)/a)])/(2*(a + b*x^3)^(2/3)) - (2^(1/3)*a^(2/3)*Log[2^(2
/3) - (a^(1/3) + b^(1/3)*x)/(a + b*x^3)^(1/3)])/(3*b^(1/3)) + (2^(1/3)*a^(2
/3)*Log[1 + (2^(2/3)*(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(1/3)*
(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(3*b^(1/3)) - (2*2^(1/3)*a^(2/3)
*Log[1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(3*b^(1/3)) +
(a^(2/3)*Log[2*2^(1/3) + (a^(1/3) + b^(1/3)*x)^2/(a + b*x^3)^(2/3) + (2^(2/
3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(3*2^(2/3)*b^(1/3))
```

Rubi [C] time = 0.0261809, antiderivative size = 55, normalized size of antiderivative = 0.12, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {430, 429}

$$\frac{x\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};1,-\frac{4}{3};\frac{4}{3};\frac{bx^3}{a},-\frac{bx^3}{a}\right)}{\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(4/3)/(a - b*x^3),x]
```

```
[Out] (x*(a + b*x^3)^(1/3)*AppellF1[1/3, 1, -4/3, 4/3, (b*x^3)/a, -((b*x^3)/a)])/
(1 + (b*x^3)/a)^(1/3)
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{4/3}}{a-bx^3}\,dx = \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{a-bx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}} = \frac{x\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};1,-\frac{4}{3};\frac{4}{3};\frac{bx^3}{a},-\frac{bx^3}{a}\right)}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [C] time = 0.115087, size = 217, normalized size = 0.47

$$\frac{x\left(\frac{48a^3F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{\left(a-bx^3\right)\left(bx^3\left(3F_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)-2F_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)+4aF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)}+5bx^3\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)-\right.}{8\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(4/3)/(a - b*x^3),x]
```

```
[Out] (x*(-4*(a + b*x^3) + 5*b*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/
3, -((b*x^3)/a), (b*x^3)/a] + (48*a^3*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/
a), (b*x^3)/a])/((a - b*x^3)*(4*a*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a),
(b*x^3)/a] + b*x^3*(3*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), (b*x^3)/a] -
 2*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a])))))/(8*(a + b*x^3)^
(2/3))
```

Maple [F] time = 0.472, size = 0, normalized size = 0.

$$\int \frac{1}{-bx^3+a}\left(bx^3+a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/(-b*x^3+a),x)
```

```
[Out] int((b*x^3+a)^(4/3)/(-b*x^3+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{bx^3-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/(-b*x^3+a),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(4/3)/(b*x^3 - a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/(-b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{a\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx - \int \frac{bx^3\sqrt[3]{a+bx^3}}{-a+bx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/(-b*x**3+a),x)
```

```
[Out] -Integral(a*(a + b*x**3)**(1/3)/(-a + b*x**3), x) - Integral(b*x**3*(a + b*
x**3)**(1/3)/(-a + b*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(bx^3+a\right)^{\frac{4}{3}}}{bx^3-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/(-b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(4/3)/(b*x^3 - a), x)
```

3.36 $\int \frac{\sqrt[3]{a+bx^3}}{a-bx^3}\,dx$

Optimal. Leaf size=398

$$-\frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{3\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}}+\frac{\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{b}}-\frac{\sqrt[3]{2}\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\sqrt[3]{a}\sqrt[3]{b}}+\frac{\log\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}\right.}{\sqrt[3]{a+}}\right.}{6\ 2^{2/3}\sqrt[3]{a}\sqrt[3]{}}$$

```
[Out] -((2^(1/3)*ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))
/Sqrt[3]])/(Sqrt[3]*a^(1/3)*b^(1/3))) - ArcTan[(1 + (2^(1/3)*(a^(1/3) + b^(
1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]]/(2^(2/3)*Sqrt[3]*a^(1/3)*b^(1/3)) - Lo
g[2^(2/3) - (a^(1/3) + b^(1/3)*x)/(a + b*x^3)^(1/3)]/(3*2^(2/3)*a^(1/3)*b^(
1/3)) + Log[1 + (2^(2/3)*(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(1
/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(3*2^(2/3)*a^(1/3)*b^(1/3)) -
 (2^(1/3)*Log[1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)])/(3*a^
(1/3)*b^(1/3)) + Log[2*2^(1/3) + (a^(1/3) + b^(1/3)*x)^2/(a + b*x^3)^(2/3)
+ (2^(2/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(6*2^(2/3)*a^(1/3)*b^(
1/3))
```

Rubi [C] time = 0.0263875, antiderivative size = 58, normalized size of antiderivative = 0.15, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {430, 429}

$$\frac{x\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};1,-\frac{1}{3};\frac{4}{3};\frac{bx^3}{a},-\frac{bx^3}{a}\right)}{a\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(1/3)/(a - b*x^3),x]
```

```
[Out] (x*(a + b*x^3)^(1/3)*AppellF1[1/3, 1, -1/3, 4/3, (b*x^3)/a, -((b*x^3)/a)])/
(a*(1 + (b*x^3)/a)^(1/3))
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{a-bx^3}\,dx = \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{a-bx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= \frac{x\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};1,-\frac{1}{3};\frac{4}{3};\frac{bx^3}{a},-\frac{bx^3}{a}\right)}{a\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [C] time = 0.14327, size = 151, normalized size = 0.38

$$\frac{4ax\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{(a-bx^3)\left(bx^3\left(3F_1\left(\frac{4}{3};-\frac{1}{3},2;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)+F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)+4aF_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(a - b*x^3),x]
```

```
[Out] (4*a*x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/
a])/((a - b*x^3)*(4*a*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a]
+ b*x^3*(3*AppellF1[4/3, -1/3, 2, 7/3, -((b*x^3)/a), (b*x^3)/a] + AppellF1[
4/3, 2/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a])))
```

Maple [F] time = 0.42, size = 0, normalized size = 0.

$$\int \frac{1}{-bx^3+a}\sqrt[3]{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/(-b*x^3+a),x)
```

```
[Out] int((b*x^3+a)^(1/3)/(-b*x^3+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(bx^3+a)^{\frac{1}{3}}}{bx^3-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(-b*x^3+a),x, algorithm="maxima")
```

```
[Out] -integrate((b*x^3 + a)^(1/3)/(b*x^3 - a), x)
```

Fricas [B] time = 170.566, size = 1494, normalized size = 3.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(-b*x^3+a),x, algorithm="fricas")
```

```
[Out] -1/18*sqrt(3)*2^(1/3)*(-1/(a*b))^(1/3)*arctan(1/3*(6*sqrt(3)*2^(2/3)*(a*b^6
*x^16 + 33*a^2*b^5*x^13 + 110*a^3*b^4*x^10 + 110*a^4*b^3*x^7 + 33*a^5*b^2*x
^4 + a^6*b*x)*(b*x^3 + a)^(1/3)*(-1/(a*b))^(2/3) + 24*sqrt(3)*2^(1/3)*(a*b^
5*x^14 + 2*a^2*b^4*x^11 - 6*a^3*b^3*x^8 + 2*a^4*b^2*x^5 + a^5*b*x^2)*(b*x^3
 + a)^(2/3)*(-1/(a*b))^(1/3) - sqrt(3)*(b^6*x^18 - 42*a*b^5*x^15 - 417*a^2*
b^4*x^12 - 812*a^3*b^3*x^9 - 417*a^4*b^2*x^6 - 42*a^5*b*x^3 + a^6))/(b^6*x^
18 + 102*a*b^5*x^15 + 447*a^2*b^4*x^12 + 628*a^3*b^3*x^9 + 447*a^4*b^2*x^6
+ 102*a^5*b*x^3 + a^6)) - 1/36*2^(1/3)*(-1/(a*b))^(1/3)*log((12*2^(2/3)*(a*
b^3*x^8 + 4*a^2*b^2*x^5 + a^3*b*x^2)*(b*x^3 + a)^(2/3)*(-1/(a*b))^(2/3) - 2
^(1/3)*(b^4*x^12 + 32*a*b^3*x^9 + 78*a^2*b^2*x^6 + 32*a^3*b*x^3 + a^4)*(-1/
(a*b))^(1/3) + 6*(b^3*x^10 + 11*a*b^2*x^7 + 11*a^2*b*x^4 + a^3*x)*(b*x^3 +
a)^(1/3))/(b^4*x^12 - 4*a*b^3*x^9 + 6*a^2*b^2*x^6 - 4*a^3*b*x^3 + a^4)) + 1
/18*2^(1/3)*(-1/(a*b))^(1/3)*log(-(12*(b*x^3 + a)^(2/3)*x^2 + 2^(2/3)*(b^2*
x^6 - 2*a*b*x^3 + a^2)*(-1/(a*b))^(2/3) + 6*2^(1/3)*(b*x^4 + a*x)*(b*x^3 +
a)^(1/3)*(-1/(a*b))^(1/3))/(b^2*x^6 - 2*a*b*x^3 + a^2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt[3]{a + bx^3}}{-a + bx^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/(-b*x**3+a),x)
```

```
[Out] -Integral((a + b*x**3)**(1/3)/(-a + b*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(bx^3 + a\right)^{\frac{1}{3}}}{bx^3 - a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(-b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate(-(b*x^3 + a)^(1/3)/(b*x^3 - a), x)
```

3.37 $\int \frac{1}{(a-bx^3)(a+bx^3)^{2/3}} \, dx$

Optimal. Leaf size=452

$$-\frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{6\ 2^{2/3}a^{4/3}\sqrt[3]{b}}+\frac{\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{6\ 2^{2/3}a^{4/3}\sqrt[3]{b}}-\frac{\log\left(\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{3\ 2^{2/3}a^{4/3}\sqrt[3]{b}}+\frac{\log\left(\frac{\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}+\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}\right.}{12\ 2^{2/3}a^{4/3}\sqrt[3]{b}}$$

```
[Out] -(ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]]
/(2^(2/3)*Sqrt[3]*a^(4/3)*b^(1/3))) - ArcTan[(1 + (2^(1/3)*(a^(1/3) + b^(1/
3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]]/(2*2^(2/3)*Sqrt[3]*a^(4/3)*b^(1/3)) + (x
*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*a
*(a + b*x^3)^(2/3)) - Log[2^(2/3) - (a^(1/3) + b^(1/3)*x)/(a + b*x^3)^(1/3)
]/(6*2^(2/3)*a^(4/3)*b^(1/3)) + Log[1 + (2^(2/3)*(a^(1/3) + b^(1/3)*x)^2)/(
a + b*x^3)^(2/3) - (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(6*2^
(2/3)*a^(4/3)*b^(1/3)) - Log[1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3
)^(1/3)]/(3*2^(2/3)*a^(4/3)*b^(1/3)) + Log[2*2^(1/3) + (a^(1/3) + b^(1/3)*x
)^2/(a + b*x^3)^(2/3) + (2^(2/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/
(12*2^(2/3)*a^(4/3)*b^(1/3))
```

Rubi [C] time = 0.0276162, antiderivative size = 58, normalized size of antiderivative = 0.13, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {430, 429}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};1,\frac{2}{3};\frac{4}{3};\frac{bx^3}{a},-\frac{bx^3}{a}\right)}{a\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((a - b*x^3)*(a + b*x^3)^(2/3)),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 1, 2/3, 4/3, (b*x^3)/a, -((b*x^3)/a)
])/(a*(a + b*x^3)^(2/3))
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{2/3}}\,dx = \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{(a-bx^3)\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{(a+bx^3)^{2/3}}$$
$$= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3};1,\frac{2}{3};\frac{4}{3};\frac{bx^3}{a},-\frac{bx^3}{a}\right)}{a\,(a+bx^3)^{2/3}}$$

<u>Mathematica</u> [C] time = 0.0514956, size = 153, normalized size = 0.34

$$\frac{4axF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{(a-bx^3)(a+bx^3)^{2/3}\left(bx^3\left(3F_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)-2F_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)+4aF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - b*x^3)*(a + b*x^3)^(2/3)),x]
```

```
[Out] (4*a*x*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a])/((a - b*x^3)*(a
 + b*x^3)^(2/3)*(4*a*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a] +
b*x^3*(3*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), (b*x^3)/a] - 2*AppellF1[4
/3, 5/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a])))
```

<u>Maple</u> [F] time = 0.428, size = 0, normalized size = 0.

$$\int \frac{1}{-bx^3+a}\left(bx^3+a\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^3+a)/(b*x^3+a)^(2/3),x)
```

```
[Out] int(1/(-b*x^3+a)/(b*x^3+a)^(2/3),x)
```

<u>Maxima</u> [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(bx^3+a)^{\frac{2}{3}}(bx^3-a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^3+a)/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] -integrate(1/((b*x^3 + a)^(2/3)*(b*x^3 - a)), x)
```

<u>Fricas</u> [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^3+a)/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-a\left(a+bx^3\right)^{\frac{2}{3}}+bx^3\left(a+bx^3\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**3+a)/(b*x**3+a)**(2/3),x)
```

```
[Out] -Integral(1/(-a*(a + b*x**3)**(2/3) + b*x**3*(a + b*x**3)**(2/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}\left(bx^3-a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^3+a)/(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate(-1/((b*x^3 + a)^(2/3)*(b*x^3 - a)), x)
```

3.38 $\int \frac{1}{(a-bx^3)(a+bx^3)^{5/3}}\,dx$

Optimal. Leaf size=473

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2a^2\left(a+bx^3\right)^{2/3}}+\frac{x}{4a^2\left(a+bx^3\right)^{2/3}}-\frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{12\ 2^{2/3}a^{7/3}\sqrt[3]{b}}+\frac{\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\frac{\sqrt[3]{2}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{\sqrt[3]{a+bx^3}}+1\right)}{12\ 2^{2/3}a^{7/3}\sqrt[3]{b}}-$$

```
[Out] x/(4*a^2*(a + b*x^3)^(2/3)) - ArcTan[(1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))
/(a + b*x^3)^(1/3))/Sqrt[3]]/(2*2^(2/3)*Sqrt[3]*a^(7/3)*b^(1/3)) - ArcTan[(
1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]]/(4*2^(2/3)*
Sqrt[3]*a^(7/3)*b^(1/3)) + (x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3,
2/3, 4/3, -((b*x^3)/a)])/(2*a^2*(a + b*x^3)^(2/3)) - Log[2^(2/3) - (a^(1/3)
 + b^(1/3)*x)/(a + b*x^3)^(1/3)]/(12*2^(2/3)*a^(7/3)*b^(1/3)) + Log[1 + (2^
(2/3)*(a^(1/3) + b^(1/3)*x)^2)/(a + b*x^3)^(2/3) - (2^(1/3)*(a^(1/3) + b^(1
/3)*x))/(a + b*x^3)^(1/3)]/(12*2^(2/3)*a^(7/3)*b^(1/3)) - Log[1 + (2^(1/3)*
(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(6*2^(2/3)*a^(7/3)*b^(1/3)) + Log
[2*2^(1/3) + (a^(1/3) + b^(1/3)*x)^2/(a + b*x^3)^(2/3) + (2^(2/3)*(a^(1/3)
+ b^(1/3)*x))/(a + b*x^3)^(1/3)]/(24*2^(2/3)*a^(7/3)*b^(1/3))
```

Rubi [C] time = 0.0287143, antiderivative size = 58, normalized size of antiderivative = 0.12, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {430, 429}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};1,\frac{5}{3};\frac{4}{3};\frac{bx^3}{a},-\frac{bx^3}{a}\right)}{a^2\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((a - b*x^3)*(a + b*x^3)^(5/3)),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 1, 5/3, 4/3, (b*x^3)/a, -((b*x^3)/a)
])/(a^2*(a + b*x^3)^(2/3))
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{5/3}}\,dx = \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{(a-bx^3)\left(1+\frac{bx^3}{a}\right)^{5/3}}\,dx}{a(a+bx^3)^{2/3}}$$
$$= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3};1,\frac{5}{3};\frac{4}{3};\frac{bx^3}{a},-\frac{bx^3}{a}\right)}{a^2(a+bx^3)^{2/3}}$$

Mathematica [C] time = 0.115975, size = 213, normalized size = 0.45

$$\frac{x\left(-\frac{bx^3\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{a^3}+\frac{4}{a^2}+\frac{48F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{(a-bx^3)\left(bx^3\left(3F_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)-2F_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)+4aF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)}\right)}{16(a+bx^3)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - b*x^3)*(a + b*x^3)^(5/3)),x]
```

```
[Out] (x*(4/a^2 - (b*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^
3)/a), (b*x^3)/a])/a^3 + (48*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*x^
3)/a])/((a - b*x^3)*(4*a*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a
] + b*x^3*(3*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), (b*x^3)/a] - 2*Appell
F1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a])))))/(16*(a + b*x^3)^(2/3))
```

Maple [F] time = 0.453, size = 0, normalized size = 0.

$$\int \frac{1}{-bx^3+a}\left(bx^3+a\right)^{-\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^3+a)/(b*x^3+a)^(5/3),x)
```

```
[Out] int(1/(-b*x^3+a)/(b*x^3+a)^(5/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(bx^3+a)^{\frac{5}{3}}(bx^3-a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^3+a)/(b*x^3+a)^(5/3),x, algorithm="maxima")
```

```
[Out] -integrate(1/((b*x^3 + a)^(5/3)*(b*x^3 - a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^3+a)/(b*x^3+a)^(5/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: KeyError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**3+a)/(b*x**3+a)**(5/3),x)
```

```
[Out] Exception raised: KeyError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(bx^3+a\right)^{\frac{5}{3}}\left(bx^3-a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^3+a)/(b*x^3+a)^(5/3),x, algorithm="giac")
```

```
[Out] integrate(-1/((b*x^3 + a)^(5/3)*(b*x^3 - a)), x)
```

3.39 $\int \frac{1}{(a-bx^3)(a+bx^3)^{8/3}} \, dx$

Optimal. Leaf size=492

$$\frac{9x\left(\frac{bx^3}{a}+1\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{20a^3\left(a+bx^3\right)^{2/3}}+\frac{13x}{40a^3\left(a+bx^3\right)^{2/3}}+\frac{x}{10a^2\left(a+bx^3\right)^{5/3}}-\frac{\log\left(2^{2/3}-\frac{\sqrt[3]{a}+\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)}{24\,2^{2/3}a^{10/3}\sqrt[3]{b}}+\frac{\log\left(\frac{2^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)^2}{\left(a+bx^3\right)^{2/3}}-\right.}{24\,2^{2/3}a^{1(}}$$

```
[Out] x/(10*a^2*(a + b*x^3)^(5/3)) + (13*x)/(40*a^3*(a + b*x^3)^(2/3)) - ArcTan[(
1 - (2*2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3))/Sqrt[3]]/(4*2^(2/3
)*Sqrt[3]*a^(10/3)*b^(1/3)) - ArcTan[(1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(
a + b*x^3)^(1/3))/Sqrt[3]]/(8*2^(2/3)*Sqrt[3]*a^(10/3)*b^(1/3)) + (9*x*(1 +
 (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(20*a^3*(
a + b*x^3)^(2/3)) - Log[2^(2/3) - (a^(1/3) + b^(1/3)*x)/(a + b*x^3)^(1/3)]/
(24*2^(2/3)*a^(10/3)*b^(1/3)) + Log[1 + (2^(2/3)*(a^(1/3) + b^(1/3)*x)^2)/(
a + b*x^3)^(2/3) - (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/3)]/(24*2
^(2/3)*a^(10/3)*b^(1/3)) - Log[1 + (2^(1/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x
^3)^(1/3)]/(12*2^(2/3)*a^(10/3)*b^(1/3)) + Log[2*2^(1/3) + (a^(1/3) + b^(1/
3)*x)^2/(a + b*x^3)^(2/3) + (2^(2/3)*(a^(1/3) + b^(1/3)*x))/(a + b*x^3)^(1/
3)]/(48*2^(2/3)*a^(10/3)*b^(1/3))
```

Rubi [C] time = 0.026983, antiderivative size = 58, normalized size of antiderivative = 0.12, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {430, 429}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3} F_1\left(\frac{1}{3};1,\frac{8}{3};\frac{4}{3};\frac{bx^3}{a},-\frac{bx^3}{a}\right)}{a^3\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((a - b*x^3)*(a + b*x^3)^(8/3)),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 1, 8/3, 4/3, (b*x^3)/a, -((b*x^3)/a)
])/(a^3*(a + b*x^3)^(2/3))
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{8/3}}\,dx = \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{(a-bx^3)\left(1+\frac{bx^3}{a}\right)^{8/3}}\,dx}{a^2(a+bx^3)^{2/3}}$$
$$= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3};1,\frac{8}{3};\frac{4}{3};\frac{bx^3}{a},-\frac{bx^3}{a}\right)}{a^3(a+bx^3)^{2/3}}$$

Mathematica [C] time = 0.159652, size = 240, normalized size = 0.49

$$\frac{x\left(\frac{368a^3(a+bx^3)F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)}{(a-bx^3)\left(bx^3\left(3F_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)-2F_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)+4aF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},\frac{bx^3}{a}\right)\right)}+16a^2-13bx^3(a+bx^3)\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{4}{3};2\cdots\right.\right.}{160a^4(a+bx^3)^{5/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - b*x^3)*(a + b*x^3)^(8/3)),x]
```

```
[Out] (x*(16*a^2 + 52*a*(a + b*x^3) - 13*b*x^3*(a + b*x^3)*(1 + (b*x^3)/a)^(2/3)*
AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), (b*x^3)/a] + (368*a^3*(a + b*x^3)*
AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a])/((a - b*x^3)*(4*a*Appe
llF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), (b*x^3)/a] + b*x^3*(3*AppellF1[4/3, 2/
3, 2, 7/3, -((b*x^3)/a), (b*x^3)/a] - 2*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3
)/a), (b*x^3)/a])))))/(160*a^4*(a + b*x^3)^(5/3))
```

Maple [F] time = 0.423, size = 0, normalized size = 0.

$$\int \frac{1}{-bx^3+a}\left(bx^3+a\right)^{-\frac{8}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^3+a)/(b*x^3+a)^(8/3),x)
```

```
[Out] int(1/(-b*x^3+a)/(b*x^3+a)^(8/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{(bx^3+a)^{\frac{8}{3}}(bx^3-a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^3+a)/(b*x^3+a)^(8/3),x, algorithm="maxima")
```

```
[Out] -integrate(1/((b*x^3 + a)^(8/3)*(b*x^3 - a)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^3+a)/(b*x^3+a)^(8/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**3+a)/(b*x**3+a)**(8/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(bx^3+a\right)^{\frac{8}{3}}\left(bx^3-a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^3+a)/(b*x^3+a)^(8/3),x, algorithm="giac")
```

```
[Out] integrate(-1/((b*x^3 + a)^(8/3)*(b*x^3 - a)), x)
```

3.40 $\int \left(a-bx^3\right)^2 \left(a+bx^3\right)^{2/3} \, dx$

Optimal. Leaf size=139

$$\frac{38}{81}a^2x\left(a+bx^3\right)^{2/3}-\frac{38a^3\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{81\sqrt[3]{b}}+\frac{76a^3\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{81\sqrt{3}\sqrt[3]{b}}-\frac{8}{27}ax\left(a+bx^3\right)^{5/3}-\frac{1}{9}x\left(a-bx^3\right)\left(a+bx\right.$$

```
[Out] (38*a^2*x*(a + b*x^3)^(2/3))/81 - (8*a*x*(a + b*x^3)^(5/3))/27 - (x*(a - b*
x^3)*(a + b*x^3)^(5/3))/9 + (76*a^3*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(
1/3))/Sqrt[3]])/(81*Sqrt[3]*b^(1/3)) - (38*a^3*Log[-(b^(1/3)*x) + (a + b*x^
3)^(1/3)])/(81*b^(1/3))
```

Rubi [A] time = 0.0566985, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {416, 388, 195, 239}

$$\frac{38}{81}a^2x\left(a+bx^3\right)^{2/3}-\frac{38a^3\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{81\sqrt[3]{b}}+\frac{76a^3\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{81\sqrt{3}\sqrt[3]{b}}-\frac{8}{27}ax\left(a+bx^3\right)^{5/3}-\frac{1}{9}x\left(a-bx^3\right)\left(a+bx\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2*(a + b*x^3)^(2/3),x]
```

```
[Out] (38*a^2*x*(a + b*x^3)^(2/3))/81 - (8*a*x*(a + b*x^3)^(5/3))/27 - (x*(a - b*
x^3)*(a + b*x^3)^(5/3))/9 + (76*a^3*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(
1/3))/Sqrt[3]])/(81*Sqrt[3]*b^(1/3)) - (38*a^3*Log[-(b^(1/3)*x) + (a + b*x^
3)^(1/3)])/(81*b^(1/3))
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
```

```
Denominator[p]])
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \left(a-bx^3\right)^2 \left(a+bx^3\right)^{2/3} dx = -\frac{1}{9}x\left(a-bx^3\right)\left(a+bx^3\right)^{5/3} + \frac{\int \left(a+bx^3\right)^{2/3}\left(10a^2b-16ab^2x^3\right) dx}{9b}$$
$$= -\frac{8}{27}ax\left(a+bx^3\right)^{5/3} - \frac{1}{9}x\left(a-bx^3\right)\left(a+bx^3\right)^{5/3} + \frac{1}{27}\left(38a^2\right)\int \left(a+bx^3\right)^{2/3} dx$$
$$= \frac{38}{81}a^2x\left(a+bx^3\right)^{2/3} - \frac{8}{27}ax\left(a+bx^3\right)^{5/3} - \frac{1}{9}x\left(a-bx^3\right)\left(a+bx^3\right)^{5/3} + \frac{1}{81}\left(76a^3\right)\int \frac{1}{\sqrt[3]{a+b\ldots}}$$
$$= \frac{38}{81}a^2x\left(a+bx^3\right)^{2/3} - \frac{8}{27}ax\left(a+bx^3\right)^{5/3} - \frac{1}{9}x\left(a-bx^3\right)\left(a+bx^3\right)^{5/3} + \frac{76a^3\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bx}}{\sqrt[3]{a+bx}\ldots}}{\sqrt{3}}\right)}{81\sqrt{3}\sqrt[3]{b}}\ldots$$

Mathematica [A] time = 0.137231, size = 151, normalized size = 1.09

$$\frac{1}{243}\left(3\left(a+bx^3\right)^{2/3}\left(5a^2x-24abx^4+9b^2x^7\right) + \frac{38a^3\left(\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}} + \frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}} + 1\right) - 2\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right) + 2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{\ldots}}{\sqrt[3]{a+\ldots}}\ldots\right)\right)}{\sqrt[3]{b}}\ldots\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2*(a + b*x^3)^(2/3),x]
```

```
[Out] (3*(a + b*x^3)^(2/3)*(5*a^2*x - 24*a*b*x^4 + 9*b^2*x^7) + (38*a^3*(2*Sqrt[3
]*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]] - 2*Log[1 - (b^(1/3
)*x)/(a + b*x^3)^(1/3)] + Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3
)*x)/(a + b*x^3)^(1/3)]))/b^(1/3))/243
```

Maple [F] time = 0.233, size = 0, normalized size = 0.

$$\int \left(-bx^3+a\right)^2 \left(bx^3+a\right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2*(b*x^3+a)^(2/3),x)
```

```
[Out] int((-b*x^3+a)^2*(b*x^3+a)^(2/3),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2*(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.63047, size = 1076, normalized size = 7.74

$$\left[\frac{114\sqrt{\frac{1}{3}}a^3b\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3\,bx^3-3\left(bx^3+a\right)^{\frac{1}{3}}(-b)^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}\,bx^3-\left(bx^3+a\right)^{\frac{1}{3}}bx^2+2\left(bx^3+a\right)^{\frac{2}{3}}(-b)^{\frac{2}{3}}\,x\right)\sqrt{\frac{(-}{}}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2*(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] [1/243*(114*sqrt(1/3)*a^3*b*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^
(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*
x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) - 76*a^3*
(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + 38*a^3*(-b)^(2/3)*lo
g(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2
) + 3*(9*b^3*x^7 - 24*a*b^2*x^4 + 5*a^2*b*x)*(b*x^3 + a)^(2/3))/b, -1/243*(
228*sqrt(1/3)*a^3*b*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2
*(b*x^3 + a)^(1/3))*sqrt(-(-b)^(1/3)/b)/x) + 76*a^3*(-b)^(2/3)*log(((-b)^(1
/3)*x + (b*x^3 + a)^(1/3))/x) - 38*a^3*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*
x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 3*(9*b^3*x^7 - 24*a
*b^2*x^4 + 5*a^2*b*x)*(b*x^3 + a)^(2/3))/b]
```

Sympy [C] time = 4.77737, size = 126, normalized size = 0.91

$$\frac{a^{\frac{8}{3}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}-\frac{2a^{\frac{5}{3}}bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{a^{\frac{2}{3}}b^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{7}{3}\\\frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2*(b*x**3+a)**(2/3),x)
```

```
[Out] a**(8/3)*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(4/3)) - 2*a**(5/3)*b*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(2/3)*b**2*x**7*gamma(7/3)*hype
r((-2/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3 + a\right)^{\frac{2}{3}} \left(bx^3 - a\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2*(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)*(b*x^3 - a)^2, x)
```

3.41 $\int \frac{(a-bx^3)^2}{\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=120

$$-\frac{17a^2\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{18\sqrt[3]{b}}+\frac{17a^2\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\sqrt[3]{b}}-\frac{13}{18}ax\left(a+bx^3\right)^{2/3}-\frac{1}{6}x\left(a-bx^3\right)\left(a+bx^3\right)^{2/3}$$

```
[Out] (-13*a*x*(a + b*x^3)^(2/3))/18 - (x*(a - b*x^3)*(a + b*x^3)^(2/3))/6 + (17*
a^2*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(1/
3)) - (17*a^2*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(18*b^(1/3))
```

Rubi [A] time = 0.0417399, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {416, 388, 239}

$$-\frac{17a^2\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{18\sqrt[3]{b}}+\frac{17a^2\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\sqrt[3]{b}}-\frac{13}{18}ax\left(a+bx^3\right)^{2/3}-\frac{1}{6}x\left(a-bx^3\right)\left(a+bx^3\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(1/3),x]
```

```
[Out] (-13*a*x*(a + b*x^3)^(2/3))/18 - (x*(a - b*x^3)*(a + b*x^3)^(2/3))/6 + (17*
a^2*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(1/
3)) - (17*a^2*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(18*b^(1/3))
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{(a-bx^3)^2}{\sqrt[3]{a+bx^3}}\,dx = -\frac{1}{6}x\left(a-bx^3\right)\left(a+bx^3\right)^{2/3} + \frac{\int \frac{7a^2b-13ab^2x^3}{\sqrt[3]{a+bx^3}}\,dx}{6b}$$
$$= -\frac{13}{18}ax\left(a+bx^3\right)^{2/3} - \frac{1}{6}x\left(a-bx^3\right)\left(a+bx^3\right)^{2/3} + \frac{1}{9}\left(17a^2\right)\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx$$
$$= -\frac{13}{18}ax\left(a+bx^3\right)^{2/3} - \frac{1}{6}x\left(a-bx^3\right)\left(a+bx^3\right)^{2/3} + \frac{17a^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}\sqrt[3]{b}} - \frac{17a^2\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx\right.}}{18\sqrt[3]{b}}$$

Mathematica [A] time = 0.0596267, size = 141, normalized size = 1.18

$$\frac{17a^2\left(\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right)-2\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)\right)}{54\sqrt[3]{b}} + \left(a+bx^3\right)^{2/3}\left(\frac{bx^4}{6}-\frac{8ax}{9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(1/3),x]
```

```
[Out] (a + b*x^3)^(2/3)*((-8*a*x)/9 + (b*x^4)/6) + (17*a^2*(2*Sqrt[3]*ArcTan[(1 +
 (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]] - 2*Log[1 - (b^(1/3)*x)/(a + b*x
^3)^(1/3)] + Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x
^3)^(1/3)]))/(54*b^(1/3))
```

Maple [F] time = 0.216, size = 0, normalized size = 0.

$$\int \left(-bx^3+a\right)^2 \frac{1}{\sqrt[3]{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(1/3),x)
```

```
[Out] int((-b*x^3+a)^2/(b*x^3+a)^(1/3),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.60116, size = 1029, normalized size = 8.57

$$\left[\frac{51\sqrt{\frac{1}{3}}a^2b\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3bx^3-3\left(bx^3+a\right)^{\frac{1}{3}}(-b)^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}bx^3-\left(bx^3+a\right)^{\frac{1}{3}}bx^2+2\left(bx^3+a\right)^{\frac{2}{3}}(-b)^{\frac{2}{3}}x\right)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/54*(51*sqrt(1/3)*a^2*b*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1
/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^
2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) - 34*a^2*(-
b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + 17*a^2*(-b)^(2/3)*log(
((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2)
+ 3*(3*b^2*x^4 - 16*a*b*x)*(b*x^3 + a)^(2/3))/b, -1/54*(102*sqrt(1/3)*a^2*b
*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))
*sqrt(-(-b)^(1/3)/b)/x) + 34*a^2*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)
^(1/3))/x) - 17*a^2*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)
^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 3*(3*b^2*x^4 - 16*a*b*x)*(b*x^3 + a)^(
2/3))/b]
```

Sympy [C] time = 3.86712, size = 121, normalized size = 1.01

$$\frac{a^{\frac{5}{3}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}-\frac{2a^{\frac{2}{3}}bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{b^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{7}{3}\\\frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(1/3),x)
```

```
[Out] a**(5/3)*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(
3*gamma(4/3)) - 2*a**(2/3)*b*x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), b*x*
*3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + b**2*x**7*gamma(7/3)*hyper((1/3, 7/3
), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(10/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(bx^3-a\right)^2}{\left(bx^3+a\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(1/3), x)
```

3.42 $\int \frac{\left(a-bx^3\right)^2}{\left(a+bx^3\right)^{4/3}}\,dx$

Optimal. Leaf size=113

$$\frac{7}{3}x\left(a+bx^3\right)^{2/3}+\frac{2x\left(a-bx^3\right)}{\sqrt[3]{a+bx^3}}+\frac{5a\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{3\sqrt[3]{b}}-\frac{10a\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{b}}$$

```
[Out] (2*x*(a - b*x^3))/(a + b*x^3)^(1/3) + (7*x*(a + b*x^3)^(2/3))/3 - (10*a*Arc
Tan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(1/3)) + (
5*a*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(3*b^(1/3))
```

Rubi [A] time = 0.0416018, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {413, 388, 239}

$$\frac{7}{3}x\left(a+bx^3\right)^{2/3}+\frac{2x\left(a-bx^3\right)}{\sqrt[3]{a+bx^3}}+\frac{5a\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{3\sqrt[3]{b}}-\frac{10a\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(4/3),x]
```



```
[Out] (2*x*(a - b*x^3))/(a + b*x^3)^(1/3) + (7*x*(a + b*x^3)^(2/3))/3 - (10*a*Arc
Tan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(1/3)) + (
5*a*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(3*b^(1/3))
```


Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\left(a-bx^3\right)^2}{\left(a+bx^3\right)^{4/3}}\,dx = \frac{2x\left(a-bx^3\right)}{\sqrt[3]{a+bx^3}} + \frac{\int \frac{-a^2b+7ab^2x^3}{\sqrt[3]{a+bx^3}}\,dx}{ab}$$

$$= \frac{2x\left(a-bx^3\right)}{\sqrt[3]{a+bx^3}} + \frac{7}{3}x\left(a+bx^3\right)^{2/3} - \frac{1}{3}(10a)\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx$$

$$= \frac{2x\left(a-bx^3\right)}{\sqrt[3]{a+bx^3}} + \frac{7}{3}x\left(a+bx^3\right)^{2/3} - \frac{10a\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{b}} + \frac{5a\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{3\sqrt[3]{b}}$$

Mathematica [A] time = 0.083197, size = 137, normalized size = 1.21

$$\frac{x\left(13a+bx^3\right)}{3\sqrt[3]{a+bx^3}} - \frac{5a\left(\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right)-2\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)\right)}{9\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(4/3),x]
```

```
[Out] (x*(13*a + b*x^3))/(3*(a + b*x^3)^(1/3)) - (5*a*(2*Sqrt[3]*ArcTan[(1 + (2*b
^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]] - 2*Log[1 - (b^(1/3)*x)/(a + b*x^3)^(
1/3)] + Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(
1/3)]))/(9*b^(1/3))
```

Maple [F] time = 0.41, size = 0, normalized size = 0.

$$\int \left(-bx^3+a\right)^2\left(bx^3+a\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(4/3),x)
```

```
[Out] int((-b*x^3+a)^2/(b*x^3+a)^(4/3),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.39682, size = 1062, normalized size = 9.4

$$\left[\frac{15\sqrt{\frac{1}{3}}\left(ab^{2}x^{3}+a^{2}b\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3\,bx^{3}-3\left(bx^{3}+a\right)^{\frac{1}{3}}b^{\frac{2}{3}}x^{2}-3\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^{3}+\left(bx^{3}+a\right)^{\frac{1}{3}}bx^{2}-2\left(bx^{3}+a\right)^{\frac{2}{3}}b^{\frac{2}{3}}x\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}+2a\right.}{9\left(b^{2}x\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

```
[Out] [1/9*(15*sqrt(1/3)*(a*b^2*x^3 + a^2*b)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*
x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b
*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 10*(a*b*x^3
 + a^2)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - 5*(a*b*x^3 + a^2)
*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3)
)/x^2) + 3*(b^2*x^4 + 13*a*b*x)*(b*x^3 + a)^(2/3))/(b^2*x^3 + a*b), 1/9*(10
*(a*b*x^3 + a^2)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - 5*(a*b*x
^3 + a^2)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 +
 a)^(2/3))/x^2) + 30*sqrt(1/3)*(a*b^2*x^3 + a^2*b)*arctan(sqrt(1/3)*(b^(1/3
)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3) + 3*(b^2*x^4 + 13*a*b*x)*(b
*x^3 + a)^(2/3))/(b^2*x^3 + a*b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a+bx^{3}\right)^{2}}{\left(a+bx^{3}\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(4/3),x)
```

```
[Out] Integral((-a + b*x**3)**2/(a + b*x**3)**(4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^{3}-a\right)^{2}}{\left(bx^{3}+a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(4/3), x)
```

3.43 $\int \frac{\left(a-bx^3\right)^2}{\left(a+bx^3\right)^{7/3}} \, dx$

Optimal. Leaf size=110

$$-\frac{x}{2\sqrt[3]{a+bx^3}}+\frac{x\left(a-bx^3\right)}{2\left(a+bx^3\right)^{4/3}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}+\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}$$

```
[Out] (x*(a - b*x^3))/(2*(a + b*x^3)^(4/3)) - x/(2*(a + b*x^3)^(1/3)) + ArcTan[(1
 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1
/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))
```

Rubi [A] time = 0.0403552, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {413, 385, 239}

$$-\frac{x}{2\sqrt[3]{a+bx^3}}+\frac{x\left(a-bx^3\right)}{2\left(a+bx^3\right)^{4/3}}-\frac{\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2\sqrt[3]{b}}+\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(7/3),x]
```

```
[Out] (x*(a - b*x^3))/(2*(a + b*x^3)^(4/3)) - x/(2*(a + b*x^3)^(1/3)) + ArcTan[(1
 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*b^(1/3)) - Log[-(b^(1
/3)*x) + (a + b*x^3)^(1/3)]/(2*b^(1/3))
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a-bx^3)^2}{(a+bx^3)^{7/3}}\,dx &= \frac{x(a-bx^3)}{2(a+bx^3)^{4/3}}+\frac{\int \frac{2a^2b+4ab^2x^3}{(a+bx^3)^{4/3}}\,dx}{4ab}\\ &= \frac{x(a-bx^3)}{2(a+bx^3)^{4/3}}-\frac{x}{2\sqrt[3]{a+bx^3}}+\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx\\ &= \frac{x(a-bx^3)}{2(a+bx^3)^{4/3}}-\frac{x}{2\sqrt[3]{a+bx^3}}+\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}}-\frac{\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}}\end{aligned}$$

Mathematica [A] time = 0.067432, size = 131, normalized size = 1.19

$$\frac{-\frac{6b^{4/3}x^4}{(a+bx^3)^{4/3}}+\log\left(\frac{b^{2/3}x^2}{(a+bx^3)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right)-2\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{6\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(7/3),x]
```

```
[Out] ((-6*b^(4/3)*x^4)/(a + b*x^3)^(4/3) + 2*Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*x)/(
a + b*x^3)^(1/3))/Sqrt[3]] - 2*Log[1 - (b^(1/3)*x)/(a + b*x^3)^(1/3)] + Log
[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)])/(6*b
^(1/3))
```

Maple [F] time = 0.368, size = 0, normalized size = 0.

$$\int \left(-bx^3+a\right)^2\left(bx^3+a\right)^{-\frac{7}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(7/3),x)
```

```
[Out] int((-b*x^3+a)^2/(b*x^3+a)^(7/3),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(7/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.99205, size = 1253, normalized size = 11.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(7/3),x, algorithm="fricas")
```

```
[Out] [-1/6*(6*(b*x^3 + a)^(2/3)*b^2*x^4 - 3*sqrt(1/3)*(b^3*x^6 + 2*a*b^2*x^3 + a
^2*b)*sqrt((-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 -
 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2
/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) + 2*a) + 2*(b^2*x^6 + 2*a*b*x^3 + a^2)
*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - (b^2*x^6 + 2*a*b*x^
3 + a^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x +
(b*x^3 + a)^(2/3))/x^2))/(b^3*x^6 + 2*a*b^2*x^3 + a^2*b), -1/6*(6*(b*x^3 +
a)^(2/3)*b^2*x^4 + 6*sqrt(1/3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b)*sqrt(-(-b)^(
1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sqrt(-(-b)^(
1/3)/b)/x) + 2*(b^2*x^6 + 2*a*b*x^3 + a^2)*(-b)^(2/3)*log(((-b)^(1/3)*x + (
b*x^3 + a)^(1/3))/x) - (b^2*x^6 + 2*a*b*x^3 + a^2)*(-b)^(2/3)*log(((-b)^(2/
3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2))/(b^3*x^6
 + 2*a*b^2*x^3 + a^2*b)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a + bx^3\right)^2}{\left(a + bx^3\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(7/3),x)
```

```
[Out] Integral((-a + b*x**3)**2/(a + b*x**3)**(7/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3 - a\right)^2}{\left(bx^3 + a\right)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(7/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(7/3), x)
```

3.44 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{10/3}} \, dx$

Optimal. Leaf size=76

$$\frac{x\left(a-bx^3\right)^2}{7a\left(a+bx^3\right)^{7/3}}+\frac{3x\left(a-bx^3\right)}{14a\left(a+bx^3\right)^{4/3}}+\frac{9x}{14a\sqrt[3]{a+bx^3}}$$

```
[Out] (x*(a - b*x^3)^2)/(7*a*(a + b*x^3)^(7/3)) + (3*x*(a - b*x^3))/(14*a*(a + b*
x^3)^(4/3)) + (9*x)/(14*a*(a + b*x^3)^(1/3))
```

Rubi [A] time = 0.0212121, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {378, 191}

$$\frac{x\left(a-bx^3\right)^2}{7a\left(a+bx^3\right)^{7/3}}+\frac{3x\left(a-bx^3\right)}{14a\left(a+bx^3\right)^{4/3}}+\frac{9x}{14a\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(10/3),x]
```

```
[Out] (x*(a - b*x^3)^2)/(7*a*(a + b*x^3)^(7/3)) + (3*x*(a - b*x^3))/(14*a*(a + b*
x^3)^(4/3)) + (9*x)/(14*a*(a + b*x^3)^(1/3))
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a-bx^3\right)^2}{\left(a+bx^3\right)^{10/3}} \, dx &= \frac{x\left(a-bx^3\right)^2}{7a\left(a+bx^3\right)^{7/3}}+\frac{6}{7}\int \frac{a-bx^3}{\left(a+bx^3\right)^{7/3}} \, dx \\
&= \frac{x\left(a-bx^3\right)^2}{7a\left(a+bx^3\right)^{7/3}}+\frac{3x\left(a-bx^3\right)}{14a\left(a+bx^3\right)^{4/3}}+\frac{9}{14}\int \frac{1}{\left(a+bx^3\right)^{4/3}} \, dx \\
&= \frac{x\left(a-bx^3\right)^2}{7a\left(a+bx^3\right)^{7/3}}+\frac{3x\left(a-bx^3\right)}{14a\left(a+bx^3\right)^{4/3}}+\frac{9x}{14a\sqrt[3]{a+bx^3}}
\end{aligned}$$

Mathematica [A] time = 0.0146196, size = 40, normalized size = 0.53

$$\frac{x\left(7a^2+7abx^3+4b^2x^6\right)}{7a\left(a+bx^3\right)^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(10/3),x]
```

```
[Out] (x*(7*a^2 + 7*a*b*x^3 + 4*b^2*x^6))/(7*a*(a + b*x^3)^(7/3))
```

Maple [A] time = 0.005, size = 37, normalized size = 0.5

$$\frac{x\left(4\,b^2x^6+7\,ax^3b+7\,a^2\right)}{7\,a}\left(bx^3+a\right)^{-\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(10/3),x)
```

```
[Out] 1/7*x*(4*b^2*x^6+7*a*b*x^3+7*a^2)/(b*x^3+a)^(7/3)/a
```

Maxima [A] time = 1.00197, size = 142, normalized size = 1.87

$$\frac{\left(4\,b-\frac{7\left(bx^3+a\right)}{x^3}\right)bx^7}{14\left(bx^3+a\right)^{\frac{7}{3}}a}+\frac{b^2x^7}{7\left(bx^3+a\right)^{\frac{7}{3}}a}+\frac{\left(2\,b^2-\frac{7\left(bx^3+a\right)b}{x^3}+\frac{14\left(bx^3+a\right)^2}{x^6}\right)x^7}{14\left(bx^3+a\right)^{\frac{7}{3}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(10/3),x, algorithm="maxima")
```

```
[Out] 1/14*(4*b - 7*(b*x^3 + a)/x^3)*b*x^7/((b*x^3 + a)^(7/3)*a) + 1/7*b^2*x^7/((
b*x^3 + a)^(7/3)*a) + 1/14*(2*b^2 - 7*(b*x^3 + a)*b/x^3 + 14*(b*x^3 + a)^2/
x^6)*x^7/((b*x^3 + a)^(7/3)*a)
```

Fricas [A] time = 1.98809, size = 142, normalized size = 1.87

$$\frac{\left(4\,b^2x^7+7\,abx^4+7\,a^2x\right)\left(bx^3+a\right)^{\frac{2}{3}}}{7\left(ab^3x^9+3\,a^2b^2x^6+3\,a^3bx^3+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(10/3),x, algorithm="fricas")
```

```
[Out] 1/7*(4*b^2*x^7 + 7*a*b*x^4 + 7*a^2*x)*(b*x^3 + a)^(2/3)/(a*b^3*x^9 + 3*a^2*
b^2*x^6 + 3*a^3*b*x^3 + a^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(10/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3 - a\right)^2}{\left(bx^3 + a\right)^{\frac{10}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(10/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(10/3), x)
```

3.45 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{13/3}}\,dx$

Optimal. Leaf size=105

$$\frac{x(a-bx^3)^3}{20a^2(a+bx^3)^{10/3}}+\frac{19x(a-bx^3)^2}{140a^2(a+bx^3)^{7/3}}+\frac{57x(a-bx^3)}{280a^2(a+bx^3)^{4/3}}+\frac{171x}{280a^2\sqrt[3]{a+bx^3}}$$

```
[Out] (x*(a - b*x^3)^3)/(20*a^2*(a + b*x^3)^(10/3)) + (19*x*(a - b*x^3)^2)/(140*a
^2*(a + b*x^3)^(7/3)) + (57*x*(a - b*x^3))/(280*a^2*(a + b*x^3)^(4/3)) + (1
71*x)/(280*a^2*(a + b*x^3)^(1/3))
```

Rubi [A] time = 0.0353146, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {382, 378, 191}

$$\frac{x(a-bx^3)^3}{20a^2(a+bx^3)^{10/3}}+\frac{19x(a-bx^3)^2}{140a^2(a+bx^3)^{7/3}}+\frac{57x(a-bx^3)}{280a^2(a+bx^3)^{4/3}}+\frac{171x}{280a^2\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(13/3),x]
```

```
[Out] (x*(a - b*x^3)^3)/(20*a^2*(a + b*x^3)^(10/3)) + (19*x*(a - b*x^3)^2)/(140*a
^2*(a + b*x^3)^(7/3)) + (57*x*(a - b*x^3))/(280*a^2*(a + b*x^3)^(4/3)) + (1
71*x)/(280*a^2*(a + b*x^3)^(1/3))
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\left(a-bx^3\right)^2}{\left(a+bx^3\right)^{13/3}}\,dx = \frac{x\left(a-bx^3\right)^3}{20a^2\left(a+bx^3\right)^{10/3}} + \frac{19\int \frac{\left(a-bx^3\right)^2}{\left(a+bx^3\right)^{10/3}}\,dx}{20a}$$

$$= \frac{x\left(a-bx^3\right)^3}{20a^2\left(a+bx^3\right)^{10/3}} + \frac{19x\left(a-bx^3\right)^2}{140a^2\left(a+bx^3\right)^{7/3}} + \frac{57\int \frac{a-bx^3}{\left(a+bx^3\right)^{7/3}}\,dx}{70a}$$

$$= \frac{x\left(a-bx^3\right)^3}{20a^2\left(a+bx^3\right)^{10/3}} + \frac{19x\left(a-bx^3\right)^2}{140a^2\left(a+bx^3\right)^{7/3}} + \frac{57x\left(a-bx^3\right)}{280a^2\left(a+bx^3\right)^{4/3}} + \frac{171\int \frac{1}{\left(a+bx^3\right)^{4/3}}\,dx}{280a}$$

$$= \frac{x\left(a-bx^3\right)^3}{20a^2\left(a+bx^3\right)^{10/3}} + \frac{19x\left(a-bx^3\right)^2}{140a^2\left(a+bx^3\right)^{7/3}} + \frac{57x\left(a-bx^3\right)}{280a^2\left(a+bx^3\right)^{4/3}} + \frac{171x}{280a^2\sqrt[3]{a+bx^3}}$$

Mathematica [A] time = 0.0324967, size = 51, normalized size = 0.49

$$\frac{x\left(245a^2bx^3+140a^3+230ab^2x^6+69b^3x^9\right)}{140a^2\left(a+bx^3\right)^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(13/3),x]
```

```
[Out] (x*(140*a^3 + 245*a^2*b*x^3 + 230*a*b^2*x^6 + 69*b^3*x^9))/(140*a^2*(a + b*
x^3)^(10/3))
```

Maple [A] time = 0.007, size = 48, normalized size = 0.5

$$\frac{x\left(69\,b^3x^9+230\,b^2x^6a+245\,bx^3a^2+140\,a^3\right)}{140\,a^2}\left(bx^3+a\right)^{-\frac{10}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(13/3),x)
```

```
[Out] 1/140*x*(69*b^3*x^9+230*a*b^2*x^6+245*a^2*b*x^3+140*a^3)/(b*x^3+a)^(10/3)/a
^2
```

Maxima [A] time = 0.977094, size = 209, normalized size = 1.99

$$-\frac{\left(7\,b-\frac{10\left(bx^3+a\right)}{x^3}\right)b^2x^{10}}{70\left(bx^3+a\right)^{\frac{10}{3}}a^2}-\frac{\left(14\,b^2-\frac{40\left(bx^3+a\right)b}{x^3}+\frac{35\left(bx^3+a\right)^2}{x^6}\right)bx^{10}}{70\left(bx^3+a\right)^{\frac{10}{3}}a^2}-\frac{\left(14\,b^3-\frac{60\left(bx^3+a\right)b^2}{x^3}+\frac{105\left(bx^3+a\right)^2b}{x^6}-\frac{140\left(bx^3+a\right)^3}{x^9}\right)x^{10}}{140\left(bx^3+a\right)^{\frac{10}{3}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(13/3),x, algorithm="maxima")
```

```
[Out] -1/70*(7*b - 10*(b*x^3 + a)/x^3)*b^2*x^10/((b*x^3 + a)^(10/3)*a^2) - 1/70*(
14*b^2 - 40*(b*x^3 + a)*b/x^3 + 35*(b*x^3 + a)^2/x^6)*b*x^10/((b*x^3 + a)^(
10/3)*a^2) - 1/140*(14*b^3 - 60*(b*x^3 + a)*b^2/x^3 + 105*(b*x^3 + a)^2*b/x
^6 - 140*(b*x^3 + a)^3/x^9)*x^10/((b*x^3 + a)^(10/3)*a^2)
```

Fricas [A] time = 1.94679, size = 203, normalized size = 1.93

$$\frac{\left(69\, b^{3} x^{10} + 230\, a b^{2} x^{7} + 245\, a^{2} b x^{4} + 140\, a^{3} x\right) \left(b x^{3} + a\right)^{\frac{2}{3}}}{140 \left(a^{2} b^{4} x^{12} + 4\, a^{3} b^{3} x^{9} + 6\, a^{4} b^{2} x^{6} + 4\, a^{5} b x^{3} + a^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(13/3),x, algorithm="fricas")
```

```
[Out] 1/140*(69*b^3*x^10 + 230*a*b^2*x^7 + 245*a^2*b*x^4 + 140*a^3*x)*(b*x^3 + a)
^(2/3)/(a^2*b^4*x^12 + 4*a^3*b^3*x^9 + 6*a^4*b^2*x^6 + 4*a^5*b*x^3 + a^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(13/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b x^{3} - a\right)^{2}}{\left(b x^{3} + a\right)^{\frac{13}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(13/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(13/3), x)
```

3.46 $\int \frac{\left(a-bx^3\right)^2}{\left(a+bx^3\right)^{16/3}} \, dx$

Optimal. Leaf size=98

$$\frac{423x}{910a^3\sqrt[3]{a+bx^3}}+\frac{141x}{910a^2\left(a+bx^3\right)^{4/3}}+\frac{47x}{455a\left(a+bx^3\right)^{7/3}}+\frac{8x}{65\left(a+bx^3\right)^{10/3}}+\frac{2x\left(a-bx^3\right)}{13\left(a+bx^3\right)^{13/3}}$$

```
[Out] (2*x*(a - b*x^3))/(13*(a + b*x^3)^(13/3)) + (8*x)/(65*(a + b*x^3)^(10/3)) +
 (47*x)/(455*a*(a + b*x^3)^(7/3)) + (141*x)/(910*a^2*(a + b*x^3)^(4/3)) + (
423*x)/(910*a^3*(a + b*x^3)^(1/3))
```

Rubi [A] time = 0.0353347, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {413, 385, 192, 191}

$$\frac{423x}{910a^3\sqrt[3]{a+bx^3}}+\frac{141x}{910a^2\left(a+bx^3\right)^{4/3}}+\frac{47x}{455a\left(a+bx^3\right)^{7/3}}+\frac{8x}{65\left(a+bx^3\right)^{10/3}}+\frac{2x\left(a-bx^3\right)}{13\left(a+bx^3\right)^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(16/3),x]
```

```
[Out] (2*x*(a - b*x^3))/(13*(a + b*x^3)^(13/3)) + (8*x)/(65*(a + b*x^3)^(10/3)) +
 (47*x)/(455*a*(a + b*x^3)^(7/3)) + (141*x)/(910*a^2*(a + b*x^3)^(4/3)) + (
423*x)/(910*a^3*(a + b*x^3)^(1/3))
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a-bx^3\right)^2}{\left(a+bx^3\right)^{16/3}}\,dx &= \frac{2x\left(a-bx^3\right)}{13\left(a+bx^3\right)^{13/3}}+\frac{\int \frac{11a^2b-5ab^2x^3}{\left(a+bx^3\right)^{13/3}}\,dx}{13ab}\\
&= \frac{2x\left(a-bx^3\right)}{13\left(a+bx^3\right)^{13/3}}+\frac{8x}{65\left(a+bx^3\right)^{10/3}}+\frac{47}{65}\int \frac{1}{\left(a+bx^3\right)^{10/3}}\,dx\\
&= \frac{2x\left(a-bx^3\right)}{13\left(a+bx^3\right)^{13/3}}+\frac{8x}{65\left(a+bx^3\right)^{10/3}}+\frac{47x}{455a\left(a+bx^3\right)^{7/3}}+\frac{282\int \frac{1}{\left(a+bx^3\right)^{7/3}}\,dx}{455a}\\
&= \frac{2x\left(a-bx^3\right)}{13\left(a+bx^3\right)^{13/3}}+\frac{8x}{65\left(a+bx^3\right)^{10/3}}+\frac{47x}{455a\left(a+bx^3\right)^{7/3}}+\frac{141x}{910a^2\left(a+bx^3\right)^{4/3}}+\frac{423\int \frac{1}{\left(a+bx^3\right)^{4/3}}\,dx}{910a^2}\\
&= \frac{2x\left(a-bx^3\right)}{13\left(a+bx^3\right)^{13/3}}+\frac{8x}{65\left(a+bx^3\right)^{10/3}}+\frac{47x}{455a\left(a+bx^3\right)^{7/3}}+\frac{141x}{910a^2\left(a+bx^3\right)^{4/3}}+\frac{423x}{910a^3\sqrt[3]{a+bx^3}}\end{aligned}$$

Mathematica [A] time = 0.0351586, size = 62, normalized size = 0.63

$$\frac{x\left(3055a^2b^2x^6+2275a^3bx^3+910a^4+1833ab^3x^9+423b^4x^{12}\right)}{910a^3\left(a+bx^3\right)^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(16/3),x]
```

```
[Out] (x*(910*a^4 + 2275*a^3*b*x^3 + 3055*a^2*b^2*x^6 + 1833*a*b^3*x^9 + 423*b^4*
x^12))/(910*a^3*(a + b*x^3)^(13/3))
```

Maple [A] time = 0.007, size = 59, normalized size = 0.6

$$\frac{x\left(423\,b^4x^{12}+1833\,b^3x^9a+3055\,b^2x^6a^2+2275\,bx^3a^3+910\,a^4\right)}{910\,a^3}\left(bx^3+a\right)^{-\frac{13}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(16/3),x)
```

```
[Out] 1/910*x*(423*b^4*x^12+1833*a*b^3*x^9+3055*a^2*b^2*x^6+2275*a^3*b*x^3+910*a^
4)/(b*x^3+a)^(13/3)/a^3
```

Maxima [B] time = 0.96415, size = 278, normalized size = 2.84

$$\frac{\left(35\,b^2-\frac{91\left(bx^3+a\right)b}{x^3}+\frac{65\left(bx^3+a\right)^2}{x^6}\right)b^2x^{13}}{455\left(bx^3+a\right)^{\frac{13}{3}}a^3}+\frac{\left(140\,b^3-\frac{546\left(bx^3+a\right)b^2}{x^3}+\frac{780\left(bx^3+a\right)^2b}{x^6}-\frac{455\left(bx^3+a\right)^3}{x^9}\right)bx^{13}}{910\left(bx^3+a\right)^{\frac{13}{3}}a^3}+\frac{\left(35\,b^4-\frac{182\left(bx^3+a\right)b^3}{x^3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(16/3),x, algorithm="maxima")
```

```
[Out] 1/455*(35*b^2 - 91*(b*x^3 + a)*b/x^3 + 65*(b*x^3 + a)^2/x^6)*b^2*x^13/((b*x
^3 + a)^(13/3)*a^3) + 1/910*(140*b^3 - 546*(b*x^3 + a)*b^2/x^3 + 780*(b*x^3
 + a)^2*b/x^6 - 455*(b*x^3 + a)^3/x^9)*b*x^13/((b*x^3 + a)^(13/3)*a^3) + 1/
455*(35*b^4 - 182*(b*x^3 + a)*b^3/x^3 + 390*(b*x^3 + a)^2*b^2/x^6 - 455*(b*
x^3 + a)^3*b/x^9 + 455*(b*x^3 + a)^4/x^12)*x^13/((b*x^3 + a)^(13/3)*a^3)
```

Fricas [A] time = 2.04223, size = 259, normalized size = 2.64

$$\frac{\left(423\, b^4 x^{13} + 1833\, a b^3 x^{10} + 3055\, a^2 b^2 x^7 + 2275\, a^3 b x^4 + 910\, a^4 x\right)\left(b x^3 + a\right)^{\frac{2}{3}}}{910\left(a^3 b^5 x^{15} + 5\, a^4 b^4 x^{12} + 10\, a^5 b^3 x^9 + 10\, a^6 b^2 x^6 + 5\, a^7 b x^3 + a^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(16/3),x, algorithm="fricas")
```

```
[Out] 1/910*(423*b^4*x^13 + 1833*a*b^3*x^10 + 3055*a^2*b^2*x^7 + 2275*a^3*b*x^4 +
 910*a^4*x)*(b*x^3 + a)^(2/3)/(a^3*b^5*x^15 + 5*a^4*b^4*x^12 + 10*a^5*b^3*x
^9 + 10*a^6*b^2*x^6 + 5*a^7*b*x^3 + a^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(16/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b x^3 - a\right)^2}{\left(b x^3 + a\right)^{\frac{16}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(16/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(16/3), x)
```

3.47 $\int \frac{\left(a-bx^3\right)^2}{\left(a+bx^3\right)^{19/3}}\,dx$

Optimal. Leaf size=117

$$\frac{81x}{182a^4\sqrt[3]{a+bx^3}}+\frac{27x}{182a^3\left(a+bx^3\right)^{4/3}}+\frac{9x}{91a^2\left(a+bx^3\right)^{7/3}}+\frac{x}{13a\left(a+bx^3\right)^{10/3}}+\frac{11x}{104\left(a+bx^3\right)^{13/3}}+\frac{x\left(a-bx^3\right)}{8\left(a+bx^3\right)^{16/3}}$$

```
[Out] (x*(a - b*x^3))/(8*(a + b*x^3)^(16/3)) + (11*x)/(104*(a + b*x^3)^(13/3)) +
x/(13*a*(a + b*x^3)^(10/3)) + (9*x)/(91*a^2*(a + b*x^3)^(7/3)) + (27*x)/(18
2*a^3*(a + b*x^3)^(4/3)) + (81*x)/(182*a^4*(a + b*x^3)^(1/3))
```

Rubi [A] time = 0.0442149, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {413, 385, 192, 191}

$$\frac{81x}{182a^4\sqrt[3]{a+bx^3}}+\frac{27x}{182a^3\left(a+bx^3\right)^{4/3}}+\frac{9x}{91a^2\left(a+bx^3\right)^{7/3}}+\frac{x}{13a\left(a+bx^3\right)^{10/3}}+\frac{11x}{104\left(a+bx^3\right)^{13/3}}+\frac{x\left(a-bx^3\right)}{8\left(a+bx^3\right)^{16/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(19/3),x]
```

```
[Out] (x*(a - b*x^3))/(8*(a + b*x^3)^(16/3)) + (11*x)/(104*(a + b*x^3)^(13/3)) +
x/(13*a*(a + b*x^3)^(10/3)) + (9*x)/(91*a^2*(a + b*x^3)^(7/3)) + (27*x)/(18
2*a^3*(a + b*x^3)^(4/3)) + (81*x)/(182*a^4*(a + b*x^3)^(1/3))
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{(a-bx^3)^2}{(a+bx^3)^{19/3}}\,dx = \frac{x(a-bx^3)}{8(a+bx^3)^{16/3}} + \frac{\int \frac{14a^2b-8ab^2x^3}{(a+bx^3)^{16/3}}\,dx}{16ab}$$

$$= \frac{x(a-bx^3)}{8(a+bx^3)^{16/3}} + \frac{11x}{104(a+bx^3)^{13/3}} + \frac{10}{13}\int \frac{1}{(a+bx^3)^{13/3}}\,dx$$

$$= \frac{x(a-bx^3)}{8(a+bx^3)^{16/3}} + \frac{11x}{104(a+bx^3)^{13/3}} + \frac{x}{13a(a+bx^3)^{10/3}} + \frac{9\int \frac{1}{(a+bx^3)^{10/3}}\,dx}{13a}$$

$$= \frac{x(a-bx^3)}{8(a+bx^3)^{16/3}} + \frac{11x}{104(a+bx^3)^{13/3}} + \frac{x}{13a(a+bx^3)^{10/3}} + \frac{9x}{91a^2(a+bx^3)^{7/3}} + \frac{54\int \frac{1}{(a+bx^3)^{7/3}}\,dx}{91a^2}$$

$$= \frac{x(a-bx^3)}{8(a+bx^3)^{16/3}} + \frac{11x}{104(a+bx^3)^{13/3}} + \frac{x}{13a(a+bx^3)^{10/3}} + \frac{9x}{91a^2(a+bx^3)^{7/3}} + \frac{27x}{182a^3(a+bx^3)^{4/3}} +$$

$$= \frac{x(a-bx^3)}{8(a+bx^3)^{16/3}} + \frac{11x}{104(a+bx^3)^{13/3}} + \frac{x}{13a(a+bx^3)^{10/3}} + \frac{9x}{91a^2(a+bx^3)^{7/3}} + \frac{27x}{182a^3(a+bx^3)^{4/3}} +$$

Mathematica [A] time = 0.0373948, size = 73, normalized size = 0.62

$$\frac{x(1872a^2b^3x^9 + 2080a^3b^2x^6 + 1183a^4bx^3 + 364a^5 + 864ab^4x^{12} + 162b^5x^{15})}{364a^4(a+bx^3)^{16/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(19/3),x]
```

```
[Out] (x*(364*a^5 + 1183*a^4*b*x^3 + 2080*a^3*b^2*x^6 + 1872*a^2*b^3*x^9 + 864*a*
b^4*x^12 + 162*b^5*x^15))/(364*a^4*(a + b*x^3)^(16/3))
```

Maple [A] time = 0.007, size = 70, normalized size = 0.6

$$\frac{x(162\,b^5x^{15} + 864\,b^4x^{12}a + 1872\,b^3x^9a^2 + 2080\,a^3b^2x^6 + 1183\,bx^3a^4 + 364\,a^5)}{364\,a^4}(bx^3+a)^{-\frac{16}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(19/3),x)
```

```
[Out] 1/364*x*(162*b^5*x^15+864*a*b^4*x^12+1872*a^2*b^3*x^9+2080*a^3*b^2*x^6+1183
*a^4*b*x^3+364*a^5)/(b*x^3+a)^(16/3)/a^4
```

Maxima [B] time = 0.980965, size = 347, normalized size = 2.97

$$-\frac{\left(455\,b^3-\frac{1680\,(bx^3+a)b^2}{x^3}+\frac{2184\,(bx^3+a)^2b}{x^6}-\frac{1040\,(bx^3+a)^3}{x^9}\right)b^2x^{16}}{7280\,(bx^3+a)^{\frac{16}{3}}a^4}-\frac{\left(455\,b^4-\frac{2240\,(bx^3+a)b^3}{x^3}+\frac{4368\,(bx^3+a)^2b^2}{x^6}-\frac{4160\,(bx^3+a)^3b}{x^9}+\right.}{3640\,(bx^3+a)^{\frac{16}{3}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(19/3),x, algorithm="maxima")
```

```
[Out] -1/7280*(455*b^3 - 1680*(b*x^3 + a)*b^2/x^3 + 2184*(b*x^3 + a)^2*b/x^6 - 10
40*(b*x^3 + a)^3/x^9)*b^2*x^16/((b*x^3 + a)^(16/3)*a^4) - 1/3640*(455*b^4 -
 2240*(b*x^3 + a)*b^3/x^3 + 4368*(b*x^3 + a)^2*b^2/x^6 - 4160*(b*x^3 + a)^3
*b/x^9 + 1820*(b*x^3 + a)^4/x^12)*b*x^16/((b*x^3 + a)^(16/3)*a^4) - 1/1456*
(91*b^5 - 560*(b*x^3 + a)*b^4/x^3 + 1456*(b*x^3 + a)^2*b^3/x^6 - 2080*(b*x^
3 + a)^3*b^2/x^9 + 1820*(b*x^3 + a)^4*b/x^12 - 1456*(b*x^3 + a)^5/x^15)*x^1
6/((b*x^3 + a)^(16/3)*a^4)
```

Fricas [A] time = 2.18981, size = 311, normalized size = 2.66

$$\frac{\left(162\,b^5x^{16}+864\,ab^4x^{13}+1872\,a^2b^3x^{10}+2080\,a^3b^2x^7+1183\,a^4bx^4+364\,a^5x\right)\left(bx^3+a\right)^{\frac{2}{3}}}{364\left(a^4b^6x^{18}+6\,a^5b^5x^{15}+15\,a^6b^4x^{12}+20\,a^7b^3x^9+15\,a^8b^2x^6+6\,a^9bx^3+a^{10}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(19/3),x, algorithm="fricas")
```

```
[Out] 1/364*(162*b^5*x^16 + 864*a*b^4*x^13 + 1872*a^2*b^3*x^10 + 2080*a^3*b^2*x^7
 + 1183*a^4*b*x^4 + 364*a^5*x)*(b*x^3 + a)^(2/3)/(a^4*b^6*x^18 + 6*a^5*b^5*
x^15 + 15*a^6*b^4*x^12 + 20*a^7*b^3*x^9 + 15*a^8*b^2*x^6 + 6*a^9*b*x^3 + a^
10)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(19/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3-a\right)^2}{\left(bx^3+a\right)^{\frac{19}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(19/3),x, algorithm="giac")

[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(19/3), x)
```

3.48 $\int \left(a-bx^3\right)^2 \left(a+bx^3\right)^{4/3} dx$

Optimal. Leaf size=94

$$\frac{57a^3x\sqrt[3]{a+bx^3}\,{}_2F_1\left(-\frac{4}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{44\sqrt[3]{\frac{bx^3}{a}+1}}-\frac{9}{44}ax\left(a+bx^3\right)^{7/3}-\frac{1}{11}x\left(a-bx^3\right)\left(a+bx^3\right)^{7/3}$$

```
[Out] (-9*a*x*(a + b*x^3)^(7/3))/44 - (x*(a - b*x^3)*(a + b*x^3)^(7/3))/11 + (57*
a^3*x*(a + b*x^3)^(1/3)*Hypergeometric2F1[-4/3, 1/3, 4/3, -((b*x^3)/a)])/(4
4*(1 + (b*x^3)/a)^(1/3))
```

Rubi [A] time = 0.033539, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {416, 388, 246, 245}

$$\frac{57a^3x\sqrt[3]{a+bx^3}\,{}_2F_1\left(-\frac{4}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{44\sqrt[3]{\frac{bx^3}{a}+1}}-\frac{9}{44}ax\left(a+bx^3\right)^{7/3}-\frac{1}{11}x\left(a-bx^3\right)\left(a+bx^3\right)^{7/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2*(a + b*x^3)^(4/3),x]
```


```
[Out] (-9*a*x*(a + b*x^3)^(7/3))/44 - (x*(a - b*x^3)*(a + b*x^3)^(7/3))/11 + (57*
a^3*x*(a + b*x^3)^(1/3)*Hypergeometric2F1[-4/3, 1/3, 4/3, -((b*x^3)/a)])/(4
4*(1 + (b*x^3)/a)^(1/3))
```


Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
```

```
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \left(a-bx^3\right)^2 \left(a+bx^3\right)^{4/3}\,dx &= -\frac{1}{11}x\left(a-bx^3\right)\left(a+bx^3\right)^{7/3}+\frac{\int \left(a+bx^3\right)^{4/3}\left(12a^2b-18ab^2x^3\right)dx}{11b}\\
&= -\frac{9}{44}ax\left(a+bx^3\right)^{7/3}-\frac{1}{11}x\left(a-bx^3\right)\left(a+bx^3\right)^{7/3}+\frac{1}{44}\left(57a^2\right)\int \left(a+bx^3\right)^{4/3}dx\\
&= -\frac{9}{44}ax\left(a+bx^3\right)^{7/3}-\frac{1}{11}x\left(a-bx^3\right)\left(a+bx^3\right)^{7/3}+\frac{\left(57a^3\sqrt[3]{a+bx^3}\right)\int \left(1+\frac{bx^3}{a}\right)^{4/3}dx}{44\sqrt[3]{1+\frac{bx^3}{a}}}\\
&= -\frac{9}{44}ax\left(a+bx^3\right)^{7/3}-\frac{1}{11}x\left(a-bx^3\right)\left(a+bx^3\right)^{7/3}+\frac{57a^3x\sqrt[3]{a+bx^3}\,{}_2F_1\left(-\frac{4}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{44\sqrt[3]{1+\frac{bx^3}{a}}}
\end{aligned}$$

Mathematica [A] time = 0.0481449, size = 97, normalized size = 1.03

$$\frac{x\left(-78a^2b^2x^6+114a^4\left(\frac{bx^3}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)+53a^3bx^3+106a^4-5ab^3x^9+20b^4x^{12}\right)}{220\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2*(a + b*x^3)^(4/3),x]
```

```
[Out] (x*(106*a^4 + 53*a^3*b*x^3 - 78*a^2*b^2*x^6 - 5*a*b^3*x^9 + 20*b^4*x^12 + 1
14*a^4*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)]
))/(220*(a + b*x^3)^(2/3))
```

Maple [F] time = 0.215, size = 0, normalized size = 0.

$$\int \left(-bx^3+a\right)^2\left(bx^3+a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2*(b*x^3+a)^(4/3),x)
```

```
[Out] int((-b*x^3+a)^2*(b*x^3+a)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3+a\right)^{\frac{4}{3}}\left(bx^3-a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2*(b*x^3+a)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)*(b*x^3 - a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^3x^9 - ab^2x^6 - a^2bx^3 + a^3\right)\left(bx^3 + a\right)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2*(b*x^3+a)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((b^3*x^9 - a*b^2*x^6 - a^2*b*x^3 + a^3)*(b*x^3 + a)^(1/3), x)
```

Sympy [C] time = 5.11501, size = 168, normalized size = 1.79

$$\frac{a^{\frac{10}{3}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{a^{\frac{7}{3}}bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} - \frac{a^{\frac{4}{3}}b^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{7}{3} \\ \frac{10}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{\sqrt[3]{a}b^3x^{10}\Gamma\left(\frac{10}{3}\right){}_2F_1\left(\right.}{3\Gamma\left(\frac{13}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2*(b*x**3+a)**(4/3),x)
```

```
[Out] a**(10/3)*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)
/(3*gamma(4/3)) - a**(7/3)*b*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,), b*x
**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) - a**(4/3)*b**2*x**7*gamma(7/3)*hyper
((-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + a**(1/3)
*b**3*x**10*gamma(10/3)*hyper((-1/3, 10/3), (13/3,), b*x**3*exp_polar(I*pi)
/a)/(3*gamma(13/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3 + a\right)^{\frac{4}{3}}\left(bx^3 - a\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2*(b*x^3+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)*(b*x^3 - a)^2, x)
```

3.49 $\int \left(a-bx^3\right)^2 \sqrt[3]{a+bx^3}\,dx$

Optimal. Leaf size=94

$$\frac{3a^2x\sqrt[3]{a+bx^3}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2\sqrt[3]{\frac{bx^3}{a}+1}}-\frac{3}{8}ax\left(a+bx^3\right)^{4/3}-\frac{1}{8}x\left(a-bx^3\right)\left(a+bx^3\right)^{4/3}$$

```
[Out] (-3*a*x*(a + b*x^3)^(4/3))/8 - (x*(a - b*x^3)*(a + b*x^3)^(4/3))/8 + (3*a^2
*x*(a + b*x^3)^(1/3)*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^3)/a)])/(2*(1
 + (b*x^3)/a)^(1/3))
```

Rubi [A] time = 0.0341371, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {416, 388, 246, 245}

$$\frac{3a^2x\sqrt[3]{a+bx^3}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2\sqrt[3]{\frac{bx^3}{a}+1}}-\frac{3}{8}ax\left(a+bx^3\right)^{4/3}-\frac{1}{8}x\left(a-bx^3\right)\left(a+bx^3\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2*(a + b*x^3)^(1/3),x]
```

```
[Out] (-3*a*x*(a + b*x^3)^(4/3))/8 - (x*(a - b*x^3)*(a + b*x^3)^(4/3))/8 + (3*a^2
*x*(a + b*x^3)^(1/3)*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^3)/a)])/(2*(1
 + (b*x^3)/a)^(1/3))
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
```

```
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \left(a-bx^3\right)^2 \sqrt[3]{a+bx^3}\,dx &= -\frac{1}{8}x\left(a-bx^3\right)\left(a+bx^3\right)^{4/3}+\frac{\int \sqrt[3]{a+bx^3}\left(9a^2b-15ab^2x^3\right)dx}{8b}\\
&= -\frac{3}{8}ax\left(a+bx^3\right)^{4/3}-\frac{1}{8}x\left(a-bx^3\right)\left(a+bx^3\right)^{4/3}+\frac{1}{2}\left(3a^2\right)\int \sqrt[3]{a+bx^3}\,dx\\
&= -\frac{3}{8}ax\left(a+bx^3\right)^{4/3}-\frac{1}{8}x\left(a-bx^3\right)\left(a+bx^3\right)^{4/3}+\frac{\left(3a^2\sqrt[3]{a+bx^3}\right)\int \sqrt[3]{1+\frac{bx^3}{a}}\,dx}{2\sqrt[3]{1+\frac{bx^3}{a}}}\\
&= -\frac{3}{8}ax\left(a+bx^3\right)^{4/3}-\frac{1}{8}x\left(a-bx^3\right)\left(a+bx^3\right)^{4/3}+\frac{3a^2x\sqrt[3]{a+bx^3}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2\sqrt[3]{1+\frac{bx^3}{a}}}
\end{aligned}$$

Mathematica [A] time = 0.0381124, size = 85, normalized size = 0.9

$$\frac{x\left(6a^3\left(\frac{bx^3}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)-a^2bx^3+2a^3-2ab^2x^6+b^3x^9\right)}{8\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2*(a + b*x^3)^(1/3),x]
```

```
[Out] (x*(2*a^3 - a^2*b*x^3 - 2*a*b^2*x^6 + b^3*x^9 + 6*a^3*(1 + (b*x^3)/a)^(2/3)
*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)]))/(8*(a + b*x^3)^(2/3))
```

Maple [F] time = 0.217, size = 0, normalized size = 0.

$$\int \left(-bx^3+a\right)^2 \sqrt[3]{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2*(b*x^3+a)^(1/3),x)
```

```
[Out] int((-b*x^3+a)^2*(b*x^3+a)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3+a\right)^{\frac{1}{3}}\left(bx^3-a\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2*(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*(b*x^3 - a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2x^6 - 2\,abx^3 + a^2\right)\left(bx^3 + a\right)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2*(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 - 2*a*b*x^3 + a^2)*(b*x^3 + a)^(1/3), x)
```

Sympy [C] time = 2.983, size = 126, normalized size = 1.34

$$\frac{a^{\frac{7}{3}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{2a^{\frac{4}{3}}bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{\sqrt[3]{a}b^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2*(b*x**3+a)**(1/3),x)
```

```
[Out] a**(7/3)*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(4/3)) - 2*a**(4/3)*b*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(1/3)*b**2*x**7*gamma(7/3)*hype
r((-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3 + a\right)^{\frac{1}{3}}\left(bx^3 - a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2*(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*(b*x^3 - a)^2, x)
```

3.50 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{2/3}} \, dx$

Optimal. Leaf size=94

$$\frac{12a^2x\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5\left(a+bx^3\right)^{2/3}}-\frac{6}{5}ax\sqrt[3]{a+bx^3}-\frac{1}{5}x\left(a-bx^3\right)\sqrt[3]{a+bx^3}$$

```
[Out] (-6*a*x*(a + b*x^3)^(1/3))/5 - (x*(a - b*x^3)*(a + b*x^3)^(1/3))/5 + (12*a^
2*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(
5*(a + b*x^3)^(2/3))
```

Rubi [A] time = 0.0341232, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {416, 388, 246, 245}

$$\frac{12a^2x\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5\left(a+bx^3\right)^{2/3}}-\frac{6}{5}ax\sqrt[3]{a+bx^3}-\frac{1}{5}x\left(a-bx^3\right)\sqrt[3]{a+bx^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(2/3),x]
```

```
[Out] (-6*a*x*(a + b*x^3)^(1/3))/5 - (x*(a - b*x^3)*(a + b*x^3)^(1/3))/5 + (12*a^
2*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(
5*(a + b*x^3)^(2/3))
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a-bx^3\right)^2}{\left(a+bx^3\right)^{2/3}}\,dx &= -\frac{1}{5}x\left(a-bx^3\right)\sqrt[3]{a+bx^3}+\frac{\int \frac{6a^2b-12ab^2x^3}{\left(a+bx^3\right)^{2/3}}\,dx}{5b}\\
&= -\frac{6}{5}ax\sqrt[3]{a+bx^3}-\frac{1}{5}x\left(a-bx^3\right)\sqrt[3]{a+bx^3}+\frac{1}{5}\left(12a^2\right)\int \frac{1}{\left(a+bx^3\right)^{2/3}}\,dx\\
&= -\frac{6}{5}ax\sqrt[3]{a+bx^3}-\frac{1}{5}x\left(a-bx^3\right)\sqrt[3]{a+bx^3}+\frac{\left(12a^2\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{5\left(a+bx^3\right)^{2/3}}\\
&= -\frac{6}{5}ax\sqrt[3]{a+bx^3}-\frac{1}{5}x\left(a-bx^3\right)\sqrt[3]{a+bx^3}+\frac{12a^2x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5\left(a+bx^3\right)^{2/3}}
\end{aligned}$$

Mathematica [A] time = 0.0363123, size = 75, normalized size = 0.8

$$\frac{12a^2x\left(\frac{bx^3}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)-7a^2x-6abx^4+b^2x^7}{5\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(2/3),x]
```

```
[Out] (-7*a^2*x - 6*a*b*x^4 + b^2*x^7 + 12*a^2*x*(1 + (b*x^3)/a)^(2/3)*Hypergeome
tric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(5*(a + b*x^3)^(2/3))
```

Maple [F] time = 0.233, size = 0, normalized size = 0.

$$\int \left(-bx^3+a\right)^2\left(bx^3+a\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(2/3),x)
```

```
[Out] int((-b*x^3+a)^2/(b*x^3+a)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3-a\right)^2}{\left(bx^3+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2x^6 - 2\,abx^3 + a^2}{\left(bx^3 + a\right)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 - 2*a*b*x^3 + a^2)/(b*x^3 + a)^(2/3), x)
```

Sympy [C] time = 3.74243, size = 121, normalized size = 1.29

$$\frac{a^{\frac{4}{3}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{2\sqrt[3]{a}bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{b^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{7}{3}\\\frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(2/3),x)
```

```
[Out] a**(4/3)*x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(
3*gamma(4/3)) - 2*a**(1/3)*b*x**4*gamma(4/3)*hyper((2/3, 4/3), (7/3,), b*x*
*3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + b**2*x**7*gamma(7/3)*hyper((2/3, 7/3
), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2/3)*gamma(10/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3 - a\right)^2}{\left(bx^3 + a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(2/3), x)
```

3.51 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{5/3}}\,dx$

Optimal. Leaf size=74

$$\frac{3bx^4\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{7}{3};-\frac{bx^3}{a}\right)}{4\left(a+bx^3\right)^{2/3}}+\frac{x\left(a-bx^3\right)}{\left(a+bx^3\right)^{2/3}}$$

```
[Out] (x*(a - b*x^3))/(a + b*x^3)^(2/3) + (3*b*x^4*(1 + (b*x^3)/a)^(2/3)*Hypergeo
metric2F1[2/3, 4/3, 7/3, -((b*x^3)/a)])/(4*(a + b*x^3)^(2/3))
```

Rubi [A] time = 0.0339424, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {413, 12, 365, 364}

$$\frac{3bx^4\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{7}{3};-\frac{bx^3}{a}\right)}{4\left(a+bx^3\right)^{2/3}}+\frac{x\left(a-bx^3\right)}{\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(5/3),x]
```

```
[Out] (x*(a - b*x^3))/(a + b*x^3)^(2/3) + (3*b*x^4*(1 + (b*x^3)/a)^(2/3)*Hypergeo
metric2F1[2/3, 4/3, 7/3, -((b*x^3)/a)])/(4*(a + b*x^3)^(2/3))
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(a-bx^3\right)^2}{\left(a+bx^3\right)^{5/3}}\,dx = \frac{x\left(a-bx^3\right)}{\left(a+bx^3\right)^{2/3}} + \frac{\int \frac{6ab^2x^3}{\left(a+bx^3\right)^{2/3}}\,dx}{2ab}$$
$$= \frac{x\left(a-bx^3\right)}{\left(a+bx^3\right)^{2/3}} + (3b)\int \frac{x^3}{\left(a+bx^3\right)^{2/3}}\,dx$$
$$= \frac{x\left(a-bx^3\right)}{\left(a+bx^3\right)^{2/3}} + \frac{\left(3b\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{x^3}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{\left(a+bx^3\right)^{2/3}}$$
$$= \frac{x\left(a-bx^3\right)}{\left(a+bx^3\right)^{2/3}} + \frac{3bx^4\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{2}{3},\frac{4}{3};\frac{7}{3};-\frac{bx^3}{a}\right)}{4\left(a+bx^3\right)^{2/3}}$$

Mathematica [A] time = 0.0375889, size = 62, normalized size = 0.84

$$\frac{-3ax\left(\frac{bx^3}{a}+1\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)+5ax+bx^4}{2\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(5/3),x]
```

```
[Out] (5*a*x + b*x^4 - 3*a*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/
3, -((b*x^3)/a)])/(2*(a + b*x^3)^(2/3))
```

Maple [F] time = 0.421, size = 0, normalized size = 0.

$$\int \left(-bx^3+a\right)^2 \left(bx^3+a\right)^{-\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(5/3),x)
```

```
[Out] int((-b*x^3+a)^2/(b*x^3+a)^(5/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3-a\right)^2}{\left(bx^3+a\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(5/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^6 - 2\,abx^3 + a^2\right)\left(bx^3 + a\right)^{\frac{1}{3}}}{b^2x^6 + 2\,abx^3 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(5/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 - 2*a*b*x^3 + a^2)*(b*x^3 + a)^(1/3)/(b^2*x^6 + 2*a*b*x^3
 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-a + bx^3\right)^2}{\left(a + bx^3\right)^{\frac{5}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(5/3),x)
```

```
[Out] Integral((-a + b*x**3)**2/(a + b*x**3)**(5/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3 - a\right)^2}{\left(bx^3 + a\right)^{\frac{5}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(5/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(5/3), x)
```

3.52 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{8/3}} \, dx$

Optimal. Leaf size=74

$$\frac{3x\left(\frac{bx^3}{a}+1\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5\left(a+bx^3\right)^{2/3}}+\frac{2x\left(a-bx^3\right)}{5\left(a+bx^3\right)^{5/3}}$$

```
[Out] (2*x*(a - b*x^3))/(5*(a + b*x^3)^(5/3)) + (3*x*(1 + (b*x^3)/a)^(2/3)*Hyperg
eometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(5*(a + b*x^3)^(2/3))
```

Rubi [A] time = 0.0269495, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {413, 21, 246, 245}

$$\frac{3x\left(\frac{bx^3}{a}+1\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5\left(a+bx^3\right)^{2/3}}+\frac{2x\left(a-bx^3\right)}{5\left(a+bx^3\right)^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(8/3),x]
```

```
[Out] (2*x*(a - b*x^3))/(5*(a + b*x^3)^(5/3)) + (3*x*(1 + (b*x^3)/a)^(2/3)*Hyperg
eometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(5*(a + b*x^3)^(2/3))
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
```

```
GtQ[a, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a-bx^3\right)^2}{\left(a+bx^3\right)^{8/3}}\,dx &= \frac{2x\left(a-bx^3\right)}{5\left(a+bx^3\right)^{5/3}}+\frac{\int \frac{3a^2b+3ab^2x^3}{\left(a+bx^3\right)^{5/3}}\,dx}{5ab}\\
&= \frac{2x\left(a-bx^3\right)}{5\left(a+bx^3\right)^{5/3}}+\frac{3}{5}\int \frac{1}{\left(a+bx^3\right)^{2/3}}\,dx\\
&= \frac{2x\left(a-bx^3\right)}{5\left(a+bx^3\right)^{5/3}}+\frac{\left(3\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{5\left(a+bx^3\right)^{2/3}}\\
&= \frac{2x\left(a-bx^3\right)}{5\left(a+bx^3\right)^{5/3}}+\frac{3x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5\left(a+bx^3\right)^{2/3}}
\end{aligned}
$$

Mathematica [A] time = 0.0574889, size = 70, normalized size = 0.95

$$\frac{3x\left(a+bx^3\right)\left(\frac{bx^3}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)+2x\left(a-bx^3\right)}{5\left(a+bx^3\right)^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(8/3),x]
```

```
[Out] (2*x*(a - b*x^3) + 3*x*(a + b*x^3)*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[
1/3, 2/3, 4/3, -((b*x^3)/a)])/(5*(a + b*x^3)^(5/3))
```

Maple [F] time = 0.376, size = 0, normalized size = 0.

$$\int \left(-bx^3+a\right)^2\left(bx^3+a\right)^{-\frac{8}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(8/3),x)
```

```
[Out] int((-b*x^3+a)^2/(b*x^3+a)^(8/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3-a\right)^2}{\left(bx^3+a\right)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(8/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(8/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^6 - 2\,abx^3 + a^2\right)\left(bx^3 + a\right)^{\frac{1}{3}}}{b^3x^9 + 3\,ab^2x^6 + 3\,a^2bx^3 + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(8/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 - 2*a*b*x^3 + a^2)*(b*x^3 + a)^(1/3)/(b^3*x^9 + 3*a*b^2*x
^6 + 3*a^2*b*x^3 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(8/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3 - a\right)^2}{\left(bx^3 + a\right)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(8/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(8/3), x)
```

3.53 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{11/3}} \, dx$

Optimal. Leaf size=77

$$\frac{3x\left(\frac{bx^3}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{8}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{4a\left(a+bx^3\right)^{2/3}}+\frac{x\left(a-bx^3\right)}{4\left(a+bx^3\right)^{8/3}}$$

```
[Out] (x*(a - b*x^3))/(4*(a + b*x^3)^(8/3)) + (3*x*(1 + (b*x^3)/a)^(2/3)*Hypergeo
metric2F1[1/3, 8/3, 4/3, -((b*x^3)/a)])/(4*a*(a + b*x^3)^(2/3))
```

Rubi [A] time = 0.0257816, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {413, 12, 246, 245}

$$\frac{3x\left(\frac{bx^3}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{8}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{4a\left(a+bx^3\right)^{2/3}}+\frac{x\left(a-bx^3\right)}{4\left(a+bx^3\right)^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(11/3),x]
```

```
[Out] (x*(a - b*x^3))/(4*(a + b*x^3)^(8/3)) + (3*x*(1 + (b*x^3)/a)^(2/3)*Hypergeo
metric2F1[1/3, 8/3, 4/3, -((b*x^3)/a)])/(4*a*(a + b*x^3)^(2/3))
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a-bx^3\right)^2}{\left(a+bx^3\right)^{11/3}}\,dx &= \frac{x\left(a-bx^3\right)}{4\left(a+bx^3\right)^{8/3}}+\frac{\int \frac{6a^2b}{\left(a+bx^3\right)^{8/3}}\,dx}{8ab}\\
&= \frac{x\left(a-bx^3\right)}{4\left(a+bx^3\right)^{8/3}}+\frac{1}{4}(3a)\int \frac{1}{\left(a+bx^3\right)^{8/3}}\,dx\\
&= \frac{x\left(a-bx^3\right)}{4\left(a+bx^3\right)^{8/3}}+\frac{\left(3\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{8/3}}\,dx}{4a\left(a+bx^3\right)^{2/3}}\\
&= \frac{x\left(a-bx^3\right)}{4\left(a+bx^3\right)^{8/3}}+\frac{3x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{8}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{4a\left(a+bx^3\right)^{2/3}}
\end{aligned}$$

Mathematica [A] time = 0.0704025, size = 85, normalized size = 1.1

$$\frac{7a^2x+3x\left(a+bx^3\right)^2\left(\frac{bx^3}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)+5abx^4+3b^2x^7}{10a\left(a+bx^3\right)^{8/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(11/3),x]
```

```
[Out] (7*a^2*x + 5*a*b*x^4 + 3*b^2*x^7 + 3*x*(a + b*x^3)^2*(1 + (b*x^3)/a)^(2/3)*
Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(10*a*(a + b*x^3)^(8/3))
```

Maple [F] time = 0.358, size = 0, normalized size = 0.

$$\int \left(-bx^3+a\right)^2\left(bx^3+a\right)^{-\frac{11}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(11/3),x)
```

```
[Out] int((-b*x^3+a)^2/(b*x^3+a)^(11/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3-a\right)^2}{\left(bx^3+a\right)^{\frac{11}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(11/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(11/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b^{2}x^{6}-2\,abx^{3}+a^{2}\right)\left(bx^{3}+a\right)^{\frac{1}{3}}}{b^{4}x^{12}+4\,ab^{3}x^{9}+6\,a^{2}b^{2}x^{6}+4\,a^{3}bx^{3}+a^{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(11/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 - 2*a*b*x^3 + a^2)*(b*x^3 + a)^(1/3)/(b^4*x^12 + 4*a*b^3*
x^9 + 6*a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(11/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^{3}-a\right)^{2}}{\left(bx^{3}+a\right)^{\frac{11}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(11/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(11/3), x)
```

3.54 $\int \frac{\left(a-bx^3\right)^2}{\left(a+bx^3\right)^{14/3}}\,dx$

Optimal. Leaf size=93

$$\frac{15x\left(\frac{bx^3}{a}+1\right)^{2/3}\,_2F_1\left(\frac{1}{3},\frac{8}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{22a^2\left(a+bx^3\right)^{2/3}}+\frac{3x}{22\left(a+bx^3\right)^{8/3}}+\frac{2x\left(a-bx^3\right)}{11\left(a+bx^3\right)^{11/3}}$$

```
[Out] (2*x*(a - b*x^3))/(11*(a + b*x^3)^(11/3)) + (3*x)/(22*(a + b*x^3)^(8/3)) +
(15*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 8/3, 4/3, -((b*x^3)/a)])
/(22*a^2*(a + b*x^3)^(2/3))
```

Rubi [A] time = 0.033415, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {413, 385, 246, 245}

$$\frac{15x\left(\frac{bx^3}{a}+1\right)^{2/3}\,_2F_1\left(\frac{1}{3},\frac{8}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{22a^2\left(a+bx^3\right)^{2/3}}+\frac{3x}{22\left(a+bx^3\right)^{8/3}}+\frac{2x\left(a-bx^3\right)}{11\left(a+bx^3\right)^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(14/3),x]
```

```
[Out] (2*x*(a - b*x^3))/(11*(a + b*x^3)^(11/3)) + (3*x)/(22*(a + b*x^3)^(8/3)) +
(15*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 8/3, 4/3, -((b*x^3)/a)])
/(22*a^2*(a + b*x^3)^(2/3))
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a-bx^3)^2}{(a+bx^3)^{14/3}}\,dx &= \frac{2x(a-bx^3)}{11(a+bx^3)^{11/3}}+\frac{\int \frac{9a^2b-3ab^2x^3}{(a+bx^3)^{11/3}}\,dx}{11ab}\\ &= \frac{2x(a-bx^3)}{11(a+bx^3)^{11/3}}+\frac{3x}{22(a+bx^3)^{8/3}}+\frac{15}{22}\int \frac{1}{(a+bx^3)^{8/3}}\,dx\\ &= \frac{2x(a-bx^3)}{11(a+bx^3)^{11/3}}+\frac{3x}{22(a+bx^3)^{8/3}}+\frac{\left(15\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{8/3}}\,dx}{22a^2(a+bx^3)^{2/3}}\\ &= \frac{2x(a-bx^3)}{11(a+bx^3)^{11/3}}+\frac{3x}{22(a+bx^3)^{8/3}}+\frac{15x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{8}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{22a^2(a+bx^3)^{2/3}}\end{aligned}$$

Mathematica [A] time = 0.0842305, size = 95, normalized size = 1.02

$$\frac{x\left(23a^2bx^3+16a^3+21ab^2x^6+6(a+bx^3)^3\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)+6b^3x^9\right)}{22a^2(a+bx^3)^{11/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(14/3),x]
```

```
[Out] (x*(16*a^3 + 23*a^2*b*x^3 + 21*a*b^2*x^6 + 6*b^3*x^9 + 6*(a + b*x^3)^3*(1 +
 (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)]))/(22*a^2*
(a + b*x^3)^(11/3))
```

Maple [F] time = 0.37, size = 0, normalized size = 0.

$$\int (-bx^3+a)^2(bx^3+a)^{-\frac{14}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(14/3),x)
```

```
[Out] int((-b*x^3+a)^2/(b*x^3+a)^(14/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^3-a)^2}{(bx^3+a)^{\frac{14}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(14/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(14/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^6 - 2\,abx^3 + a^2\right)\left(bx^3 + a\right)^{\frac{1}{3}}}{b^5x^{15} + 5\,ab^4x^{12} + 10\,a^2b^3x^9 + 10\,a^3b^2x^6 + 5\,a^4bx^3 + a^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(14/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 - 2*a*b*x^3 + a^2)*(b*x^3 + a)^(1/3)/(b^5*x^15 + 5*a*b^4*
x^12 + 10*a^2*b^3*x^9 + 10*a^3*b^2*x^6 + 5*a^4*b*x^3 + a^5), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(14/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3 - a\right)^2}{\left(bx^3 + a\right)^{\frac{14}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(14/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(14/3), x)
```

3.55 $\int \frac{(a-bx^3)^2}{(a+bx^3)^{17/3}}\,dx$

Optimal. Leaf size=93

$$\frac{57x\left(\frac{bx^3}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{11}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{77a^3\left(a+bx^3\right)^{2/3}}+\frac{9x}{77\left(a+bx^3\right)^{11/3}}+\frac{x\left(a-bx^3\right)}{7\left(a+bx^3\right)^{14/3}}$$

```
[Out] (x*(a - b*x^3))/(7*(a + b*x^3)^(14/3)) + (9*x)/(77*(a + b*x^3)^(11/3)) + (5
7*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 11/3, 4/3, -((b*x^3)/a)])/
(77*a^3*(a + b*x^3)^(2/3))
```

Rubi [A] time = 0.0341936, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {413, 385, 246, 245}

$$\frac{57x\left(\frac{bx^3}{a}+1\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{11}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{77a^3\left(a+bx^3\right)^{2/3}}+\frac{9x}{77\left(a+bx^3\right)^{11/3}}+\frac{x\left(a-bx^3\right)}{7\left(a+bx^3\right)^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^3)^2/(a + b*x^3)^(17/3),x]
```

```
[Out] (x*(a - b*x^3))/(7*(a + b*x^3)^(14/3)) + (9*x)/(77*(a + b*x^3)^(11/3)) + (5
7*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 11/3, 4/3, -((b*x^3)/a)])/
(77*a^3*(a + b*x^3)^(2/3))
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a-bx^3\right)^2}{\left(a+bx^3\right)^{17/3}}\,dx &= \frac{x\left(a-bx^3\right)}{7\left(a+bx^3\right)^{14/3}}+\frac{\int \frac{12a^2b-6ab^2x^3}{\left(a+bx^3\right)^{14/3}}\,dx}{14ab}\\
&= \frac{x\left(a-bx^3\right)}{7\left(a+bx^3\right)^{14/3}}+\frac{9x}{77\left(a+bx^3\right)^{11/3}}+\frac{57}{77}\int \frac{1}{\left(a+bx^3\right)^{11/3}}\,dx\\
&= \frac{x\left(a-bx^3\right)}{7\left(a+bx^3\right)^{14/3}}+\frac{9x}{77\left(a+bx^3\right)^{11/3}}+\frac{\left(57\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{11/3}}\,dx}{77a^3\left(a+bx^3\right)^{2/3}}\\
&= \frac{x\left(a-bx^3\right)}{7\left(a+bx^3\right)^{14/3}}+\frac{9x}{77\left(a+bx^3\right)^{11/3}}+\frac{57x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{11}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{77a^3\left(a+bx^3\right)^{2/3}}
\end{aligned}$$

Mathematica [A] time = 0.103485, size = 106, normalized size = 1.14

$$\frac{x\left(6270a^2b^2x^6+4879a^3bx^3+2282a^4+3591ab^3x^9+798\left(a+bx^3\right)^4\left(\frac{bx^3}{a}+1\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)+798b^4x^{12}\right)}{3080a^3\left(a+bx^3\right)^{14/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^3)^2/(a + b*x^3)^(17/3),x]
```

```
[Out] (x*(2282*a^4 + 4879*a^3*b*x^3 + 6270*a^2*b^2*x^6 + 3591*a*b^3*x^9 + 798*b^4
*x^12 + 798*(a + b*x^3)^4*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3,
 4/3, -((b*x^3)/a)]))/(3080*a^3*(a + b*x^3)^(14/3))
```

Maple [F] time = 0.388, size = 0, normalized size = 0.

$$\int \left(-bx^3+a\right)^2\left(bx^3+a\right)^{-\frac{17}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^3+a)^2/(b*x^3+a)^(17/3),x)
```

```
[Out] int((-b*x^3+a)^2/(b*x^3+a)^(17/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3-a\right)^2}{\left(bx^3+a\right)^{\frac{17}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(17/3),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(17/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b^2x^6 - 2\,abx^3 + a^2\right)\left(bx^3 + a\right)^{\frac{1}{3}}}{b^6x^{18} + 6\,ab^5x^{15} + 15\,a^2b^4x^{12} + 20\,a^3b^3x^9 + 15\,a^4b^2x^6 + 6\,a^5bx^3 + a^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(17/3),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 - 2*a*b*x^3 + a^2)*(b*x^3 + a)^(1/3)/(b^6*x^18 + 6*a*b^5*
x^15 + 15*a^2*b^4*x^12 + 20*a^3*b^3*x^9 + 15*a^4*b^2*x^6 + 6*a^5*b*x^3 + a^
6), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(17/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3 - a\right)^2}{\left(bx^3 + a\right)^{\frac{17}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^3+a)^2/(b*x^3+a)^(17/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 - a)^2/(b*x^3 + a)^(17/3), x)
```

3.56 $\int \left(a+bx^3\right)^{5/3} \left(c+dx^3\right) dx$

Optimal. Leaf size=174

$$-\frac{5a^2(9bc-ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{162b^{4/3}}+\frac{5a^2(9bc-ad)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{81\sqrt{3}b^{4/3}}+\frac{x\left(a+bx^3\right)^{5/3}(9bc-ad)}{54b}+\frac{5ax\left(a+bx^3\right)^{2/}}{162}$$

```
[Out] (5*a*(9*b*c - a*d)*x*(a + b*x^3)^(2/3))/(162*b) + ((9*b*c - a*d)*x*(a + b*x
^3)^(5/3))/(54*b) + (d*x*(a + b*x^3)^(8/3))/(9*b) + (5*a^2*(9*b*c - a*d)*Ar
cTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(81*Sqrt[3]*b^(4/3)) -
 (5*a^2*(9*b*c - a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(162*b^(4/3))
```

Rubi [A] time = 0.0592267, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {388, 195, 239}

$$-\frac{5a^2(9bc-ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{162b^{4/3}}+\frac{5a^2(9bc-ad)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{81\sqrt{3}b^{4/3}}+\frac{x\left(a+bx^3\right)^{5/3}(9bc-ad)}{54b}+\frac{5ax\left(a+bx^3\right)^{2/}}{162}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(5/3)*(c + d*x^3),x]
```

```
[Out] (5*a*(9*b*c - a*d)*x*(a + b*x^3)^(2/3))/(162*b) + ((9*b*c - a*d)*x*(a + b*x
^3)^(5/3))/(54*b) + (d*x*(a + b*x^3)^(8/3))/(9*b) + (5*a^2*(9*b*c - a*d)*Ar
cTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(81*Sqrt[3]*b^(4/3)) -
 (5*a^2*(9*b*c - a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(162*b^(4/3))
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \left(a+bx^3\right)^{5/3}\left(c+dx^3\right)dx = \frac{dx\left(a+bx^3\right)^{8/3}}{9b} - \frac{(-9bc+ad)\int \left(a+bx^3\right)^{5/3}dx}{9b}$$
$$= \frac{(9bc-ad)x\left(a+bx^3\right)^{5/3}}{54b} + \frac{dx\left(a+bx^3\right)^{8/3}}{9b} + \frac{(5a(9bc-ad))\int \left(a+bx^3\right)^{2/3}dx}{54b}$$
$$= \frac{5a(9bc-ad)x\left(a+bx^3\right)^{2/3}}{162b} + \frac{(9bc-ad)x\left(a+bx^3\right)^{5/3}}{54b} + \frac{dx\left(a+bx^3\right)^{8/3}}{9b} + \frac{\left(5a^2(9bc-ad)\right)}{81b}$$
$$= \frac{5a(9bc-ad)x\left(a+bx^3\right)^{2/3}}{162b} + \frac{(9bc-ad)x\left(a+bx^3\right)^{5/3}}{54b} + \frac{dx\left(a+bx^3\right)^{8/3}}{9b} + \frac{5a^2(9bc-ad)\,\mathrm{ta}}{81\sqrt{}}$$

Mathematica [C] time = 0.065927, size = 75, normalized size = 0.43

$$\frac{x\left(a+bx^3\right)^{2/3}\left(d\left(a+bx^3\right)^2-\frac{a(ad-9bc)\,{}_2F_1\left(-\frac{5}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\left(\frac{bx^3}{a}+1\right)^{2/3}}\right)}{9b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(5/3)*(c + d*x^3),x]
```

```
[Out] (x*(a + b*x^3)^(2/3)*(d*(a + b*x^3)^2 - (a*(-9*b*c + a*d)*Hypergeometric2F1
[-5/3, 1/3, 4/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^(2/3)))/(9*b)
```

Maple [F] time = 0.214, size = 0, normalized size = 0.

$$\int \left(bx^3+a\right)^{\frac{5}{3}}\left(dx^3+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(5/3)*(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(5/3)*(d*x^3+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)*(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.06421, size = 1222, normalized size = 7.02

$$\left[-\frac{15\sqrt{\frac{1}{3}}\left(9\,a^2b^2c-a^3bd\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3\,bx^3-3\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^3+\left(bx^3+a\right)^{\frac{1}{3}}bx^2-2\left(bx^3+a\right)^{\frac{2}{3}}b^{\frac{2}{3}}x\right)\sqrt{-\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)*(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/486*(15*sqrt(1/3)*(9*a^2*b^2*c - a^3*b*d)*sqrt(-1/b^(2/3))*log(3*b*x^3
- 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^
(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 10*(
9*a^2*b*c - a^3*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - 5*(9*a
^2*b*c - a^3*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b
*x^3 + a)^(2/3))/x^2) - 3*(18*b^3*d*x^7 + 3*(9*b^3*c + 11*a*b^2*d)*x^4 + 2*
(36*a*b^2*c + 5*a^2*b*d)*x)*(b*x^3 + a)^(2/3))/b^2, -1/486*(10*(9*a^2*b*c -
 a^3*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - 5*(9*a^2*b*c - a^
3*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(
2/3))/x^2) + 30*sqrt(1/3)*(9*a^2*b^2*c - a^3*b*d)*arctan(sqrt(1/3)*(b^(1/3)
*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3) - 3*(18*b^3*d*x^7 + 3*(9*b^3
*c + 11*a*b^2*d)*x^4 + 2*(36*a*b^2*c + 5*a^2*b*d)*x)*(b*x^3 + a)^(2/3))/b^2
]
```

Sympy [C] time = 6.70869, size = 170, normalized size = 0.98

$$\frac{a^{\frac{5}{3}}cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{a^{\frac{5}{3}}dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{a^{\frac{2}{3}}bcx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{a^{\frac{2}{3}}bdx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\cdots\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(5/3)*(d*x**3+c),x)
```

```
[Out] a**(5/3)*c*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(4/3)) + a**(5/3)*d*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(2/3)*b*c*x**4*gamma(4/3)*hyper
((-2/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(2/3)*b
*d*x**7*gamma(7/3)*hyper((-2/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3
*gamma(10/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(bx^3+a\right)^{\frac{5}{3}}\left(dx^3+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)*(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(5/3)*(d*x^3 + c), x)
```

3.57 $\int \left(a+bx^3\right)^{2/3} \left(c+dx^3\right) dx$

Optimal. Leaf size=141

$$-\frac{a(6bc-ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{18b^{4/3}}+\frac{a(6bc-ad)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}b^{4/3}}+\frac{x\left(a+bx^3\right)^{2/3}(6bc-ad)}{18b}+\frac{dx\left(a+bx^3\right)^{5/3}}{6b}$$

```
[Out] ((6*b*c - a*d)*x*(a + b*x^3)^(2/3))/(18*b) + (d*x*(a + b*x^3)^(5/3))/(6*b)
+ (a*(6*b*c - a*d)*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(
9*Sqrt[3]*b^(4/3)) - (a*(6*b*c - a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]
)/(18*b^(4/3))
```

Rubi [A] time = 0.0450737, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {388, 195, 239}

$$-\frac{a(6bc-ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{18b^{4/3}}+\frac{a(6bc-ad)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}b^{4/3}}+\frac{x\left(a+bx^3\right)^{2/3}(6bc-ad)}{18b}+\frac{dx\left(a+bx^3\right)^{5/3}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)*(c + d*x^3),x]
```

```
[Out] ((6*b*c - a*d)*x*(a + b*x^3)^(2/3))/(18*b) + (d*x*(a + b*x^3)^(5/3))/(6*b)
+ (a*(6*b*c - a*d)*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(
9*Sqrt[3]*b^(4/3)) - (a*(6*b*c - a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)]
)/(18*b^(4/3))
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \left(a+bx^3\right)^{2/3} \left(c+dx^3\right) dx = \frac{dx\left(a+bx^3\right)^{5/3}}{6b} - \frac{(-6bc+ad)\int \left(a+bx^3\right)^{2/3} dx}{6b}$$
$$= \frac{(6bc-ad)x\left(a+bx^3\right)^{2/3}}{18b} + \frac{dx\left(a+bx^3\right)^{5/3}}{6b} + \frac{(a(6bc-ad))\int \frac{1}{\sqrt[3]{a+bx^3}}dx}{9b}$$
$$= \frac{(6bc-ad)x\left(a+bx^3\right)^{2/3}}{18b} + \frac{dx\left(a+bx^3\right)^{5/3}}{6b} + \frac{a(6bc-ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{4/3}} - \frac{a(6bc-ad)\,\mathrm{l}}{}$$

Mathematica [C] time = 0.0694077, size = 72, normalized size = 0.51

$$\frac{x\left(a+bx^3\right)^{2/3}\left(\frac{(6bc-ad)\,{}_2F_1\left(-\frac{2}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\left(\frac{bx^3}{a}+1\right)^{2/3}}+d\left(a+bx^3\right)\right)}{6b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)*(c + d*x^3),x]
```

```
[Out] (x*(a + b*x^3)^(2/3)*(d*(a + b*x^3) + ((6*b*c - a*d)*Hypergeometric2F1[-2/3
, 1/3, 4/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^(2/3)))/(6*b)
```

Maple [F] time = 0.222, size = 0, normalized size = 0.

$$\int \left(bx^3+a\right)^{\frac{2}{3}} \left(dx^3+c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)*(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(2/3)*(d*x^3+c),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)*(d*x^3+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.02218, size = 1087, normalized size = 7.71

$$\left[-\frac{3\sqrt{\frac{1}{3}}\left(6\,ab^2c - a^2bd\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3\,bx^3 - 3\left(bx^3 + a\right)^{\frac{1}{3}}b^{\frac{2}{3}}x^2 - 3\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^3 + \left(bx^3 + a\right)^{\frac{1}{3}}bx^2 - 2\left(bx^3 + a\right)^{\frac{2}{3}}b^{\frac{2}{3}}x\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)*(d*x^3+c),x, algorithm="fricas")
```

```
[Out] [-1/54*(3*sqrt(1/3)*(6*a*b^2*c - a^2*b*d)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*
(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3
)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 2*(6*a*b
*c - a^2*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - (6*a*b*c - a^
2*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(
2/3))/x^2) - 3*(3*b^2*d*x^4 + 2*(3*b^2*c + a*b*d)*x)*(b*x^3 + a)^(2/3))/b^2
, -1/54*(2*(6*a*b*c - a^2*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x
) - (6*a*b*c - a^2*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*
x + (b*x^3 + a)^(2/3))/x^2) + 6*sqrt(1/3)*(6*a*b^2*c - a^2*b*d)*arctan(sqrt
(1/3)*(b^(1/3)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3) - 3*(3*b^2*d*x
^4 + 2*(3*b^2*c + a*b*d)*x)*(b*x^3 + a)^(2/3))/b^2]
```

Sympy [C] time = 3.2304, size = 82, normalized size = 0.58

$$\frac{a^{\frac{2}{3}}cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{a^{\frac{2}{3}}dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)*(d*x**3+c),x)
```

```
[Out] a**(2/3)*c*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(4/3)) + a**(2/3)*d*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(7/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(bx^3 + a\right)^{\frac{2}{3}}\left(dx^3 + c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)*(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)*(d*x^3 + c), x)
```

3.58 $\int \frac{c+dx^3}{\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=111

$$-\frac{(3bc-ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{6b^{4/3}}+\frac{(3bc-ad)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}}+\frac{dx\left(a+bx^3\right)^{2/3}}{3b}$$

```
[Out] (d*x*(a + b*x^3)^(2/3))/(3*b) + ((3*b*c - a*d)*ArcTan[(1 + (2*b^(1/3)*x)/(a
 + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(4/3)) - ((3*b*c - a*d)*Log[-(b^(1/
3)*x) + (a + b*x^3)^(1/3)])/(6*b^(4/3))
```

Rubi [A] time = 0.0303546, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {388, 239}

$$-\frac{(3bc-ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{6b^{4/3}}+\frac{(3bc-ad)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}}+\frac{dx\left(a+bx^3\right)^{2/3}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)/(a + b*x^3)^(1/3),x]
```

```
[Out] (d*x*(a + b*x^3)^(2/3))/(3*b) + ((3*b*c - a*d)*ArcTan[(1 + (2*b^(1/3)*x)/(a
 + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(4/3)) - ((3*b*c - a*d)*Log[-(b^(1/
3)*x) + (a + b*x^3)^(1/3)])/(6*b^(4/3))
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{c+dx^3}{\sqrt[3]{a+bx^3}}\,dx=\frac{dx\left(a+bx^3\right)^{2/3}}{3b}-\frac{(-3bc+ad)\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx}{3b}$$

$$=\frac{dx\left(a+bx^3\right)^{2/3}}{3b}+\frac{(3bc-ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}}-\frac{(3bc-ad)\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{6b^{4/3}}$$

Mathematica [A] time = 0.130079, size = 141, normalized size = 1.27

$$\frac{\frac{(3bc-ad)\left(\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right)-2\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)\right)}{6\sqrt[3]{b}}+dx\left(a+bx^3\right)^{2/3}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)/(a + b*x^3)^(1/3),x]
```

```
[Out] (d*x*(a + b*x^3)^(2/3) + ((3*b*c - a*d)*(2*Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*x
)/(a + b*x^3)^(1/3))/Sqrt[3]] - 2*Log[1 - (b^(1/3)*x)/(a + b*x^3)^(1/3)] +
Log[1 + (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)]))/
(6*b^(1/3)))/(3*b)
```

Maple [F] time = 0.217, size = 0, normalized size = 0.

$$\int (dx^3+c)\frac{1}{\sqrt[3]{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)/(b*x^3+a)^(1/3),x)
```

```
[Out] int((d*x^3+c)/(b*x^3+a)^(1/3),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.97456, size = 965, normalized size = 8.69

$$\left[\frac{6\left(bx^3+a\right)^{\frac{2}{3}}bdx-3\sqrt{\frac{1}{3}}\left(3b^2c-abd\right)\sqrt{-\frac{1}{b^{\frac{2}{3}}}}\log\left(3bx^3-3\left(bx^3+a\right)^{\frac{1}{3}}b^{\frac{2}{3}}x^2-3\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^3+\left(bx^3+a\right)^{\frac{1}{3}}bx^2-2\left(bx^3+a\right.\right.\right.}{18b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/18*(6*(b*x^3 + a)^(2/3)*b*d*x - 3*sqrt(1/3)*(3*b^2*c - a*b*d)*sqrt(-1/b^
(2/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 - 3*sqrt(1/3)*(b^(4/3)
*x^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^(2/3)*x)*sqrt(-1/b^(
2/3)) + 2*a) - 2*(3*b*c - a*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))
/x) + (3*b*c - a*d)*b^(2/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x
+ (b*x^3 + a)^(2/3))/x^2))/b^2, 1/18*(6*(b*x^3 + a)^(2/3)*b*d*x - 2*(3*b*c
- a*d)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) + (3*b*c - a*d)*b^(2
/3)*log((b^(2/3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2
) - 6*sqrt(1/3)*(3*b^2*c - a*b*d)*arctan(sqrt(1/3)*(b^(1/3)*x + 2*(b*x^3 +
a)^(1/3))/(b^(1/3)*x))/b^(1/3))/b^2]
```

Sympy [C] time = 2.31708, size = 78, normalized size = 0.7

$$\frac{cx\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\right|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{4}{3}\right)}+\frac{dx^{4}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\right|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)/(b*x**3+a)**(1/3),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1
/3)*gamma(4/3)) + d*x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*a**(1/3)*gamma(7/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^3 + c}{\left(bx^3 + a\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(1/3), x)
```

3.59 $\int \frac{c+dx^3}{\left(a+bx^3\right)^{4/3}}\,dx$

Optimal. Leaf size=99

$$-\frac{d\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2b^{4/3}}+\frac{d\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{4/3}}+\frac{x(bc-ad)}{ab\sqrt[3]{a+bx^3}}$$

```
[Out] ((b*c - a*d)*x)/(a*b*(a + b*x^3)^(1/3)) + (d*ArcTan[(1 + (2*b^(1/3)*x)/(a +
 b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(4/3)) - (d*Log[-(b^(1/3)*x) + (a + b*x
^3)^(1/3)])/(2*b^(4/3))
```

Rubi [A] time = 0.0235665, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {385, 239}

$$-\frac{d\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2b^{4/3}}+\frac{d\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{4/3}}+\frac{x(bc-ad)}{ab\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)/(a + b*x^3)^(4/3),x]
```

```
[Out] ((b*c - a*d)*x)/(a*b*(a + b*x^3)^(1/3)) + (d*ArcTan[(1 + (2*b^(1/3)*x)/(a +
 b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(4/3)) - (d*Log[-(b^(1/3)*x) + (a + b*x
^3)^(1/3)])/(2*b^(4/3))
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx^3}{\left(a+bx^3\right)^{4/3}}\,dx &= \frac{(bc-ad)x}{ab\sqrt[3]{a+bx^3}}+\frac{d\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx}{b}\\
&= \frac{(bc-ad)x}{ab\sqrt[3]{a+bx^3}}+\frac{d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{4/3}}-\frac{d\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right)}{2b^{4/3}}
\end{aligned}$$

Mathematica [C] time = 0.0354563, size = 61, normalized size = 0.62

$$\frac{dx^4\sqrt[3]{\frac{bx^3}{a}+1}\,{}_2F_1\left(\frac{4}{3},\frac{4}{3};\frac{7}{3};-\frac{bx^3}{a}\right)+4cx}{4a\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)/(a + b*x^3)^(4/3),x]
```

```
[Out] (4*c*x + d*x^4*(1 + (b*x^3)/a)^(1/3)*Hypergeometric2F1[4/3, 4/3, 7/3, -((b*
x^3)/a)])/(4*a*(a + b*x^3)^(1/3))
```

Maple [F] time = 0.221, size = 0, normalized size = 0.

$$\int (dx^3+c)\left(bx^3+a\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)/(b*x^3+a)^(4/3),x)
```

```
[Out] int((d*x^3+c)/(b*x^3+a)^(4/3),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.98966, size = 1187, normalized size = 11.99

$$\left[\frac{3\sqrt{\frac{1}{3}}\left(ab^2dx^3+a^2bd\right)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3\,bx^3-3\left(bx^3+a\right)^{\frac{1}{3}}(-b)^{\frac{2}{3}}\,x^2-3\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}\,bx^3-\left(bx^3+a\right)^{\frac{1}{3}}bx^2+2\left(bx^3+a\right)^{\frac{2}{3}}(-b)^{\frac{2}{3}}\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*(a*b^2*d*x^3 + a^2*b*d)*sqrt((-b)^(1/3)/b)*log(3*b*x^3 -
3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/3)*b*x^3 - (b*x^3
```

```
 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt((-b)^(1/3)/b) +
2*a) + 6*(b*x^3 + a)^(2/3)*(b^2*c - a*b*d)*x - 2*(a*b*d*x^3 + a^2*d)*(-b)^(
2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + (a*b*d*x^3 + a^2*d)*(-b)^(
2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/
3))/x^2))/(a*b^3*x^3 + a^2*b^2), -1/6*(6*sqrt(1/3)*(a*b^2*d*x^3 + a^2*b*d)*
sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*
sqrt(-(-b)^(1/3)/b)/x) - 6*(b*x^3 + a)^(2/3)*(b^2*c - a*b*d)*x + 2*(a*b*d*x
^3 + a^2*d)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - (a*b*d*x
^3 + a^2*d)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x
 + (b*x^3 + a)^(2/3))/x^2))/(a*b^3*x^3 + a^2*b^2)]
```

Sympy [C] time = 10.4044, size = 71, normalized size = 0.72

$$\frac{cx\Gamma\left(\frac{1}{3}\right)}{3a^{\frac{4}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{4}{3}\right)}+\frac{dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)/(b*x**3+a)**(4/3),x)
```

```
[Out] c*x*gamma(1/3)/(3*a**(4/3)*(1 + b*x**3/a)**(1/3)*gamma(4/3)) + d*x**4*gamma
(4/3)*hyper((4/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(4/3)*gamma
(7/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^3+c}{\left(bx^3+a\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(4/3), x)
```

3.60 $\int \frac{c+dx^3}{\left(a+bx^3\right)^{7/3}}\,dx$

Optimal. Leaf size=47

$$\frac{3cx}{4a^2\sqrt[3]{a+bx^3}}+\frac{x\left(c+dx^3\right)}{4a\left(a+bx^3\right)^{4/3}}$$

```
[Out] (3*c*x)/(4*a^2*(a + b*x^3)^(1/3)) + (x*(c + d*x^3))/(4*a*(a + b*x^3)^(4/3))
```

Rubi [A] time = 0.0097775, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {378, 191}

$$\frac{3cx}{4a^2\sqrt[3]{a+bx^3}}+\frac{x\left(c+dx^3\right)}{4a\left(a+bx^3\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)/(a + b*x^3)^(7/3),x]
```

```
[Out] (3*c*x)/(4*a^2*(a + b*x^3)^(1/3)) + (x*(c + d*x^3))/(4*a*(a + b*x^3)^(4/3))
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^3}{\left(a+bx^3\right)^{7/3}}\,dx &= \frac{x\left(c+dx^3\right)}{4a\left(a+bx^3\right)^{4/3}}+\frac{(3c)\int \frac{1}{\left(a+bx^3\right)^{4/3}}\,dx}{4a}\\ &= \frac{3cx}{4a^2\sqrt[3]{a+bx^3}}+\frac{x\left(c+dx^3\right)}{4a\left(a+bx^3\right)^{4/3}}\end{aligned}$$

Mathematica [A] time = 0.0202497, size = 37, normalized size = 0.79

$$\frac{x\left(4ac+adx^3+3bcx^3\right)}{4a^2\left(a+bx^3\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)/(a + b*x^3)^(7/3),x]
```

```
[Out] (x*(4*a*c + 3*b*c*x^3 + a*d*x^3))/(4*a^2*(a + b*x^3)^(4/3))
```

Maple [A] time = 0.005, size = 34, normalized size = 0.7

$$\frac{x\left(adx^3+3\,bcx^3+4\,ac\right)}{4\,a^2}\left(bx^3+a\right)^{-\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)/(b*x^3+a)^(7/3),x)
```

```
[Out] 1/4*x*(a*d*x^3+3*b*c*x^3+4*a*c)/(b*x^3+a)^(4/3)/a^2
```

Maxima [A] time = 0.957957, size = 69, normalized size = 1.47

$$-\frac{\left(b-\frac{4\left(bx^3+a\right)}{x^3}\right)cx^4}{4\left(bx^3+a\right)^{\frac{4}{3}}a^2}+\frac{dx^4}{4\left(bx^3+a\right)^{\frac{4}{3}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(7/3),x, algorithm="maxima")
```

```
[Out] -1/4*(b - 4*(b*x^3 + a)/x^3)*c*x^4/((b*x^3 + a)^(4/3)*a^2) + 1/4*d*x^4/((b*
x^3 + a)^(4/3)*a)
```

Fricas [A] time = 1.66131, size = 117, normalized size = 2.49

$$\frac{\left((3\,bc+ad)x^4+4\,acx\right)\left(bx^3+a\right)^{\frac{2}{3}}}{4\left(a^2b^2x^6+2\,a^3bx^3+a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(7/3),x, algorithm="fricas")
```

```
[Out] 1/4*((3*b*c + a*d)*x^4 + 4*a*c*x)*(b*x^3 + a)^(2/3)/(a^2*b^2*x^6 + 2*a^3*b*
x^3 + a^4)
```

Sympy [B] time = 96.6526, size = 190, normalized size = 4.04

$$c\left(\frac{4ax\Gamma\left(\frac{1}{3}\right)}{9a^{\frac{10}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)+9a^{\frac{7}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)}+\frac{3bx^4\Gamma\left(\frac{1}{3}\right)}{9a^{\frac{10}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)+9a^{\frac{7}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)}\right)+\frac{dx^{\cdots}}{3a^{\frac{7}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)/(b*x**3+a)**(7/3),x)
```

```
[Out] c*(4*a*x*gamma(1/3)/(9*a**(10/3)*(1 + b*x**3/a)**(1/3)*gamma(7/3) + 9*a**(7
/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(7/3)) + 3*b*x**4*gamma(1/3)/(9*a**(1
0/3)*(1 + b*x**3/a)**(1/3)*gamma(7/3) + 9*a**(7/3)*b*x**3*(1 + b*x**3/a)**(
1/3)*gamma(7/3))) + d*x**4*gamma(4/3)/(3*a**(7/3)*(1 + b*x**3/a)**(1/3)*gam
ma(7/3) + 3*a**(4/3)*b*x**3*(1 + b*x**3/a)**(1/3)*gamma(7/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^3 + c}{\left(bx^3 + a\right)^{\frac{7}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(7/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(7/3), x)
```

3.61 $\int \frac{c+dx^3}{\left(a+bx^3\right)^{10/3}}\,dx$

Optimal. Leaf size=91

$$\frac{3x(ad+6bc)}{28a^3b\sqrt[3]{a+bx^3}}+\frac{x(ad+6bc)}{28a^2b\left(a+bx^3\right)^{4/3}}+\frac{x(bc-ad)}{7ab\left(a+bx^3\right)^{7/3}}$$

```
[Out] ((b*c - a*d)*x)/(7*a*b*(a + b*x^3)^(7/3)) + ((6*b*c + a*d)*x)/(28*a^2*b*(a
+ b*x^3)^(4/3)) + (3*(6*b*c + a*d)*x)/(28*a^3*b*(a + b*x^3)^(1/3))
```

Rubi [A] time = 0.0275407, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {385, 192, 191}

$$\frac{3x(ad+6bc)}{28a^3b\sqrt[3]{a+bx^3}}+\frac{x(ad+6bc)}{28a^2b\left(a+bx^3\right)^{4/3}}+\frac{x(bc-ad)}{7ab\left(a+bx^3\right)^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)/(a + b*x^3)^(10/3),x]
```

```
[Out] ((b*c - a*d)*x)/(7*a*b*(a + b*x^3)^(7/3)) + ((6*b*c + a*d)*x)/(28*a^2*b*(a
+ b*x^3)^(4/3)) + (3*(6*b*c + a*d)*x)/(28*a^3*b*(a + b*x^3)^(1/3))
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{c+dx^3}{\left(a+bx^3\right)^{10/3}}\,dx = \frac{(bc-ad)x}{7ab\left(a+bx^3\right)^{7/3}} + \frac{(6bc+ad)\int \frac{1}{\left(a+bx^3\right)^{7/3}}\,dx}{7ab}$$

$$= \frac{(bc-ad)x}{7ab\left(a+bx^3\right)^{7/3}} + \frac{(6bc+ad)x}{28a^2b\left(a+bx^3\right)^{4/3}} + \frac{(3(6bc+ad))\int \frac{1}{\left(a+bx^3\right)^{4/3}}\,dx}{28a^2b}$$

$$= \frac{(bc-ad)x}{7ab\left(a+bx^3\right)^{7/3}} + \frac{(6bc+ad)x}{28a^2b\left(a+bx^3\right)^{4/3}} + \frac{3(6bc+ad)x}{28a^3b\sqrt[3]{a+bx^3}}$$

Mathematica [A] time = 0.0277739, size = 59, normalized size = 0.65

$$\frac{7a^2\left(4cx+dx^4\right)+3abx^4\left(14c+dx^3\right)+18b^2cx^7}{28a^3\left(a+bx^3\right)^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)/(a + b*x^3)^(10/3),x]
```

```
[Out] (18*b^2*c*x^7 + 3*a*b*x^4*(14*c + d*x^3) + 7*a^2*(4*c*x + d*x^4))/(28*a^3*(
a + b*x^3)^(7/3))
```

Maple [A] time = 0.003, size = 57, normalized size = 0.6

$$\frac{x\left(3\,abdx^6+18\,b^2cx^6+7\,a^2dx^3+42\,ax^3cb+28\,a^2c\right)}{28\,a^3}\left(bx^3+a\right)^{-\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)/(b*x^3+a)^(10/3),x)
```

```
[Out] 1/28*x*(3*a*b*d*x^6+18*b^2*c*x^6+7*a^2*d*x^3+42*a*b*c*x^3+28*a^2*c)/(b*x^3+
a)^(7/3)/a^3
```

Maxima [A] time = 0.965358, size = 116, normalized size = 1.27

$$-\frac{\left(4\,b-\frac{7\left(bx^3+a\right)}{x^3}\right)dx^7}{28\left(bx^3+a\right)^{\frac{7}{3}}a^2}+\frac{\left(2\,b^2-\frac{7\left(bx^3+a\right)b}{x^3}+\frac{14\left(bx^3+a\right)^2}{x^6}\right)cx^7}{14\left(bx^3+a\right)^{\frac{7}{3}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(10/3),x, algorithm="maxima")
```

```
[Out] -1/28*(4*b - 7*(b*x^3 + a)/x^3)*d*x^7/((b*x^3 + a)^(7/3)*a^2) + 1/14*(2*b^2
 - 7*(b*x^3 + a)*b/x^3 + 14*(b*x^3 + a)^2/x^6)*c*x^7/((b*x^3 + a)^(7/3)*a^3
)
```

Fricas [A] time = 1.7116, size = 188, normalized size = 2.07

$$\frac{\left(3\left(6b^2c + abd\right)x^7 + 7\left(6abc + a^2d\right)x^4 + 28a^2cx\right)\left(bx^3 + a\right)^{\frac{2}{3}}}{28\left(a^3b^3x^9 + 3a^4b^2x^6 + 3a^5bx^3 + a^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(10/3),x, algorithm="fricas")
```

```
[Out] 1/28*(3*(6*b^2*c + a*b*d)*x^7 + 7*(6*a*b*c + a^2*d)*x^4 + 28*a^2*c*x)*(b*x^
3 + a)^(2/3)/(a^3*b^3*x^9 + 3*a^4*b^2*x^6 + 3*a^5*b*x^3 + a^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)/(b*x**3+a)**(10/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^3 + c}{\left(bx^3 + a\right)^{\frac{10}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(10/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(10/3), x)
```

3.62 $\int \frac{c+dx^3}{\left(a+bx^3\right)^{13/3}}\,dx$

Optimal. Leaf size=121

$$\frac{9x(ad+9bc)}{140a^4b\sqrt[3]{a+bx^3}}+\frac{3x(ad+9bc)}{140a^3b\left(a+bx^3\right)^{4/3}}+\frac{x(ad+9bc)}{70a^2b\left(a+bx^3\right)^{7/3}}+\frac{x(bc-ad)}{10ab\left(a+bx^3\right)^{10/3}}$$

```
[Out] ((b*c - a*d)*x)/(10*a*b*(a + b*x^3)^(10/3)) + ((9*b*c + a*d)*x)/(70*a^2*b*(
a + b*x^3)^(7/3)) + (3*(9*b*c + a*d)*x)/(140*a^3*b*(a + b*x^3)^(4/3)) + (9*
(9*b*c + a*d)*x)/(140*a^4*b*(a + b*x^3)^(1/3))
```

Rubi [A] time = 0.0352966, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {385, 192, 191}

$$\frac{9x(ad+9bc)}{140a^4b\sqrt[3]{a+bx^3}}+\frac{3x(ad+9bc)}{140a^3b\left(a+bx^3\right)^{4/3}}+\frac{x(ad+9bc)}{70a^2b\left(a+bx^3\right)^{7/3}}+\frac{x(bc-ad)}{10ab\left(a+bx^3\right)^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)/(a + b*x^3)^(13/3),x]
```

```
[Out] ((b*c - a*d)*x)/(10*a*b*(a + b*x^3)^(10/3)) + ((9*b*c + a*d)*x)/(70*a^2*b*(
a + b*x^3)^(7/3)) + (3*(9*b*c + a*d)*x)/(140*a^3*b*(a + b*x^3)^(4/3)) + (9*
(9*b*c + a*d)*x)/(140*a^4*b*(a + b*x^3)^(1/3))
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{c+dx^3}{\left(a+bx^3\right)^{13/3}}\,dx = \frac{(bc-ad)x}{10ab\left(a+bx^3\right)^{10/3}} + \frac{(9bc+ad)\int \frac{1}{\left(a+bx^3\right)^{10/3}}\,dx}{10ab}$$

$$= \frac{(bc-ad)x}{10ab\left(a+bx^3\right)^{10/3}} + \frac{(9bc+ad)x}{70a^2b\left(a+bx^3\right)^{7/3}} + \frac{(3(9bc+ad))\int \frac{1}{\left(a+bx^3\right)^{7/3}}\,dx}{35a^2b}$$

$$= \frac{(bc-ad)x}{10ab\left(a+bx^3\right)^{10/3}} + \frac{(9bc+ad)x}{70a^2b\left(a+bx^3\right)^{7/3}} + \frac{3(9bc+ad)x}{140a^3b\left(a+bx^3\right)^{4/3}} + \frac{(9(9bc+ad))\int \frac{1}{\left(a+bx^3\right)^{4/3}}\,dx}{140a^3b}$$

$$= \frac{(bc-ad)x}{10ab\left(a+bx^3\right)^{10/3}} + \frac{(9bc+ad)x}{70a^2b\left(a+bx^3\right)^{7/3}} + \frac{3(9bc+ad)x}{140a^3b\left(a+bx^3\right)^{4/3}} + \frac{9(9bc+ad)x}{140a^4b\sqrt[3]{a+bx^3}}$$

Mathematica [A] time = 0.0314036, size = 80, normalized size = 0.66

$$\frac{x\left(15a^2bx^3\left(21c+2dx^3\right)+35a^3\left(4c+dx^3\right)+9ab^2x^6\left(30c+dx^3\right)+81b^3cx^9\right)}{140a^4\left(a+bx^3\right)^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)/(a + b*x^3)^(13/3),x]
```

```
[Out] (x*(81*b^3*c*x^9 + 35*a^3*(4*c + d*x^3) + 9*a*b^2*x^6*(30*c + d*x^3) + 15*a
^2*b*x^3*(21*c + 2*d*x^3)))/(140*a^4*(a + b*x^3)^(10/3))
```

Maple [A] time = 0.006, size = 81, normalized size = 0.7

$$\frac{x\left(9\,ab^2dx^9+81\,b^3cx^9+30\,a^2bdx^6+270\,ab^2cx^6+35\,a^3dx^3+315\,a^2bcx^3+140\,ca^3\right)}{140\,a^4}\left(bx^3+a\right)^{-\frac{10}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)/(b*x^3+a)^(13/3),x)
```

```
[Out] 1/140*x*(9*a*b^2*d*x^9+81*b^3*c*x^9+30*a^2*b*d*x^6+270*a*b^2*c*x^6+35*a^3*d
*x^3+315*a^2*b*c*x^3+140*a^3*c)/(b*x^3+a)^(10/3)/a^4
```

Maxima [A] time = 0.94365, size = 162, normalized size = 1.34

$$\frac{\left(14\,b^2-\frac{40\left(bx^3+a\right)b}{x^3}+\frac{35\left(bx^3+a\right)^2}{x^6}\right)dx^{10}}{140\left(bx^3+a\right)^{\frac{10}{3}}a^3}-\frac{\left(14\,b^3-\frac{60\left(bx^3+a\right)b^2}{x^3}+\frac{105\left(bx^3+a\right)^2b}{x^6}-\frac{140\left(bx^3+a\right)^3}{x^9}\right)cx^{10}}{140\left(bx^3+a\right)^{\frac{10}{3}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(13/3),x, algorithm="maxima")
```

```
[Out] 1/140*(14*b^2 - 40*(b*x^3 + a)*b/x^3 + 35*(b*x^3 + a)^2/x^6)*d*x^10/((b*x^3
 + a)^(10/3)*a^3) - 1/140*(14*b^3 - 60*(b*x^3 + a)*b^2/x^3 + 105*(b*x^3 + a
)^2*b/x^6 - 140*(b*x^3 + a)^3/x^9)*c*x^10/((b*x^3 + a)^(10/3)*a^4)
```

Fricas [A] time = 1.65692, size = 263, normalized size = 2.17

$$\frac{\left(9\left(9\,b^3c+ab^2d\right)x^{10}+30\left(9\,ab^2c+a^2bd\right)x^7+140\,a^3cx+35\left(9\,a^2bc+a^3d\right)x^4\right)\left(bx^3+a\right)^{\frac{2}{3}}}{140\left(a^4b^4x^{12}+4\,a^5b^3x^9+6\,a^6b^2x^6+4\,a^7bx^3+a^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(13/3),x, algorithm="fricas")
```

```
[Out] 1/140*(9*(9*b^3*c + a*b^2*d)*x^10 + 30*(9*a*b^2*c + a^2*b*d)*x^7 + 140*a^3*
c*x + 35*(9*a^2*b*c + a^3*d)*x^4)*(b*x^3 + a)^(2/3)/(a^4*b^4*x^12 + 4*a^5*b
^3*x^9 + 6*a^6*b^2*x^6 + 4*a^7*b*x^3 + a^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)/(b*x**3+a)**(13/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^3+c}{\left(bx^3+a\right)^{\frac{13}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(13/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(13/3), x)
```

3.63 $\int \frac{c+dx^3}{\left(a+bx^3\right)^{16/3}}\,dx$

Optimal. Leaf size=151

$$\frac{81x(ad+12bc)}{1820a^5b\sqrt[3]{a+bx^3}}+\frac{27x(ad+12bc)}{1820a^4b\left(a+bx^3\right)^{4/3}}+\frac{9x(ad+12bc)}{910a^3b\left(a+bx^3\right)^{7/3}}+\frac{x(ad+12bc)}{130a^2b\left(a+bx^3\right)^{10/3}}+\frac{x(bc-ad)}{13ab\left(a+bx^3\right)^{13/3}}$$

```
[Out] ((b*c - a*d)*x)/(13*a*b*(a + b*x^3)^(13/3)) + ((12*b*c + a*d)*x)/(130*a^2*b
*(a + b*x^3)^(10/3)) + (9*(12*b*c + a*d)*x)/(910*a^3*b*(a + b*x^3)^(7/3)) +
 (27*(12*b*c + a*d)*x)/(1820*a^4*b*(a + b*x^3)^(4/3)) + (81*(12*b*c + a*d)*
x)/(1820*a^5*b*(a + b*x^3)^(1/3))
```

Rubi [A] time = 0.0469943, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {385, 192, 191}

$$\frac{81x(ad+12bc)}{1820a^5b\sqrt[3]{a+bx^3}}+\frac{27x(ad+12bc)}{1820a^4b\left(a+bx^3\right)^{4/3}}+\frac{9x(ad+12bc)}{910a^3b\left(a+bx^3\right)^{7/3}}+\frac{x(ad+12bc)}{130a^2b\left(a+bx^3\right)^{10/3}}+\frac{x(bc-ad)}{13ab\left(a+bx^3\right)^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)/(a + b*x^3)^(16/3),x]
```

```
[Out] ((b*c - a*d)*x)/(13*a*b*(a + b*x^3)^(13/3)) + ((12*b*c + a*d)*x)/(130*a^2*b
*(a + b*x^3)^(10/3)) + (9*(12*b*c + a*d)*x)/(910*a^3*b*(a + b*x^3)^(7/3)) +
 (27*(12*b*c + a*d)*x)/(1820*a^4*b*(a + b*x^3)^(4/3)) + (81*(12*b*c + a*d)*
x)/(1820*a^5*b*(a + b*x^3)^(1/3))
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{c+dx^3}{\left(a+bx^3\right)^{16/3}}\,dx = \frac{(bc-ad)x}{13ab\left(a+bx^3\right)^{13/3}} + \frac{(12bc+ad)\int \frac{1}{\left(a+bx^3\right)^{13/3}}\,dx}{13ab}$$

$$= \frac{(bc-ad)x}{13ab\left(a+bx^3\right)^{13/3}} + \frac{(12bc+ad)x}{130a^2b\left(a+bx^3\right)^{10/3}} + \frac{(9(12bc+ad))\int \frac{1}{\left(a+bx^3\right)^{10/3}}\,dx}{130a^2b}$$

$$= \frac{(bc-ad)x}{13ab\left(a+bx^3\right)^{13/3}} + \frac{(12bc+ad)x}{130a^2b\left(a+bx^3\right)^{10/3}} + \frac{9(12bc+ad)x}{910a^3b\left(a+bx^3\right)^{7/3}} + \frac{(27(12bc+ad))\int \frac{1}{\left(a+bx^3\right)^{7/3}}\,dx}{455a^3b}$$

$$= \frac{(bc-ad)x}{13ab\left(a+bx^3\right)^{13/3}} + \frac{(12bc+ad)x}{130a^2b\left(a+bx^3\right)^{10/3}} + \frac{9(12bc+ad)x}{910a^3b\left(a+bx^3\right)^{7/3}} + \frac{27(12bc+ad)x}{1820a^4b\left(a+bx^3\right)^{4/3}} + \frac{(81(12b\ldots}{\ldots}$$

$$= \frac{(bc-ad)x}{13ab\left(a+bx^3\right)^{13/3}} + \frac{(12bc+ad)x}{130a^2b\left(a+bx^3\right)^{10/3}} + \frac{9(12bc+ad)x}{910a^3b\left(a+bx^3\right)^{7/3}} + \frac{27(12bc+ad)x}{1820a^4b\left(a+bx^3\right)^{4/3}} + \frac{81(12b\ldots}{1820a^5b\ldots}$$

Mathematica [A] time = 0.0375034, size = 100, normalized size = 0.66

$$\frac{x\left(351a^2b^2x^6\left(20c+dx^3\right)+195a^3bx^3\left(28c+3dx^3\right)+455a^4\left(4c+dx^3\right)+81ab^3x^9\left(52c+dx^3\right)+972b^4cx^{12}\right)}{1820a^5\left(a+bx^3\right)^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)/(a + b*x^3)^(16/3),x]
```

```
[Out] (x*(972*b^4*c*x^12 + 455*a^4*(4*c + d*x^3) + 351*a^2*b^2*x^6*(20*c + d*x^3)
 + 81*a*b^3*x^9*(52*c + d*x^3) + 195*a^3*b*x^3*(28*c + 3*d*x^3)))/(1820*a^5
*(a + b*x^3)^(13/3))
```

Maple [A] time = 0.004, size = 105, normalized size = 0.7

$$\frac{x\left(81\,ab^3dx^{12}+972\,b^4cx^{12}+351\,a^2b^2dx^9+4212\,ab^3cx^9+585\,a^3bdx^6+7020\,a^2b^2cx^6+455\,a^4dx^3+5460\,a^3bcx^3+1820\ldots\right.}{1820\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)/(b*x^3+a)^(16/3),x)
```

```
[Out] 1/1820*x*(81*a*b^3*d*x^12+972*b^4*c*x^12+351*a^2*b^2*d*x^9+4212*a*b^3*c*x^9
+585*a^3*b*d*x^6+7020*a^2*b^2*c*x^6+455*a^4*d*x^3+5460*a^3*b*c*x^3+1820*a^4
*c)/(b*x^3+a)^(13/3)/a^5
```

Maxima [A] time = 0.973927, size = 208, normalized size = 1.38

$$-\frac{\left(140\,b^3-\frac{546\left(bx^3+a\right)b^2}{x^3}+\frac{780\left(bx^3+a\right)^2b}{x^6}-\frac{455\left(bx^3+a\right)^3}{x^9}\right)dx^{13}}{1820\left(bx^3+a\right)^{\frac{13}{3}}a^4}+\frac{\left(35\,b^4-\frac{182\left(bx^3+a\right)b^3}{x^3}+\frac{390\left(bx^3+a\right)^2b^2}{x^6}-\frac{455\left(bx^3+a\right)^3b}{x^9}+\frac{455\left(bx^3+a\right)^4}{x^{12}}\ldots\right.}{455\left(bx^3+a\right)^{\frac{13}{3}}a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(16/3),x, algorithm="maxima")
```

```
[Out] -1/1820*(140*b^3 - 546*(b*x^3 + a)*b^2/x^3 + 780*(b*x^3 + a)^2*b/x^6 - 455*
(b*x^3 + a)^3/x^9)*d*x^13/((b*x^3 + a)^(13/3)*a^4) + 1/455*(35*b^4 - 182*(b
*x^3 + a)*b^3/x^3 + 390*(b*x^3 + a)^2*b^2/x^6 - 455*(b*x^3 + a)^3*b/x^9 + 4
55*(b*x^3 + a)^4/x^12)*c*x^13/((b*x^3 + a)^(13/3)*a^5)
```

Fricas [A] time = 1.78946, size = 352, normalized size = 2.33

$$\frac{\left(81\left(12\,b^4c + ab^3d\right)x^{13} + 351\left(12\,ab^3c + a^2b^2d\right)x^{10} + 585\left(12\,a^2b^2c + a^3bd\right)x^7 + 1820\,a^4cx + 455\left(12\,a^3bc + a^4d\right)x^4\right)\left(bx\right.}{1820\left(a^5b^5x^{15} + 5\,a^6b^4x^{12} + 10\,a^7b^3x^9 + 10\,a^8b^2x^6 + 5\,a^9bx^3 + a^{10}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(16/3),x, algorithm="fricas")
```

```
[Out] 1/1820*(81*(12*b^4*c + a*b^3*d)*x^13 + 351*(12*a*b^3*c + a^2*b^2*d)*x^10 +
585*(12*a^2*b^2*c + a^3*b*d)*x^7 + 1820*a^4*c*x + 455*(12*a^3*b*c + a^4*d)*
x^4)*(b*x^3 + a)^(2/3)/(a^5*b^5*x^15 + 5*a^6*b^4*x^12 + 10*a^7*b^3*x^9 + 10
*a^8*b^2*x^6 + 5*a^9*b*x^3 + a^10)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)/(b*x**3+a)**(16/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^3 + c}{\left(bx^3 + a\right)^{\frac{16}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(16/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(16/3), x)
```

3.64 $\int \left(a+bx^3\right)^{7/3} \left(c+dx^3\right) dx$

Optimal. Leaf size=85

$$\frac{a^2 x \sqrt[3]{a+bx^3}(11bc-ad)\,{}_2F_1\left(-\frac{7}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{11b\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx\left(a+bx^3\right)^{10/3}}{11b}$$

```
[Out] (d*x*(a + b*x^3)^(10/3))/(11*b) + (a^2*(11*b*c - a*d)*x*(a + b*x^3)^(1/3)*H
ypergeometric2F1[-7/3, 1/3, 4/3, -((b*x^3)/a)])/(11*b*(1 + (b*x^3)/a)^(1/3)
)
```

Rubi [A] time = 0.0228746, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {388, 246, 245}

$$\frac{a^2 x \sqrt[3]{a+bx^3}(11bc-ad)\,{}_2F_1\left(-\frac{7}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{11b\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx\left(a+bx^3\right)^{10/3}}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(7/3)*(c + d*x^3),x]
```

```
[Out] (d*x*(a + b*x^3)^(10/3))/(11*b) + (a^2*(11*b*c - a*d)*x*(a + b*x^3)^(1/3)*H
ypergeometric2F1[-7/3, 1/3, 4/3, -((b*x^3)/a)])/(11*b*(1 + (b*x^3)/a)^(1/3)
)
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \left(a+bx^3\right)^{7/3}\left(c+dx^3\right)\,dx = \frac{dx\left(a+bx^3\right)^{10/3}}{11b} - \frac{(-11bc+ad)\int \left(a+bx^3\right)^{7/3}\,dx}{11b}$$
$$= \frac{dx\left(a+bx^3\right)^{10/3}}{11b} - \frac{\left(a^2(-11bc+ad)\sqrt[3]{a+bx^3}\right)\int \left(1+\frac{bx^3}{a}\right)^{7/3}\,dx}{11b\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= \frac{dx\left(a+bx^3\right)^{10/3}}{11b} + \frac{a^2(11bc-ad)x\sqrt[3]{a+bx^3}\,{}_2F_1\left(-\frac{7}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{11b\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [A] time = 0.037149, size = 77, normalized size = 0.91

$$\frac{x\sqrt[3]{a+bx^3}\left(d\left(a+bx^3\right)^3-\frac{a^2(ad-11bc)\,{}_2F_1\left(-\frac{7}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\sqrt[3]{\frac{bx^3}{a}+1}}\right)}{11b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(7/3)*(c + d*x^3),x]
```

```
[Out] (x*(a + b*x^3)^(1/3)*(d*(a + b*x^3)^3 - (a^2*(-11*b*c + a*d)*Hypergeometric
2F1[-7/3, 1/3, 4/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^(1/3)))/(11*b)
```

Maple [F] time = 0.22, size = 0, normalized size = 0.

$$\int \left(bx^3+a\right)^{\frac{7}{3}}\left(dx^3+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(7/3)*(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(7/3)*(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3+a\right)^{\frac{7}{3}}\left(dx^3+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(7/3)*(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(7/3)*(d*x^3 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2dx^9+\left(b^2c+2\,abd\right)x^6+\left(2\,abc+a^2d\right)x^3+a^2c\right)\left(bx^3+a\right)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(7/3)*(d*x^3+c),x, algorithm="fricas")
```

```
[Out] integral((b^2*d*x^9 + (b^2*c + 2*a*b*d)*x^6 + (2*a*b*c + a^2*d)*x^3 + a^2*c
)*(b*x^3 + a)^(1/3), x)
```

Sympy [C] time = 7.0308, size = 265, normalized size = 3.12

$$\frac{a^{\frac{7}{3}}cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{a^{\frac{7}{3}}dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{2a^{\frac{4}{3}}bcx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{2a^{\frac{4}{3}}bdx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3}\\ \frac{1}{3}\end{matrix}\right.}{3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(7/3)*(d*x**3+c),x)
```

```
[Out] a**(7/3)*c*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(4/3)) + a**(7/3)*d*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + 2*a**(4/3)*b*c*x**4*gamma(4/3)*hyp
er((-1/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + 2*a**(4/
3)*b*d*x**7*gamma(7/3)*hyper((-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(10/3)) + a**(1/3)*b**2*c*x**7*gamma(7/3)*hyper((-1/3, 7/3), (10/
3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + a**(1/3)*b**2*d*x**10*gamm
a(10/3)*hyper((-1/3, 10/3), (13/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(13/
3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(bx^3+a\right)^{\frac{7}{3}}\left(dx^3+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(7/3)*(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(7/3)*(d*x^3 + c), x)
```

3.65 $\int \left(a+bx^3\right)^{4/3} \left(c+dx^3\right) dx$

Optimal. Leaf size=83

$$\frac{ax\sqrt[3]{a+bx^3}(8bc-ad)\,{}_2F_1\left(-\frac{4}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{8b\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx\left(a+bx^3\right)^{7/3}}{8b}$$

```
[Out] (d*x*(a + b*x^3)^(7/3))/(8*b) + (a*(8*b*c - a*d)*x*(a + b*x^3)^(1/3)*Hyperg
eometric2F1[-4/3, 1/3, 4/3, -((b*x^3)/a)])/(8*b*(1 + (b*x^3)/a)^(1/3))
```

Rubi [A] time = 0.0215988, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {388, 246, 245}

$$\frac{ax\sqrt[3]{a+bx^3}(8bc-ad)\,{}_2F_1\left(-\frac{4}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{8b\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx\left(a+bx^3\right)^{7/3}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(4/3)*(c + d*x^3),x]
```

```
[Out] (d*x*(a + b*x^3)^(7/3))/(8*b) + (a*(8*b*c - a*d)*x*(a + b*x^3)^(1/3)*Hyperg
eometric2F1[-4/3, 1/3, 4/3, -((b*x^3)/a)])/(8*b*(1 + (b*x^3)/a)^(1/3))
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \left(a+bx^3\right)^{4/3}\left(c+dx^3\right)\,dx = \frac{dx\left(a+bx^3\right)^{7/3}}{8b} - \frac{(-8bc+ad)\int \left(a+bx^3\right)^{4/3}\,dx}{8b}$$
$$= \frac{dx\left(a+bx^3\right)^{7/3}}{8b} - \frac{\left(a(-8bc+ad)\sqrt[3]{a+bx^3}\right)\int \left(1+\frac{bx^3}{a}\right)^{4/3}\,dx}{8b\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= \frac{dx\left(a+bx^3\right)^{7/3}}{8b} + \frac{a(8bc-ad)x\sqrt[3]{a+bx^3}\,{}_2F_1\left(-\frac{4}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{8b\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [A] time = 0.0616462, size = 75, normalized size = 0.9

$$\frac{x\sqrt[3]{a+bx^3}\left(d\left(a+bx^3\right)^2-\frac{a(ad-8bc)\,{}_2F_1\left(-\frac{4}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\sqrt[3]{\frac{bx^3}{a}+1}}\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(4/3)*(c + d*x^3),x]
```

```
[Out] (x*(a + b*x^3)^(1/3)*(d*(a + b*x^3)^2 - (a*(-8*b*c + a*d)*Hypergeometric2F1
[-4/3, 1/3, 4/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^(1/3)))/(8*b)
```

Maple [F] time = 0.224, size = 0, normalized size = 0.

$$\int \left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)*(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(4/3)*(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)*(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)*(d*x^3 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bdx^6+(bc+ad)x^3+ac\right)\left(bx^3+a\right)^{\frac{1}{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)*(d*x^3+c),x, algorithm="fricas")
```

```
[Out] integral((b*d*x^6 + (b*c + a*d)*x^3 + a*c)*(b*x^3 + a)^(1/3), x)
```

Sympy [C] time = 3.79038, size = 170, normalized size = 2.05

$$\frac{a^{\frac{4}{3}}cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{a^{\frac{4}{3}}dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{\sqrt[3]{a}bcx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{\sqrt[3]{a}bdx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(-\right.}{3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)*(d*x**3+c),x)
```

```
[Out] a**(4/3)*c*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(4/3)) + a**(4/3)*d*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(1/3)*b*c*x**4*gamma(4/3)*hyper
((-1/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(1/3)*b
*d*x**7*gamma(7/3)*hyper((-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3
*gamma(10/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)*(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)*(d*x^3 + c), x)
```

3.66 $\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)dx$

Optimal. Leaf size=82

$$\frac{x\sqrt[3]{a+bx^3}(5bc-ad)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5b\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx\left(a+bx^3\right)^{4/3}}{5b}$$

```
[Out] (d*x*(a + b*x^3)^(4/3))/(5*b) + ((5*b*c - a*d)*x*(a + b*x^3)^(1/3)*Hypergeo
metric2F1[-1/3, 1/3, 4/3, -((b*x^3)/a)])/(5*b*(1 + (b*x^3)/a)^(1/3))
```

Rubi [A] time = 0.0210526, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {388, 246, 245}

$$\frac{x\sqrt[3]{a+bx^3}(5bc-ad)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5b\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx\left(a+bx^3\right)^{4/3}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)*(c + d*x^3),x]
```

```
[Out] (d*x*(a + b*x^3)^(4/3))/(5*b) + ((5*b*c - a*d)*x*(a + b*x^3)^(1/3)*Hypergeo
metric2F1[-1/3, 1/3, 4/3, -((b*x^3)/a)])/(5*b*(1 + (b*x^3)/a)^(1/3))
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)\,dx = \frac{dx\left(a+bx^3\right)^{4/3}}{5b} - \frac{(-5bc+ad)\int \sqrt[3]{a+bx^3}\,dx}{5b}$$
$$= \frac{dx\left(a+bx^3\right)^{4/3}}{5b} - \frac{\left((-5bc+ad)\sqrt[3]{a+bx^3}\right)\int \sqrt[3]{1+\frac{bx^3}{a}}\,dx}{5b\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$= \frac{dx\left(a+bx^3\right)^{4/3}}{5b} + \frac{(5bc-ad)x\sqrt[3]{a+bx^3}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5b\sqrt[3]{1+\frac{bx^3}{a}}}$$

Mathematica [A] time = 0.0669086, size = 72, normalized size = 0.88

$$\frac{x\sqrt[3]{a+bx^3}\left(\frac{(5bc-ad)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{\sqrt[3]{\frac{bx^3}{a}+1}}+d\left(a+bx^3\right)\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(1/3)*(c + d*x^3),x]
```

```
[Out] (x*(a + b*x^3)^(1/3)*(d*(a + b*x^3) + ((5*b*c - a*d)*Hypergeometric2F1[-1/3
, 1/3, 4/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^(1/3)))/(5*b)
```

Maple [F] time = 0.208, size = 0, normalized size = 0.

$$\int \sqrt[3]{bx^3+a}\left(dx^3+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)*(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(1/3)*(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)*(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*(d*x^3 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)*(d*x^3+c),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(1/3)*(d*x^3 + c), x)
```

Sympy [C] time = 2.0371, size = 82, normalized size = 1.

$$\frac{\sqrt[3]{a}cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{\sqrt[3]{a}dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)*(d*x**3+c),x)
```

```
[Out] a**(1/3)*c*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(4/3)) + a**(1/3)*d*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(7/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)*(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*(d*x^3 + c), x)
```

3.67 $\int \frac{c+dx^3}{\left(a+bx^3\right)^{2/3}}\,dx$

Optimal. Leaf size=82

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(2bc-ad)\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2b\left(a+bx^3\right)^{2/3}}+\frac{dx\sqrt[3]{a+bx^3}}{2b}$$

```
[Out] (d*x*(a + b*x^3)^(1/3))/(2*b) + ((2*b*c - a*d)*x*(1 + (b*x^3)/a)^(2/3)*Hype
rgeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*b*(a + b*x^3)^(2/3))
```

Rubi [A] time = 0.0219141, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {388, 246, 245}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(2bc-ad)\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2b\left(a+bx^3\right)^{2/3}}+\frac{dx\sqrt[3]{a+bx^3}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)/(a + b*x^3)^(2/3),x]
```

```
[Out] (d*x*(a + b*x^3)^(1/3))/(2*b) + ((2*b*c - a*d)*x*(1 + (b*x^3)/a)^(2/3)*Hype
rgeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*b*(a + b*x^3)^(2/3))
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \frac{c+dx^3}{\left(a+bx^3\right)^{2/3}}\,dx = \frac{dx\sqrt[3]{a+bx^3}}{2b} - \frac{(-2bc+ad)\int \frac{1}{\left(a+bx^3\right)^{2/3}}\,dx}{2b}$$

$$= \frac{dx\sqrt[3]{a+bx^3}}{2b} - \frac{\left((-2bc+ad)\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{2b\left(a+bx^3\right)^{2/3}}$$

$$= \frac{dx\sqrt[3]{a+bx^3}}{2b} + \frac{(2bc-ad)x\left(1+\frac{bx^3}{a}\right)^{2/3}\ {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2b\left(a+bx^3\right)^{2/3}}$$

Mathematica [A] time = 0.0375045, size = 73, normalized size = 0.89

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(2bc-ad)\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)+dx\left(a+bx^3\right)}{2b\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)/(a + b*x^3)^(2/3),x]
```

```
[Out] (d*x*(a + b*x^3) + (2*b*c - a*d)*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[
1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*b*(a + b*x^3)^(2/3))
```

Maple [F] time = 0.212, size = 0, normalized size = 0.

$$\int \left(dx^3+c\right)\left(bx^3+a\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)/(b*x^3+a)^(2/3),x)
```

```
[Out] int((d*x^3+c)/(b*x^3+a)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^3+c}{\left(bx^3+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dx^3+c}{\left(bx^3+a\right)^{\frac{2}{3}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((d*x^3 + c)/(b*x^3 + a)^(2/3), x)
```

Sympy [C] time = 1.73721, size = 78, normalized size = 0.95

$$\frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}+\frac{dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)/(b*x**3+a)**(2/3),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2
/3)*gamma(4/3)) + d*x**4*gamma(4/3)*hyper((2/3, 4/3), (7/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*a**(2/3)*gamma(7/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{dx^3+c}{\left(bx^3+a\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(2/3), x)
```

3.68 $\int \frac{c+dx^3}{\left(a+bx^3\right)^{5/3}}\,dx$

Optimal. Leaf size=93

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(ad+bc)\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2ab\left(a+bx^3\right)^{2/3}}+\frac{x(bc-ad)}{2ab\left(a+bx^3\right)^{2/3}}$$

```
[Out] ((b*c - a*d)*x)/(2*a*b*(a + b*x^3)^(2/3)) + ((b*c + a*d)*x*(1 + (b*x^3)/a)^
(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*a*b*(a + b*x^3)^(2
/3))
```

Rubi [A] time = 0.0252296, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {385, 246, 245}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(ad+bc)\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2ab\left(a+bx^3\right)^{2/3}}+\frac{x(bc-ad)}{2ab\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)/(a + b*x^3)^(5/3),x]
```

```
[Out] ((b*c - a*d)*x)/(2*a*b*(a + b*x^3)^(2/3)) + ((b*c + a*d)*x*(1 + (b*x^3)/a)^
(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*a*b*(a + b*x^3)^(2
/3))
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \frac{c+dx^3}{\left(a+bx^3\right)^{5/3}}\,dx = \frac{(bc-ad)x}{2ab\left(a+bx^3\right)^{2/3}} + \frac{(bc+ad)\int \frac{1}{\left(a+bx^3\right)^{2/3}}\,dx}{2ab}$$

$$= \frac{(bc-ad)x}{2ab\left(a+bx^3\right)^{2/3}} + \frac{\left((bc+ad)\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{2ab\left(a+bx^3\right)^{2/3}}$$

$$= \frac{(bc-ad)x}{2ab\left(a+bx^3\right)^{2/3}} + \frac{(bc+ad)x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2ab\left(a+bx^3\right)^{2/3}}$$

Mathematica [A] time = 0.0342796, size = 66, normalized size = 0.71

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(ad+bc)\,{}_2F_1\left(\frac{1}{3},\frac{5}{3};\frac{4}{3};-\frac{bx^3}{a}\right)-adx}{ab\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)/(a + b*x^3)^(5/3),x]
```

```
[Out] (-(a*d*x) + (b*c + a*d)*x*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 5/3,
 4/3, -((b*x^3)/a)])/(a*b*(a + b*x^3)^(2/3))
```

Maple [F] time = 0.224, size = 0, normalized size = 0.

$$\int \left(dx^3+c\right)\left(bx^3+a\right)^{-\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)/(b*x^3+a)^(5/3),x)
```

```
[Out] int((d*x^3+c)/(b*x^3+a)^(5/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^3+c}{\left(bx^3+a\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(5/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)}{b^2x^6+2\,abx^3+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(5/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(1/3)*(d*x^3 + c)/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

Sympy [C] time = 14.1314, size = 78, normalized size = 0.84

$$\frac{cx\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{5}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{3}}\Gamma\left(\frac{4}{3}\right)}+\frac{dx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{5}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{5}{3}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)/(b*x**3+a)**(5/3),x)
```

```
[Out] c*x*gamma(1/3)*hyper((1/3, 5/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(5
/3)*gamma(4/3)) + d*x**4*gamma(4/3)*hyper((4/3, 5/3), (7/3,), b*x**3*exp_po
lar(I*pi)/a)/(3*a**(5/3)*gamma(7/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{dx^3+c}{\left(bx^3+a\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(5/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(5/3), x)
```

3.69 $\int \frac{c+dx^3}{\left(a+bx^3\right)^{8/3}}\,dx$

Optimal. Leaf size=94

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(ad+4bc)\,{}_2F_1\left(\frac{1}{3},\frac{5}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5a^2b\left(a+bx^3\right)^{2/3}}+\frac{x(bc-ad)}{5ab\left(a+bx^3\right)^{5/3}}$$

```
[Out] ((b*c - a*d)*x)/(5*a*b*(a + b*x^3)^(5/3)) + ((4*b*c + a*d)*x*(1 + (b*x^3)/a
)^(2/3)*Hypergeometric2F1[1/3, 5/3, 4/3, -((b*x^3)/a)])/(5*a^2*b*(a + b*x^3
)^(2/3))
```

Rubi [A] time = 0.02768, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {385, 246, 245}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}(ad+4bc)\,{}_2F_1\left(\frac{1}{3},\frac{5}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5a^2b\left(a+bx^3\right)^{2/3}}+\frac{x(bc-ad)}{5ab\left(a+bx^3\right)^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)/(a + b*x^3)^(8/3),x]
```

```
[Out] ((b*c - a*d)*x)/(5*a*b*(a + b*x^3)^(5/3)) + ((4*b*c + a*d)*x*(1 + (b*x^3)/a
)^(2/3)*Hypergeometric2F1[1/3, 5/3, 4/3, -((b*x^3)/a)])/(5*a^2*b*(a + b*x^3
)^(2/3))
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \frac{c+dx^3}{\left(a+bx^3\right)^{8/3}}\,dx = \frac{(bc-ad)x}{5ab\left(a+bx^3\right)^{5/3}} + \frac{(4bc+ad)\int \frac{1}{\left(a+bx^3\right)^{5/3}}\,dx}{5ab}$$
$$= \frac{(bc-ad)x}{5ab\left(a+bx^3\right)^{5/3}} + \frac{\left((4bc+ad)\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{5/3}}\,dx}{5a^2b\left(a+bx^3\right)^{2/3}}$$
$$= \frac{(bc-ad)x}{5ab\left(a+bx^3\right)^{5/3}} + \frac{(4bc+ad)x\left(1+\frac{bx^3}{a}\right)^{2/3}\, {}_2F_1\left(\frac{1}{3},\frac{5}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5a^2b\left(a+bx^3\right)^{2/3}}$$

Mathematica [A] time = 0.0394755, size = 75, normalized size = 0.8

$$\frac{x\left(\frac{\left(a+bx^3\right)\left(\frac{bx^3}{a}+1\right)^{2/3}(ad+4bc)\,{}_2F_1\left(\frac{1}{3},\frac{8}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{a^2}-d\right)}{4b\left(a+bx^3\right)^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)/(a + b*x^3)^(8/3),x]
```

```
[Out] (x*(-d + ((4*b*c + a*d)*(a + b*x^3)*(1 + (b*x^3)/a)^(2/3)*Hypergeometric2F1
[1/3, 8/3, 4/3, -((b*x^3)/a)])/a^2))/(4*b*(a + b*x^3)^(5/3))
```

Maple [F] time = 0.211, size = 0, normalized size = 0.

$$\int \left(dx^3+c\right)\left(bx^3+a\right)^{-\frac{8}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)/(b*x^3+a)^(8/3),x)
```

```
[Out] int((d*x^3+c)/(b*x^3+a)^(8/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^3+c}{\left(bx^3+a\right)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(8/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(8/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)}{b^3x^9+3\,ab^2x^6+3\,a^2bx^3+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(8/3),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(1/3)*(d*x^3 + c)/(b^3*x^9 + 3*a*b^2*x^6 + 3*a^2*b*x^3
 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)/(b*x**3+a)**(8/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^3+c}{\left(bx^3+a\right)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)/(b*x^3+a)^(8/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)/(b*x^3 + a)^(8/3), x)
```

3.70 $\int \left(a+bx^3\right)^{5/3} \left(c+dx^3\right)^2 \, dx$

Optimal. Leaf size=262

$$\frac{x\left(a+bx^3\right)^{5/3}\left(a^2d^2-6abcd+27b^2c^2\right)}{162b^2}+\frac{5ax\left(a+bx^3\right)^{2/3}\left(a^2d^2-6abcd+27b^2c^2\right)}{486b^2}-\frac{5a^2\left(a^2d^2-6abcd+27b^2c^2\right)\log\left(\sqrt[3]{\ }\right.}{486b^{7/3}}$$

```
[Out] (5*a*(27*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*x*(a + b*x^3)^(2/3))/(486*b^2) + ((
27*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*x*(a + b*x^3)^(5/3))/(162*b^2) + (d*(15*b
*c - 4*a*d)*x*(a + b*x^3)^(8/3))/(108*b^2) + (d*x*(a + b*x^3)^(8/3)*(c + d*
x^3))/(12*b) + (5*a^2*(27*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*ArcTan[(1 + (2*b^(
1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(243*Sqrt[3]*b^(7/3)) - (5*a^2*(27*b^2
*c^2 - 6*a*b*c*d + a^2*d^2)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(486*b^(
7/3))
```

Rubi [A] time = 0.164922, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {416, 388, 195, 239}

$$\frac{x\left(a+bx^3\right)^{5/3}\left(a^2d^2-6abcd+27b^2c^2\right)}{162b^2}+\frac{5ax\left(a+bx^3\right)^{2/3}\left(a^2d^2-6abcd+27b^2c^2\right)}{486b^2}-\frac{5a^2\left(a^2d^2-6abcd+27b^2c^2\right)\log\left(\sqrt[3]{\ }\right.}{486b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(5/3)*(c + d*x^3)^2,x]
```


```
[Out] (5*a*(27*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*x*(a + b*x^3)^(2/3))/(486*b^2) + ((
27*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*x*(a + b*x^3)^(5/3))/(162*b^2) + (d*(15*b
*c - 4*a*d)*x*(a + b*x^3)^(8/3))/(108*b^2) + (d*x*(a + b*x^3)^(8/3)*(c + d*
x^3))/(12*b) + (5*a^2*(27*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*ArcTan[(1 + (2*b^(
1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(243*Sqrt[3]*b^(7/3)) - (5*a^2*(27*b^2
*c^2 - 6*a*b*c*d + a^2*d^2)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(486*b^(
7/3))
```


Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \left(a+bx^3\right)^{5/3}\left(c+dx^3\right)^2\,dx = \frac{dx\left(a+bx^3\right)^{8/3}\left(c+dx^3\right)}{12b} + \frac{\int \left(a+bx^3\right)^{5/3}\left(c(12bc-ad)+d(15bc-4ad)x^3\right)dx}{12b}$$

$$= \frac{d(15bc-4ad)x\left(a+bx^3\right)^{8/3}}{108b^2} + \frac{dx\left(a+bx^3\right)^{8/3}\left(c+dx^3\right)}{12b} + \frac{\left(27b^2c^2-6abcd+a^2d^2\right)\int (}{27b^2}$$

$$= \frac{\left(27b^2c^2-6abcd+a^2d^2\right)x\left(a+bx^3\right)^{5/3}}{162b^2} + \frac{d(15bc-4ad)x\left(a+bx^3\right)^{8/3}}{108b^2} + \frac{dx\left(a+bx^3\right)^{8/}}{12b}$$

$$= \frac{5a\left(27b^2c^2-6abcd+a^2d^2\right)x\left(a+bx^3\right)^{2/3}}{486b^2} + \frac{\left(27b^2c^2-6abcd+a^2d^2\right)x\left(a+bx^3\right)^{5/3}}{162b^2} +$$

$$= \frac{5a\left(27b^2c^2-6abcd+a^2d^2\right)x\left(a+bx^3\right)^{2/3}}{486b^2} + \frac{\left(27b^2c^2-6abcd+a^2d^2\right)x\left(a+bx^3\right)^{5/3}}{162b^2} +$$

Mathematica [C] time = 4.8714, size = 176, normalized size = 0.67

$$\frac{x\left(a+bx^3\right)^{2/3}\left(-9bx^3\text{Gamma}\left(-\frac{2}{3}\right)\left(c+dx^3\right)^2\text{HypergeometricPFQ}\left(\left\{-\frac{2}{3},\frac{4}{3},2\right\},\left\{1,\frac{13}{3}\right\},-\frac{bx^3}{a}\right)-3bx^3\text{Gamma}\left(-\right.\right.}{252\text{Gamma}\left(\frac{1}{3}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(5/3)*(c + d*x^3)^2,x]
```

```
[Out] (x*(a + b*x^3)^(2/3)*(20*a*(14*c^2 + 7*c*d*x^3 + 2*d^2*x^6)*Gamma[-5/3]*Hyp
ergeometric2F1[-5/3, 1/3, 10/3, -((b*x^3)/a)] - 3*b*x^3*(11*c^2 + 16*c*d*x^
3 + 5*d^2*x^6)*Gamma[-2/3]*Hypergeometric2F1[-2/3, 4/3, 13/3, -((b*x^3)/a)]
 - 9*b*x^3*(c + d*x^3)^2*Gamma[-2/3]*HypergeometricPFQ[{-2/3, 4/3, 2}, {1,
13/3}, -((b*x^3)/a)]))/(252*(1 + (b*x^3)/a)^(2/3)*Gamma[1/3])
```

Maple [F] time = 0.22, size = 0, normalized size = 0.

$$\int \left(bx^3+a\right)^{\frac{5}{3}}\left(dx^3+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(5/3)*(d*x^3+c)^2,x)
```

```
[Out] int((b*x^3+a)^(5/3)*(d*x^3+c)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)*(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.7733, size = 1713, normalized size = 6.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] [1/2916*(30*sqrt(1/3)*(27*a^2*b^3*c^2 - 6*a^3*b^2*c*d + a^4*b*d^2)*sqrt((-b
)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*(
(-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)
*x)*sqrt((-b)^(1/3)/b) + 2*a) - 20*(27*a^2*b^2*c^2 - 6*a^3*b*c*d + a^4*d^2)
*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + 10*(27*a^2*b^2*c^2
- 6*a^3*b*c*d + a^4*d^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)
*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 3*(81*b^4*d^2*x^10 + 18*(12*b^4*c
*d + 7*a*b^3*d^2)*x^7 + 3*(54*b^4*c^2 + 132*a*b^3*c*d + 5*a^2*b^2*d^2)*x^4
+ 4*(108*a*b^3*c^2 + 30*a^2*b^2*c*d - 5*a^3*b*d^2)*x)*(b*x^3 + a)^(2/3))/b^
3, -1/2916*(60*sqrt(1/3)*(27*a^2*b^3*c^2 - 6*a^3*b^2*c*d + a^4*b*d^2)*sqrt(
-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sqrt(
-(-b)^(1/3)/b)/x) + 20*(27*a^2*b^2*c^2 - 6*a^3*b*c*d + a^4*d^2)*(-b)^(2/3)*
log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - 10*(27*a^2*b^2*c^2 - 6*a^3*b*c*
d + a^4*d^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*
x + (b*x^3 + a)^(2/3))/x^2) - 3*(81*b^4*d^2*x^10 + 18*(12*b^4*c*d + 7*a*b^3
*d^2)*x^7 + 3*(54*b^4*c^2 + 132*a*b^3*c*d + 5*a^2*b^2*d^2)*x^4 + 4*(108*a*b
^3*c^2 + 30*a^2*b^2*c*d - 5*a^3*b*d^2)*x)*(b*x^3 + a)^(2/3))/b^3]
```

Sympy [C] time = 14.9286, size = 270, normalized size = 1.03

$$\frac{a^{\frac{5}{3}}c^{2}x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{2a^{\frac{5}{3}}cdx^{4}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{a^{\frac{5}{3}}d^{2}x^{7}\Gamma\left(\frac{7}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{2}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^{3}e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{a^{\frac{2}{3}}bc^{2}x^{4}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(-\right.}{3\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(5/3)*(d*x**3+c)**2,x)
```

```
[Out] a**(5/3)*c**2*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(4/3)) + 2*a**(5/3)*c*d*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/
3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(5/3)*d**2*x**7*gamma(7/
3)*hyper((-2/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) +
a**(2/3)*b*c**2*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/3,), b*x**3*exp_polar
(I*pi)/a)/(3*gamma(7/3)) + 2*a**(2/3)*b*c*d*x**7*gamma(7/3)*hyper((-2/3, 7/
3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + a**(2/3)*b*d**2*x*
*10*gamma(10/3)*hyper((-2/3, 10/3), (13/3,), b*x**3*exp_polar(I*pi)/a)/(3*g
amma(13/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3 + a\right)^{\frac{5}{3}} \left(dx^3 + c\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)*(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(5/3)*(d*x^3 + c)^2, x)
```

3.71 $\int (a+bx^3)^{2/3} (c+dx^3)^2 \, dx$

Optimal. Leaf size=219

$$\frac{x\left(a+bx^3\right)^{2/3}\left(2a^2d^2-9abcd+27b^2c^2\right)}{81b^2}-\frac{a\left(2a^2d^2-9abcd+27b^2c^2\right)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{81b^{7/3}}+\frac{2a\left(2a^2d^2-9abcd+27b\right.}{81\sqrt{3}b}$$

```
[Out] ((27*b^2*c^2 - 9*a*b*c*d + 2*a^2*d^2)*x*(a + b*x^3)^(2/3))/(81*b^2) + (2*d*
(3*b*c - a*d)*x*(a + b*x^3)^(5/3))/(27*b^2) + (d*x*(a + b*x^3)^(5/3)*(c + d
*x^3))/(9*b) + (2*a*(27*b^2*c^2 - 9*a*b*c*d + 2*a^2*d^2)*ArcTan[(1 + (2*b^(
1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(81*Sqrt[3]*b^(7/3)) - (a*(27*b^2*c^2
- 9*a*b*c*d + 2*a^2*d^2)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(81*b^(7/3)
)
```

Rubi [A] time = 0.166311, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {416, 388, 195, 239}

$$\frac{x\left(a+bx^3\right)^{2/3}\left(2a^2d^2-9abcd+27b^2c^2\right)}{81b^2}-\frac{a\left(2a^2d^2-9abcd+27b^2c^2\right)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{81b^{7/3}}+\frac{2a\left(2a^2d^2-9abcd+27b\right.}{81\sqrt{3}b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)*(c + d*x^3)^2,x]
```


```
[Out] ((27*b^2*c^2 - 9*a*b*c*d + 2*a^2*d^2)*x*(a + b*x^3)^(2/3))/(81*b^2) + (2*d*
(3*b*c - a*d)*x*(a + b*x^3)^(5/3))/(27*b^2) + (d*x*(a + b*x^3)^(5/3)*(c + d
*x^3))/(9*b) + (2*a*(27*b^2*c^2 - 9*a*b*c*d + 2*a^2*d^2)*ArcTan[(1 + (2*b^(
1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(81*Sqrt[3]*b^(7/3)) - (a*(27*b^2*c^2
- 9*a*b*c*d + 2*a^2*d^2)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(81*b^(7/3)
)
```


Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \left(a+bx^3\right)^{2/3}\left(c+dx^3\right)^2\,dx = \frac{dx\left(a+bx^3\right)^{5/3}\left(c+dx^3\right)}{9b} + \frac{\int \left(a+bx^3\right)^{2/3}\left(c(9bc-ad)+4d(3bc-ad)x^3\right)dx}{9b}$$

$$= \frac{2d(3bc-ad)x\left(a+bx^3\right)^{5/3}}{27b^2} + \frac{dx\left(a+bx^3\right)^{5/3}\left(c+dx^3\right)}{9b} - \frac{(4ad(3bc-ad)-6bc(9bc-a}{54b^2}$$

$$= \frac{\left(27b^2c^2-9abcd+2a^2d^2\right)x\left(a+bx^3\right)^{2/3}}{81b^2} + \frac{2d(3bc-ad)x\left(a+bx^3\right)^{5/3}}{27b^2} + \frac{dx\left(a+bx^3\right)^{5}}{9b}$$

$$= \frac{\left(27b^2c^2-9abcd+2a^2d^2\right)x\left(a+bx^3\right)^{2/3}}{81b^2} + \frac{2d(3bc-ad)x\left(a+bx^3\right)^{5/3}}{27b^2} + \frac{dx\left(a+bx^3\right)^{5}}{9b}$$

Mathematica [C] time = 4.1492, size = 179, normalized size = 0.82

$$\frac{x\left(a+bx^3\right)^{2/3}\left(9bx^3\text{Gamma}\left(\frac{1}{3}\right)\left(c+dx^3\right)^2\text{HypergeometricPFQ}\left(\left\{\frac{1}{3},\frac{4}{3},2\right\},\left\{1,\frac{13}{3}\right\},-\frac{bx^3}{a}\right)+3bx^3\text{Gamma}\left(\frac{1}{3}\right)(11c}{420a\text{Gamma}\left(\frac{1}{3}\right)(}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(2/3)*(c + d*x^3)^2,x]
```

```
[Out] (x*(a + b*x^3)^(2/3)*(-20*a*(14*c^2 + 7*c*d*x^3 + 2*d^2*x^6)*Gamma[-2/3]*Hy
pergeometric2F1[-2/3, 1/3, 10/3, -((b*x^3)/a)] + 3*b*x^3*(11*c^2 + 16*c*d*x
^3 + 5*d^2*x^6)*Gamma[1/3]*Hypergeometric2F1[1/3, 4/3, 13/3, -((b*x^3)/a)]
+ 9*b*x^3*(c + d*x^3)^2*Gamma[1/3]*HypergeometricPFQ[{1/3, 4/3, 2}, {1, 13/
3}, -((b*x^3)/a)]))/(420*a*(1 + (b*x^3)/a)^(2/3)*Gamma[1/3])
```

Maple [F] time = 0.219, size = 0, normalized size = 0.

$$\int \left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)*(d*x^3+c)^2,x)
```

```
[Out] int((b*x^3+a)^(2/3)*(d*x^3+c)^2,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)*(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.8091, size = 1519, normalized size = 6.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] [1/243*(3*sqrt(1/3)*(27*a*b^3*c^2 - 9*a^2*b^2*c*d + 2*a^3*b*d^2)*sqrt((-b)^
(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-
b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x
)*sqrt((-b)^(1/3)/b) + 2*a) - 2*(27*a*b^2*c^2 - 9*a^2*b*c*d + 2*a^3*d^2)*(-
b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + (27*a*b^2*c^2 - 9*a^2*
b*c*d + 2*a^3*d^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^
(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 3*(9*b^3*d^2*x^7 + 3*(9*b^3*c*d + a*b^2
*d^2)*x^4 + (27*b^3*c^2 + 18*a*b^2*c*d - 4*a^2*b*d^2)*x)*(b*x^3 + a)^(2/3))
/b^3, -1/243*(6*sqrt(1/3)*(27*a*b^3*c^2 - 9*a^2*b^2*c*d + 2*a^3*b*d^2)*sqrt
(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sqrt
(-(-b)^(1/3)/b)/x) + 2*(27*a*b^2*c^2 - 9*a^2*b*c*d + 2*a^3*d^2)*(-b)^(2/3)*
log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - (27*a*b^2*c^2 - 9*a^2*b*c*d + 2
*a^3*d^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x +
 (b*x^3 + a)^(2/3))/x^2) - 3*(9*b^3*d^2*x^7 + 3*(9*b^3*c*d + a*b^2*d^2)*x^4
 + (27*b^3*c^2 + 18*a*b^2*c*d - 4*a^2*b*d^2)*x)*(b*x^3 + a)^(2/3))/b^3]
```

Sympy [C] time = 4.88447, size = 131, normalized size = 0.6

$$\frac{a^{\frac{2}{3}}c^2x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{2a^{\frac{2}{3}}cdx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{a^{\frac{2}{3}}d^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)*(d*x**3+c)**2,x)
```

```
[Out] a**(2/3)*c**2*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(4/3)) + 2*a**(2/3)*c*d*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/
3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(2/3)*d**2*x**7*gamma(7/
3)*hyper((-2/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3 + a\right)^{\frac{2}{3}} \left(dx^3 + c\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)*(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)*(d*x^3 + c)^2, x)
```

3.72 $\int \frac{(c+dx^3)^2}{\sqrt[3]{a+bx^3}}\,dx$

Optimal. Leaf size=175

$$-\frac{(2a^2d^2-6abcd+9b^2c^2)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{18b^{7/3}}+\frac{(2a^2d^2-6abcd+9b^2c^2)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}b^{7/3}}+\frac{dx(a+bx^3)^{2/3}(9bc-4a}{18b^2}$$

```
[Out] (d*(9*b*c - 4*a*d)*x*(a + b*x^3)^(2/3))/(18*b^2) + (d*x*(a + b*x^3)^(2/3)*(
c + d*x^3))/(6*b) + ((9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^2)*ArcTan[(1 + (2*b^(
1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(7/3)) - ((9*b^2*c^2 - 6*
a*b*c*d + 2*a^2*d^2)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(18*b^(7/3))
```

Rubi [A] time = 0.096277, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {416, 388, 239}

$$-\frac{(2a^2d^2-6abcd+9b^2c^2)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{18b^{7/3}}+\frac{(2a^2d^2-6abcd+9b^2c^2)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}b^{7/3}}+\frac{dx(a+bx^3)^{2/3}(9bc-4a}{18b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^(1/3),x]
```

```
[Out] (d*(9*b*c - 4*a*d)*x*(a + b*x^3)^(2/3))/(18*b^2) + (d*x*(a + b*x^3)^(2/3)*(
c + d*x^3))/(6*b) + ((9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^2)*ArcTan[(1 + (2*b^(
1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]*b^(7/3)) - ((9*b^2*c^2 - 6*
a*b*c*d + 2*a^2*d^2)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(18*b^(7/3))
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^2}{\sqrt[3]{a+bx^3}}\,dx = \frac{dx\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}{6b} + \frac{\int \frac{c(6bc-ad)+d(9bc-4ad)x^3}{\sqrt[3]{a+bx^3}}\,dx}{6b}$$

$$= \frac{d(9bc-4ad)x\left(a+bx^3\right)^{2/3}}{18b^2} + \frac{dx\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}{6b} + \frac{\left(9b^2c^2-6abcd+2a^2d^2\right)\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx}{9b^2}$$

$$= \frac{d(9bc-4ad)x\left(a+bx^3\right)^{2/3}}{18b^2} + \frac{dx\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}{6b} + \frac{\left(9b^2c^2-6abcd+2a^2d^2\right)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{7/3}}$$

Mathematica [A] time = 5.13845, size = 172, normalized size = 0.98

$$\frac{\left(2a^2d^2-6abcd+9b^2c^2\right)\left(\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right)-2\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)\right)+3\sqrt[3]{b}dx\left(a+bx\right.}{54b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^(1/3),x]
```

```
[Out] (3*b^(1/3)*d*x*(a + b*x^3)^(2/3)*(-4*a*d + 3*b*(4*c + d*x^3)) + (9*b^2*c^2
- 6*a*b*c*d + 2*a^2*d^2)*(2*Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(
1/3))/Sqrt[3]] - 2*Log[1 - (b^(1/3)*x)/(a + b*x^3)^(1/3)] + Log[1 + (b^(2/3
)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)]))/(54*b^(7/3))
```

Maple [F] time = 0.215, size = 0, normalized size = 0.

$$\int \left(dx^3+c\right)^2 \frac{1}{\sqrt[3]{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^2/(b*x^3+a)^(1/3),x)
```

```
[Out] int((d*x^3+c)^2/(b*x^3+a)^(1/3),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(1/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.7547, size = 1341, normalized size = 7.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(1/3),x, algorithm="fricas")
```

```
[Out] [1/54*(3*sqrt(1/3)*(9*b^3*c^2 - 6*a*b^2*c*d + 2*a^2*b*d^2)*sqrt((-b)^(1/3)/
b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)*((-b)^(1/
3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/3)*x)*sqrt
((-b)^(1/3)/b) + 2*a) - 2*(9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^2)*(-b)^(2/3)*lo
g(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + (9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^
2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3
 + a)^(2/3))/x^2) + 3*(3*b^2*d^2*x^4 + 4*(3*b^2*c*d - a*b*d^2)*x)*(b*x^3 +
a)^(2/3))/b^3, -1/54*(6*sqrt(1/3)*(9*b^3*c^2 - 6*a*b^2*c*d + 2*a^2*b*d^2)*s
qrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*s
qrt(-(-b)^(1/3)/b)/x) + 2*(9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^2)*(-b)^(2/3)*lo
g(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) - (9*b^2*c^2 - 6*a*b*c*d + 2*a^2*d^
2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3
 + a)^(2/3))/x^2) - 3*(3*b^2*d^2*x^4 + 4*(3*b^2*c*d - a*b*d^2)*x)*(b*x^3 +
a)^(2/3))/b^3]
```

Sympy [C] time = 3.90301, size = 126, normalized size = 0.72

$$\frac{c^2 x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\right|\frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{4}{3}\right)} + \frac{2cdx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\right|\frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{7}{3}\right)} + \frac{d^2x^7\Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{7}{3}\\\frac{10}{3}\end{matrix}\right|\frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**(1/3),x)
```

```
[Out] c**2*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a*
*(1/3)*gamma(4/3)) + 2*c*d*x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), b*x**3
*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(7/3)) + d**2*x**7*gamma(7/3)*hyper((1
/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(10/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3 + c\right)^2}{\left(bx^3 + a\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(1/3), x)
```

3.73 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{4/3}}\,dx$

Optimal. Leaf size=159

$$-\frac{dx\left(a+bx^3\right)^{2/3}(3bc-4ad)}{3ab^2}-\frac{d(3bc-2ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{3b^{7/3}}+\frac{2d(3bc-2ad)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}b^{7/3}}+\frac{x\left(c+dx^3\right)(b}{ab\sqrt[3]{a+b}}$$

```
[Out] -(d*(3*b*c - 4*a*d)*x*(a + b*x^3)^(2/3))/(3*a*b^2) + ((b*c - a*d)*x*(c + d*
x^3))/(a*b*(a + b*x^3)^(1/3)) + (2*d*(3*b*c - 2*a*d)*ArcTan[(1 + (2*b^(1/3)
*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(7/3)) - (d*(3*b*c - 2*a*d)*L
og[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(3*b^(7/3))
```

Rubi [A] time = 0.102334, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {413, 388, 239}

$$-\frac{dx\left(a+bx^3\right)^{2/3}(3bc-4ad)}{3ab^2}-\frac{d(3bc-2ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{3b^{7/3}}+\frac{2d(3bc-2ad)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}b^{7/3}}+\frac{x\left(c+dx^3\right)(b}{ab\sqrt[3]{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^(4/3),x]
```

```
[Out] -(d*(3*b*c - 4*a*d)*x*(a + b*x^3)^(2/3))/(3*a*b^2) + ((b*c - a*d)*x*(c + d*
x^3))/(a*b*(a + b*x^3)^(1/3)) + (2*d*(3*b*c - 2*a*d)*ArcTan[(1 + (2*b^(1/3)
*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*b^(7/3)) - (d*(3*b*c - 2*a*d)*L
og[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(3*b^(7/3))
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^{4/3}}\,dx = \frac{(bc-ad)x\left(c+dx^3\right)}{ab\sqrt[3]{a+bx^3}} + \frac{\int \frac{acd-d(3bc-4ad)x^3}{\sqrt[3]{a+bx^3}}\,dx}{ab}$$

$$= -\frac{d(3bc-4ad)x\left(a+bx^3\right)^{2/3}}{3ab^2} + \frac{(bc-ad)x\left(c+dx^3\right)}{ab\sqrt[3]{a+bx^3}} + \frac{(2d(3bc-2ad))\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx}{3b^2}$$

$$= -\frac{d(3bc-4ad)x\left(a+bx^3\right)^{2/3}}{3ab^2} + \frac{(bc-ad)x\left(c+dx^3\right)}{ab\sqrt[3]{a+bx^3}} + \frac{2d(3bc-2ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{7/3}} - \frac{d(3bc-2a\ldots}{\ldots}$$

Mathematica [A] time = 5.14927, size = 168, normalized size = 1.06

$$\frac{x\left(a+bx^3\right)^{2/3}\left(\frac{3(bc-ad)^2}{a\left(a+bx^3\right)}+d^2\right)}{3b^2} + \frac{d(3bc-2ad)\left(\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right)-2\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)\right)}{9b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^(4/3),x]
```

```
[Out] (x*(a + b*x^3)^(2/3)*(d^2 + (3*(b*c - a*d)^2)/(a*(a + b*x^3))))/(3*b^2) + (
d*(3*b*c - 2*a*d)*(2*Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/S
qrt[3]] - 2*Log[1 - (b^(1/3)*x)/(a + b*x^3)^(1/3)] + Log[1 + (b^(2/3)*x^2)/
(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)]))/(9*b^(7/3))
```

Maple [F] time = 0.416, size = 0, normalized size = 0.

$$\int \left(dx^3+c\right)^2\left(bx^3+a\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^2/(b*x^3+a)^(4/3),x)
```

```
[Out] int((d*x^3+c)^2/(b*x^3+a)^(4/3),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(4/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.73604, size = 1538, normalized size = 9.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(4/3),x, algorithm="fricas")
```

```
[Out] [-1/9*(3*sqrt(1/3)*(3*a^2*b^2*c*d - 2*a^3*b*d^2 + (3*a*b^3*c*d - 2*a^2*b^2*
d^2)*x^3)*sqrt(-1/b^(2/3))*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*b^(2/3)*x^2 -
3*sqrt(1/3)*(b^(4/3)*x^3 + (b*x^3 + a)^(1/3)*b*x^2 - 2*(b*x^3 + a)^(2/3)*b^
(2/3)*x)*sqrt(-1/b^(2/3)) + 2*a) + 2*(3*a^2*b*c*d - 2*a^3*d^2 + (3*a*b^2*c*
d - 2*a^2*b*d^2)*x^3)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - (3*
a^2*b*c*d - 2*a^3*d^2 + (3*a*b^2*c*d - 2*a^2*b*d^2)*x^3)*b^(2/3)*log((b^(2/
3)*x^2 + (b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) - 3*(a*b^2*d
^2*x^4 + (3*b^3*c^2 - 6*a*b^2*c*d + 4*a^2*b*d^2)*x)*(b*x^3 + a)^(2/3))/(a*b
^4*x^3 + a^2*b^3), -1/9*(2*(3*a^2*b*c*d - 2*a^3*d^2 + (3*a*b^2*c*d - 2*a^2*
b*d^2)*x^3)*b^(2/3)*log(-(b^(1/3)*x - (b*x^3 + a)^(1/3))/x) - (3*a^2*b*c*d
- 2*a^3*d^2 + (3*a*b^2*c*d - 2*a^2*b*d^2)*x^3)*b^(2/3)*log((b^(2/3)*x^2 + (
b*x^3 + a)^(1/3)*b^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 6*sqrt(1/3)*(3*a^2*b
^2*c*d - 2*a^3*b*d^2 + (3*a*b^3*c*d - 2*a^2*b^2*d^2)*x^3)*arctan(sqrt(1/3)*
(b^(1/3)*x + 2*(b*x^3 + a)^(1/3))/(b^(1/3)*x))/b^(1/3) - 3*(a*b^2*d^2*x^4 +
 (3*b^3*c^2 - 6*a*b^2*c*d + 4*a^2*b*d^2)*x)*(b*x^3 + a)^(2/3))/(a*b^4*x^3 +
 a^2*b^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c + dx^3\right)^2}{\left(a + bx^3\right)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**(4/3),x)
```

```
[Out] Integral((c + d*x**3)**2/(a + b*x**3)**(4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3 + c\right)^2}{\left(bx^3 + a\right)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(4/3), x)
```

3.74 $\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^{7/3}}\,dx$

Optimal. Leaf size=152

$$\frac{x(bc-ad)(4ad+3bc)}{4a^2b^2\sqrt[3]{a+bx^3}}-\frac{d^2\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2b^{7/3}}+\frac{d^2\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{7/3}}+\frac{x\left(c+dx^3\right)(bc-ad)}{4ab\left(a+bx^3\right)^{4/3}}$$

```
[Out] ((b*c - a*d)*(3*b*c + 4*a*d)*x)/(4*a^2*b^2*(a + b*x^3)^(1/3)) + ((b*c - a*d
)*x*(c + d*x^3))/(4*a*b*(a + b*x^3)^(4/3)) + (d^2*ArcTan[(1 + (2*b^(1/3)*x)
/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(7/3)) - (d^2*Log[-(b^(1/3)*x) + (
a + b*x^3)^(1/3)])/(2*b^(7/3))
```

Rubi [A] time = 0.0676022, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {413, 385, 239}

$$\frac{x(bc-ad)(4ad+3bc)}{4a^2b^2\sqrt[3]{a+bx^3}}-\frac{d^2\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2b^{7/3}}+\frac{d^2\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{7/3}}+\frac{x\left(c+dx^3\right)(bc-ad)}{4ab\left(a+bx^3\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^(7/3),x]
```

```
[Out] ((b*c - a*d)*(3*b*c + 4*a*d)*x)/(4*a^2*b^2*(a + b*x^3)^(1/3)) + ((b*c - a*d
)*x*(c + d*x^3))/(4*a*b*(a + b*x^3)^(4/3)) + (d^2*ArcTan[(1 + (2*b^(1/3)*x)
/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*b^(7/3)) - (d^2*Log[-(b^(1/3)*x) + (
a + b*x^3)^(1/3)])/(2*b^(7/3))
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
```

```
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^{7/3}}\,dx = \frac{(bc-ad)x\left(c+dx^3\right)}{4ab\left(a+bx^3\right)^{4/3}} + \frac{\int \frac{c(3bc+ad)+4ad^2x^3}{\left(a+bx^3\right)^{4/3}}\,dx}{4ab}$$

$$= \frac{(bc-ad)(3bc+4ad)x}{4a^2b^2\sqrt[3]{a+bx^3}} + \frac{(bc-ad)x\left(c+dx^3\right)}{4ab\left(a+bx^3\right)^{4/3}} + \frac{d^2\int \frac{1}{\sqrt[3]{a+bx^3}}\,dx}{b^2}$$

$$= \frac{(bc-ad)(3bc+4ad)x}{4a^2b^2\sqrt[3]{a+bx^3}} + \frac{(bc-ad)x\left(c+dx^3\right)}{4ab\left(a+bx^3\right)^{4/3}} + \frac{d^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{7/3}} - \frac{d^2\log\left(-\sqrt[3]{b}x+\sqrt[3]{a+bx^3}\right.}{2b^{7/3}}$$

Mathematica [A] time = 5.2114, size = 180, normalized size = 1.18

$$\frac{x\left(\left(a+bx^3\right)\left(-5a^2d^2+2abcd+3b^2c^2\right)+a(bc-ad)^2\right)}{4a^2b^2\left(a+bx^3\right)^{4/3}} + \frac{d^2\left(\log\left(\frac{b^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1\right)-2\log\left(1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right)+2\sqrt{3}\,\mathrm{t}\right.}{6b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^(7/3),x]
```

```
[Out] (x*(a*(b*c - a*d)^2 + (3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*(a + b*x^3)))/(4*
a^2*b^2*(a + b*x^3)^(4/3)) + (d^2*(2*Sqrt[3]*ArcTan[(1 + (2*b^(1/3)*x)/(a +
 b*x^3)^(1/3))/Sqrt[3]] - 2*Log[1 - (b^(1/3)*x)/(a + b*x^3)^(1/3)] + Log[1
+ (b^(2/3)*x^2)/(a + b*x^3)^(2/3) + (b^(1/3)*x)/(a + b*x^3)^(1/3)]))/(6*b^(
7/3))
```

Maple [F] time = 0.376, size = 0, normalized size = 0.

$$\int \left(dx^3+c\right)^2\left(bx^3+a\right)^{-\frac{7}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^2/(b*x^3+a)^(7/3),x)
```

```
[Out] int((d*x^3+c)^2/(b*x^3+a)^(7/3),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(7/3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.76132, size = 1617, normalized size = 10.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(7/3),x, algorithm="fricas")
```

```
[Out] [1/12*(6*sqrt(1/3)*(a^2*b^3*d^2*x^6 + 2*a^3*b^2*d^2*x^3 + a^4*b*d^2)*sqrt((
-b)^(1/3)/b)*log(3*b*x^3 - 3*(b*x^3 + a)^(1/3)*(-b)^(2/3)*x^2 - 3*sqrt(1/3)
*((-b)^(1/3)*b*x^3 - (b*x^3 + a)^(1/3)*b*x^2 + 2*(b*x^3 + a)^(2/3)*(-b)^(2/
3)*x)*sqrt((-b)^(1/3)/b) + 2*a) - 4*(a^2*b^2*d^2*x^6 + 2*a^3*b*d^2*x^3 + a^
4*d^2)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 + a)^(1/3))/x) + 2*(a^2*b^2*d^
2*x^6 + 2*a^3*b*d^2*x^3 + a^4*d^2)*(-b)^(2/3)*log(((-b)^(2/3)*x^2 - (b*x^3
+ a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^2) + 3*((3*b^4*c^2 + 2*a*b^3
*c*d - 5*a^2*b^2*d^2)*x^4 + 4*(a*b^3*c^2 - a^3*b*d^2)*x)*(b*x^3 + a)^(2/3))
/(a^2*b^5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3), -1/12*(12*sqrt(1/3)*(a^2*b^3*d^2*
x^6 + 2*a^3*b^2*d^2*x^3 + a^4*b*d^2)*sqrt(-(-b)^(1/3)/b)*arctan(-sqrt(1/3)*
((-b)^(1/3)*x - 2*(b*x^3 + a)^(1/3))*sqrt(-(-b)^(1/3)/b)/x) + 4*(a^2*b^2*d^
2*x^6 + 2*a^3*b*d^2*x^3 + a^4*d^2)*(-b)^(2/3)*log(((-b)^(1/3)*x + (b*x^3 +
a)^(1/3))/x) - 2*(a^2*b^2*d^2*x^6 + 2*a^3*b*d^2*x^3 + a^4*d^2)*(-b)^(2/3)*l
og(((-b)^(2/3)*x^2 - (b*x^3 + a)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a)^(2/3))/x^
2) - 3*((3*b^4*c^2 + 2*a*b^3*c*d - 5*a^2*b^2*d^2)*x^4 + 4*(a*b^3*c^2 - a^3*
b*d^2)*x)*(b*x^3 + a)^(2/3))/(a^2*b^5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c + dx^3\right)^2}{\left(a + bx^3\right)^{\frac{7}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**(7/3),x)
```

```
[Out] Integral((c + d*x**3)**2/(a + b*x**3)**(7/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3 + c\right)^2}{\left(bx^3 + a\right)^{\frac{7}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(7/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(7/3), x)
```

3.75 $\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^{10/3}}\,dx$

Optimal. Leaf size=78

$$\frac{9c^2x}{14a^3\sqrt[3]{a+bx^3}}+\frac{3cx\left(c+dx^3\right)}{14a^2\left(a+bx^3\right)^{4/3}}+\frac{x\left(c+dx^3\right)^2}{7a\left(a+bx^3\right)^{7/3}}$$

```
[Out] (9*c^2*x)/(14*a^3*(a + b*x^3)^(1/3)) + (3*c*x*(c + d*x^3))/(14*a^2*(a + b*x
^3)^(4/3)) + (x*(c + d*x^3)^2)/(7*a*(a + b*x^3)^(7/3))
```

Rubi [A] time = 0.0211033, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {378, 191}

$$\frac{9c^2x}{14a^3\sqrt[3]{a+bx^3}}+\frac{3cx\left(c+dx^3\right)}{14a^2\left(a+bx^3\right)^{4/3}}+\frac{x\left(c+dx^3\right)^2}{7a\left(a+bx^3\right)^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^(10/3),x]
```

```
[Out] (9*c^2*x)/(14*a^3*(a + b*x^3)^(1/3)) + (3*c*x*(c + d*x^3))/(14*a^2*(a + b*x
^3)^(4/3)) + (x*(c + d*x^3)^2)/(7*a*(a + b*x^3)^(7/3))
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^{10/3}}\,dx &= \frac{x\left(c+dx^3\right)^2}{7a\left(a+bx^3\right)^{7/3}}+\frac{(6c)\int \frac{c+dx^3}{\left(a+bx^3\right)^{7/3}}\,dx}{7a}\\
&= \frac{3cx\left(c+dx^3\right)}{14a^2\left(a+bx^3\right)^{4/3}}+\frac{x\left(c+dx^3\right)^2}{7a\left(a+bx^3\right)^{7/3}}+\frac{\left(9c^2\right)\int \frac{1}{\left(a+bx^3\right)^{4/3}}\,dx}{14a^2}\\
&= \frac{9c^2x}{14a^3\sqrt[3]{a+bx^3}}+\frac{3cx\left(c+dx^3\right)}{14a^2\left(a+bx^3\right)^{4/3}}+\frac{x\left(c+dx^3\right)^2}{7a\left(a+bx^3\right)^{7/3}}
\end{aligned}$$

Mathematica [A] time = 0.080017, size = 126, normalized size = 1.62

$$\frac{x\sqrt[3]{\frac{bx^3}{a}+1}\left(a^2\left(14c^2+7cdx^3+2d^2x^6\right)+3abcx^3\left(7c+dx^3\right)+9b^2c^2x^6\right)}{14a^3\left(a+bx^3\right)^{7/3}\sqrt[3]{\frac{dx^3}{c}+1}\sqrt[3]{\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^(10/3),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(1/3)*(9*b^2*c^2*x^6 + 3*a*b*c*x^3*(7*c + d*x^3) + a^2*(
14*c^2 + 7*c*d*x^3 + 2*d^2*x^6)))/(14*a^3*(a + b*x^3)^(7/3)*((c*(a + b*x^3)
)/(a*(c + d*x^3)))^(1/3)*(1 + (d*x^3)/c)^(1/3))
```

Maple [A] time = 0.006, size = 76, normalized size = 1.

$$\frac{x\left(2\,a^2d^2x^6+3\,abcdx^6+9\,b^2c^2x^6+7\,a^2cdx^3+21\,abc^2x^3+14\,a^2c^2\right)}{14\,a^3}\left(bx^3+a\right)^{-\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^2/(b*x^3+a)^(10/3),x)
```

```
[Out] 1/14*x*(2*a^2*d^2*x^6+3*a*b*c*d*x^6+9*b^2*c^2*x^6+7*a^2*c*d*x^3+21*a*b*c^2*
x^3+14*a^2*c^2)/(b*x^3+a)^(7/3)/a^3
```

Maxima [A] time = 0.959703, size = 147, normalized size = 1.88

$$-\frac{\left(4\,b-\frac{7\left(bx^3+a\right)}{x^3}\right)cdx^7}{14\left(bx^3+a\right)^{\frac{7}{3}}a^2}+\frac{d^2x^7}{7\left(bx^3+a\right)^{\frac{7}{3}}a}+\frac{\left(2\,b^2-\frac{7\left(bx^3+a\right)b}{x^3}+\frac{14\left(bx^3+a\right)^2}{x^6}\right)c^2x^7}{14\left(bx^3+a\right)^{\frac{7}{3}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(10/3),x, algorithm="maxima")
```

```
[Out] -1/14*(4*b - 7*(b*x^3 + a)/x^3)*c*d*x^7/((b*x^3 + a)^(7/3)*a^2) + 1/7*d^2*x
^7/((b*x^3 + a)^(7/3)*a) + 1/14*(2*b^2 - 7*(b*x^3 + a)*b/x^3 + 14*(b*x^3 +
a)^2/x^6)*c^2*x^7/((b*x^3 + a)^(7/3)*a^3)
```

Fricas [A] time = 1.66892, size = 217, normalized size = 2.78

$$\frac{\left(\left(9\,b^2c^2+3\,abcd+2\,a^2d^2\right)x^7+14\,a^2c^2x+7\left(3\,abc^2+a^2cd\right)x^4\right)\left(bx^3+a\right)^{\frac{2}{3}}}{14\left(a^3b^3x^9+3\,a^4b^2x^6+3\,a^5bx^3+a^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(10/3),x, algorithm="fricas")
```

```
[Out] 1/14*((9*b^2*c^2 + 3*a*b*c*d + 2*a^2*d^2)*x^7 + 14*a^2*c^2*x + 7*(3*a*b*c^2
 + a^2*c*d)*x^4)*(b*x^3 + a)^(2/3)/(a^3*b^3*x^9 + 3*a^4*b^2*x^6 + 3*a^5*b*x
^3 + a^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**(10/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3 + c\right)^2}{\left(bx^3 + a\right)^{\frac{10}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(10/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(10/3), x)
```

3.76 $\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^{13/3}}\,dx$

Optimal. Leaf size=174

$$\frac{9c^2x(9bc-10ad)}{140a^4\sqrt[3]{a+bx^3}(bc-ad)}+\frac{x\left(c+dx^3\right)^2(9bc-10ad)}{70a^2\left(a+bx^3\right)^{7/3}(bc-ad)}+\frac{3cx\left(c+dx^3\right)(9bc-10ad)}{140a^3\left(a+bx^3\right)^{4/3}(bc-ad)}+\frac{bx\left(c+dx^3\right)^3}{10a\left(a+bx^3\right)^{10/3}(bc-ad)}$$

```
[Out] (9*c^2*(9*b*c - 10*a*d)*x)/(140*a^4*(b*c - a*d)*(a + b*x^3)^(1/3)) + (3*c*(
9*b*c - 10*a*d)*x*(c + d*x^3))/(140*a^3*(b*c - a*d)*(a + b*x^3)^(4/3)) + ((
9*b*c - 10*a*d)*x*(c + d*x^3)^2)/(70*a^2*(b*c - a*d)*(a + b*x^3)^(7/3)) + (
b*x*(c + d*x^3)^3)/(10*a*(b*c - a*d)*(a + b*x^3)^(10/3))
```

Rubi [A] time = 0.0726912, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {382, 378, 191}

$$\frac{9c^2x(9bc-10ad)}{140a^4\sqrt[3]{a+bx^3}(bc-ad)}+\frac{x\left(c+dx^3\right)^2(9bc-10ad)}{70a^2\left(a+bx^3\right)^{7/3}(bc-ad)}+\frac{3cx\left(c+dx^3\right)(9bc-10ad)}{140a^3\left(a+bx^3\right)^{4/3}(bc-ad)}+\frac{bx\left(c+dx^3\right)^3}{10a\left(a+bx^3\right)^{10/3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^(13/3),x]
```


```
[Out] (9*c^2*(9*b*c - 10*a*d)*x)/(140*a^4*(b*c - a*d)*(a + b*x^3)^(1/3)) + (3*c*(
9*b*c - 10*a*d)*x*(c + d*x^3))/(140*a^3*(b*c - a*d)*(a + b*x^3)^(4/3)) + ((
9*b*c - 10*a*d)*x*(c + d*x^3)^2)/(70*a^2*(b*c - a*d)*(a + b*x^3)^(7/3)) + (
b*x*(c + d*x^3)^3)/(10*a*(b*c - a*d)*(a + b*x^3)^(10/3))
```


Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{13/3}}\,dx = \frac{bx(c+dx^3)^3}{10a(bc-ad)(a+bx^3)^{10/3}} + \frac{(9bc-10ad)\int \frac{(c+dx^3)^2}{(a+bx^3)^{10/3}}\,dx}{10a(bc-ad)}$$
$$= \frac{(9bc-10ad)x(c+dx^3)^2}{70a^2(bc-ad)(a+bx^3)^{7/3}} + \frac{bx(c+dx^3)^3}{10a(bc-ad)(a+bx^3)^{10/3}} + \frac{(3c(9bc-10ad))\int \frac{c+dx^3}{(a+bx^3)^{7/3}}\,dx}{35a^2(bc-ad)}$$
$$= \frac{3c(9bc-10ad)x(c+dx^3)}{140a^3(bc-ad)(a+bx^3)^{4/3}} + \frac{(9bc-10ad)x(c+dx^3)^2}{70a^2(bc-ad)(a+bx^3)^{7/3}} + \frac{bx(c+dx^3)^3}{10a(bc-ad)(a+bx^3)^{10/3}} + \frac{(9c^2(9b}{}$$
$$= \frac{9c^2(9bc-10ad)x}{140a^4(bc-ad)\sqrt[3]{a+bx^3}} + \frac{3c(9bc-10ad)x(c+dx^3)}{140a^3(bc-ad)(a+bx^3)^{4/3}} + \frac{(9bc-10ad)x(c+dx^3)^2}{70a^2(bc-ad)(a+bx^3)^{7/3}} + \frac{bx}{10a(bc-}$$

Mathematica [A] time = 5.06123, size = 106, normalized size = 0.61

$$\frac{x\left(3a^2bx^3\left(105c^2+20cdx^3+2d^2x^6\right)+10a^3\left(14c^2+7cdx^3+2d^2x^6\right)+18ab^2cx^6\left(15c+dx^3\right)+81b^3c^2x^9\right)}{140a^4\left(a+bx^3\right)^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^(13/3),x]
```

```
[Out] (x*(81*b^3*c^2*x^9 + 18*a*b^2*c*x^6*(15*c + d*x^3) + 10*a^3*(14*c^2 + 7*c*d
*x^3 + 2*d^2*x^6) + 3*a^2*b*x^3*(105*c^2 + 20*c*d*x^3 + 2*d^2*x^6)))/(140*a
^4*(a + b*x^3)^(10/3))
```

Maple [A] time = 0.007, size = 115, normalized size = 0.7

$$\frac{x\left(6\,a^2bd^2x^9+18\,ab^2cdx^9+81\,b^3c^2x^9+20\,a^3d^2x^6+60\,a^2bcdx^6+270\,ab^2c^2x^6+70\,a^3cdx^3+315\,a^2bc^2x^3+140\,c^2a^3\right)}{140\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^2/(b*x^3+a)^(13/3),x)
```

```
[Out] 1/140*x*(6*a^2*b*d^2*x^9+18*a*b^2*c*d*x^9+81*b^3*c^2*x^9+20*a^3*d^2*x^6+60*
a^2*b*c*d*x^6+270*a*b^2*c^2*x^6+70*a^3*c*d*x^3+315*a^2*b*c^2*x^3+140*a^3*c^
2)/(b*x^3+a)^(10/3)/a^4
```

Maxima [A] time = 0.971651, size = 215, normalized size = 1.24

$$-\frac{\left(7\,b-\frac{10\,(bx^3+a)}{x^3}\right)d^2x^{10}}{70\,(bx^3+a)^{\frac{10}{3}}\,a^2}+\frac{\left(14\,b^2-\frac{40\,(bx^3+a)b}{x^3}+\frac{35\,(bx^3+a)^2}{x^6}\right)cdx^{10}}{70\,(bx^3+a)^{\frac{10}{3}}\,a^3}-\frac{\left(14\,b^3-\frac{60\,(bx^3+a)b^2}{x^3}+\frac{105\,(bx^3+a)^2b}{x^6}-\frac{140\,(bx^3+a)^3}{x^9}\right)c^2x}{140\,(bx^3+a)^{\frac{10}{3}}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(13/3),x, algorithm="maxima")
```

```
[Out] -1/70*(7*b - 10*(b*x^3 + a)/x^3)*d^2*x^10/((b*x^3 + a)^(10/3)*a^2) + 1/70*(
14*b^2 - 40*(b*x^3 + a)*b/x^3 + 35*(b*x^3 + a)^2/x^6)*c*d*x^10/((b*x^3 + a)
^(10/3)*a^3) - 1/140*(14*b^3 - 60*(b*x^3 + a)*b^2/x^3 + 105*(b*x^3 + a)^2*b
/x^6 - 140*(b*x^3 + a)^3/x^9)*c^2*x^10/((b*x^3 + a)^(10/3)*a^4)
```

Fricas [A] time = 1.73703, size = 328, normalized size = 1.89

$$\frac{\left(3\left(27\,b^3c^2+6\,ab^2cd+2\,a^2bd^2\right)x^{10}+10\left(27\,ab^2c^2+6\,a^2bcd+2\,a^3d^2\right)x^7+140\,a^3c^2x+35\left(9\,a^2bc^2+2\,a^3cd\right)x^4\right)\left(bx^3+a\right)}{140\left(a^4b^4x^{12}+4\,a^5b^3x^9+6\,a^6b^2x^6+4\,a^7bx^3+a^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(13/3),x, algorithm="fricas")
```

```
[Out] 1/140*(3*(27*b^3*c^2 + 6*a*b^2*c*d + 2*a^2*b*d^2)*x^10 + 10*(27*a*b^2*c^2 +
 6*a^2*b*c*d + 2*a^3*d^2)*x^7 + 140*a^3*c^2*x + 35*(9*a^2*b*c^2 + 2*a^3*c*d
)*x^4)*(b*x^3 + a)^(2/3)/(a^4*b^4*x^12 + 4*a^5*b^3*x^9 + 6*a^6*b^2*x^6 + 4*
a^7*b*x^3 + a^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**(13/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^2}{\left(bx^3+a\right)^{\frac{13}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(13/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(13/3), x)
```

3.77 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{16/3}}\,dx$

Optimal. Leaf size=211

$$\frac{9x(2a^2d^2+9abcd+54b^2c^2)}{910a^5b^2\sqrt[3]{a+bx^3}}+\frac{3x(2a^2d^2+9abcd+54b^2c^2)}{910a^4b^2(a+bx^3)^{4/3}}+\frac{x(2a^2d^2+9abcd+54b^2c^2)}{455a^3b^2(a+bx^3)^{7/3}}+\frac{2x(bc-ad)(ad+3bc)}{65a^2b^2(a+bx^3)^{10/3}}$$

```
[Out] (2*(b*c - a*d)*(3*b*c + a*d)*x)/(65*a^2*b^2*(a + b*x^3)^(10/3)) + ((54*b^2*
c^2 + 9*a*b*c*d + 2*a^2*d^2)*x)/(455*a^3*b^2*(a + b*x^3)^(7/3)) + (3*(54*b^
2*c^2 + 9*a*b*c*d + 2*a^2*d^2)*x)/(910*a^4*b^2*(a + b*x^3)^(4/3)) + (9*(54*
b^2*c^2 + 9*a*b*c*d + 2*a^2*d^2)*x)/(910*a^5*b^2*(a + b*x^3)^(1/3)) + ((b*c
 - a*d)*x*(c + d*x^3))/(13*a*b*(a + b*x^3)^(13/3))
```

Rubi [A] time = 0.127353, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {413, 385, 192, 191}

$$\frac{9x(2a^2d^2+9abcd+54b^2c^2)}{910a^5b^2\sqrt[3]{a+bx^3}}+\frac{3x(2a^2d^2+9abcd+54b^2c^2)}{910a^4b^2(a+bx^3)^{4/3}}+\frac{x(2a^2d^2+9abcd+54b^2c^2)}{455a^3b^2(a+bx^3)^{7/3}}+\frac{2x(bc-ad)(ad+3bc)}{65a^2b^2(a+bx^3)^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^(16/3),x]
```

```
[Out] (2*(b*c - a*d)*(3*b*c + a*d)*x)/(65*a^2*b^2*(a + b*x^3)^(10/3)) + ((54*b^2*
c^2 + 9*a*b*c*d + 2*a^2*d^2)*x)/(455*a^3*b^2*(a + b*x^3)^(7/3)) + (3*(54*b^
2*c^2 + 9*a*b*c*d + 2*a^2*d^2)*x)/(910*a^4*b^2*(a + b*x^3)^(4/3)) + (9*(54*
b^2*c^2 + 9*a*b*c*d + 2*a^2*d^2)*x)/(910*a^5*b^2*(a + b*x^3)^(1/3)) + ((b*c
 - a*d)*x*(c + d*x^3))/(13*a*b*(a + b*x^3)^(13/3))
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^{16/3}}\,dx = \frac{(bc-ad)x\left(c+dx^3\right)}{13ab\left(a+bx^3\right)^{13/3}} + \frac{\int \frac{c(12bc+ad)+d(9bc+4ad)x^3}{\left(a+bx^3\right)^{13/3}}\,dx}{13ab}$$

$$= \frac{2(bc-ad)(3bc+ad)x}{65a^2b^2\left(a+bx^3\right)^{10/3}} + \frac{(bc-ad)x\left(c+dx^3\right)}{13ab\left(a+bx^3\right)^{13/3}} + \frac{\left(54b^2c^2+9abcd+2a^2d^2\right)\int \frac{1}{\left(a+bx^3\right)^{10/3}}\,dx}{65a^2b^2}$$

$$= \frac{2(bc-ad)(3bc+ad)x}{65a^2b^2\left(a+bx^3\right)^{10/3}} + \frac{\left(54b^2c^2+9abcd+2a^2d^2\right)x}{455a^3b^2\left(a+bx^3\right)^{7/3}} + \frac{(bc-ad)x\left(c+dx^3\right)}{13ab\left(a+bx^3\right)^{13/3}} + \frac{\left(6\left(54b^2c^2+9abcd\right.\right.}{45}$$

$$= \frac{2(bc-ad)(3bc+ad)x}{65a^2b^2\left(a+bx^3\right)^{10/3}} + \frac{\left(54b^2c^2+9abcd+2a^2d^2\right)x}{455a^3b^2\left(a+bx^3\right)^{7/3}} + \frac{3\left(54b^2c^2+9abcd+2a^2d^2\right)x}{910a^4b^2\left(a+bx^3\right)^{4/3}} + \frac{(bc-ad)x}{13ab\left(a+\right.}$$

$$= \frac{2(bc-ad)(3bc+ad)x}{65a^2b^2\left(a+bx^3\right)^{10/3}} + \frac{\left(54b^2c^2+9abcd+2a^2d^2\right)x}{455a^3b^2\left(a+bx^3\right)^{7/3}} + \frac{3\left(54b^2c^2+9abcd+2a^2d^2\right)x}{910a^4b^2\left(a+bx^3\right)^{4/3}} + \frac{9\left(54b^2c^2\right.}{910a}$$

Mathematica [A] time = 5.07386, size = 138, normalized size = 0.65

$$\frac{x\left(9a^2b^2x^6\left(390c^2+39cdx^3+2d^2x^6\right)+39a^3bx^3\left(70c^2+15cdx^3+2d^2x^6\right)+65a^4\left(14c^2+7cdx^3+2d^2x^6\right)+81ab^3cx^9\left(26c\right.\right.}{910a^5\left(a+bx^3\right)^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^(16/3),x]
```

```
[Out] (x*(486*b^4*c^2*x^12 + 81*a*b^3*c*x^9*(26*c + d*x^3) + 65*a^4*(14*c^2 + 7*c
*d*x^3 + 2*d^2*x^6) + 39*a^3*b*x^3*(70*c^2 + 15*c*d*x^3 + 2*d^2*x^6) + 9*a^
2*b^2*x^6*(390*c^2 + 39*c*d*x^3 + 2*d^2*x^6)))/(910*a^5*(a + b*x^3)^(13/3))
```

Maple [A] time = 0.007, size = 156, normalized size = 0.7

$$\frac{x\left(18\,a^2b^2d^2x^{12}+81\,ab^3cdx^{12}+486\,b^4c^2x^{12}+78\,a^3bd^2x^9+351\,a^2b^2cdx^9+2106\,ab^3c^2x^9+130\,a^4d^2x^6+585\,a^3bcdx^6+3\right.}{910\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^2/(b*x^3+a)^(16/3),x)
```

```
[Out] 1/910*x*(18*a^2*b^2*d^2*x^12+81*a*b^3*c*d*x^12+486*b^4*c^2*x^12+78*a^3*b*d^
2*x^9+351*a^2*b^2*c*d*x^9+2106*a*b^3*c^2*x^9+130*a^4*d^2*x^6+585*a^3*b*c*d*
x^6+3510*a^2*b^2*c^2*x^6+455*a^4*c*d*x^3+2730*a^3*b*c^2*x^3+910*a^4*c^2)/(b
*x^3+a)^(13/3)/a^5
```

Maxima [A] time = 0.977636, size = 284, normalized size = 1.35

$$\frac{\left(35\,b^{2}-\frac{91\,(bx^{3}+a)b}{x^{3}}+\frac{65\,(bx^{3}+a)^{2}}{x^{6}}\right)d^{2}x^{13}}{455\,(bx^{3}+a)^{\frac{13}{3}}a^{3}}-\frac{\left(140\,b^{3}-\frac{546\,(bx^{3}+a)b^{2}}{x^{3}}+\frac{780\,(bx^{3}+a)^{2}b}{x^{6}}-\frac{455\,(bx^{3}+a)^{3}}{x^{9}}\right)cdx^{13}}{910\,(bx^{3}+a)^{\frac{13}{3}}a^{4}}+\frac{\left(35\,b^{4}-\frac{182\,(bx^{3}+a)b}{x^{3}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(16/3),x, algorithm="maxima")
```

```
[Out] 1/455*(35*b^2 - 91*(b*x^3 + a)*b/x^3 + 65*(b*x^3 + a)^2/x^6)*d^2*x^13/((b*x
^3 + a)^(13/3)*a^3) - 1/910*(140*b^3 - 546*(b*x^3 + a)*b^2/x^3 + 780*(b*x^3
 + a)^2*b/x^6 - 455*(b*x^3 + a)^3/x^9)*c*d*x^13/((b*x^3 + a)^(13/3)*a^4) +
1/455*(35*b^4 - 182*(b*x^3 + a)*b^3/x^3 + 390*(b*x^3 + a)^2*b^2/x^6 - 455*(
b*x^3 + a)^3*b/x^9 + 455*(b*x^3 + a)^4/x^12)*c^2*x^13/((b*x^3 + a)^(13/3)*a
^5)
```

Fricas [A] time = 1.84741, size = 433, normalized size = 2.05

$$\frac{\left(9\left(54\,b^{4}c^{2}+9\,ab^{3}cd+2\,a^{2}b^{2}d^{2}\right)x^{13}+39\left(54\,ab^{3}c^{2}+9\,a^{2}b^{2}cd+2\,a^{3}bd^{2}\right)x^{10}+65\left(54\,a^{2}b^{2}c^{2}+9\,a^{3}bcd+2\,a^{4}d^{2}\right)x^{7}+\cdots\right.}{910\left(a^{5}b^{5}x^{15}+5\,a^{6}b^{4}x^{12}+10\,a^{7}b^{3}x^{9}+10\,a^{8}b^{2}x^{6}+5\,a^{9}bx^{3}+a^{10}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(16/3),x, algorithm="fricas")
```

```
[Out] 1/910*(9*(54*b^4*c^2 + 9*a*b^3*c*d + 2*a^2*b^2*d^2)*x^13 + 39*(54*a*b^3*c^2
 + 9*a^2*b^2*c*d + 2*a^3*b*d^2)*x^10 + 65*(54*a^2*b^2*c^2 + 9*a^3*b*c*d + 2
*a^4*d^2)*x^7 + 910*a^4*c^2*x + 455*(6*a^3*b*c^2 + a^4*c*d)*x^4)*(b*x^3 + a
)^(2/3)/(a^5*b^5*x^15 + 5*a^6*b^4*x^12 + 10*a^7*b^3*x^9 + 10*a^8*b^2*x^6 +
5*a^9*b*x^3 + a^10)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**(16/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^{3}+c\right)^{2}}{\left(bx^{3}+a\right)^{\frac{16}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(16/3),x, algorithm="giac")

[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(16/3), x)
```

3.78 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{19/3}} \, dx$

Optimal. Leaf size=253

$$\frac{81x(a^2d^2+6abcd+45b^2c^2)}{7280a^6b^2\sqrt[3]{a+bx^3}}+\frac{27x(a^2d^2+6abcd+45b^2c^2)}{7280a^5b^2(a+bx^3)^{4/3}}+\frac{9x(a^2d^2+6abcd+45b^2c^2)}{3640a^4b^2(a+bx^3)^{7/3}}+\frac{x(a^2d^2+6abcd+45b^2}{520a^3b^2(a+bx^3)^{10/}}$$

```
[Out] ((b*c - a*d)*(15*b*c + 4*a*d)*x)/(208*a^2*b^2*(a + b*x^3)^(13/3)) + ((45*b^
2*c^2 + 6*a*b*c*d + a^2*d^2)*x)/(520*a^3*b^2*(a + b*x^3)^(10/3)) + (9*(45*b
^2*c^2 + 6*a*b*c*d + a^2*d^2)*x)/(3640*a^4*b^2*(a + b*x^3)^(7/3)) + (27*(45
*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x)/(7280*a^5*b^2*(a + b*x^3)^(4/3)) + (81*(
45*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x)/(7280*a^6*b^2*(a + b*x^3)^(1/3)) + ((b
*c - a*d)*x*(c + d*x^3))/(16*a*b*(a + b*x^3)^(16/3))
```

Rubi [A] time = 0.207149, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {413, 385, 192, 191}

$$\frac{81x(a^2d^2+6abcd+45b^2c^2)}{7280a^6b^2\sqrt[3]{a+bx^3}}+\frac{27x(a^2d^2+6abcd+45b^2c^2)}{7280a^5b^2(a+bx^3)^{4/3}}+\frac{9x(a^2d^2+6abcd+45b^2c^2)}{3640a^4b^2(a+bx^3)^{7/3}}+\frac{x(a^2d^2+6abcd+45b^2}{520a^3b^2(a+bx^3)^{10/}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^(19/3),x]
```

```
[Out] ((b*c - a*d)*(15*b*c + 4*a*d)*x)/(208*a^2*b^2*(a + b*x^3)^(13/3)) + ((45*b^
2*c^2 + 6*a*b*c*d + a^2*d^2)*x)/(520*a^3*b^2*(a + b*x^3)^(10/3)) + (9*(45*b
^2*c^2 + 6*a*b*c*d + a^2*d^2)*x)/(3640*a^4*b^2*(a + b*x^3)^(7/3)) + (27*(45
*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x)/(7280*a^5*b^2*(a + b*x^3)^(4/3)) + (81*(
45*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x)/(7280*a^6*b^2*(a + b*x^3)^(1/3)) + ((b
*c - a*d)*x*(c + d*x^3))/(16*a*b*(a + b*x^3)^(16/3))
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
```

```
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{(c+dx^3)^2}{(a+bx^3)^{19/3}}\,dx = \frac{(bc-ad)x(c+dx^3)}{16ab(a+bx^3)^{16/3}} + \frac{\int \frac{c(15bc+ad)+4d(3bc+ad)x^3}{(a+bx^3)^{16/3}}\,dx}{16ab}$$

$$= \frac{(bc-ad)(15bc+4ad)x}{208a^2b^2(a+bx^3)^{13/3}} + \frac{(bc-ad)x(c+dx^3)}{16ab(a+bx^3)^{16/3}} + \frac{(45b^2c^2+6abcd+a^2d^2)\int \frac{1}{(a+bx^3)^{13/3}}\,dx}{52a^2b^2}$$

$$= \frac{(bc-ad)(15bc+4ad)x}{208a^2b^2(a+bx^3)^{13/3}} + \frac{(45b^2c^2+6abcd+a^2d^2)x}{520a^3b^2(a+bx^3)^{10/3}} + \frac{(bc-ad)x(c+dx^3)}{16ab(a+bx^3)^{16/3}} + \frac{(9(45b^2c^2+6abcd}{52}$$

$$= \frac{(bc-ad)(15bc+4ad)x}{208a^2b^2(a+bx^3)^{13/3}} + \frac{(45b^2c^2+6abcd+a^2d^2)x}{520a^3b^2(a+bx^3)^{10/3}} + \frac{9(45b^2c^2+6abcd+a^2d^2)x}{3640a^4b^2(a+bx^3)^{7/3}} + \frac{(bc-ad)x(}{16ab(a+b}$$

$$= \frac{(bc-ad)(15bc+4ad)x}{208a^2b^2(a+bx^3)^{13/3}} + \frac{(45b^2c^2+6abcd+a^2d^2)x}{520a^3b^2(a+bx^3)^{10/3}} + \frac{9(45b^2c^2+6abcd+a^2d^2)x}{3640a^4b^2(a+bx^3)^{7/3}} + \frac{27(45b^2c^2}{7280a^5b}$$

$$= \frac{(bc-ad)(15bc+4ad)x}{208a^2b^2(a+bx^3)^{13/3}} + \frac{(45b^2c^2+6abcd+a^2d^2)x}{520a^3b^2(a+bx^3)^{10/3}} + \frac{9(45b^2c^2+6abcd+a^2d^2)x}{3640a^4b^2(a+bx^3)^{7/3}} + \frac{27(45b^2c^2}{7280a^5b}$$

Mathematica [A] time = 5.08619, size = 169, normalized size = 0.67

$$\frac{x\left(81a^2b^3x^9(520c^2+32cdx^3+d^2x^6)+144a^3b^2x^6(325c^2+39cdx^3+3d^2x^6)+156a^4bx^3(175c^2+40cdx^3+6d^2x^6)+520\right.}{7280a^6(a+bx^3)^{16/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^(19/3),x]
```

```
[Out] (x*(3645*b^5*c^2*x^15 + 486*a*b^4*c*x^12*(40*c + d*x^3) + 81*a^2*b^3*x^9*(5
20*c^2 + 32*c*d*x^3 + d^2*x^6) + 520*a^5*(14*c^2 + 7*c*d*x^3 + 2*d^2*x^6) +
 144*a^3*b^2*x^6*(325*c^2 + 39*c*d*x^3 + 3*d^2*x^6) + 156*a^4*b*x^3*(175*c^
2 + 40*c*d*x^3 + 6*d^2*x^6)))/(7280*a^6*(a + b*x^3)^(16/3))
```

Maple [A] time = 0.008, size = 197, normalized size = 0.8

$$\frac{x\left(81\,a^2b^3d^2x^{15}+486\,ab^4cdx^{15}+3645\,b^5c^2x^{15}+432\,a^3b^2d^2x^{12}+2592\,a^2b^3cdx^{12}+19440\,ab^4c^2x^{12}+936\,a^4bd^2x^9+561\right.}{728}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^2/(b*x^3+a)^(19/3),x)
```

```
[Out] 1/7280*x*(81*a^2*b^3*d^2*x^15+486*a*b^4*c*d*x^15+3645*b^5*c^2*x^15+432*a^3*
b^2*d^2*x^12+2592*a^2*b^3*c*d*x^12+19440*a*b^4*c^2*x^12+936*a^4*b*d^2*x^9+5
616*a^3*b^2*c*d*x^9+42120*a^2*b^3*c^2*x^9+1040*a^5*d^2*x^6+6240*a^4*b*c*d*x
^6+46800*a^3*b^2*c^2*x^6+3640*a^5*c*d*x^3+27300*a^4*b*c^2*x^3+7280*a^5*c^2)
/(b*x^3+a)^(16/3)/a^6
```

Maxima [A] time = 0.969908, size = 352, normalized size = 1.39

$$-\frac{\left(455\,b^3-\frac{1680\,(bx^3+a)b^2}{x^3}+\frac{2184\,(bx^3+a)^2b}{x^6}-\frac{1040\,(bx^3+a)^3}{x^9}\right)d^2x^{16}}{7280\,(bx^3+a)^{\frac{16}{3}}a^4}+\frac{\left(455\,b^4-\frac{2240\,(bx^3+a)b^3}{x^3}+\frac{4368\,(bx^3+a)^2b^2}{x^6}-\frac{4160\,(bx^3+a)^3b}{x^9}+\right.}{3640\,(bx^3+a)^{\frac{16}{3}}a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(19/3),x, algorithm="maxima")
```

```
[Out] -1/7280*(455*b^3 - 1680*(b*x^3 + a)*b^2/x^3 + 2184*(b*x^3 + a)^2*b/x^6 - 10
40*(b*x^3 + a)^3/x^9)*d^2*x^16/((b*x^3 + a)^(16/3)*a^4) + 1/3640*(455*b^4 -
 2240*(b*x^3 + a)*b^3/x^3 + 4368*(b*x^3 + a)^2*b^2/x^6 - 4160*(b*x^3 + a)^3
*b/x^9 + 1820*(b*x^3 + a)^4/x^12)*c*d*x^16/((b*x^3 + a)^(16/3)*a^5) - 1/145
6*(91*b^5 - 560*(b*x^3 + a)*b^4/x^3 + 1456*(b*x^3 + a)^2*b^3/x^6 - 2080*(b*
x^3 + a)^3*b^2/x^9 + 1820*(b*x^3 + a)^4*b/x^12 - 1456*(b*x^3 + a)^5/x^15)*c
^2*x^16/((b*x^3 + a)^(16/3)*a^6)
```

Fricas [A] time = 2.06553, size = 544, normalized size = 2.15

$$\frac{\left(81\left(45\,b^5c^2+6\,ab^4cd+a^2b^3d^2\right)x^{16}+432\left(45\,ab^4c^2+6\,a^2b^3cd+a^3b^2d^2\right)x^{13}+936\left(45\,a^2b^3c^2+6\,a^3b^2cd+a^4bd^2\right)x^{10}\right.}{7280\left(a^6b^6x^{18}+6\,a^7b^5x^{15}+15\,a^8b^4x^{12}+20\,a^9b^3x^9+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(19/3),x, algorithm="fricas")
```

```
[Out] 1/7280*(81*(45*b^5*c^2 + 6*a*b^4*c*d + a^2*b^3*d^2)*x^16 + 432*(45*a*b^4*c^
2 + 6*a^2*b^3*c*d + a^3*b^2*d^2)*x^13 + 936*(45*a^2*b^3*c^2 + 6*a^3*b^2*c*d
 + a^4*b*d^2)*x^10 + 7280*a^5*c^2*x + 1040*(45*a^3*b^2*c^2 + 6*a^4*b*c*d +
a^5*d^2)*x^7 + 1820*(15*a^4*b*c^2 + 2*a^5*c*d)*x^4)*(b*x^3 + a)^(2/3)/(a^6*
b^6*x^18 + 6*a^7*b^5*x^15 + 15*a^8*b^4*x^12 + 20*a^9*b^3*x^9 + 15*a^10*b^2*
x^6 + 6*a^11*b*x^3 + a^12)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**(19/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{19}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(19/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(19/3), x)
```

3.79 $\int \left(a+bx^3\right)^{7/3} \left(c+dx^3\right)^2 \, dx$

Optimal. Leaf size=135

$$\frac{a^2 x \sqrt[3]{a+bx^3} \left(2a^2d^2-14abcd+77b^2c^2\right) {}_2F_1\left(-\frac{7}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{77b^2 \sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx\left(a+bx^3\right)^{10/3}(17bc-4ad)}{154b^2}+\frac{dx\left(a+bx^3\right)^{10/3}\left(c+d\right.}{14b}$$

```
[Out] (d*(17*b*c - 4*a*d)*x*(a + b*x^3)^(10/3))/(154*b^2) + (d*x*(a + b*x^3)^(10/
3)*(c + d*x^3))/(14*b) + (a^2*(77*b^2*c^2 - 14*a*b*c*d + 2*a^2*d^2)*x*(a +
b*x^3)^(1/3)*Hypergeometric2F1[-7/3, 1/3, 4/3, -((b*x^3)/a)])/(77*b^2*(1 +
(b*x^3)/a)^(1/3))
```

Rubi [A] time = 0.0666526, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {416, 388, 246, 245}

$$\frac{a^2 x \sqrt[3]{a+bx^3} \left(2a^2d^2-14abcd+77b^2c^2\right) {}_2F_1\left(-\frac{7}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{77b^2 \sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx\left(a+bx^3\right)^{10/3}(17bc-4ad)}{154b^2}+\frac{dx\left(a+bx^3\right)^{10/3}\left(c+d\right.}{14b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(7/3)*(c + d*x^3)^2,x]
```

```
[Out] (d*(17*b*c - 4*a*d)*x*(a + b*x^3)^(10/3))/(154*b^2) + (d*x*(a + b*x^3)^(10/
3)*(c + d*x^3))/(14*b) + (a^2*(77*b^2*c^2 - 14*a*b*c*d + 2*a^2*d^2)*x*(a +
b*x^3)^(1/3)*Hypergeometric2F1[-7/3, 1/3, 4/3, -((b*x^3)/a)])/(77*b^2*(1 +
(b*x^3)/a)^(1/3))
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \left(a+bx^3\right)^{7/3}\left(c+dx^3\right)^2 dx = \frac{dx\left(a+bx^3\right)^{10/3}\left(c+dx^3\right)}{14b} + \frac{\int \left(a+bx^3\right)^{7/3}\left(c(14bc-ad)+d(17bc-4ad)x^3\right) dx}{14b}$$

$$= \frac{d(17bc-4ad)x\left(a+bx^3\right)^{10/3}}{154b^2} + \frac{dx\left(a+bx^3\right)^{10/3}\left(c+dx^3\right)}{14b} - \frac{(ad(17bc-4ad)-11bc(14bc\cdot}{154b^2}$$

$$= \frac{d(17bc-4ad)x\left(a+bx^3\right)^{10/3}}{154b^2} + \frac{dx\left(a+bx^3\right)^{10/3}\left(c+dx^3\right)}{14b} - \frac{\left(a^2(ad(17bc-4ad)-11bc(14\right.}{154}$$

$$= \frac{d(17bc-4ad)x\left(a+bx^3\right)^{10/3}}{154b^2} + \frac{dx\left(a+bx^3\right)^{10/3}\left(c+dx^3\right)}{14b} + \frac{a^2\left(77b^2c^2-14abcd+2a^2d^2\right.}{77b^2}$$

Mathematica [A] time = 2.21409, size = 177, normalized size = 1.31

$$\frac{ax\sqrt[3]{a+bx^3}\left(-9bx^3\text{Gamma}\left(-\frac{4}{3}\right)\left(c+dx^3\right)^2\text{HypergeometricPFQ}\left(\left\{-\frac{4}{3},\frac{4}{3},2\right\},\left\{1,\frac{13}{3}\right\},-\frac{bx^3}{a}\right)-3bx^3\text{Gamma}\left(-\frac{4}{3}\right)(11\right.}{280\text{Gamma}\left(-\frac{7}{3}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(7/3)*(c + d*x^3)^2,x]
```

```
[Out] (a*x*(a + b*x^3)^(1/3)*(20*a*(14*c^2 + 7*c*d*x^3 + 2*d^2*x^6)*Gamma[-7/3]*H
ypergeometric2F1[-7/3, 1/3, 10/3, -((b*x^3)/a)] - 3*b*x^3*(11*c^2 + 16*c*d*
x^3 + 5*d^2*x^6)*Gamma[-4/3]*Hypergeometric2F1[-4/3, 4/3, 13/3, -((b*x^3)/a
)] - 9*b*x^3*(c + d*x^3)^2*Gamma[-4/3]*HypergeometricPFQ[{-4/3, 4/3, 2}, {1
, 13/3}, -((b*x^3)/a)]))/(280*(1 + (b*x^3)/a)^(1/3)*Gamma[-7/3])
```

Maple [F] time = 0.216, size = 0, normalized size = 0.

$$\int \left(bx^3+a\right)^{\frac{7}{3}}\left(dx^3+c\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(7/3)*(d*x^3+c)^2,x)
```

```
[Out] int((b*x^3+a)^(7/3)*(d*x^3+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3+a\right)^{\frac{7}{3}}\left(dx^3+c\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(7/3)*(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(7/3)*(d*x^3 + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2d^2x^{12} + 2\left(b^2cd + abd^2\right)x^9 + \left(b^2c^2 + 4\,abcd + a^2d^2\right)x^6 + a^2c^2 + 2\left(abc^2 + a^2cd\right)x^3\right)\left(bx^3 + a\right)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(7/3)*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*d^2*x^12 + 2*(b^2*c*d + a*b*d^2)*x^9 + (b^2*c^2 + 4*a*b*c*d +
 a^2*d^2)*x^6 + a^2*c^2 + 2*(a*b*c^2 + a^2*c*d)*x^3)*(b*x^3 + a)^(1/3), x)
```

Sympy [C] time = 13.7693, size = 418, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(7/3)*(d*x**3+c)**2,x)
```

```
[Out] a**(7/3)*c**2*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(4/3)) + 2*a**(7/3)*c*d*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/
3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(7/3)*d**2*x**7*gamma(7/
3)*hyper((-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) +
2*a**(4/3)*b*c**2*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,), b*x**3*exp_pol
ar(I*pi)/a)/(3*gamma(7/3)) + 4*a**(4/3)*b*c*d*x**7*gamma(7/3)*hyper((-1/3,
7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + 2*a**(4/3)*b*d**
2*x**10*gamma(10/3)*hyper((-1/3, 10/3), (13/3,), b*x**3*exp_polar(I*pi)/a)/
(3*gamma(13/3)) + a**(1/3)*b**2*c**2*x**7*gamma(7/3)*hyper((-1/3, 7/3), (10
/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + 2*a**(1/3)*b**2*c*d*x**10
*gamma(10/3)*hyper((-1/3, 10/3), (13/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamm
a(13/3)) + a**(1/3)*b**2*d**2*x**13*gamma(13/3)*hyper((-1/3, 13/3), (16/3,)
, b*x**3*exp_polar(I*pi)/a)/(3*gamma(16/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3 + a\right)^{\frac{7}{3}}\left(dx^3 + c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(7/3)*(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(7/3)*(d*x^3 + c)^2, x)
```

3.80 $\int \left(a+bx^3\right)^{4/3} \left(c+dx^3\right)^2 \, dx$

Optimal. Leaf size=133

$$\frac{ax\sqrt[3]{a+bx^3}\left(2a^2d^2-11abcd+44b^2c^2\right){}_2F_1\left(-\frac{4}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{44b^2\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx\left(a+bx^3\right)^{7/3}(7bc-2ad)}{44b^2}+\frac{dx\left(a+bx^3\right)^{7/3}\left(c+dx^3\right)}{11b}$$

```
[Out] (d*(7*b*c - 2*a*d)*x*(a + b*x^3)^(7/3))/(44*b^2) + (d*x*(a + b*x^3)^(7/3)*(
c + d*x^3))/(11*b) + (a*(44*b^2*c^2 - 11*a*b*c*d + 2*a^2*d^2)*x*(a + b*x^3)
^(1/3)*Hypergeometric2F1[-4/3, 1/3, 4/3, -((b*x^3)/a)])/(44*b^2*(1 + (b*x^3
)/a)^(1/3))
```

Rubi [A] time = 0.0771186, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {416, 388, 246, 245}

$$\frac{ax\sqrt[3]{a+bx^3}\left(2a^2d^2-11abcd+44b^2c^2\right){}_2F_1\left(-\frac{4}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{44b^2\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx\left(a+bx^3\right)^{7/3}(7bc-2ad)}{44b^2}+\frac{dx\left(a+bx^3\right)^{7/3}\left(c+dx^3\right)}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(4/3)*(c + d*x^3)^2,x]
```

```
[Out] (d*(7*b*c - 2*a*d)*x*(a + b*x^3)^(7/3))/(44*b^2) + (d*x*(a + b*x^3)^(7/3)*(
c + d*x^3))/(11*b) + (a*(44*b^2*c^2 - 11*a*b*c*d + 2*a^2*d^2)*x*(a + b*x^3)
^(1/3)*Hypergeometric2F1[-4/3, 1/3, 4/3, -((b*x^3)/a)])/(44*b^2*(1 + (b*x^3
)/a)^(1/3))
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^2\,dx = \frac{dx\left(a+bx^3\right)^{7/3}\left(c+dx^3\right)}{11b} + \frac{\int \left(a+bx^3\right)^{4/3}\left(c(11bc-ad)+2d(7bc-2ad)x^3\right)dx}{11b}$$

$$= \frac{d(7bc-2ad)x\left(a+bx^3\right)^{7/3}}{44b^2} + \frac{dx\left(a+bx^3\right)^{7/3}\left(c+dx^3\right)}{11b} - \frac{(2ad(7bc-2ad)-8bc(11bc-}{88b^2}$$

$$= \frac{d(7bc-2ad)x\left(a+bx^3\right)^{7/3}}{44b^2} + \frac{dx\left(a+bx^3\right)^{7/3}\left(c+dx^3\right)}{11b} - \frac{\left(a(2ad(7bc-2ad)-8bc(11b\right.}{88b}$$

$$= \frac{d(7bc-2ad)x\left(a+bx^3\right)^{7/3}}{44b^2} + \frac{dx\left(a+bx^3\right)^{7/3}\left(c+dx^3\right)}{11b} + \frac{a\left(44b^2c^2-11abcd+2a^2d^2\right)}{44b^2\sqrt[3]{}}$$

Mathematica [A] time = 3.31623, size = 176, normalized size = 1.32

$$\frac{x\sqrt[3]{a+bx^3}\left(-9bx^3\text{Gamma}\left(-\frac{1}{3}\right)\left(c+dx^3\right)^2\text{HypergeometricPFQ}\left(\left\{-\frac{1}{3},\frac{4}{3},2\right\},\left\{1,\frac{13}{3}\right\},-\frac{bx^3}{a}\right)-3bx^3\text{Gamma}\left(-\frac{1}{3}\right)\left(\right.}{280\text{Gamma}\left(-\frac{4}{3}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(4/3)*(c + d*x^3)^2,x]
```

```
[Out] (x*(a + b*x^3)^(1/3)*(20*a*(14*c^2 + 7*c*d*x^3 + 2*d^2*x^6)*Gamma[-4/3]*Hyp
ergeometric2F1[-4/3, 1/3, 10/3, -((b*x^3)/a)] - 3*b*x^3*(11*c^2 + 16*c*d*x^
3 + 5*d^2*x^6)*Gamma[-1/3]*Hypergeometric2F1[-1/3, 4/3, 13/3, -((b*x^3)/a)]
 - 9*b*x^3*(c + d*x^3)^2*Gamma[-1/3]*HypergeometricPFQ[{-1/3, 4/3, 2}, {1,
13/3}, -((b*x^3)/a)]))/(280*(1 + (b*x^3)/a)^(1/3)*Gamma[-4/3])
```

Maple [F] time = 0.23, size = 0, normalized size = 0.

$$\int \left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)*(d*x^3+c)^2,x)
```

```
[Out] int((b*x^3+a)^(4/3)*(d*x^3+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)*(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)*(d*x^3 + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bd^2x^9 + \left(2\,bcd + ad^2\right)x^6 + \left(bc^2 + 2\,acd\right)x^3 + ac^2\right)\left(bx^3 + a\right)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] integral((b*d^2*x^9 + (2*b*c*d + a*d^2)*x^6 + (b*c^2 + 2*a*c*d)*x^3 + a*c^2
)*(b*x^3 + a)^(1/3), x)
```

Sympy [C] time = 6.09875, size = 270, normalized size = 2.03

$$\frac{a^{\frac{4}{3}}c^2x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3}, \frac{1}{3} \\ \frac{4}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{2a^{\frac{4}{3}}cdx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3}, \frac{4}{3} \\ \frac{7}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{a^{\frac{4}{3}}d^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{3}, \frac{7}{3} \\ \frac{10}{3}\end{matrix}\middle|\frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{\sqrt[3]{a}bc^2x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\cdots\right.}{3\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)*(d*x**3+c)**2,x)
```

```
[Out] a**(4/3)*c**2*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(4/3)) + 2*a**(4/3)*c*d*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/
3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(4/3)*d**2*x**7*gamma(7/
3)*hyper((-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) +
a**(1/3)*b*c**2*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,), b*x**3*exp_polar
(I*pi)/a)/(3*gamma(7/3)) + 2*a**(1/3)*b*c*d*x**7*gamma(7/3)*hyper((-1/3, 7/
3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + a**(1/3)*b*d**2*x*
*10*gamma(10/3)*hyper((-1/3, 10/3), (13/3,), b*x**3*exp_polar(I*pi)/a)/(3*g
amma(13/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3 + a\right)^{\frac{4}{3}}\left(dx^3 + c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)*(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)*(d*x^3 + c)^2, x)
```

3.81 $\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)^2\,dx$

Optimal. Leaf size=131

$$\frac{x\sqrt[3]{a+bx^3}\left(a^2d^2-4abcd+10b^2c^2\right)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{10b^2\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx\left(a+bx^3\right)^{4/3}(11bc-4ad)}{40b^2}+\frac{dx\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}{8b}$$

```
[Out] (d*(11*b*c - 4*a*d)*x*(a + b*x^3)^(4/3))/(40*b^2) + (d*x*(a + b*x^3)^(4/3)*
(c + d*x^3))/(8*b) + ((10*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x*(a + b*x^3)^(1/3
)*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^3)/a)])/(10*b^2*(1 + (b*x^3)/a)^
(1/3))
```

Rubi [A] time = 0.061771, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {416, 388, 246, 245}

$$\frac{x\sqrt[3]{a+bx^3}\left(a^2d^2-4abcd+10b^2c^2\right)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{10b^2\sqrt[3]{\frac{bx^3}{a}+1}}+\frac{dx\left(a+bx^3\right)^{4/3}(11bc-4ad)}{40b^2}+\frac{dx\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)*(c + d*x^3)^2,x]
```

```
[Out] (d*(11*b*c - 4*a*d)*x*(a + b*x^3)^(4/3))/(40*b^2) + (d*x*(a + b*x^3)^(4/3)*
(c + d*x^3))/(8*b) + ((10*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x*(a + b*x^3)^(1/3
)*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b*x^3)/a)])/(10*b^2*(1 + (b*x^3)/a)^
(1/3))
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \sqrt[3]{a+bx^3}\left(c+dx^3\right)^2\,dx = \frac{dx\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}{8b}+\frac{\int \sqrt[3]{a+bx^3}\left(c(8bc-ad)+d(11bc-4ad)x^3\right)dx}{8b}$$
$$=\frac{d(11bc-4ad)x\left(a+bx^3\right)^{4/3}}{40b^2}+\frac{dx\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}{8b}+\frac{\left(10b^2c^2-4abcd+a^2d^2\right)\int \sqrt[3]{a+b}}{10b^2}$$
$$=\frac{d(11bc-4ad)x\left(a+bx^3\right)^{4/3}}{40b^2}+\frac{dx\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}{8b}+\frac{\left(\left(10b^2c^2-4abcd+a^2d^2\right)\sqrt[3]{a+b}\right.}{10b^2\sqrt[3]{1+\frac{bx^3}{a}}}$$
$$=\frac{d(11bc-4ad)x\left(a+bx^3\right)^{4/3}}{40b^2}+\frac{dx\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}{8b}+\frac{\left(10b^2c^2-4abcd+a^2d^2\right)x\sqrt[3]{a+b}}{10b^2\sqrt[3]{1+\frac{bx}{a}}}$$

Mathematica [A] time = 2.28175, size = 179, normalized size = 1.37

$$\frac{x\sqrt[3]{a+bx^3}\left(-9bx^3\text{Gamma}\left(\frac{2}{3}\right)\left(c+dx^3\right)^2\text{HypergeometricPFQ}\left(\left\{\frac{2}{3},\frac{4}{3},2\right\},\left\{1,\frac{13}{3}\right\},-\frac{bx^3}{a}\right)-3bx^3\text{Gamma}\left(\frac{2}{3}\right)\left(11c^2+\right.\right.}{280a\text{Gamma}\left(-\frac{1}{3}\right)\sqrt[3]{\frac{b}{}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)*(c + d*x^3)^2,x]
```

```
[Out] (x*(a + b*x^3)^(1/3)*(20*a*(14*c^2 + 7*c*d*x^3 + 2*d^2*x^6)*Gamma[-1/3]*Hyp
ergeometric2F1[-1/3, 1/3, 10/3, -((b*x^3)/a)] - 3*b*x^3*(11*c^2 + 16*c*d*x^
3 + 5*d^2*x^6)*Gamma[2/3]*Hypergeometric2F1[2/3, 4/3, 13/3, -((b*x^3)/a)] -
 9*b*x^3*(c + d*x^3)^2*Gamma[2/3]*HypergeometricPFQ[{2/3, 4/3, 2}, {1, 13/3
}, -((b*x^3)/a)]))/(280*a*(1 + (b*x^3)/a)^(1/3)*Gamma[-1/3])
```

Maple [F] time = 0.222, size = 0, normalized size = 0.

$$\int \sqrt[3]{bx^3+a}\left(dx^3+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)*(d*x^3+c)^2,x)
```

```
[Out] int((b*x^3+a)^(1/3)*(d*x^3+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)*(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*(d*x^3 + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d^2x^6 + 2\,cdx^3 + c^2\right)\left(bx^3 + a\right)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] integral((d^2*x^6 + 2*c*d*x^3 + c^2)*(b*x^3 + a)^(1/3), x)
```

Sympy [C] time = 2.9032, size = 131, normalized size = 1.

$$\frac{\sqrt[3]{a}c^2x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{2\sqrt[3]{a}cdx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{\sqrt[3]{a}d^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{7}{3} \\ \frac{10}{3} \end{matrix}\middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)*(d*x**3+c)**2,x)
```

```
[Out] a**(1/3)*c**2*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi
)/a)/(3*gamma(4/3)) + 2*a**(1/3)*c*d*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/
3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(1/3)*d**2*x**7*gamma(7/
3)*hyper((-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3 + a\right)^{\frac{1}{3}}\left(dx^3 + c\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)*(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)*(d*x^3 + c)^2, x)
```

3.82 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{2/3}}\,dx$

Optimal. Leaf size=132

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\left(2a^2d^2-5abcd+5b^2c^2\right){}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5b^2\left(a+bx^3\right)^{2/3}}+\frac{2dx\sqrt[3]{a+bx^3}(2bc-ad)}{5b^2}+\frac{dx\sqrt[3]{a+bx^3}\left(c+dx^3\right)}{5b}$$

```
[Out] (2*d*(2*b*c - a*d)*x*(a + b*x^3)^(1/3))/(5*b^2) + (d*x*(a + b*x^3)^(1/3)*(c
 + d*x^3))/(5*b) + ((5*b^2*c^2 - 5*a*b*c*d + 2*a^2*d^2)*x*(1 + (b*x^3)/a)^(
2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(5*b^2*(a + b*x^3)^(2/
3))
```

Rubi [A] time = 0.0754575, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {416, 388, 246, 245}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\left(2a^2d^2-5abcd+5b^2c^2\right){}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5b^2\left(a+bx^3\right)^{2/3}}+\frac{2dx\sqrt[3]{a+bx^3}(2bc-ad)}{5b^2}+\frac{dx\sqrt[3]{a+bx^3}\left(c+dx^3\right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^(2/3),x]
```

```
[Out] (2*d*(2*b*c - a*d)*x*(a + b*x^3)^(1/3))/(5*b^2) + (d*x*(a + b*x^3)^(1/3)*(c
 + d*x^3))/(5*b) + ((5*b^2*c^2 - 5*a*b*c*d + 2*a^2*d^2)*x*(1 + (b*x^3)/a)^(
2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(5*b^2*(a + b*x^3)^(2/
3))
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^{2/3}}\,dx = \frac{dx\sqrt[3]{a+bx^3}\left(c+dx^3\right)}{5b} + \frac{\int \frac{c(5bc-ad)+4d(2bc-ad)x^3}{\left(a+bx^3\right)^{2/3}}\,dx}{5b}$$

$$= \frac{2d(2bc-ad)x\sqrt[3]{a+bx^3}}{5b^2} + \frac{dx\sqrt[3]{a+bx^3}\left(c+dx^3\right)}{5b} - \frac{\left(4ad(2bc-ad)-2bc(5bc-ad)\right)\int \frac{1}{\left(a+bx^3\right)^{2/3}}\,dx}{10b^2}$$

$$= \frac{2d(2bc-ad)x\sqrt[3]{a+bx^3}}{5b^2} + \frac{dx\sqrt[3]{a+bx^3}\left(c+dx^3\right)}{5b} - \frac{\left(\left(4ad(2bc-ad)-2bc(5bc-ad)\right)\left(1+\frac{bx^3}{a}\right)^{2/3}\right.}{10b^2\left(a+bx^3\right)^{2/3}}$$

$$= \frac{2d(2bc-ad)x\sqrt[3]{a+bx^3}}{5b^2} + \frac{dx\sqrt[3]{a+bx^3}\left(c+dx^3\right)}{5b} + \frac{\left(5b^2c^2-5abcd+2a^2d^2\right)x\left(1+\frac{bx^3}{a}\right)^{2/3}{}_2F_1\left(\frac{1}{3}\right.}{5b^2\left(a+bx^3\right)^{2/3}}$$

Mathematica [B] time = 4.10862, size = 304, normalized size = 2.3

$$-\frac{x\text{Gamma}\left(\frac{4}{3}\right)\left(\frac{bx^3}{a}+1\right)^{2/3}\left(81bx^3\text{Gamma}\left(\frac{10}{3}\right)\left(c+dx^3\right)^2\text{HypergeometricPFQ}\left(\left\{\frac{4}{3},\frac{5}{3},2\right\},\left\{1,\frac{13}{3}\right\},-\frac{bx^3}{a}\right)-270a\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^(2/3),x]
```

```
[Out] -(x*(1 + (b*x^3)/a)^(2/3)*Gamma[4/3]*(-3920*a*c^2*Gamma[1/3] - 1960*a*c*d*x
^3*Gamma[1/3] - 560*a*d^2*x^6*Gamma[1/3] + 3780*a*c^2*Gamma[10/3] + 1890*a*
c*d*x^3*Gamma[10/3] + 540*a*d^2*x^6*Gamma[10/3] - 270*a*(14*c^2 + 7*c*d*x^3
 + 2*d^2*x^6)*Gamma[10/3]*Hypergeometric2F1[1/3, 2/3, 10/3, -((b*x^3)/a)] +
 297*b*c^2*x^3*Gamma[10/3]*Hypergeometric2F1[4/3, 5/3, 13/3, -((b*x^3)/a)]
+ 432*b*c*d*x^6*Gamma[10/3]*Hypergeometric2F1[4/3, 5/3, 13/3, -((b*x^3)/a)]
 + 135*b*d^2*x^9*Gamma[10/3]*Hypergeometric2F1[4/3, 5/3, 13/3, -((b*x^3)/a)
] + 81*b*x^3*(c + d*x^3)^2*Gamma[10/3]*HypergeometricPFQ[{4/3, 5/3, 2}, {1,
 13/3}, -((b*x^3)/a)]))/(1260*a*(a + b*x^3)^(2/3)*Gamma[1/3]*Gamma[10/3])
```

Maple [F] time = 0.217, size = 0, normalized size = 0.

$$\int \left(dx^3+c\right)^2\left(bx^3+a\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^2/(b*x^3+a)^(2/3),x)
```

```
[Out] int((d*x^3+c)^2/(b*x^3+a)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{d^2x^6+2\,cdx^3+c^2}{(bx^3+a)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^6 + 2*c*d*x^3 + c^2)/(b*x^3 + a)^(2/3), x)
```

Sympy [C] time = 2.88902, size = 126, normalized size = 0.95

$$\frac{c^2x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{2}{3}\\ \frac{4}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)}+\frac{2cdx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{2}{3},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{7}{3}\right)}+\frac{d^2x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{2}{3},\frac{7}{3}\\ \frac{10}{3}\end{matrix}\right|\frac{bx^3e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**(2/3),x)
```

```
[Out] c**2*x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a*
*(2/3)*gamma(4/3)) + 2*c*d*x**4*gamma(4/3)*hyper((2/3, 4/3), (7/3,), b*x**3
*exp_polar(I*pi)/a)/(3*a**(2/3)*gamma(7/3)) + d**2*x**7*gamma(7/3)*hyper((2
/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2/3)*gamma(10/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(2/3), x)
```

3.83 $\int \frac{(c+dx^3)^2}{(a+bx^3)^{5/3}}\,dx$

Optimal. Leaf size=146

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\left(-2a^2d^2+2abcd+b^2c^2\right){}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2ab^2\left(a+bx^3\right)^{2/3}}-\frac{dx\sqrt[3]{a+bx^3}(bc-2ad)}{2ab^2}+\frac{x\left(c+dx^3\right)(bc-ad)}{2ab\left(a+bx^3\right)^{2/3}}$$

```
[Out] -(d*(b*c - 2*a*d)*x*(a + b*x^3)^(1/3))/(2*a*b^2) + ((b*c - a*d)*x*(c + d*x^
3))/(2*a*b*(a + b*x^3)^(2/3)) + ((b^2*c^2 + 2*a*b*c*d - 2*a^2*d^2)*x*(1 + (
b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*a*b^2*(a
 + b*x^3)^(2/3))
```

Rubi [A] time = 0.0951581, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {413, 388, 246, 245}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\left(-2a^2d^2+2abcd+b^2c^2\right){}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{2ab^2\left(a+bx^3\right)^{2/3}}-\frac{dx\sqrt[3]{a+bx^3}(bc-2ad)}{2ab^2}+\frac{x\left(c+dx^3\right)(bc-ad)}{2ab\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^(5/3),x]
```

```
[Out] -(d*(b*c - 2*a*d)*x*(a + b*x^3)^(1/3))/(2*a*b^2) + ((b*c - a*d)*x*(c + d*x^
3))/(2*a*b*(a + b*x^3)^(2/3)) + ((b^2*c^2 + 2*a*b*c*d - 2*a^2*d^2)*x*(1 + (
b*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*a*b^2*(a
 + b*x^3)^(2/3))
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^{5/3}}\,dx = \frac{(bc-ad)x\left(c+dx^3\right)}{2ab\left(a+bx^3\right)^{2/3}} + \frac{\int \frac{c(bc+ad)-2d(bc-2ad)x^3}{\left(a+bx^3\right)^{2/3}}\,dx}{2ab}$$

$$= -\frac{d(bc-2ad)x\sqrt[3]{a+bx^3}}{2ab^2} + \frac{(bc-ad)x\left(c+dx^3\right)}{2ab\left(a+bx^3\right)^{2/3}} + -\frac{(-2ad(bc-2ad)-2bc(bc+ad))\int \frac{1}{\left(a+bx^3\right)^{2/3}}\,d}{4ab^2}$$

$$= -\frac{d(bc-2ad)x\sqrt[3]{a+bx^3}}{2ab^2} + \frac{(bc-ad)x\left(c+dx^3\right)}{2ab\left(a+bx^3\right)^{2/3}} + -\frac{\Big((-2ad(bc-2ad)-2bc(bc+ad))\left(1+\frac{bx^3}{a}\right)^{2/3}}{4ab^2\left(a+bx^3\right)^{2/3}}$$

$$= -\frac{d(bc-2ad)x\sqrt[3]{a+bx^3}}{2ab^2} + \frac{(bc-ad)x\left(c+dx^3\right)}{2ab\left(a+bx^3\right)^{2/3}} + \frac{\left(b^2c^2+2abcd-2a^2d^2\right)x\left(1+\frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3},\frac{2}{3}\right.}{2ab^2\left(a+bx^3\right)^{2/3}}$$

Mathematica [A] time = 2.52533, size = 171, normalized size = 1.17

$$\frac{x\text{Gamma}\left(\frac{2}{3}\right)\left(\frac{bx^3}{a}+1\right)^{2/3}\left(-3bx^3\left(c+dx^3\right)^2\text{HypergeometricPFQ}\left(\left\{\frac{4}{3},2,\frac{8}{3}\right\},\left\{1,\frac{13}{3}\right\},-\frac{bx^3}{a}\right)-bx^3\left(11c^2+16cdx^3+\right.\right.}{84a^2\text{Gamma}\left(\frac{5}{3}\right)\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^(5/3),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*Gamma[2/3]*(4*a*(14*c^2 + 7*c*d*x^3 + 2*d^2*x^6)*H
ypergeometric2F1[1/3, 5/3, 10/3, -((b*x^3)/a)] - b*x^3*(11*c^2 + 16*c*d*x^3
 + 5*d^2*x^6)*Hypergeometric2F1[4/3, 8/3, 13/3, -((b*x^3)/a)] - 3*b*x^3*(c
+ d*x^3)^2*HypergeometricPFQ[{4/3, 2, 8/3}, {1, 13/3}, -((b*x^3)/a)]))/(84*
a^2*(a + b*x^3)^(2/3)*Gamma[5/3])
```

Maple [F] time = 0.427, size = 0, normalized size = 0.

$$\int \left(dx^3+c\right)^2\left(bx^3+a\right)^{-\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^2/(b*x^3+a)^(5/3),x)
```

```
[Out] int((d*x^3+c)^2/(b*x^3+a)^(5/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^2}{\left(bx^3+a\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(5/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(d^2x^6+2\,cdx^3+c^2\right)\left(bx^3+a\right)^{\frac{1}{3}}}{b^2x^6+2\,abx^3+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(5/3),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^6 + 2*c*d*x^3 + c^2)*(b*x^3 + a)^(1/3)/(b^2*x^6 + 2*a*b*x^3
 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**(5/3),x)
```

```
[Out] Integral((c + d*x**3)**2/(a + b*x**3)**(5/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^2}{\left(bx^3+a\right)^{\frac{5}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(5/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(5/3), x)
```

3.84 $\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^{8/3}}\,dx$

Optimal. Leaf size=147

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\left(2a^2d^2+abcd+2b^2c^2\right){}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5a^2b^2\left(a+bx^3\right)^{2/3}}+\frac{2x\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)}{5\left(a+bx^3\right)^{2/3}}+\frac{x\left(c+dx^3\right)(bc-ad)}{5ab\left(a+bx^3\right)^{5/3}}$$

```
[Out] (2*(c^2/a^2 - d^2/b^2)*x)/(5*(a + b*x^3)^(2/3)) + ((b*c - a*d)*x*(c + d*x^3
))/(5*a*b*(a + b*x^3)^(5/3)) + ((2*b^2*c^2 + a*b*c*d + 2*a^2*d^2)*x*(1 + (b
*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(5*a^2*b^2*(
a + b*x^3)^(2/3))
```

Rubi [A] time = 0.0877303, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {413, 385, 246, 245}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}\left(2a^2d^2+abcd+2b^2c^2\right){}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5a^2b^2\left(a+bx^3\right)^{2/3}}+\frac{2x\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)}{5\left(a+bx^3\right)^{2/3}}+\frac{x\left(c+dx^3\right)(bc-ad)}{5ab\left(a+bx^3\right)^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^2/(a + b*x^3)^(8/3),x]
```

```
[Out] (2*(c^2/a^2 - d^2/b^2)*x)/(5*(a + b*x^3)^(2/3)) + ((b*c - a*d)*x*(c + d*x^3
))/(5*a*b*(a + b*x^3)^(5/3)) + ((2*b^2*c^2 + a*b*c*d + 2*a^2*d^2)*x*(1 + (b
*x^3)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x^3)/a)])/(5*a^2*b^2*(
a + b*x^3)^(2/3))
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(c+dx^3\right)^2}{\left(a+bx^3\right)^{8/3}}\,dx &= \frac{(bc-ad)x\left(c+dx^3\right)}{5ab\left(a+bx^3\right)^{5/3}}+\frac{\int \frac{c(4bc+ad)+d(bc+4ad)x^3}{\left(a+bx^3\right)^{5/3}}\,dx}{5ab}\\
&= \frac{2\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)x}{5\left(a+bx^3\right)^{2/3}}+\frac{(bc-ad)x\left(c+dx^3\right)}{5ab\left(a+bx^3\right)^{5/3}}+\frac{\left(2b^2c^2+abcd+2a^2d^2\right)\int \frac{1}{\left(a+bx^3\right)^{2/3}}\,dx}{5a^2b^2}\\
&= \frac{2\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)x}{5\left(a+bx^3\right)^{2/3}}+\frac{(bc-ad)x\left(c+dx^3\right)}{5ab\left(a+bx^3\right)^{5/3}}+\frac{\left(\left(2b^2c^2+abcd+2a^2d^2\right)\left(1+\frac{bx^3}{a}\right)^{2/3}\right)\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}}\,dx}{5a^2b^2\left(a+bx^3\right)^{2/3}}\\
&= \frac{2\left(\frac{c^2}{a^2}-\frac{d^2}{b^2}\right)x}{5\left(a+bx^3\right)^{2/3}}+\frac{(bc-ad)x\left(c+dx^3\right)}{5ab\left(a+bx^3\right)^{5/3}}+\frac{\left(2b^2c^2+abcd+2a^2d^2\right)x\left(1+\frac{bx^3}{a}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{bx^3}{a}\right)}{5a^2b^2\left(a+bx^3\right)^{2/3}}
\end{aligned}$$

Mathematica [A] time = 4.47114, size = 171, normalized size = 1.16

$$\frac{x\text{Gamma}\left(\frac{2}{3}\right)\left(\frac{bx^3}{a}+1\right)^{2/3}\left(-6bx^3\left(c+dx^3\right)^2\text{HypergeometricPFQ}\left(\left\{\frac{4}{3},2,\frac{11}{3}\right\},\left\{1,\frac{13}{3}\right\},-\frac{bx^3}{a}\right)-2bx^3\left(11c^2+16cdx^3+\ldots\right.\right.}{63a^3\text{Gamma}\left(\frac{8}{3}\right)\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3)^(8/3),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*Gamma[2/3]*(5*a*(14*c^2 + 7*c*d*x^3 + 2*d^2*x^6)*H
ypergeometric2F1[1/3, 8/3, 10/3, -((b*x^3)/a)] - 2*b*x^3*(11*c^2 + 16*c*d*x
^3 + 5*d^2*x^6)*Hypergeometric2F1[4/3, 11/3, 13/3, -((b*x^3)/a)] - 6*b*x^3*
(c + d*x^3)^2*HypergeometricPFQ[{4/3, 2, 11/3}, {1, 13/3}, -((b*x^3)/a)]))/
(63*a^3*(a + b*x^3)^(2/3)*Gamma[8/3])
```

Maple [F] time = 0.369, size = 0, normalized size = 0.

$$\int \left(dx^3+c\right)^2\left(bx^3+a\right)^{-\frac{8}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^2/(b*x^3+a)^(8/3),x)
```

```
[Out] int((d*x^3+c)^2/(b*x^3+a)^(8/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(8/3),x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(8/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(d^2x^6+2\,cdx^3+c^2)(bx^3+a)^{\frac{1}{3}}}{b^3x^9+3\,ab^2x^6+3\,a^2bx^3+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(8/3),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^6 + 2*c*d*x^3 + c^2)*(b*x^3 + a)^(1/3)/(b^3*x^9 + 3*a*b^2*x
^6 + 3*a^2*b*x^3 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**(8/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^3+c)^2}{(bx^3+a)^{\frac{8}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^2/(b*x^3+a)^(8/3),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^2/(b*x^3 + a)^(8/3), x)
```

3.85 $\int \frac{(a+bx^3)^3}{(c+dx^3)^{13/3}}\,dx$

Optimal. Leaf size=109

$$\frac{27a^2x(a+bx^3)}{140c^3(c+dx^3)^{4/3}}+\frac{81a^3x}{140c^4\sqrt[3]{c+dx^3}}+\frac{9ax(a+bx^3)^2}{70c^2(c+dx^3)^{7/3}}+\frac{x(a+bx^3)^3}{10c(c+dx^3)^{10/3}}$$

```
[Out] (x*(a + b*x^3)^3)/(10*c*(c + d*x^3)^(10/3)) + (9*a*x*(a + b*x^3)^2)/(70*c^2
*(c + d*x^3)^(7/3)) + (27*a^2*x*(a + b*x^3))/(140*c^3*(c + d*x^3)^(4/3)) +
(81*a^3*x)/(140*c^4*(c + d*x^3)^(1/3))
```

Rubi [A] time = 0.0357897, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {378, 191}

$$\frac{27a^2x(a+bx^3)}{140c^3(c+dx^3)^{4/3}}+\frac{81a^3x}{140c^4\sqrt[3]{c+dx^3}}+\frac{9ax(a+bx^3)^2}{70c^2(c+dx^3)^{7/3}}+\frac{x(a+bx^3)^3}{10c(c+dx^3)^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^3/(c + d*x^3)^(13/3),x]
```

```
[Out] (x*(a + b*x^3)^3)/(10*c*(c + d*x^3)^(10/3)) + (9*a*x*(a + b*x^3)^2)/(70*c^2
*(c + d*x^3)^(7/3)) + (27*a^2*x*(a + b*x^3))/(140*c^3*(c + d*x^3)^(4/3)) +
(81*a^3*x)/(140*c^4*(c + d*x^3)^(1/3))
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^3}{\left(c+dx^3\right)^{13/3}}\,dx = \frac{x\left(a+bx^3\right)^3}{10c\left(c+dx^3\right)^{10/3}} + \frac{(9a)\int \frac{\left(a+bx^3\right)^2}{\left(c+dx^3\right)^{10/3}}\,dx}{10c}$$

$$= \frac{x\left(a+bx^3\right)^3}{10c\left(c+dx^3\right)^{10/3}} + \frac{9ax\left(a+bx^3\right)^2}{70c^2\left(c+dx^3\right)^{7/3}} + \frac{\left(27a^2\right)\int \frac{a+bx^3}{\left(c+dx^3\right)^{7/3}}\,dx}{35c^2}$$

$$= \frac{x\left(a+bx^3\right)^3}{10c\left(c+dx^3\right)^{10/3}} + \frac{9ax\left(a+bx^3\right)^2}{70c^2\left(c+dx^3\right)^{7/3}} + \frac{27a^2x\left(a+bx^3\right)}{140c^3\left(c+dx^3\right)^{4/3}} + \frac{\left(81a^3\right)\int \frac{1}{\left(c+dx^3\right)^{4/3}}\,dx}{140c^3}$$

$$= \frac{x\left(a+bx^3\right)^3}{10c\left(c+dx^3\right)^{10/3}} + \frac{9ax\left(a+bx^3\right)^2}{70c^2\left(c+dx^3\right)^{7/3}} + \frac{27a^2x\left(a+bx^3\right)}{140c^3\left(c+dx^3\right)^{4/3}} + \frac{81a^3x}{140c^4\sqrt[3]{c+dx^3}}$$

Mathematica [A] time = 0.0373065, size = 120, normalized size = 1.1

$$\frac{x\left(3a^2bcx^3\left(35c^2+30cdx^3+9d^2x^6\right)+a^3\left(315c^2dx^3+140c^3+270cd^2x^6+81d^3x^9\right)+6ab^2c^2x^6\left(10c+3dx^3\right)+14b^3c^3x\right.}{140c^4\left(c+dx^3\right)^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^3/(c + d*x^3)^(13/3),x]
```

```
[Out] (x*(14*b^3*c^3*x^9 + 6*a*b^2*c^2*x^6*(10*c + 3*d*x^3) + 3*a^2*b*c*x^3*(35*c
^2 + 30*c*d*x^3 + 9*d^2*x^6) + a^3*(140*c^3 + 315*c^2*d*x^3 + 270*c*d^2*x^6
 + 81*d^3*x^9)))/(140*c^4*(c + d*x^3)^(10/3))
```

Maple [A] time = 0.005, size = 134, normalized size = 1.2

$$\frac{x\left(81\,a^3d^3x^9+27\,a^2bcd^2x^9+18\,ab^2c^2dx^9+14\,b^3c^3x^9+270\,a^3cd^2x^6+90\,a^2bc^2dx^6+60\,ab^2c^3x^6+315\,a^3c^2dx^3+105\right.}{140\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^3/(d*x^3+c)^(13/3),x)
```

```
[Out] 1/140*x*(81*a^3*d^3*x^9+27*a^2*b*c*d^2*x^9+18*a*b^2*c^2*d*x^9+14*b^3*c^3*x^
9+270*a^3*c*d^2*x^6+90*a^2*b*c^2*d*x^6+60*a*b^2*c^3*x^6+315*a^3*c^2*d*x^3+1
05*a^2*b*c^3*x^3+140*a^3*c^3)/(d*x^3+c)^(10/3)/c^4
```

Maxima [A] time = 0.97642, size = 246, normalized size = 2.26

$$\frac{b^3x^{10}}{10\left(dx^3+c\right)^{\frac{10}{3}}c} - \frac{3\,ab^2\left(7\,d-\frac{10\left(dx^3+c\right)}{x^3}\right)x^{10}}{70\left(dx^3+c\right)^{\frac{10}{3}}c^2} + \frac{3\left(14\,d^2-\frac{40\left(dx^3+c\right)d}{x^3}+\frac{35\left(dx^3+c\right)^2}{x^6}\right)a^2bx^{10}}{140\left(dx^3+c\right)^{\frac{10}{3}}c^3} - \frac{\left(14\,d^3-\frac{60\left(dx^3+c\right)d^2}{x^3}+\frac{105\left(dx^3\right.}{x^6}\right.}{140\left(dx^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/(d*x^3+c)^(13/3),x, algorithm="maxima")
```

```
[Out] 1/10*b^3*x^10/((d*x^3 + c)^(10/3)*c) - 3/70*a*b^2*(7*d - 10*(d*x^3 + c)/x^3
)*x^10/((d*x^3 + c)^(10/3)*c^2) + 3/140*(14*d^2 - 40*(d*x^3 + c)*d/x^3 + 35
*(d*x^3 + c)^2/x^6)*a^2*b*x^10/((d*x^3 + c)^(10/3)*c^3) - 1/140*(14*d^3 - 6
0*(d*x^3 + c)*d^2/x^3 + 105*(d*x^3 + c)^2*d/x^6 - 140*(d*x^3 + c)^3/x^9)*a^
3*x^10/((d*x^3 + c)^(10/3)*c^4)
```

Fricas [A] time = 1.65728, size = 356, normalized size = 3.27

$$\frac{\left(\left(14\,b^3c^3+18\,ab^2c^2d+27\,a^2bcd^2+81\,a^3d^3\right)x^{10}+30\left(2\,ab^2c^3+3\,a^2bc^2d+9\,a^3cd^2\right)x^7+140\,a^3c^3x+105\left(a^2bc^3+3\,a^3c^2\right.\right.}{140\left(c^4d^4x^{12}+4\,c^5d^3x^9+6\,c^6d^2x^6+4\,c^7dx^3+c^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/(d*x^3+c)^(13/3),x, algorithm="fricas")
```

```
[Out] 1/140*((14*b^3*c^3 + 18*a*b^2*c^2*d + 27*a^2*b*c*d^2 + 81*a^3*d^3)*x^10 + 3
0*(2*a*b^2*c^3 + 3*a^2*b*c^2*d + 9*a^3*c*d^2)*x^7 + 140*a^3*c^3*x + 105*(a^
2*b*c^3 + 3*a^3*c^2*d)*x^4)*(d*x^3 + c)^(2/3)/(c^4*d^4*x^12 + 4*c^5*d^3*x^9
 + 6*c^6*d^2*x^6 + 4*c^7*d*x^3 + c^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**3/(d*x**3+c)**(13/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^3}{\left(dx^3+c\right)^{\frac{13}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^3/(d*x^3+c)^(13/3),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^3/(d*x^3 + c)^(13/3), x)
```

3.86 $\int \frac{\left(a+bx^3\right)^{8/3}}{c+dx^3}\,dx$

Optimal. Leaf size=331

$$-\frac{b^{2/3}\left(20a^2d^2-24abcd+9b^2c^2\right)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{18d^3}+\frac{b^{2/3}\left(20a^2d^2-24abcd+9b^2c^2\right)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}d^3}-\frac{(bc-a\ldots}{}$$

```
[Out] -(b*(6*b*c - 11*a*d)*x*(a + b*x^3)^(2/3))/(18*d^2) + (b*x*(a + b*x^3)^(5/3)
)/(6*d) + (b^(2/3)*(9*b^2*c^2 - 24*a*b*c*d + 20*a^2*d^2)*ArcTan[(1 + (2*b^(
1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]*d^3) - ((b*c - a*d)^(8/3)*A
rcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(
Sqrt[3]*c^(2/3)*d^3) - ((b*c - a*d)^(8/3)*Log[c + d*x^3])/(6*c^(2/3)*d^3) +
 ((b*c - a*d)^(8/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])
/(2*c^(2/3)*d^3) - (b^(2/3)*(9*b^2*c^2 - 24*a*b*c*d + 20*a^2*d^2)*Log[-(b^(
1/3)*x) + (a + b*x^3)^(1/3)])/(18*d^3)
```

Rubi [C] time = 0.0275217, antiderivative size = 62, normalized size of antiderivative = 0.19, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{a^2x\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{8}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(8/3)/(c + d*x^3),x]
```

```
[Out] (a^2*x*(a + b*x^3)^(2/3)*AppellF1[1/3, -8/3, 1, 4/3, -((b*x^3)/a), -((d*x^3
)/c)])/(c*(1 + (b*x^3)/a)^(2/3))
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(a+bx^3)^{8/3}}{c+dx^3}\,dx = \frac{\left(a^2(a+bx^3)^{2/3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{8/3}}{c+dx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$
$$= \frac{a^2x(a+bx^3)^{2/3}\,F_1\left(\frac{1}{3};-\frac{8}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] time = 0.788875, size = 655, normalized size = 1.98

$$3bx^4\sqrt[3]{\frac{bx^3}{a}+1}\sqrt[3]{bc-ad}\left(20a^2d^2-24abcd+9b^2c^2\right)F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2\sqrt[3]{c}\left(-2a\sqrt[3]{a+bx^3}\left(9a^2d^2-7abcd+3b^2c^2\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(8/3)/(c + d*x^3),x]
```

```
[Out] (3*b*(b*c - a*d)^(1/3)*(9*b^2*c^2 - 24*a*b*c*d + 20*a^2*d^2)*x^4*(1 + (b*x^
3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*c^(1
/3)*(-18*a*b^2*c^(5/3)*(b*c - a*d)^(1/3)*x + 42*a^2*b*c^(2/3)*d*(b*c - a*d)
^(1/3)*x - 18*b^3*c^(5/3)*(b*c - a*d)^(1/3)*x^4 + 51*a*b^2*c^(2/3)*d*(b*c -
 a*d)^(1/3)*x^4 + 9*b^3*c^(2/3)*d*(b*c - a*d)^(1/3)*x^7 + 2*Sqrt[3]*a*(3*b^
2*c^2 - 7*a*b*c*d + 9*a^2*d^2)*(a + b*x^3)^(1/3)*ArcTan[(1 + (2*(b*c - a*d)
^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[3]] - 2*a*(3*b^2*c^2 - 7*a*b*c*
d + 9*a^2*d^2)*(a + b*x^3)^(1/3)*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a
*x^3)^(1/3)] + 3*a*b^2*c^2*(a + b*x^3)^(1/3)*Log[c^(2/3) + ((b*c - a*d)^(2/
3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)
] - 7*a^2*b*c*d*(a + b*x^3)^(1/3)*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b
+ a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + 9*a^3*d
^2*(a + b*x^3)^(1/3)*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3
) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(108*c*d^2*(b*c - a*
d)^(1/3)*(a + b*x^3)^(1/3))
```

Maple [F] time = 0.426, size = 0, normalized size = 0.

$$\int \frac{1}{dx^3+c}\left(bx^3+a\right)^{\frac{8}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(8/3)/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(8/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{8}{3}}}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(8/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(8/3)/(d*x^3 + c), x)
```

Fricas [B] time = 61.6469, size = 1486, normalized size = 4.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(8/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/54*(18*sqrt(3)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*((b^2*c^2 - 2*a*b*c*d + a
^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b*x^3 +
a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3))/((b*c - a*d)*x)) +
2*sqrt(3)*(9*b^2*c^2 - 24*a*b*c*d + 20*a^2*d^2)*(-b^2)^(1/3)*arctan(-1/3*(s
qrt(3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)) - 18*(b^2*c^2
 - 2*a*b*c*d + a^2*d^2)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*
x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*
d))/x) - 2*(9*b^2*c^2 - 24*a*b*c*d + 20*a^2*d^2)*(-b^2)^(1/3)*log(-((-b^2)^
(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + (9*b^2*c^2 - 24*a*b*c*d + 20*a^2*d^2)*(
-b^2)^(1/3)*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (
b*x^3 + a)^(2/3)*b)/x^2) + 9*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*((b^2*c^2 - 2*
a*b*c*d + a^2*d^2)/c^2)^(1/3)*log(-((b*c - a*d)*x^2*((b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)/c^2)^(1/3) + (b*x^3 + a)^(1/3)*c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^
2)/c^2)^(2/3) + (b*x^3 + a)^(2/3)*(b*c - a*d))/x^2) - 3*(3*b^2*d^2*x^4 - 2*
(3*b^2*c*d - 7*a*b*d^2)*x)*(b*x^3 + a)^(2/3))/d^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(8/3)/(d*x**3+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{8}{3}}}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(8/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(8/3)/(d*x^3 + c), x)
```

3.87 $\int \frac{(a+bx^3)^{5/3}}{c+dx^3} dx$

Optimal. Leaf size=273

$$\frac{b^{2/3}(3bc-5ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{6d^2}-\frac{b^{2/3}(3bc-5ad)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}d^2}+\frac{(bc-ad)^{5/3}\log\left(c+dx^3\right)}{6c^{2/3}d^2}-\frac{(bc-ad)^{5/3}\log\left(\right.}{2}$$

```
[Out] (b*x*(a + b*x^3)^(2/3))/(3*d) - (b^(2/3)*(3*b*c - 5*a*d)*ArcTan[(1 + (2*b^(
1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*d^2) + ((b*c - a*d)^(5/3)*A
rcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(
Sqrt[3]*c^(2/3)*d^2) + ((b*c - a*d)^(5/3)*Log[c + d*x^3])/(6*c^(2/3)*d^2) -
 ((b*c - a*d)^(5/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])
/(2*c^(2/3)*d^2) + (b^(2/3)*(3*b*c - 5*a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^
(1/3)])/(6*d^2)
```

Rubi [C] time = 0.027235, antiderivative size = 60, normalized size of antiderivative = 0.22, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{ax\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{5}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(5/3)/(c + d*x^3),x]
```

```
[Out] (a*x*(a + b*x^3)^(2/3)*AppellF1[1/3, -5/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/
c)])/(c*(1 + (b*x^3)/a)^(2/3))
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(a+bx^3)^{5/3}}{c+dx^3}\,dx = \frac{\left(a(a+bx^3)^{2/3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{5/3}}{c+dx^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$
$$= \frac{ax(a+bx^3)^{2/3}\, F_1\left(\frac{1}{3};-\frac{5}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] time = 0.476772, size = 443, normalized size = 1.62

$$2\sqrt[3]{c}\left(3a^2d\sqrt[3]{a+bx^3}\log\left(\frac{x^2(bc-ad)^{2/3}}{(ax^3+b)^{2/3}}+\frac{\sqrt[3]{c}x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}+c^{2/3}\right)+6b^2c^{2/3}x^4\sqrt[3]{bc-ad}-abc\sqrt[3]{a+bx^3}\log\left(\frac{x^2(bc-ad)^{2/3}}{(ax^3+b)^{2/3}}+\frac{\sqrt[3]{c}x\sqrt[3]{bc-}}{\sqrt[3]{ax^3+b}}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(5/3)/(c + d*x^3),x]
```

```
[Out] (3*b*(b*c - a*d)^(1/3)*(-3*b*c + 5*a*d)*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[
4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*c^(1/3)*(6*a*b*c^(2/3)*(b
*c - a*d)^(1/3)*x + 6*b^2*c^(2/3)*(b*c - a*d)^(1/3)*x^4 + 2*Sqrt[3]*a*(-(b*
c) + 3*a*d)*(a + b*x^3)^(1/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*
(b + a*x^3)^(1/3)))/Sqrt[3]] + 2*a*(b*c - 3*a*d)*(a + b*x^3)^(1/3)*Log[c^(1
/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] - a*b*c*(a + b*x^3)^(1/3)*Lo
g[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d
)^(1/3)*x)/(b + a*x^3)^(1/3)] + 3*a^2*d*(a + b*x^3)^(1/3)*Log[c^(2/3) + ((b
*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b +
 a*x^3)^(1/3)]))/(36*c*d*(b*c - a*d)^(1/3)*(a + b*x^3)^(1/3))
```

Maple [F] time = 0.427, size = 0, normalized size = 0.

$$\int \frac{1}{dx^3+c}\left(bx^3+a\right)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(5/3)/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(5/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{5}{3}}}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(5/3)/(d*x^3 + c), x)
```

Fricas [B] time = 6.45721, size = 1258, normalized size = 4.61

$$6\left(bx^3+a\right)^{\frac{2}{3}}bdx+6\sqrt{3}(bc-ad)\left(\frac{b^2c^2-2\,abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}(bc-ad)x+2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}c\left(\frac{b^2c^2-2\,abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}}{3\,(bc-ad)x}\right)+2\sqrt{3}\left(-b^2\right)^{\frac{1}{3}}(3\,bc$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] 1/18*(6*(b*x^3 + a)^(2/3)*b*d*x + 6*sqrt(3)*(b*c - a*d)*((b^2*c^2 - 2*a*b*c
*d + a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b*
x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3))/((b*c - a*d)*
x)) + 2*sqrt(3)*(-b^2)^(1/3)*(3*b*c - 5*a*d)*arctan(-1/3*(sqrt(3)*b*x - 2*s
qrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)) - 6*(b*c - a*d)*((b^2*c^2 - 2
*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^
2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d))/x) - 2*(-b^2)^(1/3)*(3*b*c - 5*a*
d)*log(-((-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + (-b^2)^(1/3)*(3*b*c - 5
*a*d)*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (b*x^3
+ a)^(2/3)*b)/x^2) + 3*(b*c - a*d)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1
/3)*log(-((b*c - a*d)*x^2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3) + (b*
x^3 + a)^(1/3)*c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(2/3) + (b*x^3 + a
)^(2/3)*(b*c - a*d))/x^2))/d^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+bx^3\right)^{\frac{5}{3}}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(5/3)/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(5/3)/(c + d*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(bx^3+a\right)^{\frac{5}{3}}}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(5/3)/(d*x^3 + c), x)
```

3.88 $\int \frac{(a+bx^3)^{2/3}}{c+dx^3} \, dx$

Optimal. Leaf size=233

$$-\frac{b^{2/3}\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2d}+\frac{b^{2/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}d}-\frac{(bc-ad)^{2/3}\log\left(c+dx^3\right)}{6c^{2/3}d}+\frac{(bc-ad)^{2/3}\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+b\ldots}\right)}{2c^{2/3}d}$$

```
[Out] (b^(2/3)*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*d)
 - ((b*c - a*d)^(2/3)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x
^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(2/3)*d) - ((b*c - a*d)^(2/3)*Log[c + d*x^
3])/(6*c^(2/3)*d) + ((b*c - a*d)^(2/3)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) -
(a + b*x^3)^(1/3)])/(2*c^(2/3)*d) - (b^(2/3)*Log[-(b^(1/3)*x) + (a + b*x^3)
^(1/3)])/(2*d)
```

Rubi [C] time = 0.0269623, antiderivative size = 59, normalized size of antiderivative = 0.25, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(2/3)/(c + d*x^3),x]
```

```
[Out] (x*(a + b*x^3)^(2/3)*AppellF1[1/3, -2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)
])/(c*(1 + (b*x^3)/a)^(2/3))
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(a+bx^3)^{2/3}}{c+dx^3}\,dx = \frac{(a+bx^3)^{2/3}\int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{c+dx^3}dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}} = \frac{x(a+bx^3)^{2/3} F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] time = 0.15415, size = 161, normalized size = 0.69

$$\frac{4acx(a+bx^3)^{2/3} F_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(c+dx^3)\left(x^3\left(2bcF_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-3adF_1\left(\frac{4}{3};-\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)+4acF_1\left(\frac{1}{3};-\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(2/3)/(c + d*x^3),x]
```

```
[Out] (4*a*c*x*(a + b*x^3)^(2/3)*AppellF1[1/3, -2/3, 1, 4/3, -((b*x^3)/a), -((d*x
^3)/c)])/((c + d*x^3)*(4*a*c*AppellF1[1/3, -2/3, 1, 4/3, -((b*x^3)/a), -((d
*x^3)/c)] + x^3*(-3*a*d*AppellF1[4/3, -2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)
/c)] + 2*b*c*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

Maple [F] time = 0.428, size = 0, normalized size = 0.

$$\int \frac{1}{dx^3+c}\left(bx^3+a\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/(d*x^3 + c), x)
```

Fricas [B] time = 1.88723, size = 1102, normalized size = 4.73

$$-\frac{2\sqrt{3}\left(\frac{b^2c^2-2\,abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}(bc-ad)x+2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}c\left(\frac{b^2c^2-2\,abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}}{3\,(bc-ad)x}\right)+2\sqrt{3}\left(-b^2\right)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}bx-2\sqrt{3}\left(bx^3+a\right)^{\frac{1}{3}}(-}{3\,bx}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sq
rt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)/c^2)^(1/3))/((b*c - a*d)*x)) + 2*sqrt(3)*(-b^2)^(1/3)*arctan(-1/3
*(sqrt(3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)) - 2*((b^2*
c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*x*((b^2*c^2 - 2*a*b*c*d + a^2*
d^2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d))/x) - 2*(-b^2)^(1/3)*log(-(
(-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + (-b^2)^(1/3)*log(-((-b^2)^(1/3)*
b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2) + ((b^
2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log(-((b*c - a*d)*x^2*((b^2*c^2 - 2
*a*b*c*d + a^2*d^2)/c^2)^(1/3) + (b*x^3 + a)^(1/3)*c*x*((b^2*c^2 - 2*a*b*c*
d + a^2*d^2)/c^2)^(2/3) + (b*x^3 + a)^(2/3)*(b*c - a*d))/x^2))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^3\right)^{\frac{2}{3}}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(2/3)/(c + d*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/(d*x^3 + c), x)
```

3.89 $\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)} dx$

Optimal. Leaf size=148

$$\frac{\log\left(c+dx^3\right)}{6c^{2/3}\sqrt[3]{bc-ad}}-\frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}}+\frac{\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}$$

```
[Out] ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]]/(
Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c + d*x^3]/(6*c^(2/3)*(b*c - a*d)^
(1/3)) - Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)]/(2*c^(2/3)*
(b*c - a*d)^(1/3))
```

Rubi [A] time = 0.191796, antiderivative size = 207, normalized size of antiderivative = 1.4, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {377, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}\sqrt[3]{bc-ad}}+\frac{\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+c^{2/3}\right)}{6c^{2/3}\sqrt[3]{bc-ad}}+\frac{\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+\sqrt[3]{c}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] ArcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3))/(Sqrt[3]*c^(1/
3))]/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(1/3)) - Log[c^(1/3) - ((b*c - a*d)^(1/3)
*x)/(a + b*x^3)^(1/3)]/(3*c^(2/3)*(b*c - a*d)^(1/3)) + Log[c^(2/3) + ((b*c
- a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(a + b*
x^3)^(1/3)]/(6*c^(2/3)*(b*c - a*d)^(1/3))
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx = \text{Subst}\left(\int \frac{1}{c-(bc-ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}} + \frac{\text{Subst}\left(\int \frac{2\sqrt[3]{c}+\sqrt[3]{bc-ad}x}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-ad}x+(bc-ad)^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}}$$

$$= -\frac{\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}\sqrt[3]{bc-ad}} + \frac{\text{Subst}\left(\int \frac{1}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-ad}x+(bc-ad)^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{2\sqrt[3]{c}} + \frac{\text{Subst}\left(\int \frac{}{c^{2/3}}\right.}{}$$

$$= -\frac{\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}\sqrt[3]{bc-ad}} + \frac{\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{6c^{2/3}\sqrt[3]{bc-ad}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2}{\sqrt[3]{}}\right.}{c^{2/3}\sqrt[3]{bc-ad}}$$

$$= \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}} - \frac{\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}\sqrt[3]{bc-ad}} + \frac{\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{6c^{2/3}\sqrt[3]{bc-ad}}$$

Mathematica [A] time = 0.14887, size = 168, normalized size = 1.14

$$\frac{\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+c^{2/3}\right)-2\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{6c^{2/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^3)^(1/3)*(c + d*x^3)),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))
/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)] + Log[
```

```
c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^
(1/3)*x)/(a + b*x^3)^(1/3)])/(6*c^(2/3)*(b*c - a*d)^(1/3))
```

Maple [F] time = 0.414, size = 0, normalized size = 0.

$$\int \frac{1}{dx^3+c}\frac{1}{\sqrt[3]{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int(1/(b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/((a + b*x**3)**(1/3)*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)), x)
```

3.90 $\int \frac{1}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=179

$$-\frac{d\log\left(c+dx^3\right)}{6c^{2/3}(bc-ad)^{4/3}}+\frac{d\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}(bc-ad)^{4/3}}-\frac{d\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}(bc-ad)^{4/3}}+\frac{bx}{a\sqrt[3]{a+bx^3}(bc-ad)}$$

```
[Out] (b*x)/(a*(b*c - a*d)*(a + b*x^3)^(1/3)) - (d*ArcTan[(1 + (2*(b*c - a*d)^(1/
3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(
4/3)) - (d*Log[c + d*x^3])/(6*c^(2/3)*(b*c - a*d)^(4/3)) + (d*Log[((b*c - a
*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(2/3)*(b*c - a*d)^(4/3))
```

Rubi [A] time = 0.193167, antiderivative size = 238, normalized size of antiderivative = 1.33, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {382, 377, 200, 31, 634, 617, 204, 628}

$$\frac{d\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}(bc-ad)^{4/3}}-\frac{d\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+c^{2/3}\right)}{6c^{2/3}(bc-ad)^{4/3}}-\frac{d\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+\sqrt[3]{c}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}(bc-ad)^{4/3}}+\frac{bx}{a\sqrt[3]{a+bx^3}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] (b*x)/(a*(b*c - a*d)*(a + b*x^3)^(1/3)) - (d*ArcTan[(c^(1/3) + (2*(b*c - a*
d)^(1/3)*x)/(a + b*x^3)^(1/3))/(Sqrt[3]*c^(1/3))])/(Sqrt[3]*c^(2/3)*(b*c -
a*d)^(4/3)) + (d*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(3
*c^(2/3)*(b*c - a*d)^(4/3)) - (d*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a +
 b*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(6*c^(2/3
)*(b*c - a*d)^(4/3))
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

```
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, 
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, 
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)}\,dx &= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{d\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx}{bc-ad} \\
&= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{d\,\mathrm{Subst}\left(\int \frac{1}{c-(bc-ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{bc-ad} \\
&= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{d\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}(bc-ad)} - \frac{d\,\mathrm{Subst}\left(\int \frac{2\sqrt[3]{c}+\sqrt[3]{}}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-a}}\right.}{3c^{2/3}(} \\
&= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} + \frac{d\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}(bc-ad)^{4/3}} - \frac{d\,\mathrm{Subst}\left(\int \frac{\sqrt[3]{c}\sqrt[3]{bc-ad}+2(bc-ad)^{2/3}x}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-ad}x+(bc-ad)^{2/3}x^2}\,dx,\right.}{6c^{2/3}(bc-ad)^{4/3}} \\
&= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} + \frac{d\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}(bc-ad)^{4/3}} - \frac{d\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{6c^{2/3}(bc-ad)^{4/3}} + \\
&= \frac{bx}{a(bc-ad)\sqrt[3]{a+bx^3}} - \frac{d\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}(bc-ad)^{4/3}} + \frac{d\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}(bc-ad)^{4/3}} - \frac{d\log\left(c^{2/3}+\frac{(bc-}{(a+}\right.}{6c^{2/3}(}
\end{aligned}
$$

Mathematica [C] time = 0.429821, size = 256, normalized size = 1.43

$$-\frac{21c^2dx^3\left(a+bx^3\right)^2 {}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)+28c^3\left(a+bx^3\right)^2 {}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)-21c^2dx^3\left(a+bx^3\right)^2-28c^3\left(a+bx^3\right)^2}{7c^3x^2\left(a+bx^3\right)^{7/3}(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(4/3)*(c + d*x^3)),x]
```

```
[Out] -(-28*c^3*(a + b*x^3)^2 - 21*c^2*d*x^3*(a + b*x^3)^2 + 28*c^3*(a + b*x^3)^2
*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 21*c^2
*d*x^3*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a
 + b*x^3))] + 3*c*(b*c - a*d)^2*x^6*Hypergeometric2F1[2, 7/3, 10/3, ((b*c -
 a*d)*x^3)/(c*(a + b*x^3))] + 3*d*(b*c - a*d)^2*x^9*Hypergeometric2F1[2, 7/
3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(7*c^3*(-(b*c) + a*d)*x^2*(a +
 b*x^3)^(7/3))
```

Maple [F] time = 0.396, size = 0, normalized size = 0.

$$\int \frac{1}{dx^3+c}\left(bx^3+a\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int(1/(b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^3\right)^{\frac{4}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/((a + b*x**3)**(4/3)*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)), x)
```

3.91 $\int \frac{1}{\left(a+bx^3\right)^{7/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=226

$$\frac{bx(3bc-7ad)}{4a^2\sqrt[3]{a+bx^3}(bc-ad)^2}+\frac{d^2\log\left(c+dx^3\right)}{6c^{2/3}(bc-ad)^{7/3}}-\frac{d^2\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}(bc-ad)^{7/3}}+\frac{d^2\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}(bc-ad)^{7/3}}+\frac{bx}{4a\left(a+bx^3\right)^{4/3}(bc\cdots}$$

```
[Out] (b*x)/(4*a*(b*c - a*d)*(a + b*x^3)^(4/3)) + (b*(3*b*c - 7*a*d)*x)/(4*a^2*(b
*c - a*d)^2*(a + b*x^3)^(1/3)) + (d^2*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(
c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(2/3)*(b*c - a*d)^(7/3)) +
 (d^2*Log[c + d*x^3])/(6*c^(2/3)*(b*c - a*d)^(7/3)) - (d^2*Log[((b*c - a*d)
^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(2/3)*(b*c - a*d)^(7/3))
```

Rubi [C] time = 2.58034, antiderivative size = 621, normalized size of antiderivative = 2.75, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$-\frac{-9c^2x^9(bc-ad)^3\,{}_3F_2\left(2,2,\frac{10}{3};1,\frac{13}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)-9d^2x^{15}(bc-ad)^3\,{}_3F_2\left(2,2,\frac{10}{3};1,\frac{13}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)-18cdx^{12}(bc-ad)^3\,{}_3F_2\left(2,\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((a + b*x^3)^(7/3)*(c + d*x^3)),x]
```

```
[Out] -(70*c^4*(b*c - a*d)*x^3*(a + b*x^3)^2 + 105*c^3*d*(b*c - a*d)*x^6*(a + b*x
^3)^2 + 45*c^2*d^2*(b*c - a*d)*x^9*(a + b*x^3)^2 + 280*c^5*(a + b*x^3)^3 +
420*c^4*d*x^3*(a + b*x^3)^3 + 180*c^3*d^2*x^6*(a + b*x^3)^3 - 280*c^5*(a +
b*x^3)^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]
- 420*c^4*d*x^3*(a + b*x^3)^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x
^3)/(c*(a + b*x^3))] - 180*c^3*d^2*x^6*(a + b*x^3)^3*Hypergeometric2F1[1/3,
 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 33*c^2*(b*c - a*d)^3*x^9*Hype
rgeometric2F1[2, 10/3, 13/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 60*c*d*(b
*c - a*d)^3*x^12*Hypergeometric2F1[2, 10/3, 13/3, ((b*c - a*d)*x^3)/(c*(a +
 b*x^3))] - 27*d^2*(b*c - a*d)^3*x^15*Hypergeometric2F1[2, 10/3, 13/3, ((b*
c - a*d)*x^3)/(c*(a + b*x^3))] - 9*c^2*(b*c - a*d)^3*x^9*HypergeometricPFQ[
{2, 2, 10/3}, {1, 13/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 18*c*d*(b*c -
 a*d)^3*x^12*HypergeometricPFQ[{2, 2, 10/3}, {1, 13/3}, ((b*c - a*d)*x^3)/(
c*(a + b*x^3))] - 9*d^2*(b*c - a*d)^3*x^15*HypergeometricPFQ[{2, 2, 10/3},
{1, 13/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(40*c^4*(b*c - a*d)^2*x^5*(a
 + b*x^3)^(10/3))
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
```

```
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^{7/3}\left(c+dx^3\right)}dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{7/3}\left(c+dx^3\right)}dx}{a^2\sqrt[3]{a+bx^3}}$$

$$= -\frac{70c^4(bc-ad)x^3\left(a+bx^3\right)^2+105c^3d(bc-ad)x^6\left(a+bx^3\right)^2+45c^2d^2(bc-ad)x^9\left(a+b\right.}{}$$

Mathematica [C] time = 1.92663, size = 621, normalized size = 2.75

$$\frac{9c^2x^9(bc-ad)^3\text{HypergeometricPFQ}\left(\left\{2,2,\frac{10}{3}\right\},\left\{1,\frac{13}{3}\right\},\frac{x^3(bc-ad)}{c\left(a+bx^3\right)}\right)+9d^2x^{15}(bc-ad)^3\text{HypergeometricPFQ}\left(\left\{2,2\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(7/3)*(c + d*x^3)),x]
```

```
[Out] (-70*c^4*(b*c - a*d)*x^3*(a + b*x^3)^2 - 105*c^3*d*(b*c - a*d)*x^6*(a + b*x
^3)^2 - 45*c^2*d^2*(b*c - a*d)*x^9*(a + b*x^3)^2 - 280*c^5*(a + b*x^3)^3 -
420*c^4*d*x^3*(a + b*x^3)^3 - 180*c^3*d^2*x^6*(a + b*x^3)^3 + 280*c^5*(a +
b*x^3)^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]
+ 420*c^4*d*x^3*(a + b*x^3)^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x
^3)/(c*(a + b*x^3))] + 180*c^3*d^2*x^6*(a + b*x^3)^3*Hypergeometric2F1[1/3,
 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 33*c^2*(b*c - a*d)^3*x^9*Hype
rgeometric2F1[2, 10/3, 13/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 60*c*d*(b
*c - a*d)^3*x^12*Hypergeometric2F1[2, 10/3, 13/3, ((b*c - a*d)*x^3)/(c*(a +
 b*x^3))] + 27*d^2*(b*c - a*d)^3*x^15*Hypergeometric2F1[2, 10/3, 13/3, ((b*
c - a*d)*x^3)/(c*(a + b*x^3))] + 9*c^2*(b*c - a*d)^3*x^9*HypergeometricPFQ[
{2, 2, 10/3}, {1, 13/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 18*c*d*(b*c -
 a*d)^3*x^12*HypergeometricPFQ[{2, 2, 10/3}, {1, 13/3}, ((b*c - a*d)*x^3)/(
c*(a + b*x^3))] + 9*d^2*(b*c - a*d)^3*x^15*HypergeometricPFQ[{2, 2, 10/3},
{1, 13/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(40*c^4*(b*c - a*d)^2*x^5*(a
 + b*x^3)^(10/3))
```

Maple [F] time = 0.403, size = 0, normalized size = 0.

$$\int \frac{1}{dx^3+c}\left(bx^3+a\right)^{-\frac{7}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(7/3)/(d*x^3+c),x)
```

```
[Out] int(1/(b*x^3+a)^(7/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{7}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(7/3)*(d*x^3 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^3\right)^{\frac{7}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(7/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/((a + b*x**3)**(7/3)*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{7}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(7/3)*(d*x^3 + c)), x)
```

3.92 $\int \frac{1}{\left(a+bx^3\right)^{10/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=280

$$\frac{bx\left(67a^2d^2-57abcd+18b^2c^2\right)}{28a^3\sqrt[3]{a+bx^3}(bc-ad)^3}+\frac{bx(6bc-13ad)}{28a^2\left(a+bx^3\right)^{4/3}(bc-ad)^2}-\frac{d^3\log\left(c+dx^3\right)}{6c^{2/3}(bc-ad)^{10/3}}+\frac{d^3\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{2c^{2/3}(bc-ad)^{10/3}}-\ldots$$

```
[Out] (b*x)/(7*a*(b*c - a*d)*(a + b*x^3)^(7/3)) + (b*(6*b*c - 13*a*d)*x)/(28*a^2*
(b*c - a*d)^2*(a + b*x^3)^(4/3)) + (b*(18*b^2*c^2 - 57*a*b*c*d + 67*a^2*d^2
)*x)/(28*a^3*(b*c - a*d)^3*(a + b*x^3)^(1/3)) - (d^3*ArcTan[(1 + (2*(b*c -
a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Sqrt[3]*c^(2/3)*(b*c
- a*d)^(10/3)) - (d^3*Log[c + d*x^3])/(6*c^(2/3)*(b*c - a*d)^(10/3)) + (d^3
*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(2*c^(2/3)*(b*c -
a*d)^(10/3))
```

Rubi [C] time = 6.64171, antiderivative size = 1172, normalized size of antiderivative = 4.19, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$-\frac{-4158d^3(bc-ad)^4\,{}_2F_1\left(2,\frac{13}{3};\frac{16}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)x^{21}-2268d^3(bc-ad)^4\,{}_3F_2\left(2,2,\frac{13}{3};1,\frac{16}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)x^{21}-378d^3(bc-ad)^4\ldots}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((a + b*x^3)^(10/3)*(c + d*x^3)),x]
```

```
[Out] -(7280*c^5*(b*c - a*d)^2*x^6*(a + b*x^3)^2 + 16380*c^4*d*(b*c - a*d)^2*x^9*
(a + b*x^3)^2 + 14040*c^3*d^2*(b*c - a*d)^2*x^12*(a + b*x^3)^2 + 4212*c^2*d
^3*(b*c - a*d)^2*x^15*(a + b*x^3)^2 + 12740*c^6*(b*c - a*d)*x^3*(a + b*x^3)
^3 + 28665*c^5*d*(b*c - a*d)*x^6*(a + b*x^3)^3 + 24570*c^4*d^2*(b*c - a*d)*
x^9*(a + b*x^3)^3 + 7371*c^3*d^3*(b*c - a*d)*x^12*(a + b*x^3)^3 + 50960*c^7
*(a + b*x^3)^4 + 114660*c^6*d*x^3*(a + b*x^3)^4 + 98280*c^5*d^2*x^6*(a + b*
x^3)^4 + 29484*c^4*d^3*x^9*(a + b*x^3)^4 - 50960*c^7*(a + b*x^3)^4*Hypergeo
metric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 114660*c^6*d*x^
3*(a + b*x^3)^4*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*
x^3))] - 98280*c^5*d^2*x^6*(a + b*x^3)^4*Hypergeometric2F1[1/3, 1, 4/3, ((b
*c - a*d)*x^3)/(c*(a + b*x^3))] - 29484*c^4*d^3*x^9*(a + b*x^3)^4*Hypergeom
etric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 5796*c^3*(b*c -
a*d)^4*x^12*Hypergeometric2F1[2, 13/3, 16/3, ((b*c - a*d)*x^3)/(c*(a + b*x^
3))] - 15246*c^2*d*(b*c - a*d)^4*x^15*Hypergeometric2F1[2, 13/3, 16/3, ((b*
c - a*d)*x^3)/(c*(a + b*x^3))] - 13608*c*d^2*(b*c - a*d)^4*x^18*Hypergeomet
ric2F1[2, 13/3, 16/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 4158*d^3*(b*c -
a*d)^4*x^21*Hypergeometric2F1[2, 13/3, 16/3, ((b*c - a*d)*x^3)/(c*(a + b*x^
3))] - 2646*c^3*(b*c - a*d)^4*x^12*HypergeometricPFQ[{2, 2, 13/3}, {1, 16/3
}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 7560*c^2*d*(b*c - a*d)^4*x^15*Hyper
geometricPFQ[{2, 2, 13/3}, {1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] -
7182*c*d^2*(b*c - a*d)^4*x^18*HypergeometricPFQ[{2, 2, 13/3}, {1, 16/3}, ((
b*c - a*d)*x^3)/(c*(a + b*x^3))] - 2268*d^3*(b*c - a*d)^4*x^21*Hypergeometr
icPFQ[{2, 2, 13/3}, {1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 378*c^3
*(b*c - a*d)^4*x^12*HypergeometricPFQ[{2, 2, 2, 13/3}, {1, 1, 16/3}, ((b*c
- a*d)*x^3)/(c*(a + b*x^3))] - 1134*c^2*d*(b*c - a*d)^4*x^15*Hypergeometric
```

```
PFQ[{2, 2, 2, 13/3}, {1, 1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 113
4*c*d^2*(b*c - a*d)^4*x^18*HypergeometricPFQ[{2, 2, 2, 13/3}, {1, 1, 16/3},
 ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 378*d^3*(b*c - a*d)^4*x^21*Hypergeome
tricPFQ[{2, 2, 2, 13/3}, {1, 1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/
(5096*c^5*(b*c - a*d)^3*x^8*(a + b*x^3)^(13/3))
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^{10/3}\left(c+dx^3\right)}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{10/3}\left(c+dx^3\right)}\,dx}{a^3\sqrt[3]{a+bx^3}}$$

$$= -\frac{7280c^5(bc-ad)^2x^6\left(a+bx^3\right)^2+16380c^4d(bc-ad)^2x^9\left(a+bx^3\right)^2+14040c^3d^2(bc-ad)^2}{}$$

Mathematica [C] time = 4.35543, size = 1172, normalized size = 4.19

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(10/3)*(c + d*x^3)),x]
```

```
[Out] -(-7280*c^5*(b*c - a*d)^2*x^6*(a + b*x^3)^2 - 16380*c^4*d*(b*c - a*d)^2*x^9
*(a + b*x^3)^2 - 14040*c^3*d^2*(b*c - a*d)^2*x^12*(a + b*x^3)^2 - 4212*c^2*
d^3*(b*c - a*d)^2*x^15*(a + b*x^3)^2 - 12740*c^6*(b*c - a*d)*x^3*(a + b*x^3
)^3 - 28665*c^5*d*(b*c - a*d)*x^6*(a + b*x^3)^3 - 24570*c^4*d^2*(b*c - a*d)
*x^9*(a + b*x^3)^3 - 7371*c^3*d^3*(b*c - a*d)*x^12*(a + b*x^3)^3 - 50960*c^
7*(a + b*x^3)^4 - 114660*c^6*d*x^3*(a + b*x^3)^4 - 98280*c^5*d^2*x^6*(a + b
*x^3)^4 - 29484*c^4*d^3*x^9*(a + b*x^3)^4 + 50960*c^7*(a + b*x^3)^4*Hyperge
ometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 114660*c^6*d*x
^3*(a + b*x^3)^4*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b
*x^3))] + 98280*c^5*d^2*x^6*(a + b*x^3)^4*Hypergeometric2F1[1/3, 1, 4/3, ((
b*c - a*d)*x^3)/(c*(a + b*x^3))] + 29484*c^4*d^3*x^9*(a + b*x^3)^4*Hypergeo
metric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 5796*c^3*(b*c -
 a*d)^4*x^12*Hypergeometric2F1[2, 13/3, 16/3, ((b*c - a*d)*x^3)/(c*(a + b*x
^3))] + 15246*c^2*d*(b*c - a*d)^4*x^15*Hypergeometric2F1[2, 13/3, 16/3, ((b
*c - a*d)*x^3)/(c*(a + b*x^3))] + 13608*c*d^2*(b*c - a*d)^4*x^18*Hypergeome
tric2F1[2, 13/3, 16/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 4158*d^3*(b*c -
 a*d)^4*x^21*Hypergeometric2F1[2, 13/3, 16/3, ((b*c - a*d)*x^3)/(c*(a + b*x
```

```
^3))] + 2646*c^3*(b*c - a*d)^4*x^12*HypergeometricPFQ[{2, 2, 13/3}, {1, 16/
3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 7560*c^2*d*(b*c - a*d)^4*x^15*Hype
rgeometricPFQ[{2, 2, 13/3}, {1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] +
 7182*c*d^2*(b*c - a*d)^4*x^18*HypergeometricPFQ[{2, 2, 13/3}, {1, 16/3}, (
(b*c - a*d)*x^3)/(c*(a + b*x^3))] + 2268*d^3*(b*c - a*d)^4*x^21*Hypergeomet
ricPFQ[{2, 2, 13/3}, {1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 378*c^
3*(b*c - a*d)^4*x^12*HypergeometricPFQ[{2, 2, 2, 13/3}, {1, 1, 16/3}, ((b*c
 - a*d)*x^3)/(c*(a + b*x^3))] + 1134*c^2*d*(b*c - a*d)^4*x^15*Hypergeometri
cPFQ[{2, 2, 2, 13/3}, {1, 1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 11
34*c*d^2*(b*c - a*d)^4*x^18*HypergeometricPFQ[{2, 2, 2, 13/3}, {1, 1, 16/3}
, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 378*d^3*(b*c - a*d)^4*x^21*Hypergeom
etricPFQ[{2, 2, 2, 13/3}, {1, 1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])
/(5096*c^5*(-(b*c) + a*d)^3*x^8*(a + b*x^3)^(13/3))
```

Maple [F] time = 0.408, size = 0, normalized size = 0.

$$\int \frac{1}{dx^3+c}\left(bx^3+a\right)^{-\frac{10}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(10/3)/(d*x^3+c),x)
```

```
[Out] int(1/(b*x^3+a)^(10/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{10}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(10/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(10/3)*(d*x^3 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(10/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(10/3)/(d*x**3+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{10}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(10/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(10/3)*(d*x^3 + c)), x)
```

3.93 $\int \frac{\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx$

Optimal. Leaf size=60

$$\frac{ax\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{4}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[Out] (a*x*(a + b*x^3)^(1/3)*AppellF1[1/3, -4/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/
c)])/(c*(1 + (b*x^3)/a)^(1/3))
```

Rubi [A] time = 0.0283028, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{ax\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{4}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(4/3)/(c + d*x^3),x]
```

```
[Out] (a*x*(a + b*x^3)^(1/3)*AppellF1[1/3, -4/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/
c)])/(c*(1 + (b*x^3)/a)^(1/3))
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^{4/3}}{c+dx^3}\,dx &= \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}\\ &= \frac{ax\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{4}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{1+\frac{bx^3}{a}}}\end{aligned}$$

Mathematica [B] time = 0.46097, size = 346, normalized size = 5.77

$$\frac{x\left(\frac{4\left(bx^3(a+bx^3)(c+dx^3)\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4ac\left(2a^2d+abdx^3+b^2x^3(c+dx^3)\right)F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}{(c+dx^3)\left(x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}+\frac{bx^3\left(\frac{bx^3}{a}+1\right)}{}\right.}{8d\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(4/3)/(c + d*x^3),x]
```

```
[Out] (x*((b*(-2*b*c + 3*a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3
, -((b*x^3)/a), -((d*x^3)/c)])/c + (4*(-4*a*c*(2*a^2*d + a*b*d*x^3 + b^2*x^
3*(c + d*x^3))*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + b*x
^3*(a + b*x^3)*(c + d*x^3)*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a),
-((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)
])))/((c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3
)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] +
 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))))/(8*d*(a
+ b*x^3)^(2/3))
```

Maple [F] time = 0.415, size = 0, normalized size = 0.

$$\int \frac{1}{dx^3+c}\left(bx^3+a\right)^{\frac{4}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(4/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{dx^3+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/(d*x^3 + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx^3)^{\frac{4}{3}}}{c+dx^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(4/3)/(c + d*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^3+a)^{\frac{4}{3}}}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/(d*x^3 + c), x)
```

3.94 $\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx$

Optimal. Leaf size=59

$$\frac{x\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[Out] (x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)
])/(c*(1 + (b*x^3)/a)^(1/3))
```

Rubi [A] time = 0.0273635, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(c + d*x^3),x]
```

```
[Out] (x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)
])/(c*(1 + (b*x^3)/a)^(1/3))
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx^3}}{c+dx^3}\,dx &= \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{c+dx^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}\\ &= \frac{x\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{1+\frac{bx^3}{a}}}\end{aligned}$$

Mathematica [B] time = 0.150646, size = 160, normalized size = 2.71

$$\frac{4acx\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\left(c+dx^3\right)\left(x^3\left(bcF_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-3adF_1\left(\frac{4}{3};-\frac{1}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)+4acF_1\left(\frac{1}{3};-\frac{1}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(c + d*x^3),x]
```

```
[Out] (4*a*c*x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), -((d*x
^3)/c)])/((c + d*x^3)*(4*a*c*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), -((d
*x^3)/c)] + x^3*(-3*a*d*AppellF1[4/3, -1/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)
/c)] + b*c*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

Maple [F] time = 0.426, size = 0, normalized size = 0.

$$\int \frac{1}{dx^3+c}\sqrt[3]{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^(1/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/(d*x^3 + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{a + bx^3}}{c + dx^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/(d*x**3+c),x)
```

```
[Out] Integral((a + b*x**3)**(1/3)/(c + d*x**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^3 + a)^{\frac{1}{3}}}{dx^3 + c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/(d*x^3 + c), x)
```

3.95 $\int \frac{1}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=59

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3} F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\left(a+bx^3\right)^{2/3}}$$

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(c*(a + b*x^3)^(2/3))
```

Rubi [A] time = 0.0275713, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3} F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(c*(a + b*x^3)^(2/3))
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)}\,dx &= \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}\left(c+dx^3\right)}\,dx}{\left(a+bx^3\right)^{2/3}}\\ &= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\left(a+bx^3\right)^{2/3}}\end{aligned}$$

Mathematica [B] time = 0.0447395, size = 161, normalized size = 2.73

$$-\frac{4acxF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)\left(x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(2/3)*(c + d*x^3)),x]
```

```
[Out] (-4*a*c*x*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/((a + b*x
^3)^(2/3)*(c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d
*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c
)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

Maple [F] time = 0.418, size = 0, normalized size = 0.

$$\int \frac{1}{dx^3+c}\left(bx^3+a\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

```
[Out] int(1/(b*x^3+a)^(2/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^3\right)^{\frac{2}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(2/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/((a + b*x**3)**(2/3)*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)), x)
```

3.96 $\int \frac{1}{\left(a+bx^3\right)^{5/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=62

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};\frac{5}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac\left(a+bx^3\right)^{2/3}}$$

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 5/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(a*c*(a + b*x^3)^(2/3))
```

Rubi [A] time = 0.0281477, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};\frac{5}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(5/3)*(c + d*x^3)),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 5/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(a*c*(a + b*x^3)^(2/3))
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+bx^3\right)^{5/3}\left(c+dx^3\right)}\,dx &= \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{5/3}\left(c+dx^3\right)}\,dx}{a\left(a+bx^3\right)^{2/3}}\\ &= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{1}{3};\frac{5}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac\left(a+bx^3\right)^{2/3}}\end{aligned}$$

Mathematica [B] time = 0.256529, size = 332, normalized size = 5.35

$$\frac{x\left(\frac{4\left(bx^3\left(c+dx^3\right)\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)+4ac\left(2ad-b\left(2c+dx^3\right)\right)F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}{\left(c+dx^3\right)\left(4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}-\frac{bdx^3\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3}\right.}{c}\right.}{8a\left(a+bx^3\right)^{2/3}(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(5/3)*(c + d*x^3)),x]
```

```
[Out] (x*(-((b*d*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a
), -((d*x^3)/c)])/c) + (4*(4*a*c*(2*a*d - b*(2*c + d*x^3))*AppellF1[1/3, 2/
3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + b*x^3*(c + d*x^3)*(3*a*d*AppellF1[
4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1,
 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((c + d*x^3)*(4*a*c*AppellF1[1/3, 2/3,
 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3
, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/
a), -((d*x^3)/c)])))))/(8*a*(-(b*c) + a*d)*(a + b*x^3)^(2/3))
```

Maple [F] time = 0.418, size = 0, normalized size = 0.

$$\int \frac{1}{dx^3+c}\left(bx^3+a\right)^{-\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(5/3)/(d*x^3+c),x)
```

```
[Out] int(1/(b*x^3+a)^(5/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{5}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(5/3)*(d*x^3 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^3\right)^{\frac{5}{3}}\left(c+dx^3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(5/3)/(d*x**3+c),x)
```

```
[Out] Integral(1/((a + b*x**3)**(5/3)*(c + d*x**3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{5}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(5/3)*(d*x^3 + c)), x)
```

3.97 $\int \frac{1}{\left(a+bx^3\right)^{8/3}\left(c+dx^3\right)}\,dx$

Optimal. Leaf size=62

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};\frac{8}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c\left(a+bx^3\right)^{2/3}}$$

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 8/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(a^2*c*(a + b*x^3)^(2/3))
```

Rubi [A] time = 0.0288902, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};\frac{8}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(8/3)*(c + d*x^3)),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 8/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(a^2*c*(a + b*x^3)^(2/3))
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+bx^3\right)^{8/3}\left(c+dx^3\right)}\,dx &= \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{8/3}\left(c+dx^3\right)}\,dx}{a^2\left(a+bx^3\right)^{2/3}}\\
&= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{1}{3};\frac{8}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c\left(a+bx^3\right)^{2/3}}
\end{aligned}$$

Mathematica [B] time = 0.7847, size = 429, normalized size = 6.92

$$-\frac{x\left(\frac{4\left(4ac\left(-a^2bd\left(20c+dx^3\right)+10a^3d^2+ab^2\left(10c^2-12cdx^3-9d^2x^6\right)+4b^3cx^3\left(2c+dx^3\right)\right)F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+bx^3\left(c+dx^3\right)\left(11a^2d+ab\left(9dx^3-6c\right)-4b^2cx^3\right)\left(3adF_1\left(\frac{4}{3}\right.\right.\right.}{\left(a+bx^3\right)\left(c+dx^3\right)\left(x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right.\right.}\right.}{40a^2\left(a+bx^3\right)^{2/3}(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(8/3)*(c + d*x^3)),x]
```

```
[Out] -(x*((b*d*(-4*b*c + 9*a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1,
7/3, -((b*x^3)/a), -((d*x^3)/c)])/c + (4*(4*a*c*(10*a^3*d^2 + 4*b^3*c*x^3*(
2*c + d*x^3) - a^2*b*d*(20*c + d*x^3) + a*b^2*(10*c^2 - 12*c*d*x^3 - 9*d^2*
x^6))*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + b*x^3*(c + d
*x^3)*(11*a^2*d - 4*b^2*c*x^3 + a*b*(-6*c + 9*d*x^3))*(3*a*d*AppellF1[4/3,
2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3,
 -((b*x^3)/a), -((d*x^3)/c)])))/((a + b*x^3)*(c + d*x^3)*(-4*a*c*AppellF1[1
/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3
, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -(
(b*x^3)/a), -((d*x^3)/c)])))))/(40*a^2*(b*c - a*d)^2*(a + b*x^3)^(2/3))
```

Maple [F] time = 0.408, size = 0, normalized size = 0.

$$\int \frac{1}{dx^3+c}\left(bx^3+a\right)^{-\frac{8}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(8/3)/(d*x^3+c),x)
```

```
[Out] int(1/(b*x^3+a)^(8/3)/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{8}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(8/3)*(d*x^3 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(8/3)/(d*x**3+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{8}{3}}\left(dx^3+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(8/3)*(d*x^3 + c)), x)
```

3.98 $\int \frac{\left(a+bx^3\right)^{8/3}}{\left(c+dx^3\right)^2} dx$

Optimal. Leaf size=351

$$\frac{b^{5/3}(3bc-4ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{3d^3}-\frac{2b^{5/3}(3bc-4ad)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}d^3}+\frac{(bc-ad)^{5/3}(ad+3bc)\log\left(c+dx^3\right)}{9c^{5/3}d^3}-\frac{(bc-}{}$$

```
[Out] (b*(2*b*c - a*d)*x*(a + b*x^3)^(2/3))/(3*c*d^2) - ((b*c - a*d)*x*(a + b*x^3
)^(5/3))/(3*c*d*(c + d*x^3)) - (2*b^(5/3)*(3*b*c - 4*a*d)*ArcTan[(1 + (2*b^
(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[3]*d^3) + (2*(b*c - a*d)^(5/3
)*(3*b*c + a*d)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1
/3)))/Sqrt[3]])/(3*Sqrt[3]*c^(5/3)*d^3) + ((b*c - a*d)^(5/3)*(3*b*c + a*d)*
Log[c + d*x^3])/(9*c^(5/3)*d^3) - ((b*c - a*d)^(5/3)*(3*b*c + a*d)*Log[((b*
c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(3*c^(5/3)*d^3) + (b^(5/3)*
(3*b*c - 4*a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(3*d^3)
```

Rubi [C] time = 0.0286094, antiderivative size = 62, normalized size of antiderivative = 0.18, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{a^2x\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{8}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(8/3)/(c + d*x^3)^2,x]
```

```
[Out] (a^2*x*(a + b*x^3)^(2/3)*AppellF1[1/3, -8/3, 2, 4/3, -((b*x^3)/a), -((d*x^3
)/c)])/(c^2*(1 + (b*x^3)/a)^(2/3))
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^2}\,dx = \frac{\left(a^2(a+bx^3)^{2/3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{8/3}}{(c+dx^3)^2}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}} = \frac{a^2x(a+bx^3)^{2/3}F_1\left(\frac{1}{3};-\frac{8}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] time = 0.890721, size = 698, normalized size = 1.99

$$\frac{1}{18}\left(\frac{2a^2b\left(\log\left(\frac{x^2(bc-ad)^{2/3}}{(ax^3+b)^{2/3}}+\frac{\sqrt[3]{c}x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}+c^{2/3}\right)-2\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{ax^3+b}}+1}{\sqrt{3}}\right)\right)}{c^{2/3}d\sqrt[3]{bc-ad}}+\frac{2a^3\left(\log\left(\frac{x^2(bc-ad)^{2/3}}{(ax^3+b)^{2/3}}\right.\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(8/3)/(c + d*x^3)^2,x]
```

```
[Out] ((6*x*(a + b*x^3)^(2/3)*(b^2 + (b*c - a*d)^2/(c*(c + d*x^3))))/d^2 - (9*b^3
*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^
3)/c)])/(d^2*(a + b*x^3)^(1/3)) + (12*a*b^2*x^4*(1 + (b*x^3)/a)^(1/3)*Appel
lF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(c*d*(a + b*x^3)^(1/3))
+ (2*a^3*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3
)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3
)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*
c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(c^(5/3)*(b*c - a*d)^(1/3)) - (2*a*b
^2*c^(1/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x
^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1
/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(
b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(d^2*(b*c - a*d)^(1/3)) + (2*a^2*b
*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3))
)/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + Log
[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)
^(1/3)*x)/(b + a*x^3)^(1/3)]))/(c^(2/3)*d*(b*c - a*d)^(1/3)))/18
```

Maple [F] time = 0.248, size = 0, normalized size = 0.

$$\int \frac{1}{(dx^3+c)^2}(bx^3+a)^{\frac{8}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(8/3)/(d*x^3+c)^2,x)
```

```
[Out] int((b*x^3+a)^(8/3)/(d*x^3+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{8}{3}}}{\left(dx^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(8/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(8/3)/(d*x^3 + c)^2, x)
```

Fricas [B] time = 40.0811, size = 1810, normalized size = 5.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(8/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] 1/9*(2*sqrt(3)*(3*b^2*c^3 - 2*a*b*c^2*d - a^2*c*d^2 + (3*b^2*c^2*d - 2*a*b*
c*d^2 - a^2*d^3)*x^3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*arctan(-1
/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b
*c*d + a^2*d^2)/c^2)^(1/3))/((b*c - a*d)*x)) + 2*sqrt(3)*(3*b^2*c^3 - 4*a*b
*c^2*d + (3*b^2*c^2*d - 4*a*b*c*d^2)*x^3)*(-b^2)^(1/3)*arctan(-1/3*(sqrt(3)
*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)) - 2*(3*b^2*c^3 - 2*
a*b*c^2*d - a^2*c*d^2 + (3*b^2*c^2*d - 2*a*b*c*d^2 - a^2*d^3)*x^3)*((b^2*c^
2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^
2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d))/x) - 2*(3*b^2*c^3 - 4*a*b*c^
2*d + (3*b^2*c^2*d - 4*a*b*c*d^2)*x^3)*(-b^2)^(1/3)*log(-((-b^2)^(2/3)*x -
(b*x^3 + a)^(1/3)*b)/x) + (3*b^2*c^3 - 4*a*b*c^2*d + (3*b^2*c^2*d - 4*a*b*c
*d^2)*x^3)*(-b^2)^(1/3)*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)
^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2) + (3*b^2*c^3 - 2*a*b*c^2*d - a^2*c*d^2
 + (3*b^2*c^2*d - 2*a*b*c*d^2 - a^2*d^3)*x^3)*((b^2*c^2 - 2*a*b*c*d + a^2*d
^2)/c^2)^(1/3)*log(-((b*c - a*d)*x^2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^
(1/3) + (b*x^3 + a)^(1/3)*c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(2/3) +
 (b*x^3 + a)^(2/3)*(b*c - a*d))/x^2) + 3*(b^2*c*d^2*x^4 + (2*b^2*c^2*d - 2*
a*b*c*d^2 + a^2*d^3)*x)*(b*x^3 + a)^(2/3))/(c*d^4*x^3 + c^2*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(8/3)/(d*x**3+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{8}{3}}}{\left(dx^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(8/3)/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(8/3)/(d*x^3 + c)^2, x)
```

3.99 $\int \frac{\left(a+bx^3\right)^{5/3}}{\left(c+dx^3\right)^2} dx$

Optimal. Leaf size=301

$$-\frac{b^{5/3}\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{b}x\right)}{2d^2}+\frac{b^{5/3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}d^2}-\frac{(bc-ad)^{2/3}(2ad+3bc)\log\left(c+dx^3\right)}{18c^{5/3}d^2}+\frac{(bc-ad)^{2/3}(2ad+3bc)\log}{6c^{5/3}}$$

```
[Out] -((b*c - a*d)*x*(a + b*x^3)^(2/3))/(3*c*d*(c + d*x^3)) + (b^(5/3)*ArcTan[(1
 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^2) - ((b*c - a*d)^
(2/3)*(3*b*c + 2*a*d)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x
^3)^(1/3)))/Sqrt[3]])/(3*Sqrt[3]*c^(5/3)*d^2) - ((b*c - a*d)^(2/3)*(3*b*c +
 2*a*d)*Log[c + d*x^3])/(18*c^(5/3)*d^2) + ((b*c - a*d)^(2/3)*(3*b*c + 2*a*
d)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(6*c^(5/3)*d^2)
- (b^(5/3)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(2*d^2)
```

Rubi [C] time = 0.0279436, antiderivative size = 60, normalized size of antiderivative = 0.2, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{ax\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{5}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(5/3)/(c + d*x^3)^2,x]
```

```
[Out] (a*x*(a + b*x^3)^(2/3)*AppellF1[1/3, -5/3, 2, 4/3, -((b*x^3)/a), -((d*x^3)/
c)])/(c^2*(1 + (b*x^3)/a)^(2/3))
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^2}\,dx = \frac{\left(a\left(a+bx^3\right)^{2/3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{5/3}}{(c+dx^3)^2}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$
$$= \frac{ax\left(a+bx^3\right)^{2/3} F_1\left(\frac{1}{3};-\frac{5}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] time = 0.607246, size = 450, normalized size = 1.5

$$\frac{\frac{4a^2\left(\log\left(\frac{x^2(bc-ad)^{2/3}}{(ax^3+b)^{2/3}}+\frac{\sqrt[3]{c}x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}+c^{2/3}\right)-2\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{ax^3+b}}+1}{\sqrt{3}}\right)\right)}{\sqrt[3]{bc-ad}}+\frac{9b^2c^{2/3}x^4\sqrt[3]{\frac{bx^3}{a}+1}F_1\left(\frac{4}{3};\frac{1}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{d\sqrt[3]{a+bx^3}}-\frac{12c^{2/3}x(a+bx}{d(c+d}}{36c^{5/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(5/3)/(c + d*x^3)^2,x]
```

```
[Out] ((-12*c^(2/3)*(b*c - a*d)*x*(a + b*x^3)^(2/3))/(d*(c + d*x^3)) + (9*b^2*c^(
2/3)*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((
d*x^3)/c)])/(d*(a + b*x^3)^(1/3)) + (4*a^2*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c -
 a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*
c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2
)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(b
*c - a*d)^(1/3) + (2*a*b*c*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(
c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x
)/(b + a*x^3)^(1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2
/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(d*(b*c - a*d)^(1/
3)))/(36*c^(5/3))
```

Maple [F] time = 0.405, size = 0, normalized size = 0.

$$\int \frac{1}{(dx^3+c)^2}\left(bx^3+a\right)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(5/3)/(d*x^3+c)^2,x)
```

```
[Out] int((b*x^3+a)^(5/3)/(d*x^3+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^3+a)^{\frac{5}{3}}}{(dx^3+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(5/3)/(d*x^3 + c)^2, x)
```

Fricas [B] time = 3.59931, size = 1454, normalized size = 4.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] -1/18*(2*sqrt(3)*((3*b*c*d + 2*a*d^2)*x^3 + 3*b*c^2 + 2*a*c*d)*((b^2*c^2 -
2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt
(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3))/((b*c
- a*d)*x)) + 6*sqrt(3)*(b*c*d*x^3 + b*c^2)*(-b^2)^(1/3)*arctan(-1/3*(sqrt(3
)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)) + 6*(b*x^3 + a)^(2
/3)*(b*c*d - a*d^2)*x - 2*((3*b*c*d + 2*a*d^2)*x^3 + 3*b*c^2 + 2*a*c*d)*((b
^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*x*((b^2*c^2 - 2*a*b*c*d + a
^2*d^2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d))/x) - 6*(b*c*d*x^3 + b*c
^2)*(-b^2)^(1/3)*log(-((-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + 3*(b*c*d*
x^3 + b*c^2)*(-b^2)^(1/3)*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^
2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2) + ((3*b*c*d + 2*a*d^2)*x^3 + 3*b*c^2
 + 2*a*c*d)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log(-((b*c - a*d)*x
^2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3) + (b*x^3 + a)^(1/3)*c*x*((b^
2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(2/3) + (b*x^3 + a)^(2/3)*(b*c - a*d))/x^
2))/(c*d^3*x^3 + c^2*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(5/3)/(d*x**3+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3 + a\right)^{\frac{5}{3}}}{\left(dx^3 + c\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(5/3)/(d*x^3 + c)^2, x)
```

3.100 $\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^2} \, dx$

Optimal. Leaf size=182

$$\frac{a\log\left(c+dx^3\right)}{9c^{5/3}\sqrt[3]{bc-ad}}-\frac{a\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{3c^{5/3}\sqrt[3]{bc-ad}}+\frac{2a\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}c^{5/3}\sqrt[3]{bc-ad}}+\frac{x\left(a+bx^3\right)^{2/3}}{3c\left(c+dx^3\right)}$$

```
[Out] (x*(a + b*x^3)^(2/3))/(3*c*(c + d*x^3)) + (2*a*ArcTan[(1 + (2*(b*c - a*d)^(
1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(3*Sqrt[3]*c^(5/3)*(b*c - a*
d)^(1/3)) + (a*Log[c + d*x^3])/(9*c^(5/3)*(b*c - a*d)^(1/3)) - (a*Log[((b*c
 - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(3*c^(5/3)*(b*c - a*d)^(1/3)
)
```

Rubi [A] time = 0.210115, antiderivative size = 241, normalized size of antiderivative = 1.32, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {378, 377, 200, 31, 634, 617, 204, 628}

$$-\frac{2a\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{9c^{5/3}\sqrt[3]{bc-ad}}+\frac{a\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+c^{2/3}\right)}{9c^{5/3}\sqrt[3]{bc-ad}}+\frac{2a\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+\sqrt[3]{c}}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}\sqrt[3]{bc-ad}}+\frac{x\left(a+bx^3\right)^{2/3}}{3c\left(c+dx^3\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(c + d*x^3)^2,x]
```

```
[Out] (x*(a + b*x^3)^(2/3))/(3*c*(c + d*x^3)) + (2*a*ArcTan[(c^(1/3) + (2*(b*c -
a*d)^(1/3)*x)/(a + b*x^3)^(1/3))/(Sqrt[3]*c^(1/3))])/(3*Sqrt[3]*c^(5/3)*(b*
c - a*d)^(1/3)) - (2*a*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3
)])/(9*c^(5/3)*(b*c - a*d)^(1/3)) + (a*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2
)/(a + b*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(9*
c^(5/3)*(b*c - a*d)^(1/3))
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
```

```
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^2}\,dx = \frac{x(a+bx^3)^{2/3}}{3c(c+dx^3)} + \frac{(2a)\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)}\,dx}{3c}$$

$$= \frac{x(a+bx^3)^{2/3}}{3c(c+dx^3)} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^3}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c}$$

$$= \frac{x(a+bx^3)^{2/3}}{3c(c+dx^3)} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}x}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{9c^{5/3}} + \frac{(2a)\,\text{Subst}\left(\int \frac{2\sqrt[3]{c}+\sqrt[3]{bc-ad}x}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-ad}x+(bc-ad)^{2/}}\right.}{9c^{5/3}}$$

$$= \frac{x(a+bx^3)^{2/3}}{3c(c+dx^3)} - \frac{2a\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{9c^{5/3}\sqrt[3]{bc-ad}} + \frac{a\,\text{Subst}\left(\int \frac{1}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-ad}x+(bc-ad)^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c^{4/3}} + \cdots$$

$$= \frac{x(a+bx^3)^{2/3}}{3c(c+dx^3)} - \frac{2a\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{9c^{5/3}\sqrt[3]{bc-ad}} + \frac{a\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(a+bx^3)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{9c^{5/3}\sqrt[3]{bc-ad}} - \frac{(2a)\,\text{Subst}\left(\int\right.}{3\cdots}$$

$$= \frac{x(a+bx^3)^{2/3}}{3c(c+dx^3)} + \frac{2a\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c^{5/3}\sqrt[3]{bc-ad}} - \frac{2a\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{9c^{5/3}\sqrt[3]{bc-ad}} + \frac{a\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{(a+bx^3)^{2/3}}+\frac{\sqrt[3]{c}\cdots}{\cdots}\right.}{9c^{5/3}\sqrt[3]{bc-ad}}$$

Mathematica [C] time = 0.0295703, size = 78, normalized size = 0.43

$$\frac{x(a+bx^3)^{2/3}\,{}_2F_1\left(-\frac{2}{3},\frac{1}{3};\frac{4}{3};\frac{(ad-bc)x^3}{a(dx^3+c)}\right)}{c^2\left(\frac{bx^3}{a}+1\right)^{2/3}\sqrt[3]{\frac{dx^3}{c}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(2/3)/(c + d*x^3)^2,x]
```

```
[Out] (x*(a + b*x^3)^(2/3)*Hypergeometric2F1[-2/3, 1/3, 4/3, ((-(b*c) + a*d)*x^3)
/(a*(c + d*x^3))])/(c^2*(1 + (b*x^3)/a)^(2/3)*(1 + (d*x^3)/c)^(1/3))
```

Maple [F] time = 0.401, size = 0, normalized size = 0.

$$\int \frac{1}{(dx^3+c)^2}(bx^3+a)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/(d*x^3+c)^2,x)
```

```
[Out] int((b*x^3+a)^(2/3)/(d*x^3+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^3+a)^{\frac{2}{3}}}{(dx^3+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/(d*x^3 + c)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx^3\right)^{\frac{2}{3}}}{\left(c + dx^3\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/(d*x**3+c)**2,x)
```

```
[Out] Integral((a + b*x**3)**(2/3)/(c + d*x**3)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3 + a\right)^{\frac{2}{3}}}{\left(dx^3 + c\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/(d*x^3 + c)^2, x)
```

3.101 $\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)^2}\,dx$

Optimal. Leaf size=217

$$\frac{(3bc-2ad)\log\left(c+dx^3\right)}{18c^{5/3}(bc-ad)^{4/3}}-\frac{(3bc-2ad)\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{6c^{5/3}(bc-ad)^{4/3}}+\frac{(3bc-2ad)\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}c^{5/3}(bc-ad)^{4/3}}-\frac{dx\left(a+bx^3\right)}{3c\left(c+dx^3\right)(bc}$$

```
[Out] -(d*x*(a + b*x^3)^(2/3))/(3*c*(b*c - a*d)*(c + d*x^3)) + ((3*b*c - 2*a*d)*A
rcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(
3*Sqrt[3]*c^(5/3)*(b*c - a*d)^(4/3)) + ((3*b*c - 2*a*d)*Log[c + d*x^3])/(18
*c^(5/3)*(b*c - a*d)^(4/3)) - ((3*b*c - 2*a*d)*Log[((b*c - a*d)^(1/3)*x)/c^
(1/3) - (a + b*x^3)^(1/3)])/(6*c^(5/3)*(b*c - a*d)^(4/3))
```

Rubi [A] time = 0.204004, antiderivative size = 276, normalized size of antiderivative = 1.27, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {382, 377, 200, 31, 634, 617, 204, 628}

$$-\frac{(3bc-2ad)\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{9c^{5/3}(bc-ad)^{4/3}}+\frac{(3bc-2ad)\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+c^{2/3}\right)}{18c^{5/3}(bc-ad)^{4/3}}+\frac{(3bc-2ad)\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+\sqrt[3]{c}}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}(bc-ad)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(1/3)*(c + d*x^3)^2),x]
```

```
[Out] -(d*x*(a + b*x^3)^(2/3))/(3*c*(b*c - a*d)*(c + d*x^3)) + ((3*b*c - 2*a*d)*A
rcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3))/(Sqrt[3]*c^(1/3
))])/(3*Sqrt[3]*c^(5/3)*(b*c - a*d)^(4/3)) - ((3*b*c - 2*a*d)*Log[c^(1/3) -
 ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(9*c^(5/3)*(b*c - a*d)^(4/3)) +
((3*b*c - 2*a*d)*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(2/3) +
(c^(1/3)*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(18*c^(5/3)*(b*c - a*d)^(
4/3))
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
```

```
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)^2}dx = -\frac{dx\left(a+bx^3\right)^{2/3}}{3c(bc-ad)\left(c+dx^3\right)} + \frac{(3bc-2ad)\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}dx}{3c(bc-ad)}$$

$$= -\frac{dx\left(a+bx^3\right)^{2/3}}{3c(bc-ad)\left(c+dx^3\right)} + \frac{(3bc-2ad)\,\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^3}dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c(bc-ad)}$$

$$= -\frac{dx\left(a+bx^3\right)^{2/3}}{3c(bc-ad)\left(c+dx^3\right)} + \frac{(3bc-2ad)\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}x}dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{9c^{5/3}(bc-ad)} + \frac{(3bc-2ad)}{}$$

$$= -\frac{dx\left(a+bx^3\right)^{2/3}}{3c(bc-ad)\left(c+dx^3\right)} - \frac{(3bc-2ad)\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{9c^{5/3}(bc-ad)^{4/3}} + \frac{(3bc-2ad)\,\text{Subst}\left(\int \frac{\sqrt[3]{c}\sqrt[3]{}}{c^{2/3}+\sqrt[3]{c}}\right.}{18c^{5/3}(}$$

$$= -\frac{dx\left(a+bx^3\right)^{2/3}}{3c(bc-ad)\left(c+dx^3\right)} - \frac{(3bc-2ad)\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{9c^{5/3}(bc-ad)^{4/3}} + \frac{(3bc-2ad)\log\left(c^{2/3}+\frac{(bc-ad}{(a+bx}\right.}{18c^{5/3}(bc-ad}$$

$$= -\frac{dx\left(a+bx^3\right)^{2/3}}{3c(bc-ad)\left(c+dx^3\right)} + \frac{(3bc-2ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c^{5/3}(bc-ad)^{4/3}} - \frac{(3bc-2ad)\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx}}\right.}{9c^{5/3}(bc-ad)^{4/3}}$$

Mathematica [C] time = 0.106124, size = 99, normalized size = 0.46

$$\frac{x\left(\left(c+dx^3\right)(3bc-2ad)\,_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)-cd\left(a+bx^3\right)\right)}{3c^2\sqrt[3]{a+bx^3}\left(c+dx^3\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^3)^(1/3)*(c + d*x^3)^2),x]
```

```
[Out] (x*(-(c*d*(a + b*x^3)) + (3*b*c - 2*a*d)*(c + d*x^3)*Hypergeometric2F1[1/3,
 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]))/(3*c^2*(b*c - a*d)*(a + b*x^3
)^(1/3)*(c + d*x^3))
```

Maple [F] time = 0.408, size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^3+c\right)^2}\frac{1}{\sqrt[3]{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(1/3)/(d*x^3+c)^2,x)
```

```
[Out] int(1/(b*x^3+a)^(1/3)/(d*x^3+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(1/3)/(d*x**3+c)**2,x)
```

```
[Out] Exception raised: ValueError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)^2), x)
```

3.102 $\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^2}\,dx$

Optimal. Leaf size=261

$$-\frac{d(3bc-ad)\log\left(c+dx^3\right)}{9c^{5/3}(bc-ad)^{7/3}}+\frac{d(3bc-ad)\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{3c^{5/3}(bc-ad)^{7/3}}-\frac{2d(3bc-ad)\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}c^{5/3}(bc-ad)^{7/3}}+\frac{bx(ad-}{3ac\sqrt[3]{a+bx}}$$

```
[Out] (b*(3*b*c + a*d)*x)/(3*a*c*(b*c - a*d)^2*(a + b*x^3)^(1/3)) - (d*x)/(3*c*(b
*c - a*d)*(a + b*x^3)^(1/3)*(c + d*x^3)) - (2*d*(3*b*c - a*d)*ArcTan[(1 + (
2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(3*Sqrt[3]*c^
(5/3)*(b*c - a*d)^(7/3)) - (d*(3*b*c - a*d)*Log[c + d*x^3])/(9*c^(5/3)*(b*c
 - a*d)^(7/3)) + (d*(3*b*c - a*d)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a +
b*x^3)^(1/3)])/(3*c^(5/3)*(b*c - a*d)^(7/3))
```

Rubi [C] time = 1.93369, antiderivative size = 625, normalized size of antiderivative = 2.39, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$-\frac{c\left(a+bx^3\right)^{2/3}\left(-\frac{54d^2x^{15}(bc-ad)^3\,{}_3F_2\left(2,2,\frac{7}{3};1,\frac{13}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)}{c^5\left(a+bx^3\right)^3}-\frac{108dx^{12}(bc-ad)^3\,{}_3F_2\left(2,2,\frac{7}{3};1,\frac{13}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)}{c^4\left(a+bx^3\right)^3}-\frac{54x^9(bc-ad)^3\,{}_3F_2\left(2,2,\frac{7}{3};1,\frac{13}{3};\frac{(bc-ad)x^3}{c(bx^3+a)}\right)}{c^3\left(a+bx^3\right)^3}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((a + b*x^3)^(4/3)*(c + d*x^3)^2),x]
```

```
[Out] -(c*(a + b*x^3)^(2/3)*(6860 + (13720*d*x^3)/c + (6300*d^2*x^6)/c^2 - (525*(
b*c - a*d)*x^3)/(c*(a + b*x^3)) - (1890*d*(b*c - a*d)*x^6)/(c^2*(a + b*x^3)
) - (945*d^2*(b*c - a*d)*x^9)/(c^3*(a + b*x^3)) - 6860*Hypergeometric2F1[1/
3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - (13720*d*x^3*Hypergeometric
2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/c - (6300*d^2*x^6*Hype
rgeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/c^2 + (2240*
(b*c - a*d)*x^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*
x^3))])/(c*(a + b*x^3)) + (5320*d*(b*c - a*d)*x^6*Hypergeometric2F1[1/3, 1,
 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(c^2*(a + b*x^3)) + (2520*d^2*(b*
c - a*d)*x^9*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3
))])/(c^3*(a + b*x^3)) - (54*(b*c - a*d)^3*x^9*HypergeometricPFQ[{2, 2, 7/3
}, {1, 13/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(c^3*(a + b*x^3)^3) - (10
8*d*(b*c - a*d)^3*x^12*HypergeometricPFQ[{2, 2, 7/3}, {1, 13/3}, ((b*c - a*
d)*x^3)/(c*(a + b*x^3))])/(c^4*(a + b*x^3)^3) - (54*d^2*(b*c - a*d)^3*x^15*
HypergeometricPFQ[{2, 2, 7/3}, {1, 13/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))
])/(c^5*(a + b*x^3)^3)))/(420*(b*c - a*d)^2*x^5*(c + d*x^3))
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^2}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{4/3}\left(c+dx^3\right)^2}\,dx}{a\sqrt[3]{a+bx^3}}$$

$$= -\frac{c\left(a+bx^3\right)^{2/3}\left(6860+\frac{13720dx^3}{c}+\frac{6300d^2x^6}{c^2}-\frac{525(bc-ad)x^3}{c\left(a+bx^3\right)}-\frac{1890d(bc-ad)x^6}{c^2\left(a+bx^3\right)}-\frac{945d^2(bc-ad)x^9}{c^3\left(a+bx^3\right)}-\cdots\right.}{\cdots}$$

Mathematica [C] time = 1.45104, size = 625, normalized size = 2.39

$$\frac{c\left(a+bx^3\right)^{2/3}\left(\frac{54d^2x^{15}(bc-ad)^3\text{HypergeometricPFQ}\left(\left\{2,2,\frac{7}{3}\right\},\left\{1,\frac{13}{3}\right\},\frac{x^3(bc-ad)}{c\left(a+bx^3\right)}\right)}{c^5\left(a+bx^3\right)^3}+\frac{108dx^{12}(bc-ad)^3\text{HypergeometricPFQ}\left(\left\{2,2,\frac{7}{3}\right\},\left\{1,\frac{13}{3}\right\},\frac{x^3(bc-ad)}{c\left(a+bx^3\right)}\right)}{c^4\left(a+bx^3\right)^3}+\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(4/3)*(c + d*x^3)^2),x]
```

```
[Out] (c*(a + b*x^3)^(2/3)*(-6860 - (13720*d*x^3)/c - (6300*d^2*x^6)/c^2 + (525*(
b*c - a*d)*x^3)/(c*(a + b*x^3)) + (1890*d*(b*c - a*d)*x^6)/(c^2*(a + b*x^3)
) + (945*d^2*(b*c - a*d)*x^9)/(c^3*(a + b*x^3)) + 6860*Hypergeometric2F1[1/
3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + (13720*d*x^3*Hypergeometric
2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/c + (6300*d^2*x^6*Hype
rgeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/c^2 - (2240*
(b*c - a*d)*x^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*
x^3))])/(c*(a + b*x^3)) + (5320*d*(-(b*c) + a*d)*x^6*Hypergeometric2F1[1/3,
 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(c^2*(a + b*x^3)) + (2520*d^2*
(-(b*c) + a*d)*x^9*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a +
 b*x^3))])/(c^3*(a + b*x^3)) + (54*(b*c - a*d)^3*x^9*HypergeometricPFQ[{2,
2, 7/3}, {1, 13/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(c^3*(a + b*x^3)^3)
 + (108*d*(b*c - a*d)^3*x^12*HypergeometricPFQ[{2, 2, 7/3}, {1, 13/3}, ((b*
c - a*d)*x^3)/(c*(a + b*x^3))])/(c^4*(a + b*x^3)^3) + (54*d^2*(b*c - a*d)^3
*x^15*HypergeometricPFQ[{2, 2, 7/3}, {1, 13/3}, ((b*c - a*d)*x^3)/(c*(a + b
*x^3))])/(c^5*(a + b*x^3)^3)))/(420*(b*c - a*d)^2*x^5*(c + d*x^3))
```

Maple [F] time = 0.403, size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^3+c\right)^2}\left(bx^3+a\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(4/3)/(d*x^3+c)^2,x)
```

```
[Out] int(1/(b*x^3+a)^(4/3)/(d*x^3+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(4/3)/(d*x**3+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)^2), x)
```

3.103 $\int \frac{1}{\left(a+bx^3\right)^{7/3}\left(c+dx^3\right)^2}\,dx$

Optimal. Leaf size=324

$$\frac{bx\left(-4a^2d^2-33abcd+9b^2c^2\right)}{12a^2c\sqrt[3]{a+bx^3}(bc-ad)^3}+\frac{d^2(9bc-2ad)\log\left(c+dx^3\right)}{18c^{5/3}(bc-ad)^{10/3}}-\frac{d^2(9bc-2ad)\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{6c^{5/3}(bc-ad)^{10/3}}+\frac{d^2(9bc-2ad)}{3\sqrt{3}c^{5/3}}$$

```
[Out] (b*(3*b*c + 4*a*d)*x)/(12*a*c*(b*c - a*d)^2*(a + b*x^3)^(4/3)) + (b*(9*b^2*
c^2 - 33*a*b*c*d - 4*a^2*d^2)*x)/(12*a^2*c*(b*c - a*d)^3*(a + b*x^3)^(1/3))
 - (d*x)/(3*c*(b*c - a*d)*(a + b*x^3)^(4/3)*(c + d*x^3)) + (d^2*(9*b*c - 2*
a*d)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[
3]])/(3*Sqrt[3]*c^(5/3)*(b*c - a*d)^(10/3)) + (d^2*(9*b*c - 2*a*d)*Log[c +
d*x^3])/(18*c^(5/3)*(b*c - a*d)^(10/3)) - (d^2*(9*b*c - 2*a*d)*Log[((b*c -
a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(6*c^(5/3)*(b*c - a*d)^(10/3))
```

Rubi [C] time = 5.6921, antiderivative size = 1214, normalized size of antiderivative = 3.75, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{6804d^3(bc-ad)^4\,{}_3F_2\left(2,2,\frac{10}{3};1,\frac{16}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)x^{21}+1134d^3(bc-ad)^4\,{}_4F_3\left(2,2,2,\frac{10}{3};1,1,\frac{16}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)x^{21}+21546cd^2(bc-}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((a + b*x^3)^(7/3)*(c + d*x^3)^2),x]
```

```
[Out] (26130*c^5*(b*c - a*d)^2*x^6*(a + b*x^3)^2 + 89505*c^4*d*(b*c - a*d)^2*x^9*
(a + b*x^3)^2 + 84240*c^3*d^2*(b*c - a*d)^2*x^12*(a + b*x^3)^2 + 26325*c^2*
d^3*(b*c - a*d)^2*x^15*(a + b*x^3)^2 + 748020*c^6*(b*c - a*d)*x^3*(a + b*x^
3)^3 + 2113020*c^5*d*(b*c - a*d)*x^6*(a + b*x^3)^3 + 1916460*c^4*d^2*(b*c -
 a*d)*x^9*(a + b*x^3)^3 + 589680*c^3*d^3*(b*c - a*d)*x^12*(a + b*x^3)^3 - 2
002000*c^7*(a + b*x^3)^4 - 5460000*c^6*d*x^3*(a + b*x^3)^4 - 4914000*c^5*d^
2*x^6*(a + b*x^3)^4 - 1506960*c^4*d^3*x^9*(a + b*x^3)^4 - 1248520*c^6*(b*c
- a*d)*x^3*(a + b*x^3)^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(
c*(a + b*x^3))] - 3478020*c^5*d*(b*c - a*d)*x^6*(a + b*x^3)^3*Hypergeometri
c2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 3144960*c^4*d^2*(b*c
 - a*d)*x^9*(a + b*x^3)^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/
(c*(a + b*x^3))] - 966420*c^3*d^3*(b*c - a*d)*x^12*(a + b*x^3)^3*Hypergeome
tric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 2002000*c^7*(a +
b*x^3)^4*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]
+ 5460000*c^6*d*x^3*(a + b*x^3)^4*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*
d)*x^3)/(c*(a + b*x^3))] + 4914000*c^5*d^2*x^6*(a + b*x^3)^4*Hypergeometric
2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 1506960*c^4*d^3*x^9*(
a + b*x^3)^4*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3
))] + 7938*c^3*(b*c - a*d)^4*x^12*HypergeometricPFQ[{2, 2, 10/3}, {1, 16/3}
, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 22680*c^2*d*(b*c - a*d)^4*x^15*Hyper
geometricPFQ[{2, 2, 10/3}, {1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] +
21546*c*d^2*(b*c - a*d)^4*x^18*HypergeometricPFQ[{2, 2, 10/3}, {1, 16/3}, (
(b*c - a*d)*x^3)/(c*(a + b*x^3))] + 6804*d^3*(b*c - a*d)^4*x^21*Hypergeomet
ricPFQ[{2, 2, 10/3}, {1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 1134*c
^3*(b*c - a*d)^4*x^12*HypergeometricPFQ[{2, 2, 2, 10/3}, {1, 1, 16/3}, ((b*
```

```
c - a*d)*x^3)/(c*(a + b*x^3))] + 3402*c^2*d*(b*c - a*d)^4*x^15*Hypergeometr
icPFQ[{2, 2, 2, 10/3}, {1, 1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 3
402*c*d^2*(b*c - a*d)^4*x^18*HypergeometricPFQ[{2, 2, 2, 10/3}, {1, 1, 16/3
}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 1134*d^3*(b*c - a*d)^4*x^21*Hyperge
ometricPFQ[{2, 2, 2, 10/3}, {1, 1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))
])/(21840*c^5*(b*c - a*d)^3*x^8*(a + b*x^3)^(10/3)*(c + d*x^3))
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^{7/3}\left(c+dx^3\right)^2}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{7/3}\left(c+dx^3\right)^2}\,dx}{a^2\sqrt[3]{a+bx^3}}$$

$$= \frac{26130c^5(bc-ad)^2x^6\left(a+bx^3\right)^2+89505c^4d(bc-ad)^2x^9\left(a+bx^3\right)^2+84240c^3d^2(bc-}{}$$

Mathematica [C] time = 4.10342, size = 1216, normalized size = 3.75

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(7/3)*(c + d*x^3)^2),x]
```

```
[Out] -(26130*c^5*(b*c - a*d)^2*x^6*(a + b*x^3)^2 + 89505*c^4*d*(b*c - a*d)^2*x^9
*(a + b*x^3)^2 + 84240*c^3*d^2*(b*c - a*d)^2*x^12*(a + b*x^3)^2 + 26325*c^2
*d^3*(b*c - a*d)^2*x^15*(a + b*x^3)^2 + 748020*c^6*(b*c - a*d)*x^3*(a + b*x
^3)^3 + 2113020*c^5*d*(b*c - a*d)*x^6*(a + b*x^3)^3 + 1916460*c^4*d^2*(b*c
- a*d)*x^9*(a + b*x^3)^3 + 589680*c^3*d^3*(b*c - a*d)*x^12*(a + b*x^3)^3 -
2002000*c^7*(a + b*x^3)^4 - 5460000*c^6*d*x^3*(a + b*x^3)^4 - 4914000*c^5*d
^2*x^6*(a + b*x^3)^4 - 1506960*c^4*d^3*x^9*(a + b*x^3)^4 - 1248520*c^6*(b*c
 - a*d)*x^3*(a + b*x^3)^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/
(c*(a + b*x^3))] - 3478020*c^5*d*(b*c - a*d)*x^6*(a + b*x^3)^3*Hypergeometr
ic2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 3144960*c^4*d^2*(b*
c - a*d)*x^9*(a + b*x^3)^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)
/(c*(a + b*x^3))] - 966420*c^3*d^3*(b*c - a*d)*x^12*(a + b*x^3)^3*Hypergeom
etric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 2002000*c^7*(a +
 b*x^3)^4*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]
 + 5460000*c^6*d*x^3*(a + b*x^3)^4*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a
*d)*x^3)/(c*(a + b*x^3))] + 4914000*c^5*d^2*x^6*(a + b*x^3)^4*Hypergeometri
```

```
c2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 1506960*c^4*d^3*x^9*
(a + b*x^3)^4*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^
3))] + 7938*c^3*(b*c - a*d)^4*x^12*HypergeometricPFQ[{2, 2, 10/3}, {1, 16/3
}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 22680*c^2*d*(b*c - a*d)^4*x^15*Hype
rgeometricPFQ[{2, 2, 10/3}, {1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] +
 21546*c*d^2*(b*c - a*d)^4*x^18*HypergeometricPFQ[{2, 2, 10/3}, {1, 16/3},
((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 6804*d^3*(b*c - a*d)^4*x^21*Hypergeome
tricPFQ[{2, 2, 10/3}, {1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 1134*
c^3*(b*c - a*d)^4*x^12*HypergeometricPFQ[{2, 2, 2, 10/3}, {1, 1, 16/3}, ((b
*c - a*d)*x^3)/(c*(a + b*x^3))] + 3402*c^2*d*(b*c - a*d)^4*x^15*Hypergeomet
ricPFQ[{2, 2, 2, 10/3}, {1, 1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] +
3402*c*d^2*(b*c - a*d)^4*x^18*HypergeometricPFQ[{2, 2, 2, 10/3}, {1, 1, 16/
3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 1134*d^3*(b*c - a*d)^4*x^21*Hyperg
eometricPFQ[{2, 2, 2, 10/3}, {1, 1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3)
)])/(21840*c^8*(-b + (a*d)/c)^3*x^8*(a + b*x^3)^(10/3)*(c + d*x^3))
```

Maple [F] time = 0.423, size = 0, normalized size = 0.

$$\int \frac{1}{\left(d x^{3}+c\right)^{2}} \left(b x^{3}+a\right)^{-\frac{7}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(7/3)/(d*x^3+c)^2,x)
```

```
[Out] int(1/(b*x^3+a)^(7/3)/(d*x^3+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b x^{3}+a\right)^{\frac{7}{3}} \left(d x^{3}+c\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(7/3)*(d*x^3 + c)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(7/3)/(d*x**3+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{7}{3}}\left(dx^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(7/3)*(d*x^3 + c)^2), x)
```

3.104 $\int \frac{\left(a+bx^3\right)^{4/3}}{\left(c+dx^3\right)^2} \, dx$

Optimal. Leaf size=60

$$\frac{ax\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{4}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[Out] (a*x*(a + b*x^3)^(1/3)*AppellF1[1/3, -4/3, 2, 4/3, -((b*x^3)/a), -((d*x^3)/
c)])/(c^2*(1 + (b*x^3)/a)^(1/3))
```

Rubi [A] time = 0.0272188, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{ax\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{4}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(4/3)/(c + d*x^3)^2,x]
```

```
[Out] (a*x*(a + b*x^3)^(1/3)*AppellF1[1/3, -4/3, 2, 4/3, -((b*x^3)/a), -((d*x^3)/
c)])/(c^2*(1 + (b*x^3)/a)^(1/3))
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^{4/3}}{\left(c+dx^3\right)^2} \, dx &= \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{\left(c+dx^3\right)^2} \, dx}{\sqrt[3]{1+\frac{bx^3}{a}}}\\ &= \frac{ax\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{4}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\sqrt[3]{1+\frac{bx^3}{a}}}\end{aligned}$$

Mathematica [B] time = 0.333535, size = 341, normalized size = 5.68

$$\frac{x\left(\frac{4c\left(x^3\left(a+bx^3\right)(ad-bc)\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4ac\left(3a^2d+abdx^3-b^2cx^3\right)F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}{\left(c+dx^3\right)\left(x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}+bx^3\left(\frac{bx^3}{a}+1\right.\right.}{12c^2d\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(4/3)/(c + d*x^3)^2,x]
```

```
[Out] (x*(b*(2*b*c + a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -(
(b*x^3)/a), -((d*x^3)/c)] + (4*c*(-4*a*c*(3*a^2*d - b^2*c*x^3 + a*b*d*x^3)*
AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + (-(b*c) + a*d)*x^3
*(a + b*x^3)*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)]
+ 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((c + d*x
^3)*(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3
*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF
1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))))/(12*c^2*d*(a + b*x^3)^
(2/3))
```

Maple [F] time = 0.441, size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^3+c\right)^2}\left(bx^3+a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/(d*x^3+c)^2,x)
```

```
[Out] int((b*x^3+a)^(4/3)/(d*x^3+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{\left(dx^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/(d*x^3 + c)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/(d*x**3+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3 + a\right)^{\frac{4}{3}}}{\left(dx^3 + c\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/(d*x^3 + c)^2, x)
```

3.105 $\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^2}\,dx$

Optimal. Leaf size=59

$$\frac{x\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{1}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[Out] (x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 2, 4/3, -((b*x^3)/a), -((d*x^3)/c)
])/(c^2*(1 + (b*x^3)/a)^(1/3))
```

Rubi [A] time = 0.026517, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{1}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(c + d*x^3)^2,x]
```

```
[Out] (x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 2, 4/3, -((b*x^3)/a), -((d*x^3)/c)
])/(c^2*(1 + (b*x^3)/a)^(1/3))
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^2}\,dx &= \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{(c+dx^3)^2}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}\\ &= \frac{x\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{1}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\sqrt[3]{1+\frac{bx^3}{a}}}\end{aligned}$$

Mathematica [B] time = 0.202273, size = 232, normalized size = 3.93

$$\frac{x\left(\frac{4\left(\frac{a+bx^3}{c}-\frac{8a^2F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}\right)}{c+dx^3}+\frac{bx^3\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2}\right)}{12\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(c + d*x^3)^2,x]
```

```
[Out] (x*((b*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -
((d*x^3)/c)])/c^2 + (4*((a + b*x^3)/c - (8*a^2*AppellF1[1/3, 2/3, 1, 4/3, -
((b*x^3)/a), -((d*x^3)/c)])/(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a)
, -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*
x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))
/(c + d*x^3)))/(12*(a + b*x^3)^(2/3))
```

Maple [F] time = 0.417, size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^3+c\right)^2}\sqrt[3]{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/(d*x^3+c)^2,x)
```

```
[Out] int((b*x^3+a)^(1/3)/(d*x^3+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/(d*x^3 + c)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{a+bx^3}}{\left(c+dx^3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/(d*x**3+c)**2,x)
```

```
[Out] Integral((a + b*x**3)**(1/3)/(c + d*x**3)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/(d*x^3 + c)^2, x)
```

3.106 $\int \frac{1}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)^2}\,dx$

Optimal. Leaf size=59

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};\frac{2}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\left(a+bx^3\right)^{2/3}}$$

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 2/3, 2, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(c^2*(a + b*x^3)^(2/3))
```

Rubi [A] time = 0.0273377, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};\frac{2}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(2/3)*(c + d*x^3)^2),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 2/3, 2, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(c^2*(a + b*x^3)^(2/3))
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)^2}\,dx &= \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}\left(c+dx^3\right)^2}\,dx}{\left(a+bx^3\right)^{2/3}}\\ &= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{1}{3};\frac{2}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2\left(a+bx^3\right)^{2/3}}\end{aligned}$$

Mathematica [B] time = 0.272235, size = 393, normalized size = 6.66

$$\frac{4acxF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\left(bdx^3\left(\frac{bx^3}{a}+1\right)^{2/3}\left(c+dx^3\right)F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+4c\left(3ad-3bc+bdx^3\right)\right)-dx^4\left(b\right.}{12c^2\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)(bc-ad)\left(x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a}\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(2/3)*(c + d*x^3)^2),x]
```

```
[Out] (4*a*c*x*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)]*(4*c*(-3*b*
c + 3*a*d + b*d*x^3) + b*d*x^3*(1 + (b*x^3)/a)^(2/3)*(c + d*x^3)*AppellF1[4
/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]) - d*x^4*(4*c*(a + b*x^3) + b*
x^3*(1 + (b*x^3)/a)^(2/3)*(c + d*x^3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/
a), -((d*x^3)/c)])*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3
)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))/(12*
c^2*(b*c - a*d)*(a + b*x^3)^(2/3)*(c + d*x^3)*(-4*a*c*AppellF1[1/3, 2/3, 1,
 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -
((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a),
 -((d*x^3)/c)])))
```

Maple [F] time = 0.412, size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^3+c\right)^2}\left(bx^3+a\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(2/3)/(d*x^3+c)^2,x)
```

```
[Out] int(1/(b*x^3+a)^(2/3)/(d*x^3+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(2/3)/(d*x**3+c)**2,x)
```

```
[Out] Exception raised: ValueError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)^2), x)
```

3.107 $\int \frac{1}{\left(a+bx^3\right)^{5/3}\left(c+dx^3\right)^2}\,dx$

Optimal. Leaf size=62

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};\frac{5}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac^2\left(a+bx^3\right)^{2/3}}$$

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 5/3, 2, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(a*c^2*(a + b*x^3)^(2/3))
```

Rubi [A] time = 0.0284627, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};\frac{5}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac^2\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(5/3)*(c + d*x^3)^2),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 5/3, 2, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(a*c^2*(a + b*x^3)^(2/3))
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+bx^3\right)^{5/3}\left(c+dx^3\right)^2}\,dx &= \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{5/3}\left(c+dx^3\right)^2}\,dx}{a\left(a+bx^3\right)^{2/3}}\\ &= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{1}{3};\frac{5}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac^2\left(a+bx^3\right)^{2/3}}\end{aligned}$$

Mathematica [B] time = 0.504609, size = 386, normalized size = 6.23

$$\frac{x\left(\frac{c\left(16ac\left(6a^2d^2+2abd\left(dx^3-6c\right)+3b^2c\left(2c+dx^3\right)\right)F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-4x^3\left(2a^2d^2+2abd^2x^3+3b^2c\left(c+dx^3\right)\right)\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a}\cdots\right.\right.\right.}{\left(c+dx^3\right)\left(4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)\right)}\cdots\right)}{24ac^2\left(a+bx^3\right)^{2/3}(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(5/3)*(c + d*x^3)^2),x]
```

```
[Out] (x*(b*d*(3*b*c + 2*a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3
, -((b*x^3)/a), -((d*x^3)/c)] + (c*(16*a*c*(6*a^2*d^2 + 2*a*b*d*(-6*c + d*x
^3) + 3*b^2*c*(2*c + d*x^3))*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*
x^3)/c)] - 4*x^3*(2*a^2*d^2 + 2*a*b*d^2*x^3 + 3*b^2*c*(c + d*x^3))*(3*a*d*A
ppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3,
 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((c + d*x^3)*(4*a*c*AppellF1[1
/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - x^3*(3*a*d*AppellF1[4/3, 2/3
, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -(
(b*x^3)/a), -((d*x^3)/c)])))))/(24*a*c^2*(b*c - a*d)^2*(a + b*x^3)^(2/3))
```

Maple [F] time = 0.411, size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^3+c\right)^2}\left(bx^3+a\right)^{-\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(5/3)/(d*x^3+c)^2,x)
```

```
[Out] int(1/(b*x^3+a)^(5/3)/(d*x^3+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{5}{3}}\left(dx^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(5/3)*(d*x^3 + c)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(5/3)/(d*x**3+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{5}{3}}\left(dx^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(5/3)*(d*x^3 + c)^2), x)
```

3.108 $\int \frac{1}{\left(a+bx^3\right)^{8/3}\left(c+dx^3\right)^2}\,dx$

Optimal. Leaf size=62

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};\frac{8}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c^2\left(a+bx^3\right)^{2/3}}$$

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 8/3, 2, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(a^2*c^2*(a + b*x^3)^(2/3))
```

Rubi [A] time = 0.0278996, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};\frac{8}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c^2\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(8/3)*(c + d*x^3)^2),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 8/3, 2, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(a^2*c^2*(a + b*x^3)^(2/3))
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+bx^3\right)^{8/3}\left(c+dx^3\right)^2}\,dx &= \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{8/3}\left(c+dx^3\right)^2}\,dx}{a^2\left(a+bx^3\right)^{2/3}}\\ &= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{1}{3};\frac{8}{3},2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c^2\left(a+bx^3\right)^{2/3}}\end{aligned}$$

Mathematica [B] time = 0.982707, size = 550, normalized size = 8.87

$$\frac{\frac{bdx^4\left(\frac{bx^3}{a}+1\right)^{2/3}\left(5a^2d^2+21abcd-6b^2c^2\right)F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(ad-bc)^3}+\frac{4c\left(x^4\left(a^2b^2d\left(24c^2+24cdx^3+5d^2x^6\right)+10a^3bd^3x^3+5a^4d^3+3ab^3c\left(-3c^2+4cdx^3+7d^2x^6\right)-6b^4c\right.\right.}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(8/3)*(c + d*x^3)^2),x]
```

```
[Out] ((b*d*(-6*b^2*c^2 + 21*a*b*c*d + 5*a^2*d^2)*x^4*(1 + (b*x^3)/a)^(2/3)*Appel
lF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(-(b*c) + a*d)^3 + (4*c*
(-4*a*c*x*(15*a^4*d^3 - 6*b^4*c^2*x^3*(2*c + d*x^3) + 5*a^3*b*d^2*(-9*c + 4
*d*x^3) + a^2*b^2*d*(45*c^2 - 21*c*d*x^3 + 5*d^2*x^6) + 3*a*b^3*c*(-5*c^2 +
 11*c*d*x^3 + 7*d^2*x^6))*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3
)/c)] + x^4*(5*a^4*d^3 + 10*a^3*b*d^3*x^3 - 6*b^4*c^2*x^3*(c + d*x^3) + a^2
*b^2*d*(24*c^2 + 24*c*d*x^3 + 5*d^2*x^6) + 3*a*b^3*c*(-3*c^2 + 4*c*d*x^3 +
7*d^2*x^6))*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] +
 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((b*c - a*
d)^3*(a + b*x^3)*(c + d*x^3)*(4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a)
, -((d*x^3)/c)] - x^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*
x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))
/(60*a^2*c^2*(a + b*x^3)^(2/3))
```

Maple [F] time = 0.405, size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^3+c\right)^2}\left(bx^3+a\right)^{-\frac{8}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(8/3)/(d*x^3+c)^2,x)
```

```
[Out] int(1/(b*x^3+a)^(8/3)/(d*x^3+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{8}{3}}\left(dx^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(8/3)*(d*x^3 + c)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(8/3)/(d*x**3+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{8}{3}}\left(dx^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(8/3)*(d*x^3 + c)^2), x)
```

3.109 $\int \frac{(a+bx^3)^{14/3}}{(c+dx^3)^3}\,dx$

Optimal. Leaf size=541

$$\frac{bx\left(a+bx^3\right)^{5/3}\left(-5a^2d^2-10abcd+18b^2c^2\right)}{18c^2d^3}-\frac{bx\left(a+bx^3\right)^{2/3}(2bc-ad)\left(-5a^2d^2-18abcd+18b^2c^2\right)}{18c^2d^4}-\frac{(bc-ad)^{8/3}\left(5\ldots\right.}{\ldots}$$

```
[Out] -(b*(2*b*c - a*d)*(18*b^2*c^2 - 18*a*b*c*d - 5*a^2*d^2)*x*(a + b*x^3)^(2/3)
)/(18*c^2*d^4) + (b*(18*b^2*c^2 - 10*a*b*c*d - 5*a^2*d^2)*x*(a + b*x^3)^(5/
3))/(18*c^2*d^3) - ((b*c - a*d)*x*(a + b*x^3)^(11/3))/(6*c*d*(c + d*x^3)^2)
 - ((b*c - a*d)*(12*b*c + 5*a*d)*x*(a + b*x^3)^(8/3))/(18*c^2*d^2*(c + d*x^
3)) + (b^(8/3)*(54*b^2*c^2 - 126*a*b*c*d + 77*a^2*d^2)*ArcTan[(1 + (2*b^(1/
3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(9*Sqrt[3]*d^5) - ((b*c - a*d)^(8/3)*(54
*b^2*c^2 + 18*a*b*c*d + 5*a^2*d^2)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(
1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(9*Sqrt[3]*c^(8/3)*d^5) - ((b*c - a*d)^(
8/3)*(54*b^2*c^2 + 18*a*b*c*d + 5*a^2*d^2)*Log[c + d*x^3])/(54*c^(8/3)*d^5)
 + ((b*c - a*d)^(8/3)*(54*b^2*c^2 + 18*a*b*c*d + 5*a^2*d^2)*Log[((b*c - a*d
)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(18*c^(8/3)*d^5) - (b^(8/3)*(54*b^
2*c^2 - 126*a*b*c*d + 77*a^2*d^2)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(1
8*d^5)
```

Rubi [C] time = 0.0273345, antiderivative size = 62, normalized size of antiderivative = 0.11, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{a^4x\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{14}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(14/3)/(c + d*x^3)^3,x]
```

```
[Out] (a^4*x*(a + b*x^3)^(2/3)*AppellF1[1/3, -14/3, 3, 4/3, -((b*x^3)/a), -((d*x^
3)/c)])/(c^3*(1 + (b*x^3)/a)^(2/3))
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{(a+bx^3)^{14/3}}{(c+dx^3)^3}\,dx = \frac{\left(a^4(a+bx^3)^{2/3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{14/3}}{(c+dx^3)^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}} = \frac{a^4x(a+bx^3)^{2/3}F_1\left(\frac{1}{3};-\frac{14}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] time = 2.1929, size = 1171, normalized size = 2.16

$$\frac{1}{108}\left(\frac{10\left(2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{bc-adx}}{\sqrt[3]{c}\sqrt[3]{ax^3+b}}+1}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-adx}}{\sqrt[3]{ax^3+b}}\right)+\log\left(\frac{(bc-ad)^{2/3}x^2}{(ax^3+b)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-adx}}{\sqrt[3]{ax^3+b}}+c^{2/3}\right)\right)a^5}{c^{8/3}\sqrt[3]{bc-ad}}+\frac{6b\left(2\sqrt{3}\tan^{-1}\left(\cdots\right.\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(14/3)/(c + d*x^3)^3,x]
```

```
[Out] ((6*x*(a + b*x^3)^(2/3)*(-2*b^3*(9*b*c - 13*a*d) + 3*b^4*d*x^3 + (3*(b*c -
a*d)^4)/(c*(c + d*x^3)^2) - ((b*c - a*d)^3*(21*b*c + 5*a*d))/(c^2*(c + d*x^
3))))/d^4 + (162*b^5*c*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3,
 -((b*x^3)/a), -((d*x^3)/c)])/(d^4*(a + b*x^3)^(1/3)) - (378*a*b^4*x^4*(1 +
 (b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(
d^3*(a + b*x^3)^(1/3)) + (231*a^2*b^3*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[4/
3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(c*d^2*(a + b*x^3)^(1/3)) + (1
0*a^5*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(
1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]
+ Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c -
 a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(c^(8/3)*(b*c - a*d)^(1/3)) + (36*a*b^4
*c^(4/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3
)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3
)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*
c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(d^4*(b*c - a*d)^(1/3)) - (72*a^2*b^
3*c^(1/3)*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^
3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/
3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b
*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(d^3*(b*c - a*d)^(1/3)) + (30*a^3*b
^2*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3
)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + L
og[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*
d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(c^(2/3)*d^2*(b*c - a*d)^(1/3)) + (6*a^4*b
*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3))
)/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + Log
[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)
^(1/3)*x)/(b + a*x^3)^(1/3)]))/(c^(5/3)*d*(b*c - a*d)^(1/3)))/108
```

Maple [F] time = 0.263, size = 0, normalized size = 0.

$$\int \frac{1}{\left(d x^3+c\right)^3}\left(b x^3+a\right)^{\frac{14}{3}} d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(14/3)/(d*x^3+c)^3,x)
```

```
[Out] int((b*x^3+a)^(14/3)/(d*x^3+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b x^3+a\right)^{\frac{14}{3}}}{\left(d x^3+c\right)^3} d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(14/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(14/3)/(d*x^3 + c)^3, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(14/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(14/3)/(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b x^3+a\right)^{\frac{14}{3}}}{\left(d x^3+c\right)^3} d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(14/3)/(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(14/3)/(d*x^3 + c)^3, x)
```

3.110 $\int \frac{\left(a+bx^3\right)^{11/3}}{\left(c+dx^3\right)^3} \, dx$

Optimal. Leaf size=458

$$\frac{bx\left(a+bx^3\right)^{2/3}\left(-5a^2d^2-7abcd+18b^2c^2\right)}{18c^2d^3}+\frac{(bc-ad)^{5/3}\left(5a^2d^2+12abcd+27b^2c^2\right)\log\left(c+dx^3\right)}{54c^{8/3}d^4}-\frac{(bc-ad)^{5/3}\left(5a^2c\right.}{}$$

```
[Out] (b*(18*b^2*c^2 - 7*a*b*c*d - 5*a^2*d^2)*x*(a + b*x^3)^(2/3))/(18*c^2*d^3) -
 ((b*c - a*d)*x*(a + b*x^3)^(8/3))/(6*c*d*(c + d*x^3)^2) - ((b*c - a*d)*(9*
b*c + 5*a*d)*x*(a + b*x^3)^(5/3))/(18*c^2*d^2*(c + d*x^3)) - (b^(8/3)*(9*b*
c - 11*a*d)*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(3*Sqrt[
3]*d^4) + ((b*c - a*d)^(5/3)*(27*b^2*c^2 + 12*a*b*c*d + 5*a^2*d^2)*ArcTan[(
1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(9*Sqrt[
3]*c^(8/3)*d^4) + ((b*c - a*d)^(5/3)*(27*b^2*c^2 + 12*a*b*c*d + 5*a^2*d^2)*
Log[c + d*x^3])/(54*c^(8/3)*d^4) - ((b*c - a*d)^(5/3)*(27*b^2*c^2 + 12*a*b*
c*d + 5*a^2*d^2)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(1
8*c^(8/3)*d^4) + (b^(8/3)*(9*b*c - 11*a*d)*Log[-(b^(1/3)*x) + (a + b*x^3)^(
1/3)])/(6*d^4)
```

Rubi [C] time = 0.02813, antiderivative size = 62, normalized size of antiderivative = 0.14, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{a^3x\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{11}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(11/3)/(c + d*x^3)^3,x]
```

```
[Out] (a^3*x*(a + b*x^3)^(2/3)*AppellF1[1/3, -11/3, 3, 4/3, -((b*x^3)/a), -((d*x^
3)/c)])/(c^3*(1 + (b*x^3)/a)^(2/3))
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{11/3}}{\left(c+dx^3\right)^3}\,dx = \frac{\left(a^3\left(a+bx^3\right)^{2/3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{11/3}}{\left(c+dx^3\right)^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

$$= \frac{a^3x\left(a+bx^3\right)^{2/3} F_1\left(\frac{1}{3};-\frac{11}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] time = 1.74223, size = 908, normalized size = 1.98

$$\frac{1}{108}\left(\frac{10\left(2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{bc-adx}}{\sqrt[3]{c}\sqrt[3]{ax^3+b}}+1}{\sqrt{3}}\right)-2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-adx}}{\sqrt[3]{ax^3+b}}\right)+\log\left(\frac{(bc-ad)^{2/3}x^2}{\left(ax^3+b\right)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-adx}}{\sqrt[3]{ax^3+b}}+c^{2/3}\right)\right)a^4}{c^{8/3}\sqrt[3]{bc-ad}}+\frac{4b\left(2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{}}{\sqrt[3]{c}}\right.\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(11/3)/(c + d*x^3)^3,x]
```

```
[Out] ((6*x*(a + b*x^3)^(2/3)*(6*b^3 - (3*(b*c - a*d)^3)/(c*(c + d*x^3)^2) + (5*(
b*c - a*d)^2*(3*b*c + a*d))/(c^2*(c + d*x^3))))/d^3 - (81*b^4*x^4*(1 + (b*x
^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(d^3*(
a + b*x^3)^(1/3)) + (99*a*b^3*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3,
1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(c*d^2*(a + b*x^3)^(1/3)) + (10*a^4*(2
*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3)))/S
qrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[c^
(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1
/3)*x)/(b + a*x^3)^(1/3)]))/(c^(8/3)*(b*c - a*d)^(1/3)) - (18*a*b^3*c^(1/3)
*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3))
)/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + Log
[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)
^(1/3)*x)/(b + a*x^3)^(1/3)]))/(d^3*(b*c - a*d)^(1/3)) + (16*a^2*b^2*(2*Sqr
t[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[
3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[c^(2/3
) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*
x)/(b + a*x^3)^(1/3)]))/(c^(2/3)*d^2*(b*c - a*d)^(1/3)) + (4*a^3*b*(2*Sqrt[
3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[3]
] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[c^(2/3)
+ ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)
/(b + a*x^3)^(1/3)]))/(c^(5/3)*d*(b*c - a*d)^(1/3)))/108
```

Maple [F] time = 0.258, size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^3+c\right)^3}\left(bx^3+a\right)^{\frac{11}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(11/3)/(d*x^3+c)^3,x)
```

```
[Out] int((b*x^3+a)^(11/3)/(d*x^3+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{11}{3}}}{\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(11/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(11/3)/(d*x^3 + c)^3, x)
```

Fricas [B] time = 174.293, size = 2693, normalized size = 5.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(11/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] 1/54*(2*sqrt(3)*(27*b^3*c^5 - 15*a*b^2*c^4*d - 7*a^2*b*c^3*d^2 - 5*a^3*c^2*
d^3 + (27*b^3*c^3*d^2 - 15*a*b^2*c^2*d^3 - 7*a^2*b*c*d^4 - 5*a^3*d^5)*x^6 +
 2*(27*b^3*c^4*d - 15*a*b^2*c^3*d^2 - 7*a^2*b*c^2*d^3 - 5*a^3*c*d^4)*x^3)*(
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sqrt(3)*(b*c - a*d)
*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1
/3))/((b*c - a*d)*x)) + 6*sqrt(3)*(9*b^3*c^5 - 11*a*b^2*c^4*d + (9*b^3*c^3*
d^2 - 11*a*b^2*c^2*d^3)*x^6 + 2*(9*b^3*c^4*d - 11*a*b^2*c^3*d^2)*x^3)*(-b^2
)^(1/3)*arctan(-1/3*(sqrt(3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3)
)/(b*x)) - 2*(27*b^3*c^5 - 15*a*b^2*c^4*d - 7*a^2*b*c^3*d^2 - 5*a^3*c^2*d^3
 + (27*b^3*c^3*d^2 - 15*a*b^2*c^2*d^3 - 7*a^2*b*c*d^4 - 5*a^3*d^5)*x^6 + 2*
(27*b^3*c^4*d - 15*a*b^2*c^3*d^2 - 7*a^2*b*c^2*d^3 - 5*a^3*c*d^4)*x^3)*((b^
2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*x*((b^2*c^2 - 2*a*b*c*d + a^
2*d^2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d))/x) - 6*(9*b^3*c^5 - 11*a
*b^2*c^4*d + (9*b^3*c^3*d^2 - 11*a*b^2*c^2*d^3)*x^6 + 2*(9*b^3*c^4*d - 11*a
*b^2*c^3*d^2)*x^3)*(-b^2)^(1/3)*log(-((-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)
/x) + 3*(9*b^3*c^5 - 11*a*b^2*c^4*d + (9*b^3*c^3*d^2 - 11*a*b^2*c^2*d^3)*x^
6 + 2*(9*b^3*c^4*d - 11*a*b^2*c^3*d^2)*x^3)*(-b^2)^(1/3)*log(-((-b^2)^(1/3)
*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2) + (27
*b^3*c^5 - 15*a*b^2*c^4*d - 7*a^2*b*c^3*d^2 - 5*a^3*c^2*d^3 + (27*b^3*c^3*d
^2 - 15*a*b^2*c^2*d^3 - 7*a^2*b*c*d^4 - 5*a^3*d^5)*x^6 + 2*(27*b^3*c^4*d -
15*a*b^2*c^3*d^2 - 7*a^2*b*c^2*d^3 - 5*a^3*c*d^4)*x^3)*((b^2*c^2 - 2*a*b*c*
d + a^2*d^2)/c^2)^(1/3)*log(-((b*c - a*d)*x^2*((b^2*c^2 - 2*a*b*c*d + a^2*d
^2)/c^2)^(1/3) + (b*x^3 + a)^(1/3)*c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2
)^(2/3) + (b*x^3 + a)^(2/3)*(b*c - a*d))/x^2) + 3*(6*b^3*c^2*d^3*x^7 + (27*
b^3*c^3*d^2 - 25*a*b^2*c^2*d^3 + 5*a^2*b*c*d^4 + 5*a^3*d^5)*x^4 + 2*(9*b^3*
c^4*d - 8*a*b^2*c^3*d^2 - 2*a^2*b*c^2*d^3 + 4*a^3*c*d^4)*x)*(b*x^3 + a)^(2/
3))/(c^2*d^6*x^6 + 2*c^3*d^5*x^3 + c^4*d^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(11/3)/(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{11}{3}}}{\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(11/3)/(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(11/3)/(d*x^3 + c)^3, x)
```

3.111 $\int \frac{\left(a+bx^3\right)^{8/3}}{\left(c+dx^3\right)^3}\,dx$

Optimal. Leaf size=391

$$-\frac{(bc-ad)^{2/3}\left(5a^2d^2+6abcd+9b^2c^2\right)\log\left(c+dx^3\right)}{54c^{8/3}d^3}+\frac{(bc-ad)^{2/3}\left(5a^2d^2+6abcd+9b^2c^2\right)\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}d^3}$$

```
[Out] -((b*c - a*d)*x*(a + b*x^3)^(5/3))/(6*c*d*(c + d*x^3)^2) - ((b*c - a*d)*(6*
b*c + 5*a*d)*x*(a + b*x^3)^(2/3))/(18*c^2*d^2*(c + d*x^3)) + (b^(8/3)*ArcTa
n[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(Sqrt[3]*d^3) - ((b*c - a
*d)^(2/3)*(9*b^2*c^2 + 6*a*b*c*d + 5*a^2*d^2)*ArcTan[(1 + (2*(b*c - a*d)^(1
/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(9*Sqrt[3]*c^(8/3)*d^3) - ((b
*c - a*d)^(2/3)*(9*b^2*c^2 + 6*a*b*c*d + 5*a^2*d^2)*Log[c + d*x^3])/(54*c^(
8/3)*d^3) + ((b*c - a*d)^(2/3)*(9*b^2*c^2 + 6*a*b*c*d + 5*a^2*d^2)*Log[((b*
c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(18*c^(8/3)*d^3) - (b^(8/3)
*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(2*d^3)
```

Rubi [C] time = 0.0275823, antiderivative size = 62, normalized size of antiderivative = 0.16, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{a^2x\left(a+bx^3\right)^{2/3}F_1\left(\frac{1}{3};-\frac{8}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(8/3)/(c + d*x^3)^3,x]
```

```
[Out] (a^2*x*(a + b*x^3)^(2/3)*AppellF1[1/3, -8/3, 3, 4/3, -((b*x^3)/a), -((d*x^3
)/c)])/(c^3*(1 + (b*x^3)/a)^(2/3))
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{8/3}}{\left(c+dx^3\right)^3}\,dx = \frac{\left(a^2\left(a+bx^3\right)^{2/3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{8/3}}{\left(c+dx^3\right)^3}\,dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}}$$
$$= \frac{a^2x\left(a+bx^3\right)^{2/3} F_1\left(\frac{1}{3};-\frac{8}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

Mathematica [C] time = 0.902278, size = 651, normalized size = 1.66

$$\frac{\frac{10a^3\left(\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(ax^3+b\right)^{2/3}}+\frac{\sqrt[3]{c}x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}+c^{2/3}\right)-2\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{ax^3+b}}+1}{\sqrt{3}}\right)\right)}{\sqrt[3]{bc-ad}}+\frac{2a^2bc\left(\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(ax^3+b\right)^{2/3}}+\frac{\sqrt[3]{c}x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}+c^{2/3}\right)-2\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}\right)\right.}{d\sqrt[3]{bc-ad}}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(8/3)/(c + d*x^3)^3,x]
```

```
[Out] ((6*c^(2/3)*(-(b*c) + a*d)*x*(a + b*x^3)^(2/3)*(3*b*c*(2*c + 3*d*x^3) + a*d
*(8*c + 5*d*x^3)))/(d^2*(c + d*x^3)^2) + (27*b^3*c^(5/3)*x^4*(1 + (b*x^3)/a
)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(d^2*(a + b
*x^3)^(1/3)) + (10*a^3*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1
/3)*(b + a*x^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b
 + a*x^3)^(1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3)
+ (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(b*c - a*d)^(1/3) + (6
*a*b^2*c^2*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x
^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1
/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(
b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(d^2*(b*c - a*d)^(1/3)) + (2*a^2*b
*c*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3
)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + L
og[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*
d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(d*(b*c - a*d)^(1/3)))/(108*c^(8/3))
```

Maple [F] time = 0.423, size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^3+c\right)^3}\left(bx^3+a\right)^{\frac{8}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(8/3)/(d*x^3+c)^3,x)
```

```
[Out] int((b*x^3+a)^(8/3)/(d*x^3+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^3 + a)^{\frac{8}{3}}}{(dx^3 + c)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(8/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(8/3)/(d*x^3 + c)^3, x)
```

Fricas [B] time = 16.0328, size = 2080, normalized size = 5.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(8/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] -1/54*(2*sqrt(3)*((9*b^2*c^2*d^2 + 6*a*b*c*d^3 + 5*a^2*d^4)*x^6 + 9*b^2*c^4
 + 6*a*b*c^3*d + 5*a^2*c^2*d^2 + 2*(9*b^2*c^3*d + 6*a*b*c^2*d^2 + 5*a^2*c*d
^3)*x^3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sqrt(3)*(
b*c - a*d)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d + a^2*d^
2)/c^2)^(1/3))/((b*c - a*d)*x)) + 18*sqrt(3)*(b^2*c^2*d^2*x^6 + 2*b^2*c^3*d
*x^3 + b^2*c^4)*(-b^2)^(1/3)*arctan(-1/3*(sqrt(3)*b*x - 2*sqrt(3)*(b*x^3 +
a)^(1/3)*(-b^2)^(1/3))/(b*x)) - 2*((9*b^2*c^2*d^2 + 6*a*b*c*d^3 + 5*a^2*d^4
)*x^6 + 9*b^2*c^4 + 6*a*b*c^3*d + 5*a^2*c^2*d^2 + 2*(9*b^2*c^3*d + 6*a*b*c^
2*d^2 + 5*a^2*c*d^3)*x^3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((
c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c -
a*d))/x) - 18*(b^2*c^2*d^2*x^6 + 2*b^2*c^3*d*x^3 + b^2*c^4)*(-b^2)^(1/3)*lo
g(-((-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + 9*(b^2*c^2*d^2*x^6 + 2*b^2*c
^3*d*x^3 + b^2*c^4)*(-b^2)^(1/3)*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^(1/
3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2) + ((9*b^2*c^2*d^2 + 6*a*b*c*d
^3 + 5*a^2*d^4)*x^6 + 9*b^2*c^4 + 6*a*b*c^3*d + 5*a^2*c^2*d^2 + 2*(9*b^2*c^
3*d + 6*a*b*c^2*d^2 + 5*a^2*c*d^3)*x^3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^
2)^(1/3)*log(-((b*c - a*d)*x^2*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)
+ (b*x^3 + a)^(1/3)*c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(2/3) + (b*x^
3 + a)^(2/3)*(b*c - a*d))/x^2) + 3*((9*b^2*c^2*d^2 - 4*a*b*c*d^3 - 5*a^2*d^
4)*x^4 + 2*(3*b^2*c^3*d + a*b*c^2*d^2 - 4*a^2*c*d^3)*x)*(b*x^3 + a)^(2/3))/
(c^2*d^5*x^6 + 2*c^3*d^4*x^3 + c^4*d^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(8/3)/(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3 + a\right)^{\frac{8}{3}}}{\left(dx^3 + c\right)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(8/3)/(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(8/3)/(d*x^3 + c)^3, x)
```

3.112 $\int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^3}\,dx$

Optimal. Leaf size=217

$$\frac{5a^2\log\left(c+dx^3\right)}{54c^{8/3}\sqrt[3]{bc-ad}}-\frac{5a^2\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}\sqrt[3]{bc-ad}}+\frac{5a^2\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}c^{8/3}\sqrt[3]{bc-ad}}+\frac{5ax\left(a+bx^3\right)^{2/3}}{18c^2\left(c+dx^3\right)}+\frac{x\left(a+bx^3\right)^{5/3}}{6c\left(c+dx^3\right)^2}$$

```
[Out] (x*(a + b*x^3)^(5/3))/(6*c*(c + d*x^3)^2) + (5*a*x*(a + b*x^3)^(2/3))/(18*c
^2*(c + d*x^3)) + (5*a^2*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a +
b*x^3)^(1/3)))/Sqrt[3]])/(9*Sqrt[3]*c^(8/3)*(b*c - a*d)^(1/3)) + (5*a^2*Log
[c + d*x^3])/(54*c^(8/3)*(b*c - a*d)^(1/3)) - (5*a^2*Log[((b*c - a*d)^(1/3)
*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(18*c^(8/3)*(b*c - a*d)^(1/3))
```

Rubi [A] time = 0.238557, antiderivative size = 276, normalized size of antiderivative = 1.27, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {378, 377, 200, 31, 634, 617, 204, 628}

$$-\frac{5a^2\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{27c^{8/3}\sqrt[3]{bc-ad}}+\frac{5a^2\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+c^{2/3}\right)}{54c^{8/3}\sqrt[3]{bc-ad}}+\frac{5a^2\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+\sqrt[3]{c}}{\sqrt{3}\sqrt[3]{c}}\right)}{9\sqrt{3}c^{8/3}\sqrt[3]{bc-ad}}+\frac{5ax\left(a+bx^3\right)^{2/3}}{18c^2\left(c+dx^3\right)}+\frac{x}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(5/3)/(c + d*x^3)^3,x]
```

```
[Out] (x*(a + b*x^3)^(5/3))/(6*c*(c + d*x^3)^2) + (5*a*x*(a + b*x^3)^(2/3))/(18*c
^2*(c + d*x^3)) + (5*a^2*ArcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x
^3)^(1/3))/(Sqrt[3]*c^(1/3))])/(9*Sqrt[3]*c^(8/3)*(b*c - a*d)^(1/3)) - (5*a
^2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(27*c^(8/3)*(b*c
 - a*d)^(1/3)) + (5*a^2*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(
2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(54*c^(8/3)*(b*c -
 a*d)^(1/3))
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
```

```
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{5/3}}{\left(c+dx^3\right)^3}\,dx = \frac{x\left(a+bx^3\right)^{5/3}}{6c\left(c+dx^3\right)^2}+\frac{(5a)\int \frac{\left(a+bx^3\right)^{2/3}}{\left(c+dx^3\right)^2}\,dx}{6c}$$

$$= \frac{x\left(a+bx^3\right)^{5/3}}{6c\left(c+dx^3\right)^2}+\frac{5ax\left(a+bx^3\right)^{2/3}}{18c^2\left(c+dx^3\right)}+\frac{\left(5a^2\right)\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}\,dx}{9c^2}$$

$$= \frac{x\left(a+bx^3\right)^{5/3}}{6c\left(c+dx^3\right)^2}+\frac{5ax\left(a+bx^3\right)^{2/3}}{18c^2\left(c+dx^3\right)}+\frac{\left(5a^2\right)\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^3}\,dx,x,\frac{x}{\sqrt[3]{a+bx^3}}\right)}{9c^2}$$

$$= \frac{x\left(a+bx^3\right)^{5/3}}{6c\left(c+dx^3\right)^2}+\frac{5ax\left(a+bx^3\right)^{2/3}}{18c^2\left(c+dx^3\right)}+\frac{\left(5a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}x}\,dx,x,\frac{x}{\sqrt[3]{a+bx^3}}\right)}{27c^{8/3}}+\frac{\left(5a^2\right)\text{Subst}\left(\int\right.}{}$$

$$= \frac{x\left(a+bx^3\right)^{5/3}}{6c\left(c+dx^3\right)^2}+\frac{5ax\left(a+bx^3\right)^{2/3}}{18c^2\left(c+dx^3\right)}-\frac{5a^2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{27c^{8/3}\sqrt[3]{bc-ad}}+\frac{\left(5a^2\right)\text{Subst}\left(\int \frac{1}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-ad}x+(bc-}\right.}{18c^{7/3}}$$

$$= \frac{x\left(a+bx^3\right)^{5/3}}{6c\left(c+dx^3\right)^2}+\frac{5ax\left(a+bx^3\right)^{2/3}}{18c^2\left(c+dx^3\right)}-\frac{5a^2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{27c^{8/3}\sqrt[3]{bc-ad}}+\frac{5a^2\log\left(c^{2/3}+\frac{(bc-ad)^{2/3}x^2}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}\sqrt[3]{bc-}}{\sqrt[3]{a+b}}\right.}{54c^{8/3}\sqrt[3]{bc-ad}}$$

$$= \frac{x\left(a+bx^3\right)^{5/3}}{6c\left(c+dx^3\right)^2}+\frac{5ax\left(a+bx^3\right)^{2/3}}{18c^2\left(c+dx^3\right)}+\frac{5a^2\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}c^{8/3}\sqrt[3]{bc-ad}}-\frac{5a^2\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{27c^{8/3}\sqrt[3]{bc-ad}}+\frac{5a^2\log}{}$$

Mathematica [C] time = 0.0263331, size = 79, normalized size = 0.36

$$\frac{ax\left(a+bx^3\right)^{2/3}\,{}_2F_1\left(-\frac{5}{3},\frac{1}{3};\frac{4}{3};\frac{(ad-bc)x^3}{a\left(dx^3+c\right)}\right)}{c^3\left(\frac{bx^3}{a}+1\right)^{2/3}\sqrt[3]{\frac{dx^3}{c}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(5/3)/(c + d*x^3)^3,x]
```

```
[Out] (a*x*(a + b*x^3)^(2/3)*Hypergeometric2F1[-5/3, 1/3, 4/3, ((-(b*c) + a*d)*x^
3)/(a*(c + d*x^3))])/(c^3*(1 + (b*x^3)/a)^(2/3)*(1 + (d*x^3)/c)^(1/3))
```

Maple [F] time = 0.438, size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^3+c\right)^3}\left(bx^3+a\right)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(5/3)/(d*x^3+c)^3,x)
```

```
[Out] int((b*x^3+a)^(5/3)/(d*x^3+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{5}{3}}}{\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(5/3)/(d*x^3 + c)^3, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(5/3)/(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{5}{3}}}{\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)/(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(5/3)/(d*x^3 + c)^3, x)
```

3.113 $\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^3}\,dx$

Optimal. Leaf size=267

$$\frac{x\left(a+bx^3\right)^{2/3}(6bc-5ad)}{18c^2\left(c+dx^3\right)(bc-ad)}+\frac{a(6bc-5ad)\log\left(c+dx^3\right)}{54c^{8/3}(bc-ad)^{4/3}}-\frac{a(6bc-5ad)\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}(bc-ad)^{4/3}}+\frac{a(6bc-5ad)\tan^{-}}{9\sqrt{3}c^{8/3}(bc}$$

```
[Out] -(d*x*(a + b*x^3)^(5/3))/(6*c*(b*c - a*d)*(c + d*x^3)^2) + ((6*b*c - 5*a*d)
*x*(a + b*x^3)^(2/3))/(18*c^2*(b*c - a*d)*(c + d*x^3)) + (a*(6*b*c - 5*a*d)
*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])
/(9*Sqrt[3]*c^(8/3)*(b*c - a*d)^(4/3)) + (a*(6*b*c - 5*a*d)*Log[c + d*x^3])
/(54*c^(8/3)*(b*c - a*d)^(4/3)) - (a*(6*b*c - 5*a*d)*Log[((b*c - a*d)^(1/3)
*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(18*c^(8/3)*(b*c - a*d)^(4/3))
```

Rubi [A] time = 0.243439, antiderivative size = 326, normalized size of antiderivative = 1.22, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {382, 378, 377, 200, 31, 634, 617, 204, 628}

$$\frac{x\left(a+bx^3\right)^{2/3}(6bc-5ad)}{18c^2\left(c+dx^3\right)(bc-ad)}-\frac{a(6bc-5ad)\log\left(\sqrt[3]{c}-\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{27c^{8/3}(bc-ad)^{4/3}}+\frac{a(6bc-5ad)\log\left(\frac{x^2(bc-ad)^{2/3}}{\left(a+bx^3\right)^{2/3}}+\frac{\sqrt[3]{c}x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}+c^{2/3}\right)}{54c^{8/3}(bc-ad)^{4/3}}+\frac{a(6}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)/(c + d*x^3)^3,x]
```

```
[Out] -(d*x*(a + b*x^3)^(5/3))/(6*c*(b*c - a*d)*(c + d*x^3)^2) + ((6*b*c - 5*a*d)
*x*(a + b*x^3)^(2/3))/(18*c^2*(b*c - a*d)*(c + d*x^3)) + (a*(6*b*c - 5*a*d)
*ArcTan[(c^(1/3) + (2*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3))/(Sqrt[3]*c^(1
/3))])/(9*Sqrt[3]*c^(8/3)*(b*c - a*d)^(4/3)) - (a*(6*b*c - 5*a*d)*Log[c^(1/
3) - ((b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(27*c^(8/3)*(b*c - a*d)^(4/3
)) + (a*(6*b*c - 5*a*d)*Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(a + b*x^3)^(
2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(a + b*x^3)^(1/3)])/(54*c^(8/3)*(b*c -
 a*d)^(4/3))
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
```

```
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{2/3}}{\left(c+dx^3\right)^3}\,dx = -\frac{dx\left(a+bx^3\right)^{5/3}}{6c(bc-ad)\left(c+dx^3\right)^2} + \frac{(6bc-5ad)\int \frac{\left(a+bx^3\right)^{2/3}}{\left(c+dx^3\right)^2}dx}{6c(bc-ad)}$$

$$= -\frac{dx\left(a+bx^3\right)^{5/3}}{6c(bc-ad)\left(c+dx^3\right)^2} + \frac{(6bc-5ad)x\left(a+bx^3\right)^{2/3}}{18c^2(bc-ad)\left(c+dx^3\right)} + \frac{(a(6bc-5ad))\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)}dx}{9c^2(bc-ad)}$$

$$= -\frac{dx\left(a+bx^3\right)^{5/3}}{6c(bc-ad)\left(c+dx^3\right)^2} + \frac{(6bc-5ad)x\left(a+bx^3\right)^{2/3}}{18c^2(bc-ad)\left(c+dx^3\right)} + \frac{(a(6bc-5ad))\,\mathrm{Subst}\left(\int \frac{1}{c-(bc-ad)x^3}dx, x, \cdot\right.}{9c^2(bc-ad)}$$

$$= -\frac{dx\left(a+bx^3\right)^{5/3}}{6c(bc-ad)\left(c+dx^3\right)^2} + \frac{(6bc-5ad)x\left(a+bx^3\right)^{2/3}}{18c^2(bc-ad)\left(c+dx^3\right)} + \frac{(a(6bc-5ad))\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}x}dx, x,\right.}{27c^{8/3}(bc-ad)}$$

$$= -\frac{dx\left(a+bx^3\right)^{5/3}}{6c(bc-ad)\left(c+dx^3\right)^2} + \frac{(6bc-5ad)x\left(a+bx^3\right)^{2/3}}{18c^2(bc-ad)\left(c+dx^3\right)} - \frac{a(6bc-5ad)\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{27c^{8/3}(bc-ad)^{4/3}} + \frac{(a(6bc}{}$$

$$= -\frac{dx\left(a+bx^3\right)^{5/3}}{6c(bc-ad)\left(c+dx^3\right)^2} + \frac{(6bc-5ad)x\left(a+bx^3\right)^{2/3}}{18c^2(bc-ad)\left(c+dx^3\right)} - \frac{a(6bc-5ad)\log\left(\sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{27c^{8/3}(bc-ad)^{4/3}} + \frac{a(6bc}{}$$

$$= -\frac{dx\left(a+bx^3\right)^{5/3}}{6c(bc-ad)\left(c+dx^3\right)^2} + \frac{(6bc-5ad)x\left(a+bx^3\right)^{2/3}}{18c^2(bc-ad)\left(c+dx^3\right)} + \frac{a(6bc-5ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}c^{8/3}(bc-ad)^{4/3}} - \frac{a(6bc}{}$$

Mathematica [C] time = 0.160942, size = 153, normalized size = 0.57

$$\frac{x\left(c\left(-a^2d\left(8c+5dx^3\right)+ab\left(6c^2-5cdx^3-5d^2x^6\right)+3b^2cx^3\left(2c+dx^3\right)\right)-2a\left(c+dx^3\right)^2(5ad-6bc)\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)\cdot}{c\left(bx^3+\cdot\right.}\right.\right.}{18c^3\sqrt[3]{a+bx^3}\left(c+dx^3\right)^2(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(2/3)/(c + d*x^3)^3,x]
```

```
[Out] (x*(c*(3*b^2*c*x^3*(2*c + d*x^3) - a^2*d*(8*c + 5*d*x^3) + a*b*(6*c^2 - 5*c
*d*x^3 - 5*d^2*x^6)) - 2*a*(-6*b*c + 5*a*d)*(c + d*x^3)^2*Hypergeometric2F1
[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]))/(18*c^3*(b*c - a*d)*(a +
 b*x^3)^(1/3)*(c + d*x^3)^2)
```

Maple [F] time = 0.432, size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^3+c\right)^3}\left(bx^3+a\right)^{\frac{2}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(2/3)/(d*x^3+c)^3,x)
```

```
[Out] int((b*x^3+a)^(2/3)/(d*x^3+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/(d*x^3 + c)^3, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)/(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{2}{3}}}{\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(2/3)/(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(2/3)/(d*x^3 + c)^3, x)
```

3.114 $\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)^3}\,dx$

Optimal. Leaf size=307

$$\frac{\left(5a^2d^2-12abcd+9b^2c^2\right)\log\left(c+dx^3\right)}{54c^{8/3}(bc-ad)^{7/3}}-\frac{\left(5a^2d^2-12abcd+9b^2c^2\right)\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}-\sqrt[3]{a+bx^3}\right)}{18c^{8/3}(bc-ad)^{7/3}}+\frac{\left(5a^2d^2-12abcd+9\right.}{9\sqrt{3}c^{8/3}}$$

```
[Out] -(d*x*(a + b*x^3)^(2/3))/(6*c*(b*c - a*d)*(c + d*x^3)^2) - (d*(9*b*c - 5*a*
d)*x*(a + b*x^3)^(2/3))/(18*c^2*(b*c - a*d)^2*(c + d*x^3)) + ((9*b^2*c^2 -
12*a*b*c*d + 5*a^2*d^2)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(a + b
*x^3)^(1/3)))/Sqrt[3]])/(9*Sqrt[3]*c^(8/3)*(b*c - a*d)^(7/3)) + ((9*b^2*c^2
 - 12*a*b*c*d + 5*a^2*d^2)*Log[c + d*x^3])/(54*c^(8/3)*(b*c - a*d)^(7/3)) -
 ((9*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2)*Log[((b*c - a*d)^(1/3)*x)/c^(1/3) -
(a + b*x^3)^(1/3)])/(18*c^(8/3)*(b*c - a*d)^(7/3))
```

Rubi [C] time = 0.310109, antiderivative size = 167, normalized size of antiderivative = 0.54, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$-\frac{x\left(cd\left(-a^2d\left(8c+5dx^3\right)+ab\left(12c^2+cdx^3-5d^2x^6\right)+3b^2cx^3\left(4c+3dx^3\right)\right)-2\left(c+dx^3\right)^2\left(5a^2d^2-12abcd+9b^2c^2\right)\,2\right.}{18c^3\sqrt[3]{a+bx^3}\left(c+dx^3\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(1/3)*(c + d*x^3)^3),x]
```

```
[Out] -(x*(c*d*(3*b^2*c*x^3*(4*c + 3*d*x^3) - a^2*d*(8*c + 5*d*x^3) + a*b*(12*c^2
 + c*d*x^3 - 5*d^2*x^6)) - 2*(9*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2)*(c + d*x^
3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]))/(1
8*c^3*(b*c - a*d)^2*(a + b*x^3)^(1/3)*(c + d*x^3)^2)
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{a+bx^3}\left(c+dx^3\right)^3}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{\sqrt[3]{1+\frac{bx^3}{a}}\left(c+dx^3\right)^3}\,dx}{\sqrt[3]{a+bx^3}}$$

$$= -\frac{x\left(cd\left(3b^2cx^3\left(4c+3dx^3\right)-a^2d\left(8c+5dx^3\right)+ab\left(12c^2+cdx^3-5d^2x^6\right)\right)-2\left(9b^2c^2-12al\right.\right.}{18c^3(bc-ad)^2\sqrt[3]{a+bx^3}\left(c+dx^3\right)^2}$$

Mathematica [C] time = 0.225834, size = 168, normalized size = 0.55

$$\frac{x\left(2\left(c+dx^3\right)^2\left(5a^2d^2-12abcd+9b^2c^2\right)\,{}_2F_1\left(\frac{1}{3},1;\frac{4}{3};\frac{(bc-ad)x^3}{c\left(bx^3+a\right)}\right)-cd\left(-a^2d\left(8c+5dx^3\right)+ab\left(12c^2+cdx^3-5d^2x^6\right)+3b^2c\right.\right.}{18c^3\sqrt[3]{a+bx^3}\left(c+dx^3\right)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^3)^(1/3)*(c + d*x^3)^3),x]
```

```
[Out] (x*(-(c*d*(3*b^2*c*x^3*(4*c + 3*d*x^3) - a^2*d*(8*c + 5*d*x^3) + a*b*(12*c^
2 + c*d*x^3 - 5*d^2*x^6))) + 2*(9*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2)*(c + d*
x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]))/
(18*c^3*(b*c - a*d)^2*(a + b*x^3)^(1/3)*(c + d*x^3)^2)
```

Maple [F] time = 0.441, size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^3+c\right)^3}\frac{1}{\sqrt[3]{bx^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(1/3)/(d*x^3+c)^3,x)
```

```
[Out] int(1/(b*x^3+a)^(1/3)/(d*x^3+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(1/3)/(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{3}}\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/3)/(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/3)*(d*x^3 + c)^3), x)
```

3.115 $\int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^3}\,dx$

Optimal. Leaf size=377

$$\frac{dx(a+bx^3)^{2/3}(-5a^2d^2+15abcd+18b^2c^2)}{18ac^2(c+dx^3)(bc-ad)^3}-\frac{d(5a^2d^2-18abcd+27b^2c^2)\log(c+dx^3)}{54c^{8/3}(bc-ad)^{10/3}}+\frac{d(5a^2d^2-18abcd+27b^2c^2)}{18c^{8/3}(bc-}$$

```
[Out] -(d*x)/(6*c*(b*c - a*d)*(a + b*x^3)^(1/3)*(c + d*x^3)^2) + (b*(6*b*c + a*d)
*x)/(6*a*c*(b*c - a*d)^2*(a + b*x^3)^(1/3)*(c + d*x^3)) + (d*(18*b^2*c^2 +
15*a*b*c*d - 5*a^2*d^2)*x*(a + b*x^3)^(2/3))/(18*a*c^2*(b*c - a*d)^3*(c + d
*x^3)) - (d*(27*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*ArcTan[(1 + (2*(b*c - a*d
)^(1/3)*x)/(c^(1/3)*(a + b*x^3)^(1/3)))/Sqrt[3]])/(9*Sqrt[3]*c^(8/3)*(b*c -
 a*d)^(10/3)) - (d*(27*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*Log[c + d*x^3])/(5
4*c^(8/3)*(b*c - a*d)^(10/3)) + (d*(27*b^2*c^2 - 18*a*b*c*d + 5*a^2*d^2)*Lo
g[((b*c - a*d)^(1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(18*c^(8/3)*(b*c - a*
d)^(10/3))
```

Rubi [C] time = 2.74231, antiderivative size = 428, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$-\frac{65c^2(a+bx^3)^2\left(-28(c+dx^3)^2\left(a^2(843c^2dx^3+500c^3+375cd^2x^6+27d^3x^9)+9abcx^3(73c^2+104cdx^3+33d^2x^6)+27\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[1/((a + b*x^3)^(4/3)*(c + d*x^3)^3),x]
```

```
[Out] -(65*c^2*(a + b*x^3)^2*(14000*a^2*c^5 + 21896*a*b*c^5*x^3 + 48104*a^2*c^4*d
*x^3 + 8391*b^2*c^5*x^6 + 70802*a*b*c^4*d*x^6 + 60807*a^2*c^3*d^2*x^6 + 244
17*b^2*c^4*d*x^9 + 81534*a*b*c^3*d^2*x^9 + 33657*a^2*c^2*d^3*x^9 + 23409*b^
2*c^3*d^2*x^12 + 38652*a*b*c^2*d^3*x^12 + 7155*a^2*c*d^4*x^12 + 7425*b^2*c^
2*d^3*x^15 + 5940*a*b*c*d^4*x^15 + 243*a^2*d^5*x^15 - 28*(c + d*x^3)^2*(27*
b^2*c^2*x^6*(7*c + 6*d*x^3) + 9*a*b*c*x^3*(73*c^2 + 104*c*d*x^3 + 33*d^2*x^
6) + a^2*(500*c^3 + 843*c^2*d*x^3 + 375*c*d^2*x^6 + 27*d^3*x^9))*Hypergeome
tric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]) - 486*(b*c - a*d)^
4*x^12*(c + d*x^3)^3*HypergeometricPFQ[{2, 2, 2, 7/3}, {1, 1, 16/3}, ((b*c
- a*d)*x^3)/(c*(a + b*x^3))])/(16380*c^5*(b*c - a*d)^3*x^8*(a + b*x^3)^(7/3
)*(c + d*x^3)^2)
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
```

```
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^{4/3}\left(c+dx^3\right)^3}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{4/3}\left(c+dx^3\right)^3}\,dx}{a\sqrt[3]{a+bx^3}}$$

$$= -\frac{65c^2\left(a+bx^3\right)^2\left(14000a^2c^5+21896abc^5x^3+48104a^2c^4dx^3+8391b^2c^5x^6+70802ab\right.}{}$$

Mathematica [C] time = 2.26589, size = 428, normalized size = 1.14

$$-\frac{486x^{12}\left(c+dx^3\right)^3(bc-ad)^4\text{HypergeometricPFQ}\left(\left\{2,2,2,\frac{7}{3}\right\},\left\{1,1,\frac{16}{3}\right\},\frac{x^3(bc-ad)}{c\left(a+bx^3\right)}\right)+65c^2\left(a+bx^3\right)^2\left(28\left(c+dx^3\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(4/3)*(c + d*x^3)^3),x]
```

```
[Out] -(65*c^2*(a + b*x^3)^2*(-14000*a^2*c^5 - 21896*a*b*c^5*x^3 - 48104*a^2*c^4*
d*x^3 - 8391*b^2*c^5*x^6 - 70802*a*b*c^4*d*x^6 - 60807*a^2*c^3*d^2*x^6 - 24
417*b^2*c^4*d*x^9 - 81534*a*b*c^3*d^2*x^9 - 33657*a^2*c^2*d^3*x^9 - 23409*b
^2*c^3*d^2*x^12 - 38652*a*b*c^2*d^3*x^12 - 7155*a^2*c*d^4*x^12 - 7425*b^2*c
^2*d^3*x^15 - 5940*a*b*c*d^4*x^15 - 243*a^2*d^5*x^15 + 28*(c + d*x^3)^2*(27
*b^2*c^2*x^6*(7*c + 6*d*x^3) + 9*a*b*c*x^3*(73*c^2 + 104*c*d*x^3 + 33*d^2*x
^6) + a^2*(500*c^3 + 843*c^2*d*x^3 + 375*c*d^2*x^6 + 27*d^3*x^9))*Hypergeom
etric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]) + 486*(b*c - a*d)
^4*x^12*(c + d*x^3)^3*HypergeometricPFQ[{2, 2, 2, 7/3}, {1, 1, 16/3}, ((b*c
 - a*d)*x^3)/(c*(a + b*x^3))])/(16380*c^5*(-(b*c) + a*d)^3*x^8*(a + b*x^3)^
(7/3)*(c + d*x^3)^2)
```

Maple [F] time = 0.435, size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^3+c\right)^3}\left(bx^3+a\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(4/3)/(d*x^3+c)^3,x)
```

```
[Out] int(1/(b*x^3+a)^(4/3)/(d*x^3+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{4}{3}}\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(4/3)/(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3 + a\right)^{\frac{4}{3}}\left(dx^3 + c\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(4/3)/(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(4/3)*(d*x^3 + c)^3), x)
```

3.116 $\int \frac{1}{\left(a+bx^3\right)^{7/3}\left(c+dx^3\right)^3}\,dx$

Optimal. Leaf size=463

$$\frac{dx\left(a+bx^3\right)^{2/3}\left(-42a^2bcd^2+10a^3d^3-135ab^2c^2d+27b^3c^3\right)}{36a^2c^2\left(c+dx^3\right)(bc-ad)^4}+\frac{bx\left(-2a^2d^2-42abcd+9b^2c^2\right)}{12a^2c\sqrt[3]{a+bx^3}\left(c+dx^3\right)(bc-ad)^3}+\frac{d^2\left(5a^2d^2-24ab\right.}{54c^{}}$$

```
[Out] -(d*x)/(6*c*(b*c - a*d)*(a + b*x^3)^(4/3)*(c + d*x^3)^2) + (b*(3*b*c + 2*a*
d)*x)/(12*a*c*(b*c - a*d)^2*(a + b*x^3)^(4/3)*(c + d*x^3)) + (b*(9*b^2*c^2
- 42*a*b*c*d - 2*a^2*d^2)*x)/(12*a^2*c*(b*c - a*d)^3*(a + b*x^3)^(1/3)*(c +
 d*x^3)) + (d*(27*b^3*c^3 - 135*a*b^2*c^2*d - 42*a^2*b*c*d^2 + 10*a^3*d^3)*
x*(a + b*x^3)^(2/3))/(36*a^2*c^2*(b*c - a*d)^4*(c + d*x^3)) + (d^2*(54*b^2*
c^2 - 24*a*b*c*d + 5*a^2*d^2)*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*
(a + b*x^3)^(1/3)))/Sqrt[3]])/(9*Sqrt[3]*c^(8/3)*(b*c - a*d)^(13/3)) + (d^2
*(54*b^2*c^2 - 24*a*b*c*d + 5*a^2*d^2)*Log[c + d*x^3])/(54*c^(8/3)*(b*c - a
*d)^(13/3)) - (d^2*(54*b^2*c^2 - 24*a*b*c*d + 5*a^2*d^2)*Log[((b*c - a*d)^(
1/3)*x)/c^(1/3) - (a + b*x^3)^(1/3)])/(18*c^(8/3)*(b*c - a*d)^(13/3))
```

Rubi [C] time = 8.66242, antiderivative size = 1990, normalized size of antiderivative = 4.3, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Int[1/((a + b*x^3)^(7/3)*(c + d*x^3)^3),x]
```

```
[Out] -(522756*c^6*(b*c - a*d)^3*x^9*(a + b*x^3)^2 + 1516320*c^5*d*(b*c - a*d)^3*
x^12*(a + b*x^3)^2 + 2198664*c^4*d^2*(b*c - a*d)^3*x^15*(a + b*x^3)^2 + 141
5232*c^3*d^3*(b*c - a*d)^3*x^18*(a + b*x^3)^2 + 341172*c^2*d^4*(b*c - a*d)^
3*x^21*(a + b*x^3)^2 + 28042560*c^7*(b*c - a*d)^2*x^6*(a + b*x^3)^3 + 10760
2560*c^6*d*(b*c - a*d)^2*x^9*(a + b*x^3)^3 + 157697280*c^5*d^2*(b*c - a*d)^
2*x^12*(a + b*x^3)^3 + 101088000*c^4*d^3*(b*c - a*d)^2*x^15*(a + b*x^3)^3 +
 24261120*c^3*d^4*(b*c - a*d)^2*x^18*(a + b*x^3)^3 - 265470660*c^8*(b*c - a
*d)*x^3*(a + b*x^3)^4 - 1019636800*c^7*d*(b*c - a*d)*x^6*(a + b*x^3)^4 - 14
66086440*c^6*d^2*(b*c - a*d)*x^9*(a + b*x^3)^4 - 930252960*c^5*d^3*(b*c - a
*d)*x^12*(a + b*x^3)^4 - 221899860*c^4*d^4*(b*c - a*d)*x^15*(a + b*x^3)^4 +
 335877360*c^9*(a + b*x^3)^5 + 1279532800*c^8*d*x^3*(a + b*x^3)^5 + 1823334
240*c^7*d^2*x^6*(a + b*x^3)^5 + 1151579520*c^6*d^3*x^9*(a + b*x^3)^5 + 2739
39120*c^5*d^4*x^12*(a + b*x^3)^5 - 67420080*c^7*(b*c - a*d)^2*x^6*(a + b*x^
3)^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 25
9692160*c^6*d*(b*c - a*d)^2*x^9*(a + b*x^3)^3*Hypergeometric2F1[1/3, 1, 4/3
, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 377700960*c^5*d^2*(b*c - a*d)^2*x^12
*(a + b*x^3)^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x
^3))] - 241113600*c^4*d^3*(b*c - a*d)^2*x^15*(a + b*x^3)^3*Hypergeometric2F
1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 57723120*c^3*d^4*(b*c -
 a*d)^2*x^18*(a + b*x^3)^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)
/(c*(a + b*x^3))] + 349440000*c^8*(b*c - a*d)*x^3*(a + b*x^3)^4*Hypergeomet
ric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 1339520000*c^7*d*(
b*c - a*d)*x^6*(a + b*x^3)^4*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^
3)/(c*(a + b*x^3))] + 1921920000*c^6*d^2*(b*c - a*d)*x^9*(a + b*x^3)^4*Hype
```

```
rgeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 1218147840*
c^5*d^3*(b*c - a*d)*x^12*(a + b*x^3)^4*Hypergeometric2F1[1/3, 1, 4/3, ((b*c
 - a*d)*x^3)/(c*(a + b*x^3))] + 290384640*c^4*d^4*(b*c - a*d)*x^15*(a + b*x
^3)^4*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 3
35877360*c^9*(a + b*x^3)^5*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)
/(c*(a + b*x^3))] - 1279532800*c^8*d*x^3*(a + b*x^3)^5*Hypergeometric2F1[1/
3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 1823334240*c^7*d^2*x^6*(a +
 b*x^3)^5*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]
 - 1151579520*c^6*d^3*x^9*(a + b*x^3)^5*Hypergeometric2F1[1/3, 1, 4/3, ((b*
c - a*d)*x^3)/(c*(a + b*x^3))] - 273939120*c^5*d^4*x^12*(a + b*x^3)^5*Hyper
geometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 57834*c^4*(b
*c - a*d)^5*x^15*HypergeometricPFQ[{2, 2, 2, 10/3}, {1, 1, 19/3}, ((b*c - a
*d)*x^3)/(c*(a + b*x^3))] - 224532*c^3*d*(b*c - a*d)^5*x^18*HypergeometricP
FQ[{2, 2, 2, 10/3}, {1, 1, 19/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 3265
92*c^2*d^2*(b*c - a*d)^5*x^21*HypergeometricPFQ[{2, 2, 2, 10/3}, {1, 1, 19/
3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 210924*c*d^3*(b*c - a*d)^5*x^24*Hy
pergeometricPFQ[{2, 2, 2, 10/3}, {1, 1, 19/3}, ((b*c - a*d)*x^3)/(c*(a + b*
x^3))] - 51030*d^4*(b*c - a*d)^5*x^27*HypergeometricPFQ[{2, 2, 2, 10/3}, {1
, 1, 19/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 5103*c^4*(b*c - a*d)^5*x^1
5*HypergeometricPFQ[{2, 2, 2, 2, 10/3}, {1, 1, 1, 19/3}, ((b*c - a*d)*x^3)/
(c*(a + b*x^3))] - 20412*c^3*d*(b*c - a*d)^5*x^18*HypergeometricPFQ[{2, 2,
2, 2, 10/3}, {1, 1, 1, 19/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 30618*c^
2*d^2*(b*c - a*d)^5*x^21*HypergeometricPFQ[{2, 2, 2, 2, 10/3}, {1, 1, 1, 19
/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 20412*c*d^3*(b*c - a*d)^5*x^24*Hy
pergeometricPFQ[{2, 2, 2, 2, 10/3}, {1, 1, 1, 19/3}, ((b*c - a*d)*x^3)/(c*(
a + b*x^3))] - 5103*d^4*(b*c - a*d)^5*x^27*HypergeometricPFQ[{2, 2, 2, 2, 1
0/3}, {1, 1, 1, 19/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(524160*c^6*(b*c
 - a*d)^4*x^11*(a + b*x^3)^(10/3)*(c + d*x^3)^2)
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^{7/3}\left(c+dx^3\right)^3}\,dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{7/3}\left(c+dx^3\right)^3}\,dx}{a^2\sqrt[3]{a+bx^3}}$$

$$= -\frac{522756c^6(bc-ad)^3x^9\left(a+bx^3\right)^2+1516320c^5d(bc-ad)^3x^{12}\left(a+bx^3\right)^2+2198664c^4d^2(}{}$$

Mathematica [A] time = 5.84791, size = 337, normalized size = 0.73

$$\frac{1}{36}x\left(a+bx^3\right)^{2/3}\left(\frac{27b^3(bc-5ad)}{a^2\left(a+bx^3\right)(bc-ad)^4}-\frac{9b^3}{a\left(a+bx^3\right)^2(ad-bc)^3}+\frac{2d^3(5ad-21bc)}{c^2\left(c+dx^3\right)(bc-ad)^4}-\frac{6d^3}{c\left(c+dx^3\right)^2(bc-ad)^3}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(7/3)*(c + d*x^3)^3),x]
```

```
[Out] (x*(a + b*x^3)^(2/3)*((-9*b^3)/(a*(-(b*c) + a*d)^3*(a + b*x^3)^2) + (27*b^3
*(b*c - 5*a*d))/(a^2*(b*c - a*d)^4*(a + b*x^3)) - (6*d^3)/(c*(b*c - a*d)^3*
(c + d*x^3)^2) + (2*d^3*(-21*b*c + 5*a*d))/(c^2*(b*c - a*d)^4*(c + d*x^3)))
)/36 + (d^2*(54*b^2*c^2 - 24*a*b*c*d + 5*a^2*d^2)*(2*Sqrt[3]*ArcTan[(1 + (2
*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3)
 - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2
/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3
)]))/(54*c^(8/3)*(b*c - a*d)^(13/3))
```

Maple [F] time = 0.428, size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^3+c\right)^3}\left(bx^3+a\right)^{-\frac{7}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(7/3)/(d*x^3+c)^3,x)
```

```
[Out] int(1/(b*x^3+a)^(7/3)/(d*x^3+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{7}{3}}\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(7/3)*(d*x^3 + c)^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(7/3)/(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{7}{3}}\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(7/3)/(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(7/3)*(d*x^3 + c)^3), x)
```

3.117 $\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^3}\,dx$

Optimal. Leaf size=60

$$\frac{ax\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{4}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[Out] (a*x*(a + b*x^3)^(1/3)*AppellF1[1/3, -4/3, 3, 4/3, -((b*x^3)/a), -((d*x^3)/
c)])/(c^3*(1 + (b*x^3)/a)^(1/3))
```

Rubi [A] time = 0.0286702, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{ax\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{4}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(4/3)/(c + d*x^3)^3,x]
```

```
[Out] (a*x*(a + b*x^3)^(1/3)*AppellF1[1/3, -4/3, 3, 4/3, -((b*x^3)/a), -((d*x^3)/
c)])/(c^3*(1 + (b*x^3)/a)^(1/3))
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^3}\,dx &= \frac{\left(a\sqrt[3]{a+bx^3}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{(c+dx^3)^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}\\
&= \frac{ax\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{4}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\sqrt[3]{1+\frac{bx^3}{a}}}
\end{aligned}$$

Mathematica [B] time = 0.500327, size = 316, normalized size = 5.27

$$\frac{x\left(\frac{4c\left(\frac{4a^2c(c+dx^3)(10ad+bc)F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}+8a^2cd+5a^2d^2x^3-abc^2+10abcdx^3+5abd^2x^6-b^2c^2x^3+2b^2cdx^6\right)}{(c+dx^3)^2}+bx^3\right.}{72c^3d\left(a+bx^3\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(4/3)/(c + d*x^3)^3,x]
```

```
[Out] (x*(b*(2*b*c + 5*a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3,
-((b*x^3)/a), -((d*x^3)/c)] + (4*c*(-(a*b*c^2) + 8*a^2*c*d - b^2*c^2*x^3 +
10*a*b*c*d*x^3 + 5*a^2*d^2*x^3 + 2*b^2*c*d*x^6 + 5*a*b*d^2*x^6 + (4*a^2*c*(
b*c + 10*a*d)*(c + d*x^3)*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3
)/c)])/(4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - x^3*
(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*Appel
lF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))/(c + d*x^3)^2))/(72*c
^3*d*(a + b*x^3)^(2/3))
```

Maple [F] time = 0.436, size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^3+c\right)^3}\left(bx^3+a\right)^{\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)/(d*x^3+c)^3,x)
```

```
[Out] int((b*x^3+a)^(4/3)/(d*x^3+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/(d*x^3 + c)^3, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{4}{3}}}{\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)/(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(4/3)/(d*x^3 + c)^3, x)
```

3.118 $\int \frac{\sqrt[3]{a+bx^3}}{\left(c+dx^3\right)^3}\,dx$

Optimal. Leaf size=59

$$\frac{x\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{1}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\sqrt[3]{\frac{bx^3}{a}+1}}$$

```
[Out] (x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 3, 4/3, -((b*x^3)/a), -((d*x^3)/c)
])/(c^3*(1 + (b*x^3)/a)^(1/3))
```

Rubi [A] time = 0.0259295, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{1}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/3)/(c + d*x^3)^3,x]
```

```
[Out] (x*(a + b*x^3)^(1/3)*AppellF1[1/3, -1/3, 3, 4/3, -((b*x^3)/a), -((d*x^3)/c)
])/(c^3*(1 + (b*x^3)/a)^(1/3))
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[3]{a+bx^3}}{\left(c+dx^3\right)^3}\,dx &= \frac{\sqrt[3]{a+bx^3}\int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{\left(c+dx^3\right)^3}\,dx}{\sqrt[3]{1+\frac{bx^3}{a}}}\\ &= \frac{x\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{1}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\sqrt[3]{1+\frac{bx^3}{a}}}\end{aligned}$$

Mathematica [B] time = 0.540557, size = 431, normalized size = 7.31

$$\frac{\frac{c(16acx(3a^2d(6c+5dx^3)+ab(-18c^2-7cdx^3+5d^2x^6)-b^2cx^3(7c+4dx^3))F_1(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c})-4x^4(a^2d(8c+5dx^3)+ab(-7c^2+4cdx^3+5d^2x^6)-b^2cx^3(7c+4dx^3)}{(c+dx^3)^2(x^3(3adF_1(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c})+2bcF_1(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}))-4acF_1(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx}{c}}}{72c^3(a+bx^3)^{2/3}(bc}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/3)/(c + d*x^3)^3,x]
```

```
[Out] (-(b*(-4*b*c + 5*a*d)*x^4*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3,
-((b*x^3)/a), -((d*x^3)/c)]) + (c*(16*a*c*x*(-(b^2*c*x^3*(7*c + 4*d*x^3)) +
 3*a^2*d*(6*c + 5*d*x^3) + a*b*(-18*c^2 - 7*c*d*x^3 + 5*d^2*x^6))*AppellF1[
1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - 4*x^4*(-(b^2*c*x^3*(7*c + 4
*d*x^3)) + a^2*d*(8*c + 5*d*x^3) + a*b*(-7*c^2 + 4*c*d*x^3 + 5*d^2*x^6))*(3
*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF
1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((c + d*x^3)^2*(-4*a*c*A
ppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*AppellF1
[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3, 5/3, 1
, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))/(72*c^3*(b*c - a*d)*(a + b*x^3)^(2/3
))
```

Maple [F] time = 0.43, size = 0, normalized size = 0.

$$\int \frac{1}{(dx^3+c)^3}\sqrt[3]{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/3)/(d*x^3+c)^3,x)
```

```
[Out] int((b*x^3+a)^(1/3)/(d*x^3+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^3+a)^{\frac{1}{3}}}{(dx^3+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/(d*x^3 + c)^3, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/3)/(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{3}}}{\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/3)/(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/3)/(d*x^3 + c)^3, x)
```

3.119 $\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)^3}\,dx$

Optimal. Leaf size=59

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};\frac{2}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\left(a+bx^3\right)^{2/3}}$$

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 2/3, 3, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(c^3*(a + b*x^3)^(2/3))
```

Rubi [A] time = 0.0269843, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};\frac{2}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(2/3)*(c + d*x^3)^3),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 2/3, 3, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(c^3*(a + b*x^3)^(2/3))
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+bx^3\right)^{2/3}\left(c+dx^3\right)^3}\,dx &= \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{2/3}\left(c+dx^3\right)^3}\,dx}{\left(a+bx^3\right)^{2/3}}\\ &= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{1}{3};\frac{2}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\left(a+bx^3\right)^{2/3}}\end{aligned}$$

Mathematica [B] time = 0.683835, size = 442, normalized size = 7.49

$$\frac{x\left(\frac{4c\left(dx^3\left(a^2d\left(8c+5dx^3\right)+ab\left(-13c^2-2cdx^3+5d^2x^6\right)-b^2cx^3\left(13c+10dx^3\right)\right)\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4ac\left(3a^2d^2\left(6c+5dx^3\right)+abd\left(\cdots\right.\right.\right.}{\left(c+dx^3\right)^2\left(x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\cdots\right.\right.}\right.}{72c^3\left(a+bx^3\right)^{2/3}\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(2/3)*(c + d*x^3)^3),x]
```

```
[Out] (x*(5*b*d*(-2*b*c + a*d)*x^3*(1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/
3, -((b*x^3)/a), -((d*x^3)/c)] + (4*c*(-4*a*c*(3*a^2*d^2*(6*c + 5*d*x^3) +
b^2*c*(18*c^2 + 5*c*d*x^3 - 10*d^2*x^6) + a*b*d*(-36*c^2 - 25*c*d*x^3 + 5*d
^2*x^6))*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + d*x^3*(a^
2*d*(8*c + 5*d*x^3) - b^2*c*x^3*(13*c + 10*d*x^3) + a*b*(-13*c^2 - 2*c*d*x^
3 + 5*d^2*x^6))*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c
)] + 2*b*c*AppellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((c +
d*x^3)^2*(-4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x
^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*Ap
pellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))))/(72*c^3*(b*c - a*
d)^2*(a + b*x^3)^(2/3))
```

Maple [F] time = 0.422, size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^3+c\right)^3}\left(bx^3+a\right)^{-\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(2/3)/(d*x^3+c)^3,x)
```

```
[Out] int(1/(b*x^3+a)^(2/3)/(d*x^3+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(2/3)/(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{2}{3}}\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(2/3)/(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(2/3)*(d*x^3 + c)^3), x)
```

3.120 $\int \frac{1}{\left(a+bx^3\right)^{5/3}\left(c+dx^3\right)^3}\,dx$

Optimal. Leaf size=62

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3} F_1\left(\frac{1}{3};\frac{5}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac^3\left(a+bx^3\right)^{2/3}}$$

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 5/3, 3, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(a*c^3*(a + b*x^3)^(2/3))
```

Rubi [A] time = 0.0291526, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3} F_1\left(\frac{1}{3};\frac{5}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac^3\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(5/3)*(c + d*x^3)^3),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 5/3, 3, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(a*c^3*(a + b*x^3)^(2/3))
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+bx^3\right)^{5/3}\left(c+dx^3\right)^3}\,dx &= \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{5/3}\left(c+dx^3\right)^3}\,dx}{a\left(a+bx^3\right)^{2/3}}\\ &= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3};\frac{5}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{ac^3\left(a+bx^3\right)^{2/3}}\end{aligned}$$

Mathematica [B] time = 0.916664, size = 531, normalized size = 8.56

$$\frac{\frac{bdx^4\left(\frac{bx^3}{a}+1\right)^{2/3}\left(5a^2d^2-16abcd-9b^2c^2\right)F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(ad-bc)^3}-\frac{4c\left(x^4\left(a^2bd^2\left(19c^2+8cdx^3-5d^2x^6\right)-a^3d^3\left(8c+5dx^3\right)+ab^2cd^2x^3\left(19c+16dx^3\right)+9b^3c^2\left(c+dx^3\right)\right.\right.}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(5/3)*(c + d*x^3)^3),x]
```

```
[Out] ((b*d*(-9*b^2*c^2 - 16*a*b*c*d + 5*a^2*d^2)*x^4*(1 + (b*x^3)/a)^(2/3)*Appel
lF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])/(-(b*c) + a*d)^3 - (4*c*
(4*a*c*x*(3*a^3*d^3*(6*c + 5*d*x^3) + a*b^2*c*d*(54*c^2 + 35*c*d*x^3 - 16*d
^2*x^6) - 9*b^3*c^2*(2*c^2 + 3*c*d*x^3 + d^2*x^6) + a^2*b*d^2*(-54*c^2 - 43
*c*d*x^3 + 5*d^2*x^6))*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c
)] + x^4*(9*b^3*c^2*(c + d*x^3)^2 - a^3*d^3*(8*c + 5*d*x^3) + a*b^2*c*d^2*x
^3*(19*c + 16*d*x^3) + a^2*b*d^2*(19*c^2 + 8*c*d*x^3 - 5*d^2*x^6))*(3*a*d*A
ppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3,
 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))/((b*c - a*d)^3*(c + d*x^3)^2*(
4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - x^3*(3*a*d*A
ppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*AppellF1[4/3,
 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))/(72*a*c^3*(a + b*x^3)^(2/3))
```

Maple [F] time = 0.429, size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^3+c\right)^3}\left(bx^3+a\right)^{-\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(5/3)/(d*x^3+c)^3,x)
```

```
[Out] int(1/(b*x^3+a)^(5/3)/(d*x^3+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{5}{3}}\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(5/3)*(d*x^3 + c)^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(5/3)/(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{5}{3}}\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(5/3)/(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(5/3)*(d*x^3 + c)^3), x)
```

3.121 $\int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)^3}\,dx$

Optimal. Leaf size=62

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};\frac{8}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c^3\left(a+bx^3\right)^{2/3}}$$

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 8/3, 3, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(a^2*c^3*(a + b*x^3)^(2/3))
```

Rubi [A] time = 0.0289201, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {430, 429}

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{2/3}F_1\left(\frac{1}{3};\frac{8}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c^3\left(a+bx^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(8/3)*(c + d*x^3)^3),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 8/3, 3, 4/3, -((b*x^3)/a), -((d*x^3)
/c)])/(a^2*c^3*(a + b*x^3)^(2/3))
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a+bx^3\right)^{8/3}\left(c+dx^3\right)^3}\,dx &= \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}\int \frac{1}{\left(1+\frac{bx^3}{a}\right)^{8/3}\left(c+dx^3\right)^3}\,dx}{a^2\left(a+bx^3\right)^{2/3}}\\ &= \frac{x\left(1+\frac{bx^3}{a}\right)^{2/3}F_1\left(\frac{1}{3};\frac{8}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2c^3\left(a+bx^3\right)^{2/3}}\end{aligned}$$

Mathematica [B] time = 1.63346, size = 515, normalized size = 8.31

$$x\left(bdx^3\left(\frac{bx^3}{a}+1\right)^{2/3}\left(-110a^2bcd^2+25a^3d^3-171ab^2c^2d+36b^3c^3\right)F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+\frac{4c\left(\frac{4ac\left(c+dx^3\right)\left(540a^2b^2c^2d^2-235a^3bcd^3\right.}{4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-x^3\left(3ad\right.}}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(8/3)*(c + d*x^3)^3),x]
```

```
[Out] (x*(b*d*(36*b^3*c^3 - 171*a*b^2*c^2*d - 110*a^2*b*c*d^2 + 25*a^3*d^3)*x^3*(
1 + (b*x^3)/a)^(2/3)*AppellF1[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]
 + (4*c*((36*b^5*c^3*x^3*(c + d*x^3)^2 + 9*a*b^4*c^2*(6*c - 19*d*x^3)*(c +
d*x^3)^2 + 5*a^5*d^4*(8*c + 5*d*x^3) + 5*a^3*b^2*d^3*x^3*(-50*c^2 - 36*c*d*
x^3 + 5*d^2*x^6) + 5*a^4*b*d^3*(-25*c^2 - 6*c*d*x^3 + 10*d^2*x^6) - a^2*b^3
*c*d*(189*c^3 + 378*c^2*d*x^3 + 314*c*d^2*x^6 + 110*d^3*x^9))/(a + b*x^3) +
 (4*a*c*(36*b^4*c^4 - 171*a*b^3*c^3*d + 540*a^2*b^2*c^2*d^2 - 235*a^3*b*c*d
^3 + 50*a^4*d^4)*(c + d*x^3)*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*
x^3)/c)])/(4*a*c*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] - x
^3*(3*a*d*AppellF1[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*Ap
pellF1[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))/(c + d*x^3)^2))/(3
60*a^2*c^3*(b*c - a*d)^4*(a + b*x^3)^(2/3))
```

Maple [F] time = 0.45, size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^3+c\right)^3}\left(bx^3+a\right)^{-\frac{8}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(8/3)/(d*x^3+c)^3,x)
```

```
[Out] int(1/(b*x^3+a)^(8/3)/(d*x^3+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{8}{3}}\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(8/3)*(d*x^3 + c)^3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(8/3)/(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{8}{3}}\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(8/3)/(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(8/3)*(d*x^3 + c)^3), x)
```

3.122 $\int \frac{(a+bx^3)^{7/4}}{(c+dx^3)^{37/12}} \, dx$

Optimal. Leaf size=155

$$\frac{189a^2x\sqrt[4]{\frac{c(a+bx^3)}{a(c+dx^3)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{3};\frac{4}{3};-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{325c^3\sqrt[4]{a+bx^3}\sqrt[12]{c+dx^3}}+\frac{84ax\left(a+bx^3\right)^{3/4}}{325c^2\left(c+dx^3\right)^{13/12}}+\frac{4x\left(a+bx^3\right)^{7/4}}{25c\left(c+dx^3\right)^{25/12}}$$

```
[Out] (4*x*(a + b*x^3)^(7/4))/(25*c*(c + d*x^3)^(25/12)) + (84*a*x*(a + b*x^3)^(3
/4))/(325*c^2*(c + d*x^3)^(13/12)) + (189*a^2*x*((c*(a + b*x^3))/(a*(c + d*
x^3)))^(1/4)*Hypergeometric2F1[1/4, 1/3, 4/3, -(((b*c - a*d)*x^3)/(a*(c + d
*x^3)))])/(325*c^3*(a + b*x^3)^(1/4)*(c + d*x^3)^(1/12))
```

Rubi [A] time = 0.0623555, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {378, 380}

$$\frac{189a^2x\sqrt[4]{\frac{c(a+bx^3)}{a(c+dx^3)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{3};\frac{4}{3};-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{325c^3\sqrt[4]{a+bx^3}\sqrt[12]{c+dx^3}}+\frac{84ax\left(a+bx^3\right)^{3/4}}{325c^2\left(c+dx^3\right)^{13/12}}+\frac{4x\left(a+bx^3\right)^{7/4}}{25c\left(c+dx^3\right)^{25/12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(7/4)/(c + d*x^3)^(37/12),x]
```

```
[Out] (4*x*(a + b*x^3)^(7/4))/(25*c*(c + d*x^3)^(25/12)) + (84*a*x*(a + b*x^3)^(3
/4))/(325*c^2*(c + d*x^3)^(13/12)) + (189*a^2*x*((c*(a + b*x^3))/(a*(c + d*
x^3)))^(1/4)*Hypergeometric2F1[1/4, 1/3, 4/3, -(((b*c - a*d)*x^3)/(a*(c + d
*x^3)))])/(325*c^3*(a + b*x^3)^(1/4)*(c + d*x^3)^(1/12))
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(x*(a + b*x^n)^p*Hypergeometric2F1[1/n, -p, 1 + 1/n, -(((b*c - a*d)
*x^n)/(a*(c + d*x^n)))])/(c*((c*(a + b*x^n))/(a*(c + d*x^n)))^p*(c + d*x^n)
^(1/n + p)), x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(a+bx^3)^{7/4}}{(c+dx^3)^{37/12}}\,dx = \frac{4x(a+bx^3)^{7/4}}{25c(c+dx^3)^{25/12}} + \frac{(21a)\int \frac{(a+bx^3)^{3/4}}{(c+dx^3)^{25/12}}\,dx}{25c}$$
$$= \frac{4x(a+bx^3)^{7/4}}{25c(c+dx^3)^{25/12}} + \frac{84ax(a+bx^3)^{3/4}}{325c^2(c+dx^3)^{13/12}} + \frac{(189a^2)\int \frac{1}{\sqrt[4]{a+bx^3}(c+dx^3)^{13/12}}\,dx}{325c^2}$$
$$= \frac{4x(a+bx^3)^{7/4}}{25c(c+dx^3)^{25/12}} + \frac{84ax(a+bx^3)^{3/4}}{325c^2(c+dx^3)^{13/12}} + \frac{189a^2x\sqrt[4]{\frac{c(a+bx^3)}{a(c+dx^3)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{3};\frac{4}{3};-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{325c^3\sqrt[4]{a+bx^3}\sqrt[12]{c+dx^3}}$$

Mathematica [A] time = 0.0328561, size = 90, normalized size = 0.58

$$\frac{ax(a+bx^3)^{3/4}\,{}_2F_1\left(-\frac{7}{4},\frac{1}{3};\frac{4}{3};\frac{(ad-bc)x^3}{a(dx^3+c)}\right)}{c^3\left(\frac{bx^3}{a}+1\right)^{3/4}\sqrt[12]{c+dx^3}\sqrt[4]{\frac{dx^3}{c}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(7/4)/(c + d*x^3)^(37/12),x]
```

```
[Out] (a*x*(a + b*x^3)^(3/4)*Hypergeometric2F1[-7/4, 1/3, 4/3, ((-(b*c) + a*d)*x^
3)/(a*(c + d*x^3))])/(c^3*(1 + (b*x^3)/a)^(3/4)*(c + d*x^3)^(1/12)*(1 + (d*
x^3)/c)^(1/4))
```

Maple [F] time = 0.466, size = 0, normalized size = 0.

$$\int (bx^3+a)^{\frac{7}{4}}(dx^3+c)^{-\frac{37}{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(7/4)/(d*x^3+c)^(37/12),x)
```

```
[Out] int((b*x^3+a)^(7/4)/(d*x^3+c)^(37/12),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^3+a)^{\frac{7}{4}}}{(dx^3+c)^{\frac{37}{12}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(7/4)/(d*x^3+c)^(37/12),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(7/4)/(d*x^3 + c)^(37/12), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^3+a\right)^{\frac{7}{4}}\left(dx^3+c\right)^{\frac{11}{12}}}{d^4x^{12}+4\,cd^3x^9+6\,c^2d^2x^6+4\,c^3dx^3+c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(7/4)/(d*x^3+c)^(37/12),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(7/4)*(d*x^3 + c)^(11/12)/(d^4*x^12 + 4*c*d^3*x^9 + 6*
c^2*d^2*x^6 + 4*c^3*d*x^3 + c^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(7/4)/(d*x**3+c)**(37/12),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(7/4)/(d*x^3+c)^(37/12),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.123 $\int \frac{\left(a+bx^3\right)^{5/4}}{\left(c+dx^3\right)^{31/12}}\,dx$

Optimal. Leaf size=155

$$\frac{45a^2x\left(c+dx^3\right)^{5/12}\left(\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}\right)^{3/4}{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a\left(dx^3+c\right)}\right)}{133c^3\left(a+bx^3\right)^{3/4}}+\frac{60ax\sqrt[4]{a+bx^3}}{133c^2\left(c+dx^3\right)^{7/12}}+\frac{4x\left(a+bx^3\right)^{5/4}}{19c\left(c+dx^3\right)^{19/12}}$$

```
[Out] (4*x*(a + b*x^3)^(5/4))/(19*c*(c + d*x^3)^(19/12)) + (60*a*x*(a + b*x^3)^(1
/4))/(133*c^2*(c + d*x^3)^(7/12)) + (45*a^2*x*((c*(a + b*x^3))/(a*(c + d*x^
3)))^(3/4)*(c + d*x^3)^(5/12)*Hypergeometric2F1[1/3, 3/4, 4/3, -(((b*c - a*
d)*x^3)/(a*(c + d*x^3)))])/(133*c^3*(a + b*x^3)^(3/4))
```

Rubi [A] time = 0.0577109, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {378, 380}

$$\frac{45a^2x\left(c+dx^3\right)^{5/12}\left(\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}\right)^{3/4}{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a\left(dx^3+c\right)}\right)}{133c^3\left(a+bx^3\right)^{3/4}}+\frac{60ax\sqrt[4]{a+bx^3}}{133c^2\left(c+dx^3\right)^{7/12}}+\frac{4x\left(a+bx^3\right)^{5/4}}{19c\left(c+dx^3\right)^{19/12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(5/4)/(c + d*x^3)^(31/12),x]
```

```
[Out] (4*x*(a + b*x^3)^(5/4))/(19*c*(c + d*x^3)^(19/12)) + (60*a*x*(a + b*x^3)^(1
/4))/(133*c^2*(c + d*x^3)^(7/12)) + (45*a^2*x*((c*(a + b*x^3))/(a*(c + d*x^
3)))^(3/4)*(c + d*x^3)^(5/12)*Hypergeometric2F1[1/3, 3/4, 4/3, -(((b*c - a*
d)*x^3)/(a*(c + d*x^3)))])/(133*c^3*(a + b*x^3)^(3/4))
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(x*(a + b*x^n)^p*Hypergeometric2F1[1/n, -p, 1 + 1/n, -(((b*c - a*d)
*x^n)/(a*(c + d*x^n)))])/(c*((c*(a + b*x^n))/(a*(c + d*x^n)))^p*(c + d*x^n)
^(1/n + p)), x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{5/4}}{\left(c+dx^3\right)^{31/12}}\,dx = \frac{4x\left(a+bx^3\right)^{5/4}}{19c\left(c+dx^3\right)^{19/12}} + \frac{(15a)\int \frac{\sqrt[4]{a+bx^3}}{\left(c+dx^3\right)^{19/12}}\,dx}{19c}$$

$$= \frac{4x\left(a+bx^3\right)^{5/4}}{19c\left(c+dx^3\right)^{19/12}} + \frac{60ax\sqrt[4]{a+bx^3}}{133c^2\left(c+dx^3\right)^{7/12}} + \frac{\left(45a^2\right)\int \frac{1}{\left(a+bx^3\right)^{3/4}\left(c+dx^3\right)^{7/12}}\,dx}{133c^2}$$

$$= \frac{4x\left(a+bx^3\right)^{5/4}}{19c\left(c+dx^3\right)^{19/12}} + \frac{60ax\sqrt[4]{a+bx^3}}{133c^2\left(c+dx^3\right)^{7/12}} + \frac{45a^2x\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4}\left(c+dx^3\right)^{5/12}\,{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{133c^3\left(a+bx^3\right)^{3/4}}$$

Mathematica [A] time = 0.0315534, size = 90, normalized size = 0.58

$$\frac{ax\sqrt[4]{a+bx^3}\sqrt[4]{\frac{dx^3}{c}+1}\,{}_2F_1\left(-\frac{5}{4},\frac{1}{3};\frac{4}{3};\frac{(ad-bc)x^3}{a(dx^3+c)}\right)}{c^2\sqrt[4]{\frac{bx^3}{a}+1}\left(c+dx^3\right)^{7/12}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(5/4)/(c + d*x^3)^(31/12),x]
```

```
[Out] (a*x*(a + b*x^3)^(1/4)*(1 + (d*x^3)/c)^(1/4)*Hypergeometric2F1[-5/4, 1/3, 4
/3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/(c^2*(1 + (b*x^3)/a)^(1/4)*(c +
d*x^3)^(7/12))
```

Maple [F] time = 0.487, size = 0, normalized size = 0.

$$\int \left(bx^3+a\right)^{\frac{5}{4}}\left(dx^3+c\right)^{-\frac{31}{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(5/4)/(d*x^3+c)^(31/12),x)
```

```
[Out] int((b*x^3+a)^(5/4)/(d*x^3+c)^(31/12),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{5}{4}}}{\left(dx^3+c\right)^{\frac{31}{12}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/4)/(d*x^3+c)^(31/12),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(5/4)/(d*x^3 + c)^(31/12), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^3+a\right)^{\frac{5}{4}}\left(dx^3+c\right)^{\frac{5}{12}}}{d^3x^9+3\,cd^2x^6+3\,c^2dx^3+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/4)/(d*x^3+c)^(31/12),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(5/4)*(d*x^3 + c)^(5/12)/(d^3*x^9 + 3*c*d^2*x^6 + 3*c^
2*d*x^3 + c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(5/4)/(d*x**3+c)**(31/12),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{5}{4}}}{\left(dx^3+c\right)^{\frac{31}{12}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/4)/(d*x^3+c)^(31/12),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(5/4)/(d*x^3 + c)^(31/12), x)
```

3.124 $\int \frac{(a+bx^3)^{3/4}}{(c+dx^3)^{25/12}}\,dx$

Optimal. Leaf size=122

$$\frac{9ax\sqrt[4]{\frac{c(a+bx^3)}{a(c+dx^3)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{3};\frac{4}{3};-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{13c^2\sqrt[4]{a+bx^3}\,\sqrt[12]{c+dx^3}}+\frac{4x\left(a+bx^3\right)^{3/4}}{13c\left(c+dx^3\right)^{13/12}}$$

```
[Out] (4*x*(a + b*x^3)^(3/4))/(13*c*(c + d*x^3)^(13/12)) + (9*a*x*((c*(a + b*x^3)
)/(a*(c + d*x^3)))^(1/4)*Hypergeometric2F1[1/4, 1/3, 4/3, -(((b*c - a*d)*x^
3)/(a*(c + d*x^3)))])/(13*c^2*(a + b*x^3)^(1/4)*(c + d*x^3)^(1/12))
```

Rubi [A] time = 0.035532, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {378, 380}

$$\frac{9ax\sqrt[4]{\frac{c(a+bx^3)}{a(c+dx^3)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{3};\frac{4}{3};-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{13c^2\sqrt[4]{a+bx^3}\,\sqrt[12]{c+dx^3}}+\frac{4x\left(a+bx^3\right)^{3/4}}{13c\left(c+dx^3\right)^{13/12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(3/4)/(c + d*x^3)^(25/12),x]
```

```
[Out] (4*x*(a + b*x^3)^(3/4))/(13*c*(c + d*x^3)^(13/12)) + (9*a*x*((c*(a + b*x^3)
)/(a*(c + d*x^3)))^(1/4)*Hypergeometric2F1[1/4, 1/3, 4/3, -(((b*c - a*d)*x^
3)/(a*(c + d*x^3)))])/(13*c^2*(a + b*x^3)^(1/4)*(c + d*x^3)^(1/12))
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(x*(a + b*x^n)^p*Hypergeometric2F1[1/n, -p, 1 + 1/n, -(((b*c - a*d)
*x^n)/(a*(c + d*x^n)))])/(c*((c*(a + b*x^n))/(a*(c + d*x^n)))^p*(c + d*x^n)
^(1/n + p)), x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^3\right)^{3/4}}{\left(c+dx^3\right)^{25/12}}\,dx = \frac{4x\left(a+bx^3\right)^{3/4}}{13c\left(c+dx^3\right)^{13/12}} + \frac{(9a)\int \frac{1}{\sqrt[4]{a+bx^3}\left(c+dx^3\right)^{13/12}}\,dx}{13c}$$

$$= \frac{4x\left(a+bx^3\right)^{3/4}}{13c\left(c+dx^3\right)^{13/12}} + \frac{9ax\sqrt[4]{\frac{c(a+bx^3)}{a(c+dx^3)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{3};\frac{4}{3};-\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{13c^2\sqrt[4]{a+bx^3}\,\sqrt[12]{c+dx^3}}$$

Mathematica [A] time = 0.0203517, size = 89, normalized size = 0.73

$$\frac{x\left(a+bx^3\right)^{3/4}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{3};\frac{4}{3};\frac{(ad-bc)x^3}{a(dx^3+c)}\right)}{c^2\left(\frac{bx^3}{a}+1\right)^{3/4}\sqrt[12]{c+dx^3}\sqrt[4]{\frac{dx^3}{c}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(3/4)/(c + d*x^3)^(25/12),x]
```

```
[Out] (x*(a + b*x^3)^(3/4)*Hypergeometric2F1[-3/4, 1/3, 4/3, ((-(b*c) + a*d)*x^3)
/(a*(c + d*x^3))])/(c^2*(1 + (b*x^3)/a)^(3/4)*(c + d*x^3)^(1/12)*(1 + (d*x^
3)/c)^(1/4))
```

Maple [F] time = 0.444, size = 0, normalized size = 0.

$$\int \left(bx^3+a\right)^{\frac{3}{4}}\left(dx^3+c\right)^{-\frac{25}{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(3/4)/(d*x^3+c)^(25/12),x)
```

```
[Out] int((b*x^3+a)^(3/4)/(d*x^3+c)^(25/12),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{3}{4}}}{\left(dx^3+c\right)^{\frac{25}{12}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/4)/(d*x^3+c)^(25/12),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(3/4)/(d*x^3 + c)^(25/12), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^3+a\right)^{\frac{3}{4}}\left(dx^3+c\right)^{\frac{11}{12}}}{d^3x^9+3\,cd^2x^6+3\,c^2dx^3+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/4)/(d*x^3+c)^(25/12),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(3/4)*(d*x^3 + c)^(11/12)/(d^3*x^9 + 3*c*d^2*x^6 + 3*c
^2*d*x^3 + c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(3/4)/(d*x**3+c)**(25/12),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(3/4)/(d*x^3+c)^(25/12),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.125 $\int \frac{\sqrt[4]{a+bx^3}}{\left(c+dx^3\right)^{19/12}} \, dx$

Optimal. Leaf size=122

$$\frac{3ax\left(c+dx^3\right)^{5/12}\left(\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}\right)^{3/4}{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a\left(dx^3+c\right)}\right)}{7c^2\left(a+bx^3\right)^{3/4}}+\frac{4x\sqrt[4]{a+bx^3}}{7c\left(c+dx^3\right)^{7/12}}$$

```
[Out] (4*x*(a + b*x^3)^(1/4))/(7*c*(c + d*x^3)^(7/12)) + (3*a*x*((c*(a + b*x^3))/
(a*(c + d*x^3)))^(3/4)*(c + d*x^3)^(5/12)*Hypergeometric2F1[1/3, 3/4, 4/3,
-(((b*c - a*d)*x^3)/(a*(c + d*x^3)))])/(7*c^2*(a + b*x^3)^(3/4))
```

Rubi [A] time = 0.036754, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {378, 380}

$$\frac{3ax\left(c+dx^3\right)^{5/12}\left(\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}\right)^{3/4}{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a\left(dx^3+c\right)}\right)}{7c^2\left(a+bx^3\right)^{3/4}}+\frac{4x\sqrt[4]{a+bx^3}}{7c\left(c+dx^3\right)^{7/12}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(1/4)/(c + d*x^3)^(19/12),x]
```

```
[Out] (4*x*(a + b*x^3)^(1/4))/(7*c*(c + d*x^3)^(7/12)) + (3*a*x*((c*(a + b*x^3))/
(a*(c + d*x^3)))^(3/4)*(c + d*x^3)^(5/12)*Hypergeometric2F1[1/3, 3/4, 4/3,
-(((b*c - a*d)*x^3)/(a*(c + d*x^3)))])/(7*c^2*(a + b*x^3)^(3/4))
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(x*(a + b*x^n)^p*Hypergeometric2F1[1/n, -p, 1 + 1/n, -(((b*c - a*d)
*x^n)/(a*(c + d*x^n)))])/(c*((c*(a + b*x^n))/(a*(c + d*x^n)))^p*(c + d*x^n)
^(1/n + p)), x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{\sqrt[4]{a+bx^3}}{\left(c+dx^3\right)^{19/12}}\,dx = \frac{4x\sqrt[4]{a+bx^3}}{7c\left(c+dx^3\right)^{7/12}} + \frac{(3a)\int \frac{1}{\left(a+bx^3\right)^{3/4}\left(c+dx^3\right)^{7/12}}\,dx}{7c}$$

$$= \frac{4x\sqrt[4]{a+bx^3}}{7c\left(c+dx^3\right)^{7/12}} + \frac{3ax\left(\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}\right)^{3/4}\left(c+dx^3\right)^{5/12}\,{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a\left(c+dx^3\right)}\right)}{7c^2\left(a+bx^3\right)^{3/4}}$$

Mathematica [A] time = 0.0237392, size = 89, normalized size = 0.73

$$\frac{x\sqrt[4]{a+bx^3}\sqrt[4]{\frac{dx^3}{c}+1}\,{}_2F_1\left(-\frac{1}{4},\frac{1}{3};\frac{4}{3};\frac{(ad-bc)x^3}{a\left(dx^3+c\right)}\right)}{c\sqrt[4]{\frac{bx^3}{a}+1}\left(c+dx^3\right)^{7/12}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(1/4)/(c + d*x^3)^(19/12),x]
```

```
[Out] (x*(a + b*x^3)^(1/4)*(1 + (d*x^3)/c)^(1/4)*Hypergeometric2F1[-1/4, 1/3, 4/3
, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/(c*(1 + (b*x^3)/a)^(1/4)*(c + d*x^
3)^(7/12))
```

Maple [F] time = 0.453, size = 0, normalized size = 0.

$$\int \sqrt[4]{bx^3+a}\left(dx^3+c\right)^{-\frac{19}{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(1/4)/(d*x^3+c)^(19/12),x)
```

```
[Out] int((b*x^3+a)^(1/4)/(d*x^3+c)^(19/12),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{4}}}{\left(dx^3+c\right)^{\frac{19}{12}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/4)/(d*x^3+c)^(19/12),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(1/4)/(d*x^3 + c)^(19/12), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^3+a\right)^{\frac{1}{4}}\left(dx^3+c\right)^{\frac{5}{12}}}{d^2x^6+2\,cdx^3+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/4)/(d*x^3+c)^(19/12),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(1/4)*(d*x^3 + c)^(5/12)/(d^2*x^6 + 2*c*d*x^3 + c^2),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(1/4)/(d*x**3+c)**(19/12),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^{\frac{1}{4}}}{\left(dx^3+c\right)^{\frac{19}{12}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(1/4)/(d*x^3+c)^(19/12),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(1/4)/(d*x^3 + c)^(19/12), x)
```

3.126 $\int \frac{1}{\sqrt[4]{a+bx^3}\left(c+dx^3\right)^{13/12}}\,dx$

Optimal. Leaf size=87

$$\frac{x\sqrt[4]{\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{3};\frac{4}{3};-\frac{(bc-ad)x^3}{a\left(dx^3+c\right)}\right)}{c\sqrt[4]{a+bx^3}\,\sqrt[12]{c+dx^3}}$$

```
[Out] (x*((c*(a + b*x^3))/(a*(c + d*x^3)))^(1/4)*Hypergeometric2F1[1/4, 1/3, 4/3,
 -(((b*c - a*d)*x^3)/(a*(c + d*x^3)))])/(c*(a + b*x^3)^(1/4)*(c + d*x^3)^(1
/12))
```

Rubi [A] time = 0.0168134, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {380}

$$\frac{x\sqrt[4]{\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{3};\frac{4}{3};-\frac{(bc-ad)x^3}{a\left(dx^3+c\right)}\right)}{c\sqrt[4]{a+bx^3}\,\sqrt[12]{c+dx^3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(1/4)*(c + d*x^3)^(13/12)),x]
```

```
[Out] (x*((c*(a + b*x^3))/(a*(c + d*x^3)))^(1/4)*Hypergeometric2F1[1/4, 1/3, 4/3,
 -(((b*c - a*d)*x^3)/(a*(c + d*x^3)))])/(c*(a + b*x^3)^(1/4)*(c + d*x^3)^(1
/12))
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(x*(a + b*x^n)^p*Hypergeometric2F1[1/n, -p, 1 + 1/n, -(((b*c - a*d)
*x^n)/(a*(c + d*x^n)))])/(c*((c*(a + b*x^n))/(a*(c + d*x^n)))^p*(c + d*x^n)
^(1/n + p)), x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a+bx^3}\left(c+dx^3\right)^{13/12}}\,dx=\frac{x\sqrt[4]{\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}}\,{}_2F_1\left(\frac{1}{4},\frac{1}{3};\frac{4}{3};-\frac{(bc-ad)x^3}{a\left(c+dx^3\right)}\right)}{c\sqrt[4]{a+bx^3}\,\sqrt[12]{c+dx^3}}$$

Mathematica [A] time = 0.0287124, size = 86, normalized size = 0.99

$$\frac{x\sqrt[4]{\frac{bx^3}{a}+1}\left(\frac{dx^3}{c}+1\right)^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{1}{3};\frac{4}{3};\frac{(ad-bc)x^3}{a\left(dx^3+c\right)}\right)}{\sqrt[4]{a+bx^3}\left(c+dx^3\right)^{13/12}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(1/4)*(c + d*x^3)^(13/12)),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(1/4)*(1 + (d*x^3)/c)^(3/4)*Hypergeometric2F1[1/4, 1/3,
4/3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/((a + b*x^3)^(1/4)*(c + d*x^3)^
(13/12))
```

Maple [F] time = 0.446, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{b x^3 + a}} \left(d x^3 + c\right)^{-\frac{13}{12}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(1/4)/(d*x^3+c)^(13/12),x)
```

```
[Out] int(1/(b*x^3+a)^(1/4)/(d*x^3+c)^(13/12),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b x^3 + a\right)^{\frac{1}{4}} \left(d x^3 + c\right)^{\frac{13}{12}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/4)/(d*x^3+c)^(13/12),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/4)*(d*x^3 + c)^(13/12)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(b x^3 + a\right)^{\frac{3}{4}} \left(d x^3 + c\right)^{\frac{11}{12}}}{b d^2 x^9 + \left(2 \, b c d + a d^2\right) x^6 + \left(b c^2 + 2 \, a c d\right) x^3 + a c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/4)/(d*x^3+c)^(13/12),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(3/4)*(d*x^3 + c)^(11/12)/(b*d^2*x^9 + (2*b*c*d + a*d^
2)*x^6 + (b*c^2 + 2*a*c*d)*x^3 + a*c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(1/4)/(d*x**3+c)**(13/12),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{1}{4}}\left(dx^3+c\right)^{\frac{13}{12}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(1/4)/(d*x^3+c)^(13/12),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(1/4)*(d*x^3 + c)^(13/12)), x)
```

3.127 $\int \frac{1}{(a+bx^3)^{3/4}(c+dx^3)^{7/12}}\,dx$

Optimal. Leaf size=87

$$\frac{x\left(c+dx^3\right)^{5/12}\left(\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}\right)^{3/4}\,{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a\left(dx^3+c\right)}\right)}{c\left(a+bx^3\right)^{3/4}}$$

```
[Out] (x*((c*(a + b*x^3))/(a*(c + d*x^3)))^(3/4)*(c + d*x^3)^(5/12)*Hypergeometri
c2F1[1/3, 3/4, 4/3, -(((b*c - a*d)*x^3)/(a*(c + d*x^3)))])/(c*(a + b*x^3)^(
3/4))
```

Rubi [A] time = 0.0173785, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {380}

$$\frac{x\left(c+dx^3\right)^{5/12}\left(\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}\right)^{3/4}\,{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a\left(dx^3+c\right)}\right)}{c\left(a+bx^3\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(3/4)*(c + d*x^3)^(7/12)),x]
```

```
[Out] (x*((c*(a + b*x^3))/(a*(c + d*x^3)))^(3/4)*(c + d*x^3)^(5/12)*Hypergeometri
c2F1[1/3, 3/4, 4/3, -(((b*c - a*d)*x^3)/(a*(c + d*x^3)))])/(c*(a + b*x^3)^(
3/4))
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(x*(a + b*x^n)^p*Hypergeometric2F1[1/n, -p, 1 + 1/n, -(((b*c - a*d)
*x^n)/(a*(c + d*x^n)))])/(c*((c*(a + b*x^n))/(a*(c + d*x^n)))^p*(c + d*x^n)
^(1/n + p)), x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^{3/4}\left(c+dx^3\right)^{7/12}}\,dx=\frac{x\left(\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}\right)^{3/4}\left(c+dx^3\right)^{5/12}\,{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a\left(c+dx^3\right)}\right)}{c\left(a+bx^3\right)^{3/4}}$$

Mathematica [A] time = 0.0212314, size = 86, normalized size = 0.99

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{3/4}\sqrt[4]{\frac{dx^3}{c}+1}\,{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};\frac{(ad-bc)x^3}{a\left(dx^3+c\right)}\right)}{\left(a+bx^3\right)^{3/4}\left(c+dx^3\right)^{7/12}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(3/4)*(c + d*x^3)^(7/12)),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(3/4)*(1 + (d*x^3)/c)^(1/4)*Hypergeometric2F1[1/3, 3/4,
4/3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/((a + b*x^3)^(3/4)*(c + d*x^3)^
(7/12))
```

Maple [F] time = 0.433, size = 0, normalized size = 0.

$$\int \left(bx^3+a\right)^{-\frac{3}{4}} \left(dx^3+c\right)^{-\frac{7}{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(3/4)/(d*x^3+c)^(7/12),x)
```

```
[Out] int(1/(b*x^3+a)^(3/4)/(d*x^3+c)^(7/12),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{3}{4}} \left(dx^3+c\right)^{\frac{7}{12}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(3/4)/(d*x^3+c)^(7/12),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(3/4)*(d*x^3 + c)^(7/12)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bx^3+a\right)^{\frac{1}{4}} \left(dx^3+c\right)^{\frac{5}{12}}}{bdx^6+(bc+ad)x^3+ac}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(3/4)/(d*x^3+c)^(7/12),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(1/4)*(d*x^3 + c)^(5/12)/(b*d*x^6 + (b*c + a*d)*x^3 +
a*c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^3\right)^{\frac{3}{4}} \left(c+dx^3\right)^{\frac{7}{12}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(3/4)/(d*x**3+c)**(7/12),x)
```

```
[Out] Integral(1/((a + b*x**3)**(3/4)*(c + d*x**3)**(7/12)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3 + a\right)^{\frac{3}{4}}\left(dx^3 + c\right)^{\frac{7}{12}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(3/4)/(d*x^3+c)^(7/12),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(3/4)*(d*x^3 + c)^(7/12)), x)
```

3.128 $\int \frac{1}{\left(a+bx^3\right)^{5/4} \sqrt[12]{c+dx^3}} \, dx$

Optimal. Leaf size=87

$$\frac{x \left(c+dx^3\right)^{11/12} \left(\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}\right)^{5/4} {}_2F_1\left(\frac{1}{3},\frac{5}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a\left(dx^3+c\right)}\right)}{c \left(a+bx^3\right)^{5/4}}$$

```
[Out] (x*((c*(a + b*x^3))/(a*(c + d*x^3)))^(5/4)*(c + d*x^3)^(11/12)*Hypergeometr
ic2F1[1/3, 5/4, 4/3, -(((b*c - a*d)*x^3)/(a*(c + d*x^3)))])/(c*(a + b*x^3)^
(5/4))
```

Rubi [A] time = 0.0163963, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {380}

$$\frac{x \left(c+dx^3\right)^{11/12} \left(\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}\right)^{5/4} {}_2F_1\left(\frac{1}{3},\frac{5}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a\left(dx^3+c\right)}\right)}{c \left(a+bx^3\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^3)^(5/4)*(c + d*x^3)^(1/12)),x]
```

```
[Out] (x*((c*(a + b*x^3))/(a*(c + d*x^3)))^(5/4)*(c + d*x^3)^(11/12)*Hypergeometr
ic2F1[1/3, 5/4, 4/3, -(((b*c - a*d)*x^3)/(a*(c + d*x^3)))])/(c*(a + b*x^3)^
(5/4))
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(x*(a + b*x^n)^p*Hypergeometric2F1[1/n, -p, 1 + 1/n, -(((b*c - a*d)
*x^n)/(a*(c + d*x^n)))])/(c*((c*(a + b*x^n))/(a*(c + d*x^n)))^p*(c + d*x^n)
^(1/n + p)), x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^3\right)^{5/4} \sqrt[12]{c+dx^3}} \, dx = \frac{x \left(\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}\right)^{5/4} \left(c+dx^3\right)^{11/12} {}_2F_1\left(\frac{1}{3},\frac{5}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a\left(c+dx^3\right)}\right)}{c \left(a+bx^3\right)^{5/4}}$$

Mathematica [A] time = 0.0220741, size = 89, normalized size = 1.02

$$\frac{x \sqrt[4]{\frac{bx^3}{a}+1} \, {}_2F_1\left(\frac{1}{3},\frac{5}{4};\frac{4}{3};\frac{(ad-bc)x^3}{a\left(dx^3+c\right)}\right)}{a \sqrt[4]{a+bx^3} \sqrt[12]{c+dx^3} \sqrt[4]{\frac{dx^3}{c}+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(5/4)*(c + d*x^3)^(1/12)),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(1/4)*Hypergeometric2F1[1/3, 5/4, 4/3, ((-(b*c) + a*d)*x
^3)/(a*(c + d*x^3))])/(a*(a + b*x^3)^(1/4)*(c + d*x^3)^(1/12)*(1 + (d*x^3)/
c)^(1/4))
```

Maple [F] time = 0.449, size = 0, normalized size = 0.

$$\int \left(b x^{3}+a\right)^{-\frac{5}{4}} \left(d x^{3}+c\right)^{-\frac{1}{12}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^3+a)^(5/4)/(d*x^3+c)^(1/12),x)
```

```
[Out] int(1/(b*x^3+a)^(5/4)/(d*x^3+c)^(1/12),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b x^{3}+a\right)^{\frac{5}{4}} \left(d x^{3}+c\right)^{\frac{1}{12}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(5/4)/(d*x^3+c)^(1/12),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^3 + a)^(5/4)*(d*x^3 + c)^(1/12)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(b x^{3}+a\right)^{\frac{3}{4}} \left(d x^{3}+c\right)^{\frac{11}{12}}}{b^{2} d x^{9}+\left(b^{2} c+2\, a b d\right) x^{6}+\left(2\, a b c+a^{2} d\right) x^{3}+a^{2} c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(5/4)/(d*x^3+c)^(1/12),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(3/4)*(d*x^3 + c)^(11/12)/(b^2*d*x^9 + (b^2*c + 2*a*b*
d)*x^6 + (2*a*b*c + a^2*d)*x^3 + a^2*c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(5/4)/(d*x**3+c)**(1/12),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^3+a\right)^{\frac{5}{4}}\left(dx^3+c\right)^{\frac{1}{12}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^3+a)^(5/4)/(d*x^3+c)^(1/12),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^3 + a)^(5/4)*(d*x^3 + c)^(1/12)), x)
```

3.129 $\int \frac{\left(c+dx^3\right)^{5/12}}{\left(a+bx^3\right)^{7/4}}\,dx$

Optimal. Leaf size=121

$$\frac{5x\left(c+dx^3\right)^{5/12}\left(\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}\right)^{3/4}{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a\left(dx^3+c\right)}\right)}{9a\left(a+bx^3\right)^{3/4}}+\frac{4x\left(c+dx^3\right)^{5/12}}{9a\left(a+bx^3\right)^{3/4}}$$

```
[Out] (4*x*(c + d*x^3)^(5/12))/(9*a*(a + b*x^3)^(3/4)) + (5*x*((c*(a + b*x^3))/(a
*(c + d*x^3)))^(3/4)*(c + d*x^3)^(5/12)*Hypergeometric2F1[1/3, 3/4, 4/3, -(
((b*c - a*d)*x^3)/(a*(c + d*x^3)))])/(9*a*(a + b*x^3)^(3/4))
```

Rubi [A] time = 0.0373702, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {378, 380}

$$\frac{5x\left(c+dx^3\right)^{5/12}\left(\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}\right)^{3/4}{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a\left(dx^3+c\right)}\right)}{9a\left(a+bx^3\right)^{3/4}}+\frac{4x\left(c+dx^3\right)^{5/12}}{9a\left(a+bx^3\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(5/12)/(a + b*x^3)^(7/4),x]
```

```
[Out] (4*x*(c + d*x^3)^(5/12))/(9*a*(a + b*x^3)^(3/4)) + (5*x*((c*(a + b*x^3))/(a
*(c + d*x^3)))^(3/4)*(c + d*x^3)^(5/12)*Hypergeometric2F1[1/3, 3/4, 4/3, -(
((b*c - a*d)*x^3)/(a*(c + d*x^3)))])/(9*a*(a + b*x^3)^(3/4))
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(x*(a + b*x^n)^p*Hypergeometric2F1[1/n, -p, 1 + 1/n, -(((b*c - a*d)
*x^n)/(a*(c + d*x^n)))])/(c*((c*(a + b*x^n))/(a*(c + d*x^n)))^p*(c + d*x^n)
^(1/n + p)), x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^{5/12}}{\left(a+bx^3\right)^{7/4}}\,dx = \frac{4x\left(c+dx^3\right)^{5/12}}{9a\left(a+bx^3\right)^{3/4}} + \frac{(5c)\int \frac{1}{\left(a+bx^3\right)^{3/4}\left(c+dx^3\right)^{7/12}}\,dx}{9a}$$

$$= \frac{4x\left(c+dx^3\right)^{5/12}}{9a\left(a+bx^3\right)^{3/4}} + \frac{5x\left(\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}\right)^{3/4}\left(c+dx^3\right)^{5/12}\,{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a\left(c+dx^3\right)}\right)}{9a\left(a+bx^3\right)^{3/4}}$$

Mathematica [A] time = 0.0248773, size = 89, normalized size = 0.74

$$\frac{x\left(\frac{bx^3}{a}+1\right)^{3/4}\left(c+dx^3\right)^{5/12}\,{}_2F_1\left(\frac{1}{3},\frac{7}{4};\frac{4}{3};\frac{(ad-bc)x^3}{a\left(dx^3+c\right)}\right)}{a\left(a+bx^3\right)^{3/4}\left(\frac{dx^3}{c}+1\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^(5/12)/(a + b*x^3)^(7/4),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(3/4)*(c + d*x^3)^(5/12)*Hypergeometric2F1[1/3, 7/4, 4/3
, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/(a*(a + b*x^3)^(3/4)*(1 + (d*x^3)/
c)^(3/4))
```

Maple [F] time = 0.433, size = 0, normalized size = 0.

$$\int \left(dx^3+c\right)^{\frac{5}{12}}\left(bx^3+a\right)^{-\frac{7}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(5/12)/(b*x^3+a)^(7/4),x)
```

```
[Out] int((d*x^3+c)^(5/12)/(b*x^3+a)^(7/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^{\frac{5}{12}}}{\left(bx^3+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(5/12)/(b*x^3+a)^(7/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(5/12)/(b*x^3 + a)^(7/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^3+a\right)^{\frac{1}{4}}\left(dx^3+c\right)^{\frac{5}{12}}}{b^2x^6+2\,abx^3+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(5/12)/(b*x^3+a)^(7/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(1/4)*(d*x^3 + c)^(5/12)/(b^2*x^6 + 2*a*b*x^3 + a^2),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(5/12)/(b*x**3+a)**(7/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(dx^3+c\right)^{\frac{5}{12}}}{\left(bx^3+a\right)^{\frac{7}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(5/12)/(b*x^3+a)^(7/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(5/12)/(b*x^3 + a)^(7/4), x)
```

3.130 $\int \frac{\left(c+dx^3\right)^{11/12}}{\left(a+bx^3\right)^{9/4}} dx$

Optimal. Leaf size=121

$$\frac{11x\left(c+dx^3\right)^{11/12}\left(\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}\right)^{5/4}{}_2F_1\left(\frac{1}{3},\frac{5}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a\left(dx^3+c\right)}\right)}{15a\left(a+bx^3\right)^{5/4}}+\frac{4x\left(c+dx^3\right)^{11/12}}{15a\left(a+bx^3\right)^{5/4}}$$

```
[Out] (4*x*(c + d*x^3)^(11/12))/(15*a*(a + b*x^3)^(5/4)) + (11*x*((c*(a + b*x^3))
/(a*(c + d*x^3)))^(5/4)*(c + d*x^3)^(11/12)*Hypergeometric2F1[1/3, 5/4, 4/3
, -(((b*c - a*d)*x^3)/(a*(c + d*x^3)))])/(15*a*(a + b*x^3)^(5/4))
```

Rubi [A] time = 0.0349659, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {378, 380}

$$\frac{11x\left(c+dx^3\right)^{11/12}\left(\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}\right)^{5/4}{}_2F_1\left(\frac{1}{3},\frac{5}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a\left(dx^3+c\right)}\right)}{15a\left(a+bx^3\right)^{5/4}}+\frac{4x\left(c+dx^3\right)^{11/12}}{15a\left(a+bx^3\right)^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(11/12)/(a + b*x^3)^(9/4),x]
```

```
[Out] (4*x*(c + d*x^3)^(11/12))/(15*a*(a + b*x^3)^(5/4)) + (11*x*((c*(a + b*x^3))
/(a*(c + d*x^3)))^(5/4)*(c + d*x^3)^(11/12)*Hypergeometric2F1[1/3, 5/4, 4/3
, -(((b*c - a*d)*x^3)/(a*(c + d*x^3)))])/(15*a*(a + b*x^3)^(5/4))
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(x*(a + b*x^n)^p*Hypergeometric2F1[1/n, -p, 1 + 1/n, -(((b*c - a*d)
*x^n)/(a*(c + d*x^n)))])/(c*((c*(a + b*x^n))/(a*(c + d*x^n)))^p*(c + d*x^n)
^(1/n + p)), x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^{11/12}}{\left(a+bx^3\right)^{9/4}}\,dx = \frac{4x\left(c+dx^3\right)^{11/12}}{15a\left(a+bx^3\right)^{5/4}} + \frac{(11c)\int \frac{1}{\left(a+bx^3\right)^{5/4}\sqrt[12]{c+dx^3}}\,dx}{15a}$$

$$= \frac{4x\left(c+dx^3\right)^{11/12}}{15a\left(a+bx^3\right)^{5/4}} + \frac{11x\left(\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}\right)^{5/4}\left(c+dx^3\right)^{11/12}\,{}_2F_1\left(\frac{1}{3},\frac{5}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a\left(c+dx^3\right)}\right)}{15a\left(a+bx^3\right)^{5/4}}$$

Mathematica [A] time = 0.0262797, size = 89, normalized size = 0.74

$$\frac{x\sqrt[4]{\frac{bx^3}{a}+1}\left(c+dx^3\right)^{11/12}\,{}_2F_1\left(\frac{1}{3},\frac{9}{4};\frac{4}{3};\frac{(ad-bc)x^3}{a\left(dx^3+c\right)}\right)}{a^2\sqrt[4]{a+bx^3}\left(\frac{dx^3}{c}+1\right)^{5/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^(11/12)/(a + b*x^3)^(9/4),x]
```

```
[Out] (x*(1 + (b*x^3)/a)^(1/4)*(c + d*x^3)^(11/12)*Hypergeometric2F1[1/3, 9/4, 4/
3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/(a^2*(a + b*x^3)^(1/4)*(1 + (d*x^
3)/c)^(5/4))
```

Maple [F] time = 0.433, size = 0, normalized size = 0.

$$\int \left(dx^3+c\right)^{\frac{11}{12}}\left(bx^3+a\right)^{-\frac{9}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(11/12)/(b*x^3+a)^(9/4),x)
```

```
[Out] int((d*x^3+c)^(11/12)/(b*x^3+a)^(9/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^{\frac{11}{12}}}{\left(bx^3+a\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(11/12)/(b*x^3+a)^(9/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(11/12)/(b*x^3 + a)^(9/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^{3}+a\right)^{\frac{3}{4}}\left(dx^{3}+c\right)^{\frac{11}{12}}}{b^{3}x^{9}+3\,ab^{2}x^{6}+3\,a^{2}bx^{3}+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(11/12)/(b*x^3+a)^(9/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(3/4)*(d*x^3 + c)^(11/12)/(b^3*x^9 + 3*a*b^2*x^6 + 3*a
^2*b*x^3 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(11/12)/(b*x**3+a)**(9/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^{3}+c\right)^{\frac{11}{12}}}{\left(bx^{3}+a\right)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(11/12)/(b*x^3+a)^(9/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(11/12)/(b*x^3 + a)^(9/4), x)
```

3.131 $\int \frac{\left(c+dx^3\right)^{17/12}}{\left(a+bx^3\right)^{11/4}} dx$

Optimal. Leaf size=153

$$\frac{85cx\left(c+dx^3\right)^{5/12}\left(\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}\right)^{3/4}\,_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a\left(dx^3+c\right)}\right)}{189a^2\left(a+bx^3\right)^{3/4}}+\frac{68cx\left(c+dx^3\right)^{5/12}}{189a^2\left(a+bx^3\right)^{3/4}}+\frac{4x\left(c+dx^3\right)^{17/12}}{21a\left(a+bx^3\right)^{7/4}}$$

```
[Out] (68*c*x*(c + d*x^3)^(5/12))/(189*a^2*(a + b*x^3)^(3/4)) + (4*x*(c + d*x^3)^
(17/12))/(21*a*(a + b*x^3)^(7/4)) + (85*c*x*((c*(a + b*x^3))/(a*(c + d*x^3)
))^(3/4)*(c + d*x^3)^(5/12)*Hypergeometric2F1[1/3, 3/4, 4/3, -(((b*c - a*d)
*x^3)/(a*(c + d*x^3)))])/(189*a^2*(a + b*x^3)^(3/4))
```

Rubi [A] time = 0.0548961, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {378, 380}

$$\frac{85cx\left(c+dx^3\right)^{5/12}\left(\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}\right)^{3/4}\,_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a\left(dx^3+c\right)}\right)}{189a^2\left(a+bx^3\right)^{3/4}}+\frac{68cx\left(c+dx^3\right)^{5/12}}{189a^2\left(a+bx^3\right)^{3/4}}+\frac{4x\left(c+dx^3\right)^{17/12}}{21a\left(a+bx^3\right)^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(17/12)/(a + b*x^3)^(11/4),x]
```

```
[Out] (68*c*x*(c + d*x^3)^(5/12))/(189*a^2*(a + b*x^3)^(3/4)) + (4*x*(c + d*x^3)^
(17/12))/(21*a*(a + b*x^3)^(7/4)) + (85*c*x*((c*(a + b*x^3))/(a*(c + d*x^3)
))^(3/4)*(c + d*x^3)^(5/12)*Hypergeometric2F1[1/3, 3/4, 4/3, -(((b*c - a*d)
*x^3)/(a*(c + d*x^3)))])/(189*a^2*(a + b*x^3)^(3/4))
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(x*(a + b*x^n)^p*Hypergeometric2F1[1/n, -p, 1 + 1/n, -(((b*c - a*d)
*x^n)/(a*(c + d*x^n)))])/(c*((c*(a + b*x^n))/(a*(c + d*x^n)))^p*(c + d*x^n)
^(1/n + p)), x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^{17/12}}{\left(a+bx^3\right)^{11/4}}\,dx = \frac{4x\left(c+dx^3\right)^{17/12}}{21a\left(a+bx^3\right)^{7/4}} + \frac{(17c)\int \frac{\left(c+dx^3\right)^{5/12}}{\left(a+bx^3\right)^{7/4}}\,dx}{21a}$$

$$= \frac{68cx\left(c+dx^3\right)^{5/12}}{189a^2\left(a+bx^3\right)^{3/4}} + \frac{4x\left(c+dx^3\right)^{17/12}}{21a\left(a+bx^3\right)^{7/4}} + \frac{\left(85c^2\right)\int \frac{1}{\left(a+bx^3\right)^{3/4}\left(c+dx^3\right)^{7/12}}\,dx}{189a^2}$$

$$= \frac{68cx\left(c+dx^3\right)^{5/12}}{189a^2\left(a+bx^3\right)^{3/4}} + \frac{4x\left(c+dx^3\right)^{17/12}}{21a\left(a+bx^3\right)^{7/4}} + \frac{85cx\left(\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}\right)^{3/4}\left(c+dx^3\right)^{5/12}\,{}_2F_1\left(\frac{1}{3},\frac{3}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a\left(c+dx^3\right)}\right)}{189a^2\left(a+bx^3\right)^{3/4}}$$

Mathematica [A] time = 0.0296268, size = 90, normalized size = 0.59

$$\frac{cx\left(\frac{bx^3}{a}+1\right)^{3/4}\left(c+dx^3\right)^{5/12}\,{}_2F_1\left(\frac{1}{3},\frac{11}{4};\frac{4}{3};\frac{(ad-bc)x^3}{a\left(dx^3+c\right)}\right)}{a^2\left(a+bx^3\right)^{3/4}\left(\frac{dx^3}{c}+1\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^(17/12)/(a + b*x^3)^(11/4),x]
```

```
[Out] (c*x*(1 + (b*x^3)/a)^(3/4)*(c + d*x^3)^(5/12)*Hypergeometric2F1[1/3, 11/4,
4/3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/(a^2*(a + b*x^3)^(3/4)*(1 + (d*
x^3)/c)^(3/4))
```

Maple [F] time = 0.5, size = 0, normalized size = 0.

$$\int \left(dx^3+c\right)^{\frac{17}{12}}\left(bx^3+a\right)^{-\frac{11}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(17/12)/(b*x^3+a)^(11/4),x)
```

```
[Out] int((d*x^3+c)^(17/12)/(b*x^3+a)^(11/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^{\frac{17}{12}}}{\left(bx^3+a\right)^{\frac{11}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(17/12)/(b*x^3+a)^(11/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(17/12)/(b*x^3 + a)^(11/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^3+a\right)^{\frac{1}{4}}\left(dx^3+c\right)^{\frac{17}{12}}}{b^3x^9+3\,ab^2x^6+3\,a^2bx^3+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(17/12)/(b*x^3+a)^(11/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(1/4)*(d*x^3 + c)^(17/12)/(b^3*x^9 + 3*a*b^2*x^6 + 3*a
^2*b*x^3 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(17/12)/(b*x**3+a)**(11/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^{\frac{17}{12}}}{\left(bx^3+a\right)^{\frac{11}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(17/12)/(b*x^3+a)^(11/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(17/12)/(b*x^3 + a)^(11/4), x)
```

3.132 $\int \frac{(c+dx^3)^{23/12}}{(a+bx^3)^{13/4}}\,dx$

Optimal. Leaf size=153

$$\frac{253cx\left(c+dx^3\right)^{11/12}\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{5/4}{}_2F_1\left(\frac{1}{3},\frac{5}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{405a^2\left(a+bx^3\right)^{5/4}}+\frac{92cx\left(c+dx^3\right)^{11/12}}{405a^2\left(a+bx^3\right)^{5/4}}+\frac{4x\left(c+dx^3\right)^{23/12}}{27a\left(a+bx^3\right)^{9/4}}$$

```
[Out] (92*c*x*(c + d*x^3)^(11/12))/(405*a^2*(a + b*x^3)^(5/4)) + (4*x*(c + d*x^3)
^(23/12))/(27*a*(a + b*x^3)^(9/4)) + (253*c*x*((c*(a + b*x^3))/(a*(c + d*x^
3)))^(5/4)*(c + d*x^3)^(11/12)*Hypergeometric2F1[1/3, 5/4, 4/3, -(((b*c - a
*d)*x^3)/(a*(c + d*x^3)))])/(405*a^2*(a + b*x^3)^(5/4))
```

Rubi [A] time = 0.0565592, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {378, 380}

$$\frac{253cx\left(c+dx^3\right)^{11/12}\left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{5/4}{}_2F_1\left(\frac{1}{3},\frac{5}{4};\frac{4}{3};-\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{405a^2\left(a+bx^3\right)^{5/4}}+\frac{92cx\left(c+dx^3\right)^{11/12}}{405a^2\left(a+bx^3\right)^{5/4}}+\frac{4x\left(c+dx^3\right)^{23/12}}{27a\left(a+bx^3\right)^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^(23/12)/(a + b*x^3)^(13/4),x]
```


```
[Out] (92*c*x*(c + d*x^3)^(11/12))/(405*a^2*(a + b*x^3)^(5/4)) + (4*x*(c + d*x^3)
^(23/12))/(27*a*(a + b*x^3)^(9/4)) + (253*c*x*((c*(a + b*x^3))/(a*(c + d*x^
3)))^(5/4)*(c + d*x^3)^(11/12)*Hypergeometric2F1[1/3, 5/4, 4/3, -(((b*c - a
*d)*x^3)/(a*(c + d*x^3)))])/(405*a^2*(a + b*x^3)^(5/4))
```


Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(x*(a + b*x^n)^p*Hypergeometric2F1[1/n, -p, 1 + 1/n, -(((b*c - a*d)
*x^n)/(a*(c + d*x^n)))])/(c*((c*(a + b*x^n))/(a*(c + d*x^n)))^p*(c + d*x^n)
^(1/n + p)), x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int \frac{\left(c+dx^3\right)^{23/12}}{\left(a+bx^3\right)^{13/4}}\,dx = \frac{4x\left(c+dx^3\right)^{23/12}}{27a\left(a+bx^3\right)^{9/4}} + \frac{(23c)\int \frac{\left(c+dx^3\right)^{11/12}}{\left(a+bx^3\right)^{9/4}}\,dx}{27a}$$

$$= \frac{92cx\left(c+dx^3\right)^{11/12}}{405a^2\left(a+bx^3\right)^{5/4}} + \frac{4x\left(c+dx^3\right)^{23/12}}{27a\left(a+bx^3\right)^{9/4}} + \frac{\left(253c^2\right)\int \frac{1}{\left(a+bx^3\right)^{5/4}\sqrt[12]{c+dx^3}}\,dx}{405a^2}$$

$$= \frac{92cx\left(c+dx^3\right)^{11/12}}{405a^2\left(a+bx^3\right)^{5/4}} + \frac{4x\left(c+dx^3\right)^{23/12}}{27a\left(a+bx^3\right)^{9/4}} + \frac{253cx\left(\frac{c\left(a+bx^3\right)}{a\left(c+dx^3\right)}\right)^{5/4}\left(c+dx^3\right)^{11/12}{}_2F_1\left(\frac{1}{3},\frac{5}{4};\frac{4}{3};-\frac{(bc-ad}{a(c+dx}\right.}{405a^2\left(a+bx^3\right)^{5/4}}$$

Mathematica [A] time = 0.0255563, size = 90, normalized size = 0.59

$$\frac{cx\sqrt[4]{\frac{bx^3}{a}+1}\left(c+dx^3\right)^{11/12}{}_2F_1\left(\frac{1}{3},\frac{13}{4};\frac{4}{3};\frac{(ad-bc)x^3}{a\left(dx^3+c\right)}\right)}{a^3\sqrt[4]{a+bx^3}\left(\frac{dx^3}{c}+1\right)^{5/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^(23/12)/(a + b*x^3)^(13/4),x]
```

```
[Out] (c*x*(1 + (b*x^3)/a)^(1/4)*(c + d*x^3)^(11/12)*Hypergeometric2F1[1/3, 13/4,
 4/3, ((-(b*c) + a*d)*x^3)/(a*(c + d*x^3))])/(a^3*(a + b*x^3)^(1/4)*(1 + (d
*x^3)/c)^(5/4))
```

Maple [F] time = 0.439, size = 0, normalized size = 0.

$$\int \left(dx^3+c\right)^{\frac{23}{12}}\left(bx^3+a\right)^{-\frac{13}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^(23/12)/(b*x^3+a)^(13/4),x)
```

```
[Out] int((d*x^3+c)^(23/12)/(b*x^3+a)^(13/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^{\frac{23}{12}}}{\left(bx^3+a\right)^{\frac{13}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(23/12)/(b*x^3+a)^(13/4),x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^(23/12)/(b*x^3 + a)^(13/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^3+a\right)^{\frac{3}{4}}\left(dx^3+c\right)^{\frac{23}{12}}}{b^4x^{12}+4\,ab^3x^9+6\,a^2b^2x^6+4\,a^3bx^3+a^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(23/12)/(b*x^3+a)^(13/4),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^(3/4)*(d*x^3 + c)^(23/12)/(b^4*x^12 + 4*a*b^3*x^9 + 6*
a^2*b^2*x^6 + 4*a^3*b*x^3 + a^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**(23/12)/(b*x**3+a)**(13/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^{\frac{23}{12}}}{\left(bx^3+a\right)^{\frac{13}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^(23/12)/(b*x^3+a)^(13/4),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^(23/12)/(b*x^3 + a)^(13/4), x)
```

3.133 $\int \left(a+bx^3\right)^m \left(c+dx^3\right)^p \, dx$

Optimal. Leaf size=79

$$x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}\left(c+dx^3\right)^p\left(\frac{dx^3}{c}+1\right)^{-p}F_1\left(\frac{1}{3};-m,-p;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)$$

```
[Out] (x*(a + b*x^3)^m*(c + d*x^3)^p*AppellF1[1/3, -m, -p, 4/3, -((b*x^3)/a), -((
d*x^3)/c)])/((1 + (b*x^3)/a)^m*(1 + (d*x^3)/c)^p)
```

Rubi [A] time = 0.0417169, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {430, 429}

$$x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}\left(c+dx^3\right)^p\left(\frac{dx^3}{c}+1\right)^{-p}F_1\left(\frac{1}{3};-m,-p;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^m*(c + d*x^3)^p,x]
```

```
[Out] (x*(a + b*x^3)^m*(c + d*x^3)^p*AppellF1[1/3, -m, -p, 4/3, -((b*x^3)/a), -((
d*x^3)/c)])/((1 + (b*x^3)/a)^m*(1 + (d*x^3)/c)^p)
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \left(a+bx^3\right)^m\left(c+dx^3\right)^p\,dx &= \left(\left(a+bx^3\right)^m\left(1+\frac{bx^3}{a}\right)^{-m}\right)\int\left(1+\frac{bx^3}{a}\right)^m\left(c+dx^3\right)^p\,dx\\
&= \left(\left(a+bx^3\right)^m\left(1+\frac{bx^3}{a}\right)^{-m}\left(c+dx^3\right)^p\left(1+\frac{dx^3}{c}\right)^{-p}\right)\int\left(1+\frac{bx^3}{a}\right)^m\left(1+\frac{dx^3}{c}\right)^p\,dx\\
&= x\left(a+bx^3\right)^m\left(1+\frac{bx^3}{a}\right)^{-m}\left(c+dx^3\right)^p\left(1+\frac{dx^3}{c}\right)^{-p}F_1\left(\frac{1}{3};-m,-p;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)
\end{aligned}$$

Mathematica [B] time = 0.220218, size = 172, normalized size = 2.18

$$\frac{4acx\left(a+bx^3\right)^m\left(c+dx^3\right)^pF_1\left(\frac{1}{3};-m,-p;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{3x^3\left(bcmF_1\left(\frac{4}{3};1-m,-p;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+adpF_1\left(\frac{4}{3};-m,1-p;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)+4acF_1\left(\frac{1}{3};-m,-p;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^m*(c + d*x^3)^p,x]
```

```
[Out] (4*a*c*x*(a + b*x^3)^m*(c + d*x^3)^p*AppellF1[1/3, -m, -p, 4/3, -((b*x^3)/a
), -((d*x^3)/c)])/(4*a*c*AppellF1[1/3, -m, -p, 4/3, -((b*x^3)/a), -((d*x^3)
/c)] + 3*x^3*(b*c*m*AppellF1[4/3, 1 - m, -p, 7/3, -((b*x^3)/a), -((d*x^3)/c
)] + a*d*p*AppellF1[4/3, -m, 1 - p, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))
```

Maple [F] time = 0.503, size = 0, normalized size = 0.

$$\int \left(b x^3 + a\right)^m \left(d x^3 + c\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^m*(d*x^3+c)^p,x)
```

```
[Out] int((b*x^3+a)^m*(d*x^3+c)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b x^3 + a\right)^m \left(d x^3 + c\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m*(d*x^3+c)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^m*(d*x^3 + c)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b x^3 + a\right)^m \left(d x^3 + c\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m*(d*x^3+c)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^m*(d*x^3 + c)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**m*(d*x**3+c)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3+a\right)^m \left(dx^3+c\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m*(d*x^3+c)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^m*(d*x^3 + c)^p, x)
```

3.134 $\int \left(a+bx^3\right)^2 \left(c+dx^3\right)^q \, dx$

Optimal. Leaf size=167

$$\frac{x\left(c+dx^3\right)^{q+1}\left(a^2d^2\left(9q^2+33q+28\right)-2abcd(3q+7)+4b^2c^2\right){}_2F_1\left(1,q+\frac{4}{3};\frac{4}{3};-\frac{dx^3}{c}\right)}{cd^2(3q+4)(3q+7)}-\frac{bx\left(c+dx^3\right)^{q+1}(4bc-ad(3q+1}{d^2(3q+4)(3q+7)}$$

```
[Out] -((b*(4*b*c - a*d*(10 + 3*q))*x*(c + d*x^3)^(1 + q))/(d^2*(4 + 3*q)*(7 + 3*
q))) + (b*x*(a + b*x^3)*(c + d*x^3)^(1 + q))/(d*(7 + 3*q)) + ((4*b^2*c^2 -
2*a*b*c*d*(7 + 3*q) + a^2*d^2*(28 + 33*q + 9*q^2))*x*(c + d*x^3)^(1 + q)*Hy
pergeometric2F1[1, 4/3 + q, 4/3, -((d*x^3)/c)])/(c*d^2*(4 + 3*q)*(7 + 3*q))
```

Rubi [A] time = 0.125828, antiderivative size = 176, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {416, 388, 246, 245}

$$\frac{x\left(c+dx^3\right)^q\left(\frac{dx^3}{c}+1\right)^{-q}\left(a^2d^2\left(9q^2+33q+28\right)-2abcd(3q+7)+4b^2c^2\right){}_2F_1\left(\frac{1}{3},-q;\frac{4}{3};-\frac{dx^3}{c}\right)}{d^2(3q+4)(3q+7)}-\frac{bx\left(c+dx^3\right)^{q+1}(4bc-ad}{d^2(3q+4)(3q+}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^2*(c + d*x^3)^q,x]
```

```
[Out] -((b*(4*b*c - a*d*(10 + 3*q))*x*(c + d*x^3)^(1 + q))/(d^2*(4 + 3*q)*(7 + 3*
q))) + (b*x*(a + b*x^3)*(c + d*x^3)^(1 + q))/(d*(7 + 3*q)) + ((4*b^2*c^2 -
2*a*b*c*d*(7 + 3*q) + a^2*d^2*(28 + 33*q + 9*q^2))*x*(c + d*x^3)^q*Hypergeo
metric2F1[1/3, -q, 4/3, -((d*x^3)/c)])/(d^2*(4 + 3*q)*(7 + 3*q)*(1 + (d*x^3
)/c)^q)
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \left(a+bx^3\right)^2\left(c+dx^3\right)^q\,dx=\frac{bx\left(a+bx^3\right)\left(c+dx^3\right)^{1+q}}{d(7+3q)}+\frac{\int \left(c+dx^3\right)^q\left(-a(bc-ad(7+3q))-b(4bc-ad(10+3q))x^3\right.}{d(7+3q)}$$

$$=-\frac{b(4bc-ad(10+3q))x\left(c+dx^3\right)^{1+q}}{d^2(4+3q)(7+3q)}+\frac{bx\left(a+bx^3\right)\left(c+dx^3\right)^{1+q}}{d(7+3q)}+\frac{\left(4b^2c^2-2abcd(7+\right.}{}$$

$$=-\frac{b(4bc-ad(10+3q))x\left(c+dx^3\right)^{1+q}}{d^2(4+3q)(7+3q)}+\frac{bx\left(a+bx^3\right)\left(c+dx^3\right)^{1+q}}{d(7+3q)}+\frac{\left(\left(4b^2c^2-2abcd(7+\right.\right.}{}$$

$$=-\frac{b(4bc-ad(10+3q))x\left(c+dx^3\right)^{1+q}}{d^2(4+3q)(7+3q)}+\frac{bx\left(a+bx^3\right)\left(c+dx^3\right)^{1+q}}{d(7+3q)}+\frac{\left(4b^2c^2-2abcd(7+\right.}{}$$

Mathematica [A] time = 0.0506743, size = 106, normalized size = 0.63

$$\frac{1}{14}x\left(c+dx^3\right)^q\left(\frac{dx^3}{c}+1\right)^{-q}\left(14a^2\,{}_2F_1\left(\frac{1}{3},-q;\frac{4}{3};-\frac{dx^3}{c}\right)+bx^3\left(7a\,{}_2F_1\left(\frac{4}{3},-q;\frac{7}{3};-\frac{dx^3}{c}\right)+2bx^3\,{}_2F_1\left(\frac{7}{3},-q;\frac{10}{3};-\frac{dx^3}{c}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^2*(c + d*x^3)^q,x]
```

```
[Out] (x*(c + d*x^3)^q*(14*a^2*Hypergeometric2F1[1/3, -q, 4/3, -((d*x^3)/c)] + b*
x^3*(7*a*Hypergeometric2F1[4/3, -q, 7/3, -((d*x^3)/c)] + 2*b*x^3*Hypergeome
tric2F1[7/3, -q, 10/3, -((d*x^3)/c)])))/(14*(1 + (d*x^3)/c)^q)
```

Maple [F] time = 0.431, size = 0, normalized size = 0.

$$\int \left(bx^3+a\right)^2\left(dx^3+c\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^2*(d*x^3+c)^q,x)
```

```
[Out] int((b*x^3+a)^2*(d*x^3+c)^q,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3+a\right)^2\left(dx^3+c\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(d*x^3+c)^q,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^2*(d*x^3 + c)^q, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2x^6 + 2\,abx^3 + a^2\right)\left(dx^3 + c\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(d*x^3+c)^q,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^6 + 2*a*b*x^3 + a^2)*(d*x^3 + c)^q, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**2*(d*x**3+c)**q,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3 + a\right)^2 \left(dx^3 + c\right)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^2*(d*x^3+c)^q,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^2*(d*x^3 + c)^q, x)
```

3.135 $\int \left(a+bx^3\right)\left(c+dx^3\right)^q\,dx$

Optimal. Leaf size=84

$$\frac{bx\left(c+dx^3\right)^{q+1}}{d(3q+4)}-\frac{x\left(c+dx^3\right)^{q+1}(bc-ad(3q+4))\,{}_2F_1\left(1,q+\frac{4}{3};\frac{4}{3};-\frac{dx^3}{c}\right)}{cd(3q+4)}$$

```
[Out] (b*x*(c + d*x^3)^(1 + q))/(d*(4 + 3*q)) - ((b*c - a*d*(4 + 3*q))*x*(c + d*x
^3)^(1 + q)*Hypergeometric2F1[1, 4/3 + q, 4/3, -((d*x^3)/c)])/(c*d*(4 + 3*q
))
```

Rubi [A] time = 0.0388256, antiderivative size = 85, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {388, 246, 245}

$$x\left(c+dx^3\right)^q\left(\frac{dx^3}{c}+1\right)^{-q}\left(a-\frac{bc}{3dq+4d}\right){}_2F_1\left(\frac{1}{3},-q;\frac{4}{3};-\frac{dx^3}{c}\right)+\frac{bx\left(c+dx^3\right)^{q+1}}{d(3q+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)*(c + d*x^3)^q,x]
```

```
[Out] (b*x*(c + d*x^3)^(1 + q))/(d*(4 + 3*q)) + ((a - (b*c)/(4*d + 3*d*q))*x*(c +
 d*x^3)^q*Hypergeometric2F1[1/3, -q, 4/3, -((d*x^3)/c)])/(1 + (d*x^3)/c)^q
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int (a+bx^3)(c+dx^3)^q\,dx &= \frac{bx(c+dx^3)^{1+q}}{d(4+3q)} - \left(-a+\frac{bc}{4d+3dq}\right)\int (c+dx^3)^q\,dx\\ &= \frac{bx(c+dx^3)^{1+q}}{d(4+3q)} - \left(\left(-a+\frac{bc}{4d+3dq}\right)(c+dx^3)^q\left(1+\frac{dx^3}{c}\right)^{-q}\right)\int \left(1+\frac{dx^3}{c}\right)^q dx\\ &= \frac{bx(c+dx^3)^{1+q}}{d(4+3q)} + \left(a-\frac{bc}{4d+3dq}\right)x(c+dx^3)^q\left(1+\frac{dx^3}{c}\right)^{-q}{}_2F_1\left(\frac{1}{3},-q;\frac{4}{3};-\frac{dx^3}{c}\right)\end{aligned}$$

Mathematica [A] time = 0.0292039, size = 90, normalized size = 1.07

$$\frac{x(c+dx^3)^q\left(\frac{dx^3}{c}+1\right)^{-q}\left((ad(3q+4)-bc)\,{}_2F_1\left(\frac{1}{3},-q;\frac{4}{3};-\frac{dx^3}{c}\right)+b(c+dx^3)\left(\frac{dx^3}{c}+1\right)^q\right)}{d(3q+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)*(c + d*x^3)^q,x]
```

```
[Out] (x*(c + d*x^3)^q*(b*(c + d*x^3)*(1 + (d*x^3)/c)^q + (-(b*c) + a*d*(4 + 3*q)
)*Hypergeometric2F1[1/3, -q, 4/3, -((d*x^3)/c)]))/(d*(4 + 3*q)*(1 + (d*x^3)
/c)^q)
```

Maple [F] time = 0.239, size = 0, normalized size = 0.

$$\int (bx^3+a)(dx^3+c)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)*(d*x^3+c)^q,x)
```

```
[Out] int((b*x^3+a)*(d*x^3+c)^q,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^3+a)(dx^3+c)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c)^q,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)*(d*x^3 + c)^q, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^3+a)(dx^3+c)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c)^q,x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)*(d*x^3 + c)^q, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(d*x**3+c)**q,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^3 + a)(dx^3 + c)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)*(d*x^3+c)^q,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)*(d*x^3 + c)^q, x)
```

3.136 $\int \frac{(c+dx^3)^q}{a+bx^3} dx$

Optimal. Leaf size=57

$$\frac{x\left(c+dx^3\right)^q\left(\frac{dx^3}{c}+1\right)^{-q}F_1\left(\frac{1}{3};1,-q;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a}$$

```
[Out] (x*(c + d*x^3)^q*AppellF1[1/3, 1, -q, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(a*
(1 + (d*x^3)/c)^q)
```

Rubi [A] time = 0.0271866, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {430, 429}

$$\frac{x\left(c+dx^3\right)^q\left(\frac{dx^3}{c}+1\right)^{-q}F_1\left(\frac{1}{3};1,-q;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^q/(a + b*x^3),x]
```

```
[Out] (x*(c + d*x^3)^q*AppellF1[1/3, 1, -q, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(a*
(1 + (d*x^3)/c)^q)
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(c+dx^3\right)^q}{a+bx^3}\,dx &= \left(\left(c+dx^3\right)^q\left(1+\frac{dx^3}{c}\right)^{-q}\right)\int \frac{\left(1+\frac{dx^3}{c}\right)^q}{a+bx^3}\,dx\\ &= \frac{x\left(c+dx^3\right)^q\left(1+\frac{dx^3}{c}\right)^{-q}F_1\left(\frac{1}{3};1,-q;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a}\end{aligned}$$

Mathematica [B] time = 0.17228, size = 162, normalized size = 2.84

$$\frac{4acx\left(c+dx^3\right)^qF_1\left(\frac{1}{3};-q,1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)}{\left(a+bx^3\right)\left(3x^3\left(adqF_1\left(\frac{4}{3};1-q,1;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)-bcF_1\left(\frac{4}{3};-q,2;\frac{7}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)+4acF_1\left(\frac{1}{3};-q,1;\frac{4}{3};-\frac{dx^3}{c},-\frac{bx^3}{a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^q/(a + b*x^3),x]
```

```
[Out] (4*a*c*x*(c + d*x^3)^q*AppellF1[1/3, -q, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)
])/((a + b*x^3)*(4*a*c*AppellF1[1/3, -q, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)
] + 3*x^3*(a*d*q*AppellF1[4/3, 1 - q, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)] -
 b*c*AppellF1[4/3, -q, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)])))
```

Maple [F] time = 0.44, size = 0, normalized size = 0.

$$\int \frac{\left(d x^{3}+c\right)^{q}}{b x^{3}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^q/(b*x^3+a),x)
```

```
[Out] int((d*x^3+c)^q/(b*x^3+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d x^{3}+c\right)^{q}}{b x^{3}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^q/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^q/(b*x^3 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(d x^{3}+c\right)^{q}}{b x^{3}+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^q/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] integral((d*x^3 + c)^q/(b*x^3 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**q/(b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^q}{bx^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^q/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^q/(b*x^3 + a), x)
```

3.137 $\int \frac{(c+dx^3)^q}{(a+bx^3)^2}\,dx$

Optimal. Leaf size=57

$$\frac{x\left(c+dx^3\right)^q\left(\frac{dx^3}{c}+1\right)^{-q}F_1\left(\frac{1}{3};2,-q;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2}$$

```
[Out] (x*(c + d*x^3)^q*AppellF1[1/3, 2, -q, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(a^
2*(1 + (d*x^3)/c)^q)
```

Rubi [A] time = 0.0273022, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {430, 429}

$$\frac{x\left(c+dx^3\right)^q\left(\frac{dx^3}{c}+1\right)^{-q}F_1\left(\frac{1}{3};2,-q;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^3)^q/(a + b*x^3)^2,x]
```

```
[Out] (x*(c + d*x^3)^q*AppellF1[1/3, 2, -q, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(a^
2*(1 + (d*x^3)/c)^q)
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(c+dx^3\right)^q}{\left(a+bx^3\right)^2}\,dx &= \left(\left(c+dx^3\right)^q\left(1+\frac{dx^3}{c}\right)^{-q}\right)\int \frac{\left(1+\frac{dx^3}{c}\right)^q}{\left(a+bx^3\right)^2}\,dx\\ &= \frac{x\left(c+dx^3\right)^q\left(1+\frac{dx^3}{c}\right)^{-q}F_1\left(\frac{1}{3};2,-q;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{a^2}\end{aligned}$$

Mathematica [B] time = 0.193763, size = 162, normalized size = 2.84

$$\frac{4acx\left(c+dx^3\right)^qF_1\left(\frac{1}{3};2,-q;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\left(a+bx^3\right)^2\left(3x^3\left(adqF_1\left(\frac{4}{3};2,1-q;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-2bcF_1\left(\frac{4}{3};3,-q;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)+4acF_1\left(\frac{1}{3};2,-q;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^3)^q/(a + b*x^3)^2,x]
```

```
[Out] (4*a*c*x*(c + d*x^3)^q*AppellF1[1/3, 2, -q, 4/3, -((b*x^3)/a), -((d*x^3)/c)
])/((a + b*x^3)^2*(4*a*c*AppellF1[1/3, 2, -q, 4/3, -((b*x^3)/a), -((d*x^3)/
c)] + 3*x^3*(a*d*q*AppellF1[4/3, 2, 1 - q, 7/3, -((b*x^3)/a), -((d*x^3)/c)]
 - 2*b*c*AppellF1[4/3, 3, -q, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

Maple [F] time = 0.443, size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^q}{\left(bx^3+a\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^q/(b*x^3+a)^2,x)
```

```
[Out] int((d*x^3+c)^q/(b*x^3+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^3+c\right)^q}{\left(bx^3+a\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^q/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^q/(b*x^3 + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(dx^3+c\right)^q}{b^2x^6+2\,abx^3+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^q/(b*x^3+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x^3 + c)^q/(b^2*x^6 + 2*a*b*x^3 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**q/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^3+c)^q}{(bx^3+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^q/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^q/(b*x^3 + a)^2, x)
```

3.138 $\int \left(a+bx^3\right)^m \left(c+dx^3\right)^3 \, dx$

Optimal. Leaf size=298

$$-\frac{x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}\left(-12a^2bcd^2(3m+10)+28a^3d^3+3ab^2c^2d\left(9m^2+51m+70\right)-b^3c^3\left(27m^3+189m^2+414m+\right.\right.}{b^3(3m+4)(3m+7)(3m+10)}$$

```
[Out] (d*(28*a^2*d^2 - a*b*c*d*(92 + 15*m) + b^2*c^2*(118 + 60*m + 9*m^2))*x*(a +
 b*x^3)^(1 + m))/(b^3*(4 + 3*m)*(7 + 3*m)*(10 + 3*m)) - (d*(7*a*d - b*c*(16
 + 3*m))*x*(a + b*x^3)^(1 + m)*(c + d*x^3))/(b^2*(7 + 3*m)*(10 + 3*m)) + (d
*x*(a + b*x^3)^(1 + m)*(c + d*x^3)^2)/(b*(10 + 3*m)) - ((28*a^3*d^3 - 12*a^
2*b*c*d^2*(10 + 3*m) + 3*a*b^2*c^2*d*(70 + 51*m + 9*m^2) - b^3*c^3*(280 + 4
14*m + 189*m^2 + 27*m^3))*x*(a + b*x^3)^m*Hypergeometric2F1[1/3, -m, 4/3, -
((b*x^3)/a)])/(b^3*(4 + 3*m)*(7 + 3*m)*(10 + 3*m)*(1 + (b*x^3)/a)^m)
```

Rubi [A] time = 0.303879, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {416, 528, 388, 246, 245}

$$-\frac{x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}\left(-12a^2bcd^2(3m+10)+28a^3d^3+3ab^2c^2d\left(9m^2+51m+70\right)-b^3c^3\left(27m^3+189m^2+414m+\right.\right.}{b^3(3m+4)(3m+7)(3m+10)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^m*(c + d*x^3)^3,x]
```


```
[Out] (d*(28*a^2*d^2 - a*b*c*d*(92 + 15*m) + b^2*c^2*(118 + 60*m + 9*m^2))*x*(a +
 b*x^3)^(1 + m))/(b^3*(4 + 3*m)*(7 + 3*m)*(10 + 3*m)) - (d*(7*a*d - b*c*(16
 + 3*m))*x*(a + b*x^3)^(1 + m)*(c + d*x^3))/(b^2*(7 + 3*m)*(10 + 3*m)) + (d
*x*(a + b*x^3)^(1 + m)*(c + d*x^3)^2)/(b*(10 + 3*m)) - ((28*a^3*d^3 - 12*a^
2*b*c*d^2*(10 + 3*m) + 3*a*b^2*c^2*d*(70 + 51*m + 9*m^2) - b^3*c^3*(280 + 4
14*m + 189*m^2 + 27*m^3))*x*(a + b*x^3)^m*Hypergeometric2F1[1/3, -m, 4/3, -
((b*x^3)/a)])/(b^3*(4 + 3*m)*(7 + 3*m)*(10 + 3*m)*(1 + (b*x^3)/a)^m)
```


Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \left(a+bx^3\right)^m \left(c+dx^3\right)^3 \, dx &= \frac{dx\left(a+bx^3\right)^{1+m}\left(c+dx^3\right)^2}{b(10+3m)} + \frac{\int \left(a+bx^3\right)^m \left(c+dx^3\right)\left(-c(ad-bc(10+3m))-d(7ad-\right.}{b(10+3m)} \\
&= -\frac{d(7ad-bc(16+3m))x\left(a+bx^3\right)^{1+m}\left(c+dx^3\right)}{b^2(7+3m)(10+3m)} + \frac{dx\left(a+bx^3\right)^{1+m}\left(c+dx^3\right)^2}{b(10+3m)} + \frac{\int \left(a+\right.}{} \\
&= \frac{d\left(28a^2d^2-abcd(92+15m)+b^2c^2\left(118+60m+9m^2\right)\right)x\left(a+bx^3\right)^{1+m}}{b^3(4+3m)(7+3m)(10+3m)} - \frac{d(7ad-bc(1}{} \\
&= \frac{d\left(28a^2d^2-abcd(92+15m)+b^2c^2\left(118+60m+9m^2\right)\right)x\left(a+bx^3\right)^{1+m}}{b^3(4+3m)(7+3m)(10+3m)} - \frac{d(7ad-bc(1}{} \\
&= \frac{d\left(28a^2d^2-abcd(92+15m)+b^2c^2\left(118+60m+9m^2\right)\right)x\left(a+bx^3\right)^{1+m}}{b^3(4+3m)(7+3m)(10+3m)} - \frac{d(7ad-bc(1}{}
\end{aligned}$$

Mathematica [A] time = 5.06369, size = 137, normalized size = 0.46

$$\frac{1}{140}x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}\left(dx^3\left(105c^2\,{}_2F_1\left(\frac{4}{3},-m;\frac{7}{3};-\frac{bx^3}{a}\right)+2dx^3\left(30c\,{}_2F_1\left(\frac{7}{3},-m;\frac{10}{3};-\frac{bx^3}{a}\right)+7dx^3\,{}_2F_1\left(\frac{10}{3},-\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^m*(c + d*x^3)^3,x]
```

```
[Out] (x*(a + b*x^3)^m*(140*c^3*Hypergeometric2F1[1/3, -m, 4/3, -((b*x^3)/a)] + d
*x^3*(105*c^2*Hypergeometric2F1[4/3, -m, 7/3, -((b*x^3)/a)] + 2*d*x^3*(30*c
*Hypergeometric2F1[7/3, -m, 10/3, -((b*x^3)/a)] + 7*d*x^3*Hypergeometric2F1
[10/3, -m, 13/3, -((b*x^3)/a)]))))/(140*(1 + (b*x^3)/a)^m)
```

Maple [F] time = 0.41, size = 0, normalized size = 0.

$$\int \left(bx^3+a\right)^m \left(dx^3+c\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^m*(d*x^3+c)^3,x)
```

```
[Out] int((b*x^3+a)^m*(d*x^3+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(dx^3+c\right)^3 \left(bx^3+a\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m*(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^3*(b*x^3 + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d^3x^9+3\,cd^2x^6+3\,c^2dx^3+c^3\right)\left(bx^3+a\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m*(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] integral((d^3*x^9 + 3*c*d^2*x^6 + 3*c^2*d*x^3 + c^3)*(b*x^3 + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**m*(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(dx^3+c\right)^3 \left(bx^3+a\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m*(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^3*(b*x^3 + a)^m, x)
```

3.139 $\int (a+bx^3)^m (c+dx^3)^2 \, dx$

Optimal. Leaf size=176

$$\frac{x(a+bx^3)^m \left(\frac{bx^3}{a}+1\right)^{-m} \left(4a^2d^2-2abcd(3m+7)+b^2c^2\left(9m^2+33m+28\right)\right) {}_2F_1\left(\frac{1}{3},-m;\frac{4}{3};-\frac{bx^3}{a}\right)}{b^2(3m+4)(3m+7)} - \frac{dx(a+bx^3)^{m+1} (}{b^2(3m+}$$

```
[Out] -((d*(4*a*d - b*c*(10 + 3*m))*x*(a + b*x^3)^(1 + m))/(b^2*(4 + 3*m)*(7 + 3*
m))) + (d*x*(a + b*x^3)^(1 + m)*(c + d*x^3))/(b*(7 + 3*m)) + ((4*a^2*d^2 -
2*a*b*c*d*(7 + 3*m) + b^2*c^2*(28 + 33*m + 9*m^2))*x*(a + b*x^3)^m*Hypergeo
metric2F1[1/3, -m, 4/3, -((b*x^3)/a)])/(b^2*(4 + 3*m)*(7 + 3*m)*(1 + (b*x^3
)/a)^m)
```

Rubi [A] time = 0.125731, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {416, 388, 246, 245}

$$\frac{x(a+bx^3)^m \left(\frac{bx^3}{a}+1\right)^{-m} \left(4a^2d^2-2abcd(3m+7)+b^2c^2\left(9m^2+33m+28\right)\right) {}_2F_1\left(\frac{1}{3},-m;\frac{4}{3};-\frac{bx^3}{a}\right)}{b^2(3m+4)(3m+7)} - \frac{dx(a+bx^3)^{m+1} (}{b^2(3m+}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^m*(c + d*x^3)^2,x]
```

```
[Out] -((d*(4*a*d - b*c*(10 + 3*m))*x*(a + b*x^3)^(1 + m))/(b^2*(4 + 3*m)*(7 + 3*
m))) + (d*x*(a + b*x^3)^(1 + m)*(c + d*x^3))/(b*(7 + 3*m)) + ((4*a^2*d^2 -
2*a*b*c*d*(7 + 3*m) + b^2*c^2*(28 + 33*m + 9*m^2))*x*(a + b*x^3)^m*Hypergeo
metric2F1[1/3, -m, 4/3, -((b*x^3)/a)])/(b^2*(4 + 3*m)*(7 + 3*m)*(1 + (b*x^3
)/a)^m)
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right)^2 dx = \frac{dx\left(a+bx^3\right)^{1+m}\left(c+dx^3\right)}{b(7+3m)} + \frac{\int \left(a+bx^3\right)^m \left(-c(ad-bc(7+3m))-d(4ad-bc(10+3m))x^3\right.}{b(7+3m)}$$

$$= -\frac{d(4ad-bc(10+3m))x\left(a+bx^3\right)^{1+m}}{b^2(4+3m)(7+3m)} + \frac{dx\left(a+bx^3\right)^{1+m}\left(c+dx^3\right)}{b(7+3m)} + \frac{\left(4a^2d^2-2abcd(7+\right.}{}$$

$$= -\frac{d(4ad-bc(10+3m))x\left(a+bx^3\right)^{1+m}}{b^2(4+3m)(7+3m)} + \frac{dx\left(a+bx^3\right)^{1+m}\left(c+dx^3\right)}{b(7+3m)} + \frac{\left(\left(4a^2d^2-2abcd(7+\right.\right.}{}$$

$$= -\frac{d(4ad-bc(10+3m))x\left(a+bx^3\right)^{1+m}}{b^2(4+3m)(7+3m)} + \frac{dx\left(a+bx^3\right)^{1+m}\left(c+dx^3\right)}{b(7+3m)} + \frac{\left(4a^2d^2-2abcd(7+\right.}{}$$

Mathematica [A] time = 5.04246, size = 106, normalized size = 0.6

$$\frac{1}{14}x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}\left(14c^2\,{}_2F_1\left(\frac{1}{3},-m;\frac{4}{3};-\frac{bx^3}{a}\right)+dx^3\left(7c\,{}_2F_1\left(\frac{4}{3},-m;\frac{7}{3};-\frac{bx^3}{a}\right)+2dx^3\,{}_2F_1\left(\frac{7}{3},-m;\frac{10}{3};-\frac{bx^3}{a}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^m*(c + d*x^3)^2,x]
```

```
[Out] (x*(a + b*x^3)^m*(14*c^2*Hypergeometric2F1[1/3, -m, 4/3, -((b*x^3)/a)] + d*
x^3*(7*c*Hypergeometric2F1[4/3, -m, 7/3, -((b*x^3)/a)] + 2*d*x^3*Hypergeome
tric2F1[7/3, -m, 10/3, -((b*x^3)/a)])))/(14*(1 + (b*x^3)/a)^m)
```

Maple [F] time = 0.398, size = 0, normalized size = 0.

$$\int \left(bx^3+a\right)^m \left(dx^3+c\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^m*(d*x^3+c)^2,x)
```

```
[Out] int((b*x^3+a)^m*(d*x^3+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(dx^3+c\right)^2 \left(bx^3+a\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m*(d*x^3+c)^2,x, algorithm="maxima")

[Out] integrate((d*x^3 + c)^2*(b*x^3 + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d^2x^6 + 2\,cdx^3 + c^2\right)\left(bx^3 + a\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m*(d*x^3+c)^2,x, algorithm="fricas")

[Out] integral((d^2*x^6 + 2*c*d*x^3 + c^2)*(b*x^3 + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**m*(d*x**3+c)**2,x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(dx^3 + c\right)^2 \left(bx^3 + a\right)^m \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m*(d*x^3+c)^2,x, algorithm="giac")

[Out] integrate((d*x^3 + c)^2*(b*x^3 + a)^m, x)
```

3.140 $\int \left(a+bx^3\right)^m \left(c+dx^3\right) dx$

Optimal. Leaf size=93

$$\frac{dx\left(a+bx^3\right)^{m+1}}{b(3m+4)}-\frac{x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}(ad-bc(3m+4))\,{}_2F_1\left(\frac{1}{3},-m;\frac{4}{3};-\frac{bx^3}{a}\right)}{b(3m+4)}$$

```
[Out] (d*x*(a + b*x^3)^(1 + m))/(b*(4 + 3*m)) - ((a*d - b*c*(4 + 3*m))*x*(a + b*x
^3)^m*Hypergeometric2F1[1/3, -m, 4/3, -((b*x^3)/a)])/(b*(4 + 3*m)*(1 + (b*x
^3)/a)^m)
```

Rubi [A] time = 0.0424879, antiderivative size = 85, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {388, 246, 245}

$$x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}\left(c-\frac{ad}{3bm+4b}\right){}_2F_1\left(\frac{1}{3},-m;\frac{4}{3};-\frac{bx^3}{a}\right)+\frac{dx\left(a+bx^3\right)^{m+1}}{b(3m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^m*(c + d*x^3),x]
```

```
[Out] (d*x*(a + b*x^3)^(1 + m))/(b*(4 + 3*m)) + ((c - (a*d)/(4*b + 3*b*m))*x*(a +
 b*x^3)^m*Hypergeometric2F1[1/3, -m, 4/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^m
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \left(a+bx^3\right)^m \left(c+dx^3\right) dx = \frac{dx\left(a+bx^3\right)^{1+m}}{b(4+3m)} - \left(-c+\frac{ad}{4b+3bm}\right)\int \left(a+bx^3\right)^m dx$$
$$= \frac{dx\left(a+bx^3\right)^{1+m}}{b(4+3m)} - \left(\left(-c+\frac{ad}{4b+3bm}\right)\left(a+bx^3\right)^m \left(1+\frac{bx^3}{a}\right)^{-m}\right)\int \left(1+\frac{bx^3}{a}\right)^m dx$$
$$= \frac{dx\left(a+bx^3\right)^{1+m}}{b(4+3m)} + \left(c-\frac{ad}{4b+3bm}\right)x\left(a+bx^3\right)^m \left(1+\frac{bx^3}{a}\right)^{-m} {}_2F_1\left(\frac{1}{3},-m;\frac{4}{3};-\frac{bx^3}{a}\right)$$

Mathematica [A] time = 0.0308868, size = 90, normalized size = 0.97

$$\frac{x\left(a+bx^3\right)^m \left(\frac{bx^3}{a}+1\right)^{-m}\left((bc(3m+4)-ad)\,{}_2F_1\left(\frac{1}{3},-m;\frac{4}{3};-\frac{bx^3}{a}\right)+d\left(a+bx^3\right)\left(\frac{bx^3}{a}+1\right)^m\right)}{b(3m+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^m*(c + d*x^3),x]
```

```
[Out] (x*(a + b*x^3)^m*(d*(a + b*x^3)*(1 + (b*x^3)/a)^m + (-(a*d) + b*c*(4 + 3*m)
)*Hypergeometric2F1[1/3, -m, 4/3, -((b*x^3)/a)]))/(b*(4 + 3*m)*(1 + (b*x^3)
/a)^m)
```

Maple [F] time = 0.225, size = 0, normalized size = 0.

$$\int \left(bx^3+a\right)^m \left(dx^3+c\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^m*(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^m*(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(dx^3+c\right)\left(bx^3+a\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m*(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)*(b*x^3 + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(dx^3+c\right)\left(bx^3+a\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m*(d*x^3+c),x, algorithm="fricas")
```

```
[Out] integral((d*x^3 + c)*(b*x^3 + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**m*(d*x**3+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(dx^3 + c\right)\left(bx^3 + a\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m*(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)*(b*x^3 + a)^m, x)
```

3.141 $\int \left(a+bx^3\right)^m \, dx$

Optimal. Leaf size=44

$$x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}{}_2F_1\left(\frac{1}{3},-m;\frac{4}{3};-\frac{bx^3}{a}\right)$$

```
[Out] (x*(a + b*x^3)^m*Hypergeometric2F1[1/3, -m, 4/3, -((b*x^3)/a)])/(1 + (b*x^3
)/a)^m
```

Rubi [A] time = 0.0101219, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {246, 245}

$$x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}{}_2F_1\left(\frac{1}{3},-m;\frac{4}{3};-\frac{bx^3}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^m,x]
```

```
[Out] (x*(a + b*x^3)^m*Hypergeometric2F1[1/3, -m, 4/3, -((b*x^3)/a)])/(1 + (b*x^3
)/a)^m
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^3\right)^m \, dx &= \left(\left(a+bx^3\right)^m\left(1+\frac{bx^3}{a}\right)^{-m}\right)\int \left(1+\frac{bx^3}{a}\right)^m \, dx\\ &= x\left(a+bx^3\right)^m\left(1+\frac{bx^3}{a}\right)^{-m}{}_2F_1\left(\frac{1}{3},-m;\frac{4}{3};-\frac{bx^3}{a}\right)\end{aligned}$$

Mathematica [C] time = 0.155368, size = 196, normalized size = 4.45

$$\frac{2^{-m}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)\left(\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}}\right)^{-m}\left(\frac{i\left(\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1\right)}{\sqrt{3}+3i}\right)^{-m}\left(a+bx^3\right)^m F_1\left(m+1;-m,-m;m+2;-\frac{i\left(\sqrt[3]{b}x+(-1)^{2/3}\sqrt[3]{a}\right)}{\sqrt{3}\sqrt[3]{a}},\frac{-\frac{2i\sqrt[3]{b}x}{\sqrt[3]{a}}+\cdots}{3i+\cdots}\right.}{\sqrt[3]{b}(m+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^m,x]
```

```
[Out] (((-1)^(2/3)*a^(1/3) + b^(1/3)*x)*(a + b*x^3)^m*AppellF1[1 + m, -m, -m, 2 +
 m, ((-I)*((-1)^(2/3)*a^(1/3) + b^(1/3)*x))/(Sqrt[3]*a^(1/3)), (I + Sqrt[3]
 - ((2*I)*b^(1/3)*x)/a^(1/3))/(3*I + Sqrt[3])])/(2^m*b^(1/3)*(1 + m)*((a^(1
/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3)))^m*((I*(1 + (b^(1/3)
*x)/a^(1/3)))/(3*I + Sqrt[3]))^m)
```

Maple [F] time = 0.25, size = 0, normalized size = 0.

$$\int \left(bx^3+a\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^m,x)
```

```
[Out] int((b*x^3+a)^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3+a\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^3+a\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m,x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^m, x)
```

Sympy [C] time = 15.3105, size = 34, normalized size = 0.77

$$\frac{a^m x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} \frac{1}{3}, -m \\ \frac{4}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**m,x)
```

```
[Out] a**m*x*gamma(1/3)*hyper((1/3, -m), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gam
ma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3 + a\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^m, x)
```

3.142 $\int \frac{(a+bx^3)^m}{c+dx^3} \, dx$

Optimal. Leaf size=57

$$\frac{x \left(a+bx^3\right)^m \left(\frac{bx^3}{a}+1\right)^{-m} F_1\left(\frac{1}{3};-m,1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c}$$

```
[Out] (x*(a + b*x^3)^m*AppellF1[1/3, -m, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(c*
(1 + (b*x^3)/a)^m)
```

Rubi [A] time = 0.0251674, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {430, 429}

$$\frac{x \left(a+bx^3\right)^m \left(\frac{bx^3}{a}+1\right)^{-m} F_1\left(\frac{1}{3};-m,1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^m/(c + d*x^3),x]
```

```
[Out] (x*(a + b*x^3)^m*AppellF1[1/3, -m, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(c*
(1 + (b*x^3)/a)^m)
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^3\right)^m}{c+dx^3} \, dx &= \left(\left(a+bx^3\right)^m \left(1+\frac{bx^3}{a}\right)^{-m}\right) \int \frac{\left(1+\frac{bx^3}{a}\right)^m}{c+dx^3} \, dx \\
&= \frac{x \left(a+bx^3\right)^m \left(1+\frac{bx^3}{a}\right)^{-m} F_1\left(\frac{1}{3};-m,1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c}
\end{aligned}$$

Mathematica [B] time = 0.184583, size = 162, normalized size = 2.84

$$-\frac{4acx \left(a+bx^3\right)^m F_1\left(\frac{1}{3};-m,1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\left(c+dx^3\right) \left(3x^3 \left(adF_1\left(\frac{4}{3};-m,2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-bcmF_1\left(\frac{4}{3};1-m,1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4acF_1\left(\frac{1}{3};-m,1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^m/(c + d*x^3),x]
```

```
[Out] (-4*a*c*x*(a + b*x^3)^m*AppellF1[1/3, -m, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c
)])/((c + d*x^3)*(-4*a*c*AppellF1[1/3, -m, 1, 4/3, -((b*x^3)/a), -((d*x^3)/
c)] + 3*x^3*(-(b*c*m*AppellF1[4/3, 1 - m, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c
)]) + a*d*AppellF1[4/3, -m, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

Maple [F] time = 0.431, size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^m}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^m/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^m/(d*x^3+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^m}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^m/(d*x^3 + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^3+a\right)^m}{dx^3+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^m/(d*x^3 + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**m/(d*x**3+c),x)
```

```
[Out] Timed out
```

Giac [F]   time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^m}{dx^3+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^m/(d*x^3 + c), x)
```

3.143 $\int \frac{(a+bx^3)^m}{(c+dx^3)^2} dx$

Optimal. Leaf size=57

$$\frac{x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}F_1\left(\frac{1}{3};-m,2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2}$$

```
[Out] (x*(a + b*x^3)^m*AppellF1[1/3, -m, 2, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(c^
2*(1 + (b*x^3)/a)^m)
```

Rubi [A] time = 0.0249284, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {430, 429}

$$\frac{x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}F_1\left(\frac{1}{3};-m,2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^m/(c + d*x^3)^2,x]
```

```
[Out] (x*(a + b*x^3)^m*AppellF1[1/3, -m, 2, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(c^
2*(1 + (b*x^3)/a)^m)
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^m}{\left(c+dx^3\right)^2}\,dx &= \left(\left(a+bx^3\right)^m\left(1+\frac{bx^3}{a}\right)^{-m}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^m}{\left(c+dx^3\right)^2}\,dx\\ &= \frac{x\left(a+bx^3\right)^m\left(1+\frac{bx^3}{a}\right)^{-m}F_1\left(\frac{1}{3};-m,2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^2}\end{aligned}$$

Mathematica [B] time = 0.1907, size = 162, normalized size = 2.84

$$-\frac{4acx\left(a+bx^3\right)^m F_1\left(\frac{1}{3};-m,2;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\left(c+dx^3\right)^2\left(-3x^3\left(bcmF_1\left(\frac{4}{3};1-m,2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-2adF_1\left(\frac{4}{3};-m,3;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4acF_1\left(\frac{1}{3};-m,2;\frac{4}{3};-\frac{bx^3}{a},-\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^m/(c + d*x^3)^2,x]
```

```
[Out] (-4*a*c*x*(a + b*x^3)^m*AppellF1[1/3, -m, 2, 4/3, -((b*x^3)/a), -((d*x^3)/c
)])/((c + d*x^3)^2*(-4*a*c*AppellF1[1/3, -m, 2, 4/3, -((b*x^3)/a), -((d*x^3
)/c)] - 3*x^3*(b*c*m*AppellF1[4/3, 1 - m, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c
)] - 2*a*d*AppellF1[4/3, -m, 3, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

Maple [F] time = 0.461, size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^m}{\left(dx^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^m/(d*x^3+c)^2,x)
```

```
[Out] int((b*x^3+a)^m/(d*x^3+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^m}{\left(dx^3+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m/(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^m/(d*x^3 + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(bx^3+a\right)^m}{d^2x^6+2\,cdx^3+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^m/(d^2*x^6 + 2*c*d*x^3 + c^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**m/(d*x**3+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3 + a\right)^m}{\left(dx^3 + c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m/(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^m/(d*x^3 + c)^2, x)
```

3.144 $\int \frac{(a+bx^3)^m}{(c+dx^3)^3} dx$

Optimal. Leaf size=57

$$\frac{x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}F_1\left(\frac{1}{3};-m,3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3}$$

```
[Out] (x*(a + b*x^3)^m*AppellF1[1/3, -m, 3, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(c^
3*(1 + (b*x^3)/a)^m)
```

Rubi [A] time = 0.0270919, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {430, 429}

$$\frac{x\left(a+bx^3\right)^m\left(\frac{bx^3}{a}+1\right)^{-m}F_1\left(\frac{1}{3};-m,3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^m/(c + d*x^3)^3,x]
```

```
[Out] (x*(a + b*x^3)^m*AppellF1[1/3, -m, 3, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(c^
3*(1 + (b*x^3)/a)^m)
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^3\right)^m}{\left(c+dx^3\right)^3}\,dx&=\left(\left(a+bx^3\right)^m\left(1+\frac{bx^3}{a}\right)^{-m}\right)\int \frac{\left(1+\frac{bx^3}{a}\right)^m}{\left(c+dx^3\right)^3}\,dx\\&=\frac{x\left(a+bx^3\right)^m\left(1+\frac{bx^3}{a}\right)^{-m}F_1\left(\frac{1}{3};-m,3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3}\end{aligned}$$

Mathematica [B] time = 0.23782, size = 162, normalized size = 2.84

$$-\frac{4acx\left(a+bx^3\right)^m F_1\left(\frac{1}{3};-m,3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{\left(c+dx^3\right)^3\left(-3x^3\left(bcmF_1\left(\frac{4}{3};1-m,3;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)-3adF_1\left(\frac{4}{3};-m,4;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4acF_1\left(\frac{1}{3};-m,3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^m/(c + d*x^3)^3,x]
```

```
[Out] (-4*a*c*x*(a + b*x^3)^m*AppellF1[1/3, -m, 3, 4/3, -((b*x^3)/a), -((d*x^3)/c
)])/((c + d*x^3)^3*(-4*a*c*AppellF1[1/3, -m, 3, 4/3, -((b*x^3)/a), -((d*x^3
)/c)] - 3*x^3*(b*c*m*AppellF1[4/3, 1 - m, 3, 7/3, -((b*x^3)/a), -((d*x^3)/c
)] - 3*a*d*AppellF1[4/3, -m, 4, 7/3, -((b*x^3)/a), -((d*x^3)/c)])))
```

Maple [F] time = 0.442, size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^m}{\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^m/(d*x^3+c)^3,x)
```

```
[Out] int((b*x^3+a)^m/(d*x^3+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3+a\right)^m}{\left(dx^3+c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m/(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^m/(d*x^3 + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(bx^3+a\right)^m}{d^3x^9+3\,cd^2x^6+3\,c^2dx^3+c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^m/(d^3*x^9 + 3*c*d^2*x^6 + 3*c^2*d*x^3 + c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**m/(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^3 + a\right)^m}{\left(dx^3 + c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m/(d*x^3+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^m/(d*x^3 + c)^3, x)
```

3.145 $\int \left(a+bx^3\right)^{-1-\frac{bc}{3bc-3ad}} \left(c+dx^3\right)^{-1+\frac{ad}{3bc-3ad}} dx$

Optimal. Leaf size=53

$$\frac{x\left(a+bx^3\right)^{-\frac{bc}{3bc-3ad}}\left(c+dx^3\right)^{\frac{ad}{3bc-3ad}}}{ac}$$

```
[Out] (x*(c + d*x^3)^((a*d)/(3*b*c - 3*a*d)))/(a*c*(a + b*x^3)^((b*c)/(3*b*c - 3*
a*d)))
```

Rubi [A] time = 0.019047, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.02, Rules used = {381}

$$\frac{x\left(a+bx^3\right)^{-\frac{bc}{3bc-3ad}}\left(c+dx^3\right)^{\frac{ad}{3bc-3ad}}}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(-1 - (b*c)/(3*b*c - 3*a*d))*(c + d*x^3)^(-1 + (a*d)/(3*b*c
 - 3*a*d)),x]
```

```
[Out] (x*(c + d*x^3)^((a*d)/(3*b*c - 3*a*d)))/(a*c*(a + b*x^3)^((b*c)/(3*b*c - 3*
a*d)))
```

Rule 381

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c), x] /; FreeQ[{a,
b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] &&
EqQ[a*d*(p + 1) + b*c*(q + 1), 0]
```

Rubi steps

$$\int \left(a+bx^3\right)^{-1-\frac{bc}{3bc-3ad}} \left(c+dx^3\right)^{-1+\frac{ad}{3bc-3ad}} dx = \frac{x\left(a+bx^3\right)^{-\frac{bc}{3bc-3ad}}\left(c+dx^3\right)^{\frac{ad}{3bc-3ad}}}{ac}$$

Mathematica [A] time = 0.032566, size = 52, normalized size = 0.98

$$\frac{x\left(a+bx^3\right)^{\frac{bc}{3ad-3bc}}\left(c+dx^3\right)^{\frac{ad}{3bc-3ad}}}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)^(-1 - (b*c)/(3*b*c - 3*a*d))*(c + d*x^3)^(-1 + (a*d)/
(3*b*c - 3*a*d)),x]
```

```
[Out] (x*(a + b*x^3)^((b*c)/(-3*b*c + 3*a*d))*(c + d*x^3)^((a*d)/(3*b*c - 3*a*d))
)/(a*c)
```

Maple [A] time = 0.003, size = 71, normalized size = 1.3

$$\frac{x}{ac}\left(bx^3+a\right)^{1-\frac{3ad-4bc}{3ad-3bc}}\left(dx^3+c\right)^{1-\frac{4ad-3bc}{3ad-3bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(-1-b*c/(-3*a*d+3*b*c))*(d*x^3+c)^(-1+a*d/(-3*a*d+3*b*c)),x)
```

```
[Out] (b*x^3+a)^(1-1/3*(3*a*d-4*b*c)/(a*d-b*c))*(d*x^3+c)^(1-1/3*(4*a*d-3*b*c)/(a
*d-b*c))/a/c*x
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(bx^3+a\right)^{-\frac{bc}{3(bc-ad)}-1}\left(dx^3+c\right)^{\frac{ad}{3(bc-ad)}-1}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(-1-b*c/(-3*a*d+3*b*c))*(d*x^3+c)^(-1+a*d/(-3*a*d+3*b*c
)),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(-1/3*b*c/(b*c - a*d) - 1)*(d*x^3 + c)^(1/3*a*d/(b*c
- a*d) - 1), x)
```

Fricas [A] time = 1.47237, size = 182, normalized size = 3.43

$$\frac{bdx^7+(bc+ad)x^4+acx}{\left(bx^3+a\right)^{\frac{4bc-3ad}{3(bc-ad)}}\left(dx^3+c\right)^{\frac{3bc-4ad}{3(bc-ad)}}ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(-1-b*c/(-3*a*d+3*b*c))*(d*x^3+c)^(-1+a*d/(-3*a*d+3*b*c
)),x, algorithm="fricas")
```

```
[Out] (b*d*x^7 + (b*c + a*d)*x^4 + a*c*x)/((b*x^3 + a)^(1/3*(4*b*c - 3*a*d)/(b*c
- a*d))*(d*x^3 + c)^(1/3*(3*b*c - 4*a*d)/(b*c - a*d))*a*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(-1-b*c/(-3*a*d+3*b*c))*(d*x**3+c)**(-1+a*d/(-3*a*d+3
*b*c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^3+a\right)^{-\frac{bc}{3(bc-ad)}-1}\left(dx^3+c\right)^{\frac{ad}{3(bc-ad)}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(-1-b*c/(-3*a*d+3*b*c))*(d*x^3+c)^(-1+a*d/(-3*a*d+3*b*c
)),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^(-1/3*b*c/(b*c - a*d) - 1)*(d*x^3 + c)^(1/3*a*d/(b*c
- a*d) - 1), x)
```

3.146 $\int \left(a+bx^4\right)\left(c+dx^4\right)^4\,dx$

Optimal. Leaf size=94

$$\frac{2}{9}c^2dx^9(3ad+2bc)+\frac{1}{5}c^3x^5(4ad+bc)+\frac{1}{17}d^3x^{17}(ad+4bc)+\frac{2}{13}cd^2x^{13}(2ad+3bc)+ac^4x+\frac{1}{21}bd^4x^{21}$$

```
[Out] a*c^4*x + (c^3*(b*c + 4*a*d)*x^5)/5 + (2*c^2*d*(2*b*c + 3*a*d)*x^9)/9 + (2*
c*d^2*(3*b*c + 2*a*d)*x^13)/13 + (d^3*(4*b*c + a*d)*x^17)/17 + (b*d^4*x^21)
/21
```

Rubi [A] time = 0.0676504, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$\frac{2}{9}c^2dx^9(3ad+2bc)+\frac{1}{5}c^3x^5(4ad+bc)+\frac{1}{17}d^3x^{17}(ad+4bc)+\frac{2}{13}cd^2x^{13}(2ad+3bc)+ac^4x+\frac{1}{21}bd^4x^{21}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)*(c + d*x^4)^4,x]
```

```
[Out] a*c^4*x + (c^3*(b*c + 4*a*d)*x^5)/5 + (2*c^2*d*(2*b*c + 3*a*d)*x^9)/9 + (2*
c*d^2*(3*b*c + 2*a*d)*x^13)/13 + (d^3*(4*b*c + a*d)*x^17)/17 + (b*d^4*x^21)
/21
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^4\right)\left(c+dx^4\right)^4\,dx &= \int \left(ac^4+c^3(bc+4ad)x^4+2c^2d(2bc+3ad)x^8+2cd^2(3bc+2ad)x^{12}+d^3(4bc+ad)x^{16}+bd\right.\\ &= ac^4x+\frac{1}{5}c^3(bc+4ad)x^5+\frac{2}{9}c^2d(2bc+3ad)x^9+\frac{2}{13}cd^2(3bc+2ad)x^{13}+\frac{1}{17}d^3(4bc+ad)x^{17}\end{aligned}$$

Mathematica [A] time = 0.0207885, size = 94, normalized size = 1.

$$\frac{2}{9}c^2dx^9(3ad+2bc)+\frac{1}{5}c^3x^5(4ad+bc)+\frac{1}{17}d^3x^{17}(ad+4bc)+\frac{2}{13}cd^2x^{13}(2ad+3bc)+ac^4x+\frac{1}{21}bd^4x^{21}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)*(c + d*x^4)^4,x]
```

```
[Out] a*c^4*x + (c^3*(b*c + 4*a*d)*x^5)/5 + (2*c^2*d*(2*b*c + 3*a*d)*x^9)/9 + (2*
c*d^2*(3*b*c + 2*a*d)*x^13)/13 + (d^3*(4*b*c + a*d)*x^17)/17 + (b*d^4*x^21)
/21
```

Maple [A] time = 0.001, size = 97, normalized size = 1.

$$\frac{bd^4x^{21}}{21}+\frac{(ad^4+4\,bcd^3)\,x^{17}}{17}+\frac{(4\,acd^3+6\,c^2d^2b)\,x^{13}}{13}+\frac{(6\,ac^2d^2+4\,c^3db)\,x^9}{9}+\frac{(4\,ac^3d+bc^4)\,x^5}{5}+ac^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)*(d*x^4+c)^4,x)
```

```
[Out] 1/21*b*d^4*x^21+1/17*(a*d^4+4*b*c*d^3)*x^17+1/13*(4*a*c*d^3+6*b*c^2*d^2)*x^
13+1/9*(6*a*c^2*d^2+4*b*c^3*d)*x^9+1/5*(4*a*c^3*d+b*c^4)*x^5+a*c^4*x
```

Maxima [A] time = 0.942229, size = 130, normalized size = 1.38

$$\frac{1}{21}\,bd^4x^{21}+\frac{1}{17}\left(4\,bcd^3+ad^4\right)x^{17}+\frac{2}{13}\left(3\,bc^2d^2+2\,acd^3\right)x^{13}+\frac{2}{9}\left(2\,bc^3d+3\,ac^2d^2\right)x^9+ac^4x+\frac{1}{5}\left(bc^4+4\,ac^3d\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c)^4,x, algorithm="maxima")
```

```
[Out] 1/21*b*d^4*x^21 + 1/17*(4*b*c*d^3 + a*d^4)*x^17 + 2/13*(3*b*c^2*d^2 + 2*a*c
*d^3)*x^13 + 2/9*(2*b*c^3*d + 3*a*c^2*d^2)*x^9 + a*c^4*x + 1/5*(b*c^4 + 4*a
*c^3*d)*x^5
```

Fricas [A] time = 1.09905, size = 242, normalized size = 2.57

$$\frac{1}{21}x^{21}d^4b+\frac{4}{17}x^{17}d^3cb+\frac{1}{17}x^{17}d^4a+\frac{6}{13}x^{13}d^2c^2b+\frac{4}{13}x^{13}d^3ca+\frac{4}{9}x^9dc^3b+\frac{2}{3}x^9d^2c^2a+\frac{1}{5}x^5c^4b+\frac{4}{5}x^5dc^3a+xc^4a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c)^4,x, algorithm="fricas")
```

```
[Out] 1/21*x^21*d^4*b + 4/17*x^17*d^3*c*b + 1/17*x^17*d^4*a + 6/13*x^13*d^2*c^2*b
 + 4/13*x^13*d^3*c*a + 4/9*x^9*d*c^3*b + 2/3*x^9*d^2*c^2*a + 1/5*x^5*c^4*b
+ 4/5*x^5*d*c^3*a + x*c^4*a
```

Sympy [A] time = 0.079749, size = 107, normalized size = 1.14

$$ac^4x+\frac{bd^4x^{21}}{21}+x^{17}\left(\frac{ad^4}{17}+\frac{4bcd^3}{17}\right)+x^{13}\left(\frac{4acd^3}{13}+\frac{6bc^2d^2}{13}\right)+x^9\left(\frac{2ac^2d^2}{3}+\frac{4bc^3d}{9}\right)+x^5\left(\frac{4ac^3d}{5}+\frac{bc^4}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)*(d*x**4+c)**4,x)
```

```
[Out] a*c**4*x + b*d**4*x**21/21 + x**17*(a*d**4/17 + 4*b*c*d**3/17) + x**13*(4*a
*c*d**3/13 + 6*b*c**2*d**2/13) + x**9*(2*a*c**2*d**2/3 + 4*b*c**3*d/9) + x*
*5*(4*a*c**3*d/5 + b*c**4/5)
```

Giac [A] time = 1.10633, size = 132, normalized size = 1.4

$$\frac{1}{21}\,bd^{4}x^{21}+\frac{4}{17}\,bcd^{3}x^{17}+\frac{1}{17}\,ad^{4}x^{17}+\frac{6}{13}\,bc^{2}d^{2}x^{13}+\frac{4}{13}\,acd^{3}x^{13}+\frac{4}{9}\,bc^{3}dx^{9}+\frac{2}{3}\,ac^{2}d^{2}x^{9}+\frac{1}{5}\,bc^{4}x^{5}+\frac{4}{5}\,ac^{3}dx^{5}+ac^{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c)^4,x, algorithm="giac")
```

```
[Out] 1/21*b*d^4*x^21 + 4/17*b*c*d^3*x^17 + 1/17*a*d^4*x^17 + 6/13*b*c^2*d^2*x^13
 + 4/13*a*c*d^3*x^13 + 4/9*b*c^3*d*x^9 + 2/3*a*c^2*d^2*x^9 + 1/5*b*c^4*x^5
+ 4/5*a*c^3*d*x^5 + a*c^4*x
```

3.147 $\int \left(a+bx^4\right)\left(c+dx^4\right)^3\,dx$

Optimal. Leaf size=70

$$\frac{1}{5}c^2x^5(3ad+bc)+\frac{1}{13}d^2x^{13}(ad+3bc)+\frac{1}{3}cdx^9(ad+bc)+ac^3x+\frac{1}{17}bd^3x^{17}$$

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^5)/5 + (c*d*(b*c + a*d)*x^9)/3 + (d^2*(3*b*c
 + a*d)*x^13)/13 + (b*d^3*x^17)/17
```

Rubi [A] time = 0.0487288, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$\frac{1}{5}c^2x^5(3ad+bc)+\frac{1}{13}d^2x^{13}(ad+3bc)+\frac{1}{3}cdx^9(ad+bc)+ac^3x+\frac{1}{17}bd^3x^{17}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)*(c + d*x^4)^3,x]
```

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^5)/5 + (c*d*(b*c + a*d)*x^9)/3 + (d^2*(3*b*c
 + a*d)*x^13)/13 + (b*d^3*x^17)/17
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^4\right)\left(c+dx^4\right)^3\,dx &= \int \left(ac^3+c^2(bc+3ad)x^4+3cd(bc+ad)x^8+d^2(3bc+ad)x^{12}+bd^3x^{16}\right)dx\\ &= ac^3x+\frac{1}{5}c^2(bc+3ad)x^5+\frac{1}{3}cd(bc+ad)x^9+\frac{1}{13}d^2(3bc+ad)x^{13}+\frac{1}{17}bd^3x^{17}\end{aligned}$$

Mathematica [A] time = 0.0153898, size = 70, normalized size = 1.

$$\frac{1}{5}c^2x^5(3ad+bc)+\frac{1}{13}d^2x^{13}(ad+3bc)+\frac{1}{3}cdx^9(ad+bc)+ac^3x+\frac{1}{17}bd^3x^{17}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)*(c + d*x^4)^3,x]
```

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^5)/5 + (c*d*(b*c + a*d)*x^9)/3 + (d^2*(3*b*c
 + a*d)*x^13)/13 + (b*d^3*x^17)/17
```

Maple [A] time = 0., size = 73, normalized size = 1.

$$\frac{bd^3x^{17}}{17}+\frac{\left(ad^3+3\,bcd^2\right)x^{13}}{13}+\frac{\left(3\,acd^2+3\,bc^2d\right)x^9}{9}+\frac{\left(3\,ac^2d+bc^3\right)x^5}{5}+ac^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)*(d*x^4+c)^3,x)
```

```
[Out] 1/17*b*d^3*x^17+1/13*(a*d^3+3*b*c*d^2)*x^13+1/9*(3*a*c*d^2+3*b*c^2*d)*x^9+1
/5*(3*a*c^2*d+b*c^3)*x^5+a*c^3*x
```

Maxima [A] time = 0.940268, size = 95, normalized size = 1.36

$$\frac{1}{17}bd^3x^{17} + \frac{1}{13}\left(3\,bcd^2 + ad^3\right)x^{13} + \frac{1}{3}\left(bc^2d + acd^2\right)x^9 + \frac{1}{5}\left(bc^3 + 3\,ac^2d\right)x^5 + ac^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c)^3,x, algorithm="maxima")
```

```
[Out] 1/17*b*d^3*x^17 + 1/13*(3*b*c*d^2 + a*d^3)*x^13 + 1/3*(b*c^2*d + a*c*d^2)*x
^9 + 1/5*(b*c^3 + 3*a*c^2*d)*x^5 + a*c^3*x
```

Fricas [A] time = 1.02544, size = 182, normalized size = 2.6

$$\frac{1}{17}x^{17}d^3b + \frac{3}{13}x^{13}d^2cb + \frac{1}{13}x^{13}d^3a + \frac{1}{3}x^9dc^2b + \frac{1}{3}x^9d^2ca + \frac{1}{5}x^5c^3b + \frac{3}{5}x^5dc^2a + xc^3a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c)^3,x, algorithm="fricas")
```

```
[Out] 1/17*x^17*d^3*b + 3/13*x^13*d^2*c*b + 1/13*x^13*d^3*a + 1/3*x^9*d*c^2*b + 1
/3*x^9*d^2*c*a + 1/5*x^5*c^3*b + 3/5*x^5*d*c^2*a + x*c^3*a
```

Sympy [A] time = 0.073041, size = 76, normalized size = 1.09

$$ac^3x + \frac{bd^3x^{17}}{17} + x^{13}\left(\frac{ad^3}{13} + \frac{3bcd^2}{13}\right) + x^9\left(\frac{acd^2}{3} + \frac{bc^2d}{3}\right) + x^5\left(\frac{3ac^2d}{5} + \frac{bc^3}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)*(d*x**4+c)**3,x)
```

```
[Out] a*c**3*x + b*d**3*x**17/17 + x**13*(a*d**3/13 + 3*b*c*d**2/13) + x**9*(a*c*
d**2/3 + b*c**2*d/3) + x**5*(3*a*c**2*d/5 + b*c**3/5)
```

Giac [A] time = 1.10574, size = 100, normalized size = 1.43

$$\frac{1}{17}bd^3x^{17} + \frac{3}{13}bcd^2x^{13} + \frac{1}{13}ad^3x^{13} + \frac{1}{3}bc^2dx^9 + \frac{1}{3}acd^2x^9 + \frac{1}{5}bc^3x^5 + \frac{3}{5}ac^2dx^5 + ac^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c)^3,x, algorithm="giac")
```

```
[Out] 1/17*b*d^3*x^17 + 3/13*b*c*d^2*x^13 + 1/13*a*d^3*x^13 + 1/3*b*c^2*d*x^9 + 1
/3*a*c*d^2*x^9 + 1/5*b*c^3*x^5 + 3/5*a*c^2*d*x^5 + a*c^3*x
```

3.148 $\int (a+bx^4)(c+dx^4)^2 \, dx$

Optimal. Leaf size=50

$$\frac{1}{9}dx^9(ad+2bc)+\frac{1}{5}cx^5(2ad+bc)+ac^2x+\frac{1}{13}bd^2x^{13}$$

```
[Out] a*c^2*x + (c*(b*c + 2*a*d)*x^5)/5 + (d*(2*b*c + a*d)*x^9)/9 + (b*d^2*x^13)/
13
```

Rubi [A] time = 0.029443, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$\frac{1}{9}dx^9(ad+2bc)+\frac{1}{5}cx^5(2ad+bc)+ac^2x+\frac{1}{13}bd^2x^{13}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)*(c + d*x^4)^2,x]
```

```
[Out] a*c^2*x + (c*(b*c + 2*a*d)*x^5)/5 + (d*(2*b*c + a*d)*x^9)/9 + (b*d^2*x^13)/
13
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^4)(c+dx^4)^2 \, dx &= \int \left(ac^2+c(bc+2ad)x^4+d(2bc+ad)x^8+bd^2x^{12}\right) dx\\ &= ac^2x+\frac{1}{5}c(bc+2ad)x^5+\frac{1}{9}d(2bc+ad)x^9+\frac{1}{13}bd^2x^{13}\end{aligned}$$

Mathematica [A] time = 0.011794, size = 50, normalized size = 1.

$$\frac{1}{9}dx^9(ad+2bc)+\frac{1}{5}cx^5(2ad+bc)+ac^2x+\frac{1}{13}bd^2x^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)*(c + d*x^4)^2,x]
```

```
[Out] a*c^2*x + (c*(b*c + 2*a*d)*x^5)/5 + (d*(2*b*c + a*d)*x^9)/9 + (b*d^2*x^13)/
13
```

Maple [A] time = 0., size = 49, normalized size = 1.

$$\frac{bd^2x^{13}}{13}+\frac{\left(ad^2+2\,bcd\right)x^9}{9}+\frac{\left(2\,acd+bc^2\right)x^5}{5}+ac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)*(d*x^4+c)^2,x)
```

```
[Out] 1/13*b*d^2*x^13+1/9*(a*d^2+2*b*c*d)*x^9+1/5*(2*a*c*d+b*c^2)*x^5+a*c^2*x
```

Maxima [A] time = 0.948208, size = 65, normalized size = 1.3

$$\frac{1}{13}\,bd^2x^{13}+\frac{1}{9}\left(2\,bcd+ad^2\right)x^9+\frac{1}{5}\left(bc^2+2\,acd\right)x^5+ac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] 1/13*b*d^2*x^13 + 1/9*(2*b*c*d + a*d^2)*x^9 + 1/5*(b*c^2 + 2*a*c*d)*x^5 + a
*c^2*x
```

Fricas [A] time = 1.04988, size = 123, normalized size = 2.46

$$\frac{1}{13}x^{13}d^2b+\frac{2}{9}x^9dcb+\frac{1}{9}x^9d^2a+\frac{1}{5}x^5c^2b+\frac{2}{5}x^5dca+xc^2a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*d^2*b + 2/9*x^9*d*c*b + 1/9*x^9*d^2*a + 1/5*x^5*c^2*b + 2/5*x^5*d
*c*a + x*c^2*a
```

Sympy [A] time = 0.0685, size = 53, normalized size = 1.06

$$ac^2x+\frac{bd^2x^{13}}{13}+x^9\left(\frac{ad^2}{9}+\frac{2bcd}{9}\right)+x^5\left(\frac{2acd}{5}+\frac{bc^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)*(d*x**4+c)**2,x)
```

```
[Out] a*c**2*x + b*d**2*x**13/13 + x**9*(a*d**2/9 + 2*b*c*d/9) + x**5*(2*a*c*d/5
+ b*c**2/5)
```

Giac [A] time = 1.09182, size = 68, normalized size = 1.36

$$\frac{1}{13}\,bd^2x^{13}+\frac{2}{9}\,bcdx^9+\frac{1}{9}\,ad^2x^9+\frac{1}{5}\,bc^2x^5+\frac{2}{5}\,acdx^5+ac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] 1/13*b*d^2*x^13 + 2/9*b*c*d*x^9 + 1/9*a*d^2*x^9 + 1/5*b*c^2*x^5 + 2/5*a*c*d
*x^5 + a*c^2*x
```

3.149 $\int \left(a+bx^4\right)\left(c+dx^4\right)\,dx$

Optimal. Leaf size=28

$$\frac{1}{5}x^5(ad+bc)+acx+\frac{1}{9}bdx^9$$

```
[Out] a*c*x + ((b*c + a*d)*x^5)/5 + (b*d*x^9)/9
```

Rubi [A] time = 0.0144161, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {373}

$$\frac{1}{5}x^5(ad+bc)+acx+\frac{1}{9}bdx^9$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)*(c + d*x^4),x]
```

```
[Out] a*c*x + ((b*c + a*d)*x^5)/5 + (b*d*x^9)/9
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^4\right)\left(c+dx^4\right)\,dx &= \int \left(ac+(bc+ad)x^4+bdx^8\right)\,dx\\ &= acx+\frac{1}{5}(bc+ad)x^5+\frac{1}{9}bdx^9\end{aligned}$$

Mathematica [A] time = 0.005435, size = 28, normalized size = 1.

$$\frac{1}{5}x^5(ad+bc)+acx+\frac{1}{9}bdx^9$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)*(c + d*x^4),x]
```

```
[Out] a*c*x + ((b*c + a*d)*x^5)/5 + (b*d*x^9)/9
```

Maple [A] time = 0.001, size = 25, normalized size = 0.9

$$acx+\frac{(ad+bc)\,x^5}{5}+\frac{bdx^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)*(d*x^4+c),x)
```

```
[Out] a*c*x+1/5*(a*d+b*c)*x^5+1/9*b*d*x^9
```

Maxima [A] time = 0.948273, size = 32, normalized size = 1.14

$$\frac{1}{9}bdx^9 + \frac{1}{5}(bc + ad)x^5 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/9*b*d*x^9 + 1/5*(b*c + a*d)*x^5 + a*c*x
```

Fricas [A] time = 0.946154, size = 66, normalized size = 2.36

$$\frac{1}{9}x^9db + \frac{1}{5}x^5cb + \frac{1}{5}x^5da + xca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c),x, algorithm="fricas")
```

```
[Out] 1/9*x^9*d*b + 1/5*x^5*c*b + 1/5*x^5*d*a + x*c*a
```

Sympy [A] time = 0.057678, size = 26, normalized size = 0.93

$$acx + \frac{bdx^9}{9} + x^5\left(\frac{ad}{5} + \frac{bc}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)*(d*x**4+c),x)
```

```
[Out] a*c*x + b*d*x**9/9 + x**5*(a*d/5 + b*c/5)
```

Giac [A] time = 1.10344, size = 35, normalized size = 1.25

$$\frac{1}{9}bdx^9 + \frac{1}{5}bcx^5 + \frac{1}{5}adx^5 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/9*b*d*x^9 + 1/5*b*c*x^5 + 1/5*a*d*x^5 + a*c*x
```

3.150 $\int \frac{a+bx^4}{c+dx^4}\,dx$

Optimal. Leaf size=223

$$\frac{(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}d^{5/4}}-\frac{(bc-ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}d^{5/4}}+\frac{(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}d^{5/4}}-$$

```
[Out] (b*x)/d + ((b*c - a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*
c^(3/4)*d^(5/4)) - ((b*c - a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2
*Sqrt[2]*c^(3/4)*d^(5/4)) + ((b*c - a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1
/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(3/4)*d^(5/4)) - ((b*c - a*d)*Log[Sqrt[c
] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(3/4)*d^(5/4))
```

Rubi [A] time = 0.151368, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {388, 211, 1165, 628, 1162, 617, 204}

$$\frac{(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}d^{5/4}}-\frac{(bc-ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}d^{5/4}}+\frac{(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}d^{5/4}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)/(c + d*x^4),x]
```

```
[Out] (b*x)/d + ((b*c - a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*
c^(3/4)*d^(5/4)) - ((b*c - a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2
*Sqrt[2]*c^(3/4)*d^(5/4)) + ((b*c - a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1
/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(3/4)*d^(5/4)) - ((b*c - a*d)*Log[Sqrt[c
] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(3/4)*d^(5/4))
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^4}{c+dx^4}\,dx &= \frac{bx}{d} - \frac{(bc-ad)\int \frac{1}{c+dx^4}\,dx}{d}\\ &= \frac{bx}{d} - \frac{(bc-ad)\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx}{2\sqrt{cd}} - \frac{(bc-ad)\int \frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4}\,dx}{2\sqrt{cd}}\\ &= \frac{bx}{d} - \frac{(bc-ad)\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx}{4\sqrt{c}d^{3/2}} - \frac{(bc-ad)\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx}{4\sqrt{c}d^{3/2}} + \frac{(bc-ad)\int \frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{d}}+2x}{-\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}-x^2}\,dx}{4\sqrt{2}c^{3/4}d^{5/4}} + \frac{(bc-ac\ldots}{}\\ &= \frac{bx}{d} + \frac{(bc-ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}d^{5/4}} - \frac{(bc-ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}d^{5/4}} - \frac{(bc-ad)\,\mathrm{Sub}\ldots}{}\\ &= \frac{bx}{d} + \frac{(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}d^{5/4}} - \frac{(bc-ad)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}d^{5/4}} + \frac{(bc-ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}\ldots\right.}{4\sqrt{2}c^{3/4}d^{5/4}}\end{aligned}$$

Mathematica [A] time = 0.129272, size = 196, normalized size = 0.88

$$\frac{\sqrt{2}(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)-\sqrt{2}(bc-ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)+2\sqrt{2}(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\ldots}{\sqrt[4]{\ldots}}\right.}{8c^{3/4}d^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)/(c + d*x^4),x]
```

```
[Out] (8*b*c^(3/4)*d^(1/4)*x + 2*Sqrt[2]*(b*c - a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*
x)/c^(1/4)] - 2*Sqrt[2]*(b*c - a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)]
 + Sqrt[2]*(b*c - a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^
```

```
2] - Sqrt[2]*(b*c - a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*
x^2])/(8*c^(3/4)*d^(5/4))
```

Maple [A] time = 0.007, size = 266, normalized size = 1.2

$$\frac{bx}{d}+\frac{\sqrt{2}\,a}{4c}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)-\frac{\sqrt{2}\,b}{4d}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)+\frac{\sqrt{2}\,a}{4c}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)-\frac{\sqrt{2}\,b}{4d}\sqrt[4]{\frac{c}{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)/(d*x^4+c),x)
```

```
[Out] b*x/d+1/4*(c/d)^(1/4)/c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)*a-1/4/d*(c/
d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)*b+1/4*(c/d)^(1/4)/c*2^(1/2
)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*a-1/4/d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2
)/(c/d)^(1/4)*x-1)*b+1/8*(c/d)^(1/4)/c*2^(1/2)*ln((x^2+(c/d)^(1/4)*x*2^(1/2
)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))*a-1/8/d*(c/d)^(1/4)
*2^(1/2)*ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^(1
/2)+(c/d)^(1/2)))*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.36777, size = 1315, normalized size = 5.9

$$4d\left(-\frac{b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2-4a^3bcd^3+a^4d^4}{c^3d^5}\right)^{\frac{1}{4}}\arctan\left(\frac{c^2d^4x\left(-\frac{b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2-4a^3bcd^3+a^4d^4}{c^3d^5}\right)^{\frac{3}{4}}-c^2d^4\sqrt{\frac{c^2d^2\sqrt{-\frac{b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2-4\cdots}{c^3d^5}}}{b^2c^2-2ab\cdots}}}{b^3c^3-3ab^2c^2d+3a^2bc\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] 1/4*(4*d*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a
^4*d^4)/(c^3*d^5))^(1/4)*arctan((c^2*d^4*x*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a
^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5))^(3/4) - c^2*d^4*sqrt((
c^2*d^2*sqrt(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
+ a^4*d^4)/(c^3*d^5)) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^2)/(b^2*c^2 - 2*a
*b*c*d + a^2*d^2))*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b
*c*d^3 + a^4*d^4)/(c^3*d^5))^(3/4))/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^
2 - a^3*d^3)) + d*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*
```

```
c*d^3 + a^4*d^4)/(c^3*d^5))^(1/4)*log(c*d*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^
2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5))^(1/4) - (b*c - a*d)*x)
- d*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^
4)/(c^3*d^5))^(1/4)*log(-c*d*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2
 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5))^(1/4) - (b*c - a*d)*x) + 4*b*x)/d
```

Sympy [A] time = 0.589211, size = 87, normalized size = 0.39

$$\frac{bx}{d} + \operatorname{RootSum}\left(256t^4c^3d^5 + a^4d^4 - 4a^3bcd^3 + 6a^2b^2c^2d^2 - 4ab^3c^3d + b^4c^4, \left(t \mapsto t\log\left(\frac{4tcd}{ad - bc} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)/(d*x**4+c),x)
```

```
[Out] b*x/d + RootSum(256*_t**4*c**3*d**5 + a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*
b**2*c**2*d**2 - 4*a*b**3*c**3*d + b**4*c**4, Lambda(_t, _t*log(4*_t*c*d/(a
*d - b*c) + x)))
```

Giac [A] time = 1.09969, size = 331, normalized size = 1.48

$$\frac{bx}{d} - \frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}bc - \left(cd^3\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4cd^2} - \frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}bc - \left(cd^3\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4cd^2} - \frac{\sqrt{2}\left(\left(cd\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] b*x/d - 1/4*sqrt(2)*((c*d^3)^(1/4)*b*c - (c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt
(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(c*d^2) - 1/4*sqrt(2)*((c*d^3)
^(1/4)*b*c - (c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/
4))/(c/d)^(1/4))/(c*d^2) - 1/8*sqrt(2)*((c*d^3)^(1/4)*b*c - (c*d^3)^(1/4)*a
*d)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(c*d^2) + 1/8*sqrt(2)*((c*
d^3)^(1/4)*b*c - (c*d^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(
c/d))/(c*d^2)
```

3.151 $\int \frac{a+bx^4}{(c+dx^4)^2} dx$

Optimal. Leaf size=245

$$-\frac{(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}d^{5/4}}+\frac{(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}d^{5/4}}-\frac{(3ad+bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}d^{5/4}}$$

```
[Out] -((b*c - a*d)*x)/(4*c*d*(c + d*x^4)) - ((b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]*d
^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(7/4)*d^(5/4)) + ((b*c + 3*a*d)*ArcTan[1 +
 (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(7/4)*d^(5/4)) - ((b*c + 3*a*d)
*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(16*Sqrt[2]*c^(7/4
)*d^(5/4)) + ((b*c + 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[
d]*x^2])/(16*Sqrt[2]*c^(7/4)*d^(5/4))
```

Rubi [A] time = 0.146578, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {385, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}d^{5/4}}+\frac{(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}d^{5/4}}-\frac{(3ad+bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}d^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)/(c + d*x^4)^2,x]
```


```
[Out] -((b*c - a*d)*x)/(4*c*d*(c + d*x^4)) - ((b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]*d
^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(7/4)*d^(5/4)) + ((b*c + 3*a*d)*ArcTan[1 +
 (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(7/4)*d^(5/4)) - ((b*c + 3*a*d)
*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(16*Sqrt[2]*c^(7/4
)*d^(5/4)) + ((b*c + 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[
d]*x^2])/(16*Sqrt[2]*c^(7/4)*d^(5/4))
```


Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a+bx^4}{\left(c+dx^4\right)^2}\,dx = -\frac{(bc-ad)x}{4cd\left(c+dx^4\right)} + \frac{(bc+3ad)\int \frac{1}{c+dx^4}\,dx}{4cd}$$

$$= -\frac{(bc-ad)x}{4cd\left(c+dx^4\right)} + \frac{(bc+3ad)\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx}{8c^{3/2}d} + \frac{(bc+3ad)\int \frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4}\,dx}{8c^{3/2}d}$$

$$= -\frac{(bc-ad)x}{4cd\left(c+dx^4\right)} + \frac{(bc+3ad)\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx}{16c^{3/2}d^{3/2}} + \frac{(bc+3ad)\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx}{16c^{3/2}d^{3/2}} - \frac{(bc+3ad)\int \frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{d}} \cdots}{-\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}}{\cdots}}}{16\sqrt{2}c^{7/4}d^{5/4}} \cdots$$

$$= -\frac{(bc-ad)x}{4cd\left(c+dx^4\right)} - \frac{(bc+3ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}d^{5/4}} + \frac{(bc+3ad)\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\cdots\right.}{16\sqrt{2}c^{7/4}d^{5/4}} \cdots$$

$$= -\frac{(bc-ad)x}{4cd\left(c+dx^4\right)} - \frac{(bc+3ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}d^{5/4}} + \frac{(bc+3ad)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}d^{5/4}} - \frac{(bc+3ad)\log\left(\sqrt{c}\cdots\right.}{16\sqrt{\cdots}} \cdots$$

Mathematica [A] time = 0.172137, size = 212, normalized size = 0.87

$$\frac{-\frac{8c^{3/4}\sqrt[4]{d}x(bc-ad)}{c+dx^4} - \sqrt{2}(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right) + \sqrt{2}(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right) - 2\sqrt{2}(3ad \cdots}{32c^{7/4}d^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)/(c + d*x^4)^2,x]
```

```
[Out] ((-8*c^(3/4)*d^(1/4)*(b*c - a*d)*x)/(c + d*x^4) - 2*Sqrt[2]*(b*c + 3*a*d)*A
rcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 2*Sqrt[2]*(b*c + 3*a*d)*ArcTan[1 +
 (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - Sqrt[2]*(b*c + 3*a*d)*Log[Sqrt[c] - Sqrt[2]
*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2] + Sqrt[2]*(b*c + 3*a*d)*Log[Sqrt[c] + Sqr
t[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(32*c^(7/4)*d^(5/4))
```

Maple [A] time = 0.008, size = 295, normalized size = 1.2

$$\frac{(ad-bc)\,x}{4\,cd\left(dx^4+c\right)}+\frac{3\,\sqrt{2}a}{16\,c^2}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)+\frac{\sqrt{2}b}{16\,cd}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)+\frac{3\,\sqrt{2}a}{32\,c^2}\sqrt[4]{\frac{c}{d}}\ln\left(\left(x^2+\sqrt[4]{\frac{c}{d}}x\sqrt{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)/(d*x^4+c)^2,x)
```

```
[Out] 1/4*(a*d-b*c)/c/d*x/(d*x^4+c)+3/16/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
c/d)^(1/4)*x-1)*a+1/16/c/d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x
-1)*b+3/32/c^2*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2
))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))*a+1/32/c/d*(c/d)^(1/4)*2^(1/2)*
ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)
^(1/2)))*b+3/16/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)*a+1
/16/c/d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.41881, size = 1526, normalized size = 6.23

$$4\left(cd^2x^4+c^2d\right)\left(-\frac{b^4c^4+12\,ab^3c^3d+54\,a^2b^2c^2d^2+108\,a^3bcd^3+81\,a^4d^4}{c^7d^5}\right)^{\frac{1}{4}}\arctan\left(-\frac{c^5d^4x\left(-\frac{b^4c^4+12\,ab^3c^3d+54\,a^2b^2c^2d^2+108\,a^3bcd^3+81\,a^4d^4}{c^7d^5}\right)^{\frac{3}{4}}-c^5d^4\sqrt{\vphantom{x}}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*(c*d^2*x^4 + c^2*d)*(-(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^
2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(c^7*d^5))^(1/4)*arctan(-(c^5*d^4*x*(-(b^
```

```
4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)
/(c^7*d^5))^(3/4) - c^5*d^4*sqrt((c^4*d^2*sqrt(-(b^4*c^4 + 12*a*b^3*c^3*d +
 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(c^7*d^5)) + (b^2*c^2 +
 6*a*b*c*d + 9*a^2*d^2)*x^2)/(b^2*c^2 + 6*a*b*c*d + 9*a^2*d^2))*(-(b^4*c^4
+ 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(c^7*
d^5))^(3/4))/(b^3*c^3 + 9*a*b^2*c^2*d + 27*a^2*b*c*d^2 + 27*a^3*d^3)) + (c*
d^2*x^4 + c^2*d)*(-(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3
*b*c*d^3 + 81*a^4*d^4)/(c^7*d^5))^(1/4)*log(c^2*d*(-(b^4*c^4 + 12*a*b^3*c^3
*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(c^7*d^5))^(1/4) +
(b*c + 3*a*d)*x) - (c*d^2*x^4 + c^2*d)*(-(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2
*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d^4)/(c^7*d^5))^(1/4)*log(-c^2*d*(-
(b^4*c^4 + 12*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 108*a^3*b*c*d^3 + 81*a^4*d
^4)/(c^7*d^5))^(1/4) + (b*c + 3*a*d)*x) - 4*(b*c - a*d)*x)/(c*d^2*x^4 + c^2
*d)
```

Sympy [A] time = 0.838773, size = 112, normalized size = 0.46

$$\frac{x\,(ad-bc)}{4c^2d+4cd^2x^4}+\operatorname{RootSum}\left(65536t^4c^7d^5+81a^4d^4+108a^3bcd^3+54a^2b^2c^2d^2+12ab^3c^3d+b^4c^4,\left(t\mapsto t\log\left(\frac{16tc^2d}{3ad+bc}+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)/(d*x**4+c)**2,x)
```

```
[Out] x*(a*d - b*c)/(4*c**2*d + 4*c*d**2*x**4) + RootSum(65536*_t**4*c**7*d**5 +
81*a**4*d**4 + 108*a**3*b*c*d**3 + 54*a**2*b**2*c**2*d**2 + 12*a*b**3*c**3*
d + b**4*c**4, Lambda(_t, _t*log(16*_t*c**2*d/(3*a*d + b*c) + x)))
```

Giac [A] time = 1.12676, size = 359, normalized size = 1.47

$$\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}bc+3\left(cd^3\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{16c^2d^2}+\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}bc+3\left(cd^3\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{16c^2d^2}+\frac{\sqrt{2}\left(\left(cd\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*((c*d^3)^(1/4)*b*c + 3*(c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(
2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(c^2*d^2) + 1/16*sqrt(2)*((c*d^3)^(
1/4)*b*c + 3*(c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/
4))/(c/d)^(1/4))/(c^2*d^2) + 1/32*sqrt(2)*((c*d^3)^(1/4)*b*c + 3*(c*d^3)^(1
/4)*a*d)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(c^2*d^2) - 1/32*sqrt
(2)*((c*d^3)^(1/4)*b*c + 3*(c*d^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x*(c/d)^(1/
4) + sqrt(c/d))/(c^2*d^2) - 1/4*(b*c*x - a*d*x)/((d*x^4 + c)*c*d)
```

3.152 $\int \frac{a+bx^4}{\left(c+dx^4\right)^3} \, dx$

Optimal. Leaf size=273

$$-\frac{3(7ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{128\sqrt{2}c^{11/4}d^{5/4}}+\frac{3(7ad+bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{128\sqrt{2}c^{11/4}d^{5/4}}-\frac{3(7ad+bc)\tan^{-1}\left(1-\right.}{64\sqrt{2}c^{11/4}d^{5/4}}$$

```
[Out] -((b*c - a*d)*x)/(8*c*d*(c + d*x^4)^2) + ((b*c + 7*a*d)*x)/(32*c^2*d*(c + d
*x^4)) - (3*(b*c + 7*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(64*Sqrt
[2]*c^(11/4)*d^(5/4)) + (3*(b*c + 7*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(
1/4)])/(64*Sqrt[2]*c^(11/4)*d^(5/4)) - (3*(b*c + 7*a*d)*Log[Sqrt[c] - Sqrt[
2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(128*Sqrt[2]*c^(11/4)*d^(5/4)) + (3*(b
*c + 7*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(128*Sq
rt[2]*c^(11/4)*d^(5/4))
```

Rubi [A] time = 0.174277, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {385, 199, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3(7ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{128\sqrt{2}c^{11/4}d^{5/4}}+\frac{3(7ad+bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{128\sqrt{2}c^{11/4}d^{5/4}}-\frac{3(7ad+bc)\tan^{-1}\left(1-\right.}{64\sqrt{2}c^{11/4}d^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)/(c + d*x^4)^3,x]
```


```
[Out] -((b*c - a*d)*x)/(8*c*d*(c + d*x^4)^2) + ((b*c + 7*a*d)*x)/(32*c^2*d*(c + d
*x^4)) - (3*(b*c + 7*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(64*Sqrt
[2]*c^(11/4)*d^(5/4)) + (3*(b*c + 7*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(
1/4)])/(64*Sqrt[2]*c^(11/4)*d^(5/4)) - (3*(b*c + 7*a*d)*Log[Sqrt[c] - Sqrt[
2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(128*Sqrt[2]*c^(11/4)*d^(5/4)) + (3*(b
*c + 7*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(128*Sq
rt[2]*c^(11/4)*d^(5/4))
```


Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a+bx^4}{\left(c+dx^4\right)^3}\,dx = -\frac{(bc-ad)x}{8cd\left(c+dx^4\right)^2} + \frac{(bc+7ad)\int \frac{1}{\left(c+dx^4\right)^2}\,dx}{8cd}$$

$$= -\frac{(bc-ad)x}{8cd\left(c+dx^4\right)^2} + \frac{(bc+7ad)x}{32c^2d\left(c+dx^4\right)} + \frac{(3(bc+7ad))\int \frac{1}{c+dx^4}\,dx}{32c^2d}$$

$$= -\frac{(bc-ad)x}{8cd\left(c+dx^4\right)^2} + \frac{(bc+7ad)x}{32c^2d\left(c+dx^4\right)} + \frac{(3(bc+7ad))\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx}{64c^{5/2}d} + \frac{(3(bc+7ad))\int \frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4}\,dx}{64c^{5/2}d}$$

$$= -\frac{(bc-ad)x}{8cd\left(c+dx^4\right)^2} + \frac{(bc+7ad)x}{32c^2d\left(c+dx^4\right)} + \frac{(3(bc+7ad))\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx}{128c^{5/2}d^{3/2}} + \frac{(3(bc+7ad))\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+}}{128c^{5/2}d^{3/2}}$$

$$= -\frac{(bc-ad)x}{8cd\left(c+dx^4\right)^2} + \frac{(bc+7ad)x}{32c^2d\left(c+dx^4\right)} - \frac{3(bc+7ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{128\sqrt{2}c^{11/4}d^{5/4}} + \frac{3(bc+7ad)\log}{1}$$

$$= -\frac{(bc-ad)x}{8cd\left(c+dx^4\right)^2} + \frac{(bc+7ad)x}{32c^2d\left(c+dx^4\right)} - \frac{3(bc+7ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{64\sqrt{2}c^{11/4}d^{5/4}} + \frac{3(bc+7ad)\tan^{-1}\left(1+\frac{\sqrt{2}}{}\right.}{64\sqrt{2}c^{11/4}d^{5/4}}$$

Mathematica [A] time = 0.206871, size = 243, normalized size = 0.89

$$\frac{-\frac{32c^{7/4}\sqrt[4]{d}x(bc-ad)}{\left(c+dx^4\right)^2} + \frac{8c^{3/4}\sqrt[4]{d}x(7ad+bc)}{c+dx^4} - 3\sqrt{2}(7ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right) + 3\sqrt{2}(7ad+bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\right.}{256c^{11/4}d^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)/(c + d*x^4)^3,x]
```

```
[Out] ((-32*c^(7/4)*d^(1/4)*(b*c - a*d)*x)/(c + d*x^4)^2 + (8*c^(3/4)*d^(1/4)*(b*
c + 7*a*d)*x)/(c + d*x^4) - 6*Sqrt[2]*(b*c + 7*a*d)*ArcTan[1 - (Sqrt[2]*d^(
1/4)*x)/c^(1/4)] + 6*Sqrt[2]*(b*c + 7*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c
^(1/4)] - 3*Sqrt[2]*(b*c + 7*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x +
 Sqrt[d]*x^2] + 3*Sqrt[2]*(b*c + 7*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/
4)*x + Sqrt[d]*x^2])/(256*c^(11/4)*d^(5/4))
```

Maple [A] time = 0.01, size = 314, normalized size = 1.2

$$\frac{1}{\left(dx^4+c\right)^2}\left(\frac{(7\,ad+bc)\,x^5}{32\,c^2} + \frac{(11\,ad-3\,bc)\,x}{32\,cd}\right) + \frac{21\,\sqrt{2}a}{128\,c^3}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right) + \frac{3\,\sqrt{2}b}{128\,c^2d}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)/(d*x^4+c)^3,x)
```

```
[Out] (1/32*(7*a*d+b*c)/c^2*x^5+1/32*(11*a*d-3*b*c)/c/d*x)/(d*x^4+c)^2+21/128/c^3
*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)*a+3/128/c^2/d*(c/d)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)*b+21/128/c^3*(c/d)^(1/4)*2^(1/2
```

```
)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*a+3/128/c^2/d*(c/d)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(c/d)^(1/4)*x-1)*b+21/256/c^3*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/
4)*x*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))*a+3/256/
c^2/d*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2-(
c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/(d*x^4+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.39843, size = 1752, normalized size = 6.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/(d*x^4+c)^3,x, algorithm="fricas")
```

```
[Out] 1/128*(4*(b*c*d + 7*a*d^2)*x^5 + 12*(c^2*d^3*x^8 + 2*c^3*d^2*x^4 + c^4*d)*(
-(b^4*c^4 + 28*a*b^3*c^3*d + 294*a^2*b^2*c^2*d^2 + 1372*a^3*b*c*d^3 + 2401*
a^4*d^4)/(c^11*d^5))^(1/4)*arctan(-(c^8*d^4*x*(-(b^4*c^4 + 28*a*b^3*c^3*d +
 294*a^2*b^2*c^2*d^2 + 1372*a^3*b*c*d^3 + 2401*a^4*d^4)/(c^11*d^5))^(3/4) -
 c^8*d^4*sqrt((c^6*d^2*sqrt(-(b^4*c^4 + 28*a*b^3*c^3*d + 294*a^2*b^2*c^2*d^
2 + 1372*a^3*b*c*d^3 + 2401*a^4*d^4)/(c^11*d^5)) + (b^2*c^2 + 14*a*b*c*d +
49*a^2*d^2)*x^2)/(b^2*c^2 + 14*a*b*c*d + 49*a^2*d^2))*(-(b^4*c^4 + 28*a*b^3
*c^3*d + 294*a^2*b^2*c^2*d^2 + 1372*a^3*b*c*d^3 + 2401*a^4*d^4)/(c^11*d^5))
^(3/4))/(b^3*c^3 + 21*a*b^2*c^2*d + 147*a^2*b*c*d^2 + 343*a^3*d^3)) + 3*(c^
2*d^3*x^8 + 2*c^3*d^2*x^4 + c^4*d)*(-(b^4*c^4 + 28*a*b^3*c^3*d + 294*a^2*b^
2*c^2*d^2 + 1372*a^3*b*c*d^3 + 2401*a^4*d^4)/(c^11*d^5))^(1/4)*log(3*c^3*d*
(-(b^4*c^4 + 28*a*b^3*c^3*d + 294*a^2*b^2*c^2*d^2 + 1372*a^3*b*c*d^3 + 2401
*a^4*d^4)/(c^11*d^5))^(1/4) + 3*(b*c + 7*a*d)*x) - 3*(c^2*d^3*x^8 + 2*c^3*d
^2*x^4 + c^4*d)*(-(b^4*c^4 + 28*a*b^3*c^3*d + 294*a^2*b^2*c^2*d^2 + 1372*a^
3*b*c*d^3 + 2401*a^4*d^4)/(c^11*d^5))^(1/4)*log(-3*c^3*d*(-(b^4*c^4 + 28*a*
b^3*c^3*d + 294*a^2*b^2*c^2*d^2 + 1372*a^3*b*c*d^3 + 2401*a^4*d^4)/(c^11*d^
5))^(1/4) + 3*(b*c + 7*a*d)*x) - 4*(3*b*c^2 - 11*a*c*d)*x)/(c^2*d^3*x^8 + 2
*c^3*d^2*x^4 + c^4*d)
```

Sympy [A] time = 1.6505, size = 151, normalized size = 0.55

$$\frac{x^5\left(7ad^2+bcd\right)+x\left(11acd-3bc^2\right)}{32c^4d+64c^3d^2x^4+32c^2d^3x^8}+\operatorname{RootSum}\Big(268435456t^4c^{11}d^5+194481a^4d^4+111132a^3bcd^3+23814a^2b^2c^2d^2+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)/(d*x**4+c)**3,x)
```

```
[Out] (x**5*(7*a*d**2 + b*c*d) + x*(11*a*c*d - 3*b*c**2))/(32*c**4*d + 64*c**3*d*
*2*x**4 + 32*c**2*d**3*x**8) + RootSum(268435456*_t**4*c**11*d**5 + 194481*
a**4*d**4 + 111132*a**3*b*c*d**3 + 23814*a**2*b**2*c**2*d**2 + 2268*a*b**3*
c**3*d + 81*b**4*c**4, Lambda(_t, _t*log(128*_t*c**3*d/(21*a*d + 3*b*c) + x
)))
```

Giac [A] time = 1.13543, size = 386, normalized size = 1.41

$$\frac{3\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}}bc+7\left(cd^{3}\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{128\,c^{3}d^{2}}+\frac{3\sqrt{2}\left(\left(cd^{3}\right)^{\frac{1}{4}}bc+7\left(cd^{3}\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{128\,c^{3}d^{2}}+\frac{3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/(d*x^4+c)^3,x, algorithm="giac")
```

```
[Out] 3/128*sqrt(2)*((c*d^3)^(1/4)*b*c + 7*(c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*
(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(c^3*d^2) + 3/128*sqrt(2)*((c*d^3)
^(1/4)*b*c + 7*(c*d^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(
1/4))/(c/d)^(1/4))/(c^3*d^2) + 3/256*sqrt(2)*((c*d^3)^(1/4)*b*c + 7*(c*d^3)
^(1/4)*a*d)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(c^3*d^2) - 3/256*
sqrt(2)*((c*d^3)^(1/4)*b*c + 7*(c*d^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x*(c/d)
^(1/4) + sqrt(c/d))/(c^3*d^2) + 1/32*(b*c*d*x^5 + 7*a*d^2*x^5 - 3*b*c^2*x +
 11*a*c*d*x)/((d*x^4 + c)^2*c^2*d)
```

3.153 $\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^4 \, dx$

Optimal. Leaf size=154

$$\frac{1}{17}d^2x^{17}\left(a^2d^2+8abcd+6b^2c^2\right)+\frac{4}{13}cdx^{13}\left(a^2d^2+3abcd+b^2c^2\right)+\frac{1}{9}c^2x^9\left(6a^2d^2+8abcd+b^2c^2\right)+a^2c^4x+\frac{2}{5}ac^3x^5(2ad$$

```
[Out] a^2*c^4*x + (2*a*c^3*(b*c + 2*a*d)*x^5)/5 + (c^2*(b^2*c^2 + 8*a*b*c*d + 6*a
^2*d^2)*x^9)/9 + (4*c*d*(b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^13)/13 + (d^2*(6*
b^2*c^2 + 8*a*b*c*d + a^2*d^2)*x^17)/17 + (2*b*d^3*(2*b*c + a*d)*x^21)/21 +
 (b^2*d^4*x^25)/25
```

Rubi [A] time = 0.113943, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {373}

$$\frac{1}{17}d^2x^{17}\left(a^2d^2+8abcd+6b^2c^2\right)+\frac{4}{13}cdx^{13}\left(a^2d^2+3abcd+b^2c^2\right)+\frac{1}{9}c^2x^9\left(6a^2d^2+8abcd+b^2c^2\right)+a^2c^4x+\frac{2}{5}ac^3x^5(2ad$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^2*(c + d*x^4)^4,x]
```

```
[Out] a^2*c^4*x + (2*a*c^3*(b*c + 2*a*d)*x^5)/5 + (c^2*(b^2*c^2 + 8*a*b*c*d + 6*a
^2*d^2)*x^9)/9 + (4*c*d*(b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^13)/13 + (d^2*(6*
b^2*c^2 + 8*a*b*c*d + a^2*d^2)*x^17)/17 + (2*b*d^3*(2*b*c + a*d)*x^21)/21 +
 (b^2*d^4*x^25)/25
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^4 \, dx &= \int \left(a^2c^4+2ac^3(bc+2ad)x^4+c^2\left(b^2c^2+8abcd+6a^2d^2\right)x^8+4cd\left(b^2c^2+3abcd+a^2d^2\right)x^{1}\right.\\ &= a^2c^4x+\frac{2}{5}ac^3(bc+2ad)x^5+\frac{1}{9}c^2\left(b^2c^2+8abcd+6a^2d^2\right)x^9+\frac{4}{13}cd\left(b^2c^2+3abcd+a^2d^2\right)x\end{aligned}$$

Mathematica [A] time = 0.0314645, size = 154, normalized size = 1.

$$\frac{1}{17}d^2x^{17}\left(a^2d^2+8abcd+6b^2c^2\right)+\frac{4}{13}cdx^{13}\left(a^2d^2+3abcd+b^2c^2\right)+\frac{1}{9}c^2x^9\left(6a^2d^2+8abcd+b^2c^2\right)+a^2c^4x+\frac{2}{5}ac^3x^5(2ad$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^2*(c + d*x^4)^4,x]
```

```
[Out] a^2*c^4*x + (2*a*c^3*(b*c + 2*a*d)*x^5)/5 + (c^2*(b^2*c^2 + 8*a*b*c*d + 6*a
^2*d^2)*x^9)/9 + (4*c*d*(b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^13)/13 + (d^2*(6*
b^2*c^2 + 8*a*b*c*d + a^2*d^2)*x^17)/17 + (2*b*d^3*(2*b*c + a*d)*x^21)/21 +
 (b^2*d^4*x^25)/25
```

Maple [A] time = 0., size = 163, normalized size = 1.1

$$\frac{b^2d^4x^{25}}{25}+\frac{\left(2\,abd^4+4\,b^2cd^3\right)x^{21}}{21}+\frac{\left(a^2d^4+8\,abcd^3+6\,b^2c^2d^2\right)x^{17}}{17}+\frac{\left(4\,a^2cd^3+12\,abc^2d^2+4\,b^2c^3d\right)x^{13}}{13}+\frac{\left(6\,a^2c^2d^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^2*(d*x^4+c)^4,x)
```

```
[Out] 1/25*b^2*d^4*x^25+1/21*(2*a*b*d^4+4*b^2*c*d^3)*x^21+1/17*(a^2*d^4+8*a*b*c*d
^3+6*b^2*c^2*d^2)*x^17+1/13*(4*a^2*c*d^3+12*a*b*c^2*d^2+4*b^2*c^3*d)*x^13+1
/9*(6*a^2*c^2*d^2+8*a*b*c^3*d+b^2*c^4)*x^9+1/5*(4*a^2*c^3*d+2*a*b*c^4)*x^5+
a^2*c^4*x
```

Maxima [A] time = 0.949895, size = 213, normalized size = 1.38

$$\frac{1}{25}\,b^2d^4x^{25}+\frac{2}{21}\left(2\,b^2cd^3+abd^4\right)x^{21}+\frac{1}{17}\left(6\,b^2c^2d^2+8\,abcd^3+a^2d^4\right)x^{17}+\frac{4}{13}\left(b^2c^3d+3\,abc^2d^2+a^2cd^3\right)x^{13}+\frac{1}{9}\left(b^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^4,x, algorithm="maxima")
```

```
[Out] 1/25*b^2*d^4*x^25 + 2/21*(2*b^2*c*d^3 + a*b*d^4)*x^21 + 1/17*(6*b^2*c^2*d^2
 + 8*a*b*c*d^3 + a^2*d^4)*x^17 + 4/13*(b^2*c^3*d + 3*a*b*c^2*d^2 + a^2*c*d^
3)*x^13 + 1/9*(b^2*c^4 + 8*a*b*c^3*d + 6*a^2*c^2*d^2)*x^9 + a^2*c^4*x + 2/5
*(a*b*c^4 + 2*a^2*c^3*d)*x^5
```

Fricas [A] time = 1.11083, size = 413, normalized size = 2.68

$$\frac{1}{25}x^{25}d^4b^2+\frac{4}{21}x^{21}d^3cb^2+\frac{2}{21}x^{21}d^4ba+\frac{6}{17}x^{17}d^2c^2b^2+\frac{8}{17}x^{17}d^3cba+\frac{1}{17}x^{17}d^4a^2+\frac{4}{13}x^{13}dc^3b^2+\frac{12}{13}x^{13}d^2c^2ba+\frac{4}{13}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^4,x, algorithm="fricas")
```

```
[Out] 1/25*x^25*d^4*b^2 + 4/21*x^21*d^3*c*b^2 + 2/21*x^21*d^4*b*a + 6/17*x^17*d^2
*c^2*b^2 + 8/17*x^17*d^3*c*b*a + 1/17*x^17*d^4*a^2 + 4/13*x^13*d*c^3*b^2 +
12/13*x^13*d^2*c^2*b*a + 4/13*x^13*d^3*c*a^2 + 1/9*x^9*c^4*b^2 + 8/9*x^9*d*
c^3*b*a + 2/3*x^9*d^2*c^2*a^2 + 2/5*x^5*c^4*b*a + 4/5*x^5*d*c^3*a^2 + x*c^4
*a^2
```

Sympy [A] time = 0.090721, size = 185, normalized size = 1.2

$$a^2c^4x+\frac{b^2d^4x^{25}}{25}+x^{21}\left(\frac{2abd^4}{21}+\frac{4b^2cd^3}{21}\right)+x^{17}\left(\frac{a^2d^4}{17}+\frac{8abcd^3}{17}+\frac{6b^2c^2d^2}{17}\right)+x^{13}\left(\frac{4a^2cd^3}{13}+\frac{12abc^2d^2}{13}+\frac{4b^2c^3d}{13}\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**2*(d*x**4+c)**4,x)
```

```
[Out] a**2*c**4*x + b**2*d**4*x**25/25 + x**21*(2*a*b*d**4/21 + 4*b**2*c*d**3/21)
 + x**17*(a**2*d**4/17 + 8*a*b*c*d**3/17 + 6*b**2*c**2*d**2/17) + x**13*(4*
a**2*c*d**3/13 + 12*a*b*c**2*d**2/13 + 4*b**2*c**3*d/13) + x**9*(2*a**2*c**
2*d**2/3 + 8*a*b*c**3*d/9 + b**2*c**4/9) + x**5*(4*a**2*c**3*d/5 + 2*a*b*c*
*4/5)
```

Giac [A] time = 1.11187, size = 234, normalized size = 1.52

$$\frac{1}{25}b^2d^4x^{25} + \frac{4}{21}b^2cd^3x^{21} + \frac{2}{21}abd^4x^{21} + \frac{6}{17}b^2c^2d^2x^{17} + \frac{8}{17}abcd^3x^{17} + \frac{1}{17}a^2d^4x^{17} + \frac{4}{13}b^2c^3dx^{13} + \frac{12}{13}abc^2d^2x^{13} + \frac{4}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^4,x, algorithm="giac")
```

```
[Out] 1/25*b^2*d^4*x^25 + 4/21*b^2*c*d^3*x^21 + 2/21*a*b*d^4*x^21 + 6/17*b^2*c^2*
d^2*x^17 + 8/17*a*b*c*d^3*x^17 + 1/17*a^2*d^4*x^17 + 4/13*b^2*c^3*d*x^13 +
12/13*a*b*c^2*d^2*x^13 + 4/13*a^2*c*d^3*x^13 + 1/9*b^2*c^4*x^9 + 8/9*a*b*c^
3*d*x^9 + 2/3*a^2*c^2*d^2*x^9 + 2/5*a*b*c^4*x^5 + 4/5*a^2*c^3*d*x^5 + a^2*c
^4*x
```

3.154 $\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^3 \, dx$

Optimal. Leaf size=122

$$\frac{1}{13}dx^{13}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{9}cx^9\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{5}ac^2x^5(3ad+2bc)+\frac{1}{17}bd^2x^{17}(2ad+3bc)+$$

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^5)/5 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2
*d^2)*x^9)/9 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^13)/13 + (b*d^2*(3*b*
c + 2*a*d)*x^17)/17 + (b^2*d^3*x^21)/21
```

Rubi [A] time = 0.0766227, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {373}

$$\frac{1}{13}dx^{13}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{9}cx^9\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{5}ac^2x^5(3ad+2bc)+\frac{1}{17}bd^2x^{17}(2ad+3bc)+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^2*(c + d*x^4)^3,x]
```

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^5)/5 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2
*d^2)*x^9)/9 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^13)/13 + (b*d^2*(3*b*
c + 2*a*d)*x^17)/17 + (b^2*d^3*x^21)/21
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \left(a+bx^4\right)^2\left(c+dx^4\right)^3 dx=\int \left(a^2c^3+ac^2(2bc+3ad)x^4+c\left(b^2c^2+6abcd+3a^2d^2\right)x^8+d\left(3b^2c^2+6abcd+a^2d^2\right)x$$
$$=a^2c^3x+\frac{1}{5}ac^2(2bc+3ad)x^5+\frac{1}{9}c\left(b^2c^2+6abcd+3a^2d^2\right)x^9+\frac{1}{13}d\left(3b^2c^2+6abcd+a^2d\right.$$

Mathematica [A] time = 0.0222812, size = 122, normalized size = 1.

$$\frac{1}{13}dx^{13}\left(a^2d^2+6abcd+3b^2c^2\right)+\frac{1}{9}cx^9\left(3a^2d^2+6abcd+b^2c^2\right)+a^2c^3x+\frac{1}{5}ac^2x^5(3ad+2bc)+\frac{1}{17}bd^2x^{17}(2ad+3bc)+$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^2*(c + d*x^4)^3,x]
```

```
[Out] a^2*c^3*x + (a*c^2*(2*b*c + 3*a*d)*x^5)/5 + (c*(b^2*c^2 + 6*a*b*c*d + 3*a^2
*d^2)*x^9)/9 + (d*(3*b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^13)/13 + (b*d^2*(3*b*
c + 2*a*d)*x^17)/17 + (b^2*d^3*x^21)/21
```

Maple [A] time = 0.001, size = 125, normalized size = 1.

$$\frac{b^2d^3x^{21}}{21}+\frac{\left(2\,abd^3+3\,b^2cd^2\right)x^{17}}{17}+\frac{\left(a^2d^3+6\,abcd^2+3\,b^2c^2d\right)x^{13}}{13}+\frac{\left(3\,a^2cd^2+6\,abc^2d+b^2c^3\right)x^9}{9}+\frac{\left(3\,a^2c^2d+2\,abc^3\right)x}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^2*(d*x^4+c)^3,x)
```

```
[Out] 1/21*b^2*d^3*x^21+1/17*(2*a*b*d^3+3*b^2*c*d^2)*x^17+1/13*(a^2*d^3+6*a*b*c*d
^2+3*b^2*c^2*d)*x^13+1/9*(3*a^2*c*d^2+6*a*b*c^2*d+b^2*c^3)*x^9+1/5*(3*a^2*c
^2*d+2*a*b*c^3)*x^5+a^2*c^3*x
```

Maxima [A] time = 0.95612, size = 167, normalized size = 1.37

$$\frac{1}{21}\,b^2d^3x^{21}+\frac{1}{17}\left(3\,b^2cd^2+2\,abd^3\right)x^{17}+\frac{1}{13}\left(3\,b^2c^2d+6\,abcd^2+a^2d^3\right)x^{13}+\frac{1}{9}\left(b^2c^3+6\,abc^2d+3\,a^2cd^2\right)x^9+a^2c^3x+\frac{1}{5}\,($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^3,x, algorithm="maxima")
```

```
[Out] 1/21*b^2*d^3*x^21 + 1/17*(3*b^2*c*d^2 + 2*a*b*d^3)*x^17 + 1/13*(3*b^2*c^2*d
 + 6*a*b*c*d^2 + a^2*d^3)*x^13 + 1/9*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*
x^9 + a^2*c^3*x + 1/5*(2*a*b*c^3 + 3*a^2*c^2*d)*x^5
```

Fricas [A] time = 1.02337, size = 315, normalized size = 2.58

$$\frac{1}{21}x^{21}d^3b^2+\frac{3}{17}x^{17}d^2cb^2+\frac{2}{17}x^{17}d^3ba+\frac{3}{13}x^{13}dc^2b^2+\frac{6}{13}x^{13}d^2cba+\frac{1}{13}x^{13}d^3a^2+\frac{1}{9}x^9c^3b^2+\frac{2}{3}x^9dc^2ba+\frac{1}{3}x^9d^2ca^2+\frac{2}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^3,x, algorithm="fricas")
```

```
[Out] 1/21*x^21*d^3*b^2 + 3/17*x^17*d^2*c*b^2 + 2/17*x^17*d^3*b*a + 3/13*x^13*d*c
^2*b^2 + 6/13*x^13*d^2*c*b*a + 1/13*x^13*d^3*a^2 + 1/9*x^9*c^3*b^2 + 2/3*x^
9*d*c^2*b*a + 1/3*x^9*d^2*c*a^2 + 2/5*x^5*c^3*b*a + 3/5*x^5*d*c^2*a^2 + x*c
^3*a^2
```

Sympy [A] time = 0.085057, size = 139, normalized size = 1.14

$$a^2c^3x+\frac{b^2d^3x^{21}}{21}+x^{17}\left(\frac{2abd^3}{17}+\frac{3b^2cd^2}{17}\right)+x^{13}\left(\frac{a^2d^3}{13}+\frac{6abcd^2}{13}+\frac{3b^2c^2d}{13}\right)+x^9\left(\frac{a^2cd^2}{3}+\frac{2abc^2d}{3}+\frac{b^2c^3}{9}\right)+x^5\left(\frac{3a^2c^2d}{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**2*(d*x**4+c)**3,x)
```

```
[Out] a**2*c**3*x + b**2*d**3*x**21/21 + x**17*(2*a*b*d**3/17 + 3*b**2*c*d**2/17)
 + x**13*(a**2*d**3/13 + 6*a*b*c*d**2/13 + 3*b**2*c**2*d/13) + x**9*(a**2*c
*d**2/3 + 2*a*b*c**2*d/3 + b**2*c**3/9) + x**5*(3*a**2*c**2*d/5 + 2*a*b*c**
```

```
3/5)
```

Giac [A] time = 1.10903, size = 178, normalized size = 1.46

$$\frac{1}{21}b^2d^3x^{21} + \frac{3}{17}b^2cd^2x^{17} + \frac{2}{17}abd^3x^{17} + \frac{3}{13}b^2c^2dx^{13} + \frac{6}{13}abcd^2x^{13} + \frac{1}{13}a^2d^3x^{13} + \frac{1}{9}b^2c^3x^9 + \frac{2}{3}abc^2dx^9 + \frac{1}{3}a^2cd^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^3,x, algorithm="giac")
```

```
[Out] 1/21*b^2*d^3*x^21 + 3/17*b^2*c*d^2*x^17 + 2/17*a*b*d^3*x^17 + 3/13*b^2*c^2*
d*x^13 + 6/13*a*b*c*d^2*x^13 + 1/13*a^2*d^3*x^13 + 1/9*b^2*c^3*x^9 + 2/3*a*
b*c^2*d*x^9 + 1/3*a^2*c*d^2*x^9 + 2/5*a*b*c^3*x^5 + 3/5*a^2*c^2*d*x^5 + a^2
*c^3*x
```

3.155 $\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^2 \, dx$

Optimal. Leaf size=82

$$\frac{1}{9}x^9\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{2}{13}bdx^{13}(ad+bc)+\frac{2}{5}acx^5(ad+bc)+\frac{1}{17}b^2d^2x^{17}$$

```
[Out] a^2*c^2*x + (2*a*c*(b*c + a*d)*x^5)/5 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^
9)/9 + (2*b*d*(b*c + a*d)*x^13)/13 + (b^2*d^2*x^17)/17
```

Rubi [A] time = 0.0491069, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {373}

$$\frac{1}{9}x^9\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{2}{13}bdx^{13}(ad+bc)+\frac{2}{5}acx^5(ad+bc)+\frac{1}{17}b^2d^2x^{17}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^2*(c + d*x^4)^2,x]
```

```
[Out] a^2*c^2*x + (2*a*c*(b*c + a*d)*x^5)/5 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^
9)/9 + (2*b*d*(b*c + a*d)*x^13)/13 + (b^2*d^2*x^17)/17
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^4\right)^2\left(c+dx^4\right)^2 dx &= \int \left(a^2c^2+2ac(bc+ad)x^4+\left(b^2c^2+4abcd+a^2d^2\right)x^8+2bd(bc+ad)x^{12}+b^2d^2x^{16}\right) dx\\ &= a^2c^2x+\frac{2}{5}ac(bc+ad)x^5+\frac{1}{9}\left(b^2c^2+4abcd+a^2d^2\right)x^9+\frac{2}{13}bd(bc+ad)x^{13}+\frac{1}{17}b^2d^2x^{17}\end{aligned}$$

Mathematica [A] time = 0.0166878, size = 82, normalized size = 1.

$$\frac{1}{9}x^9\left(a^2d^2+4abcd+b^2c^2\right)+a^2c^2x+\frac{2}{13}bdx^{13}(ad+bc)+\frac{2}{5}acx^5(ad+bc)+\frac{1}{17}b^2d^2x^{17}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^2*(c + d*x^4)^2,x]
```

```
[Out] a^2*c^2*x + (2*a*c*(b*c + a*d)*x^5)/5 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^
9)/9 + (2*b*d*(b*c + a*d)*x^13)/13 + (b^2*d^2*x^17)/17
```

Maple [A] time = 0.001, size = 87, normalized size = 1.1

$$\frac{b^2d^2x^{17}}{17}+\frac{\left(2\,abd^2+2\,b^2cd\right)x^{13}}{13}+\frac{\left(a^2d^2+4\,cabd+b^2c^2\right)x^9}{9}+\frac{\left(2\,a^2cd+2\,abc^2\right)x^5}{5}+a^2c^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^2*(d*x^4+c)^2,x)
```

```
[Out] 1/17*b^2*d^2*x^17+1/13*(2*a*b*d^2+2*b^2*c*d)*x^13+1/9*(a^2*d^2+4*a*b*c*d+b^
2*c^2)*x^9+1/5*(2*a^2*c*d+2*a*b*c^2)*x^5+a^2*c^2*x
```

Maxima [A] time = 0.94481, size = 111, normalized size = 1.35

$$\frac{1}{17}\,b^{2}d^{2}x^{17}+\frac{2}{13}\left(b^{2}cd+abd^{2}\right)x^{13}+\frac{1}{9}\left(b^{2}c^{2}+4\,abcd+a^{2}d^{2}\right)x^{9}+\frac{2}{5}\left(abc^{2}+a^{2}cd\right)x^{5}+a^{2}c^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] 1/17*b^2*d^2*x^17 + 2/13*(b^2*c*d + a*b*d^2)*x^13 + 1/9*(b^2*c^2 + 4*a*b*c*
d + a^2*d^2)*x^9 + 2/5*(a*b*c^2 + a^2*c*d)*x^5 + a^2*c^2*x
```

Fricas [A] time = 1.08849, size = 217, normalized size = 2.65

$$\frac{1}{17}x^{17}d^{2}b^{2}+\frac{2}{13}x^{13}dcb^{2}+\frac{2}{13}x^{13}d^{2}ba+\frac{1}{9}x^{9}c^{2}b^{2}+\frac{4}{9}x^{9}dcba+\frac{1}{9}x^{9}d^{2}a^{2}+\frac{2}{5}x^{5}c^{2}ba+\frac{2}{5}x^{5}dca^{2}+xc^{2}a^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] 1/17*x^17*d^2*b^2 + 2/13*x^13*d*c*b^2 + 2/13*x^13*d^2*b*a + 1/9*x^9*c^2*b^2
 + 4/9*x^9*d*c*b*a + 1/9*x^9*d^2*a^2 + 2/5*x^5*c^2*b*a + 2/5*x^5*d*c*a^2 +
x*c^2*a^2
```

Sympy [A] time = 0.076386, size = 97, normalized size = 1.18

$$a^{2}c^{2}x+\frac{b^{2}d^{2}x^{17}}{17}+x^{13}\left(\frac{2abd^{2}}{13}+\frac{2b^{2}cd}{13}\right)+x^{9}\left(\frac{a^{2}d^{2}}{9}+\frac{4abcd}{9}+\frac{b^{2}c^{2}}{9}\right)+x^{5}\left(\frac{2a^{2}cd}{5}+\frac{2abc^{2}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**2*(d*x**4+c)**2,x)
```

```
[Out] a**2*c**2*x + b**2*d**2*x**17/17 + x**13*(2*a*b*d**2/13 + 2*b**2*c*d/13) +
x**9*(a**2*d**2/9 + 4*a*b*c*d/9 + b**2*c**2/9) + x**5*(2*a**2*c*d/5 + 2*a*b
*c**2/5)
```

Giac [A] time = 1.09573, size = 123, normalized size = 1.5

$$\frac{1}{17}\,b^{2}d^{2}x^{17}+\frac{2}{13}\,b^{2}cdx^{13}+\frac{2}{13}\,abd^{2}x^{13}+\frac{1}{9}\,b^{2}c^{2}x^{9}+\frac{4}{9}\,abcdx^{9}+\frac{1}{9}\,a^{2}d^{2}x^{9}+\frac{2}{5}\,abc^{2}x^{5}+\frac{2}{5}\,a^{2}cdx^{5}+a^{2}c^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] 1/17*b^2*d^2*x^17 + 2/13*b^2*c*d*x^13 + 2/13*a*b*d^2*x^13 + 1/9*b^2*c^2*x^9
 + 4/9*a*b*c*d*x^9 + 1/9*a^2*d^2*x^9 + 2/5*a*b*c^2*x^5 + 2/5*a^2*c*d*x^5 +
a^2*c^2*x
```

3.156 $\int (a+bx^4)^2 (c+dx^4)\, dx$

Optimal. Leaf size=50

$$a^2cx+\frac{1}{9}bx^9(2ad+bc)+\frac{1}{5}ax^5(ad+2bc)+\frac{1}{13}b^2dx^{13}$$

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^5)/5 + (b*(b*c + 2*a*d)*x^9)/9 + (b^2*d*x^13)/
13
```

Rubi [A] time = 0.0302635, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$a^2cx+\frac{1}{9}bx^9(2ad+bc)+\frac{1}{5}ax^5(ad+2bc)+\frac{1}{13}b^2dx^{13}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^2*(c + d*x^4),x]
```

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^5)/5 + (b*(b*c + 2*a*d)*x^9)/9 + (b^2*d*x^13)/
13
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^4)^2 (c+dx^4)\, dx &= \int \left(a^2c+a(2bc+ad)x^4+b(bc+2ad)x^8+b^2dx^{12}\right) dx\\ &= a^2cx+\frac{1}{5}a(2bc+ad)x^5+\frac{1}{9}b(bc+2ad)x^9+\frac{1}{13}b^2dx^{13}\end{aligned}$$

Mathematica [A] time = 0.007936, size = 50, normalized size = 1.

$$a^2cx+\frac{1}{9}bx^9(2ad+bc)+\frac{1}{5}ax^5(ad+2bc)+\frac{1}{13}b^2dx^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^2*(c + d*x^4),x]
```

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^5)/5 + (b*(b*c + 2*a*d)*x^9)/9 + (b^2*d*x^13)/
13
```

Maple [A] time = 0.001, size = 49, normalized size = 1.

$$\frac{b^2dx^{13}}{13}+\frac{\left(2\,abd+b^2c\right)x^9}{9}+\frac{\left(a^2d+2\,abc\right)x^5}{5}+a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^2*(d*x^4+c),x)
```

```
[Out] 1/13*b^2*d*x^13+1/9*(2*a*b*d+b^2*c)*x^9+1/5*(a^2*d+2*a*b*c)*x^5+a^2*c*x
```

Maxima [A] time = 0.971098, size = 65, normalized size = 1.3

$$\frac{1}{13}b^2dx^{13} + \frac{1}{9}\left(b^2c + 2\,abd\right)x^9 + \frac{1}{5}\left(2\,abc + a^2d\right)x^5 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c),x, algorithm="maxima")
```

```
[Out] 1/13*b^2*d*x^13 + 1/9*(b^2*c + 2*a*b*d)*x^9 + 1/5*(2*a*b*c + a^2*d)*x^5 + a
^2*c*x
```

Fricas [A] time = 1.07308, size = 123, normalized size = 2.46

$$\frac{1}{13}x^{13}db^2 + \frac{1}{9}x^9cb^2 + \frac{2}{9}x^9dba + \frac{2}{5}x^5cba + \frac{1}{5}x^5da^2 + xca^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c),x, algorithm="fricas")
```

```
[Out] 1/13*x^13*d*b^2 + 1/9*x^9*c*b^2 + 2/9*x^9*d*b*a + 2/5*x^5*c*b*a + 1/5*x^5*d
*a^2 + x*c*a^2
```

Sympy [A] time = 0.130334, size = 53, normalized size = 1.06

$$a^2cx + \frac{b^2dx^{13}}{13} + x^9\left(\frac{2abd}{9} + \frac{b^2c}{9}\right) + x^5\left(\frac{a^2d}{5} + \frac{2abc}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**2*(d*x**4+c),x)
```

```
[Out] a**2*c*x + b**2*d*x**13/13 + x**9*(2*a*b*d/9 + b**2*c/9) + x**5*(a**2*d/5 +
 2*a*b*c/5)
```

Giac [A] time = 1.09141, size = 68, normalized size = 1.36

$$\frac{1}{13}b^2dx^{13} + \frac{1}{9}b^2cx^9 + \frac{2}{9}abdx^9 + \frac{2}{5}abcx^5 + \frac{1}{5}a^2dx^5 + a^2cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/13*b^2*d*x^13 + 1/9*b^2*c*x^9 + 2/9*a*b*d*x^9 + 2/5*a*b*c*x^5 + 1/5*a^2*d
*x^5 + a^2*c*x
```

3.157 $\int \frac{(a+bx^4)^2}{c+dx^4}\,dx$

Optimal. Leaf size=253

$$-\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}d^{9/4}}+\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}d^{9/4}}-\frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}d^{9/4}}-$$

```
[Out] -((b*(b*c - 2*a*d)*x)/d^2) + (b^2*x^5)/(5*d) - ((b*c - a*d)^2*ArcTan[1 - (S
qrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(3/4)*d^(9/4)) + ((b*c - a*d)^2*Ar
cTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(3/4)*d^(9/4)) - ((b*c
- a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]
*c^(3/4)*d^(9/4)) + ((b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x
+ Sqrt[d]*x^2])/(4*Sqrt[2]*c^(3/4)*d^(9/4))
```

Rubi [A] time = 0.193657, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {390, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}d^{9/4}}+\frac{(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}d^{9/4}}-\frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}d^{9/4}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^2/(c + d*x^4),x]
```

```
[Out] -((b*(b*c - 2*a*d)*x)/d^2) + (b^2*x^5)/(5*d) - ((b*c - a*d)^2*ArcTan[1 - (S
qrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(3/4)*d^(9/4)) + ((b*c - a*d)^2*Ar
cTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[2]*c^(3/4)*d^(9/4)) - ((b*c
- a*d)^2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]
*c^(3/4)*d^(9/4)) + ((b*c - a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x
+ Sqrt[d]*x^2])/(4*Sqrt[2]*c^(3/4)*d^(9/4))
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^4\right)^2}{c+dx^4}\,dx = \int \left(-\frac{b(bc-2ad)}{d^2}+\frac{b^2x^4}{d}+\frac{b^2c^2-2abcd+a^2d^2}{d^2\left(c+dx^4\right)}\right)dx$$

$$= -\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^5}{5d}+\frac{(bc-ad)^2\int\frac{1}{c+dx^4}\,dx}{d^2}$$

$$= -\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^5}{5d}+\frac{(bc-ad)^2\int\frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx}{2\sqrt{c}d^2}+\frac{(bc-ad)^2\int\frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4}\,dx}{2\sqrt{c}d^2}$$

$$= -\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^5}{5d}+\frac{(bc-ad)^2\int\frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx}{4\sqrt{c}d^{5/2}}+\frac{(bc-ad)^2\int\frac{1}{\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx}{4\sqrt{c}d^{5/2}}-\frac{(bc-ad)^2\ \cdots}{4\cdots}$$

$$= -\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^5}{5d}-\frac{(bc-ad)^2\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}d^{9/4}}+\frac{(bc-ad)^2\log\left(\sqrt{c}+\sqrt{2}\sqrt[4]{c}\sqrt[4]{\cdots}\right.}{4\sqrt{2}c^{3/4}d^{9/4}}$$

$$= -\frac{b(bc-2ad)x}{d^2}+\frac{b^2x^5}{5d}-\frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}d^{9/4}}+\frac{(bc-ad)^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}d^{9/4}}-\frac{(bc-ad\cdots}{\cdots}$$

Mathematica [A] time = 0.109429, size = 231, normalized size = 0.91

$$\frac{-40bc^{3/4}\sqrt[4]{d}x(bc-2ad)-5\sqrt{2}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)+5\sqrt{2}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x\cdots\right.}{40c^{3/4}d^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^2/(c + d*x^4),x]
```

```
[Out] (-40*b*c^(3/4)*d^(1/4)*(b*c - 2*a*d)*x + 8*b^2*c^(3/4)*d^(5/4)*x^5 - 10*Sqr
t[2]*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 10*Sqrt[2]*(b*
c - a*d)^2*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - 5*Sqrt[2]*(b*c - a*d)^
2*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2] + 5*Sqrt[2]*(b*c -
 a*d)^2*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(40*c^(3/4)
*d^(9/4))
```

Maple [B] time = 0.003, size = 436, normalized size = 1.7

$$\frac{b^2x^5}{5d}+2\frac{xab}{d}-\frac{b^2xc}{d^2}+\frac{\sqrt{2}a^2}{4c}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)-\frac{\sqrt{2}ab}{2d}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)+\frac{c\sqrt{2}b^2}{4d^2}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^2/(d*x^4+c),x)
```

```
[Out] 1/5*b^2*x^5/d+2*b/d*a*x-b^2/d^2*x*c+1/4*(c/d)^(1/4)/c*2^(1/2)*arctan(2^(1/2
)/(c/d)^(1/4)*x+1)*a^2-1/2/d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)
*x+1)*a*b+1/4/d^2*(c/d)^(1/4)*c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)*b^2
+1/4*(c/d)^(1/4)/c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*a^2-1/2/d*(c/d)^
(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*a*b+1/4/d^2*(c/d)^(1/4)*c*2^(
1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*b^2+1/8*(c/d)^(1/4)/c*2^(1/2)*ln((x^2+
(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))
*a^2-1/4/d*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(
x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))*a*b+1/8/d^2*(c/d)^(1/4)*c*2^(1/2)*l
n((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^
(1/2)))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.50696, size = 2547, normalized size = 10.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] 1/20*(4*b^2*d*x^5 + 20*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2
 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2
*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(1/4)*arctan(-(c^2*d^7*x*(-(
b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*
b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8
```

```
*d^8)/(c^3*d^9))^(3/4) - c^2*d^7*sqrt((c^2*d^4*sqrt(-(b^8*c^8 - 8*a*b^7*c^7
*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*
b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9)) + (b
^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*x^2)/
(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4))*(-
(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4
*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^
8*d^8)/(c^3*d^9))^(3/4))/(b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20
*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)) + 5*d^2*(
-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^
4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a
^8*d^8)/(c^3*d^9))^(1/4)*log(c*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*
c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28
*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(1/4) + (b^2*c^2 - 2
*a*b*c*d + a^2*d^2)*x) - 5*d^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*
d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6
*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(c^3*d^9))^(1/4)*log(-c*d^2*(-(b^8*
c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*
c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8
)/(c^3*d^9))^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x) - 20*(b^2*c - 2*a*b
*d)*x)/d^2
```

Sympy [A] time = 1.41845, size = 187, normalized size = 0.74

$$\frac{b^2x^5}{5d} + \text{RootSum}\left(256t^4c^3d^9 + a^8d^8 - 8a^7bcd^7 + 28a^6b^2c^2d^6 - 56a^5b^3c^3d^5 + 70a^4b^4c^4d^4 - 56a^3b^5c^5d^3 + 28a^2b^6c^6d^2 \cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**2/(d*x**4+c),x)
```

```
[Out] b**2*x**5/(5*d) + RootSum(256*_t**4*c**3*d**9 + a**8*d**8 - 8*a**7*b*c*d**7
 + 28*a**6*b**2*c**2*d**6 - 56*a**5*b**3*c**3*d**5 + 70*a**4*b**4*c**4*d**4
 - 56*a**3*b**5*c**5*d**3 + 28*a**2*b**6*c**6*d**2 - 8*a*b**7*c**7*d + b**8
*c**8, Lambda(_t, _t*log(4*_t*c*d**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) +
x))) + x*(2*a*b*d - b**2*c)/d**2
```

Giac [A] time = 1.09653, size = 477, normalized size = 1.89

$$\frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}b^2c^2 - 2\left(cd^3\right)^{\frac{1}{4}}abcd + \left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{4cd^3} + \frac{\sqrt{2}\left(\left(cd^3\right)^{\frac{1}{4}}b^2c^2 - 2\left(cd^3\right)^{\frac{1}{4}}abcd + \left(cd^3\right)^{\frac{1}{4}}a \cdots\right.}{4cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(1/4
)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(c*d
^3) + 1/4*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3
)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4)
)/(c*d^3) + 1/8*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d +
(c*d^3)^(1/4)*a^2*d^2)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(c*d^3)
```

```
 - 1/8*sqrt(2)*((c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d + (c*d^3)^(
1/4)*a^2*d^2)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(c*d^3) + 1/5*(b
^2*d^4*x^5 - 5*b^2*c*d^3*x + 10*a*b*d^4*x)/d^5
```

3.158 $\int \frac{(a+bx^4)^2}{(c+dx^4)^2}\,dx$

Optimal. Leaf size=291

$$\frac{(bc-ad)(3ad+5bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}d^{9/4}}-\frac{(bc-ad)(3ad+5bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}d^{9/4}}+\frac{(bc-ad}{}$$

```
[Out] (b^2*x)/d^2 + ((b*c - a*d)^2*x)/(4*c*d^2*(c + d*x^4)) + ((b*c - a*d)*(5*b*c
 + 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(7/4)*d^(9/
4)) - ((b*c - a*d)*(5*b*c + 3*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])
/(8*Sqrt[2]*c^(7/4)*d^(9/4)) + ((b*c - a*d)*(5*b*c + 3*a*d)*Log[Sqrt[c] - S
qrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(16*Sqrt[2]*c^(7/4)*d^(9/4)) - ((b
*c - a*d)*(5*b*c + 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]
*x^2])/(16*Sqrt[2]*c^(7/4)*d^(9/4))
```

Rubi [A] time = 0.365765, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {390, 385, 211, 1165, 628, 1162, 617, 204}

$$\frac{(bc-ad)(3ad+5bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}d^{9/4}}-\frac{(bc-ad)(3ad+5bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}d^{9/4}}+\frac{(bc-ad}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^2/(c + d*x^4)^2,x]
```


```
[Out] (b^2*x)/d^2 + ((b*c - a*d)^2*x)/(4*c*d^2*(c + d*x^4)) + ((b*c - a*d)*(5*b*c
 + 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(7/4)*d^(9/
4)) - ((b*c - a*d)*(5*b*c + 3*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)])
/(8*Sqrt[2]*c^(7/4)*d^(9/4)) + ((b*c - a*d)*(5*b*c + 3*a*d)*Log[Sqrt[c] - S
qrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(16*Sqrt[2]*c^(7/4)*d^(9/4)) - ((b
*c - a*d)*(5*b*c + 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]
*x^2])/(16*Sqrt[2]*c^(7/4)*d^(9/4))
```


Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(a+bx^4\right)^2}{\left(c+dx^4\right)^2}\,dx = \int \left(\frac{b^2}{d^2}-\frac{b^2c^2-a^2d^2+2bd(bc-ad)x^4}{d^2\left(c+dx^4\right)^2}\right)dx$$

$$= \frac{b^2x}{d^2}-\frac{\int \frac{b^2c^2-a^2d^2+2bd(bc-ad)x^4}{\left(c+dx^4\right)^2}\,dx}{d^2}$$

$$= \frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{4cd^2\left(c+dx^4\right)}-\frac{((bc-ad)(5bc+3ad))\int \frac{1}{c+dx^4}\,dx}{4cd^2}$$

$$= \frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{4cd^2\left(c+dx^4\right)}-\frac{((bc-ad)(5bc+3ad))\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx}{8c^{3/2}d^2}-\frac{((bc-ad)(5bc+3ad))\int \frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4}\,dx}{8c^{3/2}d^2}$$

$$= \frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{4cd^2\left(c+dx^4\right)}-\frac{((bc-ad)(5bc+3ad))\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx}{16c^{3/2}d^{5/2}}-\frac{((bc-ad)(5bc+3ad))\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}}{}}}{16c^{3/2}d^{5/2}}$$

$$= \frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{4cd^2\left(c+dx^4\right)}+\frac{(bc-ad)(5bc+3ad)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{d}x^2\right)}{16\sqrt{2}c^{7/4}d^{9/4}}-\frac{(bc-ad)(5bc+3ad}{}$$

$$= \frac{b^2x}{d^2}+\frac{(bc-ad)^2x}{4cd^2\left(c+dx^4\right)}+\frac{(bc-ad)(5bc+3ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}d^{9/4}}-\frac{(bc-ad)(5bc+3ad)\tan^{-1}\left(1+\right.}{8\sqrt{2}c^{7/4}d^{9/4}}$$

Mathematica [A] time = 0.172948, size = 298, normalized size = 1.02

$$\frac{\frac{\sqrt{2}\left(-3a^2d^2-2abcd+5b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{c^{7/4}}-\frac{\sqrt{2}\left(-3a^2d^2-2abcd+5b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{c^{7/4}}+\frac{2\sqrt{2}\left(-3a^2d^2-2abcd+5b^2c^2\right)\tan^{-1}\left(1\right.}{c^{7/4}}}{32d^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^2/(c + d*x^4)^2,x]
```

```
[Out] (32*b^2*d^(1/4)*x + (8*d^(1/4)*(b*c - a*d)^2*x)/(c*(c + d*x^4)) + (2*Sqrt[2
]*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4
)])/c^(7/4) - (2*Sqrt[2]*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*ArcTan[1 + (Sq
rt[2]*d^(1/4)*x)/c^(1/4)])/c^(7/4) + (Sqrt[2]*(5*b^2*c^2 - 2*a*b*c*d - 3*a^
2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/c^(7/4) - (S
qrt[2]*(5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^
(1/4)*x + Sqrt[d]*x^2])/c^(7/4))/(32*d^(9/4))
```

Maple [B] time = 0.01, size = 475, normalized size = 1.6

$$\frac{b^2x}{d^2}+\frac{a^2x}{4c\left(dx^4+c\right)}-\frac{xab}{2d\left(dx^4+c\right)}+\frac{cxb^2}{4d^2\left(dx^4+c\right)}+\frac{3\sqrt{2}a^2}{16c^2}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right)+\frac{\sqrt{2}ab}{8cd}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^2/(d*x^4+c)^2,x)
```

```
[Out] b^2*x/d^2+1/4/c*x/(d*x^4+c)*a^2-1/2/d*x/(d*x^4+c)*a*b+1/4/d^2*c*x/(d*x^4+c)
*b^2+3/16/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*a^2+1/8/d
/c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*a*b-5/16/d^2*(c/d)^(
1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*b^2+3/32/c^2*(c/d)^(1/4)*2^(1/
2)*ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c
/d)^(1/2)))*a^2+1/16/d/c*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*x*2^(1/2)+
(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))*a*b-5/32/d^2*(c/d)^(1
/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2
^(1/2)+(c/d)^(1/2)))*b^2+3/16/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^
(1/4)*x+1)*a^2+1/8/d/c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)*
a*b-5/16/d^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.57329, size = 2909, normalized size = 10.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(16*b^2*c*d*x^5 + 4*(c*d^3*x^4 + c^2*d^2)*(-(625*b^8*c^8 - 1000*a*b^7*
c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 -
984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(
c^7*d^9))^(1/4)*arctan((c^5*d^7*x*(-(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a
^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c
^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9))^(3/
4) - c^5*d^7*sqrt((c^4*d^4*sqrt(-(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*
b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*
d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9)) + (25*
b^4*c^4 - 20*a*b^3*c^3*d - 26*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + 9*a^4*d^4)
*x^2)/(25*b^4*c^4 - 20*a*b^3*c^3*d - 26*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 +
9*a^4*d^4))*(-(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*
a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c
^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9))^(3/4))/(125*b^6*c^6 - 150
*a*b^5*c^5*d - 165*a^2*b^4*c^4*d^2 + 172*a^3*b^3*c^3*d^3 + 99*a^4*b^2*c^2*d
^4 - 54*a^5*b*c*d^5 - 27*a^6*d^6)) + (c*d^3*x^4 + c^2*d^2)*(-(625*b^8*c^8 -
 1000*a*b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^
4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 8
1*a^8*d^8)/(c^7*d^9))^(1/4)*log(c^2*d^2*(-(625*b^8*c^8 - 1000*a*b^7*c^7*d -
 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5
*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9
))^(1/4) - (5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*x) - (c*d^3*x^4 + c^2*d^2)*(
-(625*b^8*c^8 - 1000*a*b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d
^3 + 646*a^4*b^4*c^4*d^4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*
a^7*b*c*d^7 + 81*a^8*d^8)/(c^7*d^9))^(1/4)*log(-c^2*d^2*(-(625*b^8*c^8 - 10
```

```
00*a*b^7*c^7*d - 900*a^2*b^6*c^6*d^2 + 1640*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c
^4*d^4 - 984*a^5*b^3*c^3*d^5 - 324*a^6*b^2*c^2*d^6 + 216*a^7*b*c*d^7 + 81*a
^8*d^8)/(c^7*d^9))^(1/4) - (5*b^2*c^2 - 2*a*b*c*d - 3*a^2*d^2)*x) + 4*(5*b^
2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/(c*d^3*x^4 + c^2*d^2)
```

Sympy [A] time = 2.74498, size = 219, normalized size = 0.75

$$\frac{b^2x}{d^2} + \frac{x\left(a^2d^2 - 2abcd + b^2c^2\right)}{4c^2d^2 + 4cd^3x^4} + \mathrm{RootSum}\left(65536t^4c^7d^9 + 81a^8d^8 + 216a^7bcd^7 - 324a^6b^2c^2d^6 - 984a^5b^3c^3d^5 + 646a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**2/(d*x**4+c)**2,x)
```

```
[Out] b**2*x/d**2 + x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(4*c**2*d**2 + 4*c*d**3
*x**4) + RootSum(65536*_t**4*c**7*d**9 + 81*a**8*d**8 + 216*a**7*b*c*d**7 -
 324*a**6*b**2*c**2*d**6 - 984*a**5*b**3*c**3*d**5 + 646*a**4*b**4*c**4*d**
4 + 1640*a**3*b**5*c**5*d**3 - 900*a**2*b**6*c**6*d**2 - 1000*a*b**7*c**7*d
 + 625*b**8*c**8, Lambda(_t, _t*log(16*_t*c**2*d**2/(3*a**2*d**2 + 2*a*b*c*
d - 5*b**2*c**2) + x)))
```

Giac [A] time = 1.13484, size = 508, normalized size = 1.75

$$\frac{b^2x}{d^2} - \frac{\sqrt{2}\left(5\left(cd^3\right)^{\frac{1}{4}}b^2c^2 - 2\left(cd^3\right)^{\frac{1}{4}}abcd - 3\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{16\,c^2d^3} - \frac{\sqrt{2}\left(5\left(cd^3\right)^{\frac{1}{4}}b^2c^2 - 2\left(cd^3\right)^{\frac{1}{4}}abc\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] b^2*x/d^2 - 1/16*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^(1/4)*a*b*c*d
 - 3*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/
(c/d)^(1/4))/(c^2*d^3) - 1/16*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 - 2*(c*d^3)^
(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*
(c/d)^(1/4))/(c/d)^(1/4))/(c^2*d^3) - 1/32*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2
 - 2*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*log(x^2 + sqrt(2)*x*(
c/d)^(1/4) + sqrt(c/d))/(c^2*d^3) + 1/32*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 -
 2*(c*d^3)^(1/4)*a*b*c*d - 3*(c*d^3)^(1/4)*a^2*d^2)*log(x^2 - sqrt(2)*x*(c/
d)^(1/4) + sqrt(c/d))/(c^2*d^3) + 1/4*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)
/((d*x^4 + c)*c*d^2)
```

3.159 $\int \frac{(a+bx^4)^2}{(c+dx^4)^3} dx$

Optimal. Leaf size=349

$$-\frac{\left(21a^2d^2+6abcd+5b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{128\sqrt{2}c^{11/4}d^{9/4}}+\frac{\left(21a^2d^2+6abcd+5b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{128\sqrt{2}c^{11/4}d^{9/4}}-$$

```
[Out] -((b*c - a*d)*x*(a + b*x^4))/(8*c*d*(c + d*x^4)^2) - ((b*c - a*d)*(5*b*c +
7*a*d)*x)/(32*c^2*d^2*(c + d*x^4)) - ((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*
ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(64*Sqrt[2]*c^(11/4)*d^(9/4)) + ((
5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)]
)/(64*Sqrt[2]*c^(11/4)*d^(9/4)) - ((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*Log
[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(128*Sqrt[2]*c^(11/4)*
d^(9/4)) + ((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1
/4)*d^(1/4)*x + Sqrt[d]*x^2])/(128*Sqrt[2]*c^(11/4)*d^(9/4))
```

Rubi [A] time = 0.265617, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {413, 385, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\left(21a^2d^2+6abcd+5b^2c^2\right)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{128\sqrt{2}c^{11/4}d^{9/4}}+\frac{\left(21a^2d^2+6abcd+5b^2c^2\right)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{128\sqrt{2}c^{11/4}d^{9/4}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^2/(c + d*x^4)^3,x]
```


```
[Out] -((b*c - a*d)*x*(a + b*x^4))/(8*c*d*(c + d*x^4)^2) - ((b*c - a*d)*(5*b*c +
7*a*d)*x)/(32*c^2*d^2*(c + d*x^4)) - ((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*
ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(64*Sqrt[2]*c^(11/4)*d^(9/4)) + ((
5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)]
)/(64*Sqrt[2]*c^(11/4)*d^(9/4)) - ((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*Log
[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(128*Sqrt[2]*c^(11/4)*
d^(9/4)) + ((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*Log[Sqrt[c] + Sqrt[2]*c^(1
/4)*d^(1/4)*x + Sqrt[d]*x^2])/(128*Sqrt[2]*c^(11/4)*d^(9/4))
```


Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
```

```
p, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(a+bx^4)^2}{(c+dx^4)^3}\,dx = -\frac{(bc-ad)x(a+bx^4)}{8cd(c+dx^4)^2} + \frac{\int \frac{a(bc+7ad)+b(5bc+3ad)x^4}{(c+dx^4)^2}\,dx}{8cd}$$

$$= -\frac{(bc-ad)x(a+bx^4)}{8cd(c+dx^4)^2} - \frac{(bc-ad)(5bc+7ad)x}{32c^2d^2(c+dx^4)} + \frac{(5b^2c^2+6abcd+21a^2d^2)\int \frac{1}{c+dx^4}\,dx}{32c^2d^2}$$

$$= -\frac{(bc-ad)x(a+bx^4)}{8cd(c+dx^4)^2} - \frac{(bc-ad)(5bc+7ad)x}{32c^2d^2(c+dx^4)} + \frac{(5b^2c^2+6abcd+21a^2d^2)\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx}{64c^{5/2}d^2} + \frac{(5b^2c^2+}{}$$

$$= -\frac{(bc-ad)x(a+bx^4)}{8cd(c+dx^4)^2} - \frac{(bc-ad)(5bc+7ad)x}{32c^2d^2(c+dx^4)} + \frac{(5b^2c^2+6abcd+21a^2d^2)\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx}{128c^{5/2}d^{5/2}} + \frac{(5b^2}{}$$

$$= -\frac{(bc-ad)x(a+bx^4)}{8cd(c+dx^4)^2} - \frac{(bc-ad)(5bc+7ad)x}{32c^2d^2(c+dx^4)} - \frac{(5b^2c^2+6abcd+21a^2d^2)\log\left(\sqrt{c}-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{}\right.}{128\sqrt{2}c^{11/4}d^{9/4}}$$

$$= -\frac{(bc-ad)x(a+bx^4)}{8cd(c+dx^4)^2} - \frac{(bc-ad)(5bc+7ad)x}{32c^2d^2(c+dx^4)} - \frac{(5b^2c^2+6abcd+21a^2d^2)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{64\sqrt{2}c^{11/4}d^{9/4}} + \frac{(5b}{}$$

Mathematica [A] time = 0.186252, size = 319, normalized size = 0.91

$$\frac{-\frac{8c^{3/4}\sqrt[4]{d}x(-7a^2d^2-2abcd+9b^2c^2)}{c+dx^4} - \sqrt{2}(21a^2d^2+6abcd+5b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right) + \sqrt{2}(21a^2d^2+6abcd+5b^2c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^2/(c + d*x^4)^3,x]
```

```
[Out] ((32*c^(7/4)*d^(1/4)*(b*c - a*d)^2*x)/(c + d*x^4)^2 - (8*c^(3/4)*d^(1/4)*(9
*b^2*c^2 - 2*a*b*c*d - 7*a^2*d^2)*x)/(c + d*x^4) - 2*Sqrt[2]*(5*b^2*c^2 + 6
*a*b*c*d + 21*a^2*d^2)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 2*Sqrt[2]*
(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)
] - Sqrt[2]*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*Log[Sqrt[c] - Sqrt[2]*c^(1
/4)*d^(1/4)*x + Sqrt[d]*x^2] + Sqrt[2]*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)
*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(256*c^(11/4)*d^(9
/4))
```

Maple [A] time = 0.008, size = 499, normalized size = 1.4

$$\frac{1}{(dx^4+c)^2}\left(\frac{(7a^2d^2+2cabd-9b^2c^2)x^5}{32c^2d} + \frac{(11a^2d^2-6cabd-5b^2c^2)x}{32d^2c}\right) + \frac{21\sqrt{2}a^2}{128c^3}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}-1\right) + \frac{3\sqrt{2}a}{64c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^2/(d*x^4+c)^3,x)
```

```
[Out] (1/32*(7*a^2*d^2+2*a*b*c*d-9*b^2*c^2)/c^2/d*x^5+1/32*(11*a^2*d^2-6*a*b*c*d-
5*b^2*c^2)/d^2/c*x)/(d*x^4+c)^2+21/128/c^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/
2)/(c/d)^(1/4)*x-1)*a^2+3/64/c^2/d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)
^(1/4)*x-1)*a*b+5/128/c/d^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*
x-1)*b^2+21/256/c^3*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)
^(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))*a^2+3/128/c^2/d*(c/d)^(1/4
)*2^(1/2)*ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^(
1/2)+(c/d)^(1/2)))*a*b+5/256/c/d^2*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*
x*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))*b^2+21/128/
c^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)*a^2+3/64/c^2/d*(c/d
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)*a*b+5/128/c/d^2*(c/d)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/(d*x^4+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.5754, size = 3302, normalized size = 9.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/(d*x^4+c)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(4*(9*b^2*c^2*d - 2*a*b*c*d^2 - 7*a^2*d^3)*x^5 - 4*(c^2*d^4*x^8 + 2*
c^3*d^3*x^4 + c^4*d^2)*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^
6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3
*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^11*
d^9))^(1/4)*arctan(-(c^8*d^7*x*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^
2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5
*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8
)/(c^11*d^9))^(3/4) - c^8*d^7*sqrt((c^6*d^4*sqrt(-(625*b^8*c^8 + 3000*a*b^7
*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4
*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7
 + 194481*a^8*d^8)/(c^11*d^9)) + (25*b^4*c^4 + 60*a*b^3*c^3*d + 246*a^2*b^2
*c^2*d^2 + 252*a^3*b*c*d^3 + 441*a^4*d^4)*x^2)/(25*b^4*c^4 + 60*a*b^3*c^3*d
 + 246*a^2*b^2*c^2*d^2 + 252*a^3*b*c*d^3 + 441*a^4*d^4))*(-(625*b^8*c^8 + 3
000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^
4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^
7*b*c*d^7 + 194481*a^8*d^8)/(c^11*d^9))^(3/4))/(125*b^6*c^6 + 450*a*b^5*c^5
*d + 2115*a^2*b^4*c^4*d^2 + 3996*a^3*b^3*c^3*d^3 + 8883*a^4*b^2*c^2*d^4 + 7
938*a^5*b*c*d^5 + 9261*a^6*d^6)) - (c^2*d^4*x^8 + 2*c^3*d^3*x^4 + c^4*d^2)*
(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c
^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c
^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^11*d^9))^(1/4)*log(c^3*d^2
*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*
c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*
c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^11*d^9))^(1/4) + (5*b^2*c
```

```
^2 + 6*a*b*c*d + 21*a^2*d^2)*x) + (c^2*d^4*x^8 + 2*c^3*d^3*x^4 + c^4*d^2)*(
-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^
5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^
2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^11*d^9))^(1/4)*log(-c^3*d^2
*(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*
c^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*
c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^11*d^9))^(1/4) + (5*b^2*c
^2 + 6*a*b*c*d + 21*a^2*d^2)*x) + 4*(5*b^2*c^3 + 6*a*b*c^2*d - 11*a^2*c*d^2
)*x)/(c^2*d^4*x^8 + 2*c^3*d^3*x^4 + c^4*d^2)
```

Sympy [A] time = 8.0575, size = 264, normalized size = 0.76

$$\frac{x^5\left(7a^2d^3+2abcd^2-9b^2c^2d\right)+x\left(11a^2cd^2-6abc^2d-5b^2c^3\right)}{32c^4d^2+64c^3d^3x^4+32c^2d^4x^8}+\operatorname{RootSum}\left(268435456t^4c^{11}d^9+194481a^8d^8+222264a^7\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**2/(d*x**4+c)**3,x)
```

```
[Out] (x**5*(7*a**2*d**3 + 2*a*b*c*d**2 - 9*b**2*c**2*d) + x*(11*a**2*c*d**2 - 6*
a*b*c**2*d - 5*b**2*c**3))/(32*c**4*d**2 + 64*c**3*d**3*x**4 + 32*c**2*d**4
*x**8) + RootSum(268435456*_t**4*c**11*d**9 + 194481*a**8*d**8 + 222264*a**
7*b*c*d**7 + 280476*a**6*b**2*c**2*d**6 + 176904*a**5*b**3*c**3*d**5 + 1128
06*a**4*b**4*c**4*d**4 + 42120*a**3*b**5*c**5*d**3 + 15900*a**2*b**6*c**6*d
**2 + 3000*a*b**7*c**7*d + 625*b**8*c**8, Lambda(_t, _t*log(128*_t*c**3*d**
2/(21*a**2*d**2 + 6*a*b*c*d + 5*b**2*c**2) + x)))
```

Giac [A] time = 1.14108, size = 549, normalized size = 1.57

$$\frac{\sqrt{2}\left(5\left(cd^3\right)^{\frac{1}{4}}b^2c^2+6\left(cd^3\right)^{\frac{1}{4}}abcd+21\left(cd^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{128\,c^3d^3}+\frac{\sqrt{2}\left(5\left(cd^3\right)^{\frac{1}{4}}b^2c^2+6\left(cd^3\right)^{\frac{1}{4}}abcd+21\right.}{128\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/(d*x^4+c)^3,x, algorithm="giac")
```

```
[Out] 1/128*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 + 6*(c*d^3)^(1/4)*a*b*c*d + 21*(c*d^
3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4
))/(c^3*d^3) + 1/128*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 + 6*(c*d^3)^(1/4)*a*b
*c*d + 21*(c*d^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1
/4))/(c/d)^(1/4))/(c^3*d^3) + 1/256*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 + 6*(c
*d^3)^(1/4)*a*b*c*d + 21*(c*d^3)^(1/4)*a^2*d^2)*log(x^2 + sqrt(2)*x*(c/d)^(
1/4) + sqrt(c/d))/(c^3*d^3) - 1/256*sqrt(2)*(5*(c*d^3)^(1/4)*b^2*c^2 + 6*(c
*d^3)^(1/4)*a*b*c*d + 21*(c*d^3)^(1/4)*a^2*d^2)*log(x^2 - sqrt(2)*x*(c/d)^(
1/4) + sqrt(c/d))/(c^3*d^3) - 1/32*(9*b^2*c^2*d*x^5 - 2*a*b*c*d^2*x^5 - 7*a
^2*d^3*x^5 + 5*b^2*c^3*x + 6*a*b*c^2*d*x - 11*a^2*c*d^2*x)/((d*x^4 + c)^2*c
^2*d^2)
```

3.160 $\int \frac{(c+dx^4)^4}{a+bx^4} dx$

Optimal. Leaf size=332

$$\frac{d^2x^5\left(a^2d^2-4abcd+6b^2c^2\right)}{5b^3}+\frac{dx(2bc-ad)\left(a^2d^2-2abcd+2b^2c^2\right)}{b^4}-\frac{(bc-ad)^4\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{17/4}}+$$

```
[Out] (d*(2*b*c - a*d)*(2*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/b^4 + (d^2*(6*b^2*c^2
 - 4*a*b*c*d + a^2*d^2)*x^5)/(5*b^3) + (d^3*(4*b*c - a*d)*x^9)/(9*b^2) + (d
^4*x^13)/(13*b) - ((b*c - a*d)^4*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(
2*Sqrt[2]*a^(3/4)*b^(17/4)) + ((b*c - a*d)^4*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)
/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(17/4)) - ((b*c - a*d)^4*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(17/4)) + ((b*c
- a*d)^4*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]
*a^(3/4)*b^(17/4))
```

Rubi [A] time = 0.266637, antiderivative size = 332, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {390, 211, 1165, 628, 1162, 617, 204}

$$\frac{d^2x^5\left(a^2d^2-4abcd+6b^2c^2\right)}{5b^3}+\frac{dx(2bc-ad)\left(a^2d^2-2abcd+2b^2c^2\right)}{b^4}-\frac{(bc-ad)^4\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{17/4}}+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^4)^4/(a + b*x^4),x]
```



```
[Out] (d*(2*b*c - a*d)*(2*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/b^4 + (d^2*(6*b^2*c^2
 - 4*a*b*c*d + a^2*d^2)*x^5)/(5*b^3) + (d^3*(4*b*c - a*d)*x^9)/(9*b^2) + (d
^4*x^13)/(13*b) - ((b*c - a*d)^4*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(
2*Sqrt[2]*a^(3/4)*b^(17/4)) + ((b*c - a*d)^4*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)
/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(17/4)) - ((b*c - a*d)^4*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(17/4)) + ((b*c
- a*d)^4*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]
*a^(3/4)*b^(17/4))
```


Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(c+dx^4\right)^4}{a+bx^4}\,dx = \int \left(\frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)}{b^4}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^4}{b^3}+\frac{d^3(4bc-ad)x^8}{b^2}+\frac{d^4x^{12}}{b}+\cdots\right.$$

$$=\frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^5}{5b^3}+\frac{d^3(4bc-ad)x^9}{9b^2}+\frac{d^4x^{13}}{13b}+\frac{(bc\cdots}{\cdots}$$

$$=\frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^5}{5b^3}+\frac{d^3(4bc-ad)x^9}{9b^2}+\frac{d^4x^{13}}{13b}+\frac{(bc\cdots}{\cdots}$$

$$=\frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^5}{5b^3}+\frac{d^3(4bc-ad)x^9}{9b^2}+\frac{d^4x^{13}}{13b}+\frac{(bc\cdots}{\cdots}$$

$$=\frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^5}{5b^3}+\frac{d^3(4bc-ad)x^9}{9b^2}+\frac{d^4x^{13}}{13b}-\frac{(bc\cdots}{\cdots}$$

$$=\frac{d(2bc-ad)\left(2b^2c^2-2abcd+a^2d^2\right)x}{b^4}+\frac{d^2\left(6b^2c^2-4abcd+a^2d^2\right)x^5}{5b^3}+\frac{d^3(4bc-ad)x^9}{9b^2}+\frac{d^4x^{13}}{13b}-\frac{(bc\cdots}{\cdots}$$

Mathematica [A] time = 0.192085, size = 322, normalized size = 0.97

$$\frac{936b^{5/4}d^2x^5\left(a^2d^2-4abcd+6b^2c^2\right)+4680\sqrt[4]{b}dx\left(4a^2bcd^2-a^3d^3-6ab^2c^2d+4b^3c^3\right)-\frac{585\sqrt{2}(bc-ad)^4\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\cdots\right.}{a^{3/4}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^4)^4/(a + b*x^4),x]
```

```
[Out] (4680*b^(1/4)*d*(4*b^3*c^3 - 6*a*b^2*c^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*x + 9
36*b^(5/4)*d^2*(6*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x^5 + 520*b^(9/4)*d^3*(4*b
*c - a*d)*x^9 + 360*b^(13/4)*d^4*x^13 - (1170*Sqrt[2]*(b*c - a*d)^4*ArcTan[
1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/4) + (1170*Sqrt[2]*(b*c - a*d)^4*Arc
Tan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(3/4) - (585*Sqrt[2]*(b*c - a*d)^4*
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(3/4) + (585*Sqrt
[2]*(b*c - a*d)^4*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a
^(3/4))/(4680*b^(17/4))
```

Maple [B] time = 0.007, size = 837, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^4/(b*x^4+a),x)
```

```
[Out] -d^4/b^4*a^3*x+4*d/b*c^3*x-1/2/b*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4
)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))*c^3
*d+1/4/b^4*(1/b*a)^(1/4)*a^3*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)*d^4-
1/b*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)*c^3*d+1/4*(1/b*
a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*c^4+1/8*(1/b*a)^(1/4)/
a*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)
*x*2^(1/2)+(1/b*a)^(1/2)))*c^4-4/5*d^3/b^2*x^5*a*c+1/4*(1/b*a)^(1/4)/a*2^(1
/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)*c^4+1/13*d^4*x^13/b+1/4/b^4*(1/b*a)^(
1/4)*a^3*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*d^4-1/b*(1/b*a)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*c^3*d+1/8/b^4*(1/b*a)^(1/4)*a^3*2^(
1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^
(1/2)+(1/b*a)^(1/2)))*d^4+4*d^3/b^3*a^2*c*x-6*d^2/b^2*a*c^2*x-1/b^3*(1/b*a)
^(1/4)*a^2*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*c*d^3+3/2/b^2*(1/b*a)^
(1/4)*a*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*c^2*d^2-1/2/b^3*(1/b*a)^(
1/4)*a^2*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a
)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))*c*d^3+3/4/b^2*(1/b*a)^(1/4)*a*2^(1/2)*ln(
(x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1
/b*a)^(1/2)))*c^2*d^2-1/b^3*(1/b*a)^(1/4)*a^2*2^(1/2)*arctan(2^(1/2)/(1/b*a
)^(1/4)*x+1)*c*d^3+3/2/b^2*(1/b*a)^(1/4)*a*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(
1/4)*x+1)*c^2*d^2+6/5*d^2/b*x^5*c^2-1/9*d^4/b^2*x^9*a+4/9*d^3/b*x^9*c+1/5*d
^4/b^3*x^5*a^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^4/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.72795, size = 5701, normalized size = 17.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^4/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/2340*(180*b^3*d^4*x^13 + 260*(4*b^3*c*d^3 - a*b^2*d^4)*x^9 + 468*(6*b^3*c
^2*d^2 - 4*a*b^2*c*d^3 + a^2*b*d^4)*x^5 + 2340*b^4*(-(b^16*c^16 - 16*a*b^15
*c^15*d + 120*a^2*b^14*c^14*d^2 - 560*a^3*b^13*c^13*d^3 + 1820*a^4*b^12*c^1
2*d^4 - 4368*a^5*b^11*c^11*d^5 + 8008*a^6*b^10*c^10*d^6 - 11440*a^7*b^9*c^9
*d^7 + 12870*a^8*b^8*c^8*d^8 - 11440*a^9*b^7*c^7*d^9 + 8008*a^10*b^6*c^6*d^
10 - 4368*a^11*b^5*c^5*d^11 + 1820*a^12*b^4*c^4*d^12 - 560*a^13*b^3*c^3*d^1
3 + 120*a^14*b^2*c^2*d^14 - 16*a^15*b*c*d^15 + a^16*d^16)/(a^3*b^17))^(1/4)
*arctan(-(a^2*b^13*x*(-(b^16*c^16 - 16*a*b^15*c^15*d + 120*a^2*b^14*c^14*d^
2 - 560*a^3*b^13*c^13*d^3 + 1820*a^4*b^12*c^12*d^4 - 4368*a^5*b^11*c^11*d^5
 + 8008*a^6*b^10*c^10*d^6 - 11440*a^7*b^9*c^9*d^7 + 12870*a^8*b^8*c^8*d^8 -
 11440*a^9*b^7*c^7*d^9 + 8008*a^10*b^6*c^6*d^10 - 4368*a^11*b^5*c^5*d^11 +
1820*a^12*b^4*c^4*d^12 - 560*a^13*b^3*c^3*d^13 + 120*a^14*b^2*c^2*d^14 - 16
*a^15*b*c*d^15 + a^16*d^16)/(a^3*b^17))^(3/4) - a^2*b^13*sqrt((a^2*b^8*sqrt
(-(b^16*c^16 - 16*a*b^15*c^15*d + 120*a^2*b^14*c^14*d^2 - 560*a^3*b^13*c^13
*d^3 + 1820*a^4*b^12*c^12*d^4 - 4368*a^5*b^11*c^11*d^5 + 8008*a^6*b^10*c^10
*d^6 - 11440*a^7*b^9*c^9*d^7 + 12870*a^8*b^8*c^8*d^8 - 11440*a^9*b^7*c^7*d^
9 + 8008*a^10*b^6*c^6*d^10 - 4368*a^11*b^5*c^5*d^11 + 1820*a^12*b^4*c^4*d^1
2 - 560*a^13*b^3*c^3*d^13 + 120*a^14*b^2*c^2*d^14 - 16*a^15*b*c*d^15 + a^16
*d^16)/(a^3*b^17)) + (b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3
*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6
 - 8*a^7*b*c*d^7 + a^8*d^8)*x^2)/(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*
d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6
*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8))*(-(b^16*c^16 - 16*a*b^15*c^15*d +
120*a^2*b^14*c^14*d^2 - 560*a^3*b^13*c^13*d^3 + 1820*a^4*b^12*c^12*d^4 - 43
68*a^5*b^11*c^11*d^5 + 8008*a^6*b^10*c^10*d^6 - 11440*a^7*b^9*c^9*d^7 + 128
70*a^8*b^8*c^8*d^8 - 11440*a^9*b^7*c^7*d^9 + 8008*a^10*b^6*c^6*d^10 - 4368*
a^11*b^5*c^5*d^11 + 1820*a^12*b^4*c^4*d^12 - 560*a^13*b^3*c^3*d^13 + 120*a^
14*b^2*c^2*d^14 - 16*a^15*b*c*d^15 + a^16*d^16)/(a^3*b^17))^(3/4))/(b^12*c^
12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^
4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5
*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 1
2*a^11*b*c*d^11 + a^12*d^12)) + 585*b^4*(-(b^16*c^16 - 16*a*b^15*c^15*d + 1
20*a^2*b^14*c^14*d^2 - 560*a^3*b^13*c^13*d^3 + 1820*a^4*b^12*c^12*d^4 - 436
8*a^5*b^11*c^11*d^5 + 8008*a^6*b^10*c^10*d^6 - 11440*a^7*b^9*c^9*d^7 + 1287
0*a^8*b^8*c^8*d^8 - 11440*a^9*b^7*c^7*d^9 + 8008*a^10*b^6*c^6*d^10 - 4368*a
^11*b^5*c^5*d^11 + 1820*a^12*b^4*c^4*d^12 - 560*a^13*b^3*c^3*d^13 + 120*a^1
4*b^2*c^2*d^14 - 16*a^15*b*c*d^15 + a^16*d^16)/(a^3*b^17))^(1/4)*log(a*b^4*
(-(b^16*c^16 - 16*a*b^15*c^15*d + 120*a^2*b^14*c^14*d^2 - 560*a^3*b^13*c^13
*d^3 + 1820*a^4*b^12*c^12*d^4 - 4368*a^5*b^11*c^11*d^5 + 8008*a^6*b^10*c^10
*d^6 - 11440*a^7*b^9*c^9*d^7 + 12870*a^8*b^8*c^8*d^8 - 11440*a^9*b^7*c^7*d^
9 + 8008*a^10*b^6*c^6*d^10 - 4368*a^11*b^5*c^5*d^11 + 1820*a^12*b^4*c^4*d^1
2 - 560*a^13*b^3*c^3*d^13 + 120*a^14*b^2*c^2*d^14 - 16*a^15*b*c*d^15 + a^16
*d^16)/(a^3*b^17))^(1/4) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4
*a^3*b*c*d^3 + a^4*d^4)*x) - 585*b^4*(-(b^16*c^16 - 16*a*b^15*c^15*d + 120*
a^2*b^14*c^14*d^2 - 560*a^3*b^13*c^13*d^3 + 1820*a^4*b^12*c^12*d^4 - 4368*a
```

```
^5*b^11*c^11*d^5 + 8008*a^6*b^10*c^10*d^6 - 11440*a^7*b^9*c^9*d^7 + 12870*a
^8*b^8*c^8*d^8 - 11440*a^9*b^7*c^7*d^9 + 8008*a^10*b^6*c^6*d^10 - 4368*a^11
*b^5*c^5*d^11 + 1820*a^12*b^4*c^4*d^12 - 560*a^13*b^3*c^3*d^13 + 120*a^14*b
^2*c^2*d^14 - 16*a^15*b*c*d^15 + a^16*d^16)/(a^3*b^17))^(1/4)*log(-a*b^4*(-
(b^16*c^16 - 16*a*b^15*c^15*d + 120*a^2*b^14*c^14*d^2 - 560*a^3*b^13*c^13*d
^3 + 1820*a^4*b^12*c^12*d^4 - 4368*a^5*b^11*c^11*d^5 + 8008*a^6*b^10*c^10*d
^6 - 11440*a^7*b^9*c^9*d^7 + 12870*a^8*b^8*c^8*d^8 - 11440*a^9*b^7*c^7*d^9
+ 8008*a^10*b^6*c^6*d^10 - 4368*a^11*b^5*c^5*d^11 + 1820*a^12*b^4*c^4*d^12
- 560*a^13*b^3*c^3*d^13 + 120*a^14*b^2*c^2*d^14 - 16*a^15*b*c*d^15 + a^16*d
^16)/(a^3*b^17))^(1/4) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a
^3*b*c*d^3 + a^4*d^4)*x) + 2340*(4*b^3*c^3*d - 6*a*b^2*c^2*d^2 + 4*a^2*b*c*
d^3 - a^3*d^4)*x)/b^4
```

Sympy [A] time = 2.90635, size = 430, normalized size = 1.3

$$\mathrm{RootSum}\Big(256t^4a^3b^{17} + a^{16}d^{16} - 16a^{15}bcd^{15} + 120a^{14}b^2c^2d^{14} - 560a^{13}b^3c^3d^{13} + 1820a^{12}b^4c^4d^{12} - 4368a^{11}b^5c^5d^{11} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**4/(b*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**17 + a**16*d**16 - 16*a**15*b*c*d**15 + 120*a**14
*b**2*c**2*d**14 - 560*a**13*b**3*c**3*d**13 + 1820*a**12*b**4*c**4*d**12 -
 4368*a**11*b**5*c**5*d**11 + 8008*a**10*b**6*c**6*d**10 - 11440*a**9*b**7*
c**7*d**9 + 12870*a**8*b**8*c**8*d**8 - 11440*a**7*b**9*c**9*d**7 + 8008*a*
*6*b**10*c**10*d**6 - 4368*a**5*b**11*c**11*d**5 + 1820*a**4*b**12*c**12*d*
*4 - 560*a**3*b**13*c**13*d**3 + 120*a**2*b**14*c**14*d**2 - 16*a*b**15*c**
15*d + b**16*c**16, Lambda(_t, _t*log(4*_t*a*b**4/(a**4*d**4 - 4*a**3*b*c*d
**3 + 6*a**2*b**2*c**2*d**2 - 4*a*b**3*c**3*d + b**4*c**4) + x))) + d**4*x*
*13/(13*b) - x**9*(a*d**4 - 4*b*c*d**3)/(9*b**2) + x**5*(a**2*d**4 - 4*a*b*
c*d**3 + 6*b**2*c**2*d**2)/(5*b**3) - x*(a**3*d**4 - 4*a**2*b*c*d**3 + 6*a*
b**2*c**2*d**2 - 4*b**3*c**3*d)/b**4
```

Giac [B] time = 1.11049, size = 833, normalized size = 2.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^4/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((a*b^3)^(1/4)*b^4*c^4 - 4*(a*b^3)^(1/4)*a*b^3*c^3*d + 6*(a*b^3
)^(1/4)*a^2*b^2*c^2*d^2 - 4*(a*b^3)^(1/4)*a^3*b*c*d^3 + (a*b^3)^(1/4)*a^4*d
^4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^5) + 1
/4*sqrt(2)*((a*b^3)^(1/4)*b^4*c^4 - 4*(a*b^3)^(1/4)*a*b^3*c^3*d + 6*(a*b^3)
^(1/4)*a^2*b^2*c^2*d^2 - 4*(a*b^3)^(1/4)*a^3*b*c*d^3 + (a*b^3)^(1/4)*a^4*d^
4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^5) + 1/
8*sqrt(2)*((a*b^3)^(1/4)*b^4*c^4 - 4*(a*b^3)^(1/4)*a*b^3*c^3*d + 6*(a*b^3)^
(1/4)*a^2*b^2*c^2*d^2 - 4*(a*b^3)^(1/4)*a^3*b*c*d^3 + (a*b^3)^(1/4)*a^4*d^4
)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^5) - 1/8*sqrt(2)*((a*b^
3)^(1/4)*b^4*c^4 - 4*(a*b^3)^(1/4)*a*b^3*c^3*d + 6*(a*b^3)^(1/4)*a^2*b^2*c^
2*d^2 - 4*(a*b^3)^(1/4)*a^3*b*c*d^3 + (a*b^3)^(1/4)*a^4*d^4)*log(x^2 - sqrt
(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^5) + 1/585*(45*b^12*d^4*x^13 + 260*b^12
```

```
*c*d^3*x^9 - 65*a*b^11*d^4*x^9 + 702*b^12*c^2*d^2*x^5 - 468*a*b^11*c*d^3*x^
5 + 117*a^2*b^10*d^4*x^5 + 2340*b^12*c^3*d*x - 3510*a*b^11*c^2*d^2*x + 2340
*a^2*b^10*c*d^3*x - 585*a^3*b^9*d^4*x)/b^13
```

3.161 $\int \frac{(c+dx^4)^3}{a+bx^4} dx$

Optimal. Leaf size=288

$$\frac{dx\left(a^2d^2-3abcd+3b^2c^2\right)}{b^3}-\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{13/4}}+\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{13/4}}-$$

```
[Out] (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x)/b^3 + (d^2*(3*b*c - a*d)*x^5)/(5*b^
2) + (d^3*x^9)/(9*b) - ((b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4
)])/(2*Sqrt[2]*a^(3/4)*b^(13/4)) + ((b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/
4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(13/4)) - ((b*c - a*d)^3*Log[Sqrt[a] -
 Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(13/4)) + (
(b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sq
rt[2]*a^(3/4)*b^(13/4))
```

Rubi [A] time = 0.223468, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {390, 211, 1165, 628, 1162, 617, 204}

$$\frac{dx\left(a^2d^2-3abcd+3b^2c^2\right)}{b^3}-\frac{(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{13/4}}+\frac{(bc-ad)^3\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{13/4}}-$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^4)^3/(a + b*x^4),x]
```


```
[Out] (d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x)/b^3 + (d^2*(3*b*c - a*d)*x^5)/(5*b^
2) + (d^3*x^9)/(9*b) - ((b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4
)])/(2*Sqrt[2]*a^(3/4)*b^(13/4)) + ((b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/
4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(13/4)) - ((b*c - a*d)^3*Log[Sqrt[a] -
 Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(13/4)) + (
(b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sq
rt[2]*a^(3/4)*b^(13/4))
```


Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(c+dx^4\right)^3}{a+bx^4}\,dx = \int \left(\frac{d\left(3b^2c^2-3abcd+a^2d^2\right)}{b^3}+\frac{d^2(3bc-ad)x^4}{b^2}+\frac{d^3x^8}{b}+\frac{b^3c^3-3ab^2c^2d+3a^2bcd^2-a^3d^3}{b^3\left(a+bx^4\right)}\right)dx$$

$$= \frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x}{b^3}+\frac{d^2(3bc-ad)x^5}{5b^2}+\frac{d^3x^9}{9b}+\frac{(bc-ad)^3\int\frac{1}{a+bx^4}\,dx}{b^3}$$

$$= \frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x}{b^3}+\frac{d^2(3bc-ad)x^5}{5b^2}+\frac{d^3x^9}{9b}+\frac{(bc-ad)^3\int\frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{2\sqrt{a}b^3}+\frac{(bc-ad)^3\int\frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{2\sqrt{a}b^3}$$

$$= \frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x}{b^3}+\frac{d^2(3bc-ad)x^5}{5b^2}+\frac{d^3x^9}{9b}+\frac{(bc-ad)^3\int\frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}b^{7/2}}+\frac{(bc-ad)^3\int \cdots}{4\sqrt{a}\cdots}$$

$$= \frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x}{b^3}+\frac{d^2(3bc-ad)x^5}{5b^2}+\frac{d^3x^9}{9b}-\frac{(bc-ad)^3\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{13/4}}+\cdots$$

$$= \frac{d\left(3b^2c^2-3abcd+a^2d^2\right)x}{b^3}+\frac{d^2(3bc-ad)x^5}{5b^2}+\frac{d^3x^9}{9b}-\frac{(bc-ad)^3\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{13/4}}+\frac{(bc-ad)^3\tan\cdots}{2\sqrt{2}a\cdots}$$

Mathematica [A] time = 0.143268, size = 271, normalized size = 0.94

$$360a^{3/4}\sqrt[4]{b}dx\left(a^2d^2-3abcd+3b^2c^2\right)-72a^{3/4}b^{5/4}d^2x^5(ad-3bc)+40a^{3/4}b^{9/4}d^3x^9-45\sqrt{2}(bc-ad)^3\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^4)^3/(a + b*x^4),x]
```

```
[Out] (360*a^(3/4)*b^(1/4)*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x - 72*a^(3/4)*b^(
5/4)*d^2*(-3*b*c + a*d)*x^5 + 40*a^(3/4)*b^(9/4)*d^3*x^9 - 90*Sqrt[2]*(b*c
- a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 90*Sqrt[2]*(b*c - a*d)^3
*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 45*Sqrt[2]*(b*c - a*d)^3*Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 45*Sqrt[2]*(b*c - a*d)^3*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(360*a^(3/4)*b^(13/
4))
```

Maple [B] time = 0.001, size = 627, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^3/(b*x^4+a),x)
```

```
[Out] 1/9*d^3*x^9/b-1/5*d^3/b^2*x^5*a+3/5*d^2/b*x^5*c+d^3/b^3*a^2*x-3*d^2/b^2*c*a
*x+3*d/b*c^2*x-1/4/b^3*(1/b*a)^(1/4)*a^2*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/
4)*x+1)*d^3+3/4/b^2*(1/b*a)^(1/4)*a*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+
1)*c*d^2-3/4/b*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)*c^2*
d+1/4*(1/b*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)*c^3-1/4/b^3
*(1/b*a)^(1/4)*a^2*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*d^3+3/4/b^2*(1
/b*a)^(1/4)*a*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*c*d^2-3/4/b*(1/b*a)
^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*c^2*d+1/4*(1/b*a)^(1/4)/a*
2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*c^3-1/8/b^3*(1/b*a)^(1/4)*a^2*2^(
1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^
(1/2)+(1/b*a)^(1/2)))*d^3+3/8/b^2*(1/b*a)^(1/4)*a*2^(1/2)*ln((x^2+(1/b*a)^(
1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))*
c*d^2-3/8/b*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(
1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))*c^2*d+1/8*(1/b*a)^(1/4)/
a*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)
*x*2^(1/2)+(1/b*a)^(1/2)))*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^3/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.68654, size = 4034, normalized size = 14.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^3/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/180*(20*b^2*d^3*x^9 + 36*(3*b^2*c*d^2 - a*b*d^3)*x^5 - 180*b^3*(-(b^12*c^
12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^
4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5
*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 1
2*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(1/4)*arctan((a^2*b^10*x*(-(b^12*c
^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a
^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^
5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 -
12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(3/4) - a^2*b^10*sqrt((a^2*b^6*sq
rt(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*
d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792
*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*
c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13)) + (b^6*c^6 - 6*a*b^5*c
^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5
*b*c*d^5 + a^6*d^6)*x^2)/(b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20
*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6))*(-(b^12*c
^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a
^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^
5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 -
12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(3/4))/(b^9*c^9 - 9*a*b^8*c^8*d +
 36*a^2*b^7*c^7*d^2 - 84*a^3*b^6*c^6*d^3 + 126*a^4*b^5*c^5*d^4 - 126*a^5*b^
4*c^4*d^5 + 84*a^6*b^3*c^3*d^6 - 36*a^7*b^2*c^2*d^7 + 9*a^8*b*c*d^8 - a^9*d
^9)) - 45*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*
a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c
^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 +
66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(1/4)*log(
a*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*
c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 -
 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*
b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(1/4) - (b^3*c^3 -
 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x) + 45*b^3*(-(b^12*c^12 - 12*a*b
^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d
^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*
a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*
d^11 + a^12*d^12)/(a^3*b^13))^(1/4)*log(-a*b^3*(-(b^12*c^12 - 12*a*b^11*c^1
1*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 79
2*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4
*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 +
a^12*d^12)/(a^3*b^13))^(1/4) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a
^3*d^3)*x) + 180*(3*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^3)*x)/b^3
```

Sympy [A] time = 1.73625, size = 301, normalized size = 1.05

$$\operatorname{RootSum}\left(256t^4a^3b^{13} + a^{12}d^{12} - 12a^{11}bcd^{11} + 66a^{10}b^2c^2d^{10} - 220a^9b^3c^3d^9 + 495a^8b^4c^4d^8 - 792a^7b^5c^5d^7 + 924a^6b^6c^6d^6\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**3/(b*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**13 + a**12*d**12 - 12*a**11*b*c*d**11 + 66*a**10*
b**2*c**2*d**10 - 220*a**9*b**3*c**3*d**9 + 495*a**8*b**4*c**4*d**8 - 792*a
**7*b**5*c**5*d**7 + 924*a**6*b**6*c**6*d**6 - 792*a**5*b**7*c**7*d**5 + 49
5*a**4*b**8*c**8*d**4 - 220*a**3*b**9*c**9*d**3 + 66*a**2*b**10*c**10*d**2
```

```
- 12*a*b**11*c**11*d + b**12*c**12, Lambda(_t, _t*log(-4*_t*a*b**3/(a**3*d*
*3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3) + x))) + d**3*x**9/(9*b
) - x**5*(a*d**3 - 3*b*c*d**2)/(5*b**2) + x*(a**2*d**3 - 3*a*b*c*d**2 + 3*b
**2*c**2*d)/b**3
```

Giac [B] time = 1.10867, size = 649, normalized size = 2.25

$$\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3 - 3\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d + 3\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2 - \left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4\,ab^4} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3 - 3\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^3/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3
)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(2*x + sqrt
(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^4) + 1/4*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3
- 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)
*a^3*d^3)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^
4) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3)^(1/4)*a*b^2*c^2*d + 3*(
a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*log(x^2 + sqrt(2)*x*(a/b)
^(1/4) + sqrt(a/b))/(a*b^4) - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 - 3*(a*b^3
)^(1/4)*a*b^2*c^2*d + 3*(a*b^3)^(1/4)*a^2*b*c*d^2 - (a*b^3)^(1/4)*a^3*d^3)*
log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^4) + 1/45*(5*b^8*d^3*x^9
+ 27*b^8*c*d^2*x^5 - 9*a*b^7*d^3*x^5 + 135*b^8*c^2*d*x - 135*a*b^7*c*d^2*x
+ 45*a^2*b^6*d^3*x)/b^9
```

3.162 $\int \frac{\left(c+dx^4\right)^2}{a+bx^4} dx$

Optimal. Leaf size=253

$$-\frac{(bc-ad)^2 \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{9/4}}+\frac{(bc-ad)^2 \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{9/4}}-\frac{(bc-ad)^2 \tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{9/4}}$$

```
[Out] (d*(2*b*c - a*d)*x)/b^2 + (d^2*x^5)/(5*b) - ((b*c - a*d)^2*ArcTan[1 - (Sqrt
[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(9/4)) + ((b*c - a*d)^2*ArcTa
n[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(9/4)) - ((b*c - a
*d)^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^
(3/4)*b^(9/4)) + ((b*c - a*d)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + S
qrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(9/4))
```

Rubi [A] time = 0.190198, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {390, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(bc-ad)^2 \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{9/4}}+\frac{(bc-ad)^2 \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{9/4}}-\frac{(bc-ad)^2 \tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^4)^2/(a + b*x^4),x]
```

```
[Out] (d*(2*b*c - a*d)*x)/b^2 + (d^2*x^5)/(5*b) - ((b*c - a*d)^2*ArcTan[1 - (Sqrt
[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(9/4)) + ((b*c - a*d)^2*ArcTa
n[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*b^(9/4)) - ((b*c - a
*d)^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^
(3/4)*b^(9/4)) + ((b*c - a*d)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + S
qrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(9/4))
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(c+dx^4\right)^2}{a+bx^4}\,dx = \int \left(\frac{d(2bc-ad)}{b^2}+\frac{d^2x^4}{b}+\frac{b^2c^2-2abcd+a^2d^2}{b^2\left(a+bx^4\right)}\right)dx$$

$$= \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^5}{5b}+\frac{(bc-ad)^2\int \frac{1}{a+bx^4}\,dx}{b^2}$$

$$= \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^5}{5b}+\frac{(bc-ad)^2\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{2\sqrt{a}b^2}+\frac{(bc-ad)^2\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{2\sqrt{a}b^2}$$

$$= \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^5}{5b}+\frac{(bc-ad)^2\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}b^{5/2}}+\frac{(bc-ad)^2\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}b^{5/2}}-\frac{(bc-ad)^2\int}{4\sqrt{}}$$

$$= \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^5}{5b}-\frac{(bc-ad)^2\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{9/4}}+\frac{(bc-ad)^2\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}\right.}{4\sqrt{2}a^{3/4}b^{9/4}}$$

$$= \frac{d(2bc-ad)x}{b^2}+\frac{d^2x^5}{5b}-\frac{(bc-ad)^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{9/4}}+\frac{(bc-ad)^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{9/4}}-\frac{(bc-ad)^2}{}$$

Mathematica [A] time = 0.10623, size = 231, normalized size = 0.91

$$\frac{8a^{3/4}b^{5/4}d^2x^5-40a^{3/4}\sqrt[4]{b}dx(ad-2bc)-5\sqrt{2}(bc-ad)^2\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)+5\sqrt{2}(bc-ad)^2\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{}\right.}{40a^{3/4}b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^4)^2/(a + b*x^4),x]
```

```
[Out] (-40*a^(3/4)*b^(1/4)*d*(-2*b*c + a*d)*x + 8*a^(3/4)*b^(5/4)*d^2*x^5 - 10*Sq
rt[2]*(b*c - a*d)^2*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 10*Sqrt[2]*(b
*c - a*d)^2*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 5*Sqrt[2]*(b*c - a*d)
^2*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 5*Sqrt[2]*(b*c
- a*d)^2*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(40*a^(3/4
)*b^(9/4))
```

Maple [B] time = 0.003, size = 436, normalized size = 1.7

$$\frac{d^2x^5}{5b} - \frac{ad^2x}{b^2} + 2\frac{dxc}{b} + \frac{a\sqrt{2}d^2}{4b^2}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right) - \frac{\sqrt{2}cd}{2b}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right) + \frac{\sqrt{2}c^2}{4a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^2/(b*x^4+a),x)
```

```
[Out] 1/5*d^2*x^5/b-d^2/b^2*a*x+2*d/b*x*c+1/4/b^2*(1/b*a)^(1/4)*a*2^(1/2)*arctan(
2^(1/2)/(1/b*a)^(1/4)*x-1)*d^2-1/2/b*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(
1/b*a)^(1/4)*x-1)*c*d+1/4*(1/b*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1
/4)*x-1)*c^2+1/8/b^2*(1/b*a)^(1/4)*a*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2
)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))*d^2-1/4/b*(1/
b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b
*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))*c*d+1/8*(1/b*a)^(1/4)/a*2^(1/2)*ln((x^2
+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a
)^(1/2)))*c^2+1/4/b^2*(1/b*a)^(1/4)*a*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*
x+1)*d^2-1/2/b*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)*c*d+
1/4*(1/b*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^2/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.43963, size = 2547, normalized size = 10.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^2/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/20*(4*b*d^2*x^5 + 20*b^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2
- 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2
*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(a^3*b^9))^(1/4)*arctan(-(a^2*b^7*x*(-(
b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*
b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8
```

```
*d^8)/(a^3*b^9))^(3/4) - a^2*b^7*sqrt((a^2*b^4*sqrt(-(b^8*c^8 - 8*a*b^7*c^7
*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*
b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(a^3*b^9)) + (b
^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*x^2)/
(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4))*(-
(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4
*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^
8*d^8)/(a^3*b^9))^(3/4))/(b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20
*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)) + 5*b^2*(
-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^
4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a
^8*d^8)/(a^3*b^9))^(1/4)*log(a*b^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*
c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28
*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(a^3*b^9))^(1/4) + (b^2*c^2 - 2
*a*b*c*d + a^2*d^2)*x) - 5*b^2*(-(b^8*c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*
d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6
*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8)/(a^3*b^9))^(1/4)*log(-a*b^2*(-(b^8*
c^8 - 8*a*b^7*c^7*d + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*
c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a^7*b*c*d^7 + a^8*d^8
)/(a^3*b^9))^(1/4) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x) + 20*(2*b*c*d - a*d
^2)*x)/b^2
```

Sympy [A] time = 1.0775, size = 187, normalized size = 0.74

$$\mathrm{RootSum}\left(256t^4a^3b^9 + a^8d^8 - 8a^7bcd^7 + 28a^6b^2c^2d^6 - 56a^5b^3c^3d^5 + 70a^4b^4c^4d^4 - 56a^3b^5c^5d^3 + 28a^2b^6c^6d^2 - 8ab^7c^7\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**2/(b*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**9 + a**8*d**8 - 8*a**7*b*c*d**7 + 28*a**6*b**2*c*
*2*d**6 - 56*a**5*b**3*c**3*d**5 + 70*a**4*b**4*c**4*d**4 - 56*a**3*b**5*c*
*5*d**3 + 28*a**2*b**6*c**6*d**2 - 8*a*b**7*c**7*d + b**8*c**8, Lambda(_t,
_t*log(4*_t*a*b**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x))) + d**2*x**5/(
5*b) - x*(a*d**2 - 2*b*c*d)/b**2
```

Giac [A] time = 1.43738, size = 477, normalized size = 1.89

$$\frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c^2 - 2\left(ab^3\right)^{\frac{1}{4}}abcd + \left(ab^3\right)^{\frac{1}{4}}a^2d^2\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^3} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^2c^2 - 2\left(ab^3\right)^{\frac{1}{4}}abcd + \left(ab^3\right)^{\frac{1}{4}}\right.}{4ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^2/(b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((a*b^3)^(1/4)*b^2*c^2 - 2*(a*b^3)^(1/4)*a*b*c*d + (a*b^3)^(1/4
)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b
^3) + 1/4*sqrt(2)*((a*b^3)^(1/4)*b^2*c^2 - 2*(a*b^3)^(1/4)*a*b*c*d + (a*b^3
)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4)
)/(a*b^3) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c^2 - 2*(a*b^3)^(1/4)*a*b*c*d +
(a*b^3)^(1/4)*a^2*d^2)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3)
```

```
 - 1/8*sqrt(2)*((a*b^3)^(1/4)*b^2*c^2 - 2*(a*b^3)^(1/4)*a*b*c*d + (a*b^3)^(
1/4)*a^2*d^2)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^3) + 1/5*(b
^4*d^2*x^5 + 10*b^4*c*d*x - 5*a*b^3*d^2*x)/b^5
```

3.163 $\int \frac{c+dx^4}{a+bx^4} \, dx$

Optimal. Leaf size=223

$$-\frac{(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{5/4}}+\frac{(bc-ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{5/4}}-\frac{(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{5/4}}$$

```
[Out] (d*x)/b - ((b*c - a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*
a^(3/4)*b^(5/4)) + ((b*c - a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2
*Sqrt[2]*a^(3/4)*b^(5/4)) - ((b*c - a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1
/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(5/4)) + ((b*c - a*d)*Log[Sqrt[a
] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(5/4))
```

Rubi [A] time = 0.138262, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {388, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{5/4}}+\frac{(bc-ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{5/4}}-\frac{(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^4)/(a + b*x^4),x]
```


```
[Out] (d*x)/b - ((b*c - a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*
a^(3/4)*b^(5/4)) + ((b*c - a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2
*Sqrt[2]*a^(3/4)*b^(5/4)) - ((b*c - a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1
/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(5/4)) + ((b*c - a*d)*Log[Sqrt[a
] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*b^(5/4))
```


Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{c+dx^4}{a+bx^4}\,dx = \frac{dx}{b} - \frac{(-bc+ad)\int \frac{1}{a+bx^4}\,dx}{b}$$

$$= \frac{dx}{b} + \frac{(bc-ad)\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{2\sqrt{a}b} + \frac{(bc-ad)\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{2\sqrt{a}b}$$

$$= \frac{dx}{b} + \frac{(bc-ad)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}b^{3/2}} + \frac{(bc-ad)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}b^{3/2}} - \frac{(bc-ad)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}a^{3/4}b^{5/4}} - \cdots$$

$$= \frac{dx}{b} - \frac{(bc-ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{5/4}} + \frac{(bc-ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{5/4}} + \frac{(bc-ad)\,\mathrm{Su}\cdots}{\cdots}$$

$$= \frac{dx}{b} - \frac{(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{5/4}} + \frac{(bc-ad)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{5/4}} - \frac{(bc-ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}\cdots\right)}{4\sqrt{2}a^{3/4}b^{5/4}}$$

Mathematica [A] time = 0.127104, size = 196, normalized size = 0.88

$$\frac{8a^{3/4}\sqrt[4]{b}dx-\sqrt{2}(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)+\sqrt{2}(bc-ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)-2\sqrt{2}(bc-ad)\,\mathrm{t}\cdots}{8a^{3/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^4)/(a + b*x^4),x]
```

```
[Out] (8*a^(3/4)*b^(1/4)*d*x - 2*Sqrt[2]*(b*c - a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*
x)/a^(1/4)] + 2*Sqrt[2]*(b*c - a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)]
 - Sqrt[2]*(b*c - a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^
```

```
2] + Sqrt[2]*(b*c - a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*
x^2])/(8*a^(3/4)*b^(5/4))
```

Maple [A] time = 0.003, size = 266, normalized size = 1.2

$$\frac{dx}{b}-\frac{\sqrt{2}d}{4b}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{\sqrt{2}c}{4a}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)-\frac{\sqrt{2}d}{4b}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right)+\frac{\sqrt{2}c}{4a}\sqrt[4]{\frac{a}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)/(b*x^4+a),x)
```

```
[Out] d*x/b-1/4/b*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)*d+1/4*(
1/b*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)*c-1/4/b*(1/b*a)^(1
/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*d+1/4*(1/b*a)^(1/4)/a*2^(1/2)
*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*c-1/8/b*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1
/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(
1/2)))*d+1/8*(1/b*a)^(1/4)/a*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a
)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.49693, size = 1316, normalized size = 5.9

$$-\frac{4b\left(-\frac{b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2-4a^3bcd^3+a^4d^4}{a^3b^5}\right)^{\frac{1}{4}}\arctan\left(\frac{a^2b^4x\left(-\frac{b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2-4a^3bcd^3+a^4d^4}{a^3b^5}\right)^{\frac{3}{4}}-a^2b^4\sqrt{\frac{a^2b^2\sqrt{-\frac{b^4c^4-4ab^3c^3d+6a^2b^2c^2d^2}{a^3b^5}}}{b^2c^2-2}}}{b^3c^3-3ab^2c^2d+3a^2b}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/4*(4*b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 +
a^4*d^4)/(a^3*b^5))^(1/4)*arctan((a^2*b^4*x*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*
a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(a^3*b^5))^(3/4) - a^2*b^4*sqrt(
(a^2*b^2*sqrt(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
 + a^4*d^4)/(a^3*b^5)) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^2)/(b^2*c^2 - 2*
a*b*c*d + a^2*d^2))*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*
b*c*d^3 + a^4*d^4)/(a^3*b^5))^(3/4))/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d
^2 - a^3*d^3)) + b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b
```

```
*c*d^3 + a^4*d^4)/(a^3*b^5))^(1/4)*log(a*b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a
^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(a^3*b^5))^(1/4) - (b*c - a*d)*x)
 - b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d
^4)/(a^3*b^5))^(1/4)*log(-a*b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^
2 - 4*a^3*b*c*d^3 + a^4*d^4)/(a^3*b^5))^(1/4) - (b*c - a*d)*x) - 4*d*x)/b
```

Sympy [A] time = 0.633627, size = 87, normalized size = 0.39

$$\operatorname{RootSum}\left(256t^4a^3b^5 + a^4d^4 - 4a^3bcd^3 + 6a^2b^2c^2d^2 - 4ab^3c^3d + b^4c^4, \left(t \mapsto t\log\left(-\frac{4tab}{ad-bc} + x\right)\right)\right) + \frac{dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)/(b*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**5 + a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*b**2*c**
2*d**2 - 4*a*b**3*c**3*d + b**4*c**4, Lambda(_t, _t*log(-4*_t*a*b/(a*d - b*
c) + x))) + d*x/b
```

Giac [A] time = 1.13923, size = 331, normalized size = 1.48

$$\frac{dx}{b} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}bc - \left(ab^3\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^2} + \frac{\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}bc - \left(ab^3\right)^{\frac{1}{4}}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4ab^2} + \frac{\sqrt{2}\left(\left(a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)/(b*x^4+a),x, algorithm="giac")
```

```
[Out] d*x/b + 1/4*sqrt(2)*((a*b^3)^(1/4)*b*c - (a*b^3)^(1/4)*a*d)*arctan(1/2*sqrt
(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a*b^2) + 1/4*sqrt(2)*((a*b^3)
^(1/4)*b*c - (a*b^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/
4))/(a/b)^(1/4))/(a*b^2) + 1/8*sqrt(2)*((a*b^3)^(1/4)*b*c - (a*b^3)^(1/4)*a
*d)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a*b^2) - 1/8*sqrt(2)*((a*
b^3)^(1/4)*b*c - (a*b^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(
a/b))/(a*b^2)
```

3.164 $\int \frac{1}{(a+bx^4)(c+dx^4)}\,dx$

Optimal. Leaf size=449

$$-\frac{b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)}+\frac{b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)}-\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)}+\frac{b^{3/4}\tan^{-1}\left(\sqrt{2}\right.}{2\sqrt{2}a^{3/4}(bc}$$

```
[Out] -(b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(b*c
- a*d)) + (b^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3
/4)*(b*c - a*d)) + (d^(3/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqr
t[2]*c^(3/4)*(b*c - a*d)) - (d^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)
])/(2*Sqrt[2]*c^(3/4)*(b*c - a*d)) - (b^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*(b*c - a*d)) + (b^(3/4)*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*(b*c
- a*d)) + (d^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/
(4*Sqrt[2]*c^(3/4)*(b*c - a*d)) - (d^(3/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^
(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(3/4)*(b*c - a*d))
```

Rubi [A] time = 0.267522, antiderivative size = 449, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {391, 211, 1165, 628, 1162, 617, 204}

$$-\frac{b^{3/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)}+\frac{b^{3/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)}-\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)}+\frac{b^{3/4}\tan^{-1}\left(\sqrt{2}\right.}{2\sqrt{2}a^{3/4}(bc}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] -(b^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(b*c
- a*d)) + (b^(3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3
/4)*(b*c - a*d)) + (d^(3/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqr
t[2]*c^(3/4)*(b*c - a*d)) - (d^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)
])/(2*Sqrt[2]*c^(3/4)*(b*c - a*d)) - (b^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)
*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*(b*c - a*d)) + (b^(3/4)*Log[S
qrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*(b*c
- a*d)) + (d^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/
(4*Sqrt[2]*c^(3/4)*(b*c - a*d)) - (d^(3/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^
(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(3/4)*(b*c - a*d))
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+bx^4\right)\left(c+dx^4\right)}\,dx = \frac{b\int \frac{1}{a+bx^4}\,dx}{bc-ad} - \frac{d\int \frac{1}{c+dx^4}\,dx}{bc-ad}$$

$$= \frac{b\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{2\sqrt{a}(bc-ad)} + \frac{b\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{2\sqrt{a}(bc-ad)} - \frac{d\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx}{2\sqrt{c}(bc-ad)} - \frac{d\int \frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4}\,dx}{2\sqrt{c}(bc-ad)}$$

$$= \frac{\sqrt{b}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}(bc-ad)} + \frac{\sqrt{b}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}(bc-ad)} - \frac{b^{3/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}+2x}{-\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x^2}\,dx}{4\sqrt{2}a^{3/4}(bc-ad)} - \frac{b^{3/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}}-2x}{-\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}-x}{4\sqrt{2}a^{3/4}(bc-ac}$$

$$= -\frac{b^{3/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)} + \frac{b^{3/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)} + \frac{d^{3/4}\log\left(\sqrt{c}-\right.}{4\sqrt{2}c^{3/}}$$

$$= -\frac{b^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)} + \frac{b^{3/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)} + \frac{d^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}(bc-ad)} - \frac{d^{3/4}\tan^{-1}\left(1\right.}{2\sqrt{2}c^{3/4}(b}$$

Mathematica [A] time = 0.128469, size = 340, normalized size = 0.76

$$a^{3/4}d^{3/4}\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right) - a^{3/4}d^{3/4}\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right) + 2a^{3/4}d^{3/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right) - 2a^{3/4}d^{3/}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^4)*(c + d*x^4)),x]
```

```
[Out] (-2*b^(3/4)*c^(3/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*b^(3/4)*c^(
3/4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*a^(3/4)*d^(3/4)*ArcTan[1 -
 (Sqrt[2]*d^(1/4)*x)/c^(1/4)] - 2*a^(3/4)*d^(3/4)*ArcTan[1 + (Sqrt[2]*d^(1/
4)*x)/c^(1/4)] - b^(3/4)*c^(3/4)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x +
Sqrt[b]*x^2] + b^(3/4)*c^(3/4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sq
rt[b]*x^2] + a^(3/4)*d^(3/4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt
[d]*x^2] - a^(3/4)*d^(3/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d
]*x^2])/(4*Sqrt[2]*a^(3/4)*c^(3/4)*(b*c - a*d))
```

Maple [A] time = 0.006, size = 320, normalized size = 0.7

$$\frac{d\sqrt{2}}{(8\,ad-8\,bc)\,c}\sqrt[4]{\frac{c}{d}}\ln\left(\left(x^2+\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x^2-\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right)+\frac{d\sqrt{2}}{(4\,ad-4\,bc)\,c}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)/(d*x^4+c),x)
```

```
[Out] 1/8*d/(a*d-b*c)*(c/d)^(1/4)/c*2^(1/2)*ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(
1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))+1/4*d/(a*d-b*c)*(c/d)^(1/4)/
c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+1/4*d/(a*d-b*c)*(c/d)^(1/4)/c*2^(
1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)-1/8*b/(a*d-b*c)*(1/b*a)^(1/4)/a*2^(1/2
)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/
2)+(1/b*a)^(1/2)))-1/4*b/(a*d-b*c)*(1/b*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(
1/b*a)^(1/4)*x+1)-1/4*b/(a*d-b*c)*(1/b*a)^(1/4)/a*2^(1/2)*arctan(2^(1/2)/(1
/b*a)^(1/4)*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.71658, size = 2668, normalized size = 5.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] -(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 +
 a^7*d^4))^(1/4)*arctan(((a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a
```

```
^5*d^3)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*
c*d^3 + a^7*d^4))^(3/4)*x - (a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2
- a^5*d^3)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6
*b*c*d^3 + a^7*d^4))^(3/4)*sqrt((b^2*x^2 + (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4
*d^2)*sqrt(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*
b*c*d^3 + a^7*d^4)))/b^2))/b^2) + (-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^
2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)*arctan(((b^3*c^5 - 3*a*b^
2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6
*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(3/4)*x - (b^3*c^5 - 3*a
*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d
+ 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(3/4)*sqrt((d^2*x^2 +
 (b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*sqrt(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d +
 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4)))/d^2))/d^2) + 1/4*(-b^
3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*
d^4))^(1/4)*log(b*x + (a*b*c - a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d
+ 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) - 1/4*(-b^3/(a^3*b^4
*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4
)*log(b*x - (a*b*c - a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^
2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^(1/4)) - 1/4*(-d^3/(b^4*c^7 - 4*a*b^3
*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)*log(d*x
+ (b*c^2 - a*c*d)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^
3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)) + 1/4*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6
*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^(1/4)*log(d*x - (b*c^2 -
 a*c*d)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^
3 + a^4*c^3*d^4))^(1/4))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)/(d*x**4+c),x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c),x, algorithm="giac")

[Out] Timed out
```

3.165 $\int \frac{1}{(a+bx^4)(c+dx^4)^2}\,dx$

Optimal. Leaf size=513

$$-\frac{b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)^2}+\frac{b^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)^2}-\frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)^2}+\frac{b^{7/4}\tan^{-1}\left(\sqrt{2}\right.}{2\sqrt{2}a^{3/4}(bc}$$

```
[Out] -(d*x)/(4*c*(b*c - a*d)*(c + d*x^4)) - (b^(7/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)
*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(b*c - a*d)^2) + (b^(7/4)*ArcTan[1 + (Sqrt
[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(b*c - a*d)^2) + (d^(3/4)*(7*b*
c - 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(7/4)*(b*c
 - a*d)^2) - (d^(3/4)*(7*b*c - 3*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4
)])/(8*Sqrt[2]*c^(7/4)*(b*c - a*d)^2) - (b^(7/4)*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*(b*c - a*d)^2) + (b^(7/4)*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*
(b*c - a*d)^2) + (d^(3/4)*(7*b*c - 3*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(
1/4)*x + Sqrt[d]*x^2])/(16*Sqrt[2]*c^(7/4)*(b*c - a*d)^2) - (d^(3/4)*(7*b*c
 - 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(16*Sqrt[
2]*c^(7/4)*(b*c - a*d)^2)
```

Rubi [A] time = 0.419693, antiderivative size = 513, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {414, 522, 211, 1165, 628, 1162, 617, 204}

$$-\frac{b^{7/4}\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)^2}+\frac{b^{7/4}\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)^2}-\frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)^2}+\frac{b^{7/4}\tan^{-1}\left(\sqrt{2}\right.}{2\sqrt{2}a^{3/4}(bc}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)*(c + d*x^4)^2),x]
```

```
[Out] -(d*x)/(4*c*(b*c - a*d)*(c + d*x^4)) - (b^(7/4)*ArcTan[1 - (Sqrt[2]*b^(1/4)
*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(b*c - a*d)^2) + (b^(7/4)*ArcTan[1 + (Sqrt
[2]*b^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(b*c - a*d)^2) + (d^(3/4)*(7*b*
c - 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(7/4)*(b*c
 - a*d)^2) - (d^(3/4)*(7*b*c - 3*a*d)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4
)])/(8*Sqrt[2]*c^(7/4)*(b*c - a*d)^2) - (b^(7/4)*Log[Sqrt[a] - Sqrt[2]*a^(1
/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*(b*c - a*d)^2) + (b^(7/4)*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^(3/4)*
(b*c - a*d)^2) + (d^(3/4)*(7*b*c - 3*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(
1/4)*x + Sqrt[d]*x^2])/(16*Sqrt[2]*c^(7/4)*(b*c - a*d)^2) - (d^(3/4)*(7*b*c
 - 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(16*Sqrt[
2]*c^(7/4)*(b*c - a*d)^2)
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
```

```
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx^4)(c+dx^4)^2}\,dx = -\frac{dx}{4c(bc-ad)(c+dx^4)} + \frac{\int \frac{4bc-3ad-3bdx^4}{(a+bx^4)(c+dx^4)}\,dx}{4c(bc-ad)}$$

$$= -\frac{dx}{4c(bc-ad)(c+dx^4)} + \frac{b^2\int \frac{1}{a+bx^4}\,dx}{(bc-ad)^2} - \frac{(d(7bc-3ad))\int \frac{1}{c+dx^4}\,dx}{4c(bc-ad)^2}$$

$$= -\frac{dx}{4c(bc-ad)(c+dx^4)} + \frac{b^2\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{2\sqrt{a}(bc-ad)^2} + \frac{b^2\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{2\sqrt{a}(bc-ad)^2} - \frac{(d(7bc-3ad))\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}}{8c^{3/2}(bc-ad)^2}$$

$$= -\frac{dx}{4c(bc-ad)(c+dx^4)} + \frac{b^{3/2}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}(bc-ad)^2} + \frac{b^{3/2}\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{4\sqrt{a}(bc-ad)^2} - \frac{b^{7/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{}}{\sqrt[4]{b}}}{-\frac{\sqrt{a}}{\sqrt{b}}-\sqrt{2}}}{4\sqrt{2}a^{3/4}(bc}$$

$$= -\frac{dx}{4c(bc-ad)(c+dx^4)} - \frac{b^{7/4}\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}(bc-ad)^2} + \frac{b^{7/4}\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x\right.}{4\sqrt{2}a^{3/4}(bc-ad)}$$

$$= -\frac{dx}{4c(bc-ad)(c+dx^4)} - \frac{b^{7/4}\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)^2} + \frac{b^{7/4}\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)^2} + \frac{d^{3/4}(7bc-}{8\sqrt{}}$$

Mathematica [A] time = 0.31656, size = 498, normalized size = 0.97

$$8a^{3/4}c^{3/4}dx(ad-bc) + \sqrt{2}a^{3/4}d^{3/4}\left(c+dx^4\right)(7bc-3ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right) + \sqrt{2}a^{3/4}d^{3/4}\left(c+dx^4\right)(3ad-$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^4)*(c + d*x^4)^2),x]
```

```
[Out] (8*a^(3/4)*c^(3/4)*d*(-(b*c) + a*d)*x - 8*Sqrt[2]*b^(7/4)*c^(7/4)*(c + d*x^
4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 8*Sqrt[2]*b^(7/4)*c^(7/4)*(c +
 d*x^4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 2*Sqrt[2]*a^(3/4)*d^(3/4)
*(-7*b*c + 3*a*d)*(c + d*x^4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)] + 2*S
qrt[2]*a^(3/4)*d^(3/4)*(-7*b*c + 3*a*d)*(c + d*x^4)*ArcTan[1 + (Sqrt[2]*d^(
1/4)*x)/c^(1/4)] - 4*Sqrt[2]*b^(7/4)*c^(7/4)*(c + d*x^4)*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 4*Sqrt[2]*b^(7/4)*c^(7/4)*(c + d*x^4
)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*a^(3/4)*
d^(3/4)*(7*b*c - 3*a*d)*(c + d*x^4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x
 + Sqrt[d]*x^2] + Sqrt[2]*a^(3/4)*d^(3/4)*(-7*b*c + 3*a*d)*(c + d*x^4)*Log[
Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(32*a^(3/4)*c^(7/4)*(b*
c - a*d)^2*(c + d*x^4))
```

Maple [A] time = 0.01, size = 550, normalized size = 1.1

$$\frac{d^2xa}{4\,(ad-bc)^2\,c\left(dx^4+c\right)} - \frac{bdx}{4\,(ad-bc)^2\left(dx^4+c\right)} + \frac{3\,d^2\sqrt{2}a}{16\,(ad-bc)^2\,c^2}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right) - \frac{7\,d\sqrt{2}b}{16\,(ad-bc)^2\,c}\sqrt[4]{\frac{c}{d}}\,\mathrm{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)/(d*x^4+c)^2,x)
```

```
[Out] 1/4*d^2/(a*d-b*c)^2/c*x/(d*x^4+c)*a-1/4*d/(a*d-b*c)^2*x/(d*x^4+c)*b+3/16*d^
2/(a*d-b*c)^2/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)*a-7/1
6*d/(a*d-b*c)^2/c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)*b+3/1
6*d^2/(a*d-b*c)^2/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*a
-7/16*d/(a*d-b*c)^2/c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*b
+3/32*d^2/(a*d-b*c)^2/c^2*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*x*2^(1/2)
+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))*a-7/32*d/(a*d-b*c)^2
/c*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d
)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))*b+1/8*b^2/(a*d-b*c)^2*(1/b*a)^(1/4)/a*2^(1/
2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1
/2)+(1/b*a)^(1/2)))+1/4*b^2/(a*d-b*c)^2*(1/b*a)^(1/4)/a*2^(1/2)*arctan(2^(1
/2)/(1/b*a)^(1/4)*x+1)+1/4*b^2/(a*d-b*c)^2*(1/b*a)^(1/4)/a*2^(1/2)*arctan(2
^(1/2)/(1/b*a)^(1/4)*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 117.466, size = 6808, normalized size = 13.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*((b*c^2*d - a*c*d^2)*x^4 + b*c^3 - a*c^2*d)*(-(2401*b^4*c^4*d^3 - 4
116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b
^8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a
^4*b^4*c^11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^
7 + a^8*c^7*d^8))^(1/4)*arctan(((b^6*c^11 - 6*a*b^5*c^10*d + 15*a^2*b^4*c^9
*d^2 - 20*a^3*b^3*c^8*d^3 + 15*a^4*b^2*c^7*d^4 - 6*a^5*b*c^6*d^5 + a^6*c^5*
d^6)*x*(-(2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 75
6*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^
2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a^
6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8))^(3/4) - (b^6*c^11 - 6*a*b^5
*c^10*d + 15*a^2*b^4*c^9*d^2 - 20*a^3*b^3*c^8*d^3 + 15*a^4*b^2*c^7*d^4 - 6*
a^5*b*c^6*d^5 + a^6*c^5*d^6)*(-(2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 264
6*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*
d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^
5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8))^(3/4)
*sqrt(((49*b^2*c^2*d^2 - 42*a*b*c*d^3 + 9*a^2*d^4)*x^2 + (b^4*c^8 - 4*a*b^3
*c^7*d + 6*a^2*b^2*c^6*d^2 - 4*a^3*b*c^5*d^3 + a^4*c^4*d^4)*sqrt(-(2401*b^4
*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81
*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^1
2*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*
```

```
a^7*b*c^8*d^7 + a^8*c^7*d^8)))/(49*b^2*c^2*d^2 - 42*a*b*c*d^3 + 9*a^2*d^4))
)/(343*b^3*c^3*d^2 - 441*a*b^2*c^2*d^3 + 189*a^2*b*c*d^4 - 27*a^3*d^5)) + 1
6*(-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^
5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^
10*b*c*d^7 + a^11*d^8))^(1/4)*((b*c^2*d - a*c*d^2)*x^4 + b*c^3 - a*c^2*d)*a
rctan(-((a^2*b^6*c^6 - 6*a^3*b^5*c^5*d + 15*a^4*b^4*c^4*d^2 - 20*a^5*b^3*c^
3*d^3 + 15*a^6*b^2*c^2*d^4 - 6*a^7*b*c*d^5 + a^8*d^6)*(-b^7/(a^3*b^8*c^8 -
8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*
d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8))
^(3/4)*x - (a^2*b^6*c^6 - 6*a^3*b^5*c^5*d + 15*a^4*b^4*c^4*d^2 - 20*a^5*b^3
*c^3*d^3 + 15*a^6*b^2*c^2*d^4 - 6*a^7*b*c*d^5 + a^8*d^6)*(-b^7/(a^3*b^8*c^8
 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c
^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^
8))^(3/4)*sqrt((b^4*x^2 + (a^2*b^4*c^4 - 4*a^3*b^3*c^3*d + 6*a^4*b^2*c^2*d^
2 - 4*a^5*b*c*d^3 + a^6*d^4)*sqrt(-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*
a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*
d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c*d^7 + a^11*d^8)))/b^4))/b^5) + 4*(-b^
7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d^3
+ 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*b*c
*d^7 + a^11*d^8))^(1/4)*((b*c^2*d - a*c*d^2)*x^4 + b*c^3 - a*c^2*d)*log(b^2
*x + (-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5
*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8
*a^10*b*c*d^7 + a^11*d^8))^(1/4)*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)) - 4*(
-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*b^5*c^5*d
^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6 - 8*a^10*
b*c*d^7 + a^11*d^8))^(1/4)*((b*c^2*d - a*c*d^2)*x^4 + b*c^3 - a*c^2*d)*log(
b^2*x - (-b^7/(a^3*b^8*c^8 - 8*a^4*b^7*c^7*d + 28*a^5*b^6*c^6*d^2 - 56*a^6*
b^5*c^5*d^3 + 70*a^7*b^4*c^4*d^4 - 56*a^8*b^3*c^3*d^5 + 28*a^9*b^2*c^2*d^6
- 8*a^10*b*c*d^7 + a^11*d^8))^(1/4)*(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)) +
((b*c^2*d - a*c*d^2)*x^4 + b*c^3 - a*c^2*d)*(-(2401*b^4*c^4*d^3 - 4116*a*b^
3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^15
- 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c
^11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*
c^7*d^8))^(1/4)*log(-(7*b*c*d - 3*a*d^2)*x + (b^2*c^4 - 2*a*b*c^3*d + a^2*c
^2*d^2)*(-(2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 7
56*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d
^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a
^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8))^(1/4)) - ((b*c^2*d - a*c*d
^2)*x^4 + b*c^3 - a*c^2*d)*(-(2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*
a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d
+ 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^5*
b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8))^(1/4)*l
og(-(7*b*c*d - 3*a*d^2)*x - (b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*(-(2401*b
^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 +
81*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c
^12*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 -
8*a^7*b*c^8*d^7 + a^8*c^7*d^8))^(1/4)) - 4*d*x)/((b*c^2*d - a*c*d^2)*x^4 +
b*c^3 - a*c^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)/(d*x**4+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 3.01974, size = 900, normalized size = 1.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)/(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(a*b^3)^(1/4)*b*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1
/4))/(sqrt(2)*a*b^2*c^2 - 2*sqrt(2)*a^2*b*c*d + sqrt(2)*a^3*d^2) + 1/2*(a*b
^3)^(1/4)*b*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sq
rt(2)*a*b^2*c^2 - 2*sqrt(2)*a^2*b*c*d + sqrt(2)*a^3*d^2) + 1/4*(a*b^3)^(1/4
)*b*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a*b^2*c^2 - 2*sqr
t(2)*a^2*b*c*d + sqrt(2)*a^3*d^2) - 1/4*(a*b^3)^(1/4)*b*log(x^2 - sqrt(2)*x
*(a/b)^(1/4) + sqrt(a/b))/(sqrt(2)*a*b^2*c^2 - 2*sqrt(2)*a^2*b*c*d + sqrt(2
)*a^3*d^2) - 1/8*(7*(c*d^3)^(1/4)*b*c - 3*(c*d^3)^(1/4)*a*d)*arctan(1/2*sqr
t(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2)*b^2*c^4 - 2*sqrt(2)*
a*b*c^3*d + sqrt(2)*a^2*c^2*d^2) - 1/8*(7*(c*d^3)^(1/4)*b*c - 3*(c*d^3)^(1/
4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sqrt(2
)*b^2*c^4 - 2*sqrt(2)*a*b*c^3*d + sqrt(2)*a^2*c^2*d^2) - 1/16*(7*(c*d^3)^(1
/4)*b*c - 3*(c*d^3)^(1/4)*a*d)*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))
/(sqrt(2)*b^2*c^4 - 2*sqrt(2)*a*b*c^3*d + sqrt(2)*a^2*c^2*d^2) + 1/16*(7*(c
*d^3)^(1/4)*b*c - 3*(c*d^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x*(c/d)^(1/4) + sq
rt(c/d))/(sqrt(2)*b^2*c^4 - 2*sqrt(2)*a*b*c^3*d + sqrt(2)*a^2*c^2*d^2) - 1/
4*d*x/((d*x^4 + c)*(b*c^2 - a*c*d))
```

3.166 $\int \frac{(c+dx^4)^5}{(a+bx^4)^2}\,dx$

Optimal. Leaf size=407

$$\frac{d^3x^5\left(3a^2d^2-10abcd+10b^2c^2\right)}{5b^4}+\frac{d^2x\left(15a^2bcd^2-4a^3d^3-20ab^2c^2d+10b^3c^3\right)}{b^5}-\frac{(bc-ad)^4(17ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\right.}{16\sqrt{2}a^{7/4}b^{21/4}}$$

```
[Out] (d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3)*x)/b^5 + (d
^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^5)/(5*b^4) + (d^4*(5*b*c - 2*a*d
)*x^9)/(9*b^3) + (d^5*x^13)/(13*b^2) + ((b*c - a*d)^5*x)/(4*a*b^5*(a + b*x^
4)) - ((b*c - a*d)^4*(3*b*c + 17*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4
)])/(8*Sqrt[2]*a^(7/4)*b^(21/4)) + ((b*c - a*d)^4*(3*b*c + 17*a*d)*ArcTan[1
 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(21/4)) - ((b*c - a*d
)^4*(3*b*c + 17*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]
)/(16*Sqrt[2]*a^(7/4)*b^(21/4)) + ((b*c - a*d)^4*(3*b*c + 17*a*d)*Log[Sqrt[
a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(21/4)
)
```

Rubi [A] time = 0.395549, antiderivative size = 407, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {390, 385, 211, 1165, 628, 1162, 617, 204}

$$\frac{d^3x^5\left(3a^2d^2-10abcd+10b^2c^2\right)}{5b^4}+\frac{d^2x\left(15a^2bcd^2-4a^3d^3-20ab^2c^2d+10b^3c^3\right)}{b^5}-\frac{(bc-ad)^4(17ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\right.}{16\sqrt{2}a^{7/4}b^{21/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^4)^5/(a + b*x^4)^2,x]
```


```
[Out] (d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3)*x)/b^5 + (d
^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^5)/(5*b^4) + (d^4*(5*b*c - 2*a*d
)*x^9)/(9*b^3) + (d^5*x^13)/(13*b^2) + ((b*c - a*d)^5*x)/(4*a*b^5*(a + b*x^
4)) - ((b*c - a*d)^4*(3*b*c + 17*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4
)])/(8*Sqrt[2]*a^(7/4)*b^(21/4)) + ((b*c - a*d)^4*(3*b*c + 17*a*d)*ArcTan[1
 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(21/4)) - ((b*c - a*d
)^4*(3*b*c + 17*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2]
)/(16*Sqrt[2]*a^(7/4)*b^(21/4)) + ((b*c - a*d)^4*(3*b*c + 17*a*d)*Log[Sqrt[
a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(21/4)
)
```


Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
```

```
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(c+dx^4\right)^5}{\left(a+bx^4\right)^2}\,dx = \int \left(\frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)}{b^5}+\frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^4}{b^4}+\frac{d^4(5bc-2ac}{b^3}\right.$$

$$=\frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)x}{b^5}+\frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^5}{5b^4}+\frac{d^4(5bc-2ad)x}{9b^3}$$

$$=\frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)x}{b^5}+\frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^5}{5b^4}+\frac{d^4(5bc-2ad)x}{9b^3}$$

$$=\frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)x}{b^5}+\frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^5}{5b^4}+\frac{d^4(5bc-2ad)x}{9b^3}$$

$$=\frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)x}{b^5}+\frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^5}{5b^4}+\frac{d^4(5bc-2ad)x}{9b^3}$$

$$=\frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)x}{b^5}+\frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^5}{5b^4}+\frac{d^4(5bc-2ad)x}{9b^3}$$

$$=\frac{d^2\left(10b^3c^3-20ab^2c^2d+15a^2bcd^2-4a^3d^3\right)x}{b^5}+\frac{d^3\left(10b^2c^2-10abcd+3a^2d^2\right)x^5}{5b^4}+\frac{d^4(5bc-2ad)x}{9b^3}$$

Mathematica [A] time = 0.36496, size = 391, normalized size = 0.96

$$3744b^{5/4}d^3x^5\left(3a^2d^2-10abcd+10b^2c^2\right)+18720\sqrt[4]{b}d^2x\left(15a^2bcd^2-4a^3d^3-20ab^2c^2d+10b^3c^3\right)-\frac{585\sqrt{2}(bc-ad)^4(17ad+3b}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^4)^5/(a + b*x^4)^2,x]
```

```
[Out] (18720*b^(1/4)*d^2*(10*b^3*c^3 - 20*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^
3)*x + 3744*b^(5/4)*d^3*(10*b^2*c^2 - 10*a*b*c*d + 3*a^2*d^2)*x^5 + 2080*b^
(9/4)*d^4*(5*b*c - 2*a*d)*x^9 + 1440*b^(13/4)*d^5*x^13 + (4680*b^(1/4)*(b*c
 - a*d)^5*x)/(a*(a + b*x^4)) - (1170*Sqrt[2]*(b*c - a*d)^4*(3*b*c + 17*a*d)
*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (1170*Sqrt[2]*(b*c - a*
d)^4*(3*b*c + 17*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) - (5
85*Sqrt[2]*(b*c - a*d)^4*(3*b*c + 17*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(
1/4)*x + Sqrt[b]*x^2])/a^(7/4) + (585*Sqrt[2]*(b*c - a*d)^4*(3*b*c + 17*a*d
)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4))/(18720*b
^(21/4))
```

Maple [B] time = 0.012, size = 1118, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^5/(b*x^4+a)^2,x)
```

```
[Out] -20*d^3/b^3*a*c^2*x+3/16/a^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(
1/4)*x+1)*c^5+1/13*d^5*x^13/b^2+45/8/b^3*a*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(1/b*a)^(1/4)*x-1)*c^2*d^3+5/16/b/a*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/
2)/(1/b*a)^(1/4)*x-1)*c^4*d-65/32/b^4*a^2*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/
b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1
/2)))*c*d^4+45/16/b^3*a*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/
2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))*c^2*d^3+5/32
/b/a*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(
x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))*c^4*d-65/16/b^4*a^2*(1/b*a)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)*c*d^4+45/8/b^3*a*(1/b*a)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)*c^2*d^3+5/16/b/a*(1/b*a)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)*c^4*d-65/16/b^4*a^2*(1/b*a)^(1/4)*
2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*c*d^4+17/16/b^5*a^3*(1/b*a)^(1/4)
*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)*d^5-25/8/b^2*(1/b*a)^(1/4)*2^(1/
2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)*c^3*d^2+17/16/b^5*a^3*(1/b*a)^(1/4)*2^
(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*d^5+5/4/b^4*a^3*x/(b*x^4+a)*c*d^4-5
/2/b^3*a^2*x/(b*x^4+a)*c^2*d^3+5/2/b^2*a*x/(b*x^4+a)*c^3*d^2-4*d^5/b^5*a^3*
x+10*d^2/b^2*c^3*x-25/8/b^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1
/4)*x-1)*c^3*d^2+17/32/b^5*a^3*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*
x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))*d^5-2
5/16/b^2*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2
))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))*c^3*d^2-2*d^4/b^3*x^5*a*c-1
/4/b^5*a^4*x/(b*x^4+a)*d^5-5/4/b*x/(b*x^4+a)*c^4*d+3/16/a^2*(1/b*a)^(1/4)*2
^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*c^5+3/32/a^2*(1/b*a)^(1/4)*2^(1/2)
*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2
)+(1/b*a)^(1/2)))*c^5+15*d^4/b^4*a^2*c*x+5/9*d^4/b^2*x^9*c+3/5*d^5/b^4*x^5*
a^2+2*d^3/b^2*x^5*c^2+1/4/a*x/(b*x^4+a)*c^5-2/9*d^5/b^3*x^9*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^5/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.34698, size = 8400, normalized size = 20.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^5/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/9360*(720*a*b^4*d^5*x^17 + 80*(65*a*b^4*c*d^4 - 17*a^2*b^3*d^5)*x^13 + 20
8*(90*a*b^4*c^2*d^3 - 65*a^2*b^3*c*d^4 + 17*a^3*b^2*d^5)*x^9 + 1872*(50*a*b
^4*c^3*d^2 - 90*a^2*b^3*c^2*d^3 + 65*a^3*b^2*c*d^4 - 17*a^4*b*d^5)*x^5 + 23
40*(a*b^6*x^4 + a^2*b^5)*(-(81*b^20*c^20 + 540*a*b^19*c^19*d - 4050*a^2*b^1
8*c^18*d^2 - 15780*a^3*b^17*c^17*d^3 + 132205*a^4*b^16*c^16*d^4 - 13264*a^5
*b^15*c^15*d^5 - 1960920*a^6*b^14*c^14*d^6 + 6137200*a^7*b^13*c^13*d^7 - 50
0110*a^8*b^12*c^12*d^8 - 48530040*a^9*b^11*c^11*d^9 + 174873556*a^10*b^10*c
^10*d^10 - 360900280*a^11*b^9*c^9*d^11 + 517559250*a^12*b^8*c^8*d^12 - 5482
31440*a^13*b^7*c^7*d^13 + 438700840*a^14*b^6*c^6*d^14 - 266040144*a^15*b^5*
```

c^5*d^15 + 120836285*a^16*b^4*c^4*d^16 - 39944900*a^17*b^3*c^3*d^17 + 90948
30*a^18*b^2*c^2*d^18 - 1277380*a^19*b*c*d^19 + 83521*a^20*d^20)/(a^7*b^21))
^(1/4)*arctan(-(a^5*b^16*x*(-(81*b^20*c^20 + 540*a*b^19*c^19*d - 4050*a^2*b
^18*c^18*d^2 - 15780*a^3*b^17*c^17*d^3 + 132205*a^4*b^16*c^16*d^4 - 13264*a
^5*b^15*c^15*d^5 - 1960920*a^6*b^14*c^14*d^6 + 6137200*a^7*b^13*c^13*d^7 -
500110*a^8*b^12*c^12*d^8 - 48530040*a^9*b^11*c^11*d^9 + 174873556*a^10*b^10
*c^10*d^10 - 360900280*a^11*b^9*c^9*d^11 + 517559250*a^12*b^8*c^8*d^12 - 54
8231440*a^13*b^7*c^7*d^13 + 438700840*a^14*b^6*c^6*d^14 - 266040144*a^15*b^
5*c^5*d^15 + 120836285*a^16*b^4*c^4*d^16 - 39944900*a^17*b^3*c^3*d^17 + 909
4830*a^18*b^2*c^2*d^18 - 1277380*a^19*b*c*d^19 + 83521*a^20*d^20)/(a^7*b^21
))^(3/4) - a^5*b^16*sqrt((a^4*b^10*sqrt(-(81*b^20*c^20 + 540*a*b^19*c^19*d
- 4050*a^2*b^18*c^18*d^2 - 15780*a^3*b^17*c^17*d^3 + 132205*a^4*b^16*c^16*d
^4 - 13264*a^5*b^15*c^15*d^5 - 1960920*a^6*b^14*c^14*d^6 + 6137200*a^7*b^13
*c^13*d^7 - 500110*a^8*b^12*c^12*d^8 - 48530040*a^9*b^11*c^11*d^9 + 1748735
56*a^10*b^10*c^10*d^10 - 360900280*a^11*b^9*c^9*d^11 + 517559250*a^12*b^8*c
^8*d^12 - 548231440*a^13*b^7*c^7*d^13 + 438700840*a^14*b^6*c^6*d^14 - 26604
0144*a^15*b^5*c^5*d^15 + 120836285*a^16*b^4*c^4*d^16 - 39944900*a^17*b^3*c^
3*d^17 + 9094830*a^18*b^2*c^2*d^18 - 1277380*a^19*b*c*d^19 + 83521*a^20*d^2
0)/(a^7*b^21)) + (9*b^10*c^10 + 30*a*b^9*c^9*d - 275*a^2*b^8*c^8*d^2 + 40*a
^3*b^7*c^7*d^3 + 3010*a^4*b^6*c^6*d^4 - 9548*a^5*b^5*c^5*d^5 + 14770*a^6*b^
4*c^4*d^6 - 13400*a^7*b^3*c^3*d^7 + 7285*a^8*b^2*c^2*d^8 - 2210*a^9*b*c*d^9
 + 289*a^10*d^10)*x^2)/(9*b^10*c^10 + 30*a*b^9*c^9*d - 275*a^2*b^8*c^8*d^2
+ 40*a^3*b^7*c^7*d^3 + 3010*a^4*b^6*c^6*d^4 - 9548*a^5*b^5*c^5*d^5 + 14770*
a^6*b^4*c^4*d^6 - 13400*a^7*b^3*c^3*d^7 + 7285*a^8*b^2*c^2*d^8 - 2210*a^9*b
*c*d^9 + 289*a^10*d^10))*(-(81*b^20*c^20 + 540*a*b^19*c^19*d - 4050*a^2*b^1
8*c^18*d^2 - 15780*a^3*b^17*c^17*d^3 + 132205*a^4*b^16*c^16*d^4 - 13264*a^5
*b^15*c^15*d^5 - 1960920*a^6*b^14*c^14*d^6 + 6137200*a^7*b^13*c^13*d^7 - 50
0110*a^8*b^12*c^12*d^8 - 48530040*a^9*b^11*c^11*d^9 + 174873556*a^10*b^10*c
^10*d^10 - 360900280*a^11*b^9*c^9*d^11 + 517559250*a^12*b^8*c^8*d^12 - 5482
31440*a^13*b^7*c^7*d^13 + 438700840*a^14*b^6*c^6*d^14 - 266040144*a^15*b^5*
c^5*d^15 + 120836285*a^16*b^4*c^4*d^16 - 39944900*a^17*b^3*c^3*d^17 + 90948
30*a^18*b^2*c^2*d^18 - 1277380*a^19*b*c*d^19 + 83521*a^20*d^20)/(a^7*b^21))
^(3/4))/(27*b^15*c^15 + 135*a*b^14*c^14*d - 1125*a^2*b^13*c^13*d^2 - 1945*a
^3*b^12*c^12*d^3 + 25095*a^4*b^11*c^11*d^4 - 42141*a^5*b^10*c^10*d^5 - 1319
45*a^6*b^9*c^9*d^6 + 774675*a^7*b^8*c^8*d^7 - 1837935*a^8*b^7*c^7*d^8 + 270
0885*a^9*b^6*c^6*d^9 - 2702799*a^10*b^5*c^5*d^10 + 1889685*a^11*b^4*c^4*d^1
1 - 914675*a^12*b^3*c^3*d^12 + 293505*a^13*b^2*c^2*d^13 - 56355*a^14*b*c*d^
14 + 4913*a^15*d^15)) + 585*(a*b^6*x^4 + a^2*b^5)*(-(81*b^20*c^20 + 540*a*b
^19*c^19*d - 4050*a^2*b^18*c^18*d^2 - 15780*a^3*b^17*c^17*d^3 + 132205*a^4*
b^16*c^16*d^4 - 13264*a^5*b^15*c^15*d^5 - 1960920*a^6*b^14*c^14*d^6 + 61372
00*a^7*b^13*c^13*d^7 - 500110*a^8*b^12*c^12*d^8 - 48530040*a^9*b^11*c^11*d^
9 + 174873556*a^10*b^10*c^10*d^10 - 360900280*a^11*b^9*c^9*d^11 + 517559250
*a^12*b^8*c^8*d^12 - 548231440*a^13*b^7*c^7*d^13 + 438700840*a^14*b^6*c^6*d
^14 - 266040144*a^15*b^5*c^5*d^15 + 120836285*a^16*b^4*c^4*d^16 - 39944900*
a^17*b^3*c^3*d^17 + 9094830*a^18*b^2*c^2*d^18 - 1277380*a^19*b*c*d^19 + 835
21*a^20*d^20)/(a^7*b^21))^(1/4)*log(a^2*b^5*(-(81*b^20*c^20 + 540*a*b^19*c^
19*d - 4050*a^2*b^18*c^18*d^2 - 15780*a^3*b^17*c^17*d^3 + 132205*a^4*b^16*c
^16*d^4 - 13264*a^5*b^15*c^15*d^5 - 1960920*a^6*b^14*c^14*d^6 + 6137200*a^7
*b^13*c^13*d^7 - 500110*a^8*b^12*c^12*d^8 - 48530040*a^9*b^11*c^11*d^9 + 17
4873556*a^10*b^10*c^10*d^10 - 360900280*a^11*b^9*c^9*d^11 + 517559250*a^12*
b^8*c^8*d^12 - 548231440*a^13*b^7*c^7*d^13 + 438700840*a^14*b^6*c^6*d^14 -
266040144*a^15*b^5*c^5*d^15 + 120836285*a^16*b^4*c^4*d^16 - 39944900*a^17*b
^3*c^3*d^17 + 9094830*a^18*b^2*c^2*d^18 - 1277380*a^19*b*c*d^19 + 83521*a^2
0*d^20)/(a^7*b^21))^(1/4) + (3*b^5*c^5 + 5*a*b^4*c^4*d - 50*a^2*b^3*c^3*d^2
 + 90*a^3*b^2*c^2*d^3 - 65*a^4*b*c*d^4 + 17*a^5*d^5)*x) - 585*(a*b^6*x^4 +
a^2*b^5)*(-(81*b^20*c^20 + 540*a*b^19*c^19*d - 4050*a^2*b^18*c^18*d^2 - 157
80*a^3*b^17*c^17*d^3 + 132205*a^4*b^16*c^16*d^4 - 13264*a^5*b^15*c^15*d^5 -
 1960920*a^6*b^14*c^14*d^6 + 6137200*a^7*b^13*c^13*d^7 - 500110*a^8*b^12*c^
12*d^8 - 48530040*a^9*b^11*c^11*d^9 + 174873556*a^10*b^10*c^10*d^10 - 36090

```
0280*a^11*b^9*c^9*d^11 + 517559250*a^12*b^8*c^8*d^12 - 548231440*a^13*b^7*c
^7*d^13 + 438700840*a^14*b^6*c^6*d^14 - 266040144*a^15*b^5*c^5*d^15 + 12083
6285*a^16*b^4*c^4*d^16 - 39944900*a^17*b^3*c^3*d^17 + 9094830*a^18*b^2*c^2*
d^18 - 1277380*a^19*b*c*d^19 + 83521*a^20*d^20)/(a^7*b^21))^(1/4)*log(-a^2*
b^5*(-(81*b^20*c^20 + 540*a*b^19*c^19*d - 4050*a^2*b^18*c^18*d^2 - 15780*a^
3*b^17*c^17*d^3 + 132205*a^4*b^16*c^16*d^4 - 13264*a^5*b^15*c^15*d^5 - 1960
920*a^6*b^14*c^14*d^6 + 6137200*a^7*b^13*c^13*d^7 - 500110*a^8*b^12*c^12*d^
8 - 48530040*a^9*b^11*c^11*d^9 + 174873556*a^10*b^10*c^10*d^10 - 360900280*
a^11*b^9*c^9*d^11 + 517559250*a^12*b^8*c^8*d^12 - 548231440*a^13*b^7*c^7*d^
13 + 438700840*a^14*b^6*c^6*d^14 - 266040144*a^15*b^5*c^5*d^15 + 120836285*
a^16*b^4*c^4*d^16 - 39944900*a^17*b^3*c^3*d^17 + 9094830*a^18*b^2*c^2*d^18
- 1277380*a^19*b*c*d^19 + 83521*a^20*d^20)/(a^7*b^21))^(1/4) + (3*b^5*c^5 +
 5*a*b^4*c^4*d - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 - 65*a^4*b*c*d^4 +
 17*a^5*d^5)*x) + 2340*(b^5*c^5 - 5*a*b^4*c^4*d + 50*a^2*b^3*c^3*d^2 - 90*a
^3*b^2*c^2*d^3 + 65*a^4*b*c*d^4 - 17*a^5*d^5)*x)/(a*b^6*x^4 + a^2*b^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**5/(b*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.11543, size = 1077, normalized size = 2.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^5/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b^5*c^5 + 5*(a*b^3)^(1/4)*a*b^4*c^4*d - 50*(a
*b^3)^(1/4)*a^2*b^3*c^3*d^2 + 90*(a*b^3)^(1/4)*a^3*b^2*c^2*d^3 - 65*(a*b^3)
^(1/4)*a^4*b*c*d^4 + 17*(a*b^3)^(1/4)*a^5*d^5)*arctan(1/2*sqrt(2)*(2*x + sq
rt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^6) + 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b
^5*c^5 + 5*(a*b^3)^(1/4)*a*b^4*c^4*d - 50*(a*b^3)^(1/4)*a^2*b^3*c^3*d^2 + 9
0*(a*b^3)^(1/4)*a^3*b^2*c^2*d^3 - 65*(a*b^3)^(1/4)*a^4*b*c*d^4 + 17*(a*b^3)
^(1/4)*a^5*d^5)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))
/(a^2*b^6) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^5*c^5 + 5*(a*b^3)^(1/4)*a*b^4*
c^4*d - 50*(a*b^3)^(1/4)*a^2*b^3*c^3*d^2 + 90*(a*b^3)^(1/4)*a^3*b^2*c^2*d^3
 - 65*(a*b^3)^(1/4)*a^4*b*c*d^4 + 17*(a*b^3)^(1/4)*a^5*d^5)*log(x^2 + sqrt(
2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^6) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^5
*c^5 + 5*(a*b^3)^(1/4)*a*b^4*c^4*d - 50*(a*b^3)^(1/4)*a^2*b^3*c^3*d^2 + 90*
(a*b^3)^(1/4)*a^3*b^2*c^2*d^3 - 65*(a*b^3)^(1/4)*a^4*b*c*d^4 + 17*(a*b^3)^(
1/4)*a^5*d^5)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^6) + 1/4*
(b^5*c^5*x - 5*a*b^4*c^4*d*x + 10*a^2*b^3*c^3*d^2*x - 10*a^3*b^2*c^2*d^3*x
+ 5*a^4*b*c*d^4*x - a^5*d^5*x)/((b*x^4 + a)*a*b^5) + 1/585*(45*b^24*d^5*x^1
3 + 325*b^24*c*d^4*x^9 - 130*a*b^23*d^5*x^9 + 1170*b^24*c^2*d^3*x^5 - 1170*
a*b^23*c*d^4*x^5 + 351*a^2*b^22*d^5*x^5 + 5850*b^24*c^3*d^2*x - 11700*a*b^2
3*c^2*d^3*x + 8775*a^2*b^22*c*d^4*x - 2340*a^3*b^21*d^5*x)/b^26
```

3.167 $\int \frac{(c+dx^4)^4}{(a+bx^4)^2}\,dx$

Optimal. Leaf size=357

$$\frac{d^2x\left(3a^2d^2-8abcd+6b^2c^2\right)}{b^4}-\frac{(bc-ad)^3(13ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{17/4}}+\frac{(bc-ad)^3(13ad+3bc)\log\left(\right.}{16\sqrt{2}a^{7/}}$$

```
[Out] (d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x)/b^4 + (2*d^3*(2*b*c - a*d)*x^5)
/(5*b^3) + (d^4*x^9)/(9*b^2) + ((b*c - a*d)^4*x)/(4*a*b^4*(a + b*x^4)) - ((
b*c - a*d)^3*(3*b*c + 13*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*S
qrt[2]*a^(7/4)*b^(17/4)) + ((b*c - a*d)^3*(3*b*c + 13*a*d)*ArcTan[1 + (Sqrt
[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(17/4)) - ((b*c - a*d)^3*(3*b
*c + 13*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sq
rt[2]*a^(7/4)*b^(17/4)) + ((b*c - a*d)^3*(3*b*c + 13*a*d)*Log[Sqrt[a] + Sqr
t[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(17/4))
```

Rubi [A] time = 0.36656, antiderivative size = 357, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {390, 385, 211, 1165, 628, 1162, 617, 204}

$$\frac{d^2x\left(3a^2d^2-8abcd+6b^2c^2\right)}{b^4}-\frac{(bc-ad)^3(13ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{17/4}}+\frac{(bc-ad)^3(13ad+3bc)\log\left(\right.}{16\sqrt{2}a^{7/}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^4)^4/(a + b*x^4)^2,x]
```


```
[Out] (d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x)/b^4 + (2*d^3*(2*b*c - a*d)*x^5)
/(5*b^3) + (d^4*x^9)/(9*b^2) + ((b*c - a*d)^4*x)/(4*a*b^4*(a + b*x^4)) - ((
b*c - a*d)^3*(3*b*c + 13*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*S
qrt[2]*a^(7/4)*b^(17/4)) + ((b*c - a*d)^3*(3*b*c + 13*a*d)*ArcTan[1 + (Sqrt
[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(17/4)) - ((b*c - a*d)^3*(3*b
*c + 13*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sq
rt[2]*a^(7/4)*b^(17/4)) + ((b*c - a*d)^3*(3*b*c + 13*a*d)*Log[Sqrt[a] + Sqr
t[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(17/4))
```


Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(c+dx^4)^4}{(a+bx^4)^2}\,dx = \int \left(\frac{d^2(6b^2c^2-8abcd+3a^2d^2)}{b^4} + \frac{2d^3(2bc-ad)x^4}{b^3} + \frac{d^4x^8}{b^2} + \frac{(bc-ad)^3(bc+3ad)+4bd(bc-ad)^3}{b^4(a+bx^4)^2}\right.$$

$$= \frac{d^2(6b^2c^2-8abcd+3a^2d^2)x}{b^4} + \frac{2d^3(2bc-ad)x^5}{5b^3} + \frac{d^4x^9}{9b^2} + \frac{\int \frac{(bc-ad)^3(bc+3ad)+4bd(bc-ad)^3x^4}{(a+bx^4)^2}dx}{b^4}$$

$$= \frac{d^2(6b^2c^2-8abcd+3a^2d^2)x}{b^4} + \frac{2d^3(2bc-ad)x^5}{5b^3} + \frac{d^4x^9}{9b^2} + \frac{(bc-ad)^4x}{4ab^4(a+bx^4)} + \frac{((bc-ad)^3(3bc+13a}{4ab^4}$$

$$= \frac{d^2(6b^2c^2-8abcd+3a^2d^2)x}{b^4} + \frac{2d^3(2bc-ad)x^5}{5b^3} + \frac{d^4x^9}{9b^2} + \frac{(bc-ad)^4x}{4ab^4(a+bx^4)} + \frac{((bc-ad)^3(3bc+13a}{8a^{3/2}b}$$

$$= \frac{d^2(6b^2c^2-8abcd+3a^2d^2)x}{b^4} + \frac{2d^3(2bc-ad)x^5}{5b^3} + \frac{d^4x^9}{9b^2} + \frac{(bc-ad)^4x}{4ab^4(a+bx^4)} + \frac{((bc-ad)^3(3bc+13a}{16a^3}$$

$$= \frac{d^2(6b^2c^2-8abcd+3a^2d^2)x}{b^4} + \frac{2d^3(2bc-ad)x^5}{5b^3} + \frac{d^4x^9}{9b^2} + \frac{(bc-ad)^4x}{4ab^4(a+bx^4)} - \frac{(bc-ad)^3(3bc+13ad}{1}$$

$$= \frac{d^2(6b^2c^2-8abcd+3a^2d^2)x}{b^4} + \frac{2d^3(2bc-ad)x^5}{5b^3} + \frac{d^4x^9}{9b^2} + \frac{(bc-ad)^4x}{4ab^4(a+bx^4)} - \frac{(bc-ad)^3(3bc+13ad}{8\sqrt{2}a^7}$$

Mathematica [A] time = 0.28772, size = 341, normalized size = 0.96

$$1440\sqrt[4]{b}d^2x(3a^2d^2-8abcd+6b^2c^2) + \frac{45\sqrt{2}(ad-bc)^3(13ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{7/4}} + \frac{45\sqrt{2}(bc-ad)^3(13ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{}\right.}{a^{7/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^4)^4/(a + b*x^4)^2,x]
```

```
[Out] (1440*b^(1/4)*d^2*(6*b^2*c^2 - 8*a*b*c*d + 3*a^2*d^2)*x + 576*b^(5/4)*d^3*(
2*b*c - a*d)*x^5 + 160*b^(9/4)*d^4*x^9 + (360*b^(1/4)*(b*c - a*d)^4*x)/(a*(
a + b*x^4)) + (90*Sqrt[2]*(-(b*c) + a*d)^3*(3*b*c + 13*a*d)*ArcTan[1 - (Sqr
t[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (90*Sqrt[2]*(b*c - a*d)^3*(3*b*c + 13*a
*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (45*Sqrt[2]*(-(b*c)
+ a*d)^3*(3*b*c + 13*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
*x^2])/a^(7/4) + (45*Sqrt[2]*(b*c - a*d)^3*(3*b*c + 13*a*d)*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4))/(1440*b^(17/4))
```

Maple [B] time = 0.01, size = 885, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^4/(b*x^4+a)^2,x)
```

```
[Out] 1/9*d^4*x^9/b^2-8*d^3/b^3*c*a*x+1/4/b^4*a^3*x/(b*x^4+a)*d^4+1/8/b/a*(1/b*a)
^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^
```

```
(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))*c^3*d+9/4/b^3*a*(1/b*a)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(1/b*a)^(1/4)*x+1)*c*d^3+1/4/b/a*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(1/b*a)^(1/4)*x+1)*c^3*d-13/16/b^4*a^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^
(1/2)/(1/b*a)^(1/4)*x-1)*d^4-15/8/b^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(1/b*a)^(1/4)*x-1)*c^2*d^2-13/32/b^4*a^2*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b
*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/
2)))*d^4-15/16/b^2*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1
/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))*c^2*d^2-13/16/b^4
*a^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)*d^4-15/8/b^2*(
1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)*c^2*d^2-1/b^3*a^2*x/
(b*x^4+a)*c*d^3+3/2/b^2*a*x/(b*x^4+a)*c^2*d^2-1/b*x/(b*x^4+a)*c^3*d+3/16/a^
2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*c^4+3/32/a^2*(1/b
*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*
a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))*c^4+3/16/a^2*(1/b*a)^(1/4)*2^(1/2)*arcta
n(2^(1/2)/(1/b*a)^(1/4)*x+1)*c^4+9/4/b^3*a*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(
1/2)/(1/b*a)^(1/4)*x-1)*c*d^3+1/4/b/a*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/
(1/b*a)^(1/4)*x-1)*c^3*d+9/8/b^3*a*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1
/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))*c
*d^3+1/4/a*x/(b*x^4+a)*c^4-2/5*d^4/b^3*x^5*a+4/5*d^3/b^2*x^5*c+3*d^4/b^4*a^
2*x+6*d^2/b^2*c^2*x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^4/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.21363, size = 6400, normalized size = 17.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^4/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/720*(80*a*b^3*d^4*x^13 + 16*(36*a*b^3*c*d^3 - 13*a^2*b^2*d^4)*x^9 + 144*(
30*a*b^3*c^2*d^2 - 36*a^2*b^2*c*d^3 + 13*a^3*b*d^4)*x^5 - 180*(a*b^5*x^4 +
a^2*b^4)*(-(81*b^16*c^16 + 432*a*b^15*c^15*d - 2376*a^2*b^14*c^14*d^2 - 830
4*a^3*b^13*c^13*d^3 + 45724*a^4*b^12*c^12*d^4 + 20400*a^5*b^11*c^11*d^5 - 4
34808*a^6*b^10*c^10*d^6 + 772112*a^7*b^9*c^9*d^7 + 617958*a^8*b^8*c^8*d^8 -
 4810608*a^9*b^7*c^7*d^9 + 9723912*a^10*b^6*c^6*d^10 - 11486160*a^11*b^5*c^
5*d^11 + 8923164*a^12*b^4*c^4*d^12 - 4651504*a^13*b^3*c^3*d^13 + 1577784*a^
14*b^2*c^2*d^14 - 316368*a^15*b*c*d^15 + 28561*a^16*d^16)/(a^7*b^17))^(1/4)
*arctan((a^5*b^13*x*(-(81*b^16*c^16 + 432*a*b^15*c^15*d - 2376*a^2*b^14*c^1
4*d^2 - 8304*a^3*b^13*c^13*d^3 + 45724*a^4*b^12*c^12*d^4 + 20400*a^5*b^11*c
^11*d^5 - 434808*a^6*b^10*c^10*d^6 + 772112*a^7*b^9*c^9*d^7 + 617958*a^8*b^
8*c^8*d^8 - 4810608*a^9*b^7*c^7*d^9 + 9723912*a^10*b^6*c^6*d^10 - 11486160*
a^11*b^5*c^5*d^11 + 8923164*a^12*b^4*c^4*d^12 - 4651504*a^13*b^3*c^3*d^13 +
 1577784*a^14*b^2*c^2*d^14 - 316368*a^15*b*c*d^15 + 28561*a^16*d^16)/(a^7*b
^17))^(3/4) - a^5*b^13*sqrt((a^4*b^8*sqrt(-(81*b^16*c^16 + 432*a*b^15*c^15*
d - 2376*a^2*b^14*c^14*d^2 - 8304*a^3*b^13*c^13*d^3 + 45724*a^4*b^12*c^12*d
```

```
^4 + 20400*a^5*b^11*c^11*d^5 - 434808*a^6*b^10*c^10*d^6 + 772112*a^7*b^9*c^
9*d^7 + 617958*a^8*b^8*c^8*d^8 - 4810608*a^9*b^7*c^7*d^9 + 9723912*a^10*b^6
*c^6*d^10 - 11486160*a^11*b^5*c^5*d^11 + 8923164*a^12*b^4*c^4*d^12 - 465150
4*a^13*b^3*c^3*d^13 + 1577784*a^14*b^2*c^2*d^14 - 316368*a^15*b*c*d^15 + 28
561*a^16*d^16)/(a^7*b^17)) + (9*b^8*c^8 + 24*a*b^7*c^7*d - 164*a^2*b^6*c^6*
d^2 - 24*a^3*b^5*c^5*d^3 + 1110*a^4*b^4*c^4*d^4 - 2264*a^5*b^3*c^3*d^5 + 20
76*a^6*b^2*c^2*d^6 - 936*a^7*b*c*d^7 + 169*a^8*d^8)*x^2)/(9*b^8*c^8 + 24*a*
b^7*c^7*d - 164*a^2*b^6*c^6*d^2 - 24*a^3*b^5*c^5*d^3 + 1110*a^4*b^4*c^4*d^4
 - 2264*a^5*b^3*c^3*d^5 + 2076*a^6*b^2*c^2*d^6 - 936*a^7*b*c*d^7 + 169*a^8*
d^8))*(-(81*b^16*c^16 + 432*a*b^15*c^15*d - 2376*a^2*b^14*c^14*d^2 - 8304*a
^3*b^13*c^13*d^3 + 45724*a^4*b^12*c^12*d^4 + 20400*a^5*b^11*c^11*d^5 - 4348
08*a^6*b^10*c^10*d^6 + 772112*a^7*b^9*c^9*d^7 + 617958*a^8*b^8*c^8*d^8 - 48
10608*a^9*b^7*c^7*d^9 + 9723912*a^10*b^6*c^6*d^10 - 11486160*a^11*b^5*c^5*d
^11 + 8923164*a^12*b^4*c^4*d^12 - 4651504*a^13*b^3*c^3*d^13 + 1577784*a^14*
b^2*c^2*d^14 - 316368*a^15*b*c*d^15 + 28561*a^16*d^16)/(a^7*b^17))^(3/4))/(
27*b^12*c^12 + 108*a*b^11*c^11*d - 666*a^2*b^10*c^10*d^2 - 1124*a^3*b^9*c^9
*d^3 + 8901*a^4*b^8*c^8*d^4 - 7848*a^5*b^7*c^7*d^5 - 34860*a^6*b^6*c^6*d^6
+ 113688*a^7*b^5*c^5*d^7 - 161451*a^8*b^4*c^4*d^8 + 132924*a^9*b^3*c^3*d^9
- 65754*a^10*b^2*c^2*d^10 + 18252*a^11*b*c*d^11 - 2197*a^12*d^12)) - 45*(a*
b^5*x^4 + a^2*b^4)*(-(81*b^16*c^16 + 432*a*b^15*c^15*d - 2376*a^2*b^14*c^14
*d^2 - 8304*a^3*b^13*c^13*d^3 + 45724*a^4*b^12*c^12*d^4 + 20400*a^5*b^11*c^
11*d^5 - 434808*a^6*b^10*c^10*d^6 + 772112*a^7*b^9*c^9*d^7 + 617958*a^8*b^8
*c^8*d^8 - 4810608*a^9*b^7*c^7*d^9 + 9723912*a^10*b^6*c^6*d^10 - 11486160*a
^11*b^5*c^5*d^11 + 8923164*a^12*b^4*c^4*d^12 - 4651504*a^13*b^3*c^3*d^13 +
1577784*a^14*b^2*c^2*d^14 - 316368*a^15*b*c*d^15 + 28561*a^16*d^16)/(a^7*b^
17))^(1/4)*log(a^2*b^4*(-(81*b^16*c^16 + 432*a*b^15*c^15*d - 2376*a^2*b^14*
c^14*d^2 - 8304*a^3*b^13*c^13*d^3 + 45724*a^4*b^12*c^12*d^4 + 20400*a^5*b^1
1*c^11*d^5 - 434808*a^6*b^10*c^10*d^6 + 772112*a^7*b^9*c^9*d^7 + 617958*a^8
*b^8*c^8*d^8 - 4810608*a^9*b^7*c^7*d^9 + 9723912*a^10*b^6*c^6*d^10 - 114861
60*a^11*b^5*c^5*d^11 + 8923164*a^12*b^4*c^4*d^12 - 4651504*a^13*b^3*c^3*d^1
3 + 1577784*a^14*b^2*c^2*d^14 - 316368*a^15*b*c*d^15 + 28561*a^16*d^16)/(a^
7*b^17))^(1/4) - (3*b^4*c^4 + 4*a*b^3*c^3*d - 30*a^2*b^2*c^2*d^2 + 36*a^3*b
*c*d^3 - 13*a^4*d^4)*x) + 45*(a*b^5*x^4 + a^2*b^4)*(-(81*b^16*c^16 + 432*a*
b^15*c^15*d - 2376*a^2*b^14*c^14*d^2 - 8304*a^3*b^13*c^13*d^3 + 45724*a^4*b
^12*c^12*d^4 + 20400*a^5*b^11*c^11*d^5 - 434808*a^6*b^10*c^10*d^6 + 772112*
a^7*b^9*c^9*d^7 + 617958*a^8*b^8*c^8*d^8 - 4810608*a^9*b^7*c^7*d^9 + 972391
2*a^10*b^6*c^6*d^10 - 11486160*a^11*b^5*c^5*d^11 + 8923164*a^12*b^4*c^4*d^1
2 - 4651504*a^13*b^3*c^3*d^13 + 1577784*a^14*b^2*c^2*d^14 - 316368*a^15*b*c
*d^15 + 28561*a^16*d^16)/(a^7*b^17))^(1/4)*log(-a^2*b^4*(-(81*b^16*c^16 + 4
32*a*b^15*c^15*d - 2376*a^2*b^14*c^14*d^2 - 8304*a^3*b^13*c^13*d^3 + 45724*
a^4*b^12*c^12*d^4 + 20400*a^5*b^11*c^11*d^5 - 434808*a^6*b^10*c^10*d^6 + 77
2112*a^7*b^9*c^9*d^7 + 617958*a^8*b^8*c^8*d^8 - 4810608*a^9*b^7*c^7*d^9 + 9
723912*a^10*b^6*c^6*d^10 - 11486160*a^11*b^5*c^5*d^11 + 8923164*a^12*b^4*c^
4*d^12 - 4651504*a^13*b^3*c^3*d^13 + 1577784*a^14*b^2*c^2*d^14 - 316368*a^1
5*b*c*d^15 + 28561*a^16*d^16)/(a^7*b^17))^(1/4) - (3*b^4*c^4 + 4*a*b^3*c^3*
d - 30*a^2*b^2*c^2*d^2 + 36*a^3*b*c*d^3 - 13*a^4*d^4)*x) + 180*(b^4*c^4 - 4
*a*b^3*c^3*d + 30*a^2*b^2*c^2*d^2 - 36*a^3*b*c*d^3 + 13*a^4*d^4)*x)/(a*b^5*
x^4 + a^2*b^4)
```

Sympy [A] time = 37.2598, size = 466, normalized size = 1.31

$$\frac{x\left(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4\right)}{4a^2b^4+4ab^5x^4}+\mathrm{RootSum}\left(65536t^4a^7b^{17}+28561a^{16}d^{16}-316368a^{15}bcd^{15}+157\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**4/(b*x**4+a)**2,x)
```

```
[Out] x*(a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*b**2*c**2*d**2 - 4*a*b**3*c**3*d +
b**4*c**4)/(4*a**2*b**4 + 4*a*b**5*x**4) + RootSum(65536*_t**4*a**7*b**17 +
 28561*a**16*d**16 - 316368*a**15*b*c*d**15 + 1577784*a**14*b**2*c**2*d**14
 - 4651504*a**13*b**3*c**3*d**13 + 8923164*a**12*b**4*c**4*d**12 - 11486160
*a**11*b**5*c**5*d**11 + 9723912*a**10*b**6*c**6*d**10 - 4810608*a**9*b**7*
c**7*d**9 + 617958*a**8*b**8*c**8*d**8 + 772112*a**7*b**9*c**9*d**7 - 43480
8*a**6*b**10*c**10*d**6 + 20400*a**5*b**11*c**11*d**5 + 45724*a**4*b**12*c*
*12*d**4 - 8304*a**3*b**13*c**13*d**3 - 2376*a**2*b**14*c**14*d**2 + 432*a*
b**15*c**15*d + 81*b**16*c**16, Lambda(_t, _t*log(-16*_t*a**2*b**4/(13*a**4
*d**4 - 36*a**3*b*c*d**3 + 30*a**2*b**2*c**2*d**2 - 4*a*b**3*c**3*d - 3*b**
4*c**4) + x))) + d**4*x**9/(9*b**2) - x**5*(2*a*d**4 - 4*b*c*d**3)/(5*b**3)
 + x*(3*a**2*d**4 - 8*a*b*c*d**3 + 6*b**2*c**2*d**2)/b**4
```

Giac [B] time = 1.1142, size = 867, normalized size = 2.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^4/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b^4*c^4 + 4*(a*b^3)^(1/4)*a*b^3*c^3*d - 30*(a
*b^3)^(1/4)*a^2*b^2*c^2*d^2 + 36*(a*b^3)^(1/4)*a^3*b*c*d^3 - 13*(a*b^3)^(1/
4)*a^4*d^4)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^
2*b^5) + 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b^4*c^4 + 4*(a*b^3)^(1/4)*a*b^3*c^3*
d - 30*(a*b^3)^(1/4)*a^2*b^2*c^2*d^2 + 36*(a*b^3)^(1/4)*a^3*b*c*d^3 - 13*(a
*b^3)^(1/4)*a^4*d^4)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(
1/4))/(a^2*b^5) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^4*c^4 + 4*(a*b^3)^(1/4)*a
*b^3*c^3*d - 30*(a*b^3)^(1/4)*a^2*b^2*c^2*d^2 + 36*(a*b^3)^(1/4)*a^3*b*c*d^
3 - 13*(a*b^3)^(1/4)*a^4*d^4)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/
(a^2*b^5) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^4*c^4 + 4*(a*b^3)^(1/4)*a*b^3*c
^3*d - 30*(a*b^3)^(1/4)*a^2*b^2*c^2*d^2 + 36*(a*b^3)^(1/4)*a^3*b*c*d^3 - 13
*(a*b^3)^(1/4)*a^4*d^4)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b
^5) + 1/4*(b^4*c^4*x - 4*a*b^3*c^3*d*x + 6*a^2*b^2*c^2*d^2*x - 4*a^3*b*c*d^
3*x + a^4*d^4*x)/((b*x^4 + a)*a*b^4) + 1/45*(5*b^16*d^4*x^9 + 36*b^16*c*d^3
*x^5 - 18*a*b^15*d^4*x^5 + 270*b^16*c^2*d^2*x - 360*a*b^15*c*d^3*x + 135*a^
2*b^14*d^4*x)/b^18
```

3.168 $\int \frac{(c+dx^4)^3}{(a+bx^4)^2} dx$

Optimal. Leaf size=317

$$-\frac{3(bc-ad)^2(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{13/4}}+\frac{3(bc-ad)^2(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{13/4}}-\frac{3(b}{}$$

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^5)/(5*b^2) + ((b*c - a*d)^3*x)/(4*a*b^
3*(a + b*x^4)) - (3*(b*c - a*d)^2*(b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)
*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(13/4)) + (3*(b*c - a*d)^2*(b*c + 3*a*d)
*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(13/4)) - (3
*(b*c - a*d)^2*(b*c + 3*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt
[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(13/4)) + (3*(b*c - a*d)^2*(b*c + 3*a*d)*Lo
g[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b
^(13/4))
```

Rubi [A] time = 0.317178, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {390, 385, 211, 1165, 628, 1162, 617, 204}

$$-\frac{3(bc-ad)^2(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{13/4}}+\frac{3(bc-ad)^2(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{13/4}}-\frac{3(b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^4)^3/(a + b*x^4)^2,x]
```

```
[Out] (d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^5)/(5*b^2) + ((b*c - a*d)^3*x)/(4*a*b^
3*(a + b*x^4)) - (3*(b*c - a*d)^2*(b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)
*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(13/4)) + (3*(b*c - a*d)^2*(b*c + 3*a*d)
*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(13/4)) - (3
*(b*c - a*d)^2*(b*c + 3*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt
[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(13/4)) + (3*(b*c - a*d)^2*(b*c + 3*a*d)*Lo
g[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b
^(13/4))
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(c+dx^4\right)^3}{\left(a+bx^4\right)^2}\,dx = \int \left(\frac{d^2(3bc-2ad)}{b^3}+\frac{d^3x^4}{b^2}+\frac{(bc-ad)^2(bc+2ad)+3bd(bc-ad)^2x^4}{b^3\left(a+bx^4\right)^2}\right)dx$$

$$= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^5}{5b^2}+\frac{\int \frac{(bc-ad)^2(bc+2ad)+3bd(bc-ad)^2x^4}{\left(a+bx^4\right)^2}dx}{b^3}$$

$$= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^5}{5b^2}+\frac{(bc-ad)^3x}{4ab^3\left(a+bx^4\right)}+\frac{\left(3(bc-ad)^2(bc+3ad)\right)\int \frac{1}{a+bx^4}dx}{4ab^3}$$

$$= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^5}{5b^2}+\frac{(bc-ad)^3x}{4ab^3\left(a+bx^4\right)}+\frac{\left(3(bc-ad)^2(bc+3ad)\right)\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}dx}{8a^{3/2}b^3}+\frac{\left(3(bc-ad)^2(b\right.}{}$$

$$= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^5}{5b^2}+\frac{(bc-ad)^3x}{4ab^3\left(a+bx^4\right)}+\frac{\left(3(bc-ad)^2(bc+3ad)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}dx}{16a^{3/2}b^{7/2}}+\frac{\left(3(bc-a\right.}{}$$

$$= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^5}{5b^2}+\frac{(bc-ad)^3x}{4ab^3\left(a+bx^4\right)}-\frac{3(bc-ad)^2(bc+3ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{13/4}}+$$

$$= \frac{d^2(3bc-2ad)x}{b^3}+\frac{d^3x^5}{5b^2}+\frac{(bc-ad)^3x}{4ab^3\left(a+bx^4\right)}-\frac{3(bc-ad)^2(bc+3ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{13/4}}+\frac{3(bc-ad)^2}{}$$

Mathematica [A] time = 0.222286, size = 301, normalized size = 0.95

$$\frac{-\frac{15\sqrt{2}(bc-ad)^2(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{7/4}}+\frac{15\sqrt{2}(bc-ad)^2(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{7/4}}-\frac{30\sqrt{2}(bc-ad)^2(3ad+bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{7/4}}}{160b^{13/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^4)^3/(a + b*x^4)^2,x]
```

```
[Out] (160*b^(1/4)*d^2*(3*b*c - 2*a*d)*x + 32*b^(5/4)*d^3*x^5 + (40*b^(1/4)*(b*c
- a*d)^3*x)/(a*(a + b*x^4)) - (30*Sqrt[2]*(b*c - a*d)^2*(b*c + 3*a*d)*ArcTa
n[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (30*Sqrt[2]*(b*c - a*d)^2*(b*
c + 3*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) - (15*Sqrt[2]*(
b*c - a*d)^2*(b*c + 3*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b
]*x^2])/a^(7/4) + (15*Sqrt[2]*(b*c - a*d)^2*(b*c + 3*a*d)*Log[Sqrt[a] + Sqr
t[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4))/(160*b^(13/4))
```

Maple [B] time = 0.009, size = 669, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^3/(b*x^4+a)^2,x)
```

```
[Out] 1/5*d^3*x^5/b^2-2*d^3/b^3*a*x+3*d^2/b^2*x*c-1/4/b^3*a^2*x/(b*x^4+a)*d^3+3/4
/b^2*a*x/(b*x^4+a)*c*d^2-3/4/b*x/(b*x^4+a)*c^2*d+1/4/a*x/(b*x^4+a)*c^3+9/16
```

```
/b^3*a*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*d^3-15/16/b^
2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*c*d^2+3/16/b/a*(1
/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*c^2*d+3/16/a^2*(1/b*a
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*c^3+9/32/b^3*a*(1/b*a)^(1
/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/
4)*x*2^(1/2)+(1/b*a)^(1/2)))*d^3-15/32/b^2*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1
/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(
1/2)))*c*d^2+3/32/b/a*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)
+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))*c^2*d+3/32/a^2
*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-
(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))*c^3+9/16/b^3*a*(1/b*a)^(1/4)*2^(1/2
)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)*d^3-15/16/b^2*(1/b*a)^(1/4)*2^(1/2)*arc
tan(2^(1/2)/(1/b*a)^(1/4)*x+1)*c*d^2+3/16/b/a*(1/b*a)^(1/4)*2^(1/2)*arctan(
2^(1/2)/(1/b*a)^(1/4)*x+1)*c^2*d+3/16/a^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1
/2)/(1/b*a)^(1/4)*x+1)*c^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^3/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.08882, size = 4251, normalized size = 13.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^3/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/80*(16*a*b^2*d^3*x^9 + 48*(5*a*b^2*c*d^2 - 3*a^2*b*d^3)*x^5 + 60*(a*b^4*x
^4 + a^2*b^3)*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^
3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6
*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 +
1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))^(1/4
)*arctan(-(a^5*b^10*x*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2
 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6
*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^
3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^1
3))^(3/4) - a^5*b^10*sqrt((a^4*b^6*sqrt(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*
a^2*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*
c^7*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8
- 1932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^
12*d^12)/(a^7*b^13)) + (b^6*c^6 + 2*a*b^5*c^5*d - 9*a^2*b^4*c^4*d^2 - 4*a^3
*b^3*c^3*d^3 + 31*a^4*b^2*c^2*d^4 - 30*a^5*b*c*d^5 + 9*a^6*d^6)*x^2)/(b^6*c
^6 + 2*a*b^5*c^5*d - 9*a^2*b^4*c^4*d^2 - 4*a^3*b^3*c^3*d^3 + 31*a^4*b^2*c^2
*d^4 - 30*a^5*b*c*d^5 + 9*a^6*d^6))*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2
*b^10*c^10*d^2 - 44*a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7
*d^5 - 644*a^6*b^6*c^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1
932*a^9*b^3*c^3*d^9 + 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*
d^12)/(a^7*b^13))^(3/4))/(b^9*c^9 + 3*a*b^8*c^8*d - 12*a^2*b^7*c^7*d^2 - 20
```

```
*a^3*b^6*c^6*d^3 + 78*a^4*b^5*c^5*d^4 - 6*a^5*b^4*c^4*d^5 - 188*a^6*b^3*c^3
*d^6 + 252*a^7*b^2*c^2*d^7 - 135*a^8*b*c*d^8 + 27*a^9*d^9)) + 15*(a*b^4*x^4
 + a^2*b^3)*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^3*
b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*d
^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 + 14
58*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))^(1/4)*
log(3*a^2*b^3*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^
3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6
*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 +
1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))^(1/4
) + 3*(b^3*c^3 + a*b^2*c^2*d - 5*a^2*b*c*d^2 + 3*a^3*d^3)*x) - 15*(a*b^4*x^
4 + a^2*b^3)*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*a^3
*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c^6*
d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9 + 1
458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))^(1/4)
*log(-3*a^2*b^3*(-(b^12*c^12 + 4*a*b^11*c^11*d - 14*a^2*b^10*c^10*d^2 - 44*
a^3*b^9*c^9*d^3 + 127*a^4*b^8*c^8*d^4 + 136*a^5*b^7*c^7*d^5 - 644*a^6*b^6*c
^6*d^6 + 328*a^7*b^5*c^5*d^7 + 1039*a^8*b^4*c^4*d^8 - 1932*a^9*b^3*c^3*d^9
+ 1458*a^10*b^2*c^2*d^10 - 540*a^11*b*c*d^11 + 81*a^12*d^12)/(a^7*b^13))^(1
/4) + 3*(b^3*c^3 + a*b^2*c^2*d - 5*a^2*b*c*d^2 + 3*a^3*d^3)*x) + 20*(b^3*c^
3 - 3*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 9*a^3*d^3)*x)/(a*b^4*x^4 + a^2*b^3)
```

Sympy [A] time = 6.68376, size = 335, normalized size = 1.06

$$-\frac{x\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}{4a^2b^3+4ab^4x^4}+\operatorname{RootSum}\left(65536t^4a^7b^{13}+6561a^{12}d^{12}-43740a^{11}bcd^{11}+118098a^{10}b^2c^2d^{10}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**3/(b*x**4+a)**2,x)
```

```
[Out] -x*(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3)/(4*a**2*b**3
 + 4*a*b**4*x**4) + RootSum(65536*_t**4*a**7*b**13 + 6561*a**12*d**12 - 437
40*a**11*b*c*d**11 + 118098*a**10*b**2*c**2*d**10 - 156492*a**9*b**3*c**3*d
**9 + 84159*a**8*b**4*c**4*d**8 + 26568*a**7*b**5*c**5*d**7 - 52164*a**6*b*
*6*c**6*d**6 + 11016*a**5*b**7*c**7*d**5 + 10287*a**4*b**8*c**8*d**4 - 3564
*a**3*b**9*c**9*d**3 - 1134*a**2*b**10*c**10*d**2 + 324*a*b**11*c**11*d + 8
1*b**12*c**12, Lambda(_t, _t*log(16*_t*a**2*b**3/(9*a**3*d**3 - 15*a**2*b*c
*d**2 + 3*a*b**2*c**2*d + 3*b**3*c**3) + x))) + d**3*x**5/(5*b**2) - x*(2*a
*d**3 - 3*b*c*d**2)/b**3
```

Giac [A] time = 1.1066, size = 670, normalized size = 2.11

$$\frac{3\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3+\left(ab^3\right)^{\frac{1}{4}}ab^2c^2d-5\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2+3\left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16\,a^2b^4}+\frac{3\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^3/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 3/16*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 + (a*b^3)^(1/4)*a*b^2*c^2*d - 5*(a*b^3)
^(1/4)*a^2*b*c*d^2 + 3*(a*b^3)^(1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(2*x + sqr
```

```
t(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^4) + 3/16*sqrt(2)*((a*b^3)^(1/4)*b^3*
c^3 + (a*b^3)^(1/4)*a*b^2*c^2*d - 5*(a*b^3)^(1/4)*a^2*b*c*d^2 + 3*(a*b^3)^(
1/4)*a^3*d^3)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(
a^2*b^4) + 3/32*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3 + (a*b^3)^(1/4)*a*b^2*c^2*d
- 5*(a*b^3)^(1/4)*a^2*b*c*d^2 + 3*(a*b^3)^(1/4)*a^3*d^3)*log(x^2 + sqrt(2)*
x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^4) - 3/32*sqrt(2)*((a*b^3)^(1/4)*b^3*c^3
+ (a*b^3)^(1/4)*a*b^2*c^2*d - 5*(a*b^3)^(1/4)*a^2*b*c*d^2 + 3*(a*b^3)^(1/4)
*a^3*d^3)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^4) + 1/4*(b^3
*c^3*x - 3*a*b^2*c^2*d*x + 3*a^2*b*c*d^2*x - a^3*d^3*x)/((b*x^4 + a)*a*b^3)
 + 1/5*(b^8*d^3*x^5 + 15*b^8*c*d^2*x - 10*a*b^7*d^3*x)/b^10
```

3.169 $\int \frac{(c+dx^4)^2}{(a+bx^4)^2}\,dx$

Optimal. Leaf size=291

$$-\frac{(bc-ad)(5ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{9/4}}+\frac{(bc-ad)(5ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{9/4}}-\frac{(bc-}{}$$

```
[Out] (d^2*x)/b^2 + ((b*c - a*d)^2*x)/(4*a*b^2*(a + b*x^4)) - ((b*c - a*d)*(3*b*c
 + 5*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(9/
4)) + ((b*c - a*d)*(3*b*c + 5*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])
/(8*Sqrt[2]*a^(7/4)*b^(9/4)) - ((b*c - a*d)*(3*b*c + 5*a*d)*Log[Sqrt[a] - S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(9/4)) + ((b
*c - a*d)*(3*b*c + 5*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
*x^2])/(16*Sqrt[2]*a^(7/4)*b^(9/4))
```

Rubi [A] time = 0.377291, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {390, 385, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(bc-ad)(5ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{9/4}}+\frac{(bc-ad)(5ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{9/4}}-\frac{(bc-}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^4)^2/(a + b*x^4)^2,x]
```



```
[Out] (d^2*x)/b^2 + ((b*c - a*d)^2*x)/(4*a*b^2*(a + b*x^4)) - ((b*c - a*d)*(3*b*c
 + 5*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(9/
4)) + ((b*c - a*d)*(3*b*c + 5*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])
/(8*Sqrt[2]*a^(7/4)*b^(9/4)) - ((b*c - a*d)*(3*b*c + 5*a*d)*Log[Sqrt[a] - S
qrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(9/4)) + ((b
*c - a*d)*(3*b*c + 5*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]
*x^2])/(16*Sqrt[2]*a^(7/4)*b^(9/4))
```


Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
```

```
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(c+dx^4)^2}{(a+bx^4)^2}\,dx = \int \left(\frac{d^2}{b^2} + \frac{b^2c^2 - a^2d^2 + 2bd(bc-ad)x^4}{b^2(a+bx^4)^2}\right) dx$$

$$= \frac{d^2x}{b^2} + \frac{\int \frac{b^2c^2-a^2d^2+2bd(bc-ad)x^4}{(a+bx^4)^2}\,dx}{b^2}$$

$$= \frac{d^2x}{b^2} + \frac{(bc-ad)^2x}{4ab^2(a+bx^4)} + \frac{((bc-ad)(3bc+5ad))\int \frac{1}{a+bx^4}\,dx}{4ab^2}$$

$$= \frac{d^2x}{b^2} + \frac{(bc-ad)^2x}{4ab^2(a+bx^4)} + \frac{((bc-ad)(3bc+5ad))\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{8a^{3/2}b^2} + \frac{((bc-ad)(3bc+5ad))\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,d}{8a^{3/2}b^2}$$

$$= \frac{d^2x}{b^2} + \frac{(bc-ad)^2x}{4ab^2(a+bx^4)} + \frac{((bc-ad)(3bc+5ad))\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{16a^{3/2}b^{5/2}} + \frac{((bc-ad)(3bc+5ad))\int \frac{}{\frac{\sqrt{a}}{\sqrt{b}}+\sqrt{}}}{16a^{3/2}b^{5/2}}$$

$$= \frac{d^2x}{b^2} + \frac{(bc-ad)^2x}{4ab^2(a+bx^4)} - \frac{(bc-ad)(3bc+5ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{9/4}} + \frac{(bc-ad)(3bc+5a}{}$$

$$= \frac{d^2x}{b^2} + \frac{(bc-ad)^2x}{4ab^2(a+bx^4)} - \frac{(bc-ad)(3bc+5ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{9/4}} + \frac{(bc-ad)(3bc+5ad)\tan^{-1}\left(1-\right.}{8\sqrt{2}a^{7/4}b^{9/4}}$$

Mathematica [A] time = 0.176814, size = 297, normalized size = 1.02

$$\frac{\frac{\sqrt{2}(5a^2d^2-2abcd-3b^2c^2)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{7/4}} + \frac{\sqrt{2}(-5a^2d^2+2abcd+3b^2c^2)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{7/4}} + \frac{2\sqrt{2}(5a^2d^2-2abcd-3b^2c^2)\tan^{-1}\left(1-\right.}{a^{7/4}}}{32b^{9/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^4)^2/(a + b*x^4)^2,x]
```

```
[Out] (32*b^(1/4)*d^2*x + (8*b^(1/4)*(b*c - a*d)^2*x)/(a*(a + b*x^4)) + (2*Sqrt[2
]*(-3*b^2*c^2 - 2*a*b*c*d + 5*a^2*d^2)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/
4)])/a^(7/4) + (2*Sqrt[2]*(3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*ArcTan[1 + (S
qrt[2]*b^(1/4)*x)/a^(1/4)])/a^(7/4) + (Sqrt[2]*(-3*b^2*c^2 - 2*a*b*c*d + 5*
a^2*d^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4) +
(Sqrt[2]*(3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*
b^(1/4)*x + Sqrt[b]*x^2])/a^(7/4))/(32*b^(9/4))
```

Maple [B] time = 0.009, size = 475, normalized size = 1.6

$$\frac{d^2x}{b^2} + \frac{axd^2}{4\,b^2(bx^4+a)} - \frac{cxd}{2\,b(bx^4+a)} + \frac{xc^2}{4\,a(bx^4+a)} - \frac{5\sqrt{2}d^2}{16\,b^2}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right) + \frac{\sqrt{2}cd}{8\,ab}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^2/(b*x^4+a)^2,x)
```

```
[Out] d^2*x/b^2+1/4/b^2*a*x/(b*x^4+a)*d^2-1/2/b*x/(b*x^4+a)*c*d+1/4/a*x/(b*x^4+a)
*c^2-5/16/b^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*d^2+1
/8/b/a*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*c*d+3/16/a^2
*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*c^2-5/32/b^2*(1/b*
a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a
)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))*d^2+1/16/b/a*(1/b*a)^(1/4)*2^(1/2)*ln((x^
2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*
a)^(1/2)))*c*d+3/32/a^2*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/
2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))*c^2-5/16/b^2
*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)*d^2+1/8/b/a*(1/b*a
)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)*c*d+3/16/a^2*(1/b*a)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)*c^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^2/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.92587, size = 2909, normalized size = 10.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^2/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(16*a*b*d^2*x^5 - 4*(a*b^3*x^4 + a^2*b^2)*(-(81*b^8*c^8 + 216*a*b^7*c^
7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 164
0*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(
a^7*b^9))^(1/4)*arctan((a^5*b^7*x*(-(81*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2
*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3
*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(a^7*b^9))^(3/
4) - a^5*b^7*sqrt((a^4*b^4*sqrt(-(81*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^
6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^
5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(a^7*b^9)) + (9*b
^4*c^4 + 12*a*b^3*c^3*d - 26*a^2*b^2*c^2*d^2 - 20*a^3*b*c*d^3 + 25*a^4*d^4)
*x^2)/(9*b^4*c^4 + 12*a*b^3*c^3*d - 26*a^2*b^2*c^2*d^2 - 20*a^3*b*c*d^3 + 2
5*a^4*d^4))*(-(81*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3
*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2
*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(a^7*b^9))^(3/4))/(27*b^6*c^6 + 54*a
*b^5*c^5*d - 99*a^2*b^4*c^4*d^2 - 172*a^3*b^3*c^3*d^3 + 165*a^4*b^2*c^2*d^4
 + 150*a^5*b*c*d^5 - 125*a^6*d^6)) - (a*b^3*x^4 + a^2*b^2)*(-(81*b^8*c^8 +
216*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c
^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 62
5*a^8*d^8)/(a^7*b^9))^(1/4)*log(a^2*b^2*(-(81*b^8*c^8 + 216*a*b^7*c^7*d - 3
24*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*d^4 + 1640*a^5*b
^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a^8*d^8)/(a^7*b^9
))^(1/4) - (3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*x) + (a*b^3*x^4 + a^2*b^2)*(
-(81*b^8*c^8 + 216*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3
+ 646*a^4*b^4*c^4*d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a
```

```
^7*b*c*d^7 + 625*a^8*d^8)/(a^7*b^9))^(1/4)*log(-a^2*b^2*(-(81*b^8*c^8 + 216
*a*b^7*c^7*d - 324*a^2*b^6*c^6*d^2 - 984*a^3*b^5*c^5*d^3 + 646*a^4*b^4*c^4*
d^4 + 1640*a^5*b^3*c^3*d^5 - 900*a^6*b^2*c^2*d^6 - 1000*a^7*b*c*d^7 + 625*a
^8*d^8)/(a^7*b^9))^(1/4) - (3*b^2*c^2 + 2*a*b*c*d - 5*a^2*d^2)*x) + 4*(b^2*
c^2 - 2*a*b*c*d + 5*a^2*d^2)*x)/(a*b^3*x^4 + a^2*b^2)
```

Sympy [A] time = 2.06516, size = 219, normalized size = 0.75

$$\frac{x\left(a^2d^2-2abcd+b^2c^2\right)}{4a^2b^2+4ab^3x^4}+\operatorname{RootSum}\Big(65536t^4a^7b^9+625a^8d^8-1000a^7bcd^7-900a^6b^2c^2d^6+1640a^5b^3c^3d^5+646a^4b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**2/(b*x**4+a)**2,x)
```

```
[Out] x*(a**2*d**2 - 2*a*b*c*d + b**2*c**2)/(4*a**2*b**2 + 4*a*b**3*x**4) + RootS
um(65536*_t**4*a**7*b**9 + 625*a**8*d**8 - 1000*a**7*b*c*d**7 - 900*a**6*b*
*2*c**2*d**6 + 1640*a**5*b**3*c**3*d**5 + 646*a**4*b**4*c**4*d**4 - 984*a**
3*b**5*c**5*d**3 - 324*a**2*b**6*c**6*d**2 + 216*a*b**7*c**7*d + 81*b**8*c*
*8, Lambda(_t, _t*log(-16*_t*a**2*b**2/(5*a**2*d**2 - 2*a*b*c*d - 3*b**2*c*
*2) + x))) + d**2*x/b**2
```

Giac [A] time = 1.12698, size = 508, normalized size = 1.75

$$\frac{d^2x}{b^2}+\frac{\sqrt{2}\Big(3\left(ab^3\right)^{\frac{1}{4}}b^2c^2+2\left(ab^3\right)^{\frac{1}{4}}abcd-5\left(ab^3\right)^{\frac{1}{4}}a^2d^2\Big)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16a^2b^3}+\frac{\sqrt{2}\Big(3\left(ab^3\right)^{\frac{1}{4}}b^2c^2+2\left(ab^3\right)^{\frac{1}{4}}al}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^2/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] d^2*x/b^2 + 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c^2 + 2*(a*b^3)^(1/4)*a*b*c*d
 - 5*(a*b^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/b)^(1/4))/
(a/b)^(1/4))/(a^2*b^3) + 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c^2 + 2*(a*b^3)^
(1/4)*a*b*c*d - 5*(a*b^3)^(1/4)*a^2*d^2)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*
(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^3) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c^2
 + 2*(a*b^3)^(1/4)*a*b*c*d - 5*(a*b^3)^(1/4)*a^2*d^2)*log(x^2 + sqrt(2)*x*(
a/b)^(1/4) + sqrt(a/b))/(a^2*b^3) - 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b^2*c^2 +
 2*(a*b^3)^(1/4)*a*b*c*d - 5*(a*b^3)^(1/4)*a^2*d^2)*log(x^2 - sqrt(2)*x*(a/
b)^(1/4) + sqrt(a/b))/(a^2*b^3) + 1/4*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)
/((b*x^4 + a)*a*b^2)
```

3.170 $\int \frac{c+dx^4}{\left(a+bx^4\right)^2}\,dx$

Optimal. Leaf size=245

$$-\frac{(ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{5/4}}+\frac{(ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{5/4}}-\frac{(ad+3bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{5/4}}$$

```
[Out] ((b*c - a*d)*x)/(4*a*b*(a + b*x^4)) - ((3*b*c + a*d)*ArcTan[1 - (Sqrt[2]*b^
(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(5/4)) + ((3*b*c + a*d)*ArcTan[1 +
(Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(5/4)) - ((3*b*c + a*d)*
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)
*b^(5/4)) + ((3*b*c + a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b
]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(5/4))
```

Rubi [A] time = 0.152838, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {385, 211, 1165, 628, 1162, 617, 204}

$$-\frac{(ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{5/4}}+\frac{(ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{5/4}}-\frac{(ad+3bc)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^4)/(a + b*x^4)^2,x]
```


```
[Out] ((b*c - a*d)*x)/(4*a*b*(a + b*x^4)) - ((3*b*c + a*d)*ArcTan[1 - (Sqrt[2]*b^
(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(5/4)) + ((3*b*c + a*d)*ArcTan[1 +
(Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*b^(5/4)) - ((3*b*c + a*d)*
Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)
*b^(5/4)) + ((3*b*c + a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b
]*x^2])/(16*Sqrt[2]*a^(7/4)*b^(5/4))
```


Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
```

```
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{c+dx^4}{\left(a+bx^4\right)^2}\,dx = \frac{(bc-ad)x}{4ab\left(a+bx^4\right)}+\frac{(3bc+ad)\int \frac{1}{a+bx^4}\,dx}{4ab}$$

$$= \frac{(bc-ad)x}{4ab\left(a+bx^4\right)}+\frac{(3bc+ad)\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{8a^{3/2}b}+\frac{(3bc+ad)\int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4}\,dx}{8a^{3/2}b}$$

$$= \frac{(bc-ad)x}{4ab\left(a+bx^4\right)}+\frac{(3bc+ad)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{16a^{3/2}b^{3/2}}+\frac{(3bc+ad)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}+\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}+x^2}\,dx}{16a^{3/2}b^{3/2}}-\frac{(3bc+ad)\int \cdots}{16\sqrt{2}a^{7/4}\cdots}$$

$$= \frac{(bc-ad)x}{4ab\left(a+bx^4\right)}-\frac{(3bc+ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{5/4}}+\frac{(3bc+ad)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\cdots\right)}{16\sqrt{2}a^{7/4}b^{5/4}}$$

$$= \frac{(bc-ad)x}{4ab\left(a+bx^4\right)}-\frac{(3bc+ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{5/4}}+\frac{(3bc+ad)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{5/4}}-\frac{(3bc+ad)\log\left(\cdots\right)}{16\cdots}$$

Mathematica [A] time = 0.166296, size = 212, normalized size = 0.87

$$\frac{-\frac{8a^{3/4}\sqrt[4]{b}x(ad-bc)}{a+bx^4}-\sqrt{2}(ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)+\sqrt{2}(ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)-2\sqrt{2}\cdots}{32a^{7/4}b^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^4)/(a + b*x^4)^2,x]
```

```
[Out] ((-8*a^(3/4)*b^(1/4)*(-(b*c) + a*d)*x)/(a + b*x^4) - 2*Sqrt[2]*(3*b*c + a*d
)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*Sqrt[2]*(3*b*c + a*d)*ArcTan[
1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - Sqrt[2]*(3*b*c + a*d)*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*(3*b*c + a*d)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(32*a^(7/4)*b^(5/4))
```

Maple [A] time = 0.007, size = 295, normalized size = 1.2

$$-\frac{(ad-bc)\,x}{4\,ab\left(bx^4+a\right)}+\frac{\sqrt{2}d}{16\,ab}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{3\sqrt{2}c}{16\,a^2}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}+1\right)+\frac{\sqrt{2}d}{16\,ab}\sqrt[4]{\frac{a}{b}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{a}{b}}}-1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)/(b*x^4+a)^2,x)
```

```
[Out] -1/4*(a*d-b*c)/a/b*x/(b*x^4+a)+1/16/a/b*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2
)/(1/b*a)^(1/4)*x+1)*d+3/16/a^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a
)^(1/4)*x+1)*c+1/16/a/b*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*
x-1)*d+3/16/a^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*c+1
/32/a/b*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)
)/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))*d+3/32/a^2*(1/b*a)^(1/4)*2^(
1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^
(1/2)+(1/b*a)^(1/2)))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.6461, size = 1526, normalized size = 6.23

$$\frac{4\left(ab^2x^4+a^2b\right)\left(-\frac{81\,b^4c^4+108\,ab^3c^3d+54\,a^2b^2c^2d^2+12\,a^3bcd^3+a^4d^4}{a^7b^5}\right)^{\frac{1}{4}}\arctan\left(-\frac{a^5b^4x\left(-\frac{81\,b^4c^4+108\,ab^3c^3d+54\,a^2b^2c^2d^2+12\,a^3bcd^3+a^4d^4}{a^7b^5}\right)^{\frac{3}{4}}-a^5b^4\sqrt{\frac{a^4b^2}{}}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*(a*b^2*x^4 + a^2*b)*(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^
2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(a^7*b^5))^(1/4)*arctan(-(a^5*b^4*x*(-(81
```

```
*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)
/(a^7*b^5))^(3/4) - a^5*b^4*sqrt((a^4*b^2*sqrt(-(81*b^4*c^4 + 108*a*b^3*c^3
*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(a^7*b^5)) + (9*b^2*c^2
 + 6*a*b*c*d + a^2*d^2)*x^2)/(9*b^2*c^2 + 6*a*b*c*d + a^2*d^2))*(-(81*b^4*c
^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(a^7*
b^5))^(3/4))/(27*b^3*c^3 + 27*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)) + (a*
b^2*x^4 + a^2*b)*(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*
a^3*b*c*d^3 + a^4*d^4)/(a^7*b^5))^(1/4)*log(a^2*b*(-(81*b^4*c^4 + 108*a*b^3
*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(a^7*b^5))^(1/4) +
(3*b*c + a*d)*x) - (a*b^2*x^4 + a^2*b)*(-(81*b^4*c^4 + 108*a*b^3*c^3*d + 54
*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d^4)/(a^7*b^5))^(1/4)*log(-a^2*b*(-
(81*b^4*c^4 + 108*a*b^3*c^3*d + 54*a^2*b^2*c^2*d^2 + 12*a^3*b*c*d^3 + a^4*d
^4)/(a^7*b^5))^(1/4) + (3*b*c + a*d)*x) + 4*(b*c - a*d)*x)/(a*b^2*x^4 + a^2
*b)
```

Sympy [A] time = 0.885544, size = 112, normalized size = 0.46

$$-\frac{x(ad-bc)}{4a^2b+4ab^2x^4}+\text{RootSum}\left(65536t^4a^7b^5+a^4d^4+12a^3bcd^3+54a^2b^2c^2d^2+108ab^3c^3d+81b^4c^4,\left(t\mapsto t\log\left(\frac{16ta}{ad+3}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)/(b*x**4+a)**2,x)
```

```
[Out] -x*(a*d - b*c)/(4*a**2*b + 4*a*b**2*x**4) + RootSum(65536*_t**4*a**7*b**5 +
 a**4*d**4 + 12*a**3*b*c*d**3 + 54*a**2*b**2*c**2*d**2 + 108*a*b**3*c**3*d
+ 81*b**4*c**4, Lambda(_t, _t*log(16*_t*a**2*b/(a*d + 3*b*c) + x)))
```

Giac [A] time = 1.10899, size = 359, normalized size = 1.47

$$\frac{\sqrt{2}\left(3\left(ab^3\right)^{\frac14}bc+\left(ab^3\right)^{\frac14}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{16a^2b^2}+\frac{\sqrt{2}\left(3\left(ab^3\right)^{\frac14}bc+\left(ab^3\right)^{\frac14}ad\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{b}\right)^{\frac14}\right)}{2\left(\frac{a}{b}\right)^{\frac14}}\right)}{16a^2b^2}+\frac{\sqrt{2}(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*(3*(a*b^3)^(1/4)*b*c + (a*b^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(
2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(a^2*b^2) + 1/16*sqrt(2)*(3*(a*b^3)
^(1/4)*b*c + (a*b^3)^(1/4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/
4))/(a/b)^(1/4))/(a^2*b^2) + 1/32*sqrt(2)*(3*(a*b^3)^(1/4)*b*c + (a*b^3)^(1
/4)*a*d)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))/(a^2*b^2) - 1/32*sqrt
(2)*(3*(a*b^3)^(1/4)*b*c + (a*b^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x*(a/b)^(1/
4) + sqrt(a/b))/(a^2*b^2) + 1/4*(b*c*x - a*d*x)/((b*x^4 + a)*a*b)
```

3.171 $\int \frac{1}{\left(a+bx^4\right)^2\left(c+dx^4\right)} dx$

Optimal. Leaf size=513

$$-\frac{b^{3/4}(3bc-7ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^2}+\frac{b^{3/4}(3bc-7ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^2}-\frac{b^{3/4}(3bc-7ad)\tan^{-1}}{8\sqrt{2}a^{7/4}(bc}$$

```
[Out] (b*x)/(4*a*(b*c - a*d)*(a + b*x^4)) - (b^(3/4)*(3*b*c - 7*a*d)*ArcTan[1 - (
Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(b*c - a*d)^2) + (b^(3/4)*(
3*b*c - 7*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*
(b*c - a*d)^2) - (d^(7/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[
2]*c^(3/4)*(b*c - a*d)^2) + (d^(7/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)
])/(2*Sqrt[2]*c^(3/4)*(b*c - a*d)^2) - (b^(3/4)*(3*b*c - 7*a*d)*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*(b*c - a*d
)^2) + (b^(3/4)*(3*b*c - 7*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + S
qrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*(b*c - a*d)^2) - (d^(7/4)*Log[Sqrt[c] - Sq
rt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(3/4)*(b*c - a*d)^2) +
 (d^(7/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2
]*c^(3/4)*(b*c - a*d)^2)
```

Rubi [A] time = 0.428292, antiderivative size = 513, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {414, 522, 211, 1165, 628, 1162, 617, 204}

$$-\frac{b^{3/4}(3bc-7ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^2}+\frac{b^{3/4}(3bc-7ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^2}-\frac{b^{3/4}(3bc-7ad)\tan^{-1}}{8\sqrt{2}a^{7/4}(bc}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^2*(c + d*x^4)),x]
```


```
[Out] (b*x)/(4*a*(b*c - a*d)*(a + b*x^4)) - (b^(3/4)*(3*b*c - 7*a*d)*ArcTan[1 - (
Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(b*c - a*d)^2) + (b^(3/4)*(
3*b*c - 7*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*
(b*c - a*d)^2) - (d^(7/4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(2*Sqrt[
2]*c^(3/4)*(b*c - a*d)^2) + (d^(7/4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*x)/c^(1/4)
])/(2*Sqrt[2]*c^(3/4)*(b*c - a*d)^2) - (b^(3/4)*(3*b*c - 7*a*d)*Log[Sqrt[a]
 - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*(b*c - a*d
)^2) + (b^(3/4)*(3*b*c - 7*a*d)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + S
qrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*(b*c - a*d)^2) - (d^(7/4)*Log[Sqrt[c] - Sq
rt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2]*c^(3/4)*(b*c - a*d)^2) +
 (d^(7/4)*Log[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(4*Sqrt[2
]*c^(3/4)*(b*c - a*d)^2)
```


Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
```

```
d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx^4)^2(c+dx^4)}\,dx = \frac{bx}{4a(bc-ad)(a+bx^4)} - \frac{\int \frac{-3bc+4ad-3bdx^4}{(a+bx^4)(c+dx^4)}\,dx}{4a(bc-ad)}$$

$$= \frac{bx}{4a(bc-ad)(a+bx^4)} + \frac{d^2\int \frac{1}{c+dx^4}\,dx}{(bc-ad)^2} + \frac{(b(3bc-7ad))\int \frac{1}{a+bx^4}\,dx}{4a(bc-ad)^2}$$

$$= \frac{bx}{4a(bc-ad)(a+bx^4)} + \frac{d^2\int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4}\,dx}{2\sqrt{c}(bc-ad)^2} + \frac{d^2\int \frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4}\,dx}{2\sqrt{c}(bc-ad)^2} + \frac{(b(3bc-7ad))\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{8a^{3/2}(bc-ad)^2}$$

$$= \frac{bx}{4a(bc-ad)(a+bx^4)} + \frac{d^{3/2}\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx}{4\sqrt{c}(bc-ad)^2} + \frac{d^{3/2}\int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}}+\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}+x^2}\,dx}{4\sqrt{c}(bc-ad)^2} - \frac{d^{7/4}\int \frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{d}}+2x}{-\frac{\sqrt{c}}{\sqrt{d}}-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}}-}}{4\sqrt{2}c^{3/4}(bc-a}$$

$$= \frac{bx}{4a(bc-ad)(a+bx^4)} - \frac{b^{3/4}(3bc-7ad)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^2} + \frac{b^{3/4}(3bc-7ad)\log}{16\sqrt{}}$$

$$= \frac{bx}{4a(bc-ad)(a+bx^4)} - \frac{b^{3/4}(3bc-7ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(bc-ad)^2} + \frac{b^{3/4}(3bc-7ad)\tan^{-1}\left(1+\frac{\sqrt{2}}{}\right.}{8\sqrt{2}a^{7/4}(bc-ad)^2}$$

Mathematica [A] time = 0.32449, size = 499, normalized size = 0.97

$$\frac{8a^{3/4}bc^{3/4}x(bc-ad) - 4\sqrt{2}a^{7/4}d^{7/4}(a+bx^4)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right) + 4\sqrt{2}a^{7/4}d^{7/4}(a+bx^4)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^4)^2*(c + d*x^4)),x]
```

```
[Out] (8*a^(3/4)*b*c^(3/4)*(b*c - a*d)*x - 2*Sqrt[2]*b^(3/4)*c^(3/4)*(3*b*c - 7*a
*d)*(a + b*x^4)*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 2*Sqrt[2]*b^(3/4)
*c^(3/4)*(3*b*c - 7*a*d)*(a + b*x^4)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)
] - 8*Sqrt[2]*a^(7/4)*d^(7/4)*(a + b*x^4)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^
(1/4)] + 8*Sqrt[2]*a^(7/4)*d^(7/4)*(a + b*x^4)*ArcTan[1 + (Sqrt[2]*d^(1/4)*
x)/c^(1/4)] - Sqrt[2]*b^(3/4)*c^(3/4)*(3*b*c - 7*a*d)*(a + b*x^4)*Log[Sqrt[
a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + Sqrt[2]*b^(3/4)*c^(3/4)*(3*
b*c - 7*a*d)*(a + b*x^4)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*
x^2] - 4*Sqrt[2]*a^(7/4)*d^(7/4)*(a + b*x^4)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*
d^(1/4)*x + Sqrt[d]*x^2] + 4*Sqrt[2]*a^(7/4)*d^(7/4)*(a + b*x^4)*Log[Sqrt[c
] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(32*a^(7/4)*c^(3/4)*(b*c - a*
d)^2*(a + b*x^4))
```

Maple [A] time = 0.01, size = 550, normalized size = 1.1

$$\frac{d^2\sqrt{2}}{8(ad-bc)^2c}\sqrt[4]{\frac{c}{d}}\ln\left(\left(x^2+\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)\left(x^2-\sqrt[4]{\frac{c}{d}}x\sqrt{2}+\sqrt{\frac{c}{d}}\right)^{-1}\right) + \frac{d^2\sqrt{2}}{4(ad-bc)^2c}\sqrt[4]{\frac{c}{d}}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{\frac{c}{d}}}+1\right) + \frac{}{4(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^2/(d*x^4+c),x)
```

```
[Out] 1/8*d^2/(a*d-b*c)^2*(c/d)^(1/4)/c*2^(1/2)*ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/
d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))+1/4*d^2/(a*d-b*c)^2*(c/d
)^(1/4)/c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)+1/4*d^2/(a*d-b*c)^2*(c/d)
^(1/4)/c*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)-1/4*b/(a*d-b*c)^2*x/(b*x^4
+a)*d+1/4*b^2/(a*d-b*c)^2/a*x/(b*x^4+a)*c-7/16*b/(a*d-b*c)^2/a*(1/b*a)^(1/4
)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*d+3/16*b^2/(a*d-b*c)^2/a^2*(1/b
*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*c-7/32*b/(a*d-b*c)^2/a*
(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(
1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))*d+3/32*b^2/(a*d-b*c)^2/a^2*(1/b*a)^(
1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2))/(x^2-(1/b*a)^(1
/4)*x*2^(1/2)+(1/b*a)^(1/2)))*c-7/16*b/(a*d-b*c)^2/a*(1/b*a)^(1/4)*2^(1/2)*
arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)*d+3/16*b^2/(a*d-b*c)^2/a^2*(1/b*a)^(1/4)*
2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^2/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 120.692, size = 6809, normalized size = 13.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^2/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] -1/16*(4*((a*b^2*c - a^2*b*d)*x^4 + a^2*b*c - a^3*d)*(-(81*b^7*c^4 - 756*a*
b^6*c^3*d + 2646*a^2*b^5*c^2*d^2 - 4116*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/(
a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b^5*c^5*d^3 +
70*a^11*b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d^6 - 8*a^14*b*
c*d^7 + a^15*d^8))^(1/4)*arctan(((a^5*b^6*c^6 - 6*a^6*b^5*c^5*d + 15*a^7*b^
4*c^4*d^2 - 20*a^8*b^3*c^3*d^3 + 15*a^9*b^2*c^2*d^4 - 6*a^10*b*c*d^5 + a^11
*d^6)*x*(-(81*b^7*c^4 - 756*a*b^6*c^3*d + 2646*a^2*b^5*c^2*d^2 - 4116*a^3*b
^4*c*d^3 + 2401*a^4*b^3*d^4)/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^
6*d^2 - 56*a^10*b^5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^5 + 2
8*a^13*b^2*c^2*d^6 - 8*a^14*b*c*d^7 + a^15*d^8))^(3/4) - (a^5*b^6*c^6 - 6*a
^6*b^5*c^5*d + 15*a^7*b^4*c^4*d^2 - 20*a^8*b^3*c^3*d^3 + 15*a^9*b^2*c^2*d^4
 - 6*a^10*b*c*d^5 + a^11*d^6)*(-(81*b^7*c^4 - 756*a*b^6*c^3*d + 2646*a^2*b^
5*c^2*d^2 - 4116*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/(a^7*b^8*c^8 - 8*a^8*b^7
*c^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b^5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 5
6*a^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d^6 - 8*a^14*b*c*d^7 + a^15*d^8))^(3/4
)*sqrt(((9*b^4*c^2 - 42*a*b^3*c*d + 49*a^2*b^2*d^2)*x^2 + (a^4*b^4*c^4 - 4*
a^5*b^3*c^3*d + 6*a^6*b^2*c^2*d^2 - 4*a^7*b*c*d^3 + a^8*d^4)*sqrt(-(81*b^7*
c^4 - 756*a*b^6*c^3*d + 2646*a^2*b^5*c^2*d^2 - 4116*a^3*b^4*c*d^3 + 2401*a^
4*b^3*d^4)/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b^
5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d^6
```

```
 - 8*a^14*b*c*d^7 + a^15*d^8)))/(9*b^4*c^2 - 42*a*b^3*c*d + 49*a^2*b^2*d^2)
))/(27*b^5*c^3 - 189*a*b^4*c^2*d + 441*a^2*b^3*c*d^2 - 343*a^3*b^2*d^3)) -
16*(-d^7/(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d
^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b
*c^4*d^7 + a^8*c^3*d^8))^(1/4)*((a*b^2*c - a^2*b*d)*x^4 + a^2*b*c - a^3*d)*
arctan(-((b^6*c^8 - 6*a*b^5*c^7*d + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*d^3
 + 15*a^4*b^2*c^4*d^4 - 6*a^5*b*c^3*d^5 + a^6*c^2*d^6)*(-d^7/(b^8*c^11 - 8*
a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4
 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8)
)^(3/4)*x - (b^6*c^8 - 6*a*b^5*c^7*d + 15*a^2*b^4*c^6*d^2 - 20*a^3*b^3*c^5*
d^3 + 15*a^4*b^2*c^4*d^4 - 6*a^5*b*c^3*d^5 + a^6*c^2*d^6)*(-d^7/(b^8*c^11 -
 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*
d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d
^8))^(3/4)*sqrt((d^4*x^2 + (b^4*c^6 - 4*a*b^3*c^5*d + 6*a^2*b^2*c^4*d^2 - 4
*a^3*b*c^3*d^3 + a^4*c^2*d^4)*sqrt(-d^7/(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2
*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5
 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d^7 + a^8*c^3*d^8)))/d^4))/d^5) - 4*(-d
^7/(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3 + 7
0*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^4*d
^7 + a^8*c^3*d^8))^(1/4)*((a*b^2*c - a^2*b*d)*x^4 + a^2*b*c - a^3*d)*log(d^
2*x + (-d^7/(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^
8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^
7*b*c^4*d^7 + a^8*c^3*d^8))^(1/4)*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)) + 4*
(-d^7/(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5*c^8*d^3
+ 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8*a^7*b*c^
4*d^7 + a^8*c^3*d^8))^(1/4)*((a*b^2*c - a^2*b*d)*x^4 + a^2*b*c - a^3*d)*log
(d^2*x - (-d^7/(b^8*c^11 - 8*a*b^7*c^10*d + 28*a^2*b^6*c^9*d^2 - 56*a^3*b^5
*c^8*d^3 + 70*a^4*b^4*c^7*d^4 - 56*a^5*b^3*c^6*d^5 + 28*a^6*b^2*c^5*d^6 - 8
*a^7*b*c^4*d^7 + a^8*c^3*d^8))^(1/4)*(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)) +
 ((a*b^2*c - a^2*b*d)*x^4 + a^2*b*c - a^3*d)*(-(81*b^7*c^4 - 756*a*b^6*c^3*
d + 2646*a^2*b^5*c^2*d^2 - 4116*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/(a^7*b^8*
c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b^5*c^5*d^3 + 70*a^11*
b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d^6 - 8*a^14*b*c*d^7 +
a^15*d^8))^(1/4)*log(-(3*b^2*c - 7*a*b*d)*x + (a^2*b^2*c^2 - 2*a^3*b*c*d +
a^4*d^2)*(-(81*b^7*c^4 - 756*a*b^6*c^3*d + 2646*a^2*b^5*c^2*d^2 - 4116*a^3*
b^4*c*d^3 + 2401*a^4*b^3*d^4)/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c
^6*d^2 - 56*a^10*b^5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^5 +
28*a^13*b^2*c^2*d^6 - 8*a^14*b*c*d^7 + a^15*d^8))^(1/4)) - ((a*b^2*c - a^2*
b*d)*x^4 + a^2*b*c - a^3*d)*(-(81*b^7*c^4 - 756*a*b^6*c^3*d + 2646*a^2*b^5*
c^2*d^2 - 4116*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/(a^7*b^8*c^8 - 8*a^8*b^7*c
^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b^5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 56*
a^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d^6 - 8*a^14*b*c*d^7 + a^15*d^8))^(1/4)*
log(-(3*b^2*c - 7*a*b*d)*x - (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*(-(81*b^
7*c^4 - 756*a*b^6*c^3*d + 2646*a^2*b^5*c^2*d^2 - 4116*a^3*b^4*c*d^3 + 2401*
a^4*b^3*d^4)/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*
b^5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d
^6 - 8*a^14*b*c*d^7 + a^15*d^8))^(1/4)) - 4*b*x)/((a*b^2*c - a^2*b*d)*x^4 +
 a^2*b*c - a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**2/(d*x**4+c),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13741, size = 900, normalized size = 1.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^2/(d*x^4+c),x, algorithm="giac")
```

```
[Out] 1/2*(c*d^3)^(1/4)*d*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(c/d)^(1/4))/(c/d)^(1
/4))/(sqrt(2)*b^2*c^3 - 2*sqrt(2)*a*b*c^2*d + sqrt(2)*a^2*c*d^2) + 1/2*(c*d
^3)^(1/4)*d*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(c/d)^(1/4))/(c/d)^(1/4))/(sq
rt(2)*b^2*c^3 - 2*sqrt(2)*a*b*c^2*d + sqrt(2)*a^2*c*d^2) + 1/4*(c*d^3)^(1/4
)*d*log(x^2 + sqrt(2)*x*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b^2*c^3 - 2*sqrt(
2)*a*b*c^2*d + sqrt(2)*a^2*c*d^2) - 1/4*(c*d^3)^(1/4)*d*log(x^2 - sqrt(2)*x
*(c/d)^(1/4) + sqrt(c/d))/(sqrt(2)*b^2*c^3 - 2*sqrt(2)*a*b*c^2*d + sqrt(2)*
a^2*c*d^2) + 1/8*(3*(a*b^3)^(1/4)*b*c - 7*(a*b^3)^(1/4)*a*d)*arctan(1/2*sqr
t(2)*(2*x + sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2)*a^2*b^2*c^2 - 2*sqrt
(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) + 1/8*(3*(a*b^3)^(1/4)*b*c - 7*(a*b^3)^(1/
4)*a*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/b)^(1/4))/(a/b)^(1/4))/(sqrt(2
)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) + 1/16*(3*(a*b^3)^(1
/4)*b*c - 7*(a*b^3)^(1/4)*a*d)*log(x^2 + sqrt(2)*x*(a/b)^(1/4) + sqrt(a/b))
/(sqrt(2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) - 1/16*(3*(a
*b^3)^(1/4)*b*c - 7*(a*b^3)^(1/4)*a*d)*log(x^2 - sqrt(2)*x*(a/b)^(1/4) + sq
rt(a/b))/(sqrt(2)*a^2*b^2*c^2 - 2*sqrt(2)*a^3*b*c*d + sqrt(2)*a^4*d^2) + 1/
4*b*x/((b*x^4 + a)*(a*b*c - a^2*d))
```

3.172 $\int \frac{1}{\left(a+bx^4\right)^2 \left(c+dx^4\right)^2} dx$

Optimal. Leaf size=596

$$-\frac{b^{7/4}(3bc-11ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^3}+\frac{b^{7/4}(3bc-11ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^3}-\frac{b^{7/4}(3bc-11ad)\,\mathrm{ta}}{8\sqrt{2}a^{7/4}(}$$

```
[Out] (d*(b*c + a*d)*x)/(4*a*c*(b*c - a*d)^2*(c + d*x^4)) + (b*x)/(4*a*(b*c - a*d
)*(a + b*x^4)*(c + d*x^4)) - (b^(7/4)*(3*b*c - 11*a*d)*ArcTan[1 - (Sqrt[2]*
b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) + (b^(7/4)*(3*b*c -
11*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(b*c -
a*d)^3) - (d^(7/4)*(11*b*c - 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)]
)/(8*Sqrt[2]*c^(7/4)*(b*c - a*d)^3) + (d^(7/4)*(11*b*c - 3*a*d)*ArcTan[1 +
(Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(7/4)*(b*c - a*d)^3) - (b^(7/4)*
(3*b*c - 11*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(1
6*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) + (b^(7/4)*(3*b*c - 11*a*d)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*(b*c - a*d)^3
) - (d^(7/4)*(11*b*c - 3*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqr
t[d]*x^2])/(16*Sqrt[2]*c^(7/4)*(b*c - a*d)^3) + (d^(7/4)*(11*b*c - 3*a*d)*L
og[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(16*Sqrt[2]*c^(7/4)*
(b*c - a*d)^3)
```

Rubi [A] time = 0.738621, antiderivative size = 596, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {414, 527, 522, 211, 1165, 628, 1162, 617, 204}

$$-\frac{b^{7/4}(3bc-11ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^3}+\frac{b^{7/4}(3bc-11ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}(bc-ad)^3}-\frac{b^{7/4}(3bc-11ad)\,\mathrm{ta}}{8\sqrt{2}a^{7/4}(}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^2*(c + d*x^4)^2),x]
```

```
[Out] (d*(b*c + a*d)*x)/(4*a*c*(b*c - a*d)^2*(c + d*x^4)) + (b*x)/(4*a*(b*c - a*d
)*(a + b*x^4)*(c + d*x^4)) - (b^(7/4)*(3*b*c - 11*a*d)*ArcTan[1 - (Sqrt[2]*
b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) + (b^(7/4)*(3*b*c -
11*a*d)*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(b*c -
a*d)^3) - (d^(7/4)*(11*b*c - 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)]
)/(8*Sqrt[2]*c^(7/4)*(b*c - a*d)^3) + (d^(7/4)*(11*b*c - 3*a*d)*ArcTan[1 +
(Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(8*Sqrt[2]*c^(7/4)*(b*c - a*d)^3) - (b^(7/4)*
(3*b*c - 11*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(1
6*Sqrt[2]*a^(7/4)*(b*c - a*d)^3) + (b^(7/4)*(3*b*c - 11*a*d)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(16*Sqrt[2]*a^(7/4)*(b*c - a*d)^3
) - (d^(7/4)*(11*b*c - 3*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqr
t[d]*x^2])/(16*Sqrt[2]*c^(7/4)*(b*c - a*d)^3) + (d^(7/4)*(11*b*c - 3*a*d)*L
og[Sqrt[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(16*Sqrt[2]*c^(7/4)*
(b*c - a*d)^3)
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
```

```
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
```

```
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+bx^4\right)^2\left(c+dx^4\right)^2}\,dx = \frac{bx}{4a(bc-ad)\left(a+bx^4\right)\left(c+dx^4\right)} - \frac{\int \frac{-3bc+4ad-7bdx^4}{\left(a+bx^4\right)\left(c+dx^4\right)^2}\,dx}{4a(bc-ad)}$$

$$= \frac{d(bc+ad)x}{4ac(bc-ad)^2\left(c+dx^4\right)} + \frac{bx}{4a(bc-ad)\left(a+bx^4\right)\left(c+dx^4\right)} - \frac{\int \frac{-4\left(3b^2c^2-8abcd+3a^2d^2\right)-12bd(bc+a}{\left(a+bx^4\right)\left(c+dx^4\right)}}{16ac(bc-ad)^2}$$

$$= \frac{d(bc+ad)x}{4ac(bc-ad)^2\left(c+dx^4\right)} + \frac{bx}{4a(bc-ad)\left(a+bx^4\right)\left(c+dx^4\right)} + \frac{\left(b^2(3bc-11ad)\right)\int \frac{1}{a+bx^4}\,dx}{4a(bc-ad)^3} +$$

$$= \frac{d(bc+ad)x}{4ac(bc-ad)^2\left(c+dx^4\right)} + \frac{bx}{4a(bc-ad)\left(a+bx^4\right)\left(c+dx^4\right)} + \frac{\left(b^2(3bc-11ad)\right)\int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4}\,dx}{8a^{3/2}(bc-ad)^3}$$

$$= \frac{d(bc+ad)x}{4ac(bc-ad)^2\left(c+dx^4\right)} + \frac{bx}{4a(bc-ad)\left(a+bx^4\right)\left(c+dx^4\right)} + \frac{\left(b^{3/2}(3bc-11ad)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}}-\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}}}}{16a^{3/2}(bc-ad)^3}$$

$$= \frac{d(bc+ad)x}{4ac(bc-ad)^2\left(c+dx^4\right)} + \frac{bx}{4a(bc-ad)\left(a+bx^4\right)\left(c+dx^4\right)} - \frac{b^{7/4}(3bc-11ad)\log\left(\sqrt{a}-\sqrt{2}\right.}{16\sqrt{2}a^{7/4}(bc-a}$$

$$= \frac{d(bc+ad)x}{4ac(bc-ad)^2\left(c+dx^4\right)} + \frac{bx}{4a(bc-ad)\left(a+bx^4\right)\left(c+dx^4\right)} - \frac{b^{7/4}(3bc-11ad)\tan^{-1}\left(1-\frac{\sqrt{2}}{}\right.}{8\sqrt{2}a^{7/4}(bc-ad)^3}$$

Mathematica [A] time = 1.4117, size = 561, normalized size = 0.94

$$\frac{1}{32}\left(\frac{\sqrt{2}b^{7/4}(11ad-3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{7/4}(bc-ad)^3} + \frac{\sqrt{2}b^{7/4}(11ad-3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{7/4}(ad-bc)^3} + \frac{2\sqrt{2}b^{7/4}}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^4)^2*(c + d*x^4)^2),x]
```

```
[Out] ((8*b^2*x)/(a*(b*c - a*d)^2*(a + b*x^4)) + (8*d^2*x)/(c*(b*c - a*d)^2*(c +
d*x^4)) + (2*Sqrt[2]*b^(7/4)*(-3*b*c + 11*a*d)*ArcTan[1 - (Sqrt[2]*b^(1/4)*
x)/a^(1/4)])/(a^(7/4)*(b*c - a*d)^3) + (2*Sqrt[2]*b^(7/4)*(-3*b*c + 11*a*d)
*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)])/(a^(7/4)*(-(b*c) + a*d)^3) + (2*S
qrt[2]*d^(7/4)*(-11*b*c + 3*a*d)*ArcTan[1 - (Sqrt[2]*d^(1/4)*x)/c^(1/4)])/(
c^(7/4)*(b*c - a*d)^3) + (2*Sqrt[2]*d^(7/4)*(11*b*c - 3*a*d)*ArcTan[1 + (Sq
rt[2]*d^(1/4)*x)/c^(1/4)])/(c^(7/4)*(b*c - a*d)^3) + (Sqrt[2]*b^(7/4)*(-3*b
*c + 11*a*d)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(a^(7/
4)*(b*c - a*d)^3) + (Sqrt[2]*b^(7/4)*(-3*b*c + 11*a*d)*Log[Sqrt[a] + Sqrt[2
]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(a^(7/4)*(-(b*c) + a*d)^3) + (Sqrt[2]*d
^(7/4)*(11*b*c - 3*a*d)*Log[Sqrt[c] - Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x
^2])/(c^(7/4)*(-(b*c) + a*d)^3) + (Sqrt[2]*d^(7/4)*(11*b*c - 3*a*d)*Log[Sqr
t[c] + Sqrt[2]*c^(1/4)*d^(1/4)*x + Sqrt[d]*x^2])/(c^(7/4)*(b*c - a*d)^3))/3
2
```

Maple [A] time = 0.015, size = 784, normalized size = 1.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^2/(d*x^4+c)^2,x)
```

```
[Out] 1/4*d^3/(a*d-b*c)^3/c*x/(d*x^4+c)*a-1/4*d^2/(a*d-b*c)^3*x/(d*x^4+c)*b+3/16*
d^3/(a*d-b*c)^3/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)*a-1
1/16*d^2/(a*d-b*c)^3/c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)*
b+3/16*d^3/(a*d-b*c)^3/c^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x
-1)*a-11/16*d^2/(a*d-b*c)^3/c*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4
)*x-1)*b+3/32*d^3/(a*d-b*c)^3/c^2*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*x
*2^(1/2)+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))*a-11/32*d^2/
(a*d-b*c)^3/c*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)
)/(x^2-(c/d)^(1/4)*x*2^(1/2)+(c/d)^(1/2)))*b+1/4*b^2/(a*d-b*c)^3*x/(b*x^4+a
)*d-1/4*b^3/(a*d-b*c)^3/a*x/(b*x^4+a)*c+11/16*b^2/(a*d-b*c)^3/a*(1/b*a)^(1/
4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)*d-3/16*b^3/(a*d-b*c)^3/a^2*(1/
b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x+1)*c+11/16*b^2/(a*d-b*c)^
3/a*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*d-3/16*b^3/(a*d
-b*c)^3/a^2*(1/b*a)^(1/4)*2^(1/2)*arctan(2^(1/2)/(1/b*a)^(1/4)*x-1)*c+11/32
*b^2/(a*d-b*c)^3/a*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1
/b*a)^(1/2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))*d-3/32*b^3/(a*d-b
*c)^3/a^2*(1/b*a)^(1/4)*2^(1/2)*ln((x^2+(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/
2))/(x^2-(1/b*a)^(1/4)*x*2^(1/2)+(1/b*a)^(1/2)))*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^2/(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^2/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**2/(d*x**4+c)**2,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^2/(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.173 $\int \frac{(a-bx^4)^{5/2}}{c-dx^4}\,dx$

Optimal. Leaf size=321

$$\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(47a^2d^2-56abcd+21b^2c^2\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{21d^3\sqrt{a-bx^4}}-\frac{bx\sqrt{a-bx^4}(7bc-13ad)}{21d^2}-\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(b\cdots}{\cdots}$$

```
[Out] -(b*(7*b*c - 13*a*d)*x*Sqrt[a - b*x^4])/(21*d^2) + (b*x*(a - b*x^4)^(3/2))/
(7*d) + (a^(1/4)*b^(3/4)*(21*b^2*c^2 - 56*a*b*c*d + 47*a^2*d^2)*Sqrt[1 - (b
*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(21*d^3*Sqrt[a - b*x^4
]) - (a^(1/4)*(b*c - a*d)^3*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[
d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*d^3*
Sqrt[a - b*x^4]) - (a^(1/4)*(b*c - a*d)^3*Sqrt[1 - (b*x^4)/a]*EllipticPi[(S
qrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(
1/4)*c*d^3*Sqrt[a - b*x^4])
```

Rubi [A] time = 0.382883, antiderivative size = 321, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {416, 528, 523, 224, 221, 409, 1219, 1218}

$$\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(47a^2d^2-56abcd+21b^2c^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{21d^3\sqrt{a-bx^4}}-\frac{bx\sqrt{a-bx^4}(7bc-13ad)}{21d^2}-\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(bc-ad)^3\cdots}{2\sqrt[4]{b}c\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^4)^(5/2)/(c - d*x^4),x]
```

```
[Out] -(b*(7*b*c - 13*a*d)*x*Sqrt[a - b*x^4])/(21*d^2) + (b*x*(a - b*x^4)^(3/2))/
(7*d) + (a^(1/4)*b^(3/4)*(21*b^2*c^2 - 56*a*b*c*d + 47*a^2*d^2)*Sqrt[1 - (b
*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(21*d^3*Sqrt[a - b*x^4
]) - (a^(1/4)*(b*c - a*d)^3*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[
d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*d^3*
Sqrt[a - b*x^4]) - (a^(1/4)*(b*c - a*d)^3*Sqrt[1 - (b*x^4)/a]*EllipticPi[(S
qrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(
1/4)*c*d^3*Sqrt[a - b*x^4])
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
```

```
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a-bx^4)^{5/2}}{c-dx^4}dx = \frac{bx(a-bx^4)^{3/2}}{7d} - \frac{\int \frac{\sqrt{a-bx^4}(a(bc-7ad)-b(7bc-13ad)x^4)}{c-dx^4}dx}{7d}$$

$$= -\frac{b(7bc-13ad)x\sqrt{a-bx^4}}{21d^2} + \frac{bx(a-bx^4)^{3/2}}{7d} + \frac{\int \frac{a(7b^2c^2-16abcd+21a^2d^2)-b(21b^2c^2-56abcd+47a^2d^2)x^4}{\sqrt{a-bx^4}(c-dx^4)}dx}{21d^2}$$

$$= -\frac{b(7bc-13ad)x\sqrt{a-bx^4}}{21d^2} + \frac{bx(a-bx^4)^{3/2}}{7d} - \frac{(bc-ad)^3\int \frac{1}{\sqrt{a-bx^4}(c-dx^4)}dx}{d^3} + \frac{(b(21b^2c^2-56abc}{}$$

$$= -\frac{b(7bc-13ad)x\sqrt{a-bx^4}}{21d^2} + \frac{bx(a-bx^4)^{3/2}}{7d} - \frac{(bc-ad)^3\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}dx}{2cd^3} - \frac{(bc-ad)^3\int \frac{}{\left(1+\frac{\sqrt{}}{}\right)}}{2cd^3}$$

$$= -\frac{b(7bc-13ad)x\sqrt{a-bx^4}}{21d^2} + \frac{bx(a-bx^4)^{3/2}}{7d} + \frac{\sqrt[4]{a}b^{3/4}(21b^2c^2-56abcd+47a^2d^2)\sqrt{1-\frac{bx^4}{a}}F(\text{si}}{21d^3\sqrt{a-bx^4}}$$

$$= -\frac{b(7bc-13ad)x\sqrt{a-bx^4}}{21d^2} + \frac{bx(a-bx^4)^{3/2}}{7d} + \frac{\sqrt[4]{a}b^{3/4}(21b^2c^2-56abcd+47a^2d^2)\sqrt{1-\frac{bx^4}{a}}F(\text{si}}{21d^3\sqrt{a-bx^4}}$$

Mathematica [C] time = 0.715833, size = 290, normalized size = 0.9

$$\frac{x\left(-\frac{bx^4\sqrt{1-\frac{bx^4}{a}}(47a^2d^2-56abcd+21b^2c^2)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)}{c} + \frac{25a^2c(21a^2d^2-16abcd+7b^2c^2)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)}{(c-dx^4)\left(2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)+5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\right.\right.}\right.}{105d^2\sqrt{a-bx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - b*x^4)^(5/2)/(c - d*x^4),x]
```

```
[Out] (x*(5*b*(-a + b*x^4)*(7*b*c - 16*a*d + 3*b*d*x^4) - (b*(21*b^2*c^2 - 56*a*b
*c*d + 47*a^2*d^2)*x^4*Sqrt[1 - (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9/4, (b*x^
4)/a, (d*x^4)/c])/c + (25*a^2*c*(7*b^2*c^2 - 16*a*b*c*d + 21*a^2*d^2)*Appel
lF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c])/((c - d*x^4)*(5*a*c*AppellF1[1
/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2,
 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*
x^4)/c])))))/(105*d^2*Sqrt[a - b*x^4])
```

Maple [C] time = 0.051, size = 408, normalized size = 1.3

$$-\frac{b^2x^5}{7d}\sqrt{-bx^4+a} + \frac{x}{3b}\left(\frac{b^2(3ad-bc)}{d^2} - \frac{5b^2a}{7d}\right)\sqrt{-bx^4+a} - \left(-\frac{b(3a^2d^2-3cabd+b^2c^2)}{d^3} + \frac{a}{3b}\left(\frac{b^2(3ad-bc)}{d^2} - \frac{5b^2}{7d}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^4+a)^(5/2)/(-d*x^4+c),x)
```

```
[Out] -1/7*b^2/d*x^5*(-b*x^4+a)^(1/2)+1/3*(b^2/d^2*(3*a*d-b*c)-5/7*b^2/d*a)/b*x*(
-b*x^4+a)^(1/2)-(-b*(3*a^2*d^2-3*a*b*c*d+b^2*c^2)/d^3+1/3*(b^2/d^2*(3*a*d-b
```

```
*c)-5/7*b^2/d*a)/b*a)/(1/a^(1/2)*b^(1/2))^(1/2)*(1-x^2*b^(1/2)/a^(1/2))^(1/
2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^
(1/2))^(1/2),I)+1/8/d^4*sum((-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/
_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*_alpha^2*b*x^2+2*a)/((a*d-
b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^(1/2)*_alpha^3*d/c*(1
-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*
EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^(1/2)*_alpha^2/c*d,(-1/a^(
1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_alpha=RootOf(_Z^4*d-c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(-bx^4+a\right)^{\frac{5}{2}}}{dx^4-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(5/2)/(-d*x^4+c),x, algorithm="maxima")
```

```
[Out] -integrate((-b*x^4 + a)^(5/2)/(d*x^4 - c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(5/2)/(-d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{a^2\sqrt{a-bx^4}}{-c+dx^4}\,dx-\int \frac{b^2x^8\sqrt{a-bx^4}}{-c+dx^4}\,dx-\int -\frac{2abx^4\sqrt{a-bx^4}}{-c+dx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**4+a)**(5/2)/(-d*x**4+c),x)
```

```
[Out] -Integral(a**2*sqrt(a - b*x**4)/(-c + d*x**4), x) - Integral(b**2*x**8*sqrt
(a - b*x**4)/(-c + d*x**4), x) - Integral(-2*a*b*x**4*sqrt(a - b*x**4)/(-c
+ d*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(-bx^4+a\right)^{\frac{5}{2}}}{dx^4-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(5/2)/(-d*x^4+c),x, algorithm="giac")

[Out] integrate(-(-b*x^4 + a)^(5/2)/(d*x^4 - c), x)
```

3.174 $\int \frac{(a-bx^4)^{3/2}}{c-dx^4}\,dx$

Optimal. Leaf size=277

$$-\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(3bc-5ad)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{3d^2\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(bc-ad)^2\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd^2\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1}\ldots}{}$$

```
[Out] (b*x*Sqrt[a - b*x^4])/(3*d) - (a^(1/4)*b^(3/4)*(3*b*c - 5*a*d)*Sqrt[1 - (b*
x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(3*d^2*Sqrt[a - b*x^4])
 + (a^(1/4)*(b*c - a*d)^2*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d]
)/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*d^2*Sq
rt[a - b*x^4]) + (a^(1/4)*(b*c - a*d)^2*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqr
t[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/
4)*c*d^2*Sqrt[a - b*x^4])
```

Rubi [A] time = 0.262202, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {416, 523, 224, 221, 409, 1219, 1218}

$$-\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(3bc-5ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{3d^2\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(bc-ad)^2\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd^2\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(b\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^4)^(3/2)/(c - d*x^4),x]
```

```
[Out] (b*x*Sqrt[a - b*x^4])/(3*d) - (a^(1/4)*b^(3/4)*(3*b*c - 5*a*d)*Sqrt[1 - (b*
x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(3*d^2*Sqrt[a - b*x^4])
 + (a^(1/4)*(b*c - a*d)^2*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d]
)/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*d^2*Sq
rt[a - b*x^4]) + (a^(1/4)*(b*c - a*d)^2*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqr
t[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/
4)*c*d^2*Sqrt[a - b*x^4])
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\left(a-bx^4\right)^{3/2}}{c-dx^4}\,dx = \frac{bx\sqrt{a-bx^4}}{3d} - \frac{\int \frac{a(bc-3ad)-b(3bc-5ad)x^4}{\sqrt{a-bx^4}\left(c-dx^4\right)}\,dx}{3d}$$

$$= \frac{bx\sqrt{a-bx^4}}{3d} - \frac{(b(3bc-5ad))\int \frac{1}{\sqrt{a-bx^4}}\,dx}{3d^2} + \frac{(bc-ad)^2\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}\,dx}{d^2}$$

$$= \frac{bx\sqrt{a-bx^4}}{3d} + \frac{(bc-ad)^2\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{2cd^2} + \frac{(bc-ad)^2\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{2cd^2} - \frac{\Big(b(3bc-5ad)\sqrt{}}{3d^2}$$

$$= \frac{bx\sqrt{a-bx^4}}{3d} - \frac{\sqrt[4]{a}b^{3/4}(3bc-5ad)\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{3d^2\sqrt{a-bx^4}} + \frac{\left((bc-ad)^2\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right.}}{2cd^2\sqrt{a-bx^4}}$$

$$= \frac{bx\sqrt{a-bx^4}}{3d} - \frac{\sqrt[4]{a}b^{3/4}(3bc-5ad)\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{3d^2\sqrt{a-bx^4}} + \frac{\sqrt[4]{a}(bc-ad)^2\sqrt{1-\frac{bx^4}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{}}{\sqrt{b}\sqrt{}}\right.}{2\sqrt[4]{b}cd^2\sqrt{a-}}$$

Mathematica [C] time = 0.35143, size = 341, normalized size = 1.23

$$-\frac{x\left(\frac{5\left(5ac\left(3a^2d-abdx^4+b^2x^4\left(dx^4-c\right)\right)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+2bx^4\left(a-bx^4\right)\left(c-dx^4\right)\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)\right)}{\left(dx^4-c\right)\left(2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)+5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)}+\frac{bx^4\sqrt{1-\frac{bx^4}{a}}(5a}{}\right.}{15d\sqrt{a-bx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - b*x^4)^(3/2)/(c - d*x^4),x]
```

```
[Out] -(x*((b*(-3*b*c + 5*a*d)*x^4*Sqrt[1 - (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9/4,
 (b*x^4)/a, (d*x^4)/c])/c + (5*(5*a*c*(3*a^2*d - a*b*d*x^4 + b^2*x^4*(-c +
d*x^4))*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*b*x^4*(a - b*x
^4)*(c - d*x^4)*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b
*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))/((-c + d*x^4)*(5*a*c
*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5
/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*
x^4)/a, (d*x^4)/c])))))/(15*d*Sqrt[a - b*x^4])
```

Maple [C] time = 0.019, size = 311, normalized size = 1.1

$$\frac{bx}{3d}\sqrt{-bx^4+a}-\left(-\frac{b(2ad-bc)}{d^2}+\frac{ab}{3d}\right)\sqrt{1-x^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+x^2\sqrt{b}\frac{1}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{-bx^4+a}}+\frac{1}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^4+a)^(3/2)/(-d*x^4+c),x)
```

```
[Out] 1/3*b*x*(-b*x^4+a)^(1/2)/d-(-b*(2*a*d-b*c)/d^2+1/3/d*b*a)/(1/a^(1/2)*b^(1/2
))^(1/2)*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/(-b*x^
4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)+1/8/d^3*sum((-a^2*d^2+2
*a*b*c*d-b^2*c^2)/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*_alpha^2
*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^(1/
2)*_alpha^3*d/c*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)
/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^(1/2)*_a
lpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_alpha=Ro
otOf(_Z^4*d-c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\left(-bx^4+a\right)^{\frac{3}{2}}}{dx^4-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(3/2)/(-d*x^4+c),x, algorithm="maxima")
```

```
[Out] -integrate((-b*x^4 + a)^(3/2)/(d*x^4 - c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(3/2)/(-d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{a\sqrt{a-bx^4}}{-c+dx^4}\,dx - \int -\frac{bx^4\sqrt{a-bx^4}}{-c+dx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**4+a)**(3/2)/(-d*x**4+c),x)
```

```
[Out] -Integral(a*sqrt(a - b*x**4)/(-c + d*x**4), x) - Integral(-b*x**4*sqrt(a -
b*x**4)/(-c + d*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(-bx^4+a\right)^{\frac{3}{2}}}{dx^4-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(3/2)/(-d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate(-(-b*x^4 + a)^(3/2)/(d*x^4 - c), x)
```

3.175 $\int \frac{\sqrt{a-bx^4}}{c-dx^4} \, dx$

Optimal. Leaf size=240

$$\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{d\sqrt{a-bx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(bc-ad)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd\sqrt{a-bx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(bc-ad)}{2\sqrt[4]{b}}$$

```
[Out] (a^(1/4)*b^(3/4)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)],
 -1])/(d*Sqrt[a - b*x^4]) - (a^(1/4)*(b*c - a*d)*Sqrt[1 - (b*x^4)/a]*Ellipt
icPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -
1])/(2*b^(1/4)*c*d*Sqrt[a - b*x^4]) - (a^(1/4)*(b*c - a*d)*Sqrt[1 - (b*x^4)
/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1
/4)], -1])/(2*b^(1/4)*c*d*Sqrt[a - b*x^4])
```

Rubi [A] time = 0.163473, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {406, 224, 221, 409, 1219, 1218}

$$\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{d\sqrt{a-bx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(bc-ad)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd\sqrt{a-bx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(bc-ad)\Pi\left(\frac{\sqrt{a}\sqrt{}}{\sqrt{b}\sqrt{}}\right.}{2\sqrt[4]{b}cd\sqrt{a-}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - b*x^4]/(c - d*x^4),x]
```


```
[Out] (a^(1/4)*b^(3/4)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)],
 -1])/(d*Sqrt[a - b*x^4]) - (a^(1/4)*(b*c - a*d)*Sqrt[1 - (b*x^4)/a]*Ellipt
icPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -
1])/(2*b^(1/4)*c*d*Sqrt[a - b*x^4]) - (a^(1/4)*(b*c - a*d)*Sqrt[1 - (b*x^4)
/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1
/4)], -1])/(2*b^(1/4)*c*d*Sqrt[a - b*x^4])
```


Rule 406

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/d,
 Int[1/Sqrt[a + b*x^4], x], x] - Dist[(b*c - a*d)/d, Int[1/(Sqrt[a + b*x^4]
*(c + d*x^4)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
```

```
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt{a-bx^4}}{c-dx^4}\,dx = \frac{b\int \frac{1}{\sqrt{a-bx^4}}\,dx}{d} + \frac{(-bc+ad)\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}\,dx}{d}$$

$$= \frac{(-bc+ad)\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{2cd} + \frac{(-bc+ad)\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{2cd} + \frac{\left(b\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1}{\sqrt{1-\frac{bx^4}{a}}}\,dx}{d\sqrt{a-bx^4}}$$

$$= \frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{d\sqrt{a-bx^4}} + \frac{\left((-bc+ad)\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{1-\frac{bx^4}{a}}}\,dx}{2cd\sqrt{a-bx^4}} + \frac{\left((-bc+ad)\sqrt{\cdots}\right.}{\cdots}$$

$$= \frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{d\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}(bc-ad)\sqrt{1-\frac{bx^4}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}(bc\cdots}{\cdots}$$

Mathematica [C] time = 0.164489, size = 155, normalized size = 0.65

$$-\frac{5acx\sqrt{a-bx^4}F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)}{\left(c-dx^4\right)\left(2x^4\left(bcF_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)-2adF_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)-5acF_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a - b*x^4]/(c - d*x^4),x]
```

```
[Out] (-5*a*c*x*Sqrt[a - b*x^4]*AppellF1[1/4, -1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c]
)/((c - d*x^4)*(-5*a*c*AppellF1[1/4, -1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] +
2*x^4*(-2*a*d*AppellF1[5/4, -1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*Appel
lF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))
```

Maple [C] time = 0.017, size = 259, normalized size = 1.1

$$\frac{b}{d}\sqrt{1-x^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+x^2\sqrt{b}\frac{1}{\sqrt{a}}}\text{EllipticF}\left(x\sqrt{\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{-bx^4+a}}+\frac{1}{8d^2}\sum_{\_alpha=RootOf\left(\_Z^4d-c\right)}\frac{-ad+bc}{\_alpha^3}\left(\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^4+a)^(1/2)/(-d*x^4+c),x)
```

```
[Out] 1/d*b/(1/a^(1/2)*b^(1/2))^(1/2)*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2
)/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)+
1/8/d^2*sum((-a*d+b*c)/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*_al
pha^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2)
)^(1/2)*_alpha^3*d/c*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^
(1/2)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^(1/
2)*_alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_alp
ha=RootOf(_Z^4*d-c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{-bx^4+a}}{dx^4-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/2)/(-d*x^4+c),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(-b*x^4 + a)/(d*x^4 - c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/2)/(-d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{a-bx^4}}{-c+dx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**4+a)**(1/2)/(-d*x**4+c),x)
```

```
[Out] -Integral(sqrt(a - b*x**4)/(-c + d*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{-bx^4+a}}{dx^4-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/2)/(-d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(-b*x^4 + a)/(d*x^4 - c), x)
```

3.176 $\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}\,dx$

Optimal. Leaf size=162

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}}$$

```
[Out] (a^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c
])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*Sqrt[a - b*x^4]) + (a^(
1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), Ar
cSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*Sqrt[a - b*x^4])
```

Rubi [A] time = 0.114994, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {409, 1219, 1218}

$$\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\,\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a - b*x^4]*(c - d*x^4)),x]
```

```
[Out] (a^(1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c
])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*Sqrt[a - b*x^4]) + (a^(
1/4)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), Ar
cSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*Sqrt[a - b*x^4])
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}dx = \frac{\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}dx}{2c} + \frac{\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}dx}{2c}$$

$$= \frac{\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{1-\frac{bx^4}{a}}}dx}{2c\sqrt{a-bx^4}} + \frac{\sqrt{1-\frac{bx^4}{a}}\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{1-\frac{bx^4}{a}}}dx}{2c\sqrt{a-bx^4}}$$

$$= \frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}} + \frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}\Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}}$$

Mathematica [C] time = 0.129923, size = 156, normalized size = 0.96

$$-\frac{5acxF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)}{\sqrt{a-bx^4}\left(dx^4-c\right)\left(2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)+5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a - b*x^4]*(c - d*x^4)),x]
```

```
[Out] (-5*a*c*x*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c])/(Sqrt[a - b*x^4
]*(-c + d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*
x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[
5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))
```

Maple [C] time = 0.016, size = 183, normalized size = 1.1

$$-\frac{1}{8d}\sum_{\_alpha=RootOf\left(\_Z^4d-c\right)}\frac{1}{\_alpha^3}\left(-\mathrm{Artanh}\left(\frac{-2\_alpha^2bx^2+2a}{2}\frac{1}{\sqrt{\frac{ad-bc}{d}}}\frac{1}{\sqrt{-bx^4+a}}\right)\frac{1}{\sqrt{\frac{ad-bc}{d}}}-2\frac{\_alpha^3d}{c\sqrt{-bx^4+a}}\sqrt{1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^4+a)^(1/2)/(-d*x^4+c),x)
```

```
[Out] -1/8/d*sum(1/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*_alpha^2*b*x^
2+2*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^(1/2)*_a
lpha^3*d/c*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/(-b*
x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^(1/2)*_alpha^
2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_alpha=RootOf(
_Z^4*d-c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\sqrt{-bx^4+a}\left(dx^4-c\right)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(1/2)/(-d*x^4+c),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(-b*x^4 + a)*(d*x^4 - c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(1/2)/(-d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-c\sqrt{a-bx^4}+dx^4\sqrt{a-bx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**4+a)**(1/2)/(-d*x**4+c),x)
```

```
[Out] -Integral(1/(-c*sqrt(a - b*x**4) + d*x**4*sqrt(a - b*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\sqrt{-bx^4+a}\left(dx^4-c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(1/2)/(-d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate(-1/(sqrt(-b*x^4 + a)*(d*x^4 - c)), x)
```

3.177 $\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)}\,dx$

Optimal. Leaf size=281

$$\frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{2a^{3/4}\sqrt{a-bx^4}(bc-ad)}+\frac{bx}{2a\sqrt{a-bx^4}(bc-ad)}-\frac{\sqrt[4]{a}d\sqrt{1-\frac{bx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}(bc-ad)}-\frac{\sqrt[4]{a}d}{}$$

```
[Out] (b*x)/(2*a*(b*c - a*d)*Sqrt[a - b*x^4]) + (b^(3/4)*Sqrt[1 - (b*x^4)/a]*Elli
pticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*a^(3/4)*(b*c - a*d)*Sqrt[a - b*x
^4]) - (a^(1/4)*d*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[
b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*(b*c - a*d)*Sq
rt[a - b*x^4]) - (a^(1/4)*d*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d]
)/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*(b*c -
a*d)*Sqrt[a - b*x^4])
```

Rubi [A] time = 0.222203, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {414, 523, 224, 221, 409, 1219, 1218}

$$\frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2a^{3/4}\sqrt{a-bx^4}(bc-ad)}+\frac{bx}{2a\sqrt{a-bx^4}(bc-ad)}-\frac{\sqrt[4]{a}d\sqrt{1-\frac{bx^4}{a}}\,\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}(bc-ad)}-\frac{\sqrt[4]{a}d\sqrt{1-\frac{bx^4}{a}}}{2\sqrt[4]{}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - b*x^4)^(3/2)*(c - d*x^4)),x]
```

```
[Out] (b*x)/(2*a*(b*c - a*d)*Sqrt[a - b*x^4]) + (b^(3/4)*Sqrt[1 - (b*x^4)/a]*Elli
pticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*a^(3/4)*(b*c - a*d)*Sqrt[a - b*x
^4]) - (a^(1/4)*d*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[
b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*(b*c - a*d)*Sq
rt[a - b*x^4]) - (a^(1/4)*d*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d]
)/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*(b*c -
a*d)*Sqrt[a - b*x^4])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)}\,dx = \frac{bx}{2a(bc-ad)\sqrt{a-bx^4}} + \frac{\int \frac{bc-2ad-bdx^4}{\sqrt{a-bx^4}\left(c-dx^4\right)}\,dx}{2a(bc-ad)}$$

$$= \frac{bx}{2a(bc-ad)\sqrt{a-bx^4}} + \frac{b\int \frac{1}{\sqrt{a-bx^4}}\,dx}{2a(bc-ad)} - \frac{d\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}\,dx}{bc-ad}$$

$$= \frac{bx}{2a(bc-ad)\sqrt{a-bx^4}} - \frac{d\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{2c(bc-ad)} - \frac{d\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{2c(bc-ad)} + \frac{\left(b\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1}{\sqrt{1\cdots}}}{2a(bc-ad)\sqrt{a-\cdots}}$$

$$= \frac{bx}{2a(bc-ad)\sqrt{a-bx^4}} + \frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2a^{3/4}(bc-ad)\sqrt{a-bx^4}} - \frac{\left(d\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{1-\frac{bx^4}{a}}}\cdots}{2c(bc-ad)\sqrt{a-bx^4}}$$

$$= \frac{bx}{2a(bc-ad)\sqrt{a-bx^4}} + \frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{2a^{3/4}(bc-ad)\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}d\sqrt{1-\frac{bx^4}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\cdots\right.}{2\sqrt[4]{b}c(bc-ad)\sqrt{a-\cdots}}$$

Mathematica [C] time = 0.230621, size = 381, normalized size = 1.36

$$\frac{2bx^5\left(c-dx^4\right)\left(5c-dx^4\sqrt{1-\frac{bx^4}{a}}F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)+5acxF_1\cdots}{10ac\sqrt{a-bx^4}\left(dx^4-c\right)(ad-bc)\left(2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4}\cdots\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - b*x^4)^(3/2)*(c - d*x^4)),x]
```

```
[Out] (5*a*c*x*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c]*(-5*c*(-2*b*c + 2
*a*d + b*d*x^4) + b*d*x^4*Sqrt[1 - (b*x^4)/a]*(-c + d*x^4)*AppellF1[5/4, 1/
2, 1, 9/4, (b*x^4)/a, (d*x^4)/c]) + 2*b*x^5*(c - d*x^4)*(5*c - d*x^4*Sqrt[1
 - (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])*(2*a*d*Appe
llF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/
4, (b*x^4)/a, (d*x^4)/c]))/(10*a*c*(-(b*c) + a*d)*Sqrt[a - b*x^4]*(-c + d*x
^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*
AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1
, 9/4, (b*x^4)/a, (d*x^4)/c])))
```

Maple [C] time = 0.052, size = 301, normalized size = 1.1

$$-\frac{bx}{2a(ad-bc)}\frac{1}{\sqrt{-\left(x^4-\frac{a}{b}\right)b}}-\frac{b}{2a(ad-bc)}\sqrt{1-x^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+x^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathrm{EllipticF}\left(x\sqrt{\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{-bx^4+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^4+a)^(3/2)/(-d*x^4+c),x)
```

```
[Out] -1/2*b*x/a/(a*d-b*c)/(-(x^4-1/b*a)*b)^(1/2)-1/2*b/a/(a*d-b*c)/(1/a^(1/2)*b^
(1/2))^(1/2)*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/(-
b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-1/8*sum(1/(a*d-b*c)
/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*_alpha^2*b*x^2+2*a)/((a*d
-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^(1/2)*_alpha^3*d/c*(
1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)
*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^(1/2)*_alpha^2/c*d,(-1/a^
(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_alpha=RootOf(_Z^4*d-c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(-bx^4+a\right)^{\frac{3}{2}}\left(dx^4-c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(3/2)/(-d*x^4+c),x, algorithm="maxima")
```

```
[Out] -integrate(1/((-b*x^4 + a)^(3/2)*(d*x^4 - c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(3/2)/(-d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{-ac\sqrt{a-bx^4}+adx^4\sqrt{a-bx^4}+bcx^4\sqrt{a-bx^4}-bdx^8\sqrt{a-bx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**4+a)**(3/2)/(-d*x**4+c),x)
```

```
[Out] -Integral(1/(-a*c*sqrt(a - b*x**4) + a*d*x**4*sqrt(a - b*x**4) + b*c*x**4*s
qrt(a - b*x**4) - b*d*x**8*sqrt(a - b*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(-bx^4+a\right)^{\frac{3}{2}}\left(dx^4-c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(3/2)/(-d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate(-1/((-b*x^4 + a)^(3/2)*(d*x^4 - c)), x)
```

3.178 $\int \frac{1}{\left(a-bx^4\right)^{5/2}\left(c-dx^4\right)}\,dx$

Optimal. Leaf size=334

$$\frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}(5bc-11ad)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{12a^{7/4}\sqrt{a-bx^4}(bc-ad)^2}+\frac{bx(5bc-11ad)}{12a^2\sqrt{a-bx^4}(bc-ad)^2}+\frac{\sqrt[4]{a}d^2\sqrt{1-\frac{bx^4}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\right.\right.}{2\sqrt[4]{b}c\sqrt{a-bx^4}(bc-ad)^2}$$

```
[Out] (b*x)/(6*a*(b*c - a*d)*(a - b*x^4)^(3/2)) + (b*(5*b*c - 11*a*d)*x)/(12*a^2*
(b*c - a*d)^2*Sqrt[a - b*x^4]) + (b^(3/4)*(5*b*c - 11*a*d)*Sqrt[1 - (b*x^4)
/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(12*a^(7/4)*(b*c - a*d)^2*S
qrt[a - b*x^4]) + (a^(1/4)*d^2*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sq
rt[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*(
b*c - a*d)^2*Sqrt[a - b*x^4]) + (a^(1/4)*d^2*Sqrt[1 - (b*x^4)/a]*EllipticPi
[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*
b^(1/4)*c*(b*c - a*d)^2*Sqrt[a - b*x^4])
```

Rubi [A] time = 0.399788, antiderivative size = 334, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {414, 527, 523, 224, 221, 409, 1219, 1218}

$$\frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}(5bc-11ad)F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{12a^{7/4}\sqrt{a-bx^4}(bc-ad)^2}+\frac{bx(5bc-11ad)}{12a^2\sqrt{a-bx^4}(bc-ad)^2}+\frac{\sqrt[4]{a}d^2\sqrt{1-\frac{bx^4}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right.}{2\sqrt[4]{b}c\sqrt{a-bx^4}(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - b*x^4)^(5/2)*(c - d*x^4)),x]
```

```
[Out] (b*x)/(6*a*(b*c - a*d)*(a - b*x^4)^(3/2)) + (b*(5*b*c - 11*a*d)*x)/(12*a^2*
(b*c - a*d)^2*Sqrt[a - b*x^4]) + (b^(3/4)*(5*b*c - 11*a*d)*Sqrt[1 - (b*x^4)
/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(12*a^(7/4)*(b*c - a*d)^2*S
qrt[a - b*x^4]) + (a^(1/4)*d^2*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sq
rt[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*b^(1/4)*c*(
b*c - a*d)^2*Sqrt[a - b*x^4]) + (a^(1/4)*d^2*Sqrt[1 - (b*x^4)/a]*EllipticPi
[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(2*
b^(1/4)*c*(b*c - a*d)^2*Sqrt[a - b*x^4])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
```

```
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\left(a-bx^4\right)^{5/2}\left(c-dx^4\right)}\,dx = \frac{bx}{6a(bc-ad)\left(a-bx^4\right)^{3/2}} + \frac{\int \frac{5bc-6ad-5bdx^4}{\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)}\,dx}{6a(bc-ad)}$$

$$= \frac{bx}{6a(bc-ad)\left(a-bx^4\right)^{3/2}} + \frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{a-bx^4}} + \frac{\int \frac{5b^2c^2-11abcd+12a^2d^2-bd(5bc-11ad)x^4}{\sqrt{a-bx^4}\left(c-dx^4\right)}}{12a^2(bc-ad)^2}$$

$$= \frac{bx}{6a(bc-ad)\left(a-bx^4\right)^{3/2}} + \frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{a-bx^4}} + \frac{d^2\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}\,dx}{(bc-ad)^2} + \frac{(b(5bc-1}{12a}$$

$$= \frac{bx}{6a(bc-ad)\left(a-bx^4\right)^{3/2}} + \frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{a-bx^4}} + \frac{d^2\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{2c(bc-ad)^2} + \frac{d^2\int}{2c(}$$

$$= \frac{bx}{6a(bc-ad)\left(a-bx^4\right)^{3/2}} + \frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{a-bx^4}} + \frac{b^{3/4}(5bc-11ad)\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\right.}{12a^{7/4}(bc-ad)^2\sqrt{a-b}}$$

$$= \frac{bx}{6a(bc-ad)\left(a-bx^4\right)^{3/2}} + \frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{a-bx^4}} + \frac{b^{3/4}(5bc-11ad)\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\right.}{12a^{7/4}(bc-ad)^2\sqrt{a-b}}$$

Mathematica [C] time = 0.72477, size = 422, normalized size = 1.26

$$\frac{x\left(\frac{bdx^4\sqrt{1-\frac{bx^4}{a}}(11ad-5bc)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)}{c} - \frac{5\left(5ac\left(a^2bd\left(dx^4-24c\right)+12a^3d^2+ab^2\left(12c^2+15cdx^4-11d^2x^8\right)+5b^3cx^4\left(dx^4-2c\right)\right)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+2b}{\left(a-bx^4\right)\left(dx^4-c\right)\left(2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\right.\right.}}{60a^2\sqrt{a-bx^4}(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - b*x^4)^(5/2)*(c - d*x^4)),x]
```

```
[Out] (x*((b*d*(-5*b*c + 11*a*d)*x^4*Sqrt[1 - (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9/
4, (b*x^4)/a, (d*x^4)/c])/c - (5*(5*a*c*(12*a^3*d^2 + a^2*b*d*(-24*c + d*x^
4) + 5*b^3*c*x^4*(-2*c + d*x^4) + a*b^2*(12*c^2 + 15*c*d*x^4 - 11*d^2*x^8))
*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*b*x^4*(-c + d*x^4)*(1
3*a^2*d + 5*b^2*c*x^4 - a*b*(7*c + 11*d*x^4))*(2*a*d*AppellF1[5/4, 1/2, 2,
9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x
^4)/c])))/((a - b*x^4)*(-c + d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^
4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^
4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))))/(60*a^2*(
b*c - a*d)^2*Sqrt[a - b*x^4])
```

Maple [C] time = 0.032, size = 361, normalized size = 1.1

$$-\frac{x}{6\,ab\,(ad-bc)}\sqrt{-bx^4+a}\left(x^4-\frac{a}{b}\right)^{-2} - \frac{bx\,(11\,ad-5\,bc)}{12\,a^2\,(ad-bc)^2}\frac{1}{\sqrt{-\left(x^4-\frac{a}{b}\right)b}} - \frac{b\,(11\,ad-5\,bc)}{12\,a^2\,(ad-bc)^2}\sqrt{1-x^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+x^2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^4+a)^(5/2)/(-d*x^4+c),x)
```

```
[Out] -1/6*x/a/b/(a*d-b*c)*(-b*x^4+a)^(1/2)/(x^4-1/b*a)^2-1/12*b*x/a^2*(11*a*d-5*
b*c)/(a*d-b*c)^2/(-(x^4-1/b*a)*b)^(1/2)-1/12*b/a^2*(11*a*d-5*b*c)/(a*d-b*c)
^2/(1/a^(1/2)*b^(1/2))^(1/2)*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a
^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-1/8
*d*sum(1/(a*d-b*c)^2/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*_alph
a^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^
(1/2)*_alpha^3*d/c*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1
/2)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^(1/2)
*_alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_alpha
=RootOf(_Z^4*d-c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left(-bx^4+a\right)^{\frac{5}{2}}\left(dx^4-c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(5/2)/(-d*x^4+c),x, algorithm="maxima")
```

```
[Out] -integrate(1/((-b*x^4 + a)^(5/2)*(d*x^4 - c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(5/2)/(-d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**4+a)**(5/2)/(-d*x**4+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(-bx^4+a\right)^{\frac{5}{2}}\left(dx^4-c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(5/2)/(-d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate(-1/((-b*x^4 + a)^(5/2)*(d*x^4 - c)), x)
```

3.179 $\int \frac{(a+bx^4)^{3/2}}{c+dx^4}\,dx$

Optimal. Leaf size=926

result too large to display

```
[Out] (b*x*Sqrt[a + b*x^4])/(3*d) - ((b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)
/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*x^4])])/(4*(-c)^(3/4)*d^(7/4)) - ((-(b*c) +
 a*d)^(3/2)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*x^
4])])/(4*(-c)^(3/4)*d^(7/4)) - (b^(3/4)*(3*b*c - 5*a*d)*(Sqrt[a] + Sqrt[b]*
x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4
)*x)/a^(1/4)], 1/2])/(6*a^(1/4)*d^2*Sqrt[a + b*x^4]) + (b^(1/4)*(Sqrt[b]*Sq
rt[-c] - Sqrt[a]*Sqrt[d])*(b*c - a*d)^2*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b
*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1
/2])/(4*a^(1/4)*Sqrt[-c]*d^2*(b*c + a*d)*Sqrt[a + b*x^4]) + (b^(1/4)*(Sqrt[
b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])*(b*c - a*d)^2*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(
a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4
)], 1/2])/(4*a^(1/4)*Sqrt[-c]*d^2*(b*c + a*d)*Sqrt[a + b*x^4]) + ((Sqrt[b]*
Sqrt[-c] + Sqrt[a]*Sqrt[d])^2*(b*c - a*d)^2*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a
 + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqrt[-c] - Sqrt[a
]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(
1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*d^2*(b*c + a*d)*Sqrt[a + b*x^4]) + ((Sqrt
[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])^2*(b*c - a*d)^2*(Sqrt[a] + Sqrt[b]*x^2)*Sqr
t[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[-c] + Sqr
t[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/
a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*d^2*(b*c + a*d)*Sqrt[a + b*x^4])
```

Rubi [A] time = 1.65511, antiderivative size = 926, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {416, 523, 220, 409, 1217, 1707}

$$\frac{\sqrt[4]{b}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)(bc-ad)^2}{4\sqrt[4]{a}\sqrt{-c}d^2(bc+ad)\sqrt{bx^4+a}}+\frac{\sqrt[4]{b}\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{}}{4\sqrt[4]{a}\sqrt{-c}d^2(b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(3/2)/(c + d*x^4),x]
```

```
[Out] (b*x*Sqrt[a + b*x^4])/(3*d) - ((b*c - a*d)^(3/2)*ArcTan[(Sqrt[b*c - a*d]*x)
/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*x^4])])/(4*(-c)^(3/4)*d^(7/4)) - ((-(b*c) +
 a*d)^(3/2)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*x^
4])])/(4*(-c)^(3/4)*d^(7/4)) - (b^(3/4)*(3*b*c - 5*a*d)*(Sqrt[a] + Sqrt[b]*
x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4
)*x)/a^(1/4)], 1/2])/(6*a^(1/4)*d^2*Sqrt[a + b*x^4]) + (b^(1/4)*(Sqrt[b]*Sq
rt[-c] - Sqrt[a]*Sqrt[d])*(b*c - a*d)^2*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b
*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1
/2])/(4*a^(1/4)*Sqrt[-c]*d^2*(b*c + a*d)*Sqrt[a + b*x^4]) + (b^(1/4)*(Sqrt[
b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])*(b*c - a*d)^2*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(
a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4
)], 1/2])/(4*a^(1/4)*Sqrt[-c]*d^2*(b*c + a*d)*Sqrt[a + b*x^4]) + ((Sqrt[b]*
Sqrt[-c] + Sqrt[a]*Sqrt[d])^2*(b*c - a*d)^2*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a
 + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqrt[-c] - Sqrt[a
]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(
1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*d^2*(b*c + a*d)*Sqrt[a + b*x^4]) + ((Sqrt
```

```
[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])^2*(b*c - a*d)^2*(Sqrt[a] + Sqrt[b]*x^2)*Sqr
t[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[-c] + Sqr
t[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/
a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*d^2*(b*c + a*d)*Sqrt[a + b*x^4])
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{(a+bx^4)^{3/2}}{c+dx^4}\,dx = \frac{bx\sqrt{a+bx^4}}{3d} + \frac{\int \frac{-a(bc-3ad)-b(3bc-5ad)x^4}{\sqrt{a+bx^4}(c+dx^4)}\,dx}{3d}$$

$$= \frac{bx\sqrt{a+bx^4}}{3d} - \frac{(b(3bc-5ad))\int \frac{1}{\sqrt{a+bx^4}}\,dx}{3d^2} + \frac{(bc-ad)^2\int \frac{1}{\sqrt{a+bx^4}(c+dx^4)}\,dx}{d^2}$$

$$= \frac{bx\sqrt{a+bx^4}}{3d} - \frac{b^{3/4}(3bc-5ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6\sqrt[4]{a}d^2\sqrt{a+bx^4}} + \frac{(bc-ad)^2\int \frac{}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-c}}\right.}}{2cd^2}$$

$$= \frac{bx\sqrt{a+bx^4}}{3d} - \frac{b^{3/4}(3bc-5ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{6\sqrt[4]{a}d^2\sqrt{a+bx^4}} + \frac{\left(\sqrt{b}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\right.\right.}{2\sqrt{-}}$$

$$= \frac{bx\sqrt{a+bx^4}}{3d} - \frac{(bc-ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}d^{7/4}} - \frac{(-bc+ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}d^{7/4}} - \frac{b^{3/4}(3bc-}{}$$

Mathematica [C] time = 0.462111, size = 346, normalized size = 0.37

$$\frac{x\left(\frac{5\left(2bx^4(a+bx^4)(c+dx^4)\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)-5ac\left(3a^2d+abdx^4+b^2x^4(c+dx^4)\right)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)}{(c+dx^4)\left(2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)-5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)} + \frac{bx^4\sqrt{\frac{bx^4}{a}+1}}{}\right.}{15d\sqrt{a+bx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^4)^(3/2)/(c + d*x^4),x]
```

```
[Out] (x*((b*(-3*b*c + 5*a*d)*x^4*Sqrt[1 + (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9/4,
-((b*x^4)/a), -((d*x^4)/c)])/c + (5*(-5*a*c*(3*a^2*d + a*b*d*x^4 + b^2*x^4*
(c + d*x^4))*AppellF1[1/4, 1/2, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + 2*b*x
^4*(a + b*x^4)*(c + d*x^4)*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, -((b*x^4)/a),
-((d*x^4)/c)] + b*c*AppellF1[5/4, 3/2, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])
))/((c + d*x^4)*(-5*a*c*AppellF1[1/4, 1/2, 1, 5/4, -((b*x^4)/a), -((d*x^4)/
c)] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] +
 b*c*AppellF1[5/4, 3/2, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))))/(15*d*Sqrt
[a + b*x^4])
```

Maple [C] time = 0.026, size = 322, normalized size = 0.4

$$\frac{bx}{3d}\sqrt{bx^4+a} + \left(\frac{b(2ad-bc)}{d^2} - \frac{ab}{3d}\right)\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathrm{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}} - \frac{1}{8d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(3/2)/(d*x^4+c),x)
```

```
[Out] 1/3*b*x*(b*x^4+a)^(1/2)/d+(b*(2*a*d-b*c)/d^2-1/3/d*b*a)/(I/a^(1/2)*b^(1/2))
^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x
```

```
^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/8/d^3*sum((-a^2*d^2+
2*a*b*c*d-b^2*c^2)/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(2*_alpha^2
*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(b*x^4+a)^(1/2))+2/(I/a^(1/2)*b^(1/2))^(1/2
)*_alpha^3*d/c*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1
/2)/(b*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*b^(1/2))^(1/2),I*a^(1/2)/b^(1/2
)*_alpha^2/c*d,(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))),_alph
a=RootOf(_Z^4*d+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)^{\frac{3}{2}}}{dx^4+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/2)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/2)/(d*x^4 + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/2)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^4\right)^{\frac{3}{2}}}{c+dx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(3/2)/(d*x**4+c),x)
```

```
[Out] Integral((a + b*x**4)**(3/2)/(c + d*x**4), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/2)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.180 $\int \frac{\sqrt{a+bx^4}}{c+dx^4}\,dx$

Optimal. Leaf size=881

$$-\frac{(bc-ad)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2}{8\sqrt[4]{a}\sqrt[4]{b}cd(bc+ad)\sqrt{bx^4+a}}-\frac{\sqrt[4]{b}(bc-ad)\left(\sqrt{b}x^2+\sqrt{a}\right.}{}$$

```
[Out] (Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*
x^4])])/(4*(-c)^(3/4)*d^(3/4)) - (Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[-(b*c) +
a*d]*x)/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*x^4])])/(4*(-c)^(3/4)*d^(3/4)) + (b^
(3/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*E
llipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*d*Sqrt[a + b*x^4])
 - (b^(1/4)*(Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])*(b*c - a*d)*(Sqrt[a] + Sqr
t[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b
^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*Sqrt[-c]*d*(b*c + a*d)*Sqrt[a + b*x^4]
) - (b^(1/4)*(Sqrt[b] + (Sqrt[a]*Sqrt[d])/Sqrt[-c])*(b*c - a*d)*(Sqrt[a] +
Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan
[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*d*(b*c + a*d)*Sqrt[a + b*x^4]) - ((
Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])^2*(b*c - a*d)*(Sqrt[a] + Sqrt[b]*x^2)*S
qrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqrt[-c] -
Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*
x)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*d*(b*c + a*d)*Sqrt[a + b*x^4]) - ((
Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])^2*(b*c - a*d)*(Sqrt[a] + Sqrt[b]*x^2)*S
qrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[-c] + S
qrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x
)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*d*(b*c + a*d)*Sqrt[a + b*x^4])
```

Rubi [A] time = 0.879899, antiderivative size = 881, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {406, 220, 409, 1217, 1707}

$$-\frac{(bc-ad)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2}{8\sqrt[4]{a}\sqrt[4]{b}cd(bc+ad)\sqrt{bx^4+a}}-\frac{\sqrt[4]{b}(bc-ad)\left(\sqrt{b}x^2+\sqrt{a}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^4]/(c + d*x^4),x]
```

```
[Out] (Sqrt[b*c - a*d]*ArcTan[(Sqrt[b*c - a*d]*x)/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*
x^4])])/(4*(-c)^(3/4)*d^(3/4)) - (Sqrt[-(b*c) + a*d]*ArcTan[(Sqrt[-(b*c) +
a*d]*x)/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*x^4])])/(4*(-c)^(3/4)*d^(3/4)) + (b^
(3/4)*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*E
llipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(2*a^(1/4)*d*Sqrt[a + b*x^4])
 - (b^(1/4)*(Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])*(b*c - a*d)*(Sqrt[a] + Sqr
t[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b
^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*Sqrt[-c]*d*(b*c + a*d)*Sqrt[a + b*x^4]
) - (b^(1/4)*(Sqrt[b] + (Sqrt[a]*Sqrt[d])/Sqrt[-c])*(b*c - a*d)*(Sqrt[a] +
Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan
[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4)*d*(b*c + a*d)*Sqrt[a + b*x^4]) - ((
Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])^2*(b*c - a*d)*(Sqrt[a] + Sqrt[b]*x^2)*S
qrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqrt[-c] -
Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*
```

```
x)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*d*(b*c + a*d)*Sqrt[a + b*x^4]) - ((
Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])^2*(b*c - a*d)*(Sqrt[a] + Sqrt[b]*x^2)*S
qrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[-c] + S
qrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x
)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*d*(b*c + a*d)*Sqrt[a + b*x^4])
```

Rule 406

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/d,
 Int[1/Sqrt[a + b*x^4], x], x] - Dist[(b*c - a*d)/d, Int[1/(Sqrt[a + b*x^4]
*(c + d*x^4)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^4}}{c+dx^4}\,dx = \frac{b\int \frac{1}{\sqrt{a+bx^4}}\,dx}{d} - \frac{(bc-ad)\int \frac{1}{\sqrt{a+bx^4}(c+dx^4)}\,dx}{d}$$

$$= \frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{a+bx^4}} - \frac{(bc-ad)\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{a+bx^4}}\,dx}{2cd} - \frac{(bc-ad)\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\cdots}}{2cd}$$

$$= \frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{a}d\sqrt{a+bx^4}} - \frac{\left(\sqrt{b}\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)(bc-ad)\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{2\sqrt{-c}d(bc+ad)} - \cdots$$

$$= \frac{\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}d^{3/4}} - \frac{\sqrt{-bc+ad}\tan^{-1}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}d^{3/4}} + \frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(\cdots\right.}{2\sqrt[4]{a}d\sqrt{a+bx^4}}$$

Mathematica [C] time = 0.151986, size = 161, normalized size = 0.18

$$\frac{5acx\sqrt{a+bx^4}F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{\left(c+dx^4\right)\left(2x^4\left(bcF_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)-2adF_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)+5acF_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*x^4]/(c + d*x^4),x]
```

```
[Out] (5*a*c*x*Sqrt[a + b*x^4]*AppellF1[1/4, -1/2, 1, 5/4, -((b*x^4)/a), -((d*x^4
)/c)])/((c + d*x^4)*(5*a*c*AppellF1[1/4, -1/2, 1, 5/4, -((b*x^4)/a), -((d*x
^4)/c)] + 2*x^4*(-2*a*d*AppellF1[5/4, -1/2, 2, 9/4, -((b*x^4)/a), -((d*x^4)
/c)] + b*c*AppellF1[5/4, 1/2, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))
```

Maple [C] time = 0.017, size = 273, normalized size = 0.3

$$\frac{b}{d}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathrm{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}-\frac{1}{8\,d^2}\sum_{\_alpha=RootOf(\_Z^4d+c)}\frac{-ad+bc}{\_alpha^3}\left(-A\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(1/2)/(d*x^4+c),x)
```

```
[Out] 1/d*b/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2
)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),
I)-1/8/d^2*sum((-a*d+b*c)/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(2*_
alpha^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(b*x^4+a)^(1/2))+2/(I/a^(1/2)*b^(1/2
))^(1/2)*_alpha^3*d/c*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*
x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*b^(1/2))^(1/2),I*a^(1/2)
/b^(1/2)*_alpha^2/c*d,(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))
),_alpha=RootOf(_Z^4*d+c))
```

Maximia **[F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{bx^4 + a}}{dx^4 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/2)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^4 + a)/(d*x^4 + c), x)
```

Fricas **[F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/2)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy **[F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx^4}}{c + dx^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(1/2)/(d*x**4+c),x)
```

```
[Out] Integral(sqrt(a + b*x**4)/(c + d*x**4), x)
```

Giac **[F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/2)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] Timed out
```

3.181 $\int \frac{1}{\sqrt{a+bx^4}\left(c+dx^4\right)}\,dx$

Optimal. Leaf size=742

$$\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}+\sqrt{b}\right)\text{EllipticF}\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),\frac{1}{2}\right)}{4\sqrt[4]{a}\sqrt{a+bx^4}(ad+bc)}+\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\sqrt{a}\sqrt{-c}\sqrt{d}+\right.}{4\sqrt[4]{a}c\sqrt{a+bx^4}(a}$$

```
[Out] -(d^(1/4)*ArcTan[(Sqrt[b*c - a*d]*x)/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*x^4])])
/(4*(-c)^(3/4)*Sqrt[b*c - a*d]) - (d^(1/4)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((
-c)^(1/4)*d^(1/4)*Sqrt[a + b*x^4])])/(4*(-c)^(3/4)*Sqrt[-(b*c) + a*d]) + (b
^(1/4)*(Sqrt[b] + (Sqrt[a]*Sqrt[d])/Sqrt[-c])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[
(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/
4)], 1/2])/(4*a^(1/4)*(b*c + a*d)*Sqrt[a + b*x^4]) + (b^(1/4)*(Sqrt[b]*c +
Sqrt[a]*Sqrt[-c]*Sqrt[d])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a]
 + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4
)*c*(b*c + a*d)*Sqrt[a + b*x^4]) + ((Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])^2*
(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellipti
cPi[-(Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqr
t[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*(b*c + a*d
)*Sqrt[a + b*x^4]) + ((Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])^2*(Sqrt[a] + Sqr
t[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[(Sqrt[b]*S
qrt[-c] + Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan
[(b^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*(b*c + a*d)*Sqrt[a + b*x^
4])
```

Rubi [A] time = 0.633017, antiderivative size = 742, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {409, 1217, 220, 1707}

$$-\frac{\sqrt[4]{d}\tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}\sqrt{bc-ad}}-\frac{\sqrt[4]{d}\tan^{-1}\left(\frac{x\sqrt{ad-bc}}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}\sqrt{ad-bc}}+\frac{\sqrt[4]{b}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}+\sqrt{b}\right)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right.}{4\sqrt[4]{a}\sqrt{a+bx^4}(ad+bc)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x^4]*(c + d*x^4)),x]
```

```
[Out] -(d^(1/4)*ArcTan[(Sqrt[b*c - a*d]*x)/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*x^4])])
/(4*(-c)^(3/4)*Sqrt[b*c - a*d]) - (d^(1/4)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((
-c)^(1/4)*d^(1/4)*Sqrt[a + b*x^4])])/(4*(-c)^(3/4)*Sqrt[-(b*c) + a*d]) + (b
^(1/4)*(Sqrt[b] + (Sqrt[a]*Sqrt[d])/Sqrt[-c])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[
(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/
4)], 1/2])/(4*a^(1/4)*(b*c + a*d)*Sqrt[a + b*x^4]) + (b^(1/4)*(Sqrt[b]*c +
Sqrt[a]*Sqrt[-c]*Sqrt[d])*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a]
 + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1/4
)*c*(b*c + a*d)*Sqrt[a + b*x^4]) + ((Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])^2*
(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*Ellipti
cPi[-(Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqr
t[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*(b*c + a*d
)*Sqrt[a + b*x^4]) + ((Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])^2*(Sqrt[a] + Sqr
t[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[(Sqrt[b]*S
qrt[-c] + Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan
[(b^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*(b*c + a*d)*Sqrt[a + b*x^
```

```
4])
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx^4}\left(c+dx^4\right)}\,dx = \frac{\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{a+bx^4}}\,dx}{2c} + \frac{\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{a+bx^4}}\,dx}{2c}$$

$$= \frac{\left(\sqrt{b}\left(\sqrt{b}+\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}\right)\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{2(bc+ad)} + \frac{\left(\sqrt{b}\left(\sqrt{b}c+\sqrt{a}\sqrt{-c}\sqrt{d}\right)\right)\int \frac{1}{\sqrt{a+bx^4}}\,dx}{2c(bc+ad)} - \frac{\left(\sqrt{a}\left(\sqrt{b}\sqrt{-c}-\right.\right.}{}$$

$$= -\frac{\sqrt[4]{d}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}\sqrt{bc-ad}} - \frac{\sqrt[4]{d}\tan^{-1}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}\sqrt{-bc+ad}} + \frac{\sqrt[4]{b}\left(\sqrt{b}+\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}\right)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{}}{4\sqrt[4]{a}(bc+a}$$

Mathematica [C] time = 0.0505894, size = 161, normalized size = 0.22

$$-\frac{5acxF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{\sqrt{a+bx^4}\left(c+dx^4\right)\left(2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)-5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + b*x^4]*(c + d*x^4)),x]
```

```
[Out] (-5*a*c*x*AppellF1[1/4, 1/2, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/(Sqrt[a +
 b*x^4]*(c + d*x^4)*(-5*a*c*AppellF1[1/4, 1/2, 1, 5/4, -((b*x^4)/a), -((d*x
^4)/c)] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c
)] + b*c*AppellF1[5/4, 3/2, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))
```

Maple [C] time = 0.016, size = 191, normalized size = 0.3

$$\frac{1}{8d}\sum_{\_alpha=\mathrm{RootOf}\left(\_Z^4d+c\right)}\frac{1}{\_alpha^3}\left(-\mathrm{Artanh}\left(\frac{2\_alpha^2bx^2+2a}{2}\frac{1}{\sqrt{\frac{ad-bc}{d}}}\frac{1}{\sqrt{bx^4+a}}\right)\frac{1}{\sqrt{\frac{ad-bc}{d}}}+2\frac{\_alpha^3d}{c\sqrt{bx^4+a}}\sqrt{1-\frac{i\sqrt{b}x^2}{\sqrt{a}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(1/2)/(d*x^4+c),x)
```

```
[Out] 1/8/d*sum(1/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(2*_alpha^2*b*x^2+
2*a)/((a*d-b*c)/d)^(1/2)/(b*x^4+a)^(1/2))+2/(I/a^(1/2)*b^(1/2))^(1/2)*_alph
a^3*d/c*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*
x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*b^(1/2))^(1/2),I*a^(1/2)/b^(1/2)*_alph
a^2/c*d,(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))),_alpha=RootO
f(_Z^4*d+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{bx^4+a}\left(dx^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(1/2)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b*x^4 + a)*(d*x^4 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(1/2)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+bx^4}\left(c+dx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(1/2)/(d*x**4+c),x)
```

```
[Out] Integral(1/(sqrt(a + b*x**4)*(c + d*x**4)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(1/2)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] Timed out
```

3.182 $\int \frac{1}{\left(a+bx^4\right)^{3/2}\left(c+dx^4\right)}\,dx$

Optimal. Leaf size=913

result too large to display

```
[Out] (b*x)/(2*a*(b*c - a*d)*Sqrt[a + b*x^4]) + (d^(5/4)*ArcTan[(Sqrt[b*c - a*d]*
x)/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*x^4])])/(4*(-c)^(3/4)*(b*c - a*d)^(3/2))
- (d^(5/4)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*x^4
])])/(4*(-c)^(3/4)*(-(b*c) + a*d)^(3/2)) + (b^(3/4)*(Sqrt[a] + Sqrt[b]*x^2)
*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)
/a^(1/4)], 1/2])/(4*a^(5/4)*(b*c - a*d)*Sqrt[a + b*x^4]) - (b^(1/4)*(Sqrt[b
] + (Sqrt[a]*Sqrt[d])/Sqrt[-c])*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/
(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(
4*a^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[a + b*x^4]) - (b^(1/4)*(Sqrt[b]*c +
Sqrt[a]*Sqrt[-c]*Sqrt[d])*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[
a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1
/4)*c*(b^2*c^2 - a^2*d^2)*Sqrt[a + b*x^4]) - ((Sqrt[b]*Sqrt[-c] + Sqrt[a]*S
qrt[d])^2*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2
)^2]*EllipticPi[-(Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*
Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*
c*(b*c - a*d)*(b*c + a*d)*Sqrt[a + b*x^4]) - ((Sqrt[b]*Sqrt[-c] - Sqrt[a]*S
qrt[d])^2*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2
)^2]*EllipticPi[(Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*S
qrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c
*(b*c - a*d)*(b*c + a*d)*Sqrt[a + b*x^4])
```

Rubi [A] time = 1.13196, antiderivative size = 913, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {414, 523, 220, 409, 1217, 1707}

$$-\frac{d\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}\Pi\left(\frac{\left(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d}\right)^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}};2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)\left(\sqrt{b}\sqrt{-c}-\sqrt{a}\sqrt{d}\right)^2}{8\sqrt[4]{a}\sqrt[4]{b}c(bc-ad)(bc+ad)\sqrt{bx^4+a}}+\frac{d^{5/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{bx^4+a}}\right)}{4(-c)^{3/4}(bc-ad)^{3/2}}-\frac{d^{5/4}}{4}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^(3/2)*(c + d*x^4)),x]
```

```
[Out] (b*x)/(2*a*(b*c - a*d)*Sqrt[a + b*x^4]) + (d^(5/4)*ArcTan[(Sqrt[b*c - a*d]*
x)/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*x^4])])/(4*(-c)^(3/4)*(b*c - a*d)^(3/2))
- (d^(5/4)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*x^4
])])/(4*(-c)^(3/4)*(-(b*c) + a*d)^(3/2)) + (b^(3/4)*(Sqrt[a] + Sqrt[b]*x^2)
*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)
/a^(1/4)], 1/2])/(4*a^(5/4)*(b*c - a*d)*Sqrt[a + b*x^4]) - (b^(1/4)*(Sqrt[b
] + (Sqrt[a]*Sqrt[d])/Sqrt[-c])*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/
(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(
4*a^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[a + b*x^4]) - (b^(1/4)*(Sqrt[b]*c +
Sqrt[a]*Sqrt[-c]*Sqrt[d])*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[
a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(4*a^(1
/4)*c*(b^2*c^2 - a^2*d^2)*Sqrt[a + b*x^4]) - ((Sqrt[b]*Sqrt[-c] + Sqrt[a]*S
qrt[d])^2*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2
)^2]*EllipticPi[-(Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*
Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*
c*(b*c - a*d)*(b*c + a*d)*Sqrt[a + b*x^4]) - ((Sqrt[b]*Sqrt[-c] - Sqrt[a]*S
```

```
qrt[d])^2*d*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2
)^2]*EllipticPi[(Sqrt[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*S
qrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4)], 1/2])/(8*a^(1/4)*b^(1/4)*c
*(b*c - a*d)*(b*c + a*d)*Sqrt[a + b*x^4])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^4\right)^{3/2}\left(c+dx^4\right)}dx = \frac{bx}{2a(bc-ad)\sqrt{a+bx^4}} - \frac{\int \frac{-bc+2ad-bdx^4}{\sqrt{a+bx^4}\left(c+dx^4\right)}dx}{2a(bc-ad)}$$

$$= \frac{bx}{2a(bc-ad)\sqrt{a+bx^4}} + \frac{b\int \frac{1}{\sqrt{a+bx^4}}dx}{2a(bc-ad)} - \frac{d\int \frac{1}{\sqrt{a+bx^4}\left(c+dx^4\right)}dx}{bc-ad}$$

$$= \frac{bx}{2a(bc-ad)\sqrt{a+bx^4}} + \frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}(bc-ad)\sqrt{a+bx^4}} - \frac{d\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\cdots}}{2c(bc-\cdots}$$

$$= \frac{bx}{2a(bc-ad)\sqrt{a+bx^4}} + \frac{b^{3/4}\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a+bx^4}{\left(\sqrt{a}+\sqrt{b}x^2\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)|\frac{1}{2}\right)}{4a^{5/4}(bc-ad)\sqrt{a+bx^4}} - \frac{\left(\sqrt{b}\left(\sqrt{b}+\cdots\right.\right.}{2(bc-\cdots}$$

$$= \frac{bx}{2a(bc-ad)\sqrt{a+bx^4}} + \frac{d^{5/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}(bc-ad)^{3/2}} - \frac{d^{5/4}\tan^{-1}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}(-bc+ad)^{3/2}} + \frac{b^{3/4}\left(\sqrt{a}\cdots\right.}{\cdots}$$

Mathematica [C] time = 0.245768, size = 331, normalized size = 0.36

$$\frac{x\left(\frac{5\left(2bx^4\left(c+dx^4\right)\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)+5ac\left(2ad-b\left(2c+dx^4\right)\right)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)}{\left(c+dx^4\right)\left(5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)-2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)\right)}-\frac{bdx^4\sqrt{\frac{bx^4}{a}+1}F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{bx^4}{a},\cdots\right.}{c}\cdots\right)}{10a\sqrt{a+bx^4}(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^4)^(3/2)*(c + d*x^4)),x]
```

```
[Out] (x*(-((b*d*x^4*Sqrt[1 + (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9/4, -((b*x^4)/a),
 -((d*x^4)/c)])/c) + (5*(5*a*c*(2*a*d - b*(2*c + d*x^4))*AppellF1[1/4, 1/2,
 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + 2*b*x^4*(c + d*x^4)*(2*a*d*AppellF1[
5/4, 1/2, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + b*c*AppellF1[5/4, 3/2, 1, 9
/4, -((b*x^4)/a), -((d*x^4)/c)])))/((c + d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1
, 5/4, -((b*x^4)/a), -((d*x^4)/c)] - 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4
, -((b*x^4)/a), -((d*x^4)/c)] + b*c*AppellF1[5/4, 3/2, 1, 9/4, -((b*x^4)/a)
, -((d*x^4)/c)])))))/(10*a*(-(b*c) + a*d)*Sqrt[a + b*x^4])
```

Maple [C] time = 0.025, size = 313, normalized size = 0.3

$$-\frac{bx}{2a(ad-bc)}\frac{1}{\sqrt{\left(x^4+\frac{a}{b}\right)b}}-\frac{b}{2a(ad-bc)}\sqrt{1-ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathrm{EllipticF}\left(x\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{i\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{bx^4+a}}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(3/2)/(d*x^4+c),x)
```

```
[Out] -1/2*b*x/a/(a*d-b*c)/((x^4+1/b*a)*b)^(1/2)-1/2*b/a/(a*d-b*c)/(I/a^(1/2)*b^(
1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)
```

```
/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/8*sum(1/(a*d-b*
c)/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(2*_alpha^2*b*x^2+2*a)/((a*
d-b*c)/d)^(1/2)/(b*x^4+a)^(1/2))+2/(I/a^(1/2)*b^(1/2))^(1/2)*_alpha^3*d/c*(
1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)^(1/2)/(b*x^4+a)^(1
/2)*EllipticPi(x*(I/a^(1/2)*b^(1/2))^(1/2),I*a^(1/2)/b^(1/2)*_alpha^2/c*d,(
-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))),_alpha=RootOf(_Z^4*d+
c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{3}{2}}\left(dx^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(3/2)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(3/2)*(d*x^4 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(3/2)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^4\right)^{\frac{3}{2}}\left(c+dx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(3/2)/(d*x**4+c),x)
```

```
[Out] Integral(1/((a + b*x**4)**(3/2)*(c + d*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{3}{2}}\left(dx^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(3/2)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(3/2)*(d*x^4 + c)), x)
```

3.183 $\int \frac{1}{(a+bx^4)^{5/2}(c+dx^4)}\,dx$

Optimal. Leaf size=976

result too large to display

```
[Out] (b*x)/(6*a*(b*c - a*d)*(a + b*x^4)^(3/2)) + (b*(5*b*c - 11*a*d)*x)/(12*a^2*
(b*c - a*d)^2*Sqrt[a + b*x^4]) - (d^(9/4)*ArcTan[(Sqrt[b*c - a*d]*x)/((-c)^
(1/4)*d^(1/4)*Sqrt[a + b*x^4])])/(4*(-c)^(3/4)*(b*c - a*d)^(5/2)) - (d^(9/4
)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*x^4])])/(4*(
-c)^(3/4)*(-(b*c) + a*d)^(5/2)) + (b^(3/4)*(5*b*c - 11*a*d)*(Sqrt[a] + Sqrt
[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^
(1/4)*x)/a^(1/4)], 1/2])/(24*a^(9/4)*(b*c - a*d)^2*Sqrt[a + b*x^4]) + (b^(1
/4)*(Sqrt[b]*c - Sqrt[a]*Sqrt[-c]*Sqrt[d])*d^2*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt
[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1
/4)], 1/2])/(4*a^(1/4)*c*(b*c - a*d)^2*(b*c + a*d)*Sqrt[a + b*x^4]) + (b^(1
/4)*(Sqrt[b]*c + Sqrt[a]*Sqrt[-c]*Sqrt[d])*d^2*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt
[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1
/4)], 1/2])/(4*a^(1/4)*c*(b*c - a*d)^2*(b*c + a*d)*Sqrt[a + b*x^4]) + ((Sqr
t[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])^2*d^2*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*
x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqr
t[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4)]
, 1/2])/(8*a^(1/4)*b^(1/4)*c*(b*c - a*d)^2*(b*c + a*d)*Sqrt[a + b*x^4]) + (
(Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])^2*d^2*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a
+ b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[-c] + Sqrt[a]*
Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/
4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*(b*c - a*d)^2*(b*c + a*d)*Sqrt[a + b*x^4])
```

Rubi [A] time = 1.68251, antiderivative size = 976, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {414, 527, 523, 220, 409, 1217, 1707}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{bc-ad}\,x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{bx^4+a}}\right)d^{9/4}}{4(-c)^{3/4}(bc-ad)^{5/2}}-\frac{\tan^{-1}\left(\frac{\sqrt{ad-bc}\,x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{bx^4+a}}\right)d^{9/4}}{4(-c)^{3/4}(ad-bc)^{5/2}}+\frac{\sqrt[4]{b}\left(\sqrt{b}c-\sqrt{a}\sqrt{-c}\sqrt{d}\right)\left(\sqrt{b}x^2+\sqrt{a}\right)\sqrt{\frac{bx^4+a}{\left(\sqrt{b}x^2+\sqrt{a}\right)^2}}F\left(2\tan^{-1}\left(\ldots\right.\right.}{4\sqrt[4]{a}c(bc-ad)^2(bc+ad)\sqrt{bx^4+a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^(5/2)*(c + d*x^4)),x]
```

```
[Out] (b*x)/(6*a*(b*c - a*d)*(a + b*x^4)^(3/2)) + (b*(5*b*c - 11*a*d)*x)/(12*a^2*
(b*c - a*d)^2*Sqrt[a + b*x^4]) - (d^(9/4)*ArcTan[(Sqrt[b*c - a*d]*x)/((-c)^
(1/4)*d^(1/4)*Sqrt[a + b*x^4])])/(4*(-c)^(3/4)*(b*c - a*d)^(5/2)) - (d^(9/4
)*ArcTan[(Sqrt[-(b*c) + a*d]*x)/((-c)^(1/4)*d^(1/4)*Sqrt[a + b*x^4])])/(4*(
-c)^(3/4)*(-(b*c) + a*d)^(5/2)) + (b^(3/4)*(5*b*c - 11*a*d)*(Sqrt[a] + Sqrt
[b]*x^2)*Sqrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^
(1/4)*x)/a^(1/4)], 1/2])/(24*a^(9/4)*(b*c - a*d)^2*Sqrt[a + b*x^4]) + (b^(1
/4)*(Sqrt[b]*c - Sqrt[a]*Sqrt[-c]*Sqrt[d])*d^2*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt
[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1
/4)], 1/2])/(4*a^(1/4)*c*(b*c - a*d)^2*(b*c + a*d)*Sqrt[a + b*x^4]) + (b^(1
/4)*(Sqrt[b]*c + Sqrt[a]*Sqrt[-c]*Sqrt[d])*d^2*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt
[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticF[2*ArcTan[(b^(1/4)*x)/a^(1
/4)], 1/2])/(4*a^(1/4)*c*(b*c - a*d)^2*(b*c + a*d)*Sqrt[a + b*x^4]) + ((Sqr
t[b]*Sqrt[-c] + Sqrt[a]*Sqrt[d])^2*d^2*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a + b*
```

```
x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqr
t[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/4)]
, 1/2])/(8*a^(1/4)*b^(1/4)*c*(b*c - a*d)^2*(b*c + a*d)*Sqrt[a + b*x^4]) + (
(Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])^2*d^2*(Sqrt[a] + Sqrt[b]*x^2)*Sqrt[(a
+ b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[-c] + Sqrt[a]*
Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x)/a^(1/
4)], 1/2])/(8*a^(1/4)*b^(1/4)*c*(b*c - a*d)^2*(b*c + a*d)*Sqrt[a + b*x^4])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2])/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
```

```
 + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^4\right)^{5/2}\left(c+dx^4\right)}\,dx = \frac{bx}{6a(bc-ad)\left(a+bx^4\right)^{3/2}} - \frac{\int \frac{-5bc+6ad-5bdx^4}{\left(a+bx^4\right)^{3/2}\left(c+dx^4\right)}\,dx}{6a(bc-ad)}$$

$$= \frac{bx}{6a(bc-ad)\left(a+bx^4\right)^{3/2}} + \frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{a+bx^4}} + \frac{\int \frac{5b^2c^2-11abcd+12a^2d^2+bd(5bc-11ad)x^4}{\sqrt{a+bx^4}\left(c+dx^4\right)}\,dx}{12a^2(bc-ad)^2}$$

$$= \frac{bx}{6a(bc-ad)\left(a+bx^4\right)^{3/2}} + \frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{a+bx^4}} + \frac{d^2\int \frac{1}{\sqrt{a+bx^4}\left(c+dx^4\right)}\,dx}{(bc-ad)^2} + \frac{(b(5bc-11a}{12a^2(}$$

$$= \frac{bx}{6a(bc-ad)\left(a+bx^4\right)^{3/2}} + \frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{a+bx^4}} + \frac{b^{3/4}(5bc-11ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a}{(\sqrt{a}}}}{24a^{9/4}(bc-ad)^2}$$

$$= \frac{bx}{6a(bc-ad)\left(a+bx^4\right)^{3/2}} + \frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{a+bx^4}} + \frac{b^{3/4}(5bc-11ad)\left(\sqrt{a}+\sqrt{b}x^2\right)\sqrt{\frac{a}{(\sqrt{a}}}}{24a^{9/4}(bc-ad)^2}$$

$$= \frac{bx}{6a(bc-ad)\left(a+bx^4\right)^{3/2}} + \frac{b(5bc-11ad)x}{12a^2(bc-ad)^2\sqrt{a+bx^4}} - \frac{d^{9/4}\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}(bc-ad)^{5/2}} - \frac{d^{9/4}\tan}{4(-c)}$$

Mathematica [C] time = 0.77853, size = 429, normalized size = 0.44

$$-\frac{x\left(\frac{5\left(5ac\left(-a^2bd\left(24c+dx^4\right)+12a^3d^2+ab^2\left(12c^2-15cdx^4-11d^2x^8\right)+5b^3cx^4\left(2c+dx^4\right)\right)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+2bx^4\left(c+dx^4\right)\left(13a^2d+ab\left(11dx^4-7c\right)-5b^2cx^4\right)\left(2adF\right.\right.}{\left(a+bx^4\right)\left(c+dx^4\right)\left(2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)-5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{d}{c}\right.\right.}\right.}{60a^2\sqrt{a+bx^4}(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^4)^(5/2)*(c + d*x^4)),x]
```

```
[Out] -(x*((b*d*(-5*b*c + 11*a*d)*x^4*Sqrt[1 + (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9
/4, -((b*x^4)/a), -((d*x^4)/c)])/c + (5*(5*a*c*(12*a^3*d^2 + 5*b^3*c*x^4*(2
*c + d*x^4) - a^2*b*d*(24*c + d*x^4) + a*b^2*(12*c^2 - 15*c*d*x^4 - 11*d^2*
x^8))*AppellF1[1/4, 1/2, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + 2*b*x^4*(c +
 d*x^4)*(13*a^2*d - 5*b^2*c*x^4 + a*b*(-7*c + 11*d*x^4))*(2*a*d*AppellF1[5/
4, 1/2, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + b*c*AppellF1[5/4, 3/2, 1, 9/4
, -((b*x^4)/a), -((d*x^4)/c)])))/((a + b*x^4)*(c + d*x^4)*(-5*a*c*AppellF1[
1/4, 1/2, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + 2*x^4*(2*a*d*AppellF1[5/4,
1/2, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + b*c*AppellF1[5/4, 3/2, 1, 9/4, -
((b*x^4)/a), -((d*x^4)/c)])))))/(60*a^2*(b*c - a*d)^2*Sqrt[a + b*x^4])
```

Maple [C] time = 0.03, size = 371, normalized size = 0.4

$$-\frac{x}{6ab\left(ad-bc\right)}\sqrt{bx^{4}+a}\left(x^{4}+\frac{a}{b}\right)^{-2}-\frac{bx\left(11\,ad-5\,bc\right)}{12\,a^{2}\left(ad-bc\right)^{2}}\frac{1}{\sqrt{\left(x^{4}+\frac{a}{b}\right)b}}-\frac{b\left(11\,ad-5\,bc\right)}{12\,a^{2}\left(ad-bc\right)^{2}}\sqrt{1-ix^{2}\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+ix^{2}\sqrt{b}\frac{1}{\sqrt{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(5/2)/(d*x^4+c),x)
```

```
[Out] -1/6*x/a/b/(a*d-b*c)*(b*x^4+a)^(1/2)/(x^4+1/b*a)^2-1/12*b*x/a^2*(11*a*d-5*b
*c)/(a*d-b*c)^2/((x^4+1/b*a)*b)^(1/2)-1/12*b/a^2*(11*a*d-5*b*c)/(a*d-b*c)^2
/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(
1/2)*x^2)^(1/2)/(b*x^4+a)^(1/2)*EllipticF(x*(I/a^(1/2)*b^(1/2))^(1/2),I)+1/
8*d*sum(1/(a*d-b*c)^2/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(2*_alph
a^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(b*x^4+a)^(1/2))+2/(I/a^(1/2)*b^(1/2))^(
1/2)*_alpha^3*d/c*(1-I/a^(1/2)*b^(1/2)*x^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*x^2)
^(1/2)/(b*x^4+a)^(1/2)*EllipticPi(x*(I/a^(1/2)*b^(1/2))^(1/2),I*a^(1/2)/b^(
1/2)*_alpha^2/c*d,(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))),_a
lpha=RootOf(_Z^4*d+c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^{4}+a\right)^{\frac{5}{2}}\left(dx^{4}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(5/2)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(5/2)*(d*x^4 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(5/2)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^{4}\right)^{\frac{5}{2}}\left(c+dx^{4}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(5/2)/(d*x**4+c),x)
```

```
[Out] Integral(1/((a + b*x**4)**(5/2)*(c + d*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{5}{2}}\left(dx^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(5/2)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(5/2)*(d*x^4 + c)), x)
```

3.184 $\int \frac{(a-bx^4)^{7/2}}{(c-dx^4)^2}\,dx$

Optimal. Leaf size=426

$$\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(349a^2bcd^2+21a^3d^3-553ab^2c^2d+231b^3c^3\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{84cd^4\sqrt{a-bx^4}}-\frac{bx\sqrt{a-bx^4}\left(21a^2d^2-12\right.}{84cd^3}$$

```
[Out] -(b*(77*b^2*c^2 - 122*a*b*c*d + 21*a^2*d^2)*x*Sqrt[a - b*x^4])/(84*c*d^3) +
 (b*(11*b*c - 7*a*d)*x*(a - b*x^4)^(3/2))/(28*c*d^2) - ((b*c - a*d)*x*(a -
b*x^4)^(5/2))/(4*c*d*(c - d*x^4)) + (a^(1/4)*b^(3/4)*(231*b^3*c^3 - 553*a*b
^2*c^2*d + 349*a^2*b*c*d^2 + 21*a^3*d^3)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcS
in[(b^(1/4)*x)/a^(1/4)], -1])/(84*c*d^4*Sqrt[a - b*x^4]) - (a^(1/4)*(b*c -
a*d)^3*(11*b*c + 3*a*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/
(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*d^4*Sq
rt[a - b*x^4]) - (a^(1/4)*(b*c - a*d)^3*(11*b*c + 3*a*d)*Sqrt[1 - (b*x^4)/a
]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4
)], -1])/(8*b^(1/4)*c^2*d^4*Sqrt[a - b*x^4])
```

Rubi [A] time = 0.535672, antiderivative size = 426, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {413, 528, 523, 224, 221, 409, 1219, 1218}

$$-\frac{bx\sqrt{a-bx^4}\left(21a^2d^2-122abcd+77b^2c^2\right)}{84cd^3}+\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(349a^2bcd^2+21a^3d^3-553ab^2c^2d+231b^3c^3\right)F\left(\sin^{-1}\right.}{84cd^4\sqrt{a-bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^4)^(7/2)/(c - d*x^4)^2,x]
```


```
[Out] -(b*(77*b^2*c^2 - 122*a*b*c*d + 21*a^2*d^2)*x*Sqrt[a - b*x^4])/(84*c*d^3) +
 (b*(11*b*c - 7*a*d)*x*(a - b*x^4)^(3/2))/(28*c*d^2) - ((b*c - a*d)*x*(a -
b*x^4)^(5/2))/(4*c*d*(c - d*x^4)) + (a^(1/4)*b^(3/4)*(231*b^3*c^3 - 553*a*b
^2*c^2*d + 349*a^2*b*c*d^2 + 21*a^3*d^3)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcS
in[(b^(1/4)*x)/a^(1/4)], -1])/(84*c*d^4*Sqrt[a - b*x^4]) - (a^(1/4)*(b*c -
a*d)^3*(11*b*c + 3*a*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/
(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*d^4*Sq
rt[a - b*x^4]) - (a^(1/4)*(b*c - a*d)^3*(11*b*c + 3*a*d)*Sqrt[1 - (b*x^4)/a
]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4
)], -1])/(8*b^(1/4)*c^2*d^4*Sqrt[a - b*x^4])
```


Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
```

```
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\left(a-bx^4\right)^{7/2}}{\left(c-dx^4\right)^2}\,dx = -\frac{(bc-ad)x\left(a-bx^4\right)^{5/2}}{4cd\left(c-dx^4\right)} - \frac{\int \frac{\left(a-bx^4\right)^{3/2}\left(-a(bc+3ad)+b(11bc-7ad)x^4\right)}{c-dx^4}dx}{4cd}$$

$$= \frac{b(11bc-7ad)x\left(a-bx^4\right)^{3/2}}{28cd^2} - \frac{(bc-ad)x\left(a-bx^4\right)^{5/2}}{4cd\left(c-dx^4\right)} + \frac{\int \frac{\sqrt{a-bx^4}\left(-a\left(11b^2c^2-14abcd-21a^2d^2\right)+b\left(77b^2c^2-12\right.\right.}{c-dx^4}}{28cd^2}$$

$$= -\frac{b\left(77b^2c^2-122abcd+21a^2d^2\right)x\sqrt{a-bx^4}}{84cd^3} + \frac{b(11bc-7ad)x\left(a-bx^4\right)^{3/2}}{28cd^2} - \frac{(bc-ad)x\left(a-bx^4\right)^{5}}{4cd\left(c-dx^4\right)}$$

$$= -\frac{b\left(77b^2c^2-122abcd+21a^2d^2\right)x\sqrt{a-bx^4}}{84cd^3} + \frac{b(11bc-7ad)x\left(a-bx^4\right)^{3/2}}{28cd^2} - \frac{(bc-ad)x\left(a-bx^4\right)^{5}}{4cd\left(c-dx^4\right)}$$

$$= -\frac{b\left(77b^2c^2-122abcd+21a^2d^2\right)x\sqrt{a-bx^4}}{84cd^3} + \frac{b(11bc-7ad)x\left(a-bx^4\right)^{3/2}}{28cd^2} - \frac{(bc-ad)x\left(a-bx^4\right)^{5}}{4cd\left(c-dx^4\right)}$$

$$= -\frac{b\left(77b^2c^2-122abcd+21a^2d^2\right)x\sqrt{a-bx^4}}{84cd^3} + \frac{b(11bc-7ad)x\left(a-bx^4\right)^{3/2}}{28cd^2} - \frac{(bc-ad)x\left(a-bx^4\right)^{5}}{4cd\left(c-dx^4\right)}$$

$$= -\frac{b\left(77b^2c^2-122abcd+21a^2d^2\right)x\sqrt{a-bx^4}}{84cd^3} + \frac{b(11bc-7ad)x\left(a-bx^4\right)^{3/2}}{28cd^2} - \frac{(bc-ad)x\left(a-bx^4\right)^{5}}{4cd\left(c-dx^4\right)}$$

Mathematica [C] time = 0.821353, size = 477, normalized size = 1.12

$$-\frac{bx^5\sqrt{1-\frac{bx^4}{a}}\left(349a^2bcd^2+21a^3d^3-553ab^2c^2d+231b^3c^3\right)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+\frac{5c\left(2x^5\left(bx^4-a\right)\left(-63a^2bcd^2+21a^3d^3+ab^2cd\right.\right.}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - b*x^4)^(7/2)/(c - d*x^4)^2,x]
```

```
[Out] -(b*(231*b^3*c^3 - 553*a*b^2*c^2*d + 349*a^2*b*c*d^2 + 21*a^3*d^3)*x^5*Sqrt
[1 - (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c] + (5*c*(5*
a*c*x*(-84*a^4*d^3 + 29*a^2*b^2*c*d^2*x^4 + 21*a^3*b*d^3*x^4 + a*b^3*c*d*x^
4*(111*c - 104*d*x^4) + b^4*c*x^4*(-77*c^2 + 44*c*d*x^4 + 12*d^2*x^8))*Appe
llF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^5*(-a + b*x^4)*(-63*a^2*
b*c*d^2 + 21*a^3*d^3 + a*b^2*c*d*(155*c - 92*d*x^4) + b^3*c*(-77*c^2 + 44*c
*d*x^4 + 12*d^2*x^8))*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/
c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))/((c - d*x^4)*(
5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*Appel
lF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4
, (b*x^4)/a, (d*x^4)/c]))))/(420*c^2*d^3*Sqrt[a - b*x^4])
```

Maple [C] time = 0.029, size = 540, normalized size = 1.3

$$-\frac{\left(a^3d^3-3\,cba^2d^2+3\,ab^2c^2d-b^3c^3\right)x}{4\,cd^3\left(dx^4-c\right)}\sqrt{-bx^4+a}-\frac{b^3x^5}{7\,d^2}\sqrt{-bx^4+a}-\frac{x}{3\,b}\left(-2\,\frac{b^3\left(2\,ad-bc\right)}{d^3}+\frac{5\,ab^3}{7\,d^2}\right)\sqrt{-bx^4+a}+\Bigg(\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^4+a)^(7/2)/(-d*x^4+c)^2,x)
```

```
[Out] -1/4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/c/d^3*x*(-b*x^4+a)^(1/2)
/(d*x^4-c)-1/7*b^3/d^2*x^5*(-b*x^4+a)^(1/2)-1/3*(-2*b^3/d^3*(2*a*d-b*c)+5/7
*b^3/d^2*a)/b*x*(-b*x^4+a)^(1/2)+(b^2*(6*a^2*d^2-8*a*b*c*d+3*b^2*c^2)/d^4+1
/4*b/d^4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/c+1/3*(-2*b^3/d^3*(2
*a*d-b*c)+5/7*b^3/d^2*a)/b*a)/(1/a^(1/2)*b^(1/2))^(1/2)*(1-x^2*b^(1/2)/a^(1
/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^
(1/2)*b^(1/2))^(1/2),I)-1/32/d^5/c*sum((3*a^4*d^4+2*a^3*b*c*d^3-24*a^2*b^2*
c^2*d^2+30*a*b^3*c^3*d-11*b^4*c^4)/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh
(1/2*(-2*_alpha^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(
1/2)*b^(1/2))^(1/2)*_alpha^3*d/c*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/
2)/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a
^(1/2)/b^(1/2)*_alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^
(1/2))),_alpha=RootOf(_Z^4*d-c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^4 + a\right)^{\frac{7}{2}}}{\left(dx^4 - c\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(7/2)/(-d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((-b*x^4 + a)^(7/2)/(d*x^4 - c)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(7/2)/(-d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**4+a)**(7/2)/(-d*x**4+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^4 + a\right)^{\frac{7}{2}}}{\left(dx^4 - c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(7/2)/(-d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] integrate((-b*x^4 + a)^(7/2)/(d*x^4 - c)^2, x)
```

3.185 $\int \frac{(a-bx^4)^{5/2}}{(c-dx^4)^2} dx$

Optimal. Leaf size=365

$$-\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(-3a^2d^2-26abcd+21b^2c^2\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{12cd^3\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(3ad+7bc)(bc-ad)^2\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}\right.}{8\sqrt[4]{b}c^2d^3\sqrt{a-bx^4}}$$

```
[Out] (b*(7*b*c - 3*a*d)*x*Sqrt[a - b*x^4])/(12*c*d^2) - ((b*c - a*d)*x*(a - b*x^
4)^(3/2))/(4*c*d*(c - d*x^4)) - (a^(1/4)*b^(3/4)*(21*b^2*c^2 - 26*a*b*c*d -
 3*a^2*d^2)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])
/(12*c*d^3*Sqrt[a - b*x^4]) + (a^(1/4)*(b*c - a*d)^2*(7*b*c + 3*a*d)*Sqrt[1
 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^
(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*d^3*Sqrt[a - b*x^4]) + (a^(1/4)*(b*c
 - a*d)^2*(7*b*c + 3*a*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/
(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*d^3*Sqr
t[a - b*x^4])
```

Rubi [A] time = 0.404563, antiderivative size = 365, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {413, 528, 523, 224, 221, 409, 1219, 1218}

$$-\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(-3a^2d^2-26abcd+21b^2c^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{12cd^3\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(3ad+7bc)(bc-ad)^2\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\right.}{8\sqrt[4]{b}c^2d^3\sqrt{a-bx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^4)^(5/2)/(c - d*x^4)^2,x]
```


```
[Out] (b*(7*b*c - 3*a*d)*x*Sqrt[a - b*x^4])/(12*c*d^2) - ((b*c - a*d)*x*(a - b*x^
4)^(3/2))/(4*c*d*(c - d*x^4)) - (a^(1/4)*b^(3/4)*(21*b^2*c^2 - 26*a*b*c*d -
 3*a^2*d^2)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])
/(12*c*d^3*Sqrt[a - b*x^4]) + (a^(1/4)*(b*c - a*d)^2*(7*b*c + 3*a*d)*Sqrt[1
 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^
(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*d^3*Sqrt[a - b*x^4]) + (a^(1/4)*(b*c
 - a*d)^2*(7*b*c + 3*a*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/
(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*d^3*Sqr
t[a - b*x^4])
```


Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
```

```
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a-bx^4)^{5/2}}{(c-dx^4)^2}\,dx = -\frac{(bc-ad)x(a-bx^4)^{3/2}}{4cd(c-dx^4)} - \frac{\int \frac{\sqrt{a-bx^4}(-a(bc+3ad)+b(7bc-3ad)x^4)}{c-dx^4}dx}{4cd}$$

$$= \frac{b(7bc-3ad)x\sqrt{a-bx^4}}{12cd^2} - \frac{(bc-ad)x(a-bx^4)^{3/2}}{4cd(c-dx^4)} + \frac{\int \frac{-a(7b^2c^2-6abcd-9a^2d^2)+b(21b^2c^2-26abcd-3a^2d^2)x^4}{\sqrt{a-bx^4}(c-dx^4)}dx}{12cd^2}$$

$$= \frac{b(7bc-3ad)x\sqrt{a-bx^4}}{12cd^2} - \frac{(bc-ad)x(a-bx^4)^{3/2}}{4cd(c-dx^4)} + \frac{((bc-ad)^2(7bc+3ad))\int \frac{1}{\sqrt{a-bx^4}(c-dx^4)}dx}{4cd^3} - \frac{(b(}{}$$

$$= \frac{b(7bc-3ad)x\sqrt{a-bx^4}}{12cd^2} - \frac{(bc-ad)x(a-bx^4)^{3/2}}{4cd(c-dx^4)} + \frac{((bc-ad)^2(7bc+3ad))\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}dx}{8c^2d^3} + \frac{((}{}$$

$$= \frac{b(7bc-3ad)x\sqrt{a-bx^4}}{12cd^2} - \frac{(bc-ad)x(a-bx^4)^{3/2}}{4cd(c-dx^4)} - \frac{\sqrt[4]{a}b^{3/4}(21b^2c^2-26abcd-3a^2d^2)\sqrt{1-\frac{bx^4}{a}}F\left(\mathrm{si}\right.}{12cd^3\sqrt{a-bx^4}}$$

$$= \frac{b(7bc-3ad)x\sqrt{a-bx^4}}{12cd^2} - \frac{(bc-ad)x(a-bx^4)^{3/2}}{4cd(c-dx^4)} - \frac{\sqrt[4]{a}b^{3/4}(21b^2c^2-26abcd-3a^2d^2)\sqrt{1-\frac{bx^4}{a}}F\left(\mathrm{si}\right.}{12cd^3\sqrt{a-bx^4}}$$

Mathematica [C] time = 0.579267, size = 396, normalized size = 1.08

$$-\frac{bx^5\sqrt{1-\frac{bx^4}{a}}(3a^2d^2+26abcd-21b^2c^2)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right) + \frac{5c\left(2x^5(a-bx^4)(3a^2d^2-6abcd+b^2c(7c-4dx^4))\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+\right.\right.}{(dx^4-c)\left(2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{b}{}\right.\right.\right.}}{60c^2d^2\sqrt{a-bx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - b*x^4)^(5/2)/(c - d*x^4)^2,x]
```

```
[Out] -(b*(-21*b^2*c^2 + 26*a*b*c*d + 3*a^2*d^2)*x^5*Sqrt[1 - (b*x^4)/a]*AppellF1
[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c] + (5*c*(5*a*c*x*(12*a^3*d^2 + 2*a*
b^2*c*d*x^4 - 3*a^2*b*d^2*x^4 + b^3*c*x^4*(-7*c + 4*d*x^4))*AppellF1[1/4, 1
/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^5*(a - b*x^4)*(-6*a*b*c*d + 3*a^2*d
^2 + b^2*c*(7*c - 4*d*x^4))*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d
*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))/((-c + d
*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*
d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2,
 1, 9/4, (b*x^4)/a, (d*x^4)/c]))))/(60*c^2*d^2*Sqrt[a - b*x^4])
```

Maple [C] time = 0.028, size = 412, normalized size = 1.1

$$-\frac{(a^2d^2-2\,cabd+b^2c^2)\,x}{4\,cd^2\,(dx^4-c)}\sqrt{-bx^4+a}+\frac{b^2x}{3\,d^2}\sqrt{-bx^4+a}+\left(\frac{b^2\,(3\,ad-2\,bc)}{d^3}+\frac{b\,(a^2d^2-2\,cabd+b^2c^2)}{4\,cd^3}-\frac{b^2a}{3\,d^2}\right)\sqrt{1-x^2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x)
```

```
[Out] -1/4*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c/d^2*x*(-b*x^4+a)^(1/2)/(d*x^4-c)+1/3*b^2
/d^2*x*(-b*x^4+a)^(1/2)+(b^2*(3*a*d-2*b*c)/d^3+1/4*b/d^3*(a^2*d^2-2*a*b*c*d
+b^2*c^2)/c-1/3*b^2/d^2*a)/(1/a^(1/2)*b^(1/2))^(1/2)*(1-x^2*b^(1/2)/a^(1/2)
)^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/
2)*b^(1/2))^(1/2),I)-1/32/c/d^4*sum((3*a^3*d^3+a^2*b*c*d^2-11*a*b^2*c^2*d+7
*b^3*c^3)/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*_alpha^2*b*x^2+2
*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^(1/2)*_alph
a^3*d/c*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/(-b*x^4
+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^(1/2)*_alpha^2/c
*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_alpha=RootOf(_Z^
4*d-c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^4+a\right)^{\frac{5}{2}}}{\left(dx^4-c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((-b*x^4 + a)^(5/2)/(d*x^4 - c)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**4+a)**(5/2)/(-d*x**4+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^4+a\right)^{\frac{5}{2}}}{\left(dx^4-c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] integrate((-b*x^4 + a)^(5/2)/(d*x^4 - c)^2, x)
```

3.186 $\int \frac{\left(a-bx^4\right)^{3/2}}{\left(c-dx^4\right)^2}\,dx$

Optimal. Leaf size=309

$$\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(ad+3bc)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{4cd^2\sqrt{a-bx^4}}-\frac{3\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(bc-ad)(ad+bc)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d^2\sqrt{a-bx^4}}$$

```
[Out] -((b*c - a*d)*x*Sqrt[a - b*x^4])/(4*c*d*(c - d*x^4)) + (a^(1/4)*b^(3/4)*(3*
b*c + a*d)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/
(4*c*d^2*Sqrt[a - b*x^4]) - (3*a^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[1 - (b*
x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*
x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*d^2*Sqrt[a - b*x^4]) - (3*a^(1/4)*(b*c - a
*d)*(b*c + a*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*S
qrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*d^2*Sqrt[a - b*x^
4])
```

Rubi [A] time = 0.277241, antiderivative size = 309, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {413, 523, 224, 221, 409, 1219, 1218}

$$\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}(ad+3bc)F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^2\sqrt{a-bx^4}}-\frac{3\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(bc-ad)(ad+bc)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d^2\sqrt{a-bx^4}}-\frac{3\sqrt[4]{a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^4)^(3/2)/(c - d*x^4)^2,x]
```


```
[Out] -((b*c - a*d)*x*Sqrt[a - b*x^4])/(4*c*d*(c - d*x^4)) + (a^(1/4)*b^(3/4)*(3*
b*c + a*d)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/
(4*c*d^2*Sqrt[a - b*x^4]) - (3*a^(1/4)*(b*c - a*d)*(b*c + a*d)*Sqrt[1 - (b*
x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*
x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*d^2*Sqrt[a - b*x^4]) - (3*a^(1/4)*(b*c - a
*d)*(b*c + a*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*S
qrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*d^2*Sqrt[a - b*x^
4])
```


Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\left(a-bx^4\right)^{3/2}}{\left(c-dx^4\right)^2}\,dx = -\frac{(bc-ad)x\sqrt{a-bx^4}}{4cd\left(c-dx^4\right)} - \frac{\int \frac{-a(bc+3ad)+b(3bc+ad)x^4}{\sqrt{a-bx^4}\left(c-dx^4\right)}\,dx}{4cd}$$

$$= -\frac{(bc-ad)x\sqrt{a-bx^4}}{4cd\left(c-dx^4\right)} + \frac{\left(3\left(a^2-\frac{b^2c^2}{d^2}\right)\right)\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}\,dx}{4c} + \frac{(b(3bc+ad))\int \frac{1}{\sqrt{a-bx^4}}\,dx}{4cd^2}$$

$$= -\frac{(bc-ad)x\sqrt{a-bx^4}}{4cd\left(c-dx^4\right)} + \frac{\left(3\left(a^2-\frac{b^2c^2}{d^2}\right)\right)\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{8c^2} + \frac{\left(3\left(a^2-\frac{b^2c^2}{d^2}\right)\right)\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{8c^2} + \cdots$$

$$= -\frac{(bc-ad)x\sqrt{a-bx^4}}{4cd\left(c-dx^4\right)} + \frac{\sqrt[4]{a}b^{3/4}(3bc+ad)\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^2\sqrt{a-bx^4}} + \frac{\left(3\left(a^2-\frac{b^2c^2}{d^2}\right)\sqrt{1-\frac{bx^4}{a}}\right)\int \cdots}{8c^2\sqrt{a-b\cdots}}$$

$$= -\frac{(bc-ad)x\sqrt{a-bx^4}}{4cd\left(c-dx^4\right)} + \frac{\sqrt[4]{a}b^{3/4}(3bc+ad)\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd^2\sqrt{a-bx^4}} + \frac{3\sqrt[4]{a}\left(a^2-\frac{b^2c^2}{d^2}\right)\sqrt{1-\frac{bx^4}{a}}\,\Pi\cdots}{8\sqrt[4]{b}c^2\cdots}$$

Mathematica [C] time = 0.300378, size = 342, normalized size = 1.11

$$\frac{x\left(\frac{5c\left(-5ac\left(4a^2d-abdx^4+b^2cx^4\right)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)-2x^4\left(a-bx^4\right)(ad-bc)\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)\right)}{2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)+5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)}-bx^4\sqrt{1-\frac{bx^4}{a}}\left(dx^4\right.\cdots}{20c^2d\sqrt{a-bx^4}\left(dx^4-c\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - b*x^4)^(3/2)/(c - d*x^4)^2,x]
```

```
[Out] (x*(-(b*(3*b*c + a*d)*x^4*Sqrt[1 - (b*x^4)/a]*(-c + d*x^4)*AppellF1[5/4, 1/
2, 1, 9/4, (b*x^4)/a, (d*x^4)/c]) + (5*c*(-5*a*c*(4*a^2*d + b^2*c*x^4 - a*b
*d*x^4)*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] - 2*(-(b*c) + a*d)
*x^4*(a - b*x^4)*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] +
b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))/(5*a*c*AppellF1[1/4
, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9
/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^
4)/c]))))/(20*c^2*d*Sqrt[a - b*x^4]*(-c + d*x^4))
```

Maple [C] time = 0.026, size = 329, normalized size = 1.1

$$-\frac{(ad-bc)x}{4cd\left(dx^4-c\right)}\sqrt{-bx^4+a}+\left(\frac{b^2}{d^2}+\frac{(ad-bc)b}{4cd^2}\right)\sqrt{1-x^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+x^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathrm{EllipticF}\left(x\sqrt{\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{-bx^4+a}}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x)
```

```
[Out] -1/4*(a*d-b*c)/c/d*x*(-b*x^4+a)^(1/2)/(d*x^4-c)+(b^2/d^2+1/4*b/d^2*(a*d-b*c
)/c)/(1/a^(1/2)*b^(1/2))^(1/2)*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)
/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-3
/32/c/d^3*sum((a^2*d^2-b^2*c^2)/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/
2*(-2*_alpha^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2
)*b^(1/2))^(1/2)*_alpha^3*d/c*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/
a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1
/2)/b^(1/2)*_alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/
2))),_alpha=RootOf(_Z^4*d-c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(-bx^4+a\right)^{\frac{3}{2}}}{\left(dx^4-c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((-b*x^4 + a)^(3/2)/(d*x^4 - c)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a - bx^4\right)^{\frac{3}{2}}}{\left(-c + dx^4\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**4+a)**(3/2)/(-d*x**4+c)**2,x)
```

```
[Out] Integral((a - b*x**4)**(3/2)/(-c + d*x**4)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-bx^4 + a\right)^{\frac{3}{2}}}{\left(dx^4 - c\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] integrate((-b*x^4 + a)^(3/2)/(d*x^4 - c)^2, x)
```

3.187 $\int \frac{\sqrt{a-bx^4}}{\left(c-dx^4\right)^2}\,dx$

Optimal. Leaf size=276

$$\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{4cd\sqrt{a-bx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(bc-3ad)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d\sqrt{a-bx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(bc\cdots}{\cdots}$$

```
[Out] (x*Sqrt[a - b*x^4])/(4*c*(c - d*x^4)) + (a^(1/4)*b^(3/4)*Sqrt[1 - (b*x^4)/a
]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(4*c*d*Sqrt[a - b*x^4]) - (a^
(1/4)*(b*c - 3*a*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqr
t[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*d*Sqrt[a -
 b*x^4]) - (a^(1/4)*(b*c - 3*a*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*S
qrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2
*d*Sqrt[a - b*x^4])
```

Rubi [A] time = 0.214877, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {412, 523, 224, 221, 409, 1219, 1218}

$$\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd\sqrt{a-bx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(bc-3ad)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d\sqrt{a-bx^4}}-\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(bc-3ad)\Pi\cdots}{8\sqrt[4]{b}c^2d\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - b*x^4]/(c - d*x^4)^2,x]
```

```
[Out] (x*Sqrt[a - b*x^4])/(4*c*(c - d*x^4)) + (a^(1/4)*b^(3/4)*Sqrt[1 - (b*x^4)/a
]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(4*c*d*Sqrt[a - b*x^4]) - (a^
(1/4)*(b*c - 3*a*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqr
t[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*d*Sqrt[a -
 b*x^4]) - (a^(1/4)*(b*c - 3*a*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*S
qrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2
*d*Sqrt[a - b*x^4])
```

Rule 412

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] + Dist[1/(
a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p + 1)
 + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x]
&& NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a-bx^4}}{\left(c-dx^4\right)^2}\,dx &= \frac{x\sqrt{a-bx^4}}{4c\left(c-dx^4\right)}-\frac{\int \frac{-3a+bx^4}{\sqrt{a-bx^4}\left(c-dx^4\right)}\,dx}{4c}\\
&= \frac{x\sqrt{a-bx^4}}{4c\left(c-dx^4\right)}+\frac{b\int \frac{1}{\sqrt{a-bx^4}}\,dx}{4cd}+\frac{(-bc+3ad)\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}\,dx}{4cd}\\
&= \frac{x\sqrt{a-bx^4}}{4c\left(c-dx^4\right)}-\frac{(bc-3ad)\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{8c^2d}-\frac{(bc-3ad)\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{8c^2d}+\frac{\left(b\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1}{\sqrt{1-\frac{b}{}}}}{4cd\sqrt{a-bx^4}}\\
&= \frac{x\sqrt{a-bx^4}}{4c\left(c-dx^4\right)}+\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd\sqrt{a-bx^4}}-\frac{\left((bc-3ad)\sqrt{1-\frac{bx^4}{a}}\right)\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{1-\frac{bx^4}{a}}}\,dx}{8c^2d\sqrt{a-bx^4}}-\ldots\\
&= \frac{x\sqrt{a-bx^4}}{4c\left(c-dx^4\right)}+\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4cd\sqrt{a-bx^4}}-\frac{\sqrt[4]{a}(bc-3ad)\sqrt{1-\frac{bx^4}{a}}\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\right|}{8\sqrt[4]{b}c^2d\sqrt{a-bx^4}}
\end{aligned}
$$

Mathematica [C] time = 0.132302, size = 233, normalized size = 0.84

$$
\frac{x\left(-\frac{75a^2F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)}{2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)+5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)}+\frac{bx^4\sqrt{1-\frac{bx^4}{a}}\left(c-dx^4\right)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)}{c^2}-\frac{5\left(a-bx^4\right)}{c}\right)}{20\sqrt{a-bx^4}\left(dx^4-c\right)}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a - b*x^4]/(c - d*x^4)^2,x]
```

```
[Out] (x*((-5*(a - b*x^4))/c + (b*x^4*Sqrt[1 - (b*x^4)/a]*(c - d*x^4)*AppellF1[5/
4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])/c^2 - (75*a^2*AppellF1[1/4, 1/2, 1,
5/4, (b*x^4)/a, (d*x^4)/c])/(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d
*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] +
b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c]))))/(20*Sqrt[a - b*x^4
]*(-c + d*x^4))
```

Maple [C] time = 0.025, size = 294, normalized size = 1.1

$$-\frac{x}{4c\left(dx^4-c\right)}\sqrt{-bx^4+a}+\frac{b}{4cd}\sqrt{1-x^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+x^2\sqrt{b}\frac{1}{\sqrt{a}}}\mathrm{EllipticF}\left(x\sqrt{\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{\sqrt{b}\frac{1}{\sqrt{a}}}}\frac{1}{\sqrt{-bx^4+a}}-\frac{1}{32cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x)
```

```
[Out] -1/4/c*x*(-b*x^4+a)^(1/2)/(d*x^4-c)+1/4*b/c/d/(1/a^(1/2)*b^(1/2))^(1/2)*(1-
x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*E
llipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-1/32/c/d^2*sum((3*a*d-b*c)/_alpha^3
*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*_alpha^2*b*x^2+2*a)/((a*d-b*c)/d)^
(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^(1/2)*_alpha^3*d/c*(1-x^2*b^(
1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2)/(-b*x^4+a)^(1/2)*Elliptic
Pi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^(1/2)*_alpha^2/c*d,(-1/a^(1/2)*b^(
1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_alpha=RootOf(_Z^4*d-c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^4+a}}{\left(dx^4-c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-b*x^4 + a)/(d*x^4 - c)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a - bx^4}}{\left(-c + dx^4\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**4+a)**(1/2)/(-d*x**4+c)**2,x)
```

```
[Out] Integral(sqrt(a - b*x**4)/(-c + d*x**4)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^4 + a}}{\left(dx^4 - c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-b*x^4 + a)/(d*x^4 - c)^2, x)
```

3.188 $\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)^2}\,dx$

Optimal. Leaf size=310

$$-\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}\operatorname{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{4c\sqrt{a-bx^4}(bc-ad)}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(5bc-3ad)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2\sqrt{a-bx^4}(bc-ad)}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(}{}$$

```
[Out] -(d*x*Sqrt[a - b*x^4])/(4*c*(b*c - a*d)*(c - d*x^4)) - (a^(1/4)*b^(3/4)*Sqr
t[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(4*c*(b*c - a*
d)*Sqrt[a - b*x^4]) + (a^(1/4)*(5*b*c - 3*a*d)*Sqrt[1 - (b*x^4)/a]*Elliptic
Pi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1]
)/(8*b^(1/4)*c^2*(b*c - a*d)*Sqrt[a - b*x^4]) + (a^(1/4)*(5*b*c - 3*a*d)*Sq
rt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b
^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a*d)*Sqrt[a - b*x^4])
```

Rubi [A] time = 0.247204, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {414, 523, 224, 221, 409, 1219, 1218}

$$-\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4c\sqrt{a-bx^4}(bc-ad)}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(5bc-3ad)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2\sqrt{a-bx^4}(bc-ad)}+\frac{\sqrt[4]{a}\sqrt{1-\frac{bx^4}{a}}(5bc-3}{8\sqrt[4]{b}c^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a - b*x^4]*(c - d*x^4)^2),x]
```

```
[Out] -(d*x*Sqrt[a - b*x^4])/(4*c*(b*c - a*d)*(c - d*x^4)) - (a^(1/4)*b^(3/4)*Sqr
t[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(4*c*(b*c - a*
d)*Sqrt[a - b*x^4]) + (a^(1/4)*(5*b*c - 3*a*d)*Sqrt[1 - (b*x^4)/a]*Elliptic
Pi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1]
)/(8*b^(1/4)*c^2*(b*c - a*d)*Sqrt[a - b*x^4]) + (a^(1/4)*(5*b*c - 3*a*d)*Sq
rt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b
^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a*d)*Sqrt[a - b*x^4])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)^2}\,dx = -\frac{dx\sqrt{a-bx^4}}{4c(bc-ad)\left(c-dx^4\right)}-\frac{\int \frac{-4bc+3ad-bdx^4}{\sqrt{a-bx^4}\left(c-dx^4\right)}\,dx}{4c(bc-ad)}$$

$$= -\frac{dx\sqrt{a-bx^4}}{4c(bc-ad)\left(c-dx^4\right)}-\frac{b\int \frac{1}{\sqrt{a-bx^4}}\,dx}{4c(bc-ad)}+\frac{(5bc-3ad)\int \frac{1}{\sqrt{a-bx^4}\left(c-dx^4\right)}\,dx}{4c(bc-ad)}$$

$$= -\frac{dx\sqrt{a-bx^4}}{4c(bc-ad)\left(c-dx^4\right)}+\frac{(5bc-3ad)\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{8c^2(bc-ad)}+\frac{(5bc-3ad)\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}}\,dx}{8c^2(bc-ad)}$$

$$= -\frac{dx\sqrt{a-bx^4}}{4c(bc-ad)\left(c-dx^4\right)}-\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4c(bc-ad)\sqrt{a-bx^4}}+\frac{\left((5bc-3ad)\sqrt{1-\frac{bx^4}{a}}\right)\int \cdots}{8c^2(bc-ad)\sqrt{a \cdots}}$$

$$= -\frac{dx\sqrt{a-bx^4}}{4c(bc-ad)\left(c-dx^4\right)}-\frac{\sqrt[4]{a}b^{3/4}\sqrt{1-\frac{bx^4}{a}}F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4c(bc-ad)\sqrt{a-bx^4}}+\frac{\sqrt[4]{a}(5bc-3ad)\sqrt{1-\frac{bx^4}{a}}\Pi\left(\cdots\right.}{8\sqrt[4]{b}c^2(bc-\cdots}$$

Mathematica [C] time = 0.25887, size = 386, normalized size = 1.25

$$\frac{2dx^5\left(bx^4\sqrt{1-\frac{bx^4}{a}}\left(dx^4-c\right)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+5c\left(a-bx^4\right)\right)\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\cdots\right.}{20c^2\sqrt{a-bx^4}\left(dx^4-c\right)(bc-ad)\left(2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bc\cdots\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a - b*x^4]*(c - d*x^4)^2),x]
```

```
[Out] (5*a*c*x*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c]*(-5*c*(4*b*c - 4*
a*d + b*d*x^4) + b*d*x^4*Sqrt[1 - (b*x^4)/a]*(-c + d*x^4)*AppellF1[5/4, 1/2
, 1, 9/4, (b*x^4)/a, (d*x^4)/c]) + 2*d*x^5*(5*c*(a - b*x^4) + b*x^4*Sqrt[1
- (b*x^4)/a]*(-c + d*x^4)*AppellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])
*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4
, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c]))/(20*c^2*(b*c - a*d)*Sqrt[a - b*x^4]*
(-c + d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^
4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/
4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))
```

Maple [C] time = 0.024, size = 322, normalized size = 1.

$$-\frac{dx}{(4ad-4bc)\,c\left(dx^4-c\right)}\sqrt{-bx^4+a}+\frac{b}{(4ad-4bc)\,c}\sqrt{1-x^2\sqrt{b}\frac{1}{\sqrt{a}}}\sqrt{1+x^2\sqrt{b}\frac{1}{\sqrt{a}}}\,\mathrm{EllipticF}\left(x\sqrt{\sqrt{b}\frac{1}{\sqrt{a}}},i\right)\frac{1}{\sqrt{\sqrt{b}\frac{1}{\sqrt{a}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x)
```

```
[Out] -1/4*d/(a*d-b*c)/c*x*(-b*x^4+a)^(1/2)/(d*x^4-c)+1/4*b/(a*d-b*c)/c/(1/a^(1/2
)*b^(1/2))^(1/2)*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/2
)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-1/32/c/d*sum((3
*a*d-5*b*c)/(a*d-b*c)/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*_alp
ha^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))
^(1/2)*_alpha^3*d/c*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(
1/2)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^(1/2
)*_alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_alph
a=RootOf(_Z^4*d-c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-bx^4+a}\left(dx^4-c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-b*x^4 + a)*(d*x^4 - c)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**4+a)**(1/2)/(-d*x**4+c)**2,x)
```

```
[Out] Exception raised: ValueError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-bx^4 + a}(dx^4 - c)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(1/2)/(-d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-b*x^4 + a)*(d*x^4 - c)^2), x)
```

3.189 $\int \frac{1}{\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)^2}\,dx$

Optimal. Leaf size=362

$$\frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}(ad+2bc)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{4a^{3/4}c\sqrt{a-bx^4}(bc-ad)^2}-\frac{3\sqrt[4]{a}d\sqrt{1-\frac{bx^4}{a}}(3bc-ad)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2\sqrt{a-bx^4}(bc-ad)^2}-\frac{3\sqrt[4]{a}d\ldots}{\ldots}$$

```
[Out] (b*(2*b*c + a*d)*x)/(4*a*c*(b*c - a*d)^2*Sqrt[a - b*x^4]) - (d*x)/(4*c*(b*c
 - a*d)*Sqrt[a - b*x^4]*(c - d*x^4)) + (b^(3/4)*(2*b*c + a*d)*Sqrt[1 - (b*x
^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(4*a^(3/4)*c*(b*c - a*d)
^2*Sqrt[a - b*x^4]) - (3*a^(1/4)*d*(3*b*c - a*d)*Sqrt[1 - (b*x^4)/a]*Ellipt
icPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -
1])/(8*b^(1/4)*c^2*(b*c - a*d)^2*Sqrt[a - b*x^4]) - (3*a^(1/4)*d*(3*b*c - a
*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), Arc
Sin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a*d)^2*Sqrt[a - b*x^4]
)
```

Rubi [A] time = 0.40423, antiderivative size = 362, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {414, 527, 523, 224, 221, 409, 1219, 1218}

$$\frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}(ad+2bc)F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{4a^{3/4}c\sqrt{a-bx^4}(bc-ad)^2}-\frac{3\sqrt[4]{a}d\sqrt{1-\frac{bx^4}{a}}(3bc-ad)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2\sqrt{a-bx^4}(bc-ad)^2}-\frac{3\sqrt[4]{a}d\sqrt{1-\frac{bx^4}{a}}\ldots}{8\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - b*x^4)^(3/2)*(c - d*x^4)^2),x]
```

```
[Out] (b*(2*b*c + a*d)*x)/(4*a*c*(b*c - a*d)^2*Sqrt[a - b*x^4]) - (d*x)/(4*c*(b*c
 - a*d)*Sqrt[a - b*x^4]*(c - d*x^4)) + (b^(3/4)*(2*b*c + a*d)*Sqrt[1 - (b*x
^4)/a]*EllipticF[ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(4*a^(3/4)*c*(b*c - a*d)
^2*Sqrt[a - b*x^4]) - (3*a^(1/4)*d*(3*b*c - a*d)*Sqrt[1 - (b*x^4)/a]*Ellipt
icPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -
1])/(8*b^(1/4)*c^2*(b*c - a*d)^2*Sqrt[a - b*x^4]) - (3*a^(1/4)*d*(3*b*c - a
*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), Arc
Sin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a*d)^2*Sqrt[a - b*x^4]
)
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
```

```
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)^2}\,dx = -\frac{dx}{4c(bc-ad)\sqrt{a-bx^4}\left(c-dx^4\right)} - \frac{\int \frac{-4bc+3ad-5bdx^4}{\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)}\,dx}{4c(bc-ad)}$$

$$= \frac{b(2bc+ad)x}{4ac(bc-ad)^2\sqrt{a-bx^4}} - \frac{dx}{4c(bc-ad)\sqrt{a-bx^4}\left(c-dx^4\right)} - \frac{\int \frac{-2\left(2b^2c^2-8abcd+3a^2d^2\right)+2bd(2b}{\sqrt{a-bx^4}\left(c-dx^4\right)}}{8ac(bc-ad)^2}$$

$$= \frac{b(2bc+ad)x}{4ac(bc-ad)^2\sqrt{a-bx^4}} - \frac{dx}{4c(bc-ad)\sqrt{a-bx^4}\left(c-dx^4\right)} - \frac{(3d(3bc-ad))\int \frac{1}{\sqrt{a-bx^4}\left(c-dx\right.}}{4c(bc-ad)^2}$$

$$= \frac{b(2bc+ad)x}{4ac(bc-ad)^2\sqrt{a-bx^4}} - \frac{dx}{4c(bc-ad)\sqrt{a-bx^4}\left(c-dx^4\right)} - \frac{(3d(3bc-ad))\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a}}}{8c^2(bc-ad)^2}$$

$$= \frac{b(2bc+ad)x}{4ac(bc-ad)^2\sqrt{a-bx^4}} - \frac{dx}{4c(bc-ad)\sqrt{a-bx^4}\left(c-dx^4\right)} + \frac{b^{3/4}(2bc+ad)\sqrt{1-\frac{bx^4}{a}}F\left(\mathrm{s}\right.}{4a^{3/4}c(bc-ad)^2\sqrt{}}$$

$$= \frac{b(2bc+ad)x}{4ac(bc-ad)^2\sqrt{a-bx^4}} - \frac{dx}{4c(bc-ad)\sqrt{a-bx^4}\left(c-dx^4\right)} + \frac{b^{3/4}(2bc+ad)\sqrt{1-\frac{bx^4}{a}}F\left(\mathrm{s}\right.}{4a^{3/4}c(bc-ad)^2\sqrt{}}$$

Mathematica [C] time = 0.523988, size = 374, normalized size = 1.03

$$\frac{x\left(\frac{c\left(25ac\left(4a^2d^2-abd\left(8c+dx^4\right)+2b^2c\left(2c-dx^4\right)\right)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)-10x^4\left(-a^2d^2+abd^2x^4-2b^2c\left(c-dx^4\right)\right)\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right.\right.}{\left(c-dx^4\right)\left(2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)+5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)}\right.}{20ac^2\sqrt{a-bx^4}(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - b*x^4)^(3/2)*(c - d*x^4)^2),x]
```

```
[Out] (x*(-(b*d*(2*b*c + a*d)*x^4*Sqrt[1 - (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9/4,
(b*x^4)/a, (d*x^4)/c]) + (c*(25*a*c*(4*a^2*d^2 + 2*b^2*c*(2*c - d*x^4) - a*
b*d*(8*c + d*x^4))*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] - 10*x^
4*(-(a^2*d^2) + a*b*d^2*x^4 - 2*b^2*c*(c - d*x^4))*(2*a*d*AppellF1[5/4, 1/2
, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a,
 (d*x^4)/c])))/((c - d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d
*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] +
b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))))/(20*a*c^2*(b*c -
a*d)^2*Sqrt[a - b*x^4])
```

Maple [C] time = 0.038, size = 375, normalized size = 1.

$$-\frac{d^2x}{4\,(ad-bc)^2\,c\,\left(dx^4-c\right)}\sqrt{-bx^4+a}+\frac{b^2x}{2\,a\,(ad-bc)^2}\frac{1}{\sqrt{-\left(x^4-\frac{a}{b}\right)b}}+\left(\frac{bd}{4\,(ad-bc)^2\,c}+\frac{b^2}{2\,a\,(ad-bc)^2}\right)\sqrt{1-x^2\sqrt{b}\frac{1}{\sqrt{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x)
```

```
[Out] -1/4*d^2/(a*d-b*c)^2/c*x*(-b*x^4+a)^(1/2)/(d*x^4-c)+1/2*b^2*x/a/(a*d-b*c)^2
/(-(x^4-1/b*a)*b)^(1/2)+(1/4*d*b/(a*d-b*c)^2/c+1/2*b^2/a/(a*d-b*c)^2)/(1/a^
(1/2)*b^(1/2))^(1/2)*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^
(1/2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-3/32/c*sum(
(a*d-3*b*c)/(a*d-b*c)^2/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*_a
lpha^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2
))^(1/2)*_alpha^3*d/c*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))
^(1/2)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^(1
/2)*_alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_al
pha=RootOf(_Z^4*d-c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^4+a\right)^{\frac{3}{2}}\left(dx^4-c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^4 + a)^(3/2)*(d*x^4 - c)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**4+a)**(3/2)/(-d*x**4+c)**2,x)
```

```
[Out] Exception raised: ValueError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^4+a\right)^{\frac{3}{2}}\left(dx^4-c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(3/2)/(-d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^4 + a)^(3/2)*(d*x^4 - c)^2), x)
```

3.190 $\int \frac{1}{\left(a-bx^4\right)^{5/2}\left(c-dx^4\right)^2}\,dx$

Optimal. Leaf size=439

$$\frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(-3a^2d^2-17abcd+5b^2c^2\right)\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right),-1\right)}{12a^{7/4}c\sqrt{a-bx^4}(bc-ad)^3}+\frac{bx\left(-3a^2d^2-17abcd+5b^2c^2\right)}{12a^2c\sqrt{a-bx^4}(bc-ad)^3}+\frac{\sqrt[4]{a}d^2\sqrt{1-\frac{bx^4}{a}}(\ldots}{8\ldots}$$

```
[Out] (b*(2*b*c + 3*a*d)*x)/(12*a*c*(b*c - a*d)^2*(a - b*x^4)^(3/2)) + (b*(5*b^2*
c^2 - 17*a*b*c*d - 3*a^2*d^2)*x)/(12*a^2*c*(b*c - a*d)^3*Sqrt[a - b*x^4]) -
 (d*x)/(4*c*(b*c - a*d)*(a - b*x^4)^(3/2)*(c - d*x^4)) + (b^(3/4)*(5*b^2*c^
2 - 17*a*b*c*d - 3*a^2*d^2)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x
)/a^(1/4)], -1])/(12*a^(7/4)*c*(b*c - a*d)^3*Sqrt[a - b*x^4]) + (a^(1/4)*d^
2*(13*b*c - 3*a*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt
[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a*d)
^3*Sqrt[a - b*x^4]) + (a^(1/4)*d^2*(13*b*c - 3*a*d)*Sqrt[1 - (b*x^4)/a]*Ell
ipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -
1])/(8*b^(1/4)*c^2*(b*c - a*d)^3*Sqrt[a - b*x^4])
```

Rubi [A] time = 0.535515, antiderivative size = 439, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {414, 527, 523, 224, 221, 409, 1219, 1218}

$$\frac{bx\left(-3a^2d^2-17abcd+5b^2c^2\right)}{12a^2c\sqrt{a-bx^4}(bc-ad)^3}+\frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}\left(-3a^2d^2-17abcd+5b^2c^2\right)F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|-1\right)}{12a^{7/4}c\sqrt{a-bx^4}(bc-ad)^3}+\frac{\sqrt[4]{a}d^2\sqrt{1-\frac{bx^4}{a}}(13bc-3\ldots}{8\sqrt[4]{b}c^2\sqrt{\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - b*x^4)^(5/2)*(c - d*x^4)^2),x]
```


```
[Out] (b*(2*b*c + 3*a*d)*x)/(12*a*c*(b*c - a*d)^2*(a - b*x^4)^(3/2)) + (b*(5*b^2*
c^2 - 17*a*b*c*d - 3*a^2*d^2)*x)/(12*a^2*c*(b*c - a*d)^3*Sqrt[a - b*x^4]) -
 (d*x)/(4*c*(b*c - a*d)*(a - b*x^4)^(3/2)*(c - d*x^4)) + (b^(3/4)*(5*b^2*c^
2 - 17*a*b*c*d - 3*a^2*d^2)*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^(1/4)*x
)/a^(1/4)], -1])/(12*a^(7/4)*c*(b*c - a*d)^3*Sqrt[a - b*x^4]) + (a^(1/4)*d^
2*(13*b*c - 3*a*d)*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt
[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/a^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a*d)
^3*Sqrt[a - b*x^4]) + (a^(1/4)*d^2*(13*b*c - 3*a*d)*Sqrt[1 - (b*x^4)/a]*Ell
ipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/a^(1/4)], -
1])/(8*b^(1/4)*c^2*(b*c - a*d)^3*Sqrt[a - b*x^4])
```


Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Dist[Sqrt[1 + (b*x^4)/a]/Sqrt
[a + b*x^4], Int[1/Sqrt[1 + (b*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[
b/a] &&  !GtQ[a, 0]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[(Rt[-b,
 4]*x)/Rt[a, 4]], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] &&  !GtQ[a, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\left(a-bx^4\right)^{5/2}\left(c-dx^4\right)^2}\,dx = -\frac{dx}{4c(bc-ad)\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)} - \frac{\int \frac{-4bc+3ad-9bdx^4}{\left(a-bx^4\right)^{5/2}\left(c-dx^4\right)}dx}{4c(bc-ad)}$$

$$= \frac{b(2bc+3ad)x}{12ac(bc-ad)^2\left(a-bx^4\right)^{3/2}} - \frac{dx}{4c(bc-ad)\left(a-bx^4\right)^{3/2}\left(c-dx^4\right)} - \frac{\int \frac{-2\left(10b^2c^2-24abcd+9a^2d^2\right)+}{\left(a-bx^4\right)^{3/2}\left(c-\right.}}{24ac(bc-}$$

$$= \frac{b(2bc+3ad)x}{12ac(bc-ad)^2\left(a-bx^4\right)^{3/2}} + \frac{b\left(5b^2c^2-17abcd-3a^2d^2\right)x}{12a^2c(bc-ad)^3\sqrt{a-bx^4}} - \frac{dx}{4c(bc-ad)\left(a-bx^4\right)^{3/2}\left(c-\right.}$$

$$= \frac{b(2bc+3ad)x}{12ac(bc-ad)^2\left(a-bx^4\right)^{3/2}} + \frac{b\left(5b^2c^2-17abcd-3a^2d^2\right)x}{12a^2c(bc-ad)^3\sqrt{a-bx^4}} - \frac{dx}{4c(bc-ad)\left(a-bx^4\right)^{3/2}\left(c-\right.}$$

$$= \frac{b(2bc+3ad)x}{12ac(bc-ad)^2\left(a-bx^4\right)^{3/2}} + \frac{b\left(5b^2c^2-17abcd-3a^2d^2\right)x}{12a^2c(bc-ad)^3\sqrt{a-bx^4}} - \frac{dx}{4c(bc-ad)\left(a-bx^4\right)^{3/2}\left(c-\right.}$$

$$= \frac{b(2bc+3ad)x}{12ac(bc-ad)^2\left(a-bx^4\right)^{3/2}} + \frac{b\left(5b^2c^2-17abcd-3a^2d^2\right)x}{12a^2c(bc-ad)^3\sqrt{a-bx^4}} - \frac{dx}{4c(bc-ad)\left(a-bx^4\right)^{3/2}\left(c-\right.}$$

$$= \frac{b(2bc+3ad)x}{12ac(bc-ad)^2\left(a-bx^4\right)^{3/2}} + \frac{b\left(5b^2c^2-17abcd-3a^2d^2\right)x}{12a^2c(bc-ad)^3\sqrt{a-bx^4}} - \frac{dx}{4c(bc-ad)\left(a-bx^4\right)^{3/2}\left(c-\right.}$$

Mathematica [C] time = 0.821559, size = 382, normalized size = 0.87

$$\frac{x\left(\frac{bdx^4\sqrt{1-\frac{bx^4}{a}}\left(3a^2d^2+17abcd-5b^2c^2\right)F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)}{a^2c^2}+5\left(\frac{5\left(36a^2bcd^2-9a^3d^3-17ab^2c^2d+5b^3c^3\right)F_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)}{a\left(c-dx^4\right)\left(2x^4\left(2adF_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)+bcF_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right)\right)+5acF_1\left(\frac{1}{4};\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},\frac{dx^4}{c}\right.\right.}\right.\right.}{60\sqrt{a-bx^4}(bc-ad)^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - b*x^4)^(5/2)*(c - d*x^4)^2),x]
```

```
[Out] (x*((b*d*(-5*b^2*c^2 + 17*a*b*c*d + 3*a^2*d^2)*x^4*Sqrt[1 - (b*x^4)/a]*Appe
llF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])/(a^2*c^2) + 5*((5*b^3*c)/a^2
- (17*b^2*d)/a - (2*b^2*d)/(a - b*x^4) + (2*b^3*c)/(a^2 - a*b*x^4) - (3*a*d
^3)/(c^2 - c*d*x^4) + (3*b*d^3*x^4)/(c^2 - c*d*x^4) + (5*(5*b^3*c^3 - 17*a*
b^2*c^2*d + 36*a^2*b*c*d^2 - 9*a^3*d^3)*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/
a, (d*x^4)/c])/(a*(c - d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a,
(d*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c]
+ b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c]))))))/(60*(b*c - a*d
)^3*Sqrt[a - b*x^4])
```

Maple [C] time = 0.038, size = 484, normalized size = 1.1

$$-\frac{bd^3x}{\left(4a^2d^2-8cabd+4b^2c^2\right)(ad-bc)c\left(bdx^4-bc\right)}\sqrt{-bx^4+a}+\frac{x}{6(ad-bc)^2a}\sqrt{-bx^4+a}\left(x^4-\frac{a}{b}\right)^{-2}+\frac{b^2x(17ad-5bc)}{12a^2(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x)
```

```
[Out] -1/4*b*d^3/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)/c*x*(-b*x^4+a)^(1/2)/(b*d*
x^4-b*c)+1/6/(a*d-b*c)^2/a*x*(-b*x^4+a)^(1/2)/(x^4-1/b*a)^2+1/12*b^2*x/a^2*
(17*a*d-5*b*c)/(a*d-b*c)^3/(-(x^4-1/b*a)*b)^(1/2)+(1/4*b*d^2/(a^2*d^2-2*a*b
*c*d+b^2*c^2)/(a*d-b*c)/c+1/12*b^2/a^2*(17*a*d-5*b*c)/(a*d-b*c)^3)/(1/a^(1/
2)*b^(1/2))^(1/2)*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2))^(1/
2)/(-b*x^4+a)^(1/2)*EllipticF(x*(1/a^(1/2)*b^(1/2))^(1/2),I)-1/32*d/c*sum((
3*a*d-13*b*c)/(a*d-b*c)^3/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*
_alpha^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1
/2))^(1/2)*_alpha^3*d/c*(1-x^2*b^(1/2)/a^(1/2))^(1/2)*(1+x^2*b^(1/2)/a^(1/2
))^(1/2)/(-b*x^4+a)^(1/2)*EllipticPi(x*(1/a^(1/2)*b^(1/2))^(1/2),a^(1/2)/b^
(1/2)*_alpha^2/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_
alpha=RootOf(_Z^4*d-c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^4+a\right)^{\frac{5}{2}}\left(dx^4-c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((-b*x^4 + a)^(5/2)*(d*x^4 - c)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x**4+a)**(5/2)/(-d*x**4+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-bx^4 + a\right)^{\frac{5}{2}} \left(dx^4 - c\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(5/2)/(-d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((-b*x^4 + a)^(5/2)*(d*x^4 - c)^2), x)
```

3.191 $\int \frac{\sqrt{a+bx^4}}{ac-bcx^4}\,dx$

Optimal. Leaf size=103

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}c}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}c}$$

```
[Out] ArcTan[(Sqrt[2]*a^(1/4)*b^(1/4)*x)/Sqrt[a + b*x^4]]/(2*Sqrt[2]*a^(1/4)*b^(1
/4)*c) + ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*x)/Sqrt[a + b*x^4]]/(2*Sqrt[2]*a^
(1/4)*b^(1/4)*c)
```

Rubi [A] time = 0.0557869, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {404, 212, 206, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}c}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^4]/(a*c - b*c*x^4),x]
```

```
[Out] ArcTan[(Sqrt[2]*a^(1/4)*b^(1/4)*x)/Sqrt[a + b*x^4]]/(2*Sqrt[2]*a^(1/4)*b^(1
/4)*c) + ArcTanh[(Sqrt[2]*a^(1/4)*b^(1/4)*x)/Sqrt[a + b*x^4]]/(2*Sqrt[2]*a^
(1/4)*b^(1/4)*c)
```

Rule 404

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[a/c,
 Subst[Int[1/(1 - 4*a*b*x^4), x], x, x/Sqrt[a + b*x^4]], x] /; FreeQ[{a, b,
 c, d}, x] && EqQ[b*c + a*d, 0] && PosQ[a*b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+bx^4}}{ac-bcx^4}\,dx = \frac{\text{Subst}\left(\int \frac{1}{1-4abx^4}\,dx, x, \frac{x}{\sqrt{a+bx^4}}\right)}{c}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-2\sqrt{a}\sqrt{b}x^2}\,dx, x, \frac{x}{\sqrt{a+bx^4}}\right)}{2c} + \frac{\text{Subst}\left(\int \frac{1}{1+2\sqrt{a}\sqrt{b}x^2}\,dx, x, \frac{x}{\sqrt{a+bx^4}}\right)}{2c}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}c} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}c}$$

Mathematica [C] time = 0.146802, size = 155, normalized size = 1.5

$$\frac{5ax\sqrt{a+bx^4}F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};-\frac{bx^4}{a},\frac{bx^4}{a}\right)}{c\left(a-bx^4\right)\left(2bx^4\left(2F_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};-\frac{bx^4}{a},\frac{bx^4}{a}\right)+F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{bx^4}{a},\frac{bx^4}{a}\right)\right)+5aF_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};-\frac{bx^4}{a},\frac{bx^4}{a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*x^4]/(a*c - b*c*x^4),x]
```

```
[Out] (5*a*x*Sqrt[a + b*x^4]*AppellF1[1/4, -1/2, 1, 5/4, -((b*x^4)/a), (b*x^4)/a]
)/(c*(a - b*x^4)*(5*a*AppellF1[1/4, -1/2, 1, 5/4, -((b*x^4)/a), (b*x^4)/a]
+ 2*b*x^4*(2*AppellF1[5/4, -1/2, 2, 9/4, -((b*x^4)/a), (b*x^4)/a] + AppellF
1[5/4, 1/2, 1, 9/4, -((b*x^4)/a), (b*x^4)/a])))
```

Maple [A] time = 0.106, size = 103, normalized size = 1.

$$-\frac{\sqrt{2}}{4c}\arctan\left(\frac{\sqrt{2}}{2x}\sqrt{bx^4+a}\frac{1}{\sqrt[4]{ab}}\right)\frac{1}{\sqrt[4]{ab}}+\frac{\sqrt{2}}{8c}\ln\left(\left(\frac{\sqrt{2}}{2x}\sqrt{bx^4+a}+\sqrt[4]{ab}\right)\left(\frac{\sqrt{2}}{2x}\sqrt{bx^4+a}-\sqrt[4]{ab}\right)^{-1}\right)\frac{1}{\sqrt[4]{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(1/2)/(-b*c*x^4+a*c),x)
```

```
[Out] -1/4/c*2^(1/2)/(a*b)^(1/4)*arctan(1/2*(b*x^4+a)^(1/2)*2^(1/2)/x/(a*b)^(1/4)
)+1/8/c*2^(1/2)/(a*b)^(1/4)*ln((1/2*(b*x^4+a)^(1/2)*2^(1/2)/x+(a*b)^(1/4))/
(1/2*(b*x^4+a)^(1/2)*2^(1/2)/x-(a*b)^(1/4)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{bx^4+a}}{bcx^4-ac}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/2)/(-b*c*x^4+a*c),x, algorithm="maxima")
```

```
[Out] -integrate(sqrt(b*x^4 + a)/(b*c*x^4 - a*c), x)
```

Fricas [B] time = 5.7186, size = 787, normalized size = 7.64

$$-\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}\arctan\left(\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}\sqrt{bx^4+a}c\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}-\frac{2\left(\frac{1}{4}\right)^{\frac{3}{4}}abc^3\left(\frac{1}{abc^4}\right)^{\frac{3}{4}}+\left(\frac{1}{4}\right)^{\frac{1}{4}}bcx^2\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}}{\sqrt{b}}}{x}\right)+\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}\log\left(\frac{4\left(\frac{1}{4}\right)^{\frac{3}{4}}ab\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/2)/(-b*c*x^4+a*c),x, algorithm="fricas")
```

```
[Out] -(1/4)^(1/4)*(1/(a*b*c^4))^(1/4)*arctan(((1/4)^(1/4)*sqrt(b*x^4 + a)*c*(1/(
a*b*c^4))^(1/4) - (2*(1/4)^(3/4)*a*b*c^3*(1/(a*b*c^4))^(3/4) + (1/4)^(1/4)*
b*c*x^2*(1/(a*b*c^4))^(1/4))/sqrt(b))/x) + 1/4*(1/4)^(1/4)*(1/(a*b*c^4))^(1
/4)*log((4*(1/4)^(3/4)*a*b*c^3*x^3*(1/(a*b*c^4))^(3/4) + 2*(1/4)^(1/4)*a*c*
x*(1/(a*b*c^4))^(1/4) + sqrt(b*x^4 + a)*(a*c^2*sqrt(1/(a*b*c^4)) + x^2))/(b
*x^4 - a)) - 1/4*(1/4)^(1/4)*(1/(a*b*c^4))^(1/4)*log(-(4*(1/4)^(3/4)*a*b*c^
3*x^3*(1/(a*b*c^4))^(3/4) + 2*(1/4)^(1/4)*a*c*x*(1/(a*b*c^4))^(1/4) - sqrt(
b*x^4 + a)*(a*c^2*sqrt(1/(a*b*c^4)) + x^2))/(b*x^4 - a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{\sqrt{a+bx^4}}{-a+bx^4}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(1/2)/(-b*c*x**4+a*c),x)
```

```
[Out] -Integral(sqrt(a + b*x**4)/(-a + b*x**4), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{bx^4+a}}{bcx^4-ac}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/2)/(-b*c*x^4+a*c),x, algorithm="giac")
```

```
[Out] integrate(-sqrt(b*x^4 + a)/(b*c*x^4 - a*c), x)
```

3.192 $\int \frac{\sqrt{a-bx^4}}{ac+bcx^4}\,dx$

Optimal. Leaf size=116

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x\left(\sqrt{a}+\sqrt{b}x^2\right)}{\sqrt[4]{a}\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\sqrt[4]{b}c}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x\left(\sqrt{a}-\sqrt{b}x^2\right)}{\sqrt[4]{a}\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\sqrt[4]{b}c}$$

```
[Out] ArcTan[(b^(1/4)*x*(Sqrt[a] + Sqrt[b]*x^2))/(a^(1/4)*Sqrt[a - b*x^4])]/(2*a^
(1/4)*b^(1/4)*c) + ArcTanh[(b^(1/4)*x*(Sqrt[a] - Sqrt[b]*x^2))/(a^(1/4)*Sqr
t[a - b*x^4])]/(2*a^(1/4)*b^(1/4)*c)
```

Rubi [A] time = 0.0234927, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {405}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x\left(\sqrt{a}+\sqrt{b}x^2\right)}{\sqrt[4]{a}\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\sqrt[4]{b}c}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x\left(\sqrt{a}-\sqrt{b}x^2\right)}{\sqrt[4]{a}\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\sqrt[4]{b}c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - b*x^4]/(a*c + b*c*x^4),x]
```

```
[Out] ArcTan[(b^(1/4)*x*(Sqrt[a] + Sqrt[b]*x^2))/(a^(1/4)*Sqrt[a - b*x^4])]/(2*a^
(1/4)*b^(1/4)*c) + ArcTanh[(b^(1/4)*x*(Sqrt[a] - Sqrt[b]*x^2))/(a^(1/4)*Sqr
t[a - b*x^4])]/(2*a^(1/4)*b^(1/4)*c)
```

Rule 405

```
Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] :> With[{q =
 Rt[-(a*b), 4]}, Simp[(a*ArcTan[(q*x*(a + q^2*x^2))/(a*Sqrt[a + b*x^4])])/(
2*c*q), x] + Simp[(a*ArcTanh[(q*x*(a - q^2*x^2))/(a*Sqrt[a + b*x^4])])/(2*c
*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && NegQ[a*b]
```

Rubi steps

$$\int \frac{\sqrt{a-bx^4}}{ac+bcx^4}\,dx=\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x\left(\sqrt{a}+\sqrt{b}x^2\right)}{\sqrt[4]{a}\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\sqrt[4]{b}c}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x\left(\sqrt{a}-\sqrt{b}x^2\right)}{\sqrt[4]{a}\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\sqrt[4]{b}c}$$

Mathematica [C] time = 0.154231, size = 155, normalized size = 1.34

$$\frac{5ax\sqrt{a-bx^4}F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},-\frac{bx^4}{a}\right)}{c\left(a+bx^4\right)\left(5aF_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};\frac{bx^4}{a},-\frac{bx^4}{a}\right)-2bx^4\left(2F_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};\frac{bx^4}{a},-\frac{bx^4}{a}\right)+F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};\frac{bx^4}{a},-\frac{bx^4}{a}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a - b*x^4]/(a*c + b*c*x^4),x]
```

```
[Out] (5*a*x*Sqrt[a - b*x^4]*AppellF1[1/4, -1/2, 1, 5/4, (b*x^4)/a, -((b*x^4)/a)]
)/(c*(a + b*x^4)*(5*a*AppellF1[1/4, -1/2, 1, 5/4, (b*x^4)/a, -((b*x^4)/a)]
- 2*b*x^4*(2*AppellF1[5/4, -1/2, 2, 9/4, (b*x^4)/a, -((b*x^4)/a)] + AppellF
1[5/4, 1/2, 1, 9/4, (b*x^4)/a, -((b*x^4)/a)])))
```

Maple [A] time = 0.018, size = 158, normalized size = 1.4

$$-\frac{1}{8c}\ln\left(\left(\frac{-bx^4+a}{2x^2}-\frac{1}{x}\sqrt[4]{ab}\sqrt{-bx^4+a}+\sqrt{ab}\right)\left(\frac{-bx^4+a}{2x^2}+\frac{1}{x}\sqrt[4]{ab}\sqrt{-bx^4+a}+\sqrt{ab}\right)^{-1}\right)\frac{1}{\sqrt[4]{ab}}-\frac{1}{4c}\arctan\left(\frac{1}{x}\sqrt{-bx^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^4+a)^(1/2)/(b*c*x^4+a*c),x)
```

```
[Out] -1/8/c/(a*b)^(1/4)*ln((1/2*(-b*x^4+a)/x^2-(a*b)^(1/4)*(-b*x^4+a)^(1/2)/x+(a
*b)^(1/2))/(1/2*(-b*x^4+a)/x^2+(a*b)^(1/4)*(-b*x^4+a)^(1/2)/x+(a*b)^(1/2)))
-1/4/c/(a*b)^(1/4)*arctan(1/(a*b)^(1/4)*(-b*x^4+a)^(1/2)/x+1)+1/4/c/(a*b)^(
1/4)*arctan(-1/(a*b)^(1/4)*(-b*x^4+a)^(1/2)/x+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^4+a}}{bcx^4+ac}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/2)/(b*c*x^4+a*c),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-b*x^4 + a)/(b*c*x^4 + a*c), x)
```

Fricas [B] time = 5.00008, size = 822, normalized size = 7.09

$$-\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{abc^4}\right)^{\frac{1}{4}}\arctan\left(\frac{2\left(\frac{1}{4}\right)^{\frac{3}{4}}abc^3\sqrt{-\frac{1}{b}}\left(-\frac{1}{abc^4}\right)^{\frac{3}{4}}+\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(bcx^2\sqrt{-\frac{1}{b}}+\sqrt{-bx^4+a}c\right)\left(-\frac{1}{abc^4}\right)^{\frac{1}{4}}}{x}\right)-\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{abc^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/2)/(b*c*x^4+a*c),x, algorithm="fricas")
```

```
[Out] -(1/4)^(1/4)*(-1/(a*b*c^4))^(1/4)*arctan((2*(1/4)^(3/4)*a*b*c^3*sqrt(-1/b)*
(-1/(a*b*c^4))^(3/4) + (1/4)^(1/4)*(b*c*x^2*sqrt(-1/b) + sqrt(-b*x^4 + a)*c
)*(-1/(a*b*c^4))^(1/4))/x) - 1/4*(1/4)^(1/4)*(-1/(a*b*c^4))^(1/4)*log(-(4*(
1/4)^(3/4)*a*b*c^3*x^3*(-1/(a*b*c^4))^(3/4) + sqrt(-b*x^4 + a)*a*c^2*sqrt(-
1/(a*b*c^4)) - 2*(1/4)^(1/4)*a*c*x*(-1/(a*b*c^4))^(1/4) + sqrt(-b*x^4 + a)*
x^2)/(b*x^4 + a)) + 1/4*(1/4)^(1/4)*(-1/(a*b*c^4))^(1/4)*log((4*(1/4)^(3/4)
*a*b*c^3*x^3*(-1/(a*b*c^4))^(3/4) - sqrt(-b*x^4 + a)*a*c^2*sqrt(-1/(a*b*c^4
)) - 2*(1/4)^(1/4)*a*c*x*(-1/(a*b*c^4))^(1/4) - sqrt(-b*x^4 + a)*x^2)/(b*x^
4 + a))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{a-bx^4}}{a+bx^4}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**4+a)**(1/2)/(b*c*x**4+a*c),x)
```

```
[Out] Integral(sqrt(a - b*x**4)/(a + b*x**4), x)/c
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-bx^4+a}}{bcx^4+ac}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/2)/(b*c*x^4+a*c),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-b*x^4 + a)/(b*c*x^4 + a*c), x)
```

3.193 $\int \frac{(a+bx^4)^{7/4}}{c+dx^4}\,dx$

Optimal. Leaf size=211

$$-\frac{b^{3/4}(4bc-7ad)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^2}-\frac{b^{3/4}(4bc-7ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^2}+\frac{(bc-ad)^{7/4}\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d^2}+\frac{(bc-ad)^{7/4}\,\mathrm{ta}}{2c}$$

```
[Out] (b*x*(a + b*x^4)^(3/4))/(4*d) - (b^(3/4)*(4*b*c - 7*a*d)*ArcTan[(b^(1/4)*x)
/(a + b*x^4)^(1/4)])/(8*d^2) + ((b*c - a*d)^(7/4)*ArcTan[((b*c - a*d)^(1/4)
*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*d^2) - (b^(3/4)*(4*b*c - 7*a*d
)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(8*d^2) + ((b*c - a*d)^(7/4)*ArcT
anh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*d^2)
```

Rubi [A] time = 0.224681, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {416, 530, 240, 212, 206, 203, 377, 208, 205}

$$-\frac{b^{3/4}(4bc-7ad)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^2}-\frac{b^{3/4}(4bc-7ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^2}+\frac{(bc-ad)^{7/4}\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d^2}+\frac{(bc-ad)^{7/4}\,\mathrm{ta}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(7/4)/(c + d*x^4),x]
```


```
[Out] (b*x*(a + b*x^4)^(3/4))/(4*d) - (b^(3/4)*(4*b*c - 7*a*d)*ArcTan[(b^(1/4)*x)
/(a + b*x^4)^(1/4)])/(8*d^2) + ((b*c - a*d)^(7/4)*ArcTan[((b*c - a*d)^(1/4)
*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*d^2) - (b^(3/4)*(4*b*c - 7*a*d
)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(8*d^2) + ((b*c - a*d)^(7/4)*ArcT
anh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*d^2)
```


Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^4\right)^{7/4}}{c+dx^4}\,dx &= \frac{bx\left(a+bx^4\right)^{3/4}}{4d} + \frac{\int \frac{-a(bc-4ad)-b(4bc-7ad)x^4}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)}\,dx}{4d} \\
&= \frac{bx\left(a+bx^4\right)^{3/4}}{4d} - \frac{(b(4bc-7ad))\int \frac{1}{\sqrt[4]{a+bx^4}}\,dx}{4d^2} + \frac{(bc-ad)^2\int \frac{1}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)}\,dx}{d^2} \\
&= \frac{bx\left(a+bx^4\right)^{3/4}}{4d} - \frac{(b(4bc-7ad))\,\mathrm{Subst}\left(\int \frac{1}{1-bx^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{4d^2} + \frac{(bc-ad)^2\,\mathrm{Subst}\left(\int \frac{1}{c-(bc-ad)x^4}\,dx, \ldots\right.}{d^2} \\
&= \frac{bx\left(a+bx^4\right)^{3/4}}{4d} - \frac{(b(4bc-7ad))\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{b}x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{8d^2} - \frac{(b(4bc-7ad))\,\mathrm{Subst}\left(\int \frac{1}{1+\sqrt{b}x^2}\ldots\right.}{8d^2} \\
&= \frac{bx\left(a+bx^4\right)^{3/4}}{4d} - \frac{b^{3/4}(4bc-7ad)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^2} + \frac{(bc-ad)^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d^2} - \frac{b^{3/4}(4bc-7ad)\ldots}{8\ldots}
\end{aligned}$$

Mathematica [C] time = 0.445039, size = 364, normalized size = 1.73

$$5\sqrt[4]{c}\left(4a^2d\sqrt[4]{a+bx^4}\log\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}+\sqrt[4]{c}\right)+4b^2c^{3/4}x^5\sqrt[4]{bc-ad}+4abc^{3/4}x\sqrt[4]{bc-ad}+a\sqrt[4]{a+bx^4}(bc-4ad)\log\left(\sqrt[4]{c}-\frac{x\sqrt[4]{bc-\ldots}}{\sqrt[4]{ax^4+\ldots}}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^4)^(7/4)/(c + d*x^4),x]
```

```
[Out] (4*b*(b*c - a*d)^(1/4)*(-4*b*c + 7*a*d)*x^5*(1 + (b*x^4)/a)^(1/4)*AppellF1[
5/4, 1/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 5*c^(1/4)*(4*a*b*c^(3/4)*(b
*c - a*d)^(1/4)*x + 4*b^2*c^(3/4)*(b*c - a*d)^(1/4)*x^5 + 2*a*(-(b*c) + 4*a
*d)*(a + b*x^4)^(1/4)*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(b + a*x^4)^(1/
4))] + a*(b*c - 4*a*d)*(a + b*x^4)^(1/4)*Log[c^(1/4) - ((b*c - a*d)^(1/4)*x
)/(b + a*x^4)^(1/4)] - a*b*c*(a + b*x^4)^(1/4)*Log[c^(1/4) + ((b*c - a*d)^(
1/4)*x)/(b + a*x^4)^(1/4)] + 4*a^2*d*(a + b*x^4)^(1/4)*Log[c^(1/4) + ((b*c
- a*d)^(1/4)*x)/(b + a*x^4)^(1/4)]))/(80*c*d*(b*c - a*d)^(1/4)*(a + b*x^4)^
(1/4))
```

Maple [F] time = 0.423, size = 0, normalized size = 0.

$$\int \frac{1}{dx^4 + c} \left(bx^4 + a\right)^{\frac{7}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(7/4)/(d*x^4+c),x)
```

```
[Out] int((b*x^4+a)^(7/4)/(d*x^4+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4 + a\right)^{\frac{7}{4}}}{dx^4 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(7/4)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(7/4)/(d*x^4 + c), x)
```

Fricas [B] time = 16.5479, size = 5146, normalized size = 24.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(7/4)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] 1/16*(4*(b*x^4 + a)^(3/4)*b*x + 16*d*((b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5
*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7
*a^6*b*c*d^6 - a^7*d^7)/(c^3*d^8))^(1/4)*arctan(-(c*d^2*x*sqrt(((b^7*c^8*d^
4 - 7*a*b^6*c^7*d^5 + 21*a^2*b^5*c^6*d^6 - 35*a^3*b^4*c^5*d^7 + 35*a^4*b^3*
c^4*d^8 - 21*a^5*b^2*c^3*d^9 + 7*a^6*b*c^2*d^10 - a^7*c*d^11)*x^2*sqrt((b^7
*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3
*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7*d^7)/(c^3*d^8)) + (b^10
```

```
*c^10 - 10*a*b^9*c^9*d + 45*a^2*b^8*c^8*d^2 - 120*a^3*b^7*c^7*d^3 + 210*a^4
*b^6*c^6*d^4 - 252*a^5*b^5*c^5*d^5 + 210*a^6*b^4*c^4*d^6 - 120*a^7*b^3*c^3*
d^7 + 45*a^8*b^2*c^2*d^8 - 10*a^9*b*c*d^9 + a^10*d^10)*sqrt(b*x^4 + a))/x^2
)*((b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*
a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7*d^7)/(c^3*d^8))^
(1/4) + (b^5*c^6*d^2 - 5*a*b^4*c^5*d^3 + 10*a^2*b^3*c^4*d^4 - 10*a^3*b^2*c^
3*d^5 + 5*a^4*b*c^2*d^6 - a^5*c*d^7)*(b*x^4 + a)^(1/4)*((b^7*c^7 - 7*a*b^6*
c^6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a
^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7*d^7)/(c^3*d^8))^(1/4))/((b^7*c^7 - 7*a
*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 -
 21*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7*d^7)*x)) + 4*d*((256*b^7*c^4 - 17
92*a*b^6*c^3*d + 4704*a^2*b^5*c^2*d^2 - 5488*a^3*b^4*c*d^3 + 2401*a^4*b^3*d
^4)/d^8)^(1/4)*arctan((d^2*x*sqrt(((256*b^7*c^4*d^4 - 1792*a*b^6*c^3*d^5 +
4704*a^2*b^5*c^2*d^6 - 5488*a^3*b^4*c*d^7 + 2401*a^4*b^3*d^8)*x^2*sqrt((256
*b^7*c^4 - 1792*a*b^6*c^3*d + 4704*a^2*b^5*c^2*d^2 - 5488*a^3*b^4*c*d^3 + 2
401*a^4*b^3*d^4)/d^8) + (4096*b^10*c^6 - 43008*a*b^9*c^5*d + 188160*a^2*b^8
*c^4*d^2 - 439040*a^3*b^7*c^3*d^3 + 576240*a^4*b^6*c^2*d^4 - 403368*a^5*b^5
*c*d^5 + 117649*a^6*b^4*d^6)*sqrt(b*x^4 + a))/x^2)*((256*b^7*c^4 - 1792*a*b
^6*c^3*d + 4704*a^2*b^5*c^2*d^2 - 5488*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/d^
8)^(1/4) + (64*b^5*c^3*d^2 - 336*a*b^4*c^2*d^3 + 588*a^2*b^3*c*d^4 - 343*a^
3*b^2*d^5)*(b*x^4 + a)^(1/4)*((256*b^7*c^4 - 1792*a*b^6*c^3*d + 4704*a^2*b^
5*c^2*d^2 - 5488*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/d^8)^(1/4))/((256*b^7*c^
4 - 1792*a*b^6*c^3*d + 4704*a^2*b^5*c^2*d^2 - 5488*a^3*b^4*c*d^3 + 2401*a^4
*b^3*d^4)*x)) + 4*d*((b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3
*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^
7*d^7)/(c^3*d^8))^(1/4)*log(-(c^2*d^6*x*((b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*
b^5*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5
+ 7*a^6*b*c*d^6 - a^7*d^7)/(c^3*d^8))^(3/4) + (b^5*c^5 - 5*a*b^4*c^4*d + 10
*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*(b*x^4 + a
)^(1/4))/x) - 4*d*((b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5*c^5*d^2 - 35*a^3*b
^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7*a^6*b*c*d^6 - a^7*
d^7)/(c^3*d^8))^(1/4)*log((c^2*d^6*x*((b^7*c^7 - 7*a*b^6*c^6*d + 21*a^2*b^5
*c^5*d^2 - 35*a^3*b^4*c^4*d^3 + 35*a^4*b^3*c^3*d^4 - 21*a^5*b^2*c^2*d^5 + 7
*a^6*b*c*d^6 - a^7*d^7)/(c^3*d^8))^(3/4) - (b^5*c^5 - 5*a*b^4*c^4*d + 10*a^
2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*(b*x^4 + a)^(
1/4))/x) - d*((256*b^7*c^4 - 1792*a*b^6*c^3*d + 4704*a^2*b^5*c^2*d^2 - 5488
*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/d^8)^(1/4)*log(-(d^6*x*((256*b^7*c^4 - 1
792*a*b^6*c^3*d + 4704*a^2*b^5*c^2*d^2 - 5488*a^3*b^4*c*d^3 + 2401*a^4*b^3*
d^4)/d^8)^(3/4) + (64*b^5*c^3 - 336*a*b^4*c^2*d + 588*a^2*b^3*c*d^2 - 343*a
^3*b^2*d^3)*(b*x^4 + a)^(1/4))/x) + d*((256*b^7*c^4 - 1792*a*b^6*c^3*d + 47
04*a^2*b^5*c^2*d^2 - 5488*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/d^8)^(1/4)*log(
(d^6*x*((256*b^7*c^4 - 1792*a*b^6*c^3*d + 4704*a^2*b^5*c^2*d^2 - 5488*a^3*b
^4*c*d^3 + 2401*a^4*b^3*d^4)/d^8)^(3/4) - (64*b^5*c^3 - 336*a*b^4*c^2*d + 5
88*a^2*b^3*c*d^2 - 343*a^3*b^2*d^3)*(b*x^4 + a)^(1/4))/x))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^4\right)^{\frac{7}{4}}}{c+dx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(7/4)/(d*x**4+c),x)
```

```
[Out] Integral((a + b*x**4)**(7/4)/(c + d*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)^{\frac{7}{4}}}{dx^4+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(7/4)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(7/4)/(d*x^4 + c), x)
```

3.194 $\int \frac{(a+bx^4)^{3/4}}{c+dx^4} dx$

Optimal. Leaf size=173

$$\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d}+\frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d}-\frac{(bc-ad)^{3/4}\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d}-\frac{(bc-ad)^{3/4}\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d}$$

```
[Out] (b^(3/4)*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(2*d) - ((b*c - a*d)^(3/4)*
ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*d) +
(b^(3/4)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(2*d) - ((b*c - a*d)^(3/4)
*ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*d)
```

Rubi [A] time = 0.103124, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {408, 240, 212, 206, 203, 377, 208, 205}

$$\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d}+\frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d}-\frac{(bc-ad)^{3/4}\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d}-\frac{(bc-ad)^{3/4}\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(3/4)/(c + d*x^4),x]
```

```
[Out] (b^(3/4)*ArcTan[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(2*d) - ((b*c - a*d)^(3/4)*
ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*d) +
(b^(3/4)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(2*d) - ((b*c - a*d)^(3/4)
*ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*d)
```

Rule 408

```
Int[((a_) + (b_.)*(x_)^4)^(p_)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/d
, Int[(a + b*x^4)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^4)^(p
- 1)/(c + d*x^4), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
(EqQ[p, 3/4] || EqQ[p, 5/4])
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(a+bx^4\right)^{3/4}}{c+dx^4}\,dx &= \frac{b\int \frac{1}{\sqrt[4]{a+bx^4}}\,dx}{d} - \frac{(bc-ad)\int \frac{1}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)}\,dx}{d} \\
&= \frac{b\,\mathrm{Subst}\left(\int \frac{1}{1-bx^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{d} - \frac{(bc-ad)\,\mathrm{Subst}\left(\int \frac{1}{c-(bc-ad)x^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{d} \\
&= \frac{b\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{b}x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2d} + \frac{b\,\mathrm{Subst}\left(\int \frac{1}{1+\sqrt{b}x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2d} - \frac{(bc-ad)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{c}-\sqrt{b}}\right.}{2\sqrt{cd}} \\
&= \frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d} - \frac{(bc-ad)^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d} + \frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d} - \frac{(bc-ad)^{3/4}\tanh}{2c^{3/4}}
\end{aligned}$$

Mathematica [C] time = 0.158452, size = 161, normalized size = 0.93

$$\frac{5acx\left(a+bx^4\right)^{3/4}F_1\left(\frac{1}{4};-\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{\left(c+dx^4\right)\left(x^4\left(3bcF_1\left(\frac{5}{4};\frac{1}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)-4adF_1\left(\frac{5}{4};-\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)+5acF_1\left(\frac{1}{4};-\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^4)^(3/4)/(c + d*x^4),x]
```

```
[Out] (5*a*c*x*(a + b*x^4)^(3/4)*AppellF1[1/4, -3/4, 1, 5/4, -((b*x^4)/a), -((d*x
^4)/c)])/((c + d*x^4)*(5*a*c*AppellF1[1/4, -3/4, 1, 5/4, -((b*x^4)/a), -((d
*x^4)/c)] + x^4*(-4*a*d*AppellF1[5/4, -3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)
/c)] + 3*b*c*AppellF1[5/4, 1/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))
```

Maple [F] time = 0.415, size = 0, normalized size = 0.

$$\int \frac{1}{dx^{4}+c} \left(bx^{4}+a\right)^{\frac{3}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(3/4)/(d*x^4+c),x)
```

```
[Out] int((b*x^4+a)^(3/4)/(d*x^4+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^{4}+a\right)^{\frac{3}{4}}}{dx^{4}+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/4)/(d*x^4 + c), x)
```

Fricas [B] time = 2.1963, size = 1794, normalized size = 10.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] ((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))^(1/4)*arcta
n(-(c*d*x*sqrt(((b^3*c^4*d^2 - 3*a*b^2*c^3*d^3 + 3*a^2*b*c^2*d^4 - a^3*c*d^
5)*x^2*sqrt((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))
+ (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*s
qrt(b*x^4 + a))/x^2)*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(
c^3*d^4))^(1/4) - (b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*(b*x^4 + a)^(1/4)
*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))^(1/4))/((b
^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x)) + (b^3/d^4)^(1/4)*arc
tan(-((b*x^4 + a)^(1/4)*b^2*d*(b^3/d^4)^(1/4) - d*x*(b^3/d^4)^(1/4)*sqrt((b
^3*d^2*x^2*sqrt(b^3/d^4) + sqrt(b*x^4 + a)*b^4)/x^2))/(b^3*x)) - 1/4*((b^3*
c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))^(1/4)*log((c^2*d^
3*x*((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))^(3/4) +
 (b*x^4 + a)^(1/4)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2))/x) + 1/4*((b^3*c^3 - 3*
a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))^(1/4)*log(-(c^2*d^3*x*((b
^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)/(c^3*d^4))^(3/4) - (b*x^4
 + a)^(1/4)*(b^2*c^2 - 2*a*b*c*d + a^2*d^2))/x) + 1/4*(b^3/d^4)^(1/4)*log((
d^3*x*(b^3/d^4)^(3/4) + (b*x^4 + a)^(1/4)*b^2)/x) - 1/4*(b^3/d^4)^(1/4)*log
(-(d^3*x*(b^3/d^4)^(3/4) - (b*x^4 + a)^(1/4)*b^2)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+bx^{4}\right)^{\frac{3}{4}}}{c+dx^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(3/4)/(d*x**4+c),x)
```

```
[Out] Integral((a + b*x**4)**(3/4)/(c + d*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4 + a\right)^{\frac{3}{4}}}{dx^4 + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/4)/(d*x^4 + c), x)
```

3.195 $\int \frac{1}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)}\,dx$

Optimal. Leaf size=105

$$\frac{\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}+\frac{\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}$$

```
[Out] ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))]/(2*c^(3/4)*(b*c -
 a*d)^(1/4)) + ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))]/(
2*c^(3/4)*(b*c - a*d)^(1/4))
```

Rubi [A] time = 0.0569827, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {377, 212, 208, 205}

$$\frac{\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}+\frac{\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^(1/4)*(c + d*x^4)),x]
```

```
[Out] ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))]/(2*c^(3/4)*(b*c -
 a*d)^(1/4)) + ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))]/(
2*c^(3/4)*(b*c - a*d)^(1/4))
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{a+bx^4}\,(c+dx^4)}\,dx = \text{Subst}\left(\int \frac{1}{c-(bc-ad)x^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c}-\sqrt{bc-ad}x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt{c}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c}+\sqrt{bc-ad}x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt{c}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}$$

Mathematica [A] time = 0.0405078, size = 84, normalized size = 0.8

$$\frac{\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right) + \tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^4)^(1/4)*(c + d*x^4)),x]
```

```
[Out] (ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))] + ArcTanh[((b*c
- a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*(b*c - a*d)^(1/4))
```

Maple [F] time = 0.405, size = 0, normalized size = 0.

$$\int \frac{1}{dx^4+c}\frac{1}{\sqrt[4]{bx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(1/4)/(d*x^4+c),x)
```

```
[Out] int(1/(b*x^4+a)^(1/4)/(d*x^4+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^4+a)^{\frac{1}{4}}(dx^4+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(1/4)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(1/4)*(d*x^4 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(1/4)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(1/4)/(d*x**4+c),x)
```

```
[Out] Integral(1/((a + b*x**4)**(1/4)*(c + d*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{1}{4}}\left(dx^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(1/4)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(1/4)*(d*x^4 + c)), x)
```

3.196 $\int \frac{1}{\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)}\,dx$

Optimal. Leaf size=134

$$-\frac{d\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{5/4}}-\frac{d\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{5/4}}+\frac{bx}{a\sqrt[4]{a+bx^4}(bc-ad)}$$

```
[Out] (b*x)/(a*(b*c - a*d)*(a + b*x^4)^(1/4)) - (d*ArcTan[((b*c - a*d)^(1/4)*x)/(
c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*(b*c - a*d)^(5/4)) - (d*ArcTanh[((b
*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*(b*c - a*d)^(5/
4))
```

Rubi [A] time = 0.0961695, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {382, 377, 212, 208, 205}

$$-\frac{d\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{5/4}}-\frac{d\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{5/4}}+\frac{bx}{a\sqrt[4]{a+bx^4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^(5/4)*(c + d*x^4)),x]
```

```
[Out] (b*x)/(a*(b*c - a*d)*(a + b*x^4)^(1/4)) - (d*ArcTan[((b*c - a*d)^(1/4)*x)/(
c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*(b*c - a*d)^(5/4)) - (d*ArcTanh[((b
*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*(b*c - a*d)^(5/
4))
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)}\,dx = \frac{bx}{a(bc-ad)\sqrt[4]{a+bx^4}} - \frac{d\int \frac{1}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)}\,dx}{bc-ad}$$

$$= \frac{bx}{a(bc-ad)\sqrt[4]{a+bx^4}} - \frac{d\,\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{bc-ad}$$

$$= \frac{bx}{a(bc-ad)\sqrt[4]{a+bx^4}} - \frac{d\,\text{Subst}\left(\int \frac{1}{\sqrt{c}-\sqrt{bc-ad}x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt{c}(bc-ad)} - \frac{d\,\text{Subst}\left(\int \frac{1}{\sqrt{c}+\sqrt{bc-ad}x^2}\,dx,\right.}{2\sqrt{c}(bc-ad)}$$

$$= \frac{bx}{a(bc-ad)\sqrt[4]{a+bx^4}} - \frac{d\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{5/4}} - \frac{d\tanh^{-1}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{5/4}}$$

Mathematica [C] time = 0.34564, size = 256, normalized size = 1.91

$$-\frac{36c^2dx^4\left(a+bx^4\right)^2 {}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{(bc-ad)x^4}{c\left(bx^4+a\right)}\right)+45c^3\left(a+bx^4\right)^2 {}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{(bc-ad)x^4}{c\left(bx^4+a\right)}\right)-36c^2dx^4\left(a+bx^4\right)^2-45c^3\left(a+bx^4\right)}{9c^3x^3\left(a+bx^4\right)^{9/4}(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^4)^(5/4)*(c + d*x^4)),x]
```

```
[Out] -(-45*c^3*(a + b*x^4)^2 - 36*c^2*d*x^4*(a + b*x^4)^2 + 45*c^3*(a + b*x^4)^2
*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 36*c^2
*d*x^4*(a + b*x^4)^2*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a
 + b*x^4))] + 4*c*(b*c - a*d)^2*x^8*Hypergeometric2F1[2, 9/4, 13/4, ((b*c -
 a*d)*x^4)/(c*(a + b*x^4))] + 4*d*(b*c - a*d)^2*x^12*Hypergeometric2F1[2, 9
/4, 13/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))])/(9*c^3*(-(b*c) + a*d)*x^3*(a
+ b*x^4)^(9/4))
```

Maple [F] time = 0.411, size = 0, normalized size = 0.

$$\int \frac{1}{dx^4+c}\left(bx^4+a\right)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(5/4)/(d*x^4+c),x)
```

```
[Out] int(1/(b*x^4+a)^(5/4)/(d*x^4+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{5}{4}}\left(dx^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(5/4)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(5/4)*(d*x^4 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(5/4)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^4\right)^{\frac{5}{4}}\left(c+dx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(5/4)/(d*x**4+c),x)
```

```
[Out] Integral(1/((a + b*x**4)**(5/4)*(c + d*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{5}{4}}\left(dx^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(5/4)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(5/4)*(d*x^4 + c)), x)
```

3.197 $\int \frac{1}{\left(a+bx^4\right)^{9/4}\left(c+dx^4\right)}dx$

Optimal. Leaf size=180

$$\frac{bx(4bc-9ad)}{5a^2\sqrt[4]{a+bx^4}(bc-ad)^2}+\frac{d^2\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{9/4}}+\frac{d^2\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{9/4}}+\frac{bx}{5a\left(a+bx^4\right)^{5/4}(bc-ad)}$$

```
[Out] (b*x)/(5*a*(b*c - a*d)*(a + b*x^4)^(5/4)) + (b*(4*b*c - 9*a*d)*x)/(5*a^2*(b
*c - a*d)^2*(a + b*x^4)^(1/4)) + (d^2*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)
*(a + b*x^4)^(1/4))])/(2*c^(3/4)*(b*c - a*d)^(9/4)) + (d^2*ArcTanh[((b*c -
a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*(b*c - a*d)^(9/4))
```

Rubi [A] time = 0.202568, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {414, 527, 12, 377, 212, 208, 205}

$$\frac{bx(4bc-9ad)}{5a^2\sqrt[4]{a+bx^4}(bc-ad)^2}+\frac{d^2\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{9/4}}+\frac{d^2\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{9/4}}+\frac{bx}{5a\left(a+bx^4\right)^{5/4}(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^(9/4)*(c + d*x^4)),x]
```

```
[Out] (b*x)/(5*a*(b*c - a*d)*(a + b*x^4)^(5/4)) + (b*(4*b*c - 9*a*d)*x)/(5*a^2*(b
*c - a*d)^2*(a + b*x^4)^(1/4)) + (d^2*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)
*(a + b*x^4)^(1/4))])/(2*c^(3/4)*(b*c - a*d)^(9/4)) + (d^2*ArcTanh[((b*c -
a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*(b*c - a*d)^(9/4))
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^4\right)^{9/4}\left(c+dx^4\right)}\,dx = \frac{bx}{5a(bc-ad)\left(a+bx^4\right)^{5/4}} - \frac{\int \frac{-4bc+5ad-4bdx^4}{\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)}\,dx}{5a(bc-ad)}$$

$$= \frac{bx}{5a(bc-ad)\left(a+bx^4\right)^{5/4}} + \frac{b(4bc-9ad)x}{5a^2(bc-ad)^2\sqrt[4]{a+bx^4}} + \frac{\int \frac{5a^2d^2}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)}\,dx}{5a^2(bc-ad)^2}$$

$$= \frac{bx}{5a(bc-ad)\left(a+bx^4\right)^{5/4}} + \frac{b(4bc-9ad)x}{5a^2(bc-ad)^2\sqrt[4]{a+bx^4}} + \frac{d^2\int \frac{1}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)}\,dx}{(bc-ad)^2}$$

$$= \frac{bx}{5a(bc-ad)\left(a+bx^4\right)^{5/4}} + \frac{b(4bc-9ad)x}{5a^2(bc-ad)^2\sqrt[4]{a+bx^4}} + \frac{d^2\,\mathrm{Subst}\left(\int \frac{1}{c-(bc-ad)x^4}\,dx, x, \frac{x}{\sqrt[4]{a+b}}\right.}{(bc-ad)^2}$$

$$= \frac{bx}{5a(bc-ad)\left(a+bx^4\right)^{5/4}} + \frac{b(4bc-9ad)x}{5a^2(bc-ad)^2\sqrt[4]{a+bx^4}} + \frac{d^2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{c}-\sqrt{bc-ad}x^2}\,dx, x, \frac{x}{\sqrt[4]{a-}}\right.}{2\sqrt{c}(bc-ad)^2}$$

$$= \frac{bx}{5a(bc-ad)\left(a+bx^4\right)^{5/4}} + \frac{b(4bc-9ad)x}{5a^2(bc-ad)^2\sqrt[4]{a+bx^4}} + \frac{d^2\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{9/4}} + \frac{d^2\tanh^{-1}}{2c^{3/4}(b}$$

Mathematica [C] time = 1.66171, size = 621, normalized size = 3.45

$$\frac{80c^2x^{12}(bc-ad)^3\mathrm{HypergeometricPFQ}\left(\left\{2,2,\frac{13}{4}\right\},\left\{1,\frac{17}{4}\right\},\frac{x^4(bc-ad)}{c\left(a+bx^4\right)}\right) + 80d^2x^{20}(bc-ad)^3\mathrm{HypergeometricPFQ}\left(\left\{\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^4)^(9/4)*(c + d*x^4)),x]
```

```
[Out] (-585*c^4*(b*c - a*d)*x^4*(a + b*x^4)^2 - 936*c^3*d*(b*c - a*d)*x^8*(a + b*
x^4)^2 - 416*c^2*d^2*(b*c - a*d)*x^12*(a + b*x^4)^2 - 2925*c^5*(a + b*x^4)^
3 - 4680*c^4*d*x^4*(a + b*x^4)^3 - 2080*c^3*d^2*x^8*(a + b*x^4)^3 + 2925*c^
5*(a + b*x^4)^3*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*
x^4))] + 4680*c^4*d*x^4*(a + b*x^4)^3*Hypergeometric2F1[1/4, 1, 5/4, ((b*c
- a*d)*x^4)/(c*(a + b*x^4))] + 2080*c^3*d^2*x^8*(a + b*x^4)^3*Hypergeometri
c2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 280*c^2*(b*c - a*d)^
3*x^12*Hypergeometric2F1[2, 13/4, 17/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))]
+ 520*c*d*(b*c - a*d)^3*x^16*Hypergeometric2F1[2, 13/4, 17/4, ((b*c - a*d)*
x^4)/(c*(a + b*x^4))] + 240*d^2*(b*c - a*d)^3*x^20*Hypergeometric2F1[2, 13/
4, 17/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 80*c^2*(b*c - a*d)^3*x^12*Hyp
ergeometricPFQ[{2, 2, 13/4}, {1, 17/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))]
+ 160*c*d*(b*c - a*d)^3*x^16*HypergeometricPFQ[{2, 2, 13/4}, {1, 17/4}, ((b
*c - a*d)*x^4)/(c*(a + b*x^4))] + 80*d^2*(b*c - a*d)^3*x^20*HypergeometricP
FQ[{2, 2, 13/4}, {1, 17/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))])/(325*c^4*(b
*c - a*d)^2*x^7*(a + b*x^4)^(13/4))
```

Maple [F] time = 0.41, size = 0, normalized size = 0.

$$\int \frac{1}{d x^{4}+c}\left(b x^{4}+a\right)^{-\frac{9}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(9/4)/(d*x^4+c),x)
```

```
[Out] int(1/(b*x^4+a)^(9/4)/(d*x^4+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b x^{4}+a\right)^{\frac{9}{4}}\left(d x^{4}+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(9/4)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(9/4)*(d*x^4 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(9/4)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^4\right)^{\frac{9}{4}}\left(c+dx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(9/4)/(d*x**4+c),x)
```

```
[Out] Integral(1/((a + b*x**4)**(9/4)*(c + d*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{9}{4}}\left(dx^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(9/4)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(9/4)*(d*x^4 + c)), x)
```

3.198 $\int \frac{1}{\left(a+bx^4\right)^{13/4}\left(c+dx^4\right)}\,dx$

Optimal. Leaf size=233

$$\frac{bx\left(113a^2d^2-100abcd+32b^2c^2\right)}{45a^3\sqrt[4]{a+bx^4}(bc-ad)^3}+\frac{bx(8bc-17ad)}{45a^2\left(a+bx^4\right)^{5/4}(bc-ad)^2}-\frac{d^3\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{13/4}}-\frac{d^3\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{13/4}}+\frac{}{9a\left(a\right.}$$

```
[Out] (b*x)/(9*a*(b*c - a*d)*(a + b*x^4)^(9/4)) + (b*(8*b*c - 17*a*d)*x)/(45*a^2*
(b*c - a*d)^2*(a + b*x^4)^(5/4)) + (b*(32*b^2*c^2 - 100*a*b*c*d + 113*a^2*d
^2)*x)/(45*a^3*(b*c - a*d)^3*(a + b*x^4)^(1/4)) - (d^3*ArcTan[((b*c - a*d)^
(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*(b*c - a*d)^(13/4)) - (d^
3*ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*(b
*c - a*d)^(13/4))
```

Rubi [A] time = 0.294959, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {414, 527, 12, 377, 212, 208, 205}

$$\frac{bx\left(113a^2d^2-100abcd+32b^2c^2\right)}{45a^3\sqrt[4]{a+bx^4}(bc-ad)^3}+\frac{bx(8bc-17ad)}{45a^2\left(a+bx^4\right)^{5/4}(bc-ad)^2}-\frac{d^3\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{13/4}}-\frac{d^3\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{13/4}}+\frac{}{9a\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^(13/4)*(c + d*x^4)),x]
```

```
[Out] (b*x)/(9*a*(b*c - a*d)*(a + b*x^4)^(9/4)) + (b*(8*b*c - 17*a*d)*x)/(45*a^2*
(b*c - a*d)^2*(a + b*x^4)^(5/4)) + (b*(32*b^2*c^2 - 100*a*b*c*d + 113*a^2*d
^2)*x)/(45*a^3*(b*c - a*d)^3*(a + b*x^4)^(1/4)) - (d^3*ArcTan[((b*c - a*d)^
(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*(b*c - a*d)^(13/4)) - (d^
3*ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(2*c^(3/4)*(b
*c - a*d)^(13/4))
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^4\right)^{13/4}\left(c+dx^4\right)}\,dx = \frac{bx}{9a(bc-ad)\left(a+bx^4\right)^{9/4}} - \frac{\int \frac{-8bc+9ad-8bdx^4}{\left(a+bx^4\right)^{9/4}\left(c+dx^4\right)}\,dx}{9a(bc-ad)}$$

$$= \frac{bx}{9a(bc-ad)\left(a+bx^4\right)^{9/4}} + \frac{b(8bc-17ad)x}{45a^2(bc-ad)^2\left(a+bx^4\right)^{5/4}} + \frac{\int \frac{32b^2c^2-68abcd+45a^2d^2+4bd(8bc-\cdots}{\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)}\cdots}{45a^2(bc-ad)^2}$$

$$= \frac{bx}{9a(bc-ad)\left(a+bx^4\right)^{9/4}} + \frac{b(8bc-17ad)x}{45a^2(bc-ad)^2\left(a+bx^4\right)^{5/4}} + \frac{b\left(32b^2c^2-100abcd+113a^2\cdots\right.}{45a^3(bc-ad)^3\sqrt[4]{a+bx^4}\cdots}$$

$$= \frac{bx}{9a(bc-ad)\left(a+bx^4\right)^{9/4}} + \frac{b(8bc-17ad)x}{45a^2(bc-ad)^2\left(a+bx^4\right)^{5/4}} + \frac{b\left(32b^2c^2-100abcd+113a^2\cdots\right.}{45a^3(bc-ad)^3\sqrt[4]{a+bx^4}\cdots}$$

$$= \frac{bx}{9a(bc-ad)\left(a+bx^4\right)^{9/4}} + \frac{b(8bc-17ad)x}{45a^2(bc-ad)^2\left(a+bx^4\right)^{5/4}} + \frac{b\left(32b^2c^2-100abcd+113a^2\cdots\right.}{45a^3(bc-ad)^3\sqrt[4]{a+bx^4}\cdots}$$

$$= \frac{bx}{9a(bc-ad)\left(a+bx^4\right)^{9/4}} + \frac{b(8bc-17ad)x}{45a^2(bc-ad)^2\left(a+bx^4\right)^{5/4}} + \frac{b\left(32b^2c^2-100abcd+113a^2\cdots\right.}{45a^3(bc-ad)^3\sqrt[4]{a+bx^4}\cdots}$$

$$= \frac{bx}{9a(bc-ad)\left(a+bx^4\right)^{9/4}} + \frac{b(8bc-17ad)x}{45a^2(bc-ad)^2\left(a+bx^4\right)^{5/4}} + \frac{b\left(32b^2c^2-100abcd+113a^2\cdots\right.}{45a^3(bc-ad)^3\sqrt[4]{a+bx^4}\cdots}$$

Mathematica [C] time = 4.36266, size = 1172, normalized size = 5.03

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^4)^(13/4)*(c + d*x^4)),x]
```

```
[Out] -(-16575*c^5*(b*c - a*d)^2*x^8*(a + b*x^4)^2 - 39780*c^4*d*(b*c - a*d)^2*x^
12*(a + b*x^4)^2 - 35360*c^3*d^2*(b*c - a*d)^2*x^16*(a + b*x^4)^2 - 10880*c
^2*d^3*(b*c - a*d)^2*x^20*(a + b*x^4)^2 - 29835*c^6*(b*c - a*d)*x^4*(a + b*
x^4)^3 - 71604*c^5*d*(b*c - a*d)*x^8*(a + b*x^4)^3 - 63648*c^4*d^2*(b*c - a
*d)*x^12*(a + b*x^4)^3 - 19584*c^3*d^3*(b*c - a*d)*x^16*(a + b*x^4)^3 - 149
175*c^7*(a + b*x^4)^4 - 358020*c^6*d*x^4*(a + b*x^4)^4 - 318240*c^5*d^2*x^8
*(a + b*x^4)^4 - 97920*c^4*d^3*x^12*(a + b*x^4)^4 + 149175*c^7*(a + b*x^4)^
4*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 35802
0*c^6*d*x^4*(a + b*x^4)^4*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/
(c*(a + b*x^4))] + 318240*c^5*d^2*x^8*(a + b*x^4)^4*Hypergeometric2F1[1/4,
1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 97920*c^4*d^3*x^12*(a + b*x^4)
^4*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 1362
0*c^3*(b*c - a*d)^4*x^16*Hypergeometric2F1[2, 17/4, 21/4, ((b*c - a*d)*x^4)
/(c*(a + b*x^4))] + 36900*c^2*d*(b*c - a*d)^4*x^20*Hypergeometric2F1[2, 17/
4, 21/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 33840*c*d^2*(b*c - a*d)^4*x^2
4*Hypergeometric2F1[2, 17/4, 21/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 105
60*d^3*(b*c - a*d)^4*x^28*Hypergeometric2F1[2, 17/4, 21/4, ((b*c - a*d)*x^4
)/(c*(a + b*x^4))] + 6480*c^3*(b*c - a*d)^4*x^16*HypergeometricPFQ[{2, 2, 1
7/4}, {1, 21/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 18720*c^2*d*(b*c - a*
d)^4*x^20*HypergeometricPFQ[{2, 2, 17/4}, {1, 21/4}, ((b*c - a*d)*x^4)/(c*(
a + b*x^4))] + 18000*c*d^2*(b*c - a*d)^4*x^24*HypergeometricPFQ[{2, 2, 17/4
}, {1, 21/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 5760*d^3*(b*c - a*d)^4*x
^28*HypergeometricPFQ[{2, 2, 17/4}, {1, 21/4}, ((b*c - a*d)*x^4)/(c*(a + b*
x^4))] + 960*c^3*(b*c - a*d)^4*x^16*HypergeometricPFQ[{2, 2, 2, 17/4}, {1,
1, 21/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 2880*c^2*d*(b*c - a*d)^4*x^2
0*HypergeometricPFQ[{2, 2, 2, 17/4}, {1, 1, 21/4}, ((b*c - a*d)*x^4)/(c*(a
+ b*x^4))] + 2880*c*d^2*(b*c - a*d)^4*x^24*HypergeometricPFQ[{2, 2, 2, 17/4
}, {1, 1, 21/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 960*d^3*(b*c - a*d)^4
*x^28*HypergeometricPFQ[{2, 2, 2, 17/4}, {1, 1, 21/4}, ((b*c - a*d)*x^4)/(c
*(a + b*x^4))])/(11475*c^5*(-(b*c) + a*d)^3*x^11*(a + b*x^4)^(17/4))
```

Maple [F] time = 0.425, size = 0, normalized size = 0.

$$\int \frac{1}{dx^4+c}\left(bx^4+a\right)^{-\frac{13}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(13/4)/(d*x^4+c),x)
```

```
[Out] int(1/(b*x^4+a)^(13/4)/(d*x^4+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{13}{4}}\left(dx^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(13/4)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(13/4)*(d*x^4 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(13/4)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(13/4)/(d*x**4+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^4 + a\right)^{\frac{13}{4}} \left(dx^4 + c\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(13/4)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(13/4)*(d*x^4 + c)), x)
```

3.199 $\int \frac{\left(a+bx^4\right)^{9/4}}{c+dx^4}\,dx$

Optimal. Leaf size=316

$$\frac{\sqrt{a}b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(6bc-11ad)\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{12d^2\left(a+bx^4\right)^{3/4}}-\frac{bx\sqrt[4]{a+bx^4}(6bc-11ad)}{12d^2}+\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)^2\text{I}}{2}$$

```
[Out] -(b*(6*b*c - 11*a*d)*x*(a + b*x^4)^(1/4))/(12*d^2) + (b*x*(a + b*x^4)^(5/4)
)/(6*d) + (Sqrt[a]*b^(3/2)*(6*b*c - 11*a*d)*(1 + a/(b*x^4))^(3/4)*x^3*Ellip
ticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(12*d^2*(a + b*x^4)^(3/4)) + ((b*
c - a*d)^2*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]
/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)
*c*d^2) + ((b*c - a*d)^2*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqr
t[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])
/(2*b^(1/4)*c*d^2)
```

Rubi [A] time = 0.338013, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {416, 528, 529, 237, 335, 275, 231, 407, 409, 1218}

$$\frac{\sqrt{a}b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(6bc-11ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{12d^2\left(a+bx^4\right)^{3/4}}-\frac{bx\sqrt[4]{a+bx^4}(6bc-11ad)}{12d^2}+\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)^2\Pi\left(-\frac{\sqrt{bc-}}{\sqrt{b}}\right.}{2\sqrt[4]{b}cd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(9/4)/(c + d*x^4),x]
```

```
[Out] -(b*(6*b*c - 11*a*d)*x*(a + b*x^4)^(1/4))/(12*d^2) + (b*x*(a + b*x^4)^(5/4)
)/(6*d) + (Sqrt[a]*b^(3/2)*(6*b*c - 11*a*d)*(1 + a/(b*x^4))^(3/4)*x^3*Ellip
ticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(12*d^2*(a + b*x^4)^(3/4)) + ((b*
c - a*d)^2*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]
/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)
*c*d^2) + ((b*c - a*d)^2*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqr
t[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])
/(2*b^(1/4)*c*d^2)
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
```

```
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 529

```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^4)^(3/4), x],
 x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(a + b*x^4)^(1/4)/(c + d*x^4), x],
x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 407

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c -
 a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^4\right)^{9/4}}{c+dx^4}\,dx = \frac{bx\left(a+bx^4\right)^{5/4}}{6d} + \frac{\int \frac{\sqrt[4]{a+bx^4}\left(-a(bc-6ad)-b(6bc-11ad)x^4\right)}{c+dx^4}dx}{6d}$$

$$= -\frac{b(6bc-11ad)x\sqrt[4]{a+bx^4}}{12d^2} + \frac{bx\left(a+bx^4\right)^{5/4}}{6d} + \frac{\int \frac{a\left(6b^2c^2-13abcd+12a^2d^2\right)+b\left(12b^2c^2-30abcd+23a^2d^2\right)x^4}{\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)}dx}{12d^2}$$

$$= -\frac{b(6bc-11ad)x\sqrt[4]{a+bx^4}}{12d^2} + \frac{bx\left(a+bx^4\right)^{5/4}}{6d} - \frac{(ab(6bc-11ad))\int \frac{1}{\left(a+bx^4\right)^{3/4}}dx}{12d^2} + \frac{(bc-ad)^2\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}}{d^2}$$

$$= -\frac{b(6bc-11ad)x\sqrt[4]{a+bx^4}}{12d^2} + \frac{bx\left(a+bx^4\right)^{5/4}}{6d} - \frac{\left(ab(6bc-11ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}dx}{12d^2\left(a+bx^4\right)^{3/4}} + \frac{(}{}$$

$$= -\frac{b(6bc-11ad)x\sqrt[4]{a+bx^4}}{12d^2} + \frac{bx\left(a+bx^4\right)^{5/4}}{6d} + \frac{\left(ab(6bc-11ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{3/4}}d\right.}{12d^2\left(a+bx^4\right)^{3/4}}$$

$$= -\frac{b(6bc-11ad)x\sqrt[4]{a+bx^4}}{12d^2} + \frac{bx\left(a+bx^4\right)^{5/4}}{6d} + \frac{(bc-ad)^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right.\right.}{2\sqrt[4]{b}cd^2}$$

$$= -\frac{b(6bc-11ad)x\sqrt[4]{a+bx^4}}{12d^2} + \frac{bx\left(a+bx^4\right)^{5/4}}{6d} + \frac{\sqrt{a}b^{3/2}(6bc-11ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\right|}{12d^2\left(a+bx^4\right)^{3/4}}$$

Mathematica [C] time = 0.659961, size = 294, normalized size = 0.93

$$\frac{x\left(\frac{bx^4\left(\frac{bx^4}{a}+1\right)^{3/4}\left(23a^2d^2-30abcd+12b^2c^2\right)F_1\left(\frac{5}{4};\frac{3}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c} - \frac{25a^2c\left(12a^2d^2-13abcd+6b^2c^2\right)F_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{\left(c+dx^4\right)\left(x^4\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)-5acF_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};\right.\right.}\right.}{60d^2\left(a+bx^4\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^4)^(9/4)/(c + d*x^4),x]
```

```
[Out] (x*(5*b*(a + b*x^4)*(-6*b*c + 13*a*d + 2*b*d*x^4) + (b*(12*b^2*c^2 - 30*a*b
*c*d + 23*a^2*d^2)*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4, 1, 9/4, -((
b*x^4)/a), -((d*x^4)/c)])/c - (25*a^2*c*(6*b^2*c^2 - 13*a*b*c*d + 12*a^2*d^
2)*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/((c + d*x^4)*(-5
*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + x^4*(4*a*d*Ap
pellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4,
7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))))/(60*d^2*(a + b*x^4)^(3/4))
```

Maple [F] time = 0.431, size = 0, normalized size = 0.

$$\int \frac{1}{dx^4+c}\left(bx^4+a\right)^{\frac{9}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(9/4)/(d*x^4+c),x)
```

```
[Out] int((b*x^4+a)^(9/4)/(d*x^4+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{9}{4}}}{dx^4 + c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(9/4)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(9/4)/(d*x^4 + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(9/4)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(9/4)/(d*x**4+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{9}{4}}}{dx^4 + c} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(9/4)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(9/4)/(d*x^4 + c), x)
```

3.200 $\int \frac{(a+bx^4)^{5/4}}{c+dx^4}\,dx$

Optimal. Leaf size=274

$$-\frac{\sqrt{a}b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{2d\left(a+bx^4\right)^{3/4}}-\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd}-\frac{\sqrt{\frac{a}{a+\ldots}}}{}$$

```
[Out] (b*x*(a + b*x^4)^(1/4))/(2*d) - (Sqrt[a]*b^(3/2)*(1 + a/(b*x^4))^(3/4)*x^3*
EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(2*d*(a + b*x^4)^(3/4)) - ((
b*c - a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]
/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)
*c*d) - ((b*c - a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*
c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*
b^(1/4)*c*d)
```

Rubi [A] time = 0.155669, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {408, 195, 237, 335, 275, 231, 407, 409, 1218}

$$-\frac{\sqrt{a}b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{2d\left(a+bx^4\right)^{3/4}}-\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd}-\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+\ldots}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(5/4)/(c + d*x^4),x]
```

```
[Out] (b*x*(a + b*x^4)^(1/4))/(2*d) - (Sqrt[a]*b^(3/2)*(1 + a/(b*x^4))^(3/4)*x^3*
EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(2*d*(a + b*x^4)^(3/4)) - ((
b*c - a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]
/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)
*c*d) - ((b*c - a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*
c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*
b^(1/4)*c*d)
```

Rule 408

```
Int[((a_) + (b_.)*(x_)^4)^(p_)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[b/d
, Int[(a + b*x^4)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^4)^(p
- 1)/(c + d*x^4), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
(EqQ[p, 3/4] || EqQ[p, 5/4])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
```

```
{a, b}, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 407

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c -
 a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^4\right)^{5/4}}{c+dx^4}\,dx = \frac{b\int \sqrt[4]{a+bx^4}\,dx}{d} - \frac{(bc-ad)\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx}{d}$$

$$= \frac{bx\sqrt[4]{a+bx^4}}{2d} + \frac{(ab)\int \frac{1}{\left(a+bx^4\right)^{3/4}}\,dx}{2d} - \frac{\left((bc-ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-bx^4}\left(c-(bc-ad)x^4\right)}\,dx, x, \right.}{d}$$

$$= \frac{bx\sqrt[4]{a+bx^4}}{2d} + \frac{\left(ab\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{2d\left(a+bx^4\right)^{3/4}} - \frac{\left((bc-ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\left(\int \frac{}{\left(1-\frac{\sqrt{bc-ad}}{\sqrt{c}}\right.}\right.}{2cd}$$

$$= \frac{bx\sqrt[4]{a+bx^4}}{2d} - \frac{(bc-ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd} - \frac{(bc-ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+b}}{}$$

$$= \frac{bx\sqrt[4]{a+bx^4}}{2d} - \frac{(bc-ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{b}cd} - \frac{(bc-ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+b}}{}$$

$$= \frac{bx\sqrt[4]{a+bx^4}}{2d} - \frac{\sqrt{a}b^{3/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{2d\left(a+bx^4\right)^{3/4}} - \frac{(bc-ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\right.}{2\sqrt[4]{b}cd}$$

Mathematica [C] time = 0.321675, size = 346, normalized size = 1.26

$$\frac{x\left(\frac{5\left(bx^4\left(a+bx^4\right)\left(c+dx^4\right)\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)-5ac\left(2a^2d+abdx^4+b^2x^4\left(c+dx^4\right)\right)F_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)}{\left(c+dx^4\right)\left(x^4\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)-5acF_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)}+\frac{bx^4\left(\frac{bx^4}{a}+1\right)}{}\right.}{10d\left(a+bx^4\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^4)^(5/4)/(c + d*x^4),x]
```

```
[Out] (x*((b*(-2*b*c + 3*a*d)*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4, 1, 9/4
, -((b*x^4)/a), -((d*x^4)/c)])/c + (5*(-5*a*c*(2*a^2*d + a*b*d*x^4 + b^2*x^
4*(c + d*x^4))*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + b*x
^4*(a + b*x^4)*(c + d*x^4)*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a),
-((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)
])))/((c + d*x^4)*(-5*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4
)/c)] + x^4*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] +
 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))))/(10*d*(a
 + b*x^4)^(3/4))
```

Maple [F] time = 0.416, size = 0, normalized size = 0.

$$\int \frac{1}{dx^4+c}\left(bx^4+a\right)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(5/4)/(d*x^4+c),x)
```

```
[Out] int((b*x^4+a)^(5/4)/(d*x^4+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4+a)^{\frac{5}{4}}}{dx^4+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(5/4)/(d*x^4 + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx^4)^{\frac{5}{4}}}{c+dx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(5/4)/(d*x**4+c),x)
```

```
[Out] Integral((a + b*x**4)**(5/4)/(c + d*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4+a)^{\frac{5}{4}}}{dx^4+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(5/4)/(d*x^4 + c), x)
```

3.201 $\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx$

Optimal. Leaf size=166

$$\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c}+\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c}$$

```
[Out] (Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*
Sqrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)*c) + (Sqr
t[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c
]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)*c)
```

Rubi [A] time = 0.0885933, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {407, 409, 1218}

$$\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c}+\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(1/4)/(c + d*x^4),x]
```

```
[Out] (Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*
Sqrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)*c) + (Sqr
t[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c
]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)*c)
```

Rule 407

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c -
 a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx = \left(\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-bx^4}\left(c-(bc-ad)x^4\right)}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)$$

$$= \frac{\left(\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{bc-ad}x^2}{\sqrt{c}}\right)\sqrt{1-bx^4}}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2c} + \frac{\left(\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\left(\int \frac{1}{\left(1+\frac{\sqrt{bc}}{}\right.}\right.}{2c}$$

$$= \frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{b}c} + \frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-\right.}{2\sqrt[4]{b}c}$$

Mathematica [C] time = 0.153133, size = 160, normalized size = 0.96

$$\frac{5acx\sqrt[4]{a+bx^4}F_1\left(\frac{1}{4};-\frac{1}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{\left(c+dx^4\right)\left(x^4\left(bcF_1\left(\frac{5}{4};\frac{3}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)-4adF_1\left(\frac{5}{4};-\frac{1}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)+5acF_1\left(\frac{1}{4};-\frac{1}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^4)^(1/4)/(c + d*x^4),x]
```

```
[Out] (5*a*c*x*(a + b*x^4)^(1/4)*AppellF1[1/4, -1/4, 1, 5/4, -((b*x^4)/a), -((d*x
^4)/c)])/((c + d*x^4)*(5*a*c*AppellF1[1/4, -1/4, 1, 5/4, -((b*x^4)/a), -((d
*x^4)/c)] + x^4*(-4*a*d*AppellF1[5/4, -1/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)
/c)] + b*c*AppellF1[5/4, 3/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))
```

Maple [F] time = 0.407, size = 0, normalized size = 0.

$$\int \frac{1}{dx^4+c}\sqrt[4]{bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(1/4)/(d*x^4+c),x)
```

```
[Out] int((b*x^4+a)^(1/4)/(d*x^4+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)^{\frac{1}{4}}}{dx^4+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(1/4)/(d*x^4 + c), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(1/4)/(d*x**4+c),x)
```

```
[Out] Integral((a + b*x**4)**(1/4)/(c + d*x**4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)^{\frac{1}{4}}}{dx^4+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(1/4)/(d*x^4 + c), x)
```

3.202 $\int \frac{1}{\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)}\,dx$

Optimal. Leaf size=259

$$-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{\sqrt{a}\left(a+bx^4\right)^{3/4}(bc-ad)}-\frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c(bc-ad)}-\frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a\cdots}}{\cdots}$$

```
[Out] -((b^(3/2)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]
]/2, 2])/(Sqrt[a]*(b*c - a*d)*(a + b*x^4)^(3/4))) - (d*Sqrt[a/(a + b*x^4)]*
Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[(b^
(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)*c*(b*c - a*d)) - (d*Sqrt[a/(a
+ b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), Arc
Sin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)*c*(b*c - a*d))
```

Rubi [A] time = 0.154164, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {410, 237, 335, 275, 231, 407, 409, 1218}

$$-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}\left(a+bx^4\right)^{3/4}(bc-ad)}-\frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c(bc-ad)}-\frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\cdots}{2\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^(3/4)*(c + d*x^4)),x]
```

```
[Out] -((b^(3/2)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]
]/2, 2])/(Sqrt[a]*(b*c - a*d)*(a + b*x^4)^(3/4))) - (d*Sqrt[a/(a + b*x^4)]*
Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[(b^
(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)*c*(b*c - a*d)) - (d*Sqrt[a/(a
+ b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), Arc
Sin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)*c*(b*c - a*d))
```

Rule 410

```
Int[1/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^4)^(3/4), x], x] - Dist[d/(b*c - a*d), Int[
(a + b*x^4)^(1/4)/(c + d*x^4), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
- a*d, 0]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 407

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c -
 a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)}\,dx = \frac{b\int \frac{1}{\left(a+bx^4\right)^{3/4}}\,dx}{bc-ad} - \frac{d\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx}{bc-ad}$$

$$= \frac{\left(b\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{(bc-ad)\left(a+bx^4\right)^{3/4}} - \frac{\left(d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-bx^4}\left(c-(bc-ad)x^4\right)}\,dx, x \cdots\right.}{bc-ad}$$

$$= -\frac{\left(b\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{(bc-ad)\left(a+bx^4\right)^{3/4}} - \frac{\left(d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\left(\int \frac{1}{\left(1-\frac{\sqrt{bc-ad}\cdots}{\sqrt{c}}\right.\cdots}\cdots\right.}{2c(bc-ad)}$$

$$= -\frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{b}c(bc-ad)} - \frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\cdots\right.}{2\sqrt[4]{b}c(bc-ad)}$$

$$= -\frac{b^{3/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{a}(bc-ad)\left(a+bx^4\right)^{3/4}} - \frac{d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\right|\cdots}{2\sqrt[4]{b}c(bc-ad)}$$

Mathematica [C] time = 0.0323437, size = 161, normalized size = 0.62

$$-\frac{5acxF_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)\left(x^4\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)-5acF_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^4)^(3/4)*(c + d*x^4)),x]
```

```
[Out] (-5*a*c*x*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/((a + b*x
^4)^(3/4)*(c + d*x^4)*(-5*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d
*x^4)/c)] + x^4*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c
)] + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))
```

Maple [F] time = 0.409, size = 0, normalized size = 0.

$$\int \frac{1}{dx^4+c}\left(bx^4+a\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(3/4)/(d*x^4+c),x)
```

```
[Out] int(1/(b*x^4+a)^(3/4)/(d*x^4+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{3}{4}}\left(dx^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(3/4)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(3/4)*(d*x^4 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(3/4)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^4\right)^{\frac{3}{4}}\left(c+dx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(3/4)/(d*x**4+c),x)
```

```
[Out] Integral(1/((a + b*x**4)**(3/4)*(c + d*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{3}{4}}\left(dx^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(3/4)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(3/4)*(d*x^4 + c)), x)
```

3.203 $\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)}\,dx$

Optimal. Leaf size=304

$$-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(2bc-5ad)\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{3a^{3/2}\left(a+bx^4\right)^{3/4}(bc-ad)^2}+\frac{d^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c(bc-ad)^2}+$$

```
[Out] (b*x)/(3*a*(b*c - a*d)*(a + b*x^4)^(3/4)) - (b^(3/2)*(2*b*c - 5*a*d)*(1 + a
/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(3*a^(3/
2)*(b*c - a*d)^2*(a + b*x^4)^(3/4)) + (d^2*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x
^4]*EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a
+ b*x^4)^(1/4)], -1])/(2*b^(1/4)*c*(b*c - a*d)^2) + (d^2*Sqrt[a/(a + b*x^4)
]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(
1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)*c*(b*c - a*d)^2)
```

Rubi [A] time = 0.238478, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {414, 529, 237, 335, 275, 231, 407, 409, 1218}

$$-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(2bc-5ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{3a^{3/2}\left(a+bx^4\right)^{3/4}(bc-ad)^2}+\frac{d^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{2\sqrt[4]{b}c(bc-ad)^2}+\frac{d^2\sqrt{\frac{a}{a+}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^(7/4)*(c + d*x^4)),x]
```



```
[Out] (b*x)/(3*a*(b*c - a*d)*(a + b*x^4)^(3/4)) - (b^(3/2)*(2*b*c - 5*a*d)*(1 + a
/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(3*a^(3/
2)*(b*c - a*d)^2*(a + b*x^4)^(3/4)) + (d^2*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x
^4]*EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a
+ b*x^4)^(1/4)], -1])/(2*b^(1/4)*c*(b*c - a*d)^2) + (d^2*Sqrt[a/(a + b*x^4)
]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(
1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)*c*(b*c - a*d)^2)
```


Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 529

```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^4)^(3/4), x],
 x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(a + b*x^4)^(1/4)/(c + d*x^4), x],
x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 407

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c -
 a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^4\right)^{7/4}\left(c+dx^4\right)}\,dx = \frac{bx}{3a(bc-ad)\left(a+bx^4\right)^{3/4}} - \frac{\int \frac{-2bc+3ad-2bdx^4}{\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)}\,dx}{3a(bc-ad)}$$

$$= \frac{bx}{3a(bc-ad)\left(a+bx^4\right)^{3/4}} + \frac{d^2\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx}{(bc-ad)^2} + \frac{(b(2bc-5ad))\int \frac{1}{\left(a+bx^4\right)^{3/4}}\,dx}{3a(bc-ad)^2}$$

$$= \frac{bx}{3a(bc-ad)\left(a+bx^4\right)^{3/4}} + \frac{\left(b(2bc-5ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{3a(bc-ad)^2\left(a+bx^4\right)^{3/4}} + \frac{\left(d^2\sqrt{\frac{a}{a+bx^4}}\right.}{}$$

$$= \frac{bx}{3a(bc-ad)\left(a+bx^4\right)^{3/4}} - \frac{\left(b(2bc-5ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\operatorname{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{3a(bc-ad)^2\left(a+bx^4\right)^{3/4}} +$$

$$= \frac{bx}{3a(bc-ad)\left(a+bx^4\right)^{3/4}} + \frac{d^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{2\sqrt[4]{b}c(bc-ad)^2} + \frac{d^2\sqrt{}}{}$$

$$= \frac{bx}{3a(bc-ad)\left(a+bx^4\right)^{3/4}} - \frac{b^{3/2}(2bc-5ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{3a^{3/2}(bc-ad)^2\left(a+bx^4\right)^{3/4}} + \frac{d^2\sqrt{\frac{}{a}}}{}$$

Mathematica [C] time = 0.255581, size = 332, normalized size = 1.09

$$\frac{x\left(\frac{5\left(bx^4\left(c+dx^4\right)\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)+5ac\left(3ad-b\left(3c+dx^4\right)\right)F_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)}{\left(c+dx^4\right)\left(5acF_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)-x^4\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)\right)} - \frac{2bdx^4\left(\frac{bx^4}{a}+1\right)^{3/4}F_1\left(\frac{5}{4};\frac{3}{4},1;\right.}{c}\right.}{15a\left(a+bx^4\right)^{3/4}(ad-bc)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^4)^(7/4)*(c + d*x^4)),x]
```

```
[Out] (x*((-2*b*d*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4, 1, 9/4, -((b*x^4)/
a), -((d*x^4)/c)])/c + (5*(5*a*c*(3*a*d - b*(3*c + d*x^4))*AppellF1[1/4, 3/
4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + b*x^4*(c + d*x^4)*(4*a*d*AppellF1[
5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1,
 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/((c + d*x^4)*(5*a*c*AppellF1[1/4, 3/4,
 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] - x^4*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4
, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/
a), -((d*x^4)/c)])))))/(15*a*(-(b*c) + a*d)*(a + b*x^4)^(3/4))
```

Maple [F] time = 0.403, size = 0, normalized size = 0.

$$\int \frac{1}{dx^4+c}\left(bx^4+a\right)^{-\frac{7}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(7/4)/(d*x^4+c),x)
```

```
[Out] int(1/(b*x^4+a)^(7/4)/(d*x^4+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{7}{4}}\left(dx^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(7/4)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(7/4)*(d*x^4 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(7/4)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^4\right)^{\frac{7}{4}}\left(c+dx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(7/4)/(d*x**4+c),x)
```

```
[Out] Integral(1/((a + b*x**4)**(7/4)*(c + d*x**4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{7}{4}}\left(dx^4+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(7/4)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(7/4)*(d*x^4 + c)), x)
```

3.204 $\int \frac{1}{\left(a+bx^4\right)^{11/4}\left(c+dx^4\right)}\,dx$

Optimal. Leaf size=357

$$-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\left(47a^2d^2-38abcd+12b^2c^2\right)\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{21a^{5/2}\left(a+bx^4\right)^{3/4}(bc-ad)^3}+\frac{bx(6bc-13ad)}{21a^2\left(a+bx^4\right)^{3/4}(bc-ad)^2}-\frac{d^3\sqrt{\frac{a}{a+}}}{}$$

```
[Out] (b*x)/(7*a*(b*c - a*d)*(a + b*x^4)^(7/4)) + (b*(6*b*c - 13*a*d)*x)/(21*a^2*
(b*c - a*d)^2*(a + b*x^4)^(3/4)) - (b^(3/2)*(12*b^2*c^2 - 38*a*b*c*d + 47*a
^2*d^2)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2
, 2])/(21*a^(5/2)*(b*c - a*d)^3*(a + b*x^4)^(3/4)) - (d^3*Sqrt[a/(a + b*x^4
)]*Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[
(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)*c*(b*c - a*d)^3) - (d^3*Sqr
t[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c
]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)*c*(b*c - a*d)^3)
```

Rubi [A] time = 0.400936, antiderivative size = 357, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {414, 527, 529, 237, 335, 275, 231, 407, 409, 1218}

$$-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\left(47a^2d^2-38abcd+12b^2c^2\right)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{21a^{5/2}\left(a+bx^4\right)^{3/4}(bc-ad)^3}+\frac{bx(6bc-13ad)}{21a^2\left(a+bx^4\right)^{3/4}(bc-ad)^2}-\frac{d^3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^(11/4)*(c + d*x^4)),x]
```

```
[Out] (b*x)/(7*a*(b*c - a*d)*(a + b*x^4)^(7/4)) + (b*(6*b*c - 13*a*d)*x)/(21*a^2*
(b*c - a*d)^2*(a + b*x^4)^(3/4)) - (b^(3/2)*(12*b^2*c^2 - 38*a*b*c*d + 47*a
^2*d^2)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2
, 2])/(21*a^(5/2)*(b*c - a*d)^3*(a + b*x^4)^(3/4)) - (d^3*Sqrt[a/(a + b*x^4
)]*Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[
(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)*c*(b*c - a*d)^3) - (d^3*Sqr
t[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c
]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(2*b^(1/4)*c*(b*c - a*d)^3)
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
```

```
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 529

```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^4)^(3/4), x],
 x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(a + b*x^4)^(1/4)/(c + d*x^4), x],
x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 407

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c -
 a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx^4)^{11/4}(c+dx^4)}\,dx = \frac{bx}{7a(bc-ad)(a+bx^4)^{7/4}} - \frac{\int \frac{-6bc+7ad-6bdx^4}{(a+bx^4)^{7/4}(c+dx^4)}\,dx}{7a(bc-ad)}$$

$$= \frac{bx}{7a(bc-ad)(a+bx^4)^{7/4}} + \frac{b(6bc-13ad)x}{21a^2(bc-ad)^2(a+bx^4)^{3/4}} + \frac{\int \frac{12b^2c^2-26abcd+21a^2d^2+2bd(6bc-1}{(a+bx^4)^{3/4}(c+dx^4)}}{21a^2(bc-ad)^2}$$

$$= \frac{bx}{7a(bc-ad)(a+bx^4)^{7/4}} + \frac{b(6bc-13ad)x}{21a^2(bc-ad)^2(a+bx^4)^{3/4}} - \frac{d^3 \int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx}{(bc-ad)^3} + \frac{\left(b\left(12b^2c^2\right.\right.}{}$$

$$= \frac{bx}{7a(bc-ad)(a+bx^4)^{7/4}} + \frac{b(6bc-13ad)x}{21a^2(bc-ad)^2(a+bx^4)^{3/4}} + \frac{\left(b\left(12b^2c^2-38abcd+47a^2d\right.\right.}{21a^2(bc-a}$$

$$= \frac{bx}{7a(bc-ad)(a+bx^4)^{7/4}} + \frac{b(6bc-13ad)x}{21a^2(bc-ad)^2(a+bx^4)^{3/4}} - \frac{\left(b\left(12b^2c^2-38abcd+47a^2d\right.\right.}{21a^2(}$$

$$= \frac{bx}{7a(bc-ad)(a+bx^4)^{7/4}} + \frac{b(6bc-13ad)x}{21a^2(bc-ad)^2(a+bx^4)^{3/4}} - \frac{d^3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-a}}{\sqrt{b}\sqrt{c}}\right.}{2\sqrt[4]{b}c(bc-}$$

$$= \frac{bx}{7a(bc-ad)(a+bx^4)^{7/4}} + \frac{b(6bc-13ad)x}{21a^2(bc-ad)^2(a+bx^4)^{3/4}} - \frac{b^{3/2}\left(12b^2c^2-38abcd+47a^2\right.}{21a^{5/2}(bc}$$

Mathematica [C] time = 0.724857, size = 430, normalized size = 1.2

$$\frac{x\left(-\frac{5\left(5ac\left(a^2bd\left(5dx^4-42c\right)+21a^3d^2+ab^2\left(21c^2-30cdx^4-13d^2x^8\right)+6b^3cx^4\left(3c+dx^4\right)\right)F_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+bx^4\left(c+dx^4\right)\left(16a^2d+ab\left(13dx^4-9c\right)-6b^2cx^4\right)\left(4a\right.\right.}{\left(a+bx^4\right)\left(c+dx^4\right)\left(x^4\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)-5acF_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a}\right.\right.}\right.}{105a^2\left(a+bx^4\right)^{3/4}(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^4)^(11/4)*(c + d*x^4)),x]
```

```
[Out] (x*((-2*b*d*(-6*b*c + 13*a*d)*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4,
1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])/c - (5*(5*a*c*(21*a^3*d^2 + 6*b^3*c*x^
4*(3*c + d*x^4) + a^2*b*d*(-42*c + 5*d*x^4) + a*b^2*(21*c^2 - 30*c*d*x^4 -
13*d^2*x^8))*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + b*x^4
*(c + d*x^4)*(16*a^2*d - 6*b^2*c*x^4 + a*b*(-9*c + 13*d*x^4))*(4*a*d*Appell
F1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4,
 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/((a + b*x^4)*(c + d*x^4)*(-5*a*c*Ap
pellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + x^4*(4*a*d*AppellF1[
5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1,
 9/4, -((b*x^4)/a), -((d*x^4)/c)])))))/(105*a^2*(b*c - a*d)^2*(a + b*x^4)^(
3/4))
```

Maple [F] time = 0.402, size = 0, normalized size = 0.

$$\int \frac{1}{d x^4 + c} \left(b x^4 + a\right)^{-\frac{11}{4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(11/4)/(d*x^4+c),x)
```

```
[Out] int(1/(b*x^4+a)^(11/4)/(d*x^4+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b x^4 + a\right)^{\frac{11}{4}} \left(d x^4 + c\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(11/4)/(d*x^4+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(11/4)*(d*x^4 + c)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(11/4)/(d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(11/4)/(d*x**4+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b x^4 + a\right)^{\frac{11}{4}} \left(d x^4 + c\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(11/4)/(d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(11/4)*(d*x^4 + c)), x)
```

3.205 $\int \frac{(a+bx^4)^{11/4}}{(c+dx^4)^2} dx$

Optimal. Leaf size=280

$$-\frac{b^{7/4}(8bc-11ad)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^3}-\frac{b^{7/4}(8bc-11ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^3}+\frac{(bc-ad)^{7/4}(3ad+8bc)\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^3}+\frac{(bc}{}$$

```
[Out] (b*(2*b*c - a*d)*x*(a + b*x^4)^(3/4))/(4*c*d^2) - ((b*c - a*d)*x*(a + b*x^4
)^(7/4))/(4*c*d*(c + d*x^4)) - (b^(7/4)*(8*b*c - 11*a*d)*ArcTan[(b^(1/4)*x)
/(a + b*x^4)^(1/4)])/(8*d^3) + ((b*c - a*d)^(7/4)*(8*b*c + 3*a*d)*ArcTan[((
b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*d^3) - (b^(7/4
)*(8*b*c - 11*a*d)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(8*d^3) + ((b*c
- a*d)^(7/4)*(8*b*c + 3*a*d)*ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*
x^4)^(1/4))])/(8*c^(7/4)*d^3)
```

Rubi [A] time = 0.358998, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {413, 528, 530, 240, 212, 206, 203, 377, 208, 205}

$$-\frac{b^{7/4}(8bc-11ad)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^3}-\frac{b^{7/4}(8bc-11ad)\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^3}+\frac{(bc-ad)^{7/4}(3ad+8bc)\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^3}+\frac{(bc}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(11/4)/(c + d*x^4)^2,x]
```


```
[Out] (b*(2*b*c - a*d)*x*(a + b*x^4)^(3/4))/(4*c*d^2) - ((b*c - a*d)*x*(a + b*x^4
)^(7/4))/(4*c*d*(c + d*x^4)) - (b^(7/4)*(8*b*c - 11*a*d)*ArcTan[(b^(1/4)*x)
/(a + b*x^4)^(1/4)])/(8*d^3) + ((b*c - a*d)^(7/4)*(8*b*c + 3*a*d)*ArcTan[((
b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*d^3) - (b^(7/4
)*(8*b*c - 11*a*d)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(8*d^3) + ((b*c
- a*d)^(7/4)*(8*b*c + 3*a*d)*ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*
x^4)^(1/4))])/(8*c^(7/4)*d^3)
```


Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^4\right)^{11/4}}{\left(c+dx^4\right)^2}\,dx = -\frac{(bc-ad)x\left(a+bx^4\right)^{7/4}}{4cd\left(c+dx^4\right)} + \frac{\int \frac{\left(a+bx^4\right)^{3/4}\left(a(bc+3ad)+4b(2bc-ad)x^4\right)}{c+dx^4}dx}{4cd}$$

$$= \frac{b(2bc-ad)x\left(a+bx^4\right)^{3/4}}{4cd^2} - \frac{(bc-ad)x\left(a+bx^4\right)^{7/4}}{4cd\left(c+dx^4\right)} + \frac{\int \frac{-4a\left(2b^2c^2-2abcd-3a^2d^2\right)-4b^2c(8bc-11ad)x^4}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)}dx}{16cd^2}$$

$$= \frac{b(2bc-ad)x\left(a+bx^4\right)^{3/4}}{4cd^2} - \frac{(bc-ad)x\left(a+bx^4\right)^{7/4}}{4cd\left(c+dx^4\right)} - \frac{\left(b^2(8bc-11ad)\right)\int \frac{1}{\sqrt[4]{a+bx^4}}dx}{4d^3} + \frac{\left((bc-ad)^2(8\right.}{}$$

$$= \frac{b(2bc-ad)x\left(a+bx^4\right)^{3/4}}{4cd^2} - \frac{(bc-ad)x\left(a+bx^4\right)^{7/4}}{4cd\left(c+dx^4\right)} - \frac{\left(b^2(8bc-11ad)\right)\text{Subst}\left(\int \frac{1}{1-bx^4}dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right.}{4d^3}$$

$$= \frac{b(2bc-ad)x\left(a+bx^4\right)^{3/4}}{4cd^2} - \frac{(bc-ad)x\left(a+bx^4\right)^{7/4}}{4cd\left(c+dx^4\right)} - \frac{\left(b^2(8bc-11ad)\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{b}x^2}dx, x, \frac{x}{\sqrt[4]{a+bx}}\right.}{8d^3}$$

$$= \frac{b(2bc-ad)x\left(a+bx^4\right)^{3/4}}{4cd^2} - \frac{(bc-ad)x\left(a+bx^4\right)^{7/4}}{4cd\left(c+dx^4\right)} - \frac{b^{7/4}(8bc-11ad)\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^3} + \frac{(bc-ad)^{7/}}{}$$

Mathematica [C] time = 0.793946, size = 560, normalized size = 2.

$$\frac{1}{80}\left(\frac{10a^2b\left(-\log\left(\sqrt[4]{c}-\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}\right)+\log\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}+\sqrt[4]{c}\right)+2\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{ax^4+b}}\right)\right)}{c^{3/4}d\sqrt[4]{bc-ad}} + \frac{15a^3\left(-\log\left(\sqrt[4]{c}-\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}\right)+\log\left(\frac{x\sqrt[4]{bc}}{\sqrt[4]{ax^4}}\right.\right.}{c^{7/4}\sqrt[4]{bc-}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^4)^(11/4)/(c + d*x^4)^2,x]
```

```
[Out] ((20*x*(a + b*x^4)^(3/4)*(b^2 + (b*c - a*d)^2/(c*(c + d*x^4))))/d^2 - (32*b
^3*x^5*(1 + (b*x^4)/a)^(1/4)*AppellF1[5/4, 1/4, 1, 9/4, -((b*x^4)/a), -((d*
x^4)/c)])/(d^2*(a + b*x^4)^(1/4)) + (44*a*b^2*x^5*(1 + (b*x^4)/a)^(1/4)*App
ellF1[5/4, 1/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])/(c*d*(a + b*x^4)^(1/4)
) + (15*a^3*(2*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(b + a*x^4)^(1/4))] -
Log[c^(1/4) - ((b*c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)] + Log[c^(1/4) + ((b*
c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)]))/(c^(7/4)*(b*c - a*d)^(1/4)) - (10*a*
b^2*c^(1/4)*(2*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(b + a*x^4)^(1/4))] -
Log[c^(1/4) - ((b*c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)] + Log[c^(1/4) + ((b*
c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)]))/(d^2*(b*c - a*d)^(1/4)) + (10*a^2*b*
(2*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(b + a*x^4)^(1/4))] - Log[c^(1/4)
- ((b*c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)] + Log[c^(1/4) + ((b*c - a*d)^(1/
4)*x)/(b + a*x^4)^(1/4)]))/(c^(3/4)*d*(b*c - a*d)^(1/4)))/80
```

Maple [F] time = 0.245, size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^4+c\right)^2}\left(bx^4+a\right)^{\frac{11}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(11/4)/(d*x^4+c)^2,x)

[Out] int((b*x^4+a)^(11/4)/(d*x^4+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)^{\frac{11}{4}}}{\left(dx^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(11/4)/(d*x^4+c)^2,x, algorithm="maxima")

[Out] integrate((b*x^4 + a)^(11/4)/(d*x^4 + c)^2, x)
```

Fricas [B] time = 87.7392, size = 7871, normalized size = 28.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(11/4)/(d*x^4+c)^2,x, algorithm="fricas")

[Out] 1/16*(4*(c*d^3*x^4 + c^2*d^2)*((4096*b^11*c^11 - 22528*a*b^10*c^10*d + 4646
4*a^2*b^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a^
5*b^6*c^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3*
c^3*d^8 + 891*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d^10 - 81*a^11*d^11)/(c^7*d^12
))^(1/4)*arctan(-(c^2*d^3*x*sqrt(((4096*b^11*c^14*d^6 - 22528*a*b^10*c^13*d
^7 + 46464*a^2*b^9*c^12*d^8 - 37664*a^3*b^8*c^11*d^9 - 5071*a^4*b^7*c^10*d^
10 + 25641*a^5*b^6*c^9*d^11 - 7931*a^6*b^5*c^8*d^12 - 6259*a^7*b^4*c^7*d^13
 + 2739*a^8*b^3*c^6*d^14 + 891*a^9*b^2*c^5*d^15 - 297*a^10*b*c^4*d^16 - 81*
a^11*c^3*d^17)*x^2*sqrt((4096*b^11*c^11 - 22528*a*b^10*c^10*d + 46464*a^2*b
^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a^5*b^6*c
^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3*c^3*d^8
 + 891*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d^10 - 81*a^11*d^11)/(c^7*d^12)) + (2
62144*b^16*c^16 - 2031616*a*b^15*c^15*d + 6451200*a^2*b^14*c^14*d^2 - 10168
320*a^3*b^13*c^13*d^3 + 6467520*a^4*b^12*c^12*d^4 + 3123216*a^5*b^11*c^11*d
^5 - 7258119*a^6*b^10*c^10*d^6 + 2307030*a^7*b^9*c^9*d^7 + 2428965*a^8*b^8*
c^8*d^8 - 1607320*a^9*b^7*c^7*d^9 - 387134*a^10*b^6*c^6*d^10 + 436356*a^11*
b^5*c^5*d^11 + 40770*a^12*b^4*c^4*d^12 - 63720*a^13*b^3*c^3*d^13 - 6075*a^1
4*b^2*c^2*d^14 + 4374*a^15*b*c*d^15 + 729*a^16*d^16)*sqrt(b*x^4 + a))/x^2)*
((4096*b^11*c^11 - 22528*a*b^10*c^10*d + 46464*a^2*b^9*c^9*d^2 - 37664*a^3*
b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a^5*b^6*c^6*d^5 - 7931*a^6*b^5*c
^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3*c^3*d^8 + 891*a^9*b^2*c^2*d^9
- 297*a^10*b*c*d^10 - 81*a^11*d^11)/(c^7*d^12))^(1/4) + (512*b^8*c^10*d^3 -
 1984*a*b^7*c^9*d^4 + 2456*a^2*b^6*c^8*d^5 - 413*a^3*b^5*c^7*d^6 - 1175*a^4
*b^4*c^6*d^7 + 478*a^5*b^3*c^5*d^8 + 234*a^6*b^2*c^4*d^9 - 81*a^7*b*c^3*d^1
0 - 27*a^8*c^2*d^11)*(b*x^4 + a)^(1/4)*((4096*b^11*c^11 - 22528*a*b^10*c^10
*d + 46464*a^2*b^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 +
 25641*a^5*b^6*c^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739
*a^8*b^3*c^3*d^8 + 891*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d^10 - 81*a^11*d^11)/
(c^7*d^12))^(1/4))/((4096*b^11*c^11 - 22528*a*b^10*c^10*d + 46464*a^2*b^9*c
^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a^5*b^6*c^6*d
```

```
^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3*c^3*d^8 + 8
91*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d^10 - 81*a^11*d^11)*x)) + 4*(c*d^3*x^4 +
 c^2*d^2)*((4096*b^11*c^4 - 22528*a*b^10*c^3*d + 46464*a^2*b^9*c^2*d^2 - 42
592*a^3*b^8*c*d^3 + 14641*a^4*b^7*d^4)/d^12)^(1/4)*arctan((d^3*x*sqrt(((409
6*b^11*c^4*d^6 - 22528*a*b^10*c^3*d^7 + 46464*a^2*b^9*c^2*d^8 - 42592*a^3*b
^8*c*d^9 + 14641*a^4*b^7*d^10)*x^2*sqrt((4096*b^11*c^4 - 22528*a*b^10*c^3*d
 + 46464*a^2*b^9*c^2*d^2 - 42592*a^3*b^8*c*d^3 + 14641*a^4*b^7*d^4)/d^12) +
 (262144*b^16*c^6 - 2162688*a*b^15*c^5*d + 7434240*a^2*b^14*c^4*d^2 - 13629
440*a^3*b^13*c^3*d^3 + 14055360*a^4*b^12*c^2*d^4 - 7730448*a^5*b^11*c*d^5 +
 1771561*a^6*b^10*d^6)*sqrt(b*x^4 + a))/x^2)*((4096*b^11*c^4 - 22528*a*b^10
*c^3*d + 46464*a^2*b^9*c^2*d^2 - 42592*a^3*b^8*c*d^3 + 14641*a^4*b^7*d^4)/d
^12)^(1/4) + (512*b^8*c^3*d^3 - 2112*a*b^7*c^2*d^4 + 2904*a^2*b^6*c*d^5 - 1
331*a^3*b^5*d^6)*(b*x^4 + a)^(1/4)*((4096*b^11*c^4 - 22528*a*b^10*c^3*d + 4
6464*a^2*b^9*c^2*d^2 - 42592*a^3*b^8*c*d^3 + 14641*a^4*b^7*d^4)/d^12)^(1/4)
)/((4096*b^11*c^4 - 22528*a*b^10*c^3*d + 46464*a^2*b^9*c^2*d^2 - 42592*a^3*
b^8*c*d^3 + 14641*a^4*b^7*d^4)*x)) + (c*d^3*x^4 + c^2*d^2)*((4096*b^11*c^11
 - 22528*a*b^10*c^10*d + 46464*a^2*b^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 50
71*a^4*b^7*c^7*d^4 + 25641*a^5*b^6*c^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^
7*b^4*c^4*d^7 + 2739*a^8*b^3*c^3*d^8 + 891*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d
^10 - 81*a^11*d^11)/(c^7*d^12))^(1/4)*log(-(c^5*d^9*x*((4096*b^11*c^11 - 22
528*a*b^10*c^10*d + 46464*a^2*b^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^
4*b^7*c^7*d^4 + 25641*a^5*b^6*c^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4
*c^4*d^7 + 2739*a^8*b^3*c^3*d^8 + 891*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d^10 -
 81*a^11*d^11)/(c^7*d^12))^(3/4) + (512*b^8*c^8 - 1984*a*b^7*c^7*d + 2456*a
^2*b^6*c^6*d^2 - 413*a^3*b^5*c^5*d^3 - 1175*a^4*b^4*c^4*d^4 + 478*a^5*b^3*c
^3*d^5 + 234*a^6*b^2*c^2*d^6 - 81*a^7*b*c*d^7 - 27*a^8*d^8)*(b*x^4 + a)^(1/
4))/x) - (c*d^3*x^4 + c^2*d^2)*((4096*b^11*c^11 - 22528*a*b^10*c^10*d + 464
64*a^2*b^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a
^5*b^6*c^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3
*c^3*d^8 + 891*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d^10 - 81*a^11*d^11)/(c^7*d^1
2))^(1/4)*log((c^5*d^9*x*((4096*b^11*c^11 - 22528*a*b^10*c^10*d + 46464*a^2
*b^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a^5*b^6
*c^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3*c^3*d
^8 + 891*a^9*b^2*c^2*d^9 - 297*a^10*b*c*d^10 - 81*a^11*d^11)/(c^7*d^12))^(3
/4) - (512*b^8*c^8 - 1984*a*b^7*c^7*d + 2456*a^2*b^6*c^6*d^2 - 413*a^3*b^5*
c^5*d^3 - 1175*a^4*b^4*c^4*d^4 + 478*a^5*b^3*c^3*d^5 + 234*a^6*b^2*c^2*d^6
 - 81*a^7*b*c*d^7 - 27*a^8*d^8)*(b*x^4 + a)^(1/4))/x) - (c*d^3*x^4 + c^2*d^2
)*((4096*b^11*c^4 - 22528*a*b^10*c^3*d + 46464*a^2*b^9*c^2*d^2 - 42592*a^3*
b^8*c*d^3 + 14641*a^4*b^7*d^4)/d^12)^(1/4)*log(-(d^9*x*((4096*b^11*c^4 - 22
528*a*b^10*c^3*d + 46464*a^2*b^9*c^2*d^2 - 42592*a^3*b^8*c*d^3 + 14641*a^4*
b^7*d^4)/d^12)^(3/4) + (512*b^8*c^3 - 2112*a*b^7*c^2*d + 2904*a^2*b^6*c*d^2
 - 1331*a^3*b^5*d^3)*(b*x^4 + a)^(1/4))/x) + (c*d^3*x^4 + c^2*d^2)*((4096*b
^11*c^4 - 22528*a*b^10*c^3*d + 46464*a^2*b^9*c^2*d^2 - 42592*a^3*b^8*c*d^3
+ 14641*a^4*b^7*d^4)/d^12)^(1/4)*log((d^9*x*((4096*b^11*c^4 - 22528*a*b^10*
c^3*d + 46464*a^2*b^9*c^2*d^2 - 42592*a^3*b^8*c*d^3 + 14641*a^4*b^7*d^4)/d^
12)^(3/4) - (512*b^8*c^3 - 2112*a*b^7*c^2*d + 2904*a^2*b^6*c*d^2 - 1331*a^3
*b^5*d^3)*(b*x^4 + a)^(1/4))/x) + 4*(b^2*c*d*x^5 + (2*b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)*x)*(b*x^4 + a)^(3/4))/(c*d^3*x^4 + c^2*d^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(11/4)/(d*x**4+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)^{\frac{11}{4}}}{\left(dx^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(11/4)/(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(11/4)/(d*x^4 + c)^2, x)
```

3.206 $\int \frac{(a+bx^4)^{7/4}}{(c+dx^4)^2} \, dx$

Optimal. Leaf size=230

$$\frac{b^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d^2}+\frac{b^{7/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d^2}-\frac{(bc-ad)^{3/4}(3ad+4bc)\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^2}-\frac{(bc-ad)^{3/4}(3ad+4bc)\tanh^{-1}}{8c^{7/4}d^2}$$

```
[Out] -((b*c - a*d)*x*(a + b*x^4)^(3/4))/(4*c*d*(c + d*x^4)) + (b^(7/4)*ArcTan[(b
^(1/4)*x)/(a + b*x^4)^(1/4)])/(2*d^2) - ((b*c - a*d)^(3/4)*(4*b*c + 3*a*d)*
ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*d^2)
+ (b^(7/4)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(2*d^2) - ((b*c - a*d)^(
3/4)*(4*b*c + 3*a*d)*ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/
4))])/(8*c^(7/4)*d^2)
```

Rubi [A] time = 0.175304, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {413, 530, 240, 212, 206, 203, 377, 208, 205}

$$\frac{b^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d^2}+\frac{b^{7/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d^2}-\frac{(bc-ad)^{3/4}(3ad+4bc)\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^2}-\frac{(bc-ad)^{3/4}(3ad+4bc)\tanh^{-1}}{8c^{7/4}d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(7/4)/(c + d*x^4)^2,x]
```



```
[Out] -((b*c - a*d)*x*(a + b*x^4)^(3/4))/(4*c*d*(c + d*x^4)) + (b^(7/4)*ArcTan[(b
^(1/4)*x)/(a + b*x^4)^(1/4)])/(2*d^2) - ((b*c - a*d)^(3/4)*(4*b*c + 3*a*d)*
ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*d^2)
+ (b^(7/4)*ArcTanh[(b^(1/4)*x)/(a + b*x^4)^(1/4)])/(2*d^2) - ((b*c - a*d)^(
3/4)*(4*b*c + 3*a*d)*ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/
4))])/(8*c^(7/4)*d^2)
```


Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 530

```
Int[(((a_) + (b_.)*(x_)^(n_))^(p_)*((e_) + (f_.)*(x_)^(n_)))/((c_) + (d_.)*
(x_)^(n_)), x_Symbol] :> Dist[f/d, Int[(a + b*x^n)^p, x], x] + Dist[(d*e -
c*f)/d, Int[(a + b*x^n)^p/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p
, n}, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
```

```
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+bx^4\right)^{7/4}}{\left(c+dx^4\right)^2}\,dx = -\frac{(bc-ad)x\left(a+bx^4\right)^{3/4}}{4cd\left(c+dx^4\right)}+\frac{\int \frac{a(bc+3ad)+4b^2cx^4}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)}\,dx}{4cd}$$

$$= -\frac{(bc-ad)x\left(a+bx^4\right)^{3/4}}{4cd\left(c+dx^4\right)}+\frac{b^2\int \frac{1}{\sqrt[4]{a+bx^4}}\,dx}{d^2}-\frac{((bc-ad)(4bc+3ad))\int \frac{1}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)}\,dx}{4cd^2}$$

$$= -\frac{(bc-ad)x\left(a+bx^4\right)^{3/4}}{4cd\left(c+dx^4\right)}+\frac{b^2\,\mathrm{Subst}\left(\int \frac{1}{1-bx^4}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{d^2}-\frac{((bc-ad)(4bc+3ad))\,\mathrm{Subst}\left(\int \frac{}{c-}\right.}{4cd^2}$$

$$= -\frac{(bc-ad)x\left(a+bx^4\right)^{3/4}}{4cd\left(c+dx^4\right)}+\frac{b^2\,\mathrm{Subst}\left(\int \frac{1}{1-\sqrt{b}x^2}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{2d^2}+\frac{b^2\,\mathrm{Subst}\left(\int \frac{1}{1+\sqrt{b}x^2}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right.}{2d^2}$$

$$= -\frac{(bc-ad)x\left(a+bx^4\right)^{3/4}}{4cd\left(c+dx^4\right)}+\frac{b^{7/4}\tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d^2}-\frac{(bc-ad)^{3/4}(4bc+3ad)\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^2}+\frac{b^{7/}}{}$$

Mathematica [C] time = 0.553527, size = 358, normalized size = 1.56

$$\frac{\frac{15a^2\left(-\log\left(\sqrt[4]{c}-\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}\right)+\log\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}+\sqrt[4]{c}\right)+2\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{ax^4+b}}\right)\right)}{\sqrt[4]{bc-ad}}+\frac{16b^2c^{3/4}x^5\sqrt[4]{\frac{bx^4}{a}+1}F_1\left(\frac{5}{4};\frac{1}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{d\sqrt[4]{a+bx^4}}-\frac{20c^{3/4}x\left(a+bx^4\right)^{3/4}(bc-ad)}{d\left(c+dx^4\right)}+\frac{5abc\left(-\right.}{}}{80c^{7/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^4)^(7/4)/(c + d*x^4)^2,x]
```

```
[Out] ((-20*c^(3/4)*(b*c - a*d)*x*(a + b*x^4)^(3/4))/(d*(c + d*x^4)) + (16*b^2*c^
(3/4)*x^5*(1 + (b*x^4)/a)^(1/4)*AppellF1[5/4, 1/4, 1, 9/4, -((b*x^4)/a), -(
(d*x^4)/c)])/(d*(a + b*x^4)^(1/4)) + (15*a^2*(2*ArcTan[((b*c - a*d)^(1/4)*x
)/(c^(1/4)*(b + a*x^4)^(1/4))] - Log[c^(1/4) - ((b*c - a*d)^(1/4)*x)/(b + a
*x^4)^(1/4)] + Log[c^(1/4) + ((b*c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)]))/(b*
c - a*d)^(1/4) + (5*a*b*c*(2*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(b + a*x
^4)^(1/4))] - Log[c^(1/4) - ((b*c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)] + Log[
c^(1/4) + ((b*c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)]))/(d*(b*c - a*d)^(1/4)))
/(80*c^(7/4))
```

Maple [F] time = 0.434, size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^4+c\right)^2}\left(bx^4+a\right)^{\frac{7}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(7/4)/(d*x^4+c)^2,x)
```

```
[Out] int((b*x^4+a)^(7/4)/(d*x^4+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)^{\frac{7}{4}}}{\left(dx^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(7/4)/(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(7/4)/(d*x^4 + c)^2, x)
```

Fricas [B] time = 6.94258, size = 3656, normalized size = 15.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(7/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*(b*x^4 + a)^(3/4)*(b*c - a*d)*x - 4*(c*d^2*x^4 + c^2*d)*((256*b^7*
c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189
*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^8))^(1/4)*arctan(-(
c^2*d^2*x*sqrt(((256*b^7*c^10*d^4 - 672*a^2*b^5*c^8*d^6 - 112*a^3*b^4*c^7*d
^7 + 609*a^4*b^3*c^6*d^8 + 189*a^5*b^2*c^5*d^9 - 189*a^6*b*c^4*d^10 - 81*a^
7*c^3*d^11)*x^2*sqrt((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*c^4*d
^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*a^7*d
^7)/(c^7*d^8)) + (4096*b^10*c^10 + 2048*a*b^9*c^9*d - 14592*a^2*b^8*c^8*d^2
 - 9472*a^3*b^7*c^7*d^3 + 18928*a^4*b^6*c^6*d^4 + 15624*a^5*b^5*c^5*d^5 - 9
639*a^6*b^4*c^4*d^6 - 11124*a^7*b^3*c^3*d^7 + 486*a^8*b^2*c^2*d^8 + 2916*a^
9*b*c*d^9 + 729*a^10*d^10)*sqrt(b*x^4 + a))/x^2)*((256*b^7*c^7 - 672*a^2*b^
5*c^5*d^2 - 112*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5
 - 189*a^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^8))^(1/4) - (64*b^5*c^7*d^2 + 16*a*
b^4*c^6*d^3 - 116*a^2*b^3*c^5*d^4 - 45*a^3*b^2*c^4*d^5 + 54*a^4*b*c^3*d^6 +
 27*a^5*c^2*d^7)*(b*x^4 + a)^(1/4)*((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 11
2*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c
*d^6 - 81*a^7*d^7)/(c^7*d^8))^(1/4))/((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 -
112*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b
*c*d^6 - 81*a^7*d^7)*x)) - 16*(c*d^2*x^4 + c^2*d)*(b^7/d^8)^(1/4)*arctan(-(
(b*x^4 + a)^(1/4)*b^5*d^2*(b^7/d^8)^(1/4) - d^2*x*(b^7/d^8)^(1/4)*sqrt((b^7
*d^4*x^2*sqrt(b^7/d^8) + sqrt(b*x^4 + a)*b^10)/x^2))/(b^7*x)) + (c*d^2*x^4
+ c^2*d)*((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*c^4*d^3 + 609*a^
4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^
8))^(1/4)*log((c^5*d^6*x*((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*
c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*
a^7*d^7)/(c^7*d^8))^(3/4) + (64*b^5*c^5 + 16*a*b^4*c^4*d - 116*a^2*b^3*c^3*
d^2 - 45*a^3*b^2*c^2*d^3 + 54*a^4*b*c*d^4 + 27*a^5*d^5)*(b*x^4 + a)^(1/4))/
x) - (c*d^2*x^4 + c^2*d)*((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*
c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*
a^7*d^7)/(c^7*d^8))^(1/4)*log(-(c^5*d^6*x*((256*b^7*c^7 - 672*a^2*b^5*c^5*d
^2 - 112*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*
a^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^8))^(3/4) - (64*b^5*c^5 + 16*a*b^4*c^4*d -
 116*a^2*b^3*c^3*d^2 - 45*a^3*b^2*c^2*d^3 + 54*a^4*b*c*d^4 + 27*a^5*d^5)*(b
*x^4 + a)^(1/4))/x) - 4*(c*d^2*x^4 + c^2*d)*(b^7/d^8)^(1/4)*log((d^6*x*(b^7
/d^8)^(3/4) + (b*x^4 + a)^(1/4)*b^5)/x) + 4*(c*d^2*x^4 + c^2*d)*(b^7/d^8)^(
1/4)*log(-(d^6*x*(b^7/d^8)^(3/4) - (b*x^4 + a)^(1/4)*b^5)/x))/(c*d^2*x^4 +
c^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(7/4)/(d*x**4+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4 + a\right)^{\frac{7}{4}}}{\left(dx^4 + c\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(7/4)/(d*x^4+c)^2,x, algorithm="giac")

[Out] integrate((b*x^4 + a)^(7/4)/(d*x^4 + c)^2, x)
```

3.207 $\int \frac{(a+bx^4)^{3/4}}{(c+dx^4)^2} \, dx$

Optimal. Leaf size=135

$$\frac{3a\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}\sqrt[4]{bc-ad}}+\frac{3a\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}\sqrt[4]{bc-ad}}+\frac{x\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)}$$

```
[Out] (x*(a + b*x^4)^(3/4))/(4*c*(c + d*x^4)) + (3*a*ArcTan[((b*c - a*d)^(1/4)*x)
/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*(b*c - a*d)^(1/4)) + (3*a*ArcTanh
[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*(b*c - a*d)
^(1/4))
```

Rubi [A] time = 0.0762624, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {378, 377, 212, 208, 205}

$$\frac{3a\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}\sqrt[4]{bc-ad}}+\frac{3a\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}\sqrt[4]{bc-ad}}+\frac{x\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(3/4)/(c + d*x^4)^2,x]
```


```
[Out] (x*(a + b*x^4)^(3/4))/(4*c*(c + d*x^4)) + (3*a*ArcTan[((b*c - a*d)^(1/4)*x)
/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*(b*c - a*d)^(1/4)) + (3*a*ArcTanh
[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*(b*c - a*d)
^(1/4))
```


Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+bx^4\right)^{3/4}}{\left(c+dx^4\right)^2}\,dx &= \frac{x\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)}+\frac{(3a)\int \frac{1}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)}\,dx}{4c}\\
&= \frac{x\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)}+\frac{(3a)\,\text{Subst}\left(\int \frac{1}{c-(bc-ad)x^4}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{4c}\\
&= \frac{x\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)}+\frac{(3a)\,\text{Subst}\left(\int \frac{1}{\sqrt{c}-\sqrt{bc-ad}x^2}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{8c^{3/2}}+\frac{(3a)\,\text{Subst}\left(\int \frac{1}{\sqrt{c}+\sqrt{bc-ad}x^2}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{8c^{3/2}}\\
&= \frac{x\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)}+\frac{3a\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}\sqrt[4]{bc-ad}}+\frac{3a\tanh^{-1}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}\sqrt[4]{bc-ad}}
\end{aligned}
$$

Mathematica [C] time = 0.0329985, size = 78, normalized size = 0.58

$$
\frac{x\left(a+bx^4\right)^{3/4}\,{}_2F_1\left(-\frac{3}{4},\frac{1}{4};\frac{5}{4};\frac{(ad-bc)x^4}{a\left(dx^4+c\right)}\right)}{c^2\left(\frac{bx^4}{a}+1\right)^{3/4}\sqrt[4]{\frac{dx^4}{c}+1}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^4)^(3/4)/(c + d*x^4)^2,x]
```

```
[Out] (x*(a + b*x^4)^(3/4)*Hypergeometric2F1[-3/4, 1/4, 5/4, ((-(b*c) + a*d)*x^4)
/(a*(c + d*x^4))])/(c^2*(1 + (b*x^4)/a)^(3/4)*(1 + (d*x^4)/c)^(1/4))
```

Maple [F] time = 0.41, size = 0, normalized size = 0.

$$
\int \frac{1}{\left(dx^4+c\right)^2}\left(bx^4+a\right)^{\frac{3}{4}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(3/4)/(d*x^4+c)^2,x)
```

```
[Out] int((b*x^4+a)^(3/4)/(d*x^4+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{3}{4}}}{(dx^4 + c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(3/4)/(d*x^4 + c)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx^4)^{\frac{3}{4}}}{(c + dx^4)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(3/4)/(d*x**4+c)**2,x)
```

```
[Out] Integral((a + b*x**4)**(3/4)/(c + d*x**4)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{3}{4}}}{(dx^4 + c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(3/4)/(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(3/4)/(d*x^4 + c)^2, x)
```

3.208 $\int \frac{1}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)^2} \, dx$

Optimal. Leaf size=162

$$\frac{(4bc-3ad)\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{5/4}}+\frac{(4bc-3ad)\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{5/4}}-\frac{dx\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)(bc-ad)}$$

```
[Out] -(d*x*(a + b*x^4)^(3/4))/(4*c*(b*c - a*d)*(c + d*x^4)) + ((4*b*c - 3*a*d)*A
rcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*(b*c -
 a*d)^(5/4)) + ((4*b*c - 3*a*d)*ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a +
 b*x^4)^(1/4))])/(8*c^(7/4)*(b*c - a*d)^(5/4))
```

Rubi [A] time = 0.108515, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {382, 377, 212, 208, 205}

$$\frac{(4bc-3ad)\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{5/4}}+\frac{(4bc-3ad)\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{5/4}}-\frac{dx\left(a+bx^4\right)^{3/4}}{4c\left(c+dx^4\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^(1/4)*(c + d*x^4)^2),x]
```


```
[Out] -(d*x*(a + b*x^4)^(3/4))/(4*c*(b*c - a*d)*(c + d*x^4)) + ((4*b*c - 3*a*d)*A
rcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*(b*c -
 a*d)^(5/4)) + ((4*b*c - 3*a*d)*ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a +
 b*x^4)^(1/4))])/(8*c^(7/4)*(b*c - a*d)^(5/4))
```


Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)^2}\,dx &= -\frac{dx\left(a+bx^4\right)^{3/4}}{4c(bc-ad)\left(c+dx^4\right)}+\frac{(4bc-3ad)\int \frac{1}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)}\,dx}{4c(bc-ad)}\\
&= -\frac{dx\left(a+bx^4\right)^{3/4}}{4c(bc-ad)\left(c+dx^4\right)}+\frac{(4bc-3ad)\,\mathrm{Subst}\left(\int \frac{1}{c-(bc-ad)x^4}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{4c(bc-ad)}\\
&= -\frac{dx\left(a+bx^4\right)^{3/4}}{4c(bc-ad)\left(c+dx^4\right)}+\frac{(4bc-3ad)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{c}-\sqrt{bc-ad}x^2}\,dx,x,\frac{x}{\sqrt[4]{a+bx^4}}\right)}{8c^{3/2}(bc-ad)}+\frac{(4bc-3ad)}{}\\
&= -\frac{dx\left(a+bx^4\right)^{3/4}}{4c(bc-ad)\left(c+dx^4\right)}+\frac{(4bc-3ad)\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{5/4}}+\frac{(4bc-3ad)\tanh^{-1}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{5/4}}\end{aligned}$$

Mathematica [C] time = 0.105946, size = 99, normalized size = 0.61

$$\frac{x\left(\left(c+dx^4\right)(4bc-3ad)\,{}_2F_1\left(\frac{1}{4},1;\frac{5}{4};\frac{(bc-ad)x^4}{c\left(bx^4+a\right)}\right)-cd\left(a+bx^4\right)\right)}{4c^2\sqrt[4]{a+bx^4}\left(c+dx^4\right)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^4)^(1/4)*(c + d*x^4)^2),x]
```

```
[Out] (x*(-(c*d*(a + b*x^4)) + (4*b*c - 3*a*d)*(c + d*x^4)*Hypergeometric2F1[1/4,
 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))]))/(4*c^2*(b*c - a*d)*(a + b*x^4
)^(1/4)*(c + d*x^4))
```

Maple [F] time = 0.41, size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^4+c\right)^2}\frac{1}{\sqrt[4]{bx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(1/4)/(d*x^4+c)^2,x)
```

```
[Out] int(1/(b*x^4+a)^(1/4)/(d*x^4+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{1}{4}}\left(dx^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(1/4)/(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(1/4)*(d*x^4 + c)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(1/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(1/4)/(d*x**4+c)**2,x)
```

```
[Out] Exception raised: ValueError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{1}{4}}\left(dx^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(1/4)/(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(1/4)*(d*x^4 + c)^2), x)
```

3.209 $\int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)^2}\,dx$

Optimal. Leaf size=205

$$-\frac{d(8bc-3ad)\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{9/4}}-\frac{d(8bc-3ad)\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{9/4}}+\frac{bx(ad+4bc)}{4ac\sqrt[4]{a+bx^4}(bc-ad)^2}-\frac{dx}{4c\sqrt[4]{a+bx^4}\left(c+dx^4\right)}$$

```
[Out] (b*(4*b*c + a*d)*x)/(4*a*c*(b*c - a*d)^2*(a + b*x^4)^(1/4)) - (d*x)/(4*c*(b
*c - a*d)*(a + b*x^4)^(1/4)*(c + d*x^4)) - (d*(8*b*c - 3*a*d)*ArcTan[((b*c
- a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*(b*c - a*d)^(9/4))
 - (d*(8*b*c - 3*a*d)*ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1
/4))])/(8*c^(7/4)*(b*c - a*d)^(9/4))
```

Rubi [A] time = 0.184153, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {414, 527, 12, 377, 212, 208, 205}

$$-\frac{d(8bc-3ad)\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{9/4}}-\frac{d(8bc-3ad)\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{9/4}}+\frac{bx(ad+4bc)}{4ac\sqrt[4]{a+bx^4}(bc-ad)^2}-\frac{dx}{4c\sqrt[4]{a+bx^4}\left(c+dx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^(5/4)*(c + d*x^4)^2),x]
```


```
[Out] (b*(4*b*c + a*d)*x)/(4*a*c*(b*c - a*d)^2*(a + b*x^4)^(1/4)) - (d*x)/(4*c*(b
*c - a*d)*(a + b*x^4)^(1/4)*(c + d*x^4)) - (d*(8*b*c - 3*a*d)*ArcTan[((b*c
- a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*(b*c - a*d)^(9/4))
 - (d*(8*b*c - 3*a*d)*ArcTanh[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1
/4))])/(8*c^(7/4)*(b*c - a*d)^(9/4))
```


Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)^2}\,dx = -\frac{dx}{4c(bc-ad)\sqrt[4]{a+bx^4}\left(c+dx^4\right)} + \frac{\int \frac{4bc-3ad-4bdx^4}{\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)}\,dx}{4c(bc-ad)}$$

$$= \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt[4]{a+bx^4}} - \frac{dx}{4c(bc-ad)\sqrt[4]{a+bx^4}\left(c+dx^4\right)} - \frac{\int \frac{ad(8bc-3ad)}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)}\,dx}{4ac(bc-ad)^2}$$

$$= \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt[4]{a+bx^4}} - \frac{dx}{4c(bc-ad)\sqrt[4]{a+bx^4}\left(c+dx^4\right)} - \frac{(d(8bc-3ad))\int \frac{1}{\sqrt[4]{a+bx^4}\left(c+dx^4\right)} \cdots}{4c(bc-ad)^2}$$

$$= \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt[4]{a+bx^4}} - \frac{dx}{4c(bc-ad)\sqrt[4]{a+bx^4}\left(c+dx^4\right)} - \frac{(d(8bc-3ad))\,\mathrm{Subst}\left(\int \frac{\cdots}{c-(bc-\cdots} \cdots\right.}{4c(bc-ad)\cdots}$$

$$= \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt[4]{a+bx^4}} - \frac{dx}{4c(bc-ad)\sqrt[4]{a+bx^4}\left(c+dx^4\right)} - \frac{(d(8bc-3ad))\,\mathrm{Subst}\left(\int \frac{\cdots}{\sqrt{c}-\sqrt{\cdots}} \cdots\right.}{8c^{3/2}(bc-a\cdots}$$

$$= \frac{b(4bc+ad)x}{4ac(bc-ad)^2\sqrt[4]{a+bx^4}} - \frac{dx}{4c(bc-ad)\sqrt[4]{a+bx^4}\left(c+dx^4\right)} - \frac{d(8bc-3ad)\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}\,x}{\sqrt[4]{c}\,\sqrt[4]{a+bx^4}}\right.\cdots}{8c^{7/4}(bc-ad)^{9/4}}$$

Mathematica [C] time = 1.37312, size = 625, normalized size = 3.05

$$\frac{\cdots}{c\left(a+bx^4\right)^{3/4}\left(\frac{320d^2x^{20}(bc-ad)^3\,\mathrm{HypergeometricPFQ}\left(\left\{2,2,\frac{9}{4}\right\},\left\{1,\frac{17}{4}\right\},\frac{x^4(bc-ad)}{c\left(a+bx^4\right)}\right)}{c^5\left(a+bx^4\right)^3} + \frac{640dx^{16}(bc-ad)^3\,\mathrm{HypergeometricPFQ}\left(\left\{2,2,\frac{9}{4}\right\},\left\{1,\frac{17}{4}\right\},\frac{x^4(bc-ad)}{c\left(a+bx^4\right)}\right)}{c^4\left(a+bx^4\right)^3} - \cdots\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^4)^(5/4)*(c + d*x^4)^2),x]
```

```
[Out] (c*(a + b*x^4)^(3/4)*(-47385 - (94770*d*x^4)/c - (44460*d^2*x^8)/c^2 + (514
8*(b*c - a*d)*x^4)/(c*(a + b*x^4)) + (14976*d*(b*c - a*d)*x^8)/(c^2*(a + b*
x^4)) + (7488*d^2*(b*c - a*d)*x^12)/(c^3*(a + b*x^4)) + 47385*Hypergeometri
c2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + (94770*d*x^4*Hyperge
ometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))])/c + (44460*d^2*
x^8*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))])/c^2
- (14625*(b*c - a*d)*x^4*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(
c*(a + b*x^4))])/(c*(a + b*x^4)) + (33930*d*(-(b*c) + a*d)*x^8*Hypergeometr
ic2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))])/(c^2*(a + b*x^4)) +
(16380*d^2*(-(b*c) + a*d)*x^12*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*
x^4)/(c*(a + b*x^4))])/(c^3*(a + b*x^4)) + (320*(b*c - a*d)^3*x^12*Hypergeo
metricPFQ[{2, 2, 9/4}, {1, 17/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))])/(c^3*
(a + b*x^4)^3) + (640*d*(b*c - a*d)^3*x^16*HypergeometricPFQ[{2, 2, 9/4}, {
1, 17/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))])/(c^4*(a + b*x^4)^3) + (320*d^
2*(b*c - a*d)^3*x^20*HypergeometricPFQ[{2, 2, 9/4}, {1, 17/4}, ((b*c - a*d)
*x^4)/(c*(a + b*x^4))])/(c^5*(a + b*x^4)^3)))/(2340*(b*c - a*d)^2*x^7*(c +
d*x^4))
```

Maple [F] time = 0.409, size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^4+c\right)^2}\left(bx^4+a\right)^{-\frac{5}{4}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(5/4)/(d*x^4+c)^2,x)
```

```
[Out] int(1/(b*x^4+a)^(5/4)/(d*x^4+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{5}{4}}\left(dx^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(5/4)/(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(5/4)*(d*x^4 + c)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(5/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(5/4)/(d*x**4+c)**2,x)
```

```
[Out] Exception raised: ValueError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{5}{4}}\left(dx^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(5/4)/(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(5/4)*(d*x^4 + c)^2), x)
```

3.210 $\int \frac{1}{\left(a+bx^4\right)^{9/4}\left(c+dx^4\right)^2}\,dx$

Optimal. Leaf size=266

$$\frac{bx\left(-5a^2d^2-56abcd+16b^2c^2\right)}{20a^2c\sqrt[4]{a+bx^4}(bc-ad)^3}+\frac{3d^2(4bc-ad)\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{13/4}}+\frac{3d^2(4bc-ad)\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{13/4}}-\frac{}{4c\left(a+bx^4\right.}$$

```
[Out] (b*(4*b*c + 5*a*d)*x)/(20*a*c*(b*c - a*d)^2*(a + b*x^4)^(5/4)) + (b*(16*b^2
*c^2 - 56*a*b*c*d - 5*a^2*d^2)*x)/(20*a^2*c*(b*c - a*d)^3*(a + b*x^4)^(1/4)
) - (d*x)/(4*c*(b*c - a*d)*(a + b*x^4)^(5/4)*(c + d*x^4)) + (3*d^2*(4*b*c -
 a*d)*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)
*(b*c - a*d)^(13/4)) + (3*d^2*(4*b*c - a*d)*ArcTanh[((b*c - a*d)^(1/4)*x)/(
c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*(b*c - a*d)^(13/4))
```

Rubi [A] time = 0.292855, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {414, 527, 12, 377, 212, 208, 205}

$$\frac{bx\left(-5a^2d^2-56abcd+16b^2c^2\right)}{20a^2c\sqrt[4]{a+bx^4}(bc-ad)^3}+\frac{3d^2(4bc-ad)\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{13/4}}+\frac{3d^2(4bc-ad)\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{13/4}}-\frac{}{4c\left(a+bx^4\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^(9/4)*(c + d*x^4)^2),x]
```

```
[Out] (b*(4*b*c + 5*a*d)*x)/(20*a*c*(b*c - a*d)^2*(a + b*x^4)^(5/4)) + (b*(16*b^2
*c^2 - 56*a*b*c*d - 5*a^2*d^2)*x)/(20*a^2*c*(b*c - a*d)^3*(a + b*x^4)^(1/4)
) - (d*x)/(4*c*(b*c - a*d)*(a + b*x^4)^(5/4)*(c + d*x^4)) + (3*d^2*(4*b*c -
 a*d)*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)
*(b*c - a*d)^(13/4)) + (3*d^2*(4*b*c - a*d)*ArcTanh[((b*c - a*d)^(1/4)*x)/(
c^(1/4)*(a + b*x^4)^(1/4))])/(8*c^(7/4)*(b*c - a*d)^(13/4))
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^4\right)^{9/4}\left(c+dx^4\right)^2}\,dx = -\frac{dx}{4c(bc-ad)\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)} + \frac{\int \frac{4bc-3ad-8bdx^4}{\left(a+bx^4\right)^{9/4}\left(c+dx^4\right)}\,dx}{4c(bc-ad)}$$

$$= \frac{b(4bc+5ad)x}{20ac(bc-ad)^2\left(a+bx^4\right)^{5/4}} - \frac{dx}{4c(bc-ad)\left(a+bx^4\right)^{5/4}\left(c+dx^4\right)} - \frac{\int \frac{-16b^2c^2+40abcd-15a^2d^2-}{\left(a+bx^4\right)^{5/4}\left(c+\right.}}{20ac(bc-}$$

$$= \frac{b(4bc+5ad)x}{20ac(bc-ad)^2\left(a+bx^4\right)^{5/4}} + \frac{b\left(16b^2c^2-56abcd-5a^2d^2\right)x}{20a^2c(bc-ad)^3\sqrt[4]{a+bx^4}} - \frac{dx}{4c(bc-ad)\left(a+bx^4\right)^{5/4}\left(c\right.}$$

$$= \frac{b(4bc+5ad)x}{20ac(bc-ad)^2\left(a+bx^4\right)^{5/4}} + \frac{b\left(16b^2c^2-56abcd-5a^2d^2\right)x}{20a^2c(bc-ad)^3\sqrt[4]{a+bx^4}} - \frac{dx}{4c(bc-ad)\left(a+bx^4\right)^{5/4}\left(c\right.}$$

$$= \frac{b(4bc+5ad)x}{20ac(bc-ad)^2\left(a+bx^4\right)^{5/4}} + \frac{b\left(16b^2c^2-56abcd-5a^2d^2\right)x}{20a^2c(bc-ad)^3\sqrt[4]{a+bx^4}} - \frac{dx}{4c(bc-ad)\left(a+bx^4\right)^{5/4}\left(c\right.}$$

$$= \frac{b(4bc+5ad)x}{20ac(bc-ad)^2\left(a+bx^4\right)^{5/4}} + \frac{b\left(16b^2c^2-56abcd-5a^2d^2\right)x}{20a^2c(bc-ad)^3\sqrt[4]{a+bx^4}} - \frac{dx}{4c(bc-ad)\left(a+bx^4\right)^{5/4}\left(c\right.}$$

$$= \frac{b(4bc+5ad)x}{20ac(bc-ad)^2\left(a+bx^4\right)^{5/4}} + \frac{b\left(16b^2c^2-56abcd-5a^2d^2\right)x}{20a^2c(bc-ad)^3\sqrt[4]{a+bx^4}} - \frac{dx}{4c(bc-ad)\left(a+bx^4\right)^{5/4}\left(c\right.}$$

Mathematica [C] time = 3.81365, size = 1216, normalized size = 4.57

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^4)^(9/4)*(c + d*x^4)^2),x]
```

```
[Out] -(285532*c^5*(b*c - a*d)^2*x^8*(a + b*x^4)^2 + 933504*c^4*d*(b*c - a*d)^2*x
^12*(a + b*x^4)^2 + 891072*c^3*d^2*(b*c - a*d)^2*x^16*(a + b*x^4)^2 + 28288
0*c^2*d^3*(b*c - a*d)^2*x^20*(a + b*x^4)^2 + 9793836*c^6*(b*c - a*d)*x^4*(a
 + b*x^4)^3 + 27973296*c^5*d*(b*c - a*d)*x^8*(a + b*x^4)^3 + 25968384*c^4*d
^2*(b*c - a*d)*x^12*(a + b*x^4)^3 + 8146944*c^3*d^3*(b*c - a*d)*x^16*(a + b
*x^4)^3 - 23529870*c^7*(a + b*x^4)^4 - 65547495*c^6*d*x^4*(a + b*x^4)^4 - 6
0505380*c^5*d^2*x^8*(a + b*x^4)^4 - 18935280*c^4*d^3*x^12*(a + b*x^4)^4 - 1
4499810*c^6*(b*c - a*d)*x^4*(a + b*x^4)^3*Hypergeometric2F1[1/4, 1, 5/4, ((
b*c - a*d)*x^4)/(c*(a + b*x^4))] - 41082795*c^5*d*(b*c - a*d)*x^8*(a + b*x^
4)^3*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] - 38
069460*c^4*d^2*(b*c - a*d)*x^12*(a + b*x^4)^3*Hypergeometric2F1[1/4, 1, 5/4
, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] - 11934000*c^3*d^3*(b*c - a*d)*x^16*(a
 + b*x^4)^3*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4)
)] + 23529870*c^7*(a + b*x^4)^4*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)
*x^4)/(c*(a + b*x^4))] + 65547495*c^6*d*x^4*(a + b*x^4)^4*Hypergeometric2F1
[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 60505380*c^5*d^2*x^8*(a
+ b*x^4)^4*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))
] + 18935280*c^4*d^3*x^12*(a + b*x^4)^4*Hypergeometric2F1[1/4, 1, 5/4, ((b*
c - a*d)*x^4)/(c*(a + b*x^4))] + 77760*c^3*(b*c - a*d)^4*x^16*Hypergeometri
cPFQ[{2, 2, 13/4}, {1, 21/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 224640*c
^2*d*(b*c - a*d)^4*x^20*HypergeometricPFQ[{2, 2, 13/4}, {1, 21/4}, ((b*c -
a*d)*x^4)/(c*(a + b*x^4))] + 216000*c*d^2*(b*c - a*d)^4*x^24*Hypergeometric
PFQ[{2, 2, 13/4}, {1, 21/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 69120*d^3
*(b*c - a*d)^4*x^28*HypergeometricPFQ[{2, 2, 13/4}, {1, 21/4}, ((b*c - a*d)
*x^4)/(c*(a + b*x^4))] + 11520*c^3*(b*c - a*d)^4*x^16*HypergeometricPFQ[{2,
 2, 2, 13/4}, {1, 1, 21/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 34560*c^2*
d*(b*c - a*d)^4*x^20*HypergeometricPFQ[{2, 2, 2, 13/4}, {1, 1, 21/4}, ((b*c
 - a*d)*x^4)/(c*(a + b*x^4))] + 34560*c*d^2*(b*c - a*d)^4*x^24*Hypergeometr
icPFQ[{2, 2, 2, 13/4}, {1, 1, 21/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 1
1520*d^3*(b*c - a*d)^4*x^28*HypergeometricPFQ[{2, 2, 2, 13/4}, {1, 1, 21/4}
, ((b*c - a*d)*x^4)/(c*(a + b*x^4))])/(198900*c^8*(-b + (a*d)/c)^3*x^11*(a
+ b*x^4)^(13/4)*(c + d*x^4))
```

Maple [F] time = 0.425, size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^{4}+c\right)^{2}}\left(bx^{4}+a\right)^{-\frac{9}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(9/4)/(d*x^4+c)^2,x)
```

```
[Out] int(1/(b*x^4+a)^(9/4)/(d*x^4+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^{4}+a\right)^{\frac{9}{4}}\left(dx^{4}+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(9/4)/(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(9/4)*(d*x^4 + c)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(9/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(9/4)/(d*x**4+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^4 + a\right)^{\frac{9}{4}}\left(dx^4 + c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(9/4)/(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(9/4)*(d*x^4 + c)^2), x)
```

3.211 $\int \frac{\left(a+bx^4\right)^{9/4}}{\left(c+dx^4\right)^2}\,dx$

Optimal. Leaf size=353

$$-\frac{\sqrt{a}b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(3bc-ad)\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{4cd^2\left(a+bx^4\right)^{3/4}}-\frac{3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)(ad+2bc)\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin\right.}{8\sqrt[4]{b}c^2d^2}$$

```
[Out] (b*(3*b*c - a*d)*x*(a + b*x^4)^(1/4))/(4*c*d^2) - ((b*c - a*d)*x*(a + b*x^4
)^(5/4))/(4*c*d*(c + d*x^4)) - (Sqrt[a]*b^(3/2)*(3*b*c - a*d)*(1 + a/(b*x^4
))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(4*c*d^2*(a + b
*x^4)^(3/4)) - (3*(b*c - a*d)*(2*b*c + a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*
x^4]*EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a
 + b*x^4)^(1/4)], -1])/(8*b^(1/4)*c^2*d^2) - (3*(b*c - a*d)*(2*b*c + a*d)*S
qrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt
[c]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(8*b^(1/4)*c^2*d^2)
```

Rubi [A] time = 0.347931, antiderivative size = 353, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {413, 528, 529, 237, 335, 275, 231, 407, 409, 1218}

$$-\frac{\sqrt{a}b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(3bc-ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{4cd^2\left(a+bx^4\right)^{3/4}}-\frac{3\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(bc-ad)(ad+2bc)\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{}}{\sqrt[4]{bx}}\right.\right.}{8\sqrt[4]{b}c^2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(9/4)/(c + d*x^4)^2,x]
```

```
[Out] (b*(3*b*c - a*d)*x*(a + b*x^4)^(1/4))/(4*c*d^2) - ((b*c - a*d)*x*(a + b*x^4
)^(5/4))/(4*c*d*(c + d*x^4)) - (Sqrt[a]*b^(3/2)*(3*b*c - a*d)*(1 + a/(b*x^4
))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(4*c*d^2*(a + b
*x^4)^(3/4)) - (3*(b*c - a*d)*(2*b*c + a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*
x^4]*EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a
 + b*x^4)^(1/4)], -1])/(8*b^(1/4)*c^2*d^2) - (3*(b*c - a*d)*(2*b*c + a*d)*S
qrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt
[c]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(8*b^(1/4)*c^2*d^2)
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
```

```
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 529

```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^4)^(3/4), x],
 x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(a + b*x^4)^(1/4)/(c + d*x^4), x],
x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 407

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c -
 a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{(a+bx^4)^{9/4}}{(c+dx^4)^2}dx = -\frac{(bc-ad)x(a+bx^4)^{5/4}}{4cd(c+dx^4)} + \frac{\int \frac{\sqrt[4]{a+bx^4}(a(bc+3ad)+2b(3bc-ad)x^4)}{c+dx^4}dx}{4cd}$$

$$= \frac{b(3bc-ad)x\sqrt[4]{a+bx^4}}{4cd^2} - \frac{(bc-ad)x(a+bx^4)^{5/4}}{4cd(c+dx^4)} + \frac{\int \frac{-2a(3b^2c^2-2abcd-3a^2d^2)-4b(3b^2c^2-3abcd-a^2d^2)x^4}{(a+bx^4)^{3/4}(c+dx^4)}dx}{8cd^2}$$

$$= \frac{b(3bc-ad)x\sqrt[4]{a+bx^4}}{4cd^2} - \frac{(bc-ad)x(a+bx^4)^{5/4}}{4cd(c+dx^4)} + \frac{(ab(3bc-ad))\int \frac{1}{(a+bx^4)^{3/4}}dx}{4cd^2} - \frac{(3(bc-ad)(2b$$

$$= \frac{b(3bc-ad)x\sqrt[4]{a+bx^4}}{4cd^2} - \frac{(bc-ad)x(a+bx^4)^{5/4}}{4cd(c+dx^4)} + \frac{\left(ab(3bc-ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}dx}{4cd^2(a+bx^4)^{3/4}}$$

$$= \frac{b(3bc-ad)x\sqrt[4]{a+bx^4}}{4cd^2} - \frac{(bc-ad)x(a+bx^4)^{5/4}}{4cd(c+dx^4)} - \frac{\left(ab(3bc-ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right.}\right.}{4cd^2(a+bx^4)^{3/4}}$$

$$= \frac{b(3bc-ad)x\sqrt[4]{a+bx^4}}{4cd^2} - \frac{(bc-ad)x(a+bx^4)^{5/4}}{4cd(c+dx^4)} - \frac{3(bc-ad)(2bc+ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc}}{\sqrt{b}}\right.}{8\sqrt[4]{b}c^2d^2}$$

$$= \frac{b(3bc-ad)x\sqrt[4]{a+bx^4}}{4cd^2} - \frac{(bc-ad)x(a+bx^4)^{5/4}}{4cd(c+dx^4)} - \frac{\sqrt{a}b^{3/2}(3bc-ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}}{\sqrt{}}\right.\right.}{4cd^2(a+bx^4)^{3/4}}$$

Mathematica [C] time = 0.5029, size = 392, normalized size = 1.11

$$\frac{2bx^5\left(\frac{bx^4}{a}+1\right)^{3/4}(a^2d^2+3abcd-3b^2c^2)F_1\left(\frac{5}{4};\frac{3}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right) + \frac{5c\left(x^5(a+bx^4)(a^2d^2-2abcd+b^2c(3c+2dx^4))\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a}\right.\right.\right.}{(c+dx^4)\left(x^4\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\right.\right.\right.}}{20c^2d^2(a+bx^4)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^4)^(9/4)/(c + d*x^4)^2,x]
```

```
[Out] (2*b*(-3*b^2*c^2 + 3*a*b*c*d + a^2*d^2)*x^5*(1 + (b*x^4)/a)^(3/4)*AppellF1[
5/4, 3/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + (5*c*(-5*a*c*x*(4*a^3*d^2 +
 a^2*b*d^2*x^4 + b^3*c*x^4*(3*c + 2*d*x^4))*AppellF1[1/4, 3/4, 1, 5/4, -((b
*x^4)/a), -((d*x^4)/c)] + x^5*(a + b*x^4)*(-2*a*b*c*d + a^2*d^2 + b^2*c*(3*
c + 2*d*x^4))*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)]
 + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/((c + d*
x^4)*(-5*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + x^4*(
4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*Appell
F1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)]))))/(20*c^2*d^2*(a + b*x^4
)^(3/4))
```

Maple [F] time = 0.253, size = 0, normalized size = 0.

$$\int \frac{1}{\left(d x^{4}+c\right)^{2}}\left(b x^{4}+a\right)^{\frac{9}{4}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(9/4)/(d*x^4+c)^2,x)
```

```
[Out] int((b*x^4+a)^(9/4)/(d*x^4+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b x^{4}+a\right)^{\frac{9}{4}}}{\left(d x^{4}+c\right)^{2}}\,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(9/4)/(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(9/4)/(d*x^4 + c)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(9/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(9/4)/(d*x**4+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(b x^{4}+a\right)^{\frac{9}{4}}}{\left(d x^{4}+c\right)^{2}}\,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(9/4)/(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(9/4)/(d*x^4 + c)^2, x)
```

3.212 $\int \frac{(a+bx^4)^{5/4}}{(c+dx^4)^2} \, dx$

Optimal. Leaf size=298

$$\frac{\sqrt{a}b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{4cd\left(a+bx^4\right)^{3/4}}+\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(3ad+2bc)\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d}+\cdots$$

```
[Out] -((b*c - a*d)*x*(a + b*x^4)^(1/4))/(4*c*d*(c + d*x^4)) + (Sqrt[a]*b^(3/2)*(
1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(4*
c*d*(a + b*x^4)^(3/4)) + ((2*b*c + 3*a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^
4]*EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a +
 b*x^4)^(1/4)], -1])/(8*b^(1/4)*c^2*d) + ((2*b*c + 3*a*d)*Sqrt[a/(a + b*x^4
)]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^
(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(8*b^(1/4)*c^2*d)
```

Rubi [A] time = 0.239795, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {413, 529, 237, 335, 275, 231, 407, 409, 1218}

$$\frac{\sqrt{a}b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{4cd\left(a+bx^4\right)^{3/4}}+\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(3ad+2bc)\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(5/4)/(c + d*x^4)^2,x]
```


```
[Out] -((b*c - a*d)*x*(a + b*x^4)^(1/4))/(4*c*d*(c + d*x^4)) + (Sqrt[a]*b^(3/2)*(
1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(4*
c*d*(a + b*x^4)^(3/4)) + ((2*b*c + 3*a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^
4]*EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a +
 b*x^4)^(1/4)], -1])/(8*b^(1/4)*c^2*d) + ((2*b*c + 3*a*d)*Sqrt[a/(a + b*x^4
)]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^
(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(8*b^(1/4)*c^2*d)
```


Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 529

```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^4)^(3/4), x],
 x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(a + b*x^4)^(1/4)/(c + d*x^4), x],
x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 407

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c -
 a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\left(a+bx^4\right)^{5/4}}{\left(c+dx^4\right)^2}\,dx = -\frac{(bc-ad)x\sqrt[4]{a+bx^4}}{4cd\left(c+dx^4\right)} + \frac{\int \frac{a(bc+3ad)+2b(bc+ad)x^4}{\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)}\,dx}{4cd}$$

$$= -\frac{(bc-ad)x\sqrt[4]{a+bx^4}}{4cd\left(c+dx^4\right)} - \frac{(ab)\int \frac{1}{\left(a+bx^4\right)^{3/4}}\,dx}{4cd} - \frac{(-2bc-3ad)\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx}{4cd}$$

$$= -\frac{(bc-ad)x\sqrt[4]{a+bx^4}}{4cd\left(c+dx^4\right)} - \frac{\left(ab\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{4cd\left(a+bx^4\right)^{3/4}} - \frac{\left((-2bc-3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subs}}{4c}$$

$$= -\frac{(bc-ad)x\sqrt[4]{a+bx^4}}{4cd\left(c+dx^4\right)} + \frac{\left(ab\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{4cd\left(a+bx^4\right)^{3/4}} - \frac{\left((-2bc-3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a}\right.}{}$$

$$= -\frac{(bc-ad)x\sqrt[4]{a+bx^4}}{4cd\left(c+dx^4\right)} + \frac{(2bc+3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2d} + \frac{(2bc+3ad}{}$$

$$= -\frac{(bc-ad)x\sqrt[4]{a+bx^4}}{4cd\left(c+dx^4\right)} + \frac{\sqrt{a}b^{3/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{4cd\left(a+bx^4\right)^{3/4}} + \frac{(2bc+3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\Gamma}{8\sqrt[4]{}}$$

Mathematica [C] time = 0.324814, size = 341, normalized size = 1.14

$$\frac{x\left(\frac{5c\left(x^4\left(a+bx^4\right)(ad-bc)\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)-5ac\left(4a^2d+abdx^4-b^2cx^4\right)F_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)}{\left(c+dx^4\right)\left(x^4\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)-5acF_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)}+2bx^4\left(\frac{bx^4}{a}+1\right)^{3/}\right.}{20c^2d\left(a+bx^4\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^4)^(5/4)/(c + d*x^4)^2,x]
```

```
[Out] (x*(2*b*(b*c + a*d)*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4, 1, 9/4, -(
(b*x^4)/a), -((d*x^4)/c)] + (5*c*(-5*a*c*(4*a^2*d - b^2*c*x^4 + a*b*d*x^4)*
AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + (-(b*c) + a*d)*x^4
*(a + b*x^4)*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)]
+ 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/((c + d*x
^4)*(-5*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + x^4*(4
*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF
1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))))/(20*c^2*d*(a + b*x^4)^
(3/4))
```

Maple [F] time = 0.403, size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^4+c\right)^2}\left(bx^4+a\right)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(5/4)/(d*x^4+c)^2,x)
```

```
[Out] int((b*x^4+a)^(5/4)/(d*x^4+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{5}{4}}}{(dx^4 + c)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(5/4)/(d*x^4 + c)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(5/4)/(d*x**4+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^4 + a)^{\frac{5}{4}}}{(dx^4 + c)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(5/4)/(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(5/4)/(d*x^4 + c)^2, x)
```

3.213 $\int \frac{\sqrt[4]{a+bx^4}}{\left(c+dx^4\right)^2} dx$

Optimal. Leaf size=308

$$-\frac{\sqrt{a} b^{3/2} x^3 \left(\frac{a}{bx^4}+1\right)^{3/4} \text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{4c\left(a+bx^4\right)^{3/4}(bc-ad)}+\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(2bc-3ad)\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2(bc-ad)}+ \cdots$$

```
[Out] (x*(a + b*x^4)^(1/4))/(4*c*(c + d*x^4)) - (Sqrt[a]*b^(3/2)*(1 + a/(b*x^4))^
(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(4*c*(b*c - a*d)*(
a + b*x^4)^(3/4)) + ((2*b*c - 3*a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*El
lipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^
4)^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a*d)) + ((2*b*c - 3*a*d)*Sqrt[a/(a +
b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSi
n[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a*d))
```

Rubi [A] time = 0.203448, antiderivative size = 308, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {412, 529, 237, 335, 275, 231, 407, 409, 1218}

$$-\frac{\sqrt{a} b^{3/2} x^3 \left(\frac{a}{bx^4}+1\right)^{3/4} F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{4c\left(a+bx^4\right)^{3/4}(bc-ad)}+\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(2bc-3ad)\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2(bc-ad)}+\frac{\sqrt{\frac{a}{a+bx^4}}\sqrt{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^(1/4)/(c + d*x^4)^2,x]
```

```
[Out] (x*(a + b*x^4)^(1/4))/(4*c*(c + d*x^4)) - (Sqrt[a]*b^(3/2)*(1 + a/(b*x^4))^
(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2])/(4*c*(b*c - a*d)*(
a + b*x^4)^(3/4)) + ((2*b*c - 3*a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*El
lipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^
4)^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a*d)) + ((2*b*c - 3*a*d)*Sqrt[a/(a +
b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSi
n[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a*d))
```

Rule 412

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] + Dist[1/(
a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p + 1)
 + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x]
&& NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 529

```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^4)^(3/4), x],
 x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(a + b*x^4)^(1/4)/(c + d*x^4), x],
x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 407

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c -
 a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sqrt[4]{a+bx^4}}{\left(c+dx^4\right)^2}\,dx = \frac{x\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)} - \frac{\int \frac{-3a-2bx^4}{\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)}\,dx}{4c}$$

$$= \frac{x\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)} + \frac{(ab)\int \frac{1}{\left(a+bx^4\right)^{3/4}}\,dx}{4c(bc-ad)} + \frac{(2bc-3ad)\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx}{4c(bc-ad)}$$

$$= \frac{x\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)} + \frac{\left(ab\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{4c(bc-ad)\left(a+bx^4\right)^{3/4}} + \frac{\left((2bc-3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subst}\left(\int \frac{}{\sqrt{1-bx^4}}\right.}{4c(bc-ad)}$$

$$= \frac{x\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)} - \frac{\left(ab\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{4c(bc-ad)\left(a+bx^4\right)^{3/4}} + \frac{\left((2bc-3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right)\text{Subs}}{8c^2(b}$$

$$= \frac{x\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)} + \frac{(2bc-3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2(bc-ad)} + \frac{(2bc-3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a}}{}$$

$$= \frac{x\sqrt[4]{a+bx^4}}{4c\left(c+dx^4\right)} - \frac{\sqrt{a}b^{3/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{4c(bc-ad)\left(a+bx^4\right)^{3/4}} + \frac{(2bc-3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\right.}{8\sqrt[4]{b}c^2(bc-ad)}$$

Mathematica [C] time = 0.211929, size = 233, normalized size = 0.76

$$\frac{x\left(\frac{5\left(\frac{a+bx^4}{c}-\frac{15a^2F_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{x^4\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)-5acF_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}\right)}{c+dx^4}+\frac{2bx^4\left(\frac{bx^4}{a}+1\right)^{3/4}F_1\left(\frac{5}{4};\frac{3}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{c^2}\right)}{20\left(a+bx^4\right)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^4)^(1/4)/(c + d*x^4)^2,x]
```

```
[Out] (x*((2*b*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4, 1, 9/4, -((b*x^4)/a),
 -((d*x^4)/c)])/c^2 + (5*((a + b*x^4)/c - (15*a^2*AppellF1[1/4, 3/4, 1, 5/4
, -((b*x^4)/a), -((d*x^4)/c)])/(-5*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)
/a), -((d*x^4)/c)] + x^4*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -(
(d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])
)))/(c + d*x^4)))/(20*(a + b*x^4)^(3/4))
```

Maple [F] time = 0.435, size = 0, normalized size = 0.

$$\int \frac{1}{\left(dx^4+c\right)^2}\sqrt[4]{bx^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(1/4)/(d*x^4+c)^2,x)
```

```
[Out] int((b*x^4+a)^(1/4)/(d*x^4+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)^{\frac{1}{4}}}{\left(dx^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^(1/4)/(d*x^4 + c)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[4]{a+bx^4}}{\left(c+dx^4\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(1/4)/(d*x**4+c)**2,x)
```

```
[Out] Integral((a + b*x**4)**(1/4)/(c + d*x**4)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^4+a\right)^{\frac{1}{4}}}{\left(dx^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/4)/(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^(1/4)/(d*x^4 + c)^2, x)
```

3.214 $\int \frac{1}{\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)^2}\,dx$

Optimal. Leaf size=330

$$-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(4bc-ad)\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{4\sqrt{a}c\left(a+bx^4\right)^{3/4}(bc-ad)^2}-\frac{3d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(2bc-ad)\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\right|}{8\sqrt[4]{b}c^2(bc-ad)^2}$$

```
[Out] -(d*x*(a + b*x^4)^(1/4))/(4*c*(b*c - a*d)*(c + d*x^4)) - (b^(3/2)*(4*b*c -
a*d)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2
])/(4*Sqrt[a]*c*(b*c - a*d)^2*(a + b*x^4)^(3/4)) - (3*d*(2*b*c - a*d)*Sqrt[
a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c
])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a*d)
^2) - (3*d*(2*b*c - a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqr
t[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])
/(8*b^(1/4)*c^2*(b*c - a*d)^2)
```

Rubi [A] time = 0.249539, antiderivative size = 330, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {414, 529, 237, 335, 275, 231, 407, 409, 1218}

$$-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}(4bc-ad)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{4\sqrt{a}c\left(a+bx^4\right)^{3/4}(bc-ad)^2}-\frac{3d\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(2bc-ad)\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^2}-\frac{3}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^(3/4)*(c + d*x^4)^2),x]
```

```
[Out] -(d*x*(a + b*x^4)^(1/4))/(4*c*(b*c - a*d)*(c + d*x^4)) - (b^(3/2)*(4*b*c -
a*d)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/Sqrt[a]]/2, 2
])/(4*Sqrt[a]*c*(b*c - a*d)^2*(a + b*x^4)^(3/4)) - (3*d*(2*b*c - a*d)*Sqrt[
a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c
])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a*d)
^2) - (3*d*(2*b*c - a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[Sqr
t[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])
/(8*b^(1/4)*c^2*(b*c - a*d)^2)
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 529

```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^4)^(3/4), x],
 x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(a + b*x^4)^(1/4)/(c + d*x^4), x],
x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 407

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c -
 a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)^2}\,dx = -\frac{dx\sqrt[4]{a+bx^4}}{4c(bc-ad)(c+dx^4)} + \frac{\int \frac{4bc-3ad-2bdx^4}{(a+bx^4)^{3/4}(c+dx^4)}\,dx}{4c(bc-ad)}$$

$$= -\frac{dx\sqrt[4]{a+bx^4}}{4c(bc-ad)(c+dx^4)} - \frac{(3d(2bc-ad))\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4}\,dx}{4c(bc-ad)^2} + \frac{(b(4bc-ad))\int \frac{1}{(a+bx^4)^{3/4}}\,dx}{4c(bc-ad)^2}$$

$$= -\frac{dx\sqrt[4]{a+bx^4}}{4c(bc-ad)(c+dx^4)} + \frac{\left(b(4bc-ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3}\,dx}{4c(bc-ad)^2(a+bx^4)^{3/4}} - \frac{\left(3d(2bc-ad)\sqrt{\right.}{}$$

$$= -\frac{dx\sqrt[4]{a+bx^4}}{4c(bc-ad)(c+dx^4)} - \frac{\left(b(4bc-ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right)\text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{3/4}}\,dx, x, \frac{1}{x}\right)}{4c(bc-ad)^2(a+bx^4)^{3/4}} - \frac{(3d(}{}$$

$$= -\frac{dx\sqrt[4]{a+bx^4}}{4c(bc-ad)(c+dx^4)} - \frac{3d(2bc-ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\,\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\middle|-1\right)}{8\sqrt[4]{b}c^2(bc-ad)^2}$$

$$= -\frac{dx\sqrt[4]{a+bx^4}}{4c(bc-ad)(c+dx^4)} - \frac{b^{3/2}(4bc-ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{4\sqrt{a}c(bc-ad)^2(a+bx^4)^{3/4}} - \frac{3d(2bc-a}{}$$

Mathematica [C] time = 0.340521, size = 337, normalized size = 1.02

$$\frac{x\left(\frac{2bdx^4\left(\frac{bx^4}{a}+1\right)^{3/4}F_1\left(\frac{5}{4};\frac{3}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{ad-bc} + \frac{c\left(25ac\left(4ad-4bc+bdx^4\right)F_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)-5dx^4\left(a+bx^4\right)\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\right.\right.\right.}{\left(c+dx^4\right)(bc-ad)\left(x^4\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)-5acF_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\right.\right.}\right)}{20c^2(a+bx^4)^{3/4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^4)^(3/4)*(c + d*x^4)^2),x]
```

```
[Out] (x*((2*b*d*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4, 1, 9/4, -((b*x^4)/a
), -((d*x^4)/c)])/(-(b*c) + a*d) + (c*(25*a*c*(-4*b*c + 4*a*d + b*d*x^4)*Ap
pellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] - 5*d*x^4*(a + b*x^4)*
(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*Appel
lF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/((b*c - a*d)*(c + d*x^
4)*(-5*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] + x^4*(4*
a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1
[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))))/(20*c^2*(a + b*x^4)^(3/
4))
```

Maple [F] time = 0.441, size = 0, normalized size = 0.

$$\int \frac{1}{(dx^4+c)^2}\left(bx^4+a\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(3/4)/(d*x^4+c)^2,x)
```

```
[Out] int(1/(b*x^4+a)^(3/4)/(d*x^4+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{3}{4}}\left(dx^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(3/4)/(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(3/4)*(d*x^4 + c)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(3/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(3/4)/(d*x**4+c)**2,x)
```

```
[Out] Exception raised: ValueError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^4+a\right)^{\frac{3}{4}}\left(dx^4+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(3/4)/(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(3/4)*(d*x^4 + c)^2), x)
```

3.215 $\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)^2}\,dx$

Optimal. Leaf size=390

$$-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\left(3a^2d^2-32abcd+8b^2c^2\right)\text{EllipticF}\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right),2\right)}{12a^{3/2}c\left(a+bx^4\right)^{3/4}(bc-ad)^3}+\frac{d^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(10bc-3ad)\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}\right.}{8\sqrt[4]{b}c^2(bc-ad)^3}$$

```
[Out] (b*(4*b*c + 3*a*d)*x)/(12*a*c*(b*c - a*d)^2*(a + b*x^4)^(3/4)) - (d*x)/(4*c
*(b*c - a*d)*(a + b*x^4)^(3/4)*(c + d*x^4)) - (b^(3/2)*(8*b^2*c^2 - 32*a*b*
c*d + 3*a^2*d^2)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/S
qrt[a]]/2, 2])/(12*a^(3/2)*c*(b*c - a*d)^3*(a + b*x^4)^(3/4)) + (d^2*(10*b*
c - 3*a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]
/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(8*b^(1/4)
*c^2*(b*c - a*d)^3) + (d^2*(10*b*c - 3*a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*
x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/(a +
b*x^4)^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a*d)^3)
```

Rubi [A] time = 0.406009, antiderivative size = 390, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {414, 527, 529, 237, 335, 275, 231, 407, 409, 1218}

$$-\frac{b^{3/2}x^3\left(\frac{a}{bx^4}+1\right)^{3/4}\left(3a^2d^2-32abcd+8b^2c^2\right)F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle|2\right)}{12a^{3/2}c\left(a+bx^4\right)^{3/4}(bc-ad)^3}+\frac{d^2\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}(10bc-3ad)\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}};\sin^{-1}\right.}{8\sqrt[4]{b}c^2(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^4)^(7/4)*(c + d*x^4)^2),x]
```

```
[Out] (b*(4*b*c + 3*a*d)*x)/(12*a*c*(b*c - a*d)^2*(a + b*x^4)^(3/4)) - (d*x)/(4*c
*(b*c - a*d)*(a + b*x^4)^(3/4)*(c + d*x^4)) - (b^(3/2)*(8*b^2*c^2 - 32*a*b*
c*d + 3*a^2*d^2)*(1 + a/(b*x^4))^(3/4)*x^3*EllipticF[ArcCot[(Sqrt[b]*x^2)/S
qrt[a]]/2, 2])/(12*a^(3/2)*c*(b*c - a*d)^3*(a + b*x^4)^(3/4)) + (d^2*(10*b*
c - 3*a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*x^4]*EllipticPi[-(Sqrt[b*c - a*d]
/(Sqrt[b]*Sqrt[c])), ArcSin[(b^(1/4)*x)/(a + b*x^4)^(1/4)], -1])/(8*b^(1/4)
*c^2*(b*c - a*d)^3) + (d^2*(10*b*c - 3*a*d)*Sqrt[a/(a + b*x^4)]*Sqrt[a + b*
x^4]*EllipticPi[Sqrt[b*c - a*d]/(Sqrt[b]*Sqrt[c]), ArcSin[(b^(1/4)*x)/(a +
b*x^4)^(1/4)], -1])/(8*b^(1/4)*c^2*(b*c - a*d)^3)
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
```

d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]

Rule 529

```
Int[((e_) + (f_.)*(x_)^4)/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4
)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^4)^(3/4), x],
 x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(a + b*x^4)^(1/4)/(c + d*x^4), x],
x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 231

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcTan[
Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a
, 0] && PosQ[b/a]
```

Rule 407

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] :> Dist[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c -
 a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] :> Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+bx^4\right)^{7/4}\left(c+dx^4\right)^2}\,dx = -\frac{dx}{4c(bc-ad)\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)} + \frac{\int \frac{4bc-3ad-6bdx^4}{\left(a+bx^4\right)^{7/4}\left(c+dx^4\right)}\,dx}{4c(bc-ad)}$$

$$= \frac{b(4bc+3ad)x}{12ac(bc-ad)^2\left(a+bx^4\right)^{3/4}} - \frac{dx}{4c(bc-ad)\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)} - \frac{\int \frac{-8b^2c^2+24abcd-9a^2d^2-2b\ldots}{\left(a+bx^4\right)^{3/4}\left(c+d\ldots\right)}}{12ac(bc-a\ldots}$$

$$= \frac{b(4bc+3ad)x}{12ac(bc-ad)^2\left(a+bx^4\right)^{3/4}} - \frac{dx}{4c(bc-ad)\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)} + \frac{\left(d^2(10bc-3ad)\right)\int \frac{\sqrt[4]{a\ldots}}{c+\ldots}}{4c(bc-ad)^3}$$

$$= \frac{b(4bc+3ad)x}{12ac(bc-ad)^2\left(a+bx^4\right)^{3/4}} - \frac{dx}{4c(bc-ad)\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)} + \frac{\left(b\left(8b^2c^2-32abcd+3\ldots\right.\right.}{12ac(\ldots}$$

$$= \frac{b(4bc+3ad)x}{12ac(bc-ad)^2\left(a+bx^4\right)^{3/4}} - \frac{dx}{4c(bc-ad)\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)} - \frac{\left(b\left(8b^2c^2-32abcd+3\ldots\right.\right.}{\ldots}$$

$$= \frac{b(4bc+3ad)x}{12ac(bc-ad)^2\left(a+bx^4\right)^{3/4}} - \frac{dx}{4c(bc-ad)\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)} + \frac{d^2(10bc-3ad)\sqrt{\frac{a}{a+bx^4}}\ldots}{\ldots}$$

$$= \frac{b(4bc+3ad)x}{12ac(bc-ad)^2\left(a+bx^4\right)^{3/4}} - \frac{dx}{4c(bc-ad)\left(a+bx^4\right)^{3/4}\left(c+dx^4\right)} - \frac{b^{3/2}\left(8b^2c^2-32abcd+\ldots\right.}{12a^{3\ldots}}$$

Mathematica [C] time = 0.535327, size = 387, normalized size = 0.99

$$\frac{x\left(\frac{c\left(25ac\left(12a^2d^2+3abd\left(dx^4-8c\right)+4b^2c\left(3c+dx^4\right)\right)F_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)-5x^4\left(3a^2d^2+3abd^2x^4+4b^2c\left(c+dx^4\right)\right)\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\frac{b\ldots}{\ldots}\ldots\right.\right.}{\left(c+dx^4\right)\left(5acF_1\left(\frac{1}{4};\frac{3}{4},1;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)-x^4\left(4adF_1\left(\frac{5}{4};\frac{3}{4},2;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+3bcF_1\left(\frac{5}{4};\frac{7}{4},1;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)\right)}\ldots\right)}{60ac^2\left(a+bx^4\right)^{3/4}(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^4)^(7/4)*(c + d*x^4)^2),x]
```

```
[Out] (x*(2*b*d*(4*b*c + 3*a*d)*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4, 1, 9
/4, -((b*x^4)/a), -((d*x^4)/c)] + (c*(25*a*c*(12*a^2*d^2 + 3*a*b*d*(-8*c +
d*x^4) + 4*b^2*c*(3*c + d*x^4))*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -(
(d*x^4)/c)] - 5*x^4*(3*a^2*d^2 + 3*a*b*d^2*x^4 + 4*b^2*c*(c + d*x^4))*(4*a*
d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5
/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/((c + d*x^4)*(5*a*c*AppellF
1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] - x^4*(4*a*d*AppellF1[5/4,
3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4,
 -((b*x^4)/a), -((d*x^4)/c)])))))/(60*a*c^2*(b*c - a*d)^2*(a + b*x^4)^(3/4)
)
```

Maple [F] time = 0.431, size = 0, normalized size = 0.

$$\int \frac{1}{\left(d x^{4}+c\right)^{2}}\left(b x^{4}+a\right)^{-\frac{7}{4}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^4+a)^(7/4)/(d*x^4+c)^2,x)
```

```
[Out] int(1/(b*x^4+a)^(7/4)/(d*x^4+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b x^{4}+a\right)^{\frac{7}{4}}\left(d x^{4}+c\right)^{2}}\,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(7/4)/(d*x^4+c)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^4 + a)^(7/4)*(d*x^4 + c)^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(7/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**4+a)**(7/4)/(d*x**4+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b x^{4}+a\right)^{\frac{7}{4}}\left(d x^{4}+c\right)^{2}}\,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^4+a)^(7/4)/(d*x^4+c)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^4 + a)^(7/4)*(d*x^4 + c)^2), x)
```

3.216 $\int \frac{1}{\sqrt[4]{1+x^4}(2+x^4)}\,dx$

Optimal. Leaf size=53

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{2\ 2^{3/4}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{2\ 2^{3/4}}$$

```
[Out] ArcTan[x/(2^(1/4)*(1 + x^4)^(1/4))]/(2*2^(3/4)) + ArcTanh[x/(2^(1/4)*(1 + x
^4)^(1/4))]/(2*2^(3/4))
```

Rubi [A] time = 0.018443, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {377, 212, 206, 203}

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{2\ 2^{3/4}}+\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{2\ 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^4)^(1/4)*(2 + x^4)),x]
```

```
[Out] ArcTan[x/(2^(1/4)*(1 + x^4)^(1/4))]/(2*2^(3/4)) + ArcTanh[x/(2^(1/4)*(1 + x
^4)^(1/4))]/(2*2^(3/4))
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{1+x^4}\left(2+x^4\right)}\,dx = \text{Subst}\left(\int \frac{1}{2-x^4}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2}-x^2}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right)}{2\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{2}+x^2}\,dx, x, \frac{x}{\sqrt[4]{1+x^4}}\right)}{2\sqrt{2}}$$
$$= \frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{1+x^4}}\right)}{2\ 2^{3/4}} + \frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{1+x^4}}\right)}{2\ 2^{3/4}}$$

Mathematica [A] time = 0.0147199, size = 44, normalized size = 0.83

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right) + \tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{2\ 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^4)^(1/4)*(2 + x^4)),x]
```

```
[Out] (ArcTan[x/(2^(1/4)*(1 + x^4)^(1/4))] + ArcTanh[x/(2^(1/4)*(1 + x^4)^(1/4))]
)/(2*2^(3/4))
```

Maple [F] time = 0.388, size = 0, normalized size = 0.

$$\int \frac{1}{x^4+2}\frac{1}{\sqrt[4]{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+2)/(x^4+1)^(1/4),x)
```

```
[Out] int(1/(x^4+2)/(x^4+1)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^4+2\right)\left(x^4+1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)^(1/4)/(x^4+2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + 2)*(x^4 + 1)^(1/4)), x)
```

Fricas [B] time = 44.1397, size = 583, normalized size = 11.

$$-\frac{1}{16}\cdot 8^{\frac{3}{4}}\arctan\left(-\frac{8^{\frac{3}{4}}\left(x^4+1\right)^{\frac{1}{4}}x^3+4\cdot 8^{\frac{1}{4}}\left(x^4+1\right)^{\frac{3}{4}}x-2^{\frac{1}{4}}\left(8^{\frac{3}{4}}\sqrt{x^4+1}x^2+8^{\frac{1}{4}}\left(3x^4+2\right)\right)}{2\left(x^4+2\right)}\right)+\frac{1}{64}\cdot 8^{\frac{3}{4}}\log\left(\frac{8\sqrt{2}\left(x^4+1\right)^{\frac{1}{4}}x}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)^(1/4)/(x^4+2),x, algorithm="fricas")
```

```
[Out] -1/16*8^(3/4)*arctan(-1/2*(8^(3/4)*(x^4 + 1)^(1/4)*x^3 + 4*8^(1/4)*(x^4 + 1
)^(3/4)*x - 2^(1/4)*(8^(3/4)*sqrt(x^4 + 1)*x^2 + 8^(1/4)*(3*x^4 + 2)))/(x^4
 + 2)) + 1/64*8^(3/4)*log((8*sqrt(2)*(x^4 + 1)^(1/4)*x^3 + 8*8^(1/4)*sqrt(x
^4 + 1)*x^2 + 8^(3/4)*(3*x^4 + 2) + 16*(x^4 + 1)^(3/4)*x)/(x^4 + 2)) - 1/64
*8^(3/4)*log((8*sqrt(2)*(x^4 + 1)^(1/4)*x^3 - 8*8^(1/4)*sqrt(x^4 + 1)*x^2 -
 8^(3/4)*(3*x^4 + 2) + 16*(x^4 + 1)^(3/4)*x)/(x^4 + 2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{x^4+1}\left(x^4+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+1)**(1/4)/(x**4+2),x)
```

```
[Out] Integral(1/((x**4 + 1)**(1/4)*(x**4 + 2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^4+2\right)\left(x^4+1\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)^(1/4)/(x^4+2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + 2)*(x^4 + 1)^(1/4)), x)
```

3.217 $\int \frac{1}{\left(a-(a-b)x^4\right)\sqrt[4]{a+bx^4}}\,dx$

Optimal. Leaf size=57

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}$$

```
[Out] ArcTan[(a^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*a^(5/4)) + ArcTanh[(a^(1/4)*x)/(a
+ b*x^4)^(1/4)]/(2*a^(5/4))
```

Rubi [A] time = 0.0223042, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {377, 212, 206, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a - (a - b)*x^4)*(a + b*x^4)^(1/4)),x]
```

```
[Out] ArcTan[(a^(1/4)*x)/(a + b*x^4)^(1/4)]/(2*a^(5/4)) + ArcTanh[(a^(1/4)*x)/(a
+ b*x^4)^(1/4)]/(2*a^(5/4))
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(a-(a-b)x^4\right)\sqrt[4]{a+bx^4}}\,dx = \text{Subst}\left(\int \frac{1}{a-(ab-a(-a+b))x^4}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-\sqrt{a}x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2a} + \frac{\text{Subst}\left(\int \frac{1}{1+\sqrt{a}x^2}\,dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2a}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}$$

Mathematica [A] time = 0.0199442, size = 48, normalized size = 0.84

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{a+bx^4}}\right)+\tanh^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a - (a - b)*x^4)*(a + b*x^4)^(1/4)),x]
```

```
[Out] (ArcTan[(a^(1/4)*x)/(a + b*x^4)^(1/4)] + ArcTanh[(a^(1/4)*x)/(a + b*x^4)^(1
/4)])/(2*a^(5/4))
```

Maple [F] time = 0.457, size = 0, normalized size = 0.

$$\int \frac{1}{a-(a-b)\,x^4}\frac{1}{\sqrt[4]{bx^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-(a-b)*x^4)/(b*x^4+a)^(1/4),x)
```

```
[Out] int(1/(a-(a-b)*x^4)/(b*x^4+a)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\left((a-b)x^4-a\right)\left(bx^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-(a-b)*x^4)/(b*x^4+a)^(1/4),x, algorithm="maxima")
```

```
[Out] -integrate(1/(((a - b)*x^4 - a)*(b*x^4 + a)^(1/4)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-(a-b)*x^4)/(b*x^4+a)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{ax^4\sqrt[4]{a+bx^4}-a\sqrt[4]{a+bx^4}-bx^4\sqrt[4]{a+bx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-(a-b)*x**4)/(b*x**4+a)**(1/4),x)
```

```
[Out] -Integral(1/(a*x**4*(a + b*x**4)**(1/4) - a*(a + b*x**4)**(1/4) - b*x**4*(a
 + b*x**4)**(1/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left((a-b)x^4-a\right)\left(bx^4+a\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-(a-b)*x^4)/(b*x^4+a)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(-1/(((a - b)*x^4 - a)*(b*x^4 + a)^(1/4)), x)
```

3.218 $\int \left(a+bx^4\right)^p \left(c+dx^4\right)^q \, dx$

Optimal. Leaf size=79

$$x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\left(c+dx^4\right)^q\left(\frac{dx^4}{c}+1\right)^{-q}F_1\left(\frac{1}{4};-p,-q;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)$$

```
[Out] (x*(a + b*x^4)^p*(c + d*x^4)^q*AppellF1[1/4, -p, -q, 5/4, -((b*x^4)/a), -((
d*x^4)/c)])/((1 + (b*x^4)/a)^p*(1 + (d*x^4)/c)^q)
```

Rubi [A] time = 0.0472042, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {430, 429}

$$x\left(a+bx^4\right)^p\left(\frac{bx^4}{a}+1\right)^{-p}\left(c+dx^4\right)^q\left(\frac{dx^4}{c}+1\right)^{-q}F_1\left(\frac{1}{4};-p,-q;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^p*(c + d*x^4)^q,x]
```

```
[Out] (x*(a + b*x^4)^p*(c + d*x^4)^q*AppellF1[1/4, -p, -q, 5/4, -((b*x^4)/a), -((
d*x^4)/c)])/((1 + (b*x^4)/a)^p*(1 + (d*x^4)/c)^q)
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \left(a+bx^4\right)^p\left(c+dx^4\right)^q \, dx &= \left(\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\right)\int \left(1+\frac{bx^4}{a}\right)^p\left(c+dx^4\right)^q \, dx\\
&= \left(\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\left(c+dx^4\right)^q\left(1+\frac{dx^4}{c}\right)^{-q}\right)\int \left(1+\frac{bx^4}{a}\right)^p\left(1+\frac{dx^4}{c}\right)^q \, dx\\
&= x\left(a+bx^4\right)^p\left(1+\frac{bx^4}{a}\right)^{-p}\left(c+dx^4\right)^q\left(1+\frac{dx^4}{c}\right)^{-q}F_1\left(\frac{1}{4};-p,-q;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)
\end{aligned}$$

Mathematica [B] time = 0.227013, size = 172, normalized size = 2.18

$$\frac{5acx\left(a+bx^4\right)^p\left(c+dx^4\right)^qF_1\left(\frac{1}{4};-p,-q;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{4x^4\left(bcpF_1\left(\frac{5}{4};1-p,-q;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)+adqF_1\left(\frac{5}{4};-p,1-q;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)+5acF_1\left(\frac{1}{4};-p,-q;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^4)^p*(c + d*x^4)^q,x]
```

```
[Out] (5*a*c*x*(a + b*x^4)^p*(c + d*x^4)^q*AppellF1[1/4, -p, -q, 5/4, -((b*x^4)/a
), -((d*x^4)/c)])/(5*a*c*AppellF1[1/4, -p, -q, 5/4, -((b*x^4)/a), -((d*x^4)
/c)] + 4*x^4*(b*c*p*AppellF1[5/4, 1 - p, -q, 9/4, -((b*x^4)/a), -((d*x^4)/c
)] + a*d*q*AppellF1[5/4, -p, 1 - q, 9/4, -((b*x^4)/a), -((d*x^4)/c)]))
```

Maple [F] time = 0.477, size = 0, normalized size = 0.

$$\int \left(bx^4+a\right)^p \left(dx^4+c\right)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^p*(d*x^4+c)^q,x)
```

```
[Out] int((b*x^4+a)^p*(d*x^4+c)^q,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^4+a\right)^p \left(dx^4+c\right)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^p*(d*x^4+c)^q,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^p*(d*x^4 + c)^q, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^4+a\right)^p \left(dx^4+c\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^p*(d*x^4+c)^q,x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)^p*(d*x^4 + c)^q, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**p*(d*x**4+c)**q,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^4+a\right)^p \left(dx^4+c\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^p*(d*x^4+c)^q,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^p*(d*x^4 + c)^q, x)
```

3.219 $\int \left(a+bx^4\right)^2 \left(c+dx^4\right)^q \, dx$

Optimal. Leaf size=176

$$\frac{x\left(c+dx^4\right)^q\left(\frac{dx^4}{c}+1\right)^{-q}\left(a^2d^2\left(16q^2+56q+45\right)-2abcd(4q+9)+5b^2c^2\right){}_2F_1\left(\frac{1}{4},-q;\frac{5}{4};-\frac{dx^4}{c}\right)}{d^2(4q+5)(4q+9)}-\frac{bx\left(c+dx^4\right)^{q+1}(5bc-a}{d^2(4q+5)(4q+}$$

```
[Out] -((b*(5*b*c - a*d*(13 + 4*q))*x*(c + d*x^4)^(1 + q))/(d^2*(5 + 4*q)*(9 + 4*
q))) + (b*x*(a + b*x^4)*(c + d*x^4)^(1 + q))/(d*(9 + 4*q)) + ((5*b^2*c^2 -
2*a*b*c*d*(9 + 4*q) + a^2*d^2*(45 + 56*q + 16*q^2))*x*(c + d*x^4)^q*Hyperge
ometric2F1[1/4, -q, 5/4, -((d*x^4)/c)])/(d^2*(5 + 4*q)*(9 + 4*q)*(1 + (d*x^
4)/c)^q)
```

Rubi [A] time = 0.133064, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {416, 388, 246, 245}

$$\frac{x\left(c+dx^4\right)^q\left(\frac{dx^4}{c}+1\right)^{-q}\left(a^2d^2\left(16q^2+56q+45\right)-2abcd(4q+9)+5b^2c^2\right){}_2F_1\left(\frac{1}{4},-q;\frac{5}{4};-\frac{dx^4}{c}\right)}{d^2(4q+5)(4q+9)}-\frac{bx\left(c+dx^4\right)^{q+1}(5bc-a}{d^2(4q+5)(4q+}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)^2*(c + d*x^4)^q,x]
```


```
[Out] -((b*(5*b*c - a*d*(13 + 4*q))*x*(c + d*x^4)^(1 + q))/(d^2*(5 + 4*q)*(9 + 4*
q))) + (b*x*(a + b*x^4)*(c + d*x^4)^(1 + q))/(d*(9 + 4*q)) + ((5*b^2*c^2 -
2*a*b*c*d*(9 + 4*q) + a^2*d^2*(45 + 56*q + 16*q^2))*x*(c + d*x^4)^q*Hyperge
ometric2F1[1/4, -q, 5/4, -((d*x^4)/c)])/(d^2*(5 + 4*q)*(9 + 4*q)*(1 + (d*x^
4)/c)^q)
```


Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \left(a+bx^4\right)^2\left(c+dx^4\right)^q\,dx = \frac{bx\left(a+bx^4\right)\left(c+dx^4\right)^{1+q}}{d(9+4q)} + \frac{\int \left(c+dx^4\right)^q\left(-a(bc-ad(9+4q))-b(5bc-ad(13+4q))x^4\right.}{d(9+4q)}$$

$$= -\frac{b(5bc-ad(13+4q))x\left(c+dx^4\right)^{1+q}}{d^2(5+4q)(9+4q)} + \frac{bx\left(a+bx^4\right)\left(c+dx^4\right)^{1+q}}{d(9+4q)} + \frac{\left(5b^2c^2-2abcd(9+\right.}{}$$

$$= -\frac{b(5bc-ad(13+4q))x\left(c+dx^4\right)^{1+q}}{d^2(5+4q)(9+4q)} + \frac{bx\left(a+bx^4\right)\left(c+dx^4\right)^{1+q}}{d(9+4q)} + \frac{\left(\left(5b^2c^2-2abcd(9+\right.\right.}{}$$

$$= -\frac{b(5bc-ad(13+4q))x\left(c+dx^4\right)^{1+q}}{d^2(5+4q)(9+4q)} + \frac{bx\left(a+bx^4\right)\left(c+dx^4\right)^{1+q}}{d(9+4q)} + \frac{\left(5b^2c^2-2abcd(9+\right.}{}$$

Mathematica [A] time = 0.052535, size = 106, normalized size = 0.6

$$\frac{1}{45}x\left(c+dx^4\right)^q\left(\frac{dx^4}{c}+1\right)^{-q}\left(45a^2\,{}_2F_1\left(\frac{1}{4},-q;\frac{5}{4};-\frac{dx^4}{c}\right)+bx^4\left(18a\,{}_2F_1\left(\frac{5}{4},-q;\frac{9}{4};-\frac{dx^4}{c}\right)+5bx^4\,{}_2F_1\left(\frac{9}{4},-q;\frac{13}{4};-\frac{dx^4}{c}\right)\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)^2*(c + d*x^4)^q,x]
```

```
[Out] (x*(c + d*x^4)^q*(45*a^2*Hypergeometric2F1[1/4, -q, 5/4, -((d*x^4)/c)] + b*
x^4*(18*a*Hypergeometric2F1[5/4, -q, 9/4, -((d*x^4)/c)] + 5*b*x^4*Hypergeom
etric2F1[9/4, -q, 13/4, -((d*x^4)/c)])))/(45*(1 + (d*x^4)/c)^q)
```

Maple [F] time = 0.361, size = 0, normalized size = 0.

$$\int \left(bx^4+a\right)^2\left(dx^4+c\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^2*(d*x^4+c)^q,x)
```

```
[Out] int((b*x^4+a)^2*(d*x^4+c)^q,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^4+a\right)^2\left(dx^4+c\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^q,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)^2*(d*x^4 + c)^q, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2x^8 + 2\,abx^4 + a^2\right)\left(dx^4 + c\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^q,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^8 + 2*a*b*x^4 + a^2)*(d*x^4 + c)^q, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**2*(d*x**4+c)**q,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^4 + a\right)^2 \left(dx^4 + c\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2*(d*x^4+c)^q,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)^2*(d*x^4 + c)^q, x)
```

3.220 $\int \left(a+bx^4\right)\left(c+dx^4\right)^q\,dx$

Optimal. Leaf size=93

$$\frac{bx\left(c+dx^4\right)^{q+1}}{d(4q+5)}-\frac{x\left(c+dx^4\right)^q\left(\frac{dx^4}{c}+1\right)^{-q}(bc-ad(4q+5))\,{}_2F_1\left(\frac{1}{4},-q;\frac{5}{4};-\frac{dx^4}{c}\right)}{d(4q+5)}$$

```
[Out] (b*x*(c + d*x^4)^(1 + q))/(d*(5 + 4*q)) - ((b*c - a*d*(5 + 4*q))*x*(c + d*x
^4)^q*Hypergeometric2F1[1/4, -q, 5/4, -((d*x^4)/c)])/(d*(5 + 4*q)*(1 + (d*x
^4)/c)^q)
```

Rubi [A] time = 0.0407276, antiderivative size = 85, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {388, 246, 245}

$$x\left(c+dx^4\right)^q\left(\frac{dx^4}{c}+1\right)^{-q}\left(a-\frac{bc}{4dq+5d}\right){}_2F_1\left(\frac{1}{4},-q;\frac{5}{4};-\frac{dx^4}{c}\right)+\frac{bx\left(c+dx^4\right)^{q+1}}{d(4q+5)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^4)*(c + d*x^4)^q,x]
```

```
[Out] (b*x*(c + d*x^4)^(1 + q))/(d*(5 + 4*q)) + ((a - (b*c)/(5*d + 4*d*q))*x*(c +
 d*x^4)^q*Hypergeometric2F1[1/4, -q, 5/4, -((d*x^4)/c)])/(1 + (d*x^4)/c)^q
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \left(a+bx^4\right)\left(c+dx^4\right)^q dx = \frac{bx\left(c+dx^4\right)^{1+q}}{d(5+4q)} - \left(-a+\frac{bc}{5d+4dq}\right)\int \left(c+dx^4\right)^q dx$$
$$= \frac{bx\left(c+dx^4\right)^{1+q}}{d(5+4q)} - \left(\left(-a+\frac{bc}{5d+4dq}\right)\left(c+dx^4\right)^q\left(1+\frac{dx^4}{c}\right)^{-q}\right)\int \left(1+\frac{dx^4}{c}\right)^q dx$$
$$= \frac{bx\left(c+dx^4\right)^{1+q}}{d(5+4q)} + \left(a-\frac{bc}{5d+4dq}\right)x\left(c+dx^4\right)^q\left(1+\frac{dx^4}{c}\right)^{-q} {}_2F_1\left(\frac{1}{4},-q;\frac{5}{4};-\frac{dx^4}{c}\right)$$

Mathematica [A] time = 0.0299453, size = 90, normalized size = 0.97

$$\frac{x\left(c+dx^4\right)^q\left(\frac{dx^4}{c}+1\right)^{-q}\left((ad(4q+5)-bc)\,{}_2F_1\left(\frac{1}{4},-q;\frac{5}{4};-\frac{dx^4}{c}\right)+b\left(c+dx^4\right)\left(\frac{dx^4}{c}+1\right)^q\right)}{d(4q+5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^4)*(c + d*x^4)^q,x]
```

```
[Out] (x*(c + d*x^4)^q*(b*(c + d*x^4)*(1 + (d*x^4)/c)^q + (-(b*c) + a*d*(5 + 4*q)
)*Hypergeometric2F1[1/4, -q, 5/4, -((d*x^4)/c)]))/(d*(5 + 4*q)*(1 + (d*x^4)
/c)^q)
```

Maple [F] time = 0.212, size = 0, normalized size = 0.

$$\int \left(bx^4+a\right)\left(dx^4+c\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)*(d*x^4+c)^q,x)
```

```
[Out] int((b*x^4+a)*(d*x^4+c)^q,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^4+a\right)\left(dx^4+c\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c)^q,x, algorithm="maxima")
```

```
[Out] integrate((b*x^4 + a)*(d*x^4 + c)^q, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^4+a\right)\left(dx^4+c\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c)^q,x, algorithm="fricas")
```

```
[Out] integral((b*x^4 + a)*(d*x^4 + c)^q, x)
```

Sympy [C] time = 92.1663, size = 75, normalized size = 0.81

$$\frac{ac^q x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4}, -q\\ \frac{5}{4}\end{matrix}\middle|\frac{dx^4e^{i\pi}}{c}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{bc^q x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4}, -q\\ \frac{9}{4}\end{matrix}\middle|\frac{dx^4e^{i\pi}}{c}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)*(d*x**4+c)**q,x)
```

```
[Out] a*c**q*x*gamma(1/4)*hyper((1/4, -q), (5/4,), d*x**4*exp_polar(I*pi)/c)/(4*g
amma(5/4)) + b*c**q*x**5*gamma(5/4)*hyper((5/4, -q), (9/4,), d*x**4*exp_pol
ar(I*pi)/c)/(4*gamma(9/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx^4+a\right)\left(dx^4+c\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)*(d*x^4+c)^q,x, algorithm="giac")
```

```
[Out] integrate((b*x^4 + a)*(d*x^4 + c)^q, x)
```

3.221 $\int \frac{\left(c+dx^4\right)^q}{a+bx^4}\,dx$

Optimal. Leaf size=57

$$\frac{x\left(c+dx^4\right)^q\left(\frac{dx^4}{c}+1\right)^{-q}F_1\left(\frac{1}{4};1,-q;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a}$$

```
[Out] (x*(c + d*x^4)^q*AppellF1[1/4, 1, -q, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/(a*
(1 + (d*x^4)/c)^q)
```

Rubi [A] time = 0.0286241, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {430, 429}

$$\frac{x\left(c+dx^4\right)^q\left(\frac{dx^4}{c}+1\right)^{-q}F_1\left(\frac{1}{4};1,-q;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^4)^q/(a + b*x^4),x]
```

```
[Out] (x*(c + d*x^4)^q*AppellF1[1/4, 1, -q, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/(a*
(1 + (d*x^4)/c)^q)
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(c+dx^4\right)^q}{a+bx^4}\,dx &= \left(\left(c+dx^4\right)^q\left(1+\frac{dx^4}{c}\right)^{-q}\right)\int \frac{\left(1+\frac{dx^4}{c}\right)^q}{a+bx^4}\,dx\\ &= \frac{x\left(c+dx^4\right)^q\left(1+\frac{dx^4}{c}\right)^{-q}F_1\left(\frac{1}{4};1,-q;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a}\end{aligned}$$

Mathematica [B] time = 0.173234, size = 162, normalized size = 2.84

$$\frac{5acx\left(c+dx^4\right)^qF_1\left(\frac{1}{4};-q,1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)}{\left(a+bx^4\right)\left(4x^4\left(adqF_1\left(\frac{5}{4};1-q,1;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)-bcF_1\left(\frac{5}{4};-q,2;\frac{9}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)+5acF_1\left(\frac{1}{4};-q,1;\frac{5}{4};-\frac{dx^4}{c},-\frac{bx^4}{a}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^4)^q/(a + b*x^4),x]
```

```
[Out] (5*a*c*x*(c + d*x^4)^q*AppellF1[1/4, -q, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)
])/((a + b*x^4)*(5*a*c*AppellF1[1/4, -q, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)
] + 4*x^4*(a*d*q*AppellF1[5/4, 1 - q, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)] -
 b*c*AppellF1[5/4, -q, 2, 9/4, -((d*x^4)/c), -((b*x^4)/a)])))
```

Maple [F] time = 0.414, size = 0, normalized size = 0.

$$\int \frac{\left(d x^{4}+c\right)^{q}}{b x^{4}+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^q/(b*x^4+a),x)
```

```
[Out] int((d*x^4+c)^q/(b*x^4+a),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d x^{4}+c\right)^{q}}{b x^{4}+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^q/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^4 + c)^q/(b*x^4 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(d x^{4}+c\right)^{q}}{b x^{4}+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^q/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] integral((d*x^4 + c)^q/(b*x^4 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**q/(b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^4 + c\right)^q}{bx^4 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^q/(b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate((d*x^4 + c)^q/(b*x^4 + a), x)
```

3.222 $\int \frac{(c+dx^4)^q}{(a+bx^4)^2}\,dx$

Optimal. Leaf size=57

$$\frac{x\left(c+dx^4\right)^q\left(\frac{dx^4}{c}+1\right)^{-q}F_1\left(\frac{1}{4};2,-q;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2}$$

```
[Out] (x*(c + d*x^4)^q*AppellF1[1/4, 2, -q, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/(a^
2*(1 + (d*x^4)/c)^q)
```

Rubi [A] time = 0.0276523, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {430, 429}

$$\frac{x\left(c+dx^4\right)^q\left(\frac{dx^4}{c}+1\right)^{-q}F_1\left(\frac{1}{4};2,-q;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^4)^q/(a + b*x^4)^2,x]
```

```
[Out] (x*(c + d*x^4)^q*AppellF1[1/4, 2, -q, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/(a^
2*(1 + (d*x^4)/c)^q)
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(c+dx^4\right)^q}{\left(a+bx^4\right)^2}\,dx &= \left(\left(c+dx^4\right)^q\left(1+\frac{dx^4}{c}\right)^{-q}\right)\int \frac{\left(1+\frac{dx^4}{c}\right)^q}{\left(a+bx^4\right)^2}\,dx\\
&= \frac{x\left(c+dx^4\right)^q\left(1+\frac{dx^4}{c}\right)^{-q}F_1\left(\frac{1}{4};2,-q;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{a^2}
\end{aligned}$$

Mathematica [B] time = 0.187785, size = 162, normalized size = 2.84

$$\frac{5acx\left(c+dx^4\right)^q F_1\left(\frac{1}{4};2,-q;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)}{\left(a+bx^4\right)^2\left(4x^4\left(adqF_1\left(\frac{5}{4};2,1-q;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)-2bcF_1\left(\frac{5}{4};3,-q;\frac{9}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)+5acF_1\left(\frac{1}{4};2,-q;\frac{5}{4};-\frac{bx^4}{a},-\frac{dx^4}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^4)^q/(a + b*x^4)^2,x]
```

```
[Out] (5*a*c*x*(c + d*x^4)^q*AppellF1[1/4, 2, -q, 5/4, -((b*x^4)/a), -((d*x^4)/c)
])/((a + b*x^4)^2*(5*a*c*AppellF1[1/4, 2, -q, 5/4, -((b*x^4)/a), -((d*x^4)/
c)] + 4*x^4*(a*d*q*AppellF1[5/4, 2, 1 - q, 9/4, -((b*x^4)/a), -((d*x^4)/c)]
 - 2*b*c*AppellF1[5/4, 3, -q, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))
```

Maple [F] time = 0.413, size = 0, normalized size = 0.

$$\int \frac{\left(dx^4+c\right)^q}{\left(bx^4+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^q/(b*x^4+a)^2,x)
```

```
[Out] int((d*x^4+c)^q/(b*x^4+a)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^4+c\right)^q}{\left(bx^4+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^q/(b*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^4 + c)^q/(b*x^4 + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(dx^4+c\right)^q}{b^2x^8+2\,abx^4+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^q/(b*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x^4 + c)^q/(b^2*x^8 + 2*a*b*x^4 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**q/(b*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^4 + c)^q}{(bx^4 + a)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^q/(b*x^4+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^4 + c)^q/(b*x^4 + a)^2, x)
```

3.223 $\int \frac{1}{\sqrt[5]{a+bx^5}\left(c+dx^5\right)} dx$

Optimal. Leaf size=545

$$-\frac{\log\left(\sqrt[5]{c}-\frac{x\sqrt[5]{bc-ad}}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}}+\frac{\left(1-\sqrt{5}\right)\log\left(\frac{2c^{2/5}\left(a+bx^5\right)^{2/5}+2x^2(bc-ad)^{2/5}-\sqrt{5}\sqrt[5]{c}x\sqrt[5]{a+bx^5}\sqrt[5]{bc-ad}+\sqrt[5]{c}x\sqrt[5]{a+bx^5}\sqrt[5]{bc-ad}}{\left(a+bx^5\right)^{2/5}}\right)}{20c^{4/5}\sqrt[5]{bc-ad}}+\frac{\left(1+\sqrt{5}\right)\log\left(\frac{2c^{2/5}\left(\right.}{}\right.}{}$$

```
[Out] -(Sqrt[(5 + Sqrt[5])/2]*ArcTan[Sqrt[(5 - 2*Sqrt[5])/5] - (2*Sqrt[2/(5 + Sqr
t[5])]*(b*c - a*d)^(1/5)*x)/(c^(1/5)*(a + b*x^5)^(1/5))])/(5*c^(4/5)*(b*c -
 a*d)^(1/5)) + (Sqrt[(5 - Sqrt[5])/2]*ArcTan[Sqrt[(5 + 2*Sqrt[5])/5] + (Sqr
t[(2*(5 + Sqrt[5]))/5]*(b*c - a*d)^(1/5)*x)/(c^(1/5)*(a + b*x^5)^(1/5))])/(
5*c^(4/5)*(b*c - a*d)^(1/5)) - Log[c^(1/5) - ((b*c - a*d)^(1/5)*x)/(a + b*x
^5)^(1/5)]/(5*c^(4/5)*(b*c - a*d)^(1/5)) + ((1 - Sqrt[5])*Log[(2*(b*c - a*d
)^(2/5)*x^2 + c^(1/5)*(b*c - a*d)^(1/5)*x*(a + b*x^5)^(1/5) - Sqrt[5]*c^(1/
5)*(b*c - a*d)^(1/5)*x*(a + b*x^5)^(1/5) + 2*c^(2/5)*(a + b*x^5)^(2/5))/(a
+ b*x^5)^(2/5)])/(20*c^(4/5)*(b*c - a*d)^(1/5)) + ((1 + Sqrt[5])*Log[(2*(b*
c - a*d)^(2/5)*x^2 + c^(1/5)*(b*c - a*d)^(1/5)*x*(a + b*x^5)^(1/5) + Sqrt[5
]*c^(1/5)*(b*c - a*d)^(1/5)*x*(a + b*x^5)^(1/5) + 2*c^(2/5)*(a + b*x^5)^(2/
5))/(a + b*x^5)^(2/5)])/(20*c^(4/5)*(b*c - a*d)^(1/5))
```

Rubi [A] time = 1.08507, antiderivative size = 545, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {377, 202, 634, 618, 204, 628, 31}

$$-\frac{\log\left(\sqrt[5]{c}-\frac{x\sqrt[5]{bc-ad}}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}}+\frac{\left(1-\sqrt{5}\right)\log\left(\frac{2c^{2/5}\left(a+bx^5\right)^{2/5}+2x^2(bc-ad)^{2/5}-\sqrt{5}\sqrt[5]{c}x\sqrt[5]{a+bx^5}\sqrt[5]{bc-ad}+\sqrt[5]{c}x\sqrt[5]{a+bx^5}\sqrt[5]{bc-ad}}{\left(a+bx^5\right)^{2/5}}\right)}{20c^{4/5}\sqrt[5]{bc-ad}}+\frac{\left(1+\sqrt{5}\right)\log\left(\frac{2c^{2/5}\left(\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^5)^(1/5)*(c + d*x^5)),x]
```


```
[Out] -(Sqrt[(5 + Sqrt[5])/2]*ArcTan[Sqrt[(5 - 2*Sqrt[5])/5] - (2*Sqrt[2/(5 + Sqr
t[5])]*(b*c - a*d)^(1/5)*x)/(c^(1/5)*(a + b*x^5)^(1/5))])/(5*c^(4/5)*(b*c -
 a*d)^(1/5)) + (Sqrt[(5 - Sqrt[5])/2]*ArcTan[Sqrt[(5 + 2*Sqrt[5])/5] + (Sqr
t[(2*(5 + Sqrt[5]))/5]*(b*c - a*d)^(1/5)*x)/(c^(1/5)*(a + b*x^5)^(1/5))])/(
5*c^(4/5)*(b*c - a*d)^(1/5)) - Log[c^(1/5) - ((b*c - a*d)^(1/5)*x)/(a + b*x
^5)^(1/5)]/(5*c^(4/5)*(b*c - a*d)^(1/5)) + ((1 - Sqrt[5])*Log[(2*(b*c - a*d
)^(2/5)*x^2 + c^(1/5)*(b*c - a*d)^(1/5)*x*(a + b*x^5)^(1/5) - Sqrt[5]*c^(1/
5)*(b*c - a*d)^(1/5)*x*(a + b*x^5)^(1/5) + 2*c^(2/5)*(a + b*x^5)^(2/5))/(a
+ b*x^5)^(2/5)])/(20*c^(4/5)*(b*c - a*d)^(1/5)) + ((1 + Sqrt[5])*Log[(2*(b*
c - a*d)^(2/5)*x^2 + c^(1/5)*(b*c - a*d)^(1/5)*x*(a + b*x^5)^(1/5) + Sqrt[5
]*c^(1/5)*(b*c - a*d)^(1/5)*x*(a + b*x^5)^(1/5) + 2*c^(2/5)*(a + b*x^5)^(2/
5))/(a + b*x^5)^(2/5)])/(20*c^(4/5)*(b*c - a*d)^(1/5))
```


Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r + s*Cos[(
(2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]; (r
*Int[1/(r - s*x), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 1)/2}],
x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 3)/2, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt[5]{a+bx^5}\left(c+dx^5\right)}\,dx = \text{Subst}\left(\int \frac{1}{c-(bc-ad)x^5}\,dx, x, \frac{x}{\sqrt[5]{a+bx^5}}\right)$$

$$= \frac{2\,\text{Subst}\left(\int \frac{\sqrt[5]{c}+\frac{1}{4}\left(1-\sqrt{5}\right)\sqrt[5]{bc-ad}x}{c^{2/5}+\frac{1}{2}\left(1-\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-ad}x+(bc-ad)^{2/5}x^2}\,dx, x, \frac{x}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}} +$$

$$= -\frac{\log\left(\sqrt[5]{c}-\frac{\sqrt[5]{bc-ad}x}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}} + \frac{\left(5-\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{c^{2/5}+\frac{1}{2}\left(1+\sqrt{5}\right)\sqrt[5]{c}\sqrt[5]{bc-ad}x+(bc-ad)^{2/5}x^2}\,dx, x, \frac{x}{\sqrt[5]{a+bx^5}}\right)}{20c^{3/5}}$$

$$= -\frac{\log\left(\sqrt[5]{c}-\frac{\sqrt[5]{bc-ad}x}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}} + \frac{\left(1-\sqrt{5}\right)\log\left(2c^{2/5}+\frac{2(bc-ad)^{2/5}x^2}{\left(a+bx^5\right)^{2/5}}+\frac{\sqrt[5]{c}\sqrt[5]{bc-ad}x}{\sqrt[5]{a+bx^5}}-\frac{\sqrt{5}\sqrt[5]{c}\sqrt[5]{bc-ad}x}{\sqrt[5]{a+bx^5}}\right)}{20c^{4/5}\sqrt[5]{bc-ad}} +$$

$$= \frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{\left(1-\sqrt{5}\right)\sqrt[5]{c}+\frac{4\sqrt[5]{bc-ad}x}{\sqrt[5]{a+bx^5}}}{\sqrt{2\left(5+\sqrt{5}\right)}\sqrt[5]{c}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}} + \frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{5+\sqrt{5}}\left(\left(1+\sqrt{5}\right)\sqrt[5]{c}+\frac{4\sqrt[5]{bc-ad}x}{\sqrt[5]{a+bx^5}}\right)}{2\sqrt{10}\sqrt[5]{c}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}}$$

Mathematica [C] time = 0.0196107, size = 49, normalized size = 0.09

$$\frac{x\,{}_2F_1\left(\frac{1}{5},1;\frac{6}{5};-\frac{(ad-bc)x^5}{c(bx^5+a)}\right)}{c\sqrt[5]{a+bx^5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^5)^(1/5)*(c + d*x^5)),x]
```

```
[Out] (x*Hypergeometric2F1[1/5, 1, 6/5, -(((-(b*c) + a*d)*x^5)/(c*(a + b*x^5)))])
/(c*(a + b*x^5)^(1/5))
```

Maple [F] time = 0.427, size = 0, normalized size = 0.

$$\int \frac{1}{dx^5+c}\frac{1}{\sqrt[5]{bx^5+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^5+a)^(1/5)/(d*x^5+c),x)
```

```
[Out] int(1/(b*x^5+a)^(1/5)/(d*x^5+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^5+a\right)^{\frac{1}{5}}\left(dx^5+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5+a)^(1/5)/(d*x^5+c),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^5 + a)^(1/5)*(d*x^5 + c)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5+a)^(1/5)/(d*x^5+c),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[5]{a+bx^5}\left(c+dx^5\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**5+a)**(1/5)/(d*x**5+c),x)
```

```
[Out] Integral(1/((a + b*x**5)**(1/5)*(c + d*x**5)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^5+a\right)^{\frac{1}{5}}\left(dx^5+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^5+a)^(1/5)/(d*x^5+c),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^5 + a)^(1/5)*(d*x^5 + c)), x)
```

3.224 $\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3 dx$

Optimal. Leaf size=143

$$-\frac{d\sqrt{a+\frac{b}{x}}\left(2\left(-2a^2d^2+15abcd+57b^2c^2\right)+\frac{bd(2ad+33bc)}{x}\right)}{15b^2}+\frac{c^2(6ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}+x\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3-\frac{7}{5}d\sqrt{a+$$

```
[Out] (-7*d*Sqrt[a + b/x]*(c + d/x)^2)/5 - (d*Sqrt[a + b/x]*(2*(57*b^2*c^2 + 15*a
*b*c*d - 2*a^2*d^2) + (b*d*(33*b*c + 2*a*d))/x))/(15*b^2) + Sqrt[a + b/x]*(
c + d/x)^3*x + (c^2*(b*c + 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

Rubi [A] time = 0.123315, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {375, 97, 153, 147, 63, 208}

$$-\frac{d\sqrt{a+\frac{b}{x}}\left(2\left(-2a^2d^2+15abcd+57b^2c^2\right)+\frac{bd(2ad+33bc)}{x}\right)}{15b^2}+\frac{c^2(6ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}+x\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3-\frac{7}{5}d\sqrt{a+$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]*(c + d/x)^3,x]
```

```
[Out] (-7*d*Sqrt[a + b/x]*(c + d/x)^2)/5 - (d*Sqrt[a + b/x]*(2*(57*b^2*c^2 + 15*a
*b*c*d - 2*a^2*d^2) + (b*d*(33*b*c + 2*a*d))/x))/(15*b^2) + Sqrt[a + b/x]*(
c + d/x)^3*x + (c^2*(b*c + 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3 dx = -\text{Subst}\left(\int \frac{\sqrt{a+bx}(c+dx)^3}{x^2}\,dx, x, \frac{1}{x}\right)$$

$$= \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3 x - \text{Subst}\left(\int \frac{(c+dx)^2\left(\frac{1}{2}(bc+6ad)+\frac{7bdx}{2}\right)}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{7}{5}d\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 + \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3 x - \frac{2\,\text{Subst}\left(\int \frac{(c+dx)\left(\frac{5}{4}bc(bc+6ad)+\frac{1}{4}bd(33bc+2ad)x\right)}{x\sqrt{a+bx}}\,d\ldots\right)}{5b}$$

$$= -\frac{7}{5}d\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 - \frac{d\sqrt{a+\frac{b}{x}}\left(2\left(57b^2c^2+15abcd-2a^2d^2\right)+\frac{bd(33bc+2ad)}{x}\right)}{15b^2} + \sqrt{a+\frac{b}{x}}\left(\ldots\right.$$

$$= -\frac{7}{5}d\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 - \frac{d\sqrt{a+\frac{b}{x}}\left(2\left(57b^2c^2+15abcd-2a^2d^2\right)+\frac{bd(33bc+2ad)}{x}\right)}{15b^2} + \sqrt{a+\frac{b}{x}}\left(\ldots\right.$$

$$= -\frac{7}{5}d\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 - \frac{d\sqrt{a+\frac{b}{x}}\left(2\left(57b^2c^2+15abcd-2a^2d^2\right)+\frac{bd(33bc+2ad)}{x}\right)}{15b^2} + \sqrt{a+\frac{b}{x}}\left(\ldots\right.$$

Mathematica [A] time = 0.145502, size = 118, normalized size = 0.83

$$\frac{\sqrt{a+\frac{b}{x}}\left(4a^2d^3x^2-2abd^2x(15cx+d)-3b^2\left(30c^2dx^2-5c^3x^3+10cd^2x+2d^3\right)\right)}{15b^2x^2}+\frac{c^2(6ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]*(c + d/x)^3,x]
```

```
[Out] (Sqrt[a + b/x]*(4*a^2*d^3*x^2 - 2*a*b*d^2*x*(d + 15*c*x) - 3*b^2*(2*d^3 + 1
0*c*d^2*x + 30*c^2*d*x^2 - 5*c^3*x^3)))/(15*b^2*x^2) + (c^2*(b*c + 6*a*d)*A
rcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

Maple [A] time = 0.016, size = 248, normalized size = 1.7

$$\frac{1}{30\,b^2x^3}\sqrt{\frac{ax+b}{x}}\left(90\,\ln\left(1/2\,\frac{2\,\sqrt{ax^2+bx}\sqrt{a}+2\,ax+b}{\sqrt{a}}\right)x^4ab^2c^2d+15\,\ln\left(1/2\,\frac{2\,\sqrt{ax^2+bx}\sqrt{a}+2\,ax+b}{\sqrt{a}}\right)x^4b^3c^3+180\,$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)^3*(a+b/x)^(1/2),x)
```

```
[Out] 1/30*((a*x+b)/x)^(1/2)/x^3*(90*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)
/a^(1/2))*x^4*a*b^2*c^2*d+15*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a
^(1/2))*x^4*b^3*c^3+180*a^(3/2)*(a*x^2+b*x)^(1/2)*x^4*b*c^2*d+30*a^(1/2)*(a
*x^2+b*x)^(1/2)*x^4*b^2*c^3-180*a^(1/2)*(a*x^2+b*x)^(3/2)*x^2*b*c^2*d+8*a^(
3/2)*(a*x^2+b*x)^(3/2)*x*d^3-60*d^2*c*(a*x^2+b*x)^(3/2)*a^(1/2)*x*b-12*a^(1
/2)*(a*x^2+b*x)^(3/2)*b*d^3)/((a*x+b)*x)^(1/2)/a^(1/2)/b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^3*(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.27133, size = 672, normalized size = 4.7

$$\left[\frac{15\left(b^3c^3+6\,ab^2c^2d\right)\sqrt{a}x^2\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(15\,ab^2c^3x^3-6\,ab^2d^3-2\left(45\,ab^2c^2d+15\,a^2bcd^2-2\,a^3d^3\right)x^2\right.}{30\,ab^2x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^3*(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/30*(15*(b^3*c^3 + 6*a*b^2*c^2*d)*sqrt(a)*x^2*log(2*a*x + 2*sqrt(a)*x*sqr
t((a*x + b)/x) + b) + 2*(15*a*b^2*c^3*x^3 - 6*a*b^2*d^3 - 2*(45*a*b^2*c^2*d
 + 15*a^2*b*c*d^2 - 2*a^3*d^3)*x^2 - 2*(15*a*b^2*c*d^2 + a^2*b*d^3)*x)*sqrt
((a*x + b)/x))/(a*b^2*x^2), -1/15*(15*(b^3*c^3 + 6*a*b^2*c^2*d)*sqrt(-a)*x^
2*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (15*a*b^2*c^3*x^3 - 6*a*b^2*d^3 -
2*(45*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 2*a^3*d^3)*x^2 - 2*(15*a*b^2*c*d^2 + a
^2*b*d^3)*x)*sqrt((a*x + b)/x))/(a*b^2*x^2)]
```

Sympy [A] time = 26.27, size = 454, normalized size = 3.17

$$\frac{4a^{\frac{11}{2}}b^{\frac{3}{2}}d^3x^3\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}+\frac{2a^{\frac{9}{2}}b^{\frac{5}{2}}d^3x^2\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{8a^{\frac{7}{2}}b^{\frac{7}{2}}d^3x\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{6a^{\frac{5}{2}}b^{\frac{9}{2}}d^3\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{4a^6bd^3x^{\frac{7}{2}}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)**3*(a+b/x)**(1/2),x)
```

```
[Out] 4*a**(11/2)*b**(3/2)*d**3*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) +
 15*a**(5/2)*b**4*x**(5/2)) + 2*a**(9/2)*b**(5/2)*d**3*x**2*sqrt(a*x/b + 1)
/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 8*a**(7/2)*b**(7
/2)*d**3*x*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x*
*(5/2)) - 6*a**(5/2)*b**(9/2)*d**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/
2) + 15*a**(5/2)*b**4*x**(5/2)) - 4*a**6*b*d**3*x**(7/2)/(15*a**(7/2)*b**3*
x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 4*a**5*b**2*d**3*x**(5/2)/(15*a**(7
/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 6*a*c**2*d*atan(sqrt(a + b
/x)/sqrt(-a))/sqrt(-a) + sqrt(b)*c**3*sqrt(x)*sqrt(a*x/b + 1) - 6*c**2*d*sq
rt(a + b/x) + 3*c*d**2*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/
2)/(3*b), True)) + b*c**3*asinh(sqrt(a)*sqrt(x)/sqrt(b))/sqrt(a)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^3*(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.225 $\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 dx$

Optimal. Leaf size=99

$$\frac{c^2x\left(a+\frac{b}{x}\right)^{3/2}}{a}-\frac{c\sqrt{a+\frac{b}{x}}(4ad+bc)}{a}+\frac{c(4ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2d^2\left(a+\frac{b}{x}\right)^{3/2}}{3b}$$

```
[Out] -((c*(b*c + 4*a*d)*Sqrt[a + b/x])/a) - (2*d^2*(a + b/x)^(3/2))/(3*b) + (c^2
*(a + b/x)^(3/2)*x)/a + (c*(b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sq
rt[a]
```

Rubi [A] time = 0.0666848, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {375, 89, 80, 50, 63, 208}

$$\frac{c^2x\left(a+\frac{b}{x}\right)^{3/2}}{a}-\frac{c\sqrt{a+\frac{b}{x}}(4ad+bc)}{a}+\frac{c(4ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2d^2\left(a+\frac{b}{x}\right)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]*(c + d/x)^2,x]
```


```
[Out] -((c*(b*c + 4*a*d)*Sqrt[a + b/x])/a) - (2*d^2*(a + b/x)^(3/2))/(3*b) + (c^2
*(a + b/x)^(3/2)*x)/a + (c*(b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sq
rt[a]
```


Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 dx &= -\text{Subst}\left(\int \frac{\sqrt{a+bx}(c+dx)^2}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= \frac{c^2\left(a+\frac{b}{x}\right)^{3/2}x}{a} - \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}\left(\frac{1}{2}c(bc+4ad)+ad^2x\right)}{x}\,dx, x, \frac{1}{x}\right)}{a}\\
&= -\frac{2d^2\left(a+\frac{b}{x}\right)^{3/2}}{3b} + \frac{c^2\left(a+\frac{b}{x}\right)^{3/2}x}{a} - \frac{(c(bc+4ad))\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \frac{1}{x}\right)}{2a}\\
&= -\frac{c(bc+4ad)\sqrt{a+\frac{b}{x}}}{a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{3/2}}{3b} + \frac{c^2\left(a+\frac{b}{x}\right)^{3/2}x}{a} - \frac{1}{2}(c(bc+4ad))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+\cdots}}\right.\\
&= -\frac{c(bc+4ad)\sqrt{a+\frac{b}{x}}}{a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{3/2}}{3b} + \frac{c^2\left(a+\frac{b}{x}\right)^{3/2}x}{a} - \frac{(c(bc+4ad))\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx\cdots\right.}{b}\\
&= -\frac{c(bc+4ad)\sqrt{a+\frac{b}{x}}}{a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{3/2}}{3b} + \frac{c^2\left(a+\frac{b}{x}\right)^{3/2}x}{a} + \frac{c(bc+4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.0882163, size = 84, normalized size = 0.85

$$\frac{\sqrt{a+\frac{b}{x}}\left(b\left(3c^2x^2-12cdx-2d^2\right)-2ad^2x\right)}{3bx}+\frac{c(4ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]*(c + d/x)^2,x]
```

```
[Out] (Sqrt[a + b/x]*(-2*a*d^2*x + b*(-2*d^2 - 12*c*d*x + 3*c^2*x^2)))/(3*b*x) +
(c*(b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

Maple [B] time = 0.01, size = 191, normalized size = 1.9

$$-\frac{1}{6\,bx^{2}}\sqrt{\frac{ax+b}{x}}\left(-24\,\sqrt{ax^{2}+bx}a^{3/2}x^{3}cd-6\,\sqrt{ax^{2}+bx}\sqrt{a}x^{3}bc^{2}-12\,\ln\left(1/2\,\frac{2\,\sqrt{ax^{2}+bx}\sqrt{a}+2\,ax+b}{\sqrt{a}}\right)x^{3}abcd-3\,\ln\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)^2*(a+b/x)^(1/2),x)
```

```
[Out] -1/6*((a*x+b)/x)^(1/2)/x^2*(-24*(a*x^2+b*x)^(1/2)*a^(3/2)*x^3*c*d-6*(a*x^2+
b*x)^(1/2)*a^(1/2)*x^3*b*c^2-12*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b
)/a^(1/2))*x^3*a*b*c*d-3*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/
2))*x^3*b^2*c^2+24*(a*x^2+b*x)^(3/2)*a^(1/2)*x*c*d+4*d^2*(a*x^2+b*x)^(3/2)*
a^(1/2))/((a*x+b)*x)^(1/2)/a^(1/2)/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2*(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.33427, size = 462, normalized size = 4.67

$$\left[\frac{3\left(b^{2}c^{2}+4\,abcd\right)\sqrt{a}x\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(3\,abc^{2}x^{2}-2\,abd^{2}-2\left(6\,abcd+a^{2}d^{2}\right)x\right)\sqrt{\frac{ax+b}{x}}}{6\,abx},-\frac{3\left(b^{2}c^{2}+4\,abc\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2*(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^2*c^2 + 4*a*b*c*d)*sqrt(a)*x*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x +
 b)/x) + b) + 2*(3*a*b*c^2*x^2 - 2*a*b*d^2 - 2*(6*a*b*c*d + a^2*d^2)*x)*sqr
t((a*x + b)/x))/(a*b*x), -1/3*(3*(b^2*c^2 + 4*a*b*c*d)*sqrt(-a)*x*arctan(sq
rt(-a)*sqrt((a*x + b)/x)/a) - (3*a*b*c^2*x^2 - 2*a*b*d^2 - 2*(6*a*b*c*d + a
^2*d^2)*x)*sqrt((a*x + b)/x))/(a*b*x)]
```

Sympy [A] time = 17.7495, size = 121, normalized size = 1.22

$$-\frac{4acd\operatorname{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\sqrt{b}c^{2}\sqrt{x}\sqrt{\frac{ax}{b}+1}-4cd\sqrt{a+\frac{b}{x}}+d^{2}\left(\begin{cases}-\frac{\sqrt{a}}{x} & \text{for } b=0\\ -\frac{2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+\frac{bc^{2}\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)**2*(a+b/x)**(1/2),x)
```

```
[Out] -4*a*c*d*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a) + sqrt(b)*c**2*sqrt(x)*sqrt(
a*x/b + 1) - 4*c*d*sqrt(a + b/x) + d**2*Piecewise((-sqrt(a)/x, Eq(b, 0)), (
-2*(a + b/x)**(3/2)/(3*b), True)) + b*c**2*asinh(sqrt(a)*sqrt(x)/sqrt(b))/s
qrt(a)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2*(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.226 $\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right) dx$

Optimal. Leaf size=74

$$-\frac{\sqrt{a+\frac{b}{x}}(2ad+bc)}{a}+\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{cx\left(a+\frac{b}{x}\right)^{3/2}}{a}$$

```
[Out] -(((b*c + 2*a*d)*Sqrt[a + b/x])/a) + (c*(a + b/x)^(3/2)*x)/a + ((b*c + 2*a*
d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

Rubi [A] time = 0.0483524, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {375, 78, 50, 63, 208}

$$-\frac{\sqrt{a+\frac{b}{x}}(2ad+bc)}{a}+\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{cx\left(a+\frac{b}{x}\right)^{3/2}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]*(c + d/x),x]
```

```
[Out] -(((b*c + 2*a*d)*Sqrt[a + b/x])/a) + (c*(a + b/x)^(3/2)*x)/a + ((b*c + 2*a*
d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right) dx &= -\text{Subst}\left(\int \frac{\sqrt{a+bx}(c+dx)}{x^2} dx, x, \frac{1}{x}\right) \\
&= \frac{c\left(a+\frac{b}{x}\right)^{3/2} x}{a} - \frac{\left(\frac{bc}{2}+ad\right)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x} dx, x, \frac{1}{x}\right)}{a} \\
&= -\frac{(bc+2ad)\sqrt{a+\frac{b}{x}}}{a} + \frac{c\left(a+\frac{b}{x}\right)^{3/2} x}{a} - \frac{1}{2}(bc+2ad)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, \frac{1}{x}\right) \\
&= -\frac{(bc+2ad)\sqrt{a+\frac{b}{x}}}{a} + \frac{c\left(a+\frac{b}{x}\right)^{3/2} x}{a} - \frac{(bc+2ad)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}} dx, x, \sqrt{a+\frac{b}{x}}\right)}{b} \\
&= -\frac{(bc+2ad)\sqrt{a+\frac{b}{x}}}{a} + \frac{c\left(a+\frac{b}{x}\right)^{3/2} x}{a} + \frac{(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.0411902, size = 52, normalized size = 0.7

$$\sqrt{a+\frac{b}{x}}(cx-2d) + \frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]*(c + d/x),x]
```

```
[Out] Sqrt[a + b/x]*(-2*d + c*x) + ((b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])
/Sqrt[a]
```

Maple [B] time = 0.009, size = 163, normalized size = 2.2

$$\frac{1}{2bx}\sqrt{\frac{ax+b}{x}}\left(2\ln\left(1/2\,\frac{2\sqrt{ax^2+bx}\sqrt{a}+2\,ax+b}{\sqrt{a}}\right)x^2abd+\ln\left(\frac{1}{2}\left(2\sqrt{ax^2+bx}\sqrt{a}+2\,ax+b\right)\frac{1}{\sqrt{a}}\right)x^2b^2c+4\,a^{3/2}\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)*(a+b/x)^(1/2),x)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)/x*(2*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(
1/2))*x^2*a*b*d+ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x^2*b
^2*c+4*a^(3/2)*(a*x^2+b*x)^(1/2)*x^2*d+2*a^(1/2)*(a*x^2+b*x)^(1/2)*x^2*b*c-
```

```
4*a^(1/2)*(a*x^2+b*x)^(3/2)*d)/((a*x+b)*x)^(1/2)/b/a^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)*(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.31298, size = 300, normalized size = 4.05

$$\left[\frac{(bc+2\,ad)\sqrt{a}\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\,(acx-2\,ad)\sqrt{\frac{ax+b}{x}}}{2\,a}, -\frac{(bc+2\,ad)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)-(acx-2\,ad)\sqrt{\right.}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)*(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b*c + 2*a*d)*sqrt(a)*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b)
+ 2*(a*c*x - 2*a*d)*sqrt((a*x + b)/x))/a, -((b*c + 2*a*d)*sqrt(-a)*arctan(s
qrt(-a)*sqrt((a*x + b)/x)/a) - (a*c*x - 2*a*d)*sqrt((a*x + b)/x))/a]
```

Sympy [A] time = 20.0446, size = 87, normalized size = 1.18

$$-\frac{2ad\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\sqrt{b}c\sqrt{x}\sqrt{\frac{ax}{b}+1}-2d\sqrt{a+\frac{b}{x}}+\frac{bc\,\mathrm{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)*(a+b/x)**(1/2),x)
```

```
[Out] -2*a*d*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a) + sqrt(b)*c*sqrt(x)*sqrt(a*x/b
 + 1) - 2*d*sqrt(a + b/x) + b*c*asinh(sqrt(a)*sqrt(x)/sqrt(b))/sqrt(a)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)*(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.227 $\int \sqrt{a+\frac{b}{x}}\,dx$

Optimal. Leaf size=39

$$x\sqrt{a+\frac{b}{x}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] Sqrt[a + b/x]*x + (b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

Rubi [A] time = 0.018864, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {242, 47, 63, 208}

$$x\sqrt{a+\frac{b}{x}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x],x]
```

```
[Out] Sqrt[a + b/x]*x + (b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \sqrt{a+\frac{b}{x}}\,dx = -\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2}\,dx, x, \frac{1}{x}\right)$$
$$= \sqrt{a+\frac{b}{x}}\,x - \frac{1}{2}b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)$$
$$= \sqrt{a+\frac{b}{x}}\,x - \text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)$$
$$= \sqrt{a+\frac{b}{x}}\,x + \frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Mathematica [A] time = 0.0163468, size = 39, normalized size = 1.

$$x\sqrt{a+\frac{b}{x}} + \frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x],x]
```

```
[Out] Sqrt[a + b/x]*x + (b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

Maple [B] time = 0.003, size = 74, normalized size = 1.9

$$\frac{x}{2}\sqrt{\frac{ax+b}{x}}\left(2\sqrt{ax^2+bx}\sqrt{a}+b\ln\left(\frac{1}{2}\left(2\sqrt{ax^2+bx}\sqrt{a}+2\,ax+b\right)\frac{1}{\sqrt{a}}\right)\right)\frac{1}{\sqrt{(ax+b)\,x}}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(1/2),x)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+b*ln(1/2*(2*(a*x^2+b*x
)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2)))/((a*x+b)*x)^(1/2)/a^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.32817, size = 234, normalized size = 6.

$$\left[\frac{2\,ax\sqrt{\frac{ax+b}{x}}+\sqrt{a}b\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)}{2\,a},\ \frac{ax\sqrt{\frac{ax+b}{x}}-\sqrt{-a}b\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*x*sqrt((a*x + b)/x) + sqrt(a)*b*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x
 + b)/x) + b))/a, (a*x*sqrt((a*x + b)/x) - sqrt(-a)*b*arctan(sqrt(-a)*sqrt(
(a*x + b)/x)/a))/a]
```

Sympy [A] time = 1.82385, size = 42, normalized size = 1.08

$$\sqrt{b}\sqrt{x}\sqrt{\frac{ax}{b}+1}+\frac{b\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(1/2),x)
```

```
[Out] sqrt(b)*sqrt(x)*sqrt(a*x/b + 1) + b*asinh(sqrt(a)*sqrt(x)/sqrt(b))/sqrt(a)
```

Giac [B] time = 1.14471, size = 86, normalized size = 2.21

$$-\frac{b\log\left(\left|-2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}-b\right|\right)\operatorname{sgn}(x)}{2\sqrt{a}}+\frac{b\log\left(|b|\right)\operatorname{sgn}(x)}{2\sqrt{a}}+\sqrt{ax^2+bx}\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*b*log(abs(-2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) - b))*sgn(x)/sqrt
(a) + 1/2*b*log(abs(b))*sgn(x)/sqrt(a) + sqrt(a*x^2 + b*x)*sgn(x)
```

3.228 $\int \frac{\sqrt{a+\frac{b}{x}}}{c+\frac{d}{x}}\,dx$

Optimal. Leaf size=104

$$\frac{2\sqrt{d}\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2}+\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}c^2}+\frac{x\sqrt{a+\frac{b}{x}}}{c}$$

```
[Out] (Sqrt[a + b/x]*x)/c + (2*Sqrt[d]*Sqrt[b*c - a*d]*ArcTan[(Sqrt[d]*Sqrt[a + b
/x])/Sqrt[b*c - a*d]])/c^2 + ((b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])
/(Sqrt[a]*c^2)
```

Rubi [A] time = 0.110625, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {375, 99, 156, 63, 208, 205}

$$\frac{2\sqrt{d}\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2}+\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}c^2}+\frac{x\sqrt{a+\frac{b}{x}}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]/(c + d/x),x]
```

```
[Out] (Sqrt[a + b/x]*x)/c + (2*Sqrt[d]*Sqrt[b*c - a*d]*ArcTan[(Sqrt[d]*Sqrt[a + b
/x])/Sqrt[b*c - a*d]])/c^2 + ((b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])
/(Sqrt[a]*c^2)
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{x}}}{c+\frac{d}{x}}\,dx = -\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2(c+dx)}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt{a+\frac{b}{x}}x}{c} - \frac{\operatorname{Subst}\left(\int \frac{\frac{1}{2}(bc-2ad)-\frac{bdx}{2}}{x\sqrt{a+bx}(c+dx)}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{\sqrt{a+\frac{b}{x}}x}{c} - \frac{(bc-2ad)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2c^2} + \frac{(d(bc-ad))\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx}(c+dx)}\,dx, x, \frac{1}{x}\right)}{c^2}$$

$$= \frac{\sqrt{a+\frac{b}{x}}x}{c} - \frac{(bc-2ad)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^2} + \frac{(2d(bc-ad))\operatorname{Subst}\left(\int \frac{1}{c-\frac{ad}{b}+\frac{dx^2}{b}}\,dx, x, \sqrt{a+\right.}{bc^2}$$

$$= \frac{\sqrt{a+\frac{b}{x}}x}{c} + \frac{2\sqrt{d}\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2} + \frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}c^2}$$

Mathematica [A] time = 0.191681, size = 100, normalized size = 0.96

$$\frac{2\sqrt{d}\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right) + \frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}} + cx\sqrt{a+\frac{b}{x}}}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]/(c + d/x),x]
```

```
[Out] (c*Sqrt[a + b/x]*x + 2*Sqrt[d]*Sqrt[b*c - a*d]*ArcTan[(Sqrt[d]*Sqrt[a + b/x
])/Sqrt[b*c - a*d]] + ((b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a
])/c^2
```

Maple [B] time = 0.062, size = 287, normalized size = 2.8

$$-\frac{x}{2c^3}\sqrt{\frac{ax+b}{x}}\left(2\ln\left(1/2\,\frac{2\sqrt{(ax+b)x}\sqrt{a}+2ax+b}{\sqrt{a}}\right)\sqrt{\frac{(ad-bc)d}{c^2}}acd - \ln\left(\frac{1}{2}\left(2\sqrt{(ax+b)x}\sqrt{a}+2ax+b\right)\frac{1}{\sqrt{a}}\right)\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(1/2)/(c+d/x),x)
```

```
[Out] -1/2*((a*x+b)/x)^(1/2)*x*(2*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^
(1/2))*((a*d-b*c)*d/c^2)^(1/2)*a*c*d-ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*
a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*b*c^2+2*ln((2*((a*d-b*c)*d/c^2)^(1/
2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(3/2)*d^2-2*ln((2*((a*
d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(1/2)
*b*c*d-2*((a*x+b)*x)^(1/2)*c^2*a^(1/2)*((a*d-b*c)*d/c^2)^(1/2))/((a*x+b)*x)
^(1/2)/c^3/a^(1/2)/((a*d-b*c)*d/c^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+\frac{b}{x}}}{c+\frac{d}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x)/(c + d/x), x)
```

Fricas [A] time = 1.49723, size = 1110, normalized size = 10.67

$$\left[\frac{2\,acx\sqrt{\frac{ax+b}{x}} - (bc-2\,ad)\sqrt{a}\log\left(2\,ax - 2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}} + b\right) + 2\,\sqrt{-bcd+ad^2}a\log\left(\frac{bd-(bc-2\,ad)x+2\,\sqrt{-bcd+ad^2}x\sqrt{\frac{ax+b}{x}}}{cx+d}\right)}{2\,ac^2}, \frac{2\,acx\sqrt{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*c*x*sqrt((a*x + b)/x) - (b*c - 2*a*d)*sqrt(a)*log(2*a*x - 2*sqrt(
a)*x*sqrt((a*x + b)/x) + b) + 2*sqrt(-b*c*d + a*d^2)*a*log((b*d - (b*c - 2*
a*d)*x + 2*sqrt(-b*c*d + a*d^2)*x*sqrt((a*x + b)/x))/(c*x + d)))/(a*c^2), 1
/2*(2*a*c*x*sqrt((a*x + b)/x) - 4*sqrt(b*c*d - a*d^2)*a*arctan(sqrt(b*c*d -
 a*d^2)*x*sqrt((a*x + b)/x)/(a*d*x + b*d)) - (b*c - 2*a*d)*sqrt(a)*log(2*a*
x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b))/(a*c^2), (a*c*x*sqrt((a*x + b)/x) -
 (b*c - 2*a*d)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + sqrt(-b*c*d
+ a*d^2)*a*log((b*d - (b*c - 2*a*d)*x + 2*sqrt(-b*c*d + a*d^2)*x*sqrt((a*x
+ b)/x))/(c*x + d)))/(a*c^2), (a*c*x*sqrt((a*x + b)/x) - 2*sqrt(b*c*d - a*d
^2)*a*arctan(sqrt(b*c*d - a*d^2)*x*sqrt((a*x + b)/x)/(a*d*x + b*d)) - (b*c
- 2*a*d)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a))/(a*c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{a+\frac{b}{x}}}{cx+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(1/2)/(c+d/x),x)
```

```
[Out] Integral(x*sqrt(a + b/x)/(c*x + d), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.229 $\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^2}\,dx$

Optimal. Leaf size=147

$$\frac{2d\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)}+\frac{\sqrt{d}(3bc-4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3\sqrt{bc-ad}}+\frac{(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}c^3}+\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}$$

```
[Out] (2*d*Sqrt[a + b/x])/(c^2*(c + d/x)) + (Sqrt[a + b/x]*x)/(c*(c + d/x)) + (Sq
rt[d]*(3*b*c - 4*a*d)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c^3
*Sqrt[b*c - a*d]) + ((b*c - 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(Sqrt[a]
*c^3)
```

Rubi [A] time = 0.208627, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {375, 99, 151, 156, 63, 208, 205}

$$\frac{2d\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)}+\frac{\sqrt{d}(3bc-4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3\sqrt{bc-ad}}+\frac{(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}c^3}+\frac{x\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]/(c + d/x)^2,x]
```

```
[Out] (2*d*Sqrt[a + b/x])/(c^2*(c + d/x)) + (Sqrt[a + b/x]*x)/(c*(c + d/x)) + (Sq
rt[d]*(3*b*c - 4*a*d)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c^3
*Sqrt[b*c - a*d]) + ((b*c - 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(Sqrt[a]
*c^3)
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
```

```
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^2}\,dx = -\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2(c+dx)^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)} - \frac{\operatorname{Subst}\left(\int \frac{\frac{1}{2}(bc-4ad)-\frac{3bdx}{2}}{x\sqrt{a+bx}(c+dx)^2}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{2d\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)} + \frac{\operatorname{Subst}\left(\int \frac{-\frac{1}{2}(bc-4ad)(bc-ad)+bd(bc-ad)x}{x\sqrt{a+bx}(c+dx)}\,dx, x, \frac{1}{x}\right)}{c^2(bc-ad)}$$

$$= \frac{2d\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)} - \frac{(bc-4ad)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2c^3} + \frac{(d(3bc-4ad))\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx}(c+dx)} \cdots\right)}{2c^3}$$

$$= \frac{2d\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)} - \frac{(bc-4ad)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^3} + \frac{(d(3bc-4ad))\operatorname{Subst}\left(\int \frac{1}{c-\frac{ad}{b} \cdots} \cdots\right)}{bc^3}$$

$$= \frac{2d\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)} + \frac{\sqrt{d}(3bc-4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3\sqrt{bc-ad}} + \frac{(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}c^3}$$

Mathematica [A] time = 0.311993, size = 122, normalized size = 0.83

$$\frac{\frac{cx\sqrt{a+\frac{b}{x}}(cx+2d)}{cx+d}+\frac{\sqrt{d}(3bc-4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}}+\frac{(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}}{c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]/(c + d/x)^2,x]
```

```
[Out] ((c*Sqrt[a + b/x]*x*(2*d + c*x))/(d + c*x) + (Sqrt[d]*(3*b*c - 4*a*d)*ArcTa
n[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/Sqrt[b*c - a*d] + ((b*c - 4*a*d
)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a])/c^3
```

Maple [B] time = 0.013, size = 943, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(1/2)/(c+d/x)^2,x)
```

```
[Out] -1/2*((a*x+b)/x)^(1/2)*x*(4*a^(7/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*
x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x*c*d^3+2*a^(5/2)*((a*d-b*c)*d/c^2)^
(1/2)*((a*x+b)*x)^(1/2)*x^2*c^4+4*a^(7/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a
*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*d^4-2*a^(5/2)*((a*d-b*c)*d/c^2
)^(1/2)*((a*x+b)*x)^(1/2)*x*c^3*d-7*a^(5/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*(
(a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x*b*c^2*d^2-4*a^(5/2)*((a*d-
b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c^2*d^2-7*a^(5/2)*ln((2*((a*d-b*c)*d/c^
2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b*c*d^3-2*c^4*((a*
x+b)*x)^(3/2)*a^(3/2)*((a*d-b*c)*d/c^2)^(1/2)+4*a^(3/2)*((a*d-b*c)*d/c^2)^(
1/2)*((a*x+b)*x)^(1/2)*x*b*c^4+3*a^(3/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*
x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x*b^2*c^3*d+4*ln(1/2*(2*((a*x+b
)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*((a*d-b*c)*d/c^2)^(1/2)*x*c^2*d^2-
5*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*((a*d-b*c)*d/c^
2)^(1/2)*x*b*c^3*d+ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a*
((a*d-b*c)*d/c^2)^(1/2)*x*b^2*c^4+4*a^(3/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b
)*x)^(1/2)*b*c^3*d+3*a^(3/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2
)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^2*c^2*d^2+4*ln(1/2*(2*((a*x+b)*x)^(1/2)*a
^(1/2)+2*a*x+b)/a^(1/2))*a^3*((a*d-b*c)*d/c^2)^(1/2)*c*d^3-5*ln(1/2*(2*((a*
x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*((a*d-b*c)*d/c^2)^(1/2)*b*c^2*d
^2+ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a*((a*d-b*c)*d/c^2
)^(1/2)*b^2*c^3*d)/c^4/((a*x+b)*x)^(1/2)/(a*d-b*c)/(c*x+d)/a^(3/2)/((a*d-b*
c)*d/c^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x)/(c + d/x)^2, x)
```

Fricas [A] time = 1.50225, size = 1760, normalized size = 11.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((b*c*d - 4*a*d^2 + (b*c^2 - 4*a*c*d)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a
)*x*sqrt((a*x + b)/x) + b) + (3*a*b*c*d - 4*a^2*d^2 + (3*a*b*c^2 - 4*a^2*c*
d)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt(
(a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) - 2*(a*c^2*x^2 + 2*a*c*d*x
)*sqrt((a*x + b)/x))/(a*c^4*x + a*c^3*d), -1/2*(2*(b*c*d - 4*a*d^2 + (b*c^2
 - 4*a*c*d)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (3*a*b*c*d -
 4*a^2*d^2 + (3*a*b*c^2 - 4*a^2*c*d)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c -
 a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*
x + d)) - 2*(a*c^2*x^2 + 2*a*c*d*x)*sqrt((a*x + b)/x))/(a*c^4*x + a*c^3*d),
 1/2*(2*(3*a*b*c*d - 4*a^2*d^2 + (3*a*b*c^2 - 4*a^2*c*d)*x)*sqrt(d/(b*c - a
*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b
*d)) - (b*c*d - 4*a*d^2 + (b*c^2 - 4*a*c*d)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a
)*x*sqrt((a*x + b)/x) + b) + 2*(a*c^2*x^2 + 2*a*c*d*x)*sqrt((a*x + b)/x))/(
a*c^4*x + a*c^3*d), ((3*a*b*c*d - 4*a^2*d^2 + (3*a*b*c^2 - 4*a^2*c*d)*x)*sq
rt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/
x)/(a*d*x + b*d)) - (b*c*d - 4*a*d^2 + (b*c^2 - 4*a*c*d)*x)*sqrt(-a)*arctan
(sqrt(-a)*sqrt((a*x + b)/x)/a) + (a*c^2*x^2 + 2*a*c*d*x)*sqrt((a*x + b)/x))
/(a*c^4*x + a*c^3*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{a + \frac{b}{x}}}{(cx + d)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(1/2)/(c+d/x)**2,x)
```

```
[Out] Integral(x**2*sqrt(a + b/x)/(c*x + d)**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.230 $\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3}\,dx$

Optimal. Leaf size=213

$$\frac{\sqrt{d}\left(24a^2d^2-40abcd+15b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{3/2}}+\frac{d\sqrt{a+\frac{b}{x}}(11bc-12ad)}{4c^3\left(c+\frac{d}{x}\right)(bc-ad)}+\frac{3d\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2}+\frac{(bc-6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}c^4}+$$

```
[Out] (3*d*Sqrt[a + b/x])/(2*c^2*(c + d/x)^2) + (d*(11*b*c - 12*a*d)*Sqrt[a + b/x
])/(4*c^3*(b*c - a*d)*(c + d/x)) + (Sqrt[a + b/x]*x)/(c*(c + d/x)^2) + (Sqr
t[d]*(15*b^2*c^2 - 40*a*b*c*d + 24*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/
Sqrt[b*c - a*d]])/(4*c^4*(b*c - a*d)^(3/2)) + ((b*c - 6*a*d)*ArcTanh[Sqrt[a
 + b/x]/Sqrt[a]])/(Sqrt[a]*c^4)
```

Rubi [A] time = 0.339586, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {375, 99, 151, 156, 63, 208, 205}

$$\frac{\sqrt{d}\left(24a^2d^2-40abcd+15b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{3/2}}+\frac{d\sqrt{a+\frac{b}{x}}(11bc-12ad)}{4c^3\left(c+\frac{d}{x}\right)(bc-ad)}+\frac{3d\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2}+\frac{(bc-6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}c^4}+$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]/(c + d/x)^3,x]
```



```
[Out] (3*d*Sqrt[a + b/x])/(2*c^2*(c + d/x)^2) + (d*(11*b*c - 12*a*d)*Sqrt[a + b/x
])/(4*c^3*(b*c - a*d)*(c + d/x)) + (Sqrt[a + b/x]*x)/(c*(c + d/x)^2) + (Sqr
t[d]*(15*b^2*c^2 - 40*a*b*c*d + 24*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/
Sqrt[b*c - a*d]])/(4*c^4*(b*c - a*d)^(3/2)) + ((b*c - 6*a*d)*ArcTanh[Sqrt[a
 + b/x]/Sqrt[a]])/(Sqrt[a]*c^4)
```


Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 99

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^
(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m
 + n + p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1
] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || Integ
ersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
```

```
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3}\,dx = -\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2(c+dx)^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc-6ad)-\frac{5bdx}{2}}{x\sqrt{a+bx}(c+dx)^3}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{3d\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} + \frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} + \frac{\text{Subst}\left(\int \frac{-(bc-6ad)(bc-ad)+\frac{9}{2}bd(bc-ad)x}{x\sqrt{a+bx}(c+dx)^2}\,dx, x, \frac{1}{x}\right)}{2c^2(bc-ad)}$$

$$= \frac{3d\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} + \frac{d(11bc-12ad)\sqrt{a+\frac{b}{x}}}{4c^3(bc-ad)\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} - \frac{\text{Subst}\left(\int \frac{(bc-6ad)(bc-ad)^2-\frac{1}{4}bd(11bc-12ad)(bc-ad)x}{x\sqrt{a+bx}(c+dx)}\,dx, x\right.}{2c^3(bc-ad)^2}$$

$$= \frac{3d\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} + \frac{d(11bc-12ad)\sqrt{a+\frac{b}{x}}}{4c^3(bc-ad)\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} - \frac{(bc-6ad)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2c^4} + \frac{\left(d\left(15b^2c^2\right.\right.}{}$$

$$= \frac{3d\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} + \frac{d(11bc-12ad)\sqrt{a+\frac{b}{x}}}{4c^3(bc-ad)\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} - \frac{(bc-6ad)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^4} + \frac{\left(d\left(\right.\right.}{}$$

$$= \frac{3d\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} + \frac{d(11bc-12ad)\sqrt{a+\frac{b}{x}}}{4c^3(bc-ad)\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)^2} + \frac{\sqrt{d}\left(15b^2c^2-40abcd+24a^2d^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{3/2}}$$

Mathematica [A] time = 0.609968, size = 330, normalized size = 1.55

$$\frac{(cx+d)\left(\frac{1}{2}cd^{5/2}\sqrt{a+\frac{b}{x}}(ax+b)\left(12a^2d^2-17abcd+4b^2c^2\right)+(cx+d)\left(-\frac{1}{2}ad^2\left(24a^2d^2-40abcd+15b^2c^2\right)\left(\sqrt{d}\sqrt{a+\frac{b}{x}}-\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]/(c + d/x)^3,x]
```

```
[Out] (2*c^3*d^(3/2)*(b*c - a*d)^2*(a + b/x)^(3/2)*x^3 + c^2*d^(5/2)*(2*b*c - 3*a
*d)*(b*c - a*d)*Sqrt[a + b/x]*x*(b + a*x) + (d + c*x)*((c*d^(5/2)*(4*b^2*c^
2 - 17*a*b*c*d + 12*a^2*d^2)*Sqrt[a + b/x]*(b + a*x))/2 + (d + c*x)*(-(a*d^
2*(15*b^2*c^2 - 40*a*b*c*d + 24*a^2*d^2)*(Sqrt[d]*Sqrt[a + b/x] - Sqrt[b*c
- a*d]*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]]))/2 - d^(3/2)*(b*c -
 6*a*d)*(b*c - a*d)^2*(2*Sqrt[a + b/x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b/x]/Sq
rt[a]]))))/(2*a*c^4*d^(3/2)*(b*c - a*d)^2*(d + c*x)^2)
```

Maple [B] time = 0.015, size = 1972, normalized size = 9.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(1/2)/(c+d/x)^3,x)
```

```
[Out] -1/8*((a*x+b)/x)^(1/2)*x*(64*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a
^(1/2))*a^2*((a*d-b*c)*d/c^2)^(1/2)*x*b^2*c^4*d^2+78*a^(5/2)*((a*d-b*c)*d/c
^2)^(1/2)*((a*x+b)*x)^(1/2)*x*b*c^4*d^2+32*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1
/2)+2*a*x+b)/a^(1/2))*a^2*((a*d-b*c)*d/c^2)^(1/2)*x^2*b^2*c^5*d+18*a^(5/2)*
((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*x^2*b*c^5*d-104*ln(1/2*(2*((a*x+b
)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*((a*d-b*c)*d/c^2)^(1/2)*x*b*c^3*d^
3-52*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*((a*d-b*c)*d
/c^2)^(1/2)*x^2*b*c^4*d^2-8*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^
(1/2))*a*((a*d-b*c)*d/c^2)^(1/2)*x*b^3*c^5*d-4*ln(1/2*(2*((a*x+b)*x)^(1/2)*
a^(1/2)+2*a*x+b)/a^(1/2))*a*((a*d-b*c)*d/c^2)^(1/2)*x^2*b^3*c^6-52*ln(1/2*(
2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*((a*d-b*c)*d/c^2)^(1/2)*b
*c^2*d^4+32*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*((a*d
-b*c)*d/c^2)^(1/2)*b^2*c^3*d^3+46*a^(5/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*
x)^(1/2)*b*c^3*d^3+24*a^(9/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/
2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*d^6-4*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+
2*a*x+b)/a^(1/2))*a*((a*d-b*c)*d/c^2)^(1/2)*b^3*c^4*d^2-22*a^(3/2)*((a*d-b*
c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*b^2*c^4*d^2+12*a^(7/2)*((a*d-b*c)*d/c^2)^
(1/2)*((a*x+b)*x)^(1/2)*x^3*c^5*d-14*a^(5/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+
b)*x)^(1/2)*x^3*b*c^6+24*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/
2))*a^4*((a*d-b*c)*d/c^2)^(1/2)*x^2*c^3*d^3-12*a^(5/2)*((a*d-b*c)*d/c^2)^(1
/2)*((a*x+b)*x)^(3/2)*x*c^5*d-64*a^(7/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*
x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x^2*b*c^3*d^3+55*a^(5/2)*ln((2*
((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x^2
*b^2*c^4*d^2+48*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^4*(
(a*d-b*c)*d/c^2)^(1/2)*x*c^2*d^4-36*a^(7/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b
)*x)^(1/2)*x*c^3*d^3-128*a^(7/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^
(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x*b*c^2*d^4+110*a^(5/2)*ln((2*((a*d-b*c
)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x*b^2*c^3*d^
3-22*a^(3/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*x^2*b^2*c^6-15*a^(3/
2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*
x+d))*x^2*b^3*c^5*d-30*a^(3/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1
/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x*b^3*c^4*d^2+10*a^(3/2)*((a*d-b*c)*d/c^2
)^(1/2)*((a*x+b)*x)^(3/2)*b*c^5*d-44*a^(3/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+
b)*x)^(1/2)*x*b^2*c^5*d+24*a^(9/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x
)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x^2*c^2*d^4-8*a^(5/2)*((a*d-b*c)*d/c^
2)^(1/2)*((a*x+b)*x)^(3/2)*c^4*d^2+24*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2
*a*x+b)/a^(1/2))*a^4*((a*d-b*c)*d/c^2)^(1/2)*c*d^5-24*a^(7/2)*((a*d-b*c)*d/
c^2)^(1/2)*((a*x+b)*x)^(1/2)*c^2*d^4-64*a^(7/2)*ln((2*((a*d-b*c)*d/c^2)^(1/
2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b*c*d^5+55*a^(5/2)*ln((2
*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^
2*c^2*d^4-15*a^(3/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*
d*x+b*c*x-b*d)/(c*x+d))*b^3*c^3*d^3+48*a^(9/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2
)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x*c*d^5+14*a^(3/2)*((a*d-
b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(3/2)*x*b*c^6)/c^5/((a*x+b)*x)^(1/2)/(a*d-b*c
)^2/(c*x+d)^2/a^(3/2)/((a*d-b*c)*d/c^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x)/(c + d/x)^3, x)
```

Fricas [B] time = 2.02808, size = 3664, normalized size = 17.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^3,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(b^2*c^2*d^2 - 7*a*b*c*d^3 + 6*a^2*d^4 + (b^2*c^4 - 7*a*b*c^3*d +
6*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d - 7*a*b*c^2*d^2 + 6*a^2*c*d^3)*x)*sqrt(a)
*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (15*a*b^2*c^2*d^2 - 40*a^
2*b*c*d^3 + 24*a^3*d^4 + (15*a*b^2*c^4 - 40*a^2*b*c^3*d + 24*a^3*c^2*d^2)*x
^2 + 2*(15*a*b^2*c^3*d - 40*a^2*b*c^2*d^2 + 24*a^3*c*d^3)*x)*sqrt(-d/(b*c -
 a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d +
 (b*c - 2*a*d)*x)/(c*x + d)) - 2*(4*(a*b*c^4 - a^2*c^3*d)*x^3 + (17*a*b*c^3
*d - 18*a^2*c^2*d^2)*x^2 + (11*a*b*c^2*d^2 - 12*a^2*c*d^3)*x)*sqrt((a*x + b
)/x))/(a*b*c^5*d^2 - a^2*c^4*d^3 + (a*b*c^7 - a^2*c^6*d)*x^2 + 2*(a*b*c^6*d
 - a^2*c^5*d^2)*x), 1/4*((15*a*b^2*c^2*d^2 - 40*a^2*b*c*d^3 + 24*a^3*d^4 +
(15*a*b^2*c^4 - 40*a^2*b*c^3*d + 24*a^3*c^2*d^2)*x^2 + 2*(15*a*b^2*c^3*d -
40*a^2*b*c^2*d^2 + 24*a^3*c*d^3)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)
*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) - 2*(b^2*c^2*d^2 -
7*a*b*c*d^3 + 6*a^2*d^4 + (b^2*c^4 - 7*a*b*c^3*d + 6*a^2*c^2*d^2)*x^2 + 2*(
b^2*c^3*d - 7*a*b*c^2*d^2 + 6*a^2*c*d^3)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x
*sqrt((a*x + b)/x) + b) + (4*(a*b*c^4 - a^2*c^3*d)*x^3 + (17*a*b*c^3*d - 18
*a^2*c^2*d^2)*x^2 + (11*a*b*c^2*d^2 - 12*a^2*c*d^3)*x)*sqrt((a*x + b)/x))/(
a*b*c^5*d^2 - a^2*c^4*d^3 + (a*b*c^7 - a^2*c^6*d)*x^2 + 2*(a*b*c^6*d - a^2*
c^5*d^2)*x), -1/8*(8*(b^2*c^2*d^2 - 7*a*b*c*d^3 + 6*a^2*d^4 + (b^2*c^4 - 7*
a*b*c^3*d + 6*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d - 7*a*b*c^2*d^2 + 6*a^2*c*d^3
)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (15*a*b^2*c^2*d^2 - 40
*a^2*b*c*d^3 + 24*a^3*d^4 + (15*a*b^2*c^4 - 40*a^2*b*c^3*d + 24*a^3*c^2*d^2
)*x^2 + 2*(15*a*b^2*c^3*d - 40*a^2*b*c^2*d^2 + 24*a^3*c*d^3)*x)*sqrt(-d/(b*
c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*
d + (b*c - 2*a*d)*x)/(c*x + d)) - 2*(4*(a*b*c^4 - a^2*c^3*d)*x^3 + (17*a*b*
c^3*d - 18*a^2*c^2*d^2)*x^2 + (11*a*b*c^2*d^2 - 12*a^2*c*d^3)*x)*sqrt((a*x
+ b)/x))/(a*b*c^5*d^2 - a^2*c^4*d^3 + (a*b*c^7 - a^2*c^6*d)*x^2 + 2*(a*b*c^
6*d - a^2*c^5*d^2)*x), 1/4*((15*a*b^2*c^2*d^2 - 40*a^2*b*c*d^3 + 24*a^3*d^4
 + (15*a*b^2*c^4 - 40*a^2*b*c^3*d + 24*a^3*c^2*d^2)*x^2 + 2*(15*a*b^2*c^3*d
 - 40*a^2*b*c^2*d^2 + 24*a^3*c*d^3)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a
*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) - 4*(b^2*c^2*d^2
 - 7*a*b*c*d^3 + 6*a^2*d^4 + (b^2*c^4 - 7*a*b*c^3*d + 6*a^2*c^2*d^2)*x^2 +
2*(b^2*c^3*d - 7*a*b*c^2*d^2 + 6*a^2*c*d^3)*x)*sqrt(-a)*arctan(sqrt(-a)*sqr
t((a*x + b)/x)/a) + (4*(a*b*c^4 - a^2*c^3*d)*x^3 + (17*a*b*c^3*d - 18*a^2*c
^2*d^2)*x^2 + (11*a*b*c^2*d^2 - 12*a^2*c*d^3)*x)*sqrt((a*x + b)/x))/(a*b*c^
5*d^2 - a^2*c^4*d^3 + (a*b*c^7 - a^2*c^6*d)*x^2 + 2*(a*b*c^6*d - a^2*c^5*d^
2)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(1/2)/(c+d/x)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.33123, size = 1107, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^3,x, algorithm="giac")
```

```
[Out] -1/4*(15*sqrt(a)*b^2*c^2*d*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 40*a^(3/
2)*b*c*d^2*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 24*a^(5/2)*d^3*arctan(sq
rt(a)*d/sqrt(b*c*d - a*d^2)) - 2*sqrt(b*c*d - a*d^2)*b^2*c^2*log(abs(b)) +
14*sqrt(b*c*d - a*d^2)*a*b*c*d*log(abs(b)) - 12*sqrt(b*c*d - a*d^2)*a^2*d^2
*log(abs(b)) + 9*sqrt(b*c*d - a*d^2)*a*b*c*d - 10*sqrt(b*c*d - a*d^2)*a^2*d
^2)*sgn(x)/(sqrt(b*c*d - a*d^2)*sqrt(a)*b*c^5 - sqrt(b*c*d - a*d^2)*a^(3/2)
*c^4*d) - 1/4*(15*b^2*c^2*d*sgn(x) - 40*a*b*c*d^2*sgn(x) + 24*a^2*d^3*sgn(x
))*arctan(-((sqrt(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)/sqrt(b*c*d - a*d
^2))/((b*c^5 - a*c^4*d)*sqrt(b*c*d - a*d^2)) + sqrt(a*x^2 + b*x)*sgn(x)/c^3
 - 1/4*(9*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*sqrt(a)*b^2*c^3*d*sgn(x) - 32*(
sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a^(3/2)*b*c^2*d^2*sgn(x) + 24*(sqrt(a)*x -
 sqrt(a*x^2 + b*x))^3*a^(5/2)*c*d^3*sgn(x) + 3*(sqrt(a)*x - sqrt(a*x^2 + b*
x))^2*a*b^2*c^2*d^2*sgn(x) - 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^2*b*c*d
^3*sgn(x) + 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^3*d^4*sgn(x) + 7*(sqrt(a
)*x - sqrt(a*x^2 + b*x))*sqrt(a)*b^3*c^2*d^2*sgn(x) - 44*(sqrt(a)*x - sqrt(
a*x^2 + b*x))*a^(3/2)*b^2*c*d^3*sgn(x) + 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))
*a^(5/2)*b*d^4*sgn(x) - 9*a*b^3*c*d^3*sgn(x) + 10*a^2*b^2*d^4*sgn(x))/((sqr
t(a)*b*c^5 - a^(3/2)*c^4*d)*((sqrt(a)*x - sqrt(a*x^2 + b*x))^2*c + 2*(sqrt(
a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*d + b*d)^2) - 1/2*(b*c*sgn(x) - 6*a*d*sgn
(x))*log(abs(2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))/(sqrt(a)*c^4)
```

3.231 $\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)^3 \, dx$

Optimal. Leaf size=164

$$-\frac{d\left(a+\frac{b}{x}\right)^{3/2}\left(\frac{3bd(2ad+19bc)}{x}+2(13bc-ad)(2ad+5bc)\right)}{35b^2}-3c^2\sqrt{a+\frac{b}{x}}(2ad+bc)+3\sqrt{a}c^2(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)+x$$

```
[Out] -3*c^2*(b*c + 2*a*d)*Sqrt[a + b/x] - (9*d*(a + b/x)^(3/2)*(c + d/x)^2)/7 -
(d*(a + b/x)^(3/2)*(2*(13*b*c - a*d)*(5*b*c + 2*a*d) + (3*b*d*(19*b*c + 2*a
*d))/x))/(35*b^2) + (a + b/x)^(3/2)*(c + d/x)^3*x + 3*Sqrt[a]*c^2*(b*c + 2*
a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Rubi [A] time = 0.141866, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {375, 97, 153, 147, 50, 63, 208}

$$-\frac{d\left(a+\frac{b}{x}\right)^{3/2}\left(\frac{3bd(2ad+19bc)}{x}+2(13bc-ad)(2ad+5bc)\right)}{35b^2}-3c^2\sqrt{a+\frac{b}{x}}(2ad+bc)+3\sqrt{a}c^2(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)+x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(3/2)*(c + d/x)^3,x]
```

```
[Out] -3*c^2*(b*c + 2*a*d)*Sqrt[a + b/x] - (9*d*(a + b/x)^(3/2)*(c + d/x)^2)/7 -
(d*(a + b/x)^(3/2)*(2*(13*b*c - a*d)*(5*b*c + 2*a*d) + (3*b*d*(19*b*c + 2*a
*d))/x))/(35*b^2) + (a + b/x)^(3/2)*(c + d/x)^3*x + 3*Sqrt[a]*c^2*(b*c + 2*
a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
```

```
2, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3 dx = -\text{Subst}\left(\int \frac{(a+bx)^{3/2}(c+dx)^3}{x^2}dx, x, \frac{1}{x}\right)$$
$$= \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3 x - \text{Subst}\left(\int \frac{\sqrt{a+bx}(c+dx)^2\left(\frac{3}{2}(bc+2ad)+\frac{9bdx}{2}\right)}{x}dx, x, \frac{1}{x}\right)$$
$$= -\frac{9}{7}d\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2 + \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3 x - \frac{2\,\text{Subst}\left(\int \frac{\sqrt{a+bx}(c+dx)\left(\frac{21}{4}bc(bc+2ad)+\frac{3}{4}bd(19b\right.}{x}\right.}{7b}$$
$$= -\frac{9}{7}d\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2 - \frac{d\left(a+\frac{b}{x}\right)^{3/2}\left(2(13bc-ad)(5bc+2ad)+\frac{3bd(19bc+2ad)}{x}\right)}{35b^2} + \left(a+\frac{b}{x}\right)^3$$
$$= -3c^2(bc+2ad)\sqrt{a+\frac{b}{x}} - \frac{9}{7}d\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2 - \frac{d\left(a+\frac{b}{x}\right)^{3/2}\left(2(13bc-ad)(5bc+2ad)+\right.}{35b^2}$$
$$= -3c^2(bc+2ad)\sqrt{a+\frac{b}{x}} - \frac{9}{7}d\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2 - \frac{d\left(a+\frac{b}{x}\right)^{3/2}\left(2(13bc-ad)(5bc+2ad)+\right.}{35b^2}$$
$$= -3c^2(bc+2ad)\sqrt{a+\frac{b}{x}} - \frac{9}{7}d\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2 - \frac{d\left(a+\frac{b}{x}\right)^{3/2}\left(2(13bc-ad)(5bc+2ad)+\right.}{35b^2}$$

Mathematica [A] time = 0.168792, size = 159, normalized size = 0.97

$$\frac{\sqrt{a+\frac{b}{x}}\left(-2a^2bd^2x^2(21cx+d)+4a^3d^3x^3+ab^2x\left(-280c^2dx^2+35c^3x^3-84cd^2x-16d^3\right)-2b^3\left(35c^2dx^2+35c^3x^3+21cd^2\right.\right.}{35b^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(3/2)*(c + d/x)^3,x]
```

```
[Out] (Sqrt[a + b/x]*(4*a^3*d^3*x^3 - 2*a^2*b*d^2*x^2*(d + 21*c*x) + a*b^2*x*(-16
*d^3 - 84*c*d^2*x - 280*c^2*d*x^2 + 35*c^3*x^3) - 2*b^3*(5*d^3 + 21*c*d^2*x
 + 35*c^2*d*x^2 + 35*c^3*x^3)))/(35*b^2*x^3) + 3*Sqrt[a]*c^2*(b*c + 2*a*d)*
ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Maple [B] time = 0.012, size = 353, normalized size = 2.2

$$\frac{1}{70\,x^4b^2}\sqrt{\frac{ax+b}{x}}\left(420\,\sqrt{ax^2+bx}a^{5/2}x^5bc^2d+210\,\sqrt{ax^2+bx}a^{3/2}x^5b^2c^3+8\,\left(ax^2+bx\right)^{3/2}a^{5/2}x^2d^3-420\,\left(ax^2+bx\right)^{3/2}a^{3/}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(3/2)*(c+d/x)^3,x)
```

```
[Out] 1/70*((a*x+b)/x)^(1/2)*(420*(a*x^2+b*x)^(1/2)*a^(5/2)*x^5*b*c^2*d+210*(a*x^
2+b*x)^(1/2)*a^(3/2)*x^5*b^2*c^3+8*(a*x^2+b*x)^(3/2)*a^(5/2)*x^2*d^3-420*(a
*x^2+b*x)^(3/2)*a^(3/2)*x^3*b*c^2*d+210*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)
+2*a*x+b)/a^(1/2))*x^5*a^2*b^2*c^2*d+105*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2
```

```
)+2*a*x+b)/a^(1/2))*x^5*a*b^3*c^3-84*(a*x^2+b*x)^(3/2)*a^(3/2)*x^2*b*c*d^2-
140*(a*x^2+b*x)^(3/2)*a^(1/2)*x^3*b^2*c^3-12*(a*x^2+b*x)^(3/2)*a^(3/2)*x*b*
d^3-140*(a*x^2+b*x)^(3/2)*a^(1/2)*x^2*b^2*c^2*d-84*(a*x^2+b*x)^(3/2)*a^(1/2
)*x*b^2*c*d^2-20*(a*x^2+b*x)^(3/2)*a^(1/2)*b^2*d^3)/x^4/b^2/((a*x+b)*x)^(1/
2)/a^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*(c+d/x)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.37714, size = 845, normalized size = 5.15

$$\left[\frac{105\left(b^3c^3+2\,ab^2c^2d\right)\sqrt{a}x^3\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(35\,ab^2c^3x^4-10\,b^3d^3-2\left(35\,b^3c^3+140\,ab^2c^2d+21\,a^2b\right.\right.}{70\,b^2x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*(c+d/x)^3,x, algorithm="fricas")
```

```
[Out] [1/70*(105*(b^3*c^3 + 2*a*b^2*c^2*d)*sqrt(a)*x^3*log(2*a*x + 2*sqrt(a)*x*sq
rt((a*x + b)/x) + b) + 2*(35*a*b^2*c^3*x^4 - 10*b^3*d^3 - 2*(35*b^3*c^3 + 1
40*a*b^2*c^2*d + 21*a^2*b*c*d^2 - 2*a^3*d^3)*x^3 - 2*(35*b^3*c^2*d + 42*a*b
^2*c*d^2 + a^2*b*d^3)*x^2 - 2*(21*b^3*c*d^2 + 8*a*b^2*d^3)*x)*sqrt((a*x + b
)/x))/(b^2*x^3), -1/35*(105*(b^3*c^3 + 2*a*b^2*c^2*d)*sqrt(-a)*x^3*arctan(s
qrt(-a)*sqrt((a*x + b)/x)/a) - (35*a*b^2*c^3*x^4 - 10*b^3*d^3 - 2*(35*b^3*c
^3 + 140*a*b^2*c^2*d + 21*a^2*b*c*d^2 - 2*a^3*d^3)*x^3 - 2*(35*b^3*c^2*d +
42*a*b^2*c*d^2 + a^2*b*d^3)*x^2 - 2*(21*b^3*c*d^2 + 8*a*b^2*d^3)*x)*sqrt((a
*x + b)/x))/(b^2*x^3)]
```

Sympy [A] time = 68.4514, size = 1817, normalized size = 11.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2)*(c+d/x)**3,x)
```

```
[Out] -16*a**(19/2)*b**(11/2)*d**3*x**6*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(1
3/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(
7/2)*b**10*x**(7/2)) - 40*a**(17/2)*b**(13/2)*d**3*x**5*sqrt(a*x/b + 1)/(10
5*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b*
*9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 30*a**(15/2)*b**(15/2)*d**3*x*
*4*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(1
```

```
1/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 40*a**(1
3/2)*b**(17/2)*d**3*x**3*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 31
5*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**1
0*x**(7/2)) + 4*a**(13/2)*b**(3/2)*d**3*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b
**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 100*a**(11/2)*b**(19/2)*d**3*x*
*2*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(1
1/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 12*a**(1
1/2)*b**(5/2)*c*d**2*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a
**(5/2)*b**4*x**(5/2)) + 2*a**(11/2)*b**(5/2)*d**3*x**2*sqrt(a*x/b + 1)/(15
*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 96*a**(9/2)*b**(21/2
)*d**3*x*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8
*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 6*
a**(9/2)*b**(7/2)*c*d**2*x**2*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) +
15*a**(5/2)*b**4*x**(5/2)) - 8*a**(9/2)*b**(7/2)*d**3*x*sqrt(a*x/b + 1)/(15
*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 30*a**(7/2)*b**(23/2
)*d**3*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x
**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 24*a
**(7/2)*b**(9/2)*c*d**2*x*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a
**(5/2)*b**4*x**(5/2)) - 6*a**(7/2)*b**(9/2)*d**3*sqrt(a*x/b + 1)/(15*a**(7
/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 18*a**(5/2)*b**(11/2)*c*d*
*2*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2))
+ sqrt(a)*b*c**3*asinh(sqrt(a)*sqrt(x)/sqrt(b)) + 16*a**10*b**5*d**3*x**(13
/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(
9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 48*a**9*b**6*d**3*x**(1
1/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**
(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 48*a**8*b**7*d**3*x**(
9/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**
(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 16*a**7*b**8*d**3*x**(
7/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**
(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 4*a**7*b*d**3*x**(7/2)
/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 12*a**6*b**2*c*d
**2*x**(7/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 4*a*
*6*b**2*d**3*x**(5/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2
)) - 12*a**5*b**3*c*d**2*x**(5/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*
b**4*x**(5/2)) - 6*a**2*c**2*d*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a) + a*sq
rt(b)*c**3*sqrt(x)*sqrt(a*x/b + 1) - 2*a*b*c**3*atan(sqrt(a + b/x)/sqrt(-a)
)/sqrt(-a) - 6*a*c**2*d*sqrt(a + b/x) + 3*a*c*d**2*Piecewise((-sqrt(a)/x, E
q(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True)) - 2*b*c**3*sqrt(a + b/x) + 3*b
*c**2*d*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True)
)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*(c+d/x)^3,x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.232 $\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right)^2 \, dx$

Optimal. Leaf size=126

$$\frac{c^2x\left(a+\frac{b}{x}\right)^{5/2}}{a}-\frac{c\left(a+\frac{b}{x}\right)^{3/2}(4ad+3bc)}{3a}-c\sqrt{a+\frac{b}{x}}(4ad+3bc)+\sqrt{a}c(4ad+3bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-\frac{2d^2\left(a+\frac{b}{x}\right)^{5/2}}{5b}$$

```
[Out] -(c*(3*b*c + 4*a*d)*Sqrt[a + b/x]) - (c*(3*b*c + 4*a*d)*(a + b/x)^(3/2))/(3
*a) - (2*d^2*(a + b/x)^(5/2))/(5*b) + (c^2*(a + b/x)^(5/2)*x)/a + Sqrt[a]*c
*(3*b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Rubi [A] time = 0.0795285, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {375, 89, 80, 50, 63, 208}

$$\frac{c^2x\left(a+\frac{b}{x}\right)^{5/2}}{a}-\frac{c\left(a+\frac{b}{x}\right)^{3/2}(4ad+3bc)}{3a}-c\sqrt{a+\frac{b}{x}}(4ad+3bc)+\sqrt{a}c(4ad+3bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-\frac{2d^2\left(a+\frac{b}{x}\right)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(3/2)*(c + d/x)^2,x]
```

```
[Out] -(c*(3*b*c + 4*a*d)*Sqrt[a + b/x]) - (c*(3*b*c + 4*a*d)*(a + b/x)^(3/2))/(3
*a) - (2*d^2*(a + b/x)^(5/2))/(5*b) + (c^2*(a + b/x)^(5/2)*x)/a + Sqrt[a]*c
*(3*b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2 dx &= -\text{Subst}\left(\int \frac{(a+bx)^{3/2}(c+dx)^2}{x^2}dx, x, \frac{1}{x}\right)\\
&= \frac{c^2\left(a+\frac{b}{x}\right)^{5/2}x}{a} - \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}\left(\frac{1}{2}c(3bc+4ad)+ad^2x\right)}{x}dx, x, \frac{1}{x}\right)}{a}\\
&= -\frac{2d^2\left(a+\frac{b}{x}\right)^{5/2}}{5b} + \frac{c^2\left(a+\frac{b}{x}\right)^{5/2}x}{a} - \frac{(c(3bc+4ad))\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}dx, x, \frac{1}{x}\right)}{2a}\\
&= -\frac{c(3bc+4ad)\left(a+\frac{b}{x}\right)^{3/2}}{3a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{5/2}}{5b} + \frac{c^2\left(a+\frac{b}{x}\right)^{5/2}x}{a} - \frac{1}{2}(c(3bc+4ad))\,\text{Subst}\left(\int \frac{\sqrt{a}\right.\\
&= -c(3bc+4ad)\sqrt{a+\frac{b}{x}} - \frac{c(3bc+4ad)\left(a+\frac{b}{x}\right)^{3/2}}{3a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{5/2}}{5b} + \frac{c^2\left(a+\frac{b}{x}\right)^{5/2}x}{a} - \frac{1}{2}(ac(3\\
&= -c(3bc+4ad)\sqrt{a+\frac{b}{x}} - \frac{c(3bc+4ad)\left(a+\frac{b}{x}\right)^{3/2}}{3a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{5/2}}{5b} + \frac{c^2\left(a+\frac{b}{x}\right)^{5/2}x}{a} - \frac{(ac(3b}{}\\
&= -c(3bc+4ad)\sqrt{a+\frac{b}{x}} - \frac{c(3bc+4ad)\left(a+\frac{b}{x}\right)^{3/2}}{3a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{5/2}}{5b} + \frac{c^2\left(a+\frac{b}{x}\right)^{5/2}x}{a} + \sqrt{a}c(3
\end{aligned}$$

Mathematica [A] time = 0.176633, size = 106, normalized size = 0.84

$$-\frac{c(4ad+3bc)\left(\sqrt{a+\frac{b}{x}}(4ax+b)-3a^{3/2}x\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\right)}{3ax}+\frac{c^2x\left(a+\frac{b}{x}\right)^{5/2}}{a}-\frac{2d^2\left(a+\frac{b}{x}\right)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(3/2)*(c + d/x)^2,x]
```

```
[Out] (-2*d^2*(a + b/x)^(5/2))/(5*b) + (c^2*(a + b/x)^(5/2)*x)/a - (c*(3*b*c + 4*
a*d)*(Sqrt[a + b/x]*(b + 4*a*x) - 3*a^(3/2)*x*ArcTanh[Sqrt[a + b/x]/Sqrt[a]
]))/(3*a*x)
```

Maple [B] time = 0.011, size = 260, normalized size = 2.1

$$-\frac{1}{30\,bx^{3}}\sqrt{\frac{ax+b}{x}}\left(-120\,\sqrt{ax^{2}+bx}a^{5/2}x^{4}cd-90\,\sqrt{ax^{2}+bx}a^{3/2}x^{4}bc^{2}-60\,\ln\left(1/2\,\frac{2\,\sqrt{ax^{2}+bx}\sqrt{a}+2\,ax+b}{\sqrt{a}}\right)x^{4}a^{2}bcd$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(3/2)*(c+d/x)^2,x)
```

```
[Out] -1/30*((a*x+b)/x)^(1/2)/x^3/b*(-120*(a*x^2+b*x)^(1/2)*a^(5/2)*x^4*c*d-90*(a
*x^2+b*x)^(1/2)*a^(3/2)*x^4*b*c^2-60*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*
a*x+b)/a^(1/2))*x^4*a^2*b*c*d-45*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+
b)/a^(1/2))*x^4*a*b^2*c^2+120*(a*x^2+b*x)^(3/2)*a^(3/2)*x^2*c*d+60*(a*x^2+b
*x)^(3/2)*a^(1/2)*x^2*b*c^2+12*(a*x^2+b*x)^(3/2)*a^(3/2)*x*d^2+40*(a*x^2+b*
x)^(3/2)*a^(1/2)*x*b*c*d+12*(a*x^2+b*x)^(3/2)*a^(1/2)*b*d^2)/((a*x+b)*x)^(1
/2)/a^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*(c+d/x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.24766, size = 610, normalized size = 4.84

$$\left[\frac{15\left(3\,b^{2}c^{2}+4\,abcd\right)\sqrt{a}x^{2}\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(15\,abc^{2}x^{3}-6\,b^{2}d^{2}-2\left(15\,b^{2}c^{2}+40\,abcd+3\,a^{2}d^{2}\right)x^{2}-4}{30\,bx^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*(c+d/x)^2,x, algorithm="fricas")
```

```
[Out] [1/30*(15*(3*b^2*c^2 + 4*a*b*c*d)*sqrt(a)*x^2*log(2*a*x + 2*sqrt(a)*x*sqrt(
(a*x + b)/x) + b) + 2*(15*a*b*c^2*x^3 - 6*b^2*d^2 - 2*(15*b^2*c^2 + 40*a*b*
c*d + 3*a^2*d^2)*x^2 - 4*(5*b^2*c*d + 3*a*b*d^2)*x)*sqrt((a*x + b)/x))/(b*x
^2), -1/15*(15*(3*b^2*c^2 + 4*a*b*c*d)*sqrt(-a)*x^2*arctan(sqrt(-a)*sqrt((a
*x + b)/x)/a) - (15*a*b*c^2*x^3 - 6*b^2*d^2 - 2*(15*b^2*c^2 + 40*a*b*c*d +
3*a^2*d^2)*x^2 - 4*(5*b^2*c*d + 3*a*b*d^2)*x)*sqrt((a*x + b)/x))/(b*x^2)]
```

Sympy [A] time = 44.1276, size = 534, normalized size = 4.24

$$\frac{4a^{\frac{11}{2}}b^{\frac{5}{2}}d^2x^3\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}+\frac{2a^{\frac{9}{2}}b^{\frac{7}{2}}d^2x^2\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{8a^{\frac{7}{2}}b^{\frac{9}{2}}d^2x\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{6a^{\frac{5}{2}}b^{\frac{11}{2}}d^2\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}+\sqrt{a}bc^2\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{}}{\sqrt{b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2)*(c+d/x)**2,x)
```

```
[Out] 4*a**(11/2)*b**(5/2)*d**2*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) +
 15*a**(5/2)*b**4*x**(5/2)) + 2*a**(9/2)*b**(7/2)*d**2*x**2*sqrt(a*x/b + 1)
/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 8*a**(7/2)*b**(9
/2)*d**2*x*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x*
*(5/2)) - 6*a**(5/2)*b**(11/2)*d**2*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7
/2) + 15*a**(5/2)*b**4*x**(5/2)) + sqrt(a)*b*c**2*asinh(sqrt(a)*sqrt(x)/sqr
t(b)) - 4*a**6*b**2*d**2*x**(7/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*
b**4*x**(5/2)) - 4*a**5*b**3*d**2*x**(5/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*
a**(5/2)*b**4*x**(5/2)) - 4*a**2*c*d*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a)
+ a*sqrt(b)*c**2*sqrt(x)*sqrt(a*x/b + 1) - 2*a*b*c**2*atan(sqrt(a + b/x)/sq
rt(-a))/sqrt(-a) - 4*a*c*d*sqrt(a + b/x) + a*d**2*Piecewise((-sqrt(a)/x, Eq
(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True)) - 2*b*c**2*sqrt(a + b/x) + 2*b*
c*d*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*(c+d/x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.233 $\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right) dx$

Optimal. Leaf size=100

$$-\frac{\left(a+\frac{b}{x}\right)^{3/2}(2ad+3bc)}{3a}-\sqrt{a+\frac{b}{x}}(2ad+3bc)+\sqrt{a}(2ad+3bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)+\frac{cx\left(a+\frac{b}{x}\right)^{5/2}}{a}$$

```
[Out] -((3*b*c + 2*a*d)*Sqrt[a + b/x]) - ((3*b*c + 2*a*d)*(a + b/x)^(3/2))/(3*a)
+ (c*(a + b/x)^(5/2)*x)/a + Sqrt[a]*(3*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/S
qrt[a]]
```

Rubi [A] time = 0.0627703, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {375, 78, 50, 63, 208}

$$-\frac{\left(a+\frac{b}{x}\right)^{3/2}(2ad+3bc)}{3a}-\sqrt{a+\frac{b}{x}}(2ad+3bc)+\sqrt{a}(2ad+3bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)+\frac{cx\left(a+\frac{b}{x}\right)^{5/2}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(3/2)*(c + d/x),x]
```

```
[Out] -((3*b*c + 2*a*d)*Sqrt[a + b/x]) - ((3*b*c + 2*a*d)*(a + b/x)^(3/2))/(3*a)
+ (c*(a + b/x)^(5/2)*x)/a + Sqrt[a]*(3*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/S
qrt[a]]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)dx &= -\text{Subst}\left(\int \frac{(a+bx)^{3/2}(c+dx)}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= \frac{c\left(a+\frac{b}{x}\right)^{5/2}x}{a} - \frac{\left(\frac{3bc}{2}+ad\right)\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, \frac{1}{x}\right)}{a}\\
&= -\frac{(3bc+2ad)\left(a+\frac{b}{x}\right)^{3/2}}{3a} + \frac{c\left(a+\frac{b}{x}\right)^{5/2}x}{a} - \frac{1}{2}(3bc+2ad)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \frac{1}{x}\right)\\
&= -(3bc+2ad)\sqrt{a+\frac{b}{x}} - \frac{(3bc+2ad)\left(a+\frac{b}{x}\right)^{3/2}}{3a} + \frac{c\left(a+\frac{b}{x}\right)^{5/2}x}{a} - \frac{1}{2}(a(3bc+2ad))\,\text{Subst}\Bigg(\int\\
&= -(3bc+2ad)\sqrt{a+\frac{b}{x}} - \frac{(3bc+2ad)\left(a+\frac{b}{x}\right)^{3/2}}{3a} + \frac{c\left(a+\frac{b}{x}\right)^{5/2}x}{a} - \frac{(a(3bc+2ad))\,\text{Subst}\left(\int \frac{}{-\frac{a}{b}}\right.}{b}\\
&= -(3bc+2ad)\sqrt{a+\frac{b}{x}} - \frac{(3bc+2ad)\left(a+\frac{b}{x}\right)^{3/2}}{3a} + \frac{c\left(a+\frac{b}{x}\right)^{5/2}x}{a} + \sqrt{a}(3bc+2ad)\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{}}\right.
\end{aligned}$$

Mathematica [A] time = 0.0693263, size = 73, normalized size = 0.73

$$\frac{\sqrt{a+\frac{b}{x}}(ax(3cx-8d)-2b(3cx+d))}{3x}+\sqrt{a}(2ad+3bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(3/2)*(c + d/x),x]
```

```
[Out] (Sqrt[a + b/x]*(a*x*(-8*d + 3*c*x) - 2*b*(d + 3*c*x)))/(3*x) + Sqrt[a]*(3*b
*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Maple [B] time = 0.009, size = 205, normalized size = 2.1

$$-\frac{1}{6\,bx^2}\sqrt{\frac{ax+b}{x}}\left(-12\,\sqrt{ax^2+bx}a^{5/2}x^3d-18\,\sqrt{ax^2+bx}a^{3/2}x^3bc-6\,\ln\left(1/2\,\frac{2\,\sqrt{ax^2+bx}\sqrt{a}+2\,ax+b}{\sqrt{a}}\right)x^3a^2bd-9\,\ln\left(1/\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(3/2)*(c+d/x),x)
```

```
[Out] -1/6*((a*x+b)/x)^(1/2)/x^2*(-12*(a*x^2+b*x)^(1/2)*a^(5/2)*x^3*d-18*(a*x^2+b
*x)^(1/2)*a^(3/2)*x^3*b*c-6*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^
(1/2))*x^3*a^2*b*d-9*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*
x^3*a*b^2*c+12*(a*x^2+b*x)^(3/2)*a^(3/2)*x*d+12*(a*x^2+b*x)^(3/2)*a^(1/2)*x
*b*c+4*d*(a*x^2+b*x)^(3/2)*a^(1/2)*b)/((a*x+b)*x)^(1/2)/a^(1/2)/b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*(c+d/x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.2941, size = 392, normalized size = 3.92

$$\left[\frac{3\,(3\,bc+2\,ad)\sqrt{a}x\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(3\,acx^2-2\,bd-2\,(3\,bc+4\,ad)x\right)\sqrt{\frac{ax+b}{x}}}{6\,x},-\frac{3\,(3\,bc+2\,ad)\sqrt{-a}x\,\mathrm{a}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*(c+d/x),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(3*b*c + 2*a*d)*sqrt(a)*x*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x)
 + b) + 2*(3*a*c*x^2 - 2*b*d - 2*(3*b*c + 4*a*d)*x)*sqrt((a*x + b)/x))/x, -
1/3*(3*(3*b*c + 2*a*d)*sqrt(-a)*x*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (3
*a*c*x^2 - 2*b*d - 2*(3*b*c + 4*a*d)*x)*sqrt((a*x + b)/x))/x]
```

Sympy [A] time = 27.8709, size = 163, normalized size = 1.63

$$\sqrt{a}bc\,\mathrm{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)-\frac{2a^2d\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}+a\sqrt{b}c\sqrt{x}\sqrt{\frac{ax}{b}+1}-\frac{2abc\,\mathrm{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}}-2ad\sqrt{a+\frac{b}{x}}-2bc\sqrt{a+\frac{b}{x}}+bc$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2)*(c+d/x),x)
```

```
[Out] sqrt(a)*b*c*asinh(sqrt(a)*sqrt(x)/sqrt(b)) - 2*a**2*d*atan(sqrt(a + b/x)/sq
rt(-a))/sqrt(-a) + a*sqrt(b)*c*sqrt(x)*sqrt(a*x/b + 1) - 2*a*b*c*atan(sqrt(
a + b/x)/sqrt(-a))/sqrt(-a) - 2*a*d*sqrt(a + b/x) - 2*b*c*sqrt(a + b/x) + b
*d*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)*(c+d/x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.234 $\int \left(a+\frac{b}{x}\right)^{3/2} dx$

Optimal. Leaf size=54

$$x\left(a+\frac{b}{x}\right)^{3/2}-3b\sqrt{a+\frac{b}{x}}+3\sqrt{a}b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

```
[Out] -3*b*Sqrt[a + b/x] + (a + b/x)^(3/2)*x + 3*Sqrt[a]*b*ArcTanh[Sqrt[a + b/x]/
Sqrt[a]]
```

Rubi [A] time = 0.0253246, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {242, 47, 50, 63, 208}

$$x\left(a+\frac{b}{x}\right)^{3/2}-3b\sqrt{a+\frac{b}{x}}+3\sqrt{a}b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(3/2),x]
```

```
[Out] -3*b*Sqrt[a + b/x] + (a + b/x)^(3/2)*x + 3*Sqrt[a]*b*ArcTanh[Sqrt[a + b/x]/
Sqrt[a]]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \left(a+\frac{b}{x}\right)^{3/2} dx &= -\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= \left(a+\frac{b}{x}\right)^{3/2} x - \frac{1}{2}(3b)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \frac{1}{x}\right)\\
&= -3b\sqrt{a+\frac{b}{x}} + \left(a+\frac{b}{x}\right)^{3/2} x - \frac{1}{2}(3ab)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)\\
&= -3b\sqrt{a+\frac{b}{x}} + \left(a+\frac{b}{x}\right)^{3/2} x - (3a)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)\\
&= -3b\sqrt{a+\frac{b}{x}} + \left(a+\frac{b}{x}\right)^{3/2} x + 3\sqrt{a}b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0191561, size = 46, normalized size = 0.85

$$\sqrt{a+\frac{b}{x}}(ax-2b)+3\sqrt{a}b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(3/2),x]
```

```
[Out] Sqrt[a + b/x]*(-2*b + a*x) + 3*Sqrt[a]*b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Maple [B] time = 0.007, size = 100, normalized size = 1.9

$$-\frac{1}{2x}\sqrt{\frac{ax+b}{x}}\left(-6\sqrt{ax^2+bx}a^{3/2}x^2-3\ln\left(1/2\,\frac{2\sqrt{ax^2+bx}\sqrt{a}+2ax+b}{\sqrt{a}}\right)x^2ab+4\left(ax^2+bx\right)^{3/2}\sqrt{a}\right)\frac{1}{\sqrt{(ax+b)x}}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(3/2),x)
```

```
[Out] -1/2*((a*x+b)/x)^(1/2)/x*(-6*(a*x^2+b*x)^(1/2)*a^(3/2)*x^2-3*ln(1/2*(2*(a*x
^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x^2*a*b+4*(a*x^2+b*x)^(3/2)*a^(1/2)
)/((a*x+b)*x)^(1/2)/a^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.52256, size = 246, normalized size = 4.56

$$\left[\frac{3}{2}\sqrt{a}b\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+(ax-2\,b)\sqrt{\frac{ax+b}{x}},-3\,\sqrt{-a}b\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)+(ax-2\,b)\sqrt{\frac{ax+b}{x}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [3/2*sqrt(a)*b*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (a*x - 2*b)
*sqrt((a*x + b)/x), -3*sqrt(-a)*b*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (a
*x - 2*b)*sqrt((a*x + b)/x)]
```

Sympy [B] time = 2.37587, size = 92, normalized size = 1.7

$$3\sqrt{a}b\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)+\frac{a^2x^{\frac{3}{2}}}{\sqrt{b}\sqrt{\frac{ax}{b}+1}}-\frac{a\sqrt{b}\sqrt{x}}{\sqrt{\frac{ax}{b}+1}}-\frac{2b^{\frac{3}{2}}}{\sqrt{x}\sqrt{\frac{ax}{b}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2),x)
```

```
[Out] 3*sqrt(a)*b*asinh(sqrt(a)*sqrt(x)/sqrt(b)) + a**2*x**(3/2)/(sqrt(b)*sqrt(a*
x/b + 1)) - a*sqrt(b)*sqrt(x)/sqrt(a*x/b + 1) - 2*b**(3/2)/(sqrt(x)*sqrt(a*
x/b + 1))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.235 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{c+\frac{d}{x}}\,dx$

Optimal. Leaf size=106

$$-\frac{2(bc-ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2\sqrt{d}}+\frac{\sqrt{a}(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^2}+\frac{ax\sqrt{a+\frac{b}{x}}}{c}$$

```
[Out] (a*Sqrt[a + b/x]*x)/c - (2*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])
/Sqrt[b*c - a*d]])/(c^2*Sqrt[d]) + (Sqrt[a]*(3*b*c - 2*a*d)*ArcTanh[Sqrt[a
+ b/x]/Sqrt[a]])/c^2
```

Rubi [A] time = 0.12833, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {375, 98, 156, 63, 208, 205}

$$-\frac{2(bc-ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2\sqrt{d}}+\frac{\sqrt{a}(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^2}+\frac{ax\sqrt{a+\frac{b}{x}}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(3/2)/(c + d/x),x]
```

```
[Out] (a*Sqrt[a + b/x]*x)/c - (2*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])
/Sqrt[b*c - a*d]])/(c^2*Sqrt[d]) + (Sqrt[a]*(3*b*c - 2*a*d)*ArcTanh[Sqrt[a
+ b/x]/Sqrt[a]])/c^2
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{c+\frac{d}{x}}\,dx &= -\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2(c+dx)}\,dx, x, \frac{1}{x}\right)\\
&= \frac{a\sqrt{a+\frac{b}{x}}x}{c} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}a(3bc-2ad)-\frac{1}{2}b(2bc-ad)x}{x\sqrt{a+bx}(c+dx)}\,dx, x, \frac{1}{x}\right)}{c}\\
&= \frac{a\sqrt{a+\frac{b}{x}}x}{c} - \frac{(a(3bc-2ad))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2c^2} - \frac{(bc-ad)^2\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}(c+dx)}\,dx, x, \frac{1}{x}\right)}{c^2}\\
&= \frac{a\sqrt{a+\frac{b}{x}}x}{c} - \frac{(a(3bc-2ad))\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^2} - \frac{\left(2(bc-ad)^2\right)\text{Subst}\left(\int \frac{1}{c-\frac{ad}{b}+\frac{dx^2}{b}}\,dx,
\right.}{bc^2}\\
&= \frac{a\sqrt{a+\frac{b}{x}}x}{c} - \frac{2(bc-ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2\sqrt{d}} + \frac{\sqrt{a}(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^2}
\end{aligned}
$$

Mathematica [A] time = 0.19096, size = 102, normalized size = 0.96

$$
\frac{-\frac{2(bc-ad)^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{\sqrt{d}} + \sqrt{a}(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right) + acx\sqrt{a+\frac{b}{x}}}{c^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(3/2)/(c + d/x),x]
```

```
[Out] (a*c*Sqrt[a + b/x]*x - (2*(b*c - a*d)^(3/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/
Sqrt[b*c - a*d]])/Sqrt[d] + Sqrt[a]*(3*b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/S
qrt[a]])/c^2
```

Maple [B] time = 0.011, size = 528, normalized size = 5.

$$
\frac{x}{2\,dc^3}\sqrt{\frac{ax+b}{x}}\left(\ln\left(\frac{1}{2}\left(2\sqrt{ax^2+bx}\sqrt{a}+2\,ax+b\right)\frac{1}{\sqrt{a}}\right)\sqrt{\frac{(ad-bc)\,d}{c^2}}b^2c^3+2\sqrt{a}\sqrt{\frac{(ad-bc)\,d}{c^2}}\sqrt{ax^2+bx}bc^3+2\,a^{3/2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(3/2)/(c+d/x),x)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x*(ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/
2))*((a*d-b*c)*d/c^2)^(1/2)*b^2*c^3+2*a^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*(a*x^
2+b*x)^(1/2)*b*c^3+2*a^(3/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c^2*
d-2*a^(1/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*b*c^3-2*a^(5/2)*ln((2
*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*d^
3+4*a^(3/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x
-b*d)/(c*x+d))*b*c*d^2-2*a^(1/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^
(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^2*c^2*d-2*ln(1/2*(2*((a*x+b)*x)^(1/2)
*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*a^2*c*d^2+3*ln(1/2*(2*((
a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*a*b*c^2*d
-ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1
/2)*b^2*c^3)/((a*x+b)*x)^(1/2)/d/c^3/a^(1/2)/((a*d-b*c)*d/c^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{c+\frac{d}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/(c+d/x),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^(3/2)/(c + d/x), x)
```

Fricas [A] time = 1.76807, size = 1156, normalized size = 10.91

$$\left[\frac{2\,acx\sqrt{\frac{ax+b}{x}}-(3\,bc-2\,ad)\sqrt{a}\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)-2\,(bc-ad)\sqrt{-\frac{bc-ad}{d}}\log\left(\frac{2\,dx\sqrt{-\frac{bc-ad}{d}}\sqrt{\frac{ax+b}{x}}+bd-(bc-2\,ad)x}{cx+d}\right)}{2\,c^2},\ acx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/(c+d/x),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*c*x*sqrt((a*x + b)/x) - (3*b*c - 2*a*d)*sqrt(a)*log(2*a*x - 2*sqr
t(a)*x*sqrt((a*x + b)/x) + b) - 2*(b*c - a*d)*sqrt(-(b*c - a*d)/d)*log((2*d
*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) + b*d - (b*c - 2*a*d)*x)/(c*x + d
)))/c^2, (a*c*x*sqrt((a*x + b)/x) - (3*b*c - 2*a*d)*sqrt(-a)*arctan(sqrt(-a
)*sqrt((a*x + b)/x)/a) - (b*c - a*d)*sqrt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-
(b*c - a*d)/d)*sqrt((a*x + b)/x) + b*d - (b*c - 2*a*d)*x)/(c*x + d)))/c^2,
1/2*(2*a*c*x*sqrt((a*x + b)/x) + 4*(b*c - a*d)*sqrt((b*c - a*d)/d)*arctan(-
d*sqrt((b*c - a*d)/d)*sqrt((a*x + b)/x)/(b*c - a*d)) - (3*b*c - 2*a*d)*sqrt
(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b))/c^2, (a*c*x*sqrt((a*x +
 b)/x) + 2*(b*c - a*d)*sqrt((b*c - a*d)/d)*arctan(-d*sqrt((b*c - a*d)/d)*sq
rt((a*x + b)/x)/(b*c - a*d)) - (3*b*c - 2*a*d)*sqrt(-a)*arctan(sqrt(-a)*sqr
t((a*x + b)/x)/a))/c^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{cx+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2)/(c+d/x),x)
```

```
[Out] Integral(x*(a + b/x)**(3/2)/(c*x + d), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/(c+d/x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.236 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^2}\,dx$

Optimal. Leaf size=156

$$-\frac{\sqrt{a+\frac{b}{x}}(bc-2ad)}{c^2\left(c+\frac{d}{x}\right)}-\frac{(bc-4ad)\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3\sqrt{d}}+\frac{\sqrt{a}(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^3}+\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}$$

```
[Out] -(((b*c - 2*a*d)*Sqrt[a + b/x])/(c^2*(c + d/x))) + (a*Sqrt[a + b/x]*x)/(c*(
c + d/x)) - ((b*c - 4*a*d)*Sqrt[b*c - a*d]*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/S
qrt[b*c - a*d]])/(c^3*Sqrt[d]) + (Sqrt[a]*(3*b*c - 4*a*d)*ArcTanh[Sqrt[a +
b/x]/Sqrt[a]])/c^3
```

Rubi [A] time = 0.221831, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {375, 98, 151, 156, 63, 208, 205}

$$-\frac{\sqrt{a+\frac{b}{x}}(bc-2ad)}{c^2\left(c+\frac{d}{x}\right)}-\frac{(bc-4ad)\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3\sqrt{d}}+\frac{\sqrt{a}(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^3}+\frac{ax\sqrt{a+\frac{b}{x}}}{c\left(c+\frac{d}{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(3/2)/(c + d/x)^2,x]
```

```
[Out] -(((b*c - 2*a*d)*Sqrt[a + b/x])/(c^2*(c + d/x))) + (a*Sqrt[a + b/x]*x)/(c*(
c + d/x)) - ((b*c - 4*a*d)*Sqrt[b*c - a*d]*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/S
qrt[b*c - a*d]])/(c^3*Sqrt[d]) + (Sqrt[a]*(3*b*c - 4*a*d)*ArcTanh[Sqrt[a +
b/x]/Sqrt[a]])/c^3
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
```

```
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^2}\,dx = -\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2(c+dx)^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{a\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}a(3bc-4ad)-\frac{1}{2}b(2bc-3ad)x}{x\sqrt{a+bx}(c+dx)^2}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{(bc-2ad)\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)} + \frac{a\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}a(3bc-4ad)(bc-ad)+\frac{1}{2}b(bc-2ad)(bc-ad)x}{x\sqrt{a+bx}(c+dx)}\,dx, x, \frac{1}{x}\right)}{c^2(bc-ad)}$$

$$= -\frac{(bc-2ad)\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)} + \frac{a\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)} - \frac{(a(3bc-4ad))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2c^3} - \frac{((bc-4ad)(bc-ad))\,\text{S}}{}$$

$$= -\frac{(bc-2ad)\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)} + \frac{a\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)} - \frac{(a(3bc-4ad))\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^3} - \frac{((bc-4ad)(bc-}{}$$

$$= -\frac{(bc-2ad)\sqrt{a+\frac{b}{x}}}{c^2\left(c+\frac{d}{x}\right)} + \frac{a\sqrt{a+\frac{b}{x}}\,x}{c\left(c+\frac{d}{x}\right)} - \frac{(bc-4ad)\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3\sqrt{d}} + \frac{\sqrt{a}(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{}}\right.}{c^3}$$

Mathematica [A] time = 0.284517, size = 143, normalized size = 0.92

$$\frac{-\frac{\left(4a^2d^2-5abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{\sqrt{d}\sqrt{bc-ad}} + \frac{cx\sqrt{a+\frac{b}{x}}(acx+2ad-bc)}{cx+d} + \sqrt{a}(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(3/2)/(c + d/x)^2,x]
```

```
[Out] ((c*Sqrt[a + b/x]*x*(-(b*c) + 2*a*d + a*c*x))/(d + c*x) - ((b^2*c^2 - 5*a*b
*c*d + 4*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(Sqrt[d]
*Sqrt[b*c - a*d]) + Sqrt[a]*(3*b*c - 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])
/c^3
```

Maple [B] time = 0.01, size = 834, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(3/2)/(c+d/x)^2,x)
```

```
[Out] -1/2*(4*a^(7/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b
*c*x-b*d)/(c*x+d))*x*c*d^3+2*a^(5/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1
/2)*x^2*c^4+4*a^(7/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a
```

```
*d*x+b*c*x-b*d)/(c*x+d))*d^4-2*a^(5/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^
(1/2)*x*c^3*d-5*a^(5/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2
*a*d*x+b*c*x-b*d)/(c*x+d))*x*b*c^2*d^2-4*a^(5/2)*((a*d-b*c)*d/c^2)^(1/2)*((
a*x+b)*x)^(1/2)*c^2*d^2-5*a^(5/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)
^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b*c*d^3-2*c^4*((a*x+b)*x)^(3/2)*a^(3/2
)*((a*d-b*c)*d/c^2)^(1/2)+2*a^(3/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/
2)*x*b*c^4+a^(3/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*
x+b*c*x-b*d)/(c*x+d))*x*b^2*c^3*d+4*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a
*x+b)/a^(1/2))*a^3*((a*d-b*c)*d/c^2)^(1/2)*x*c^2*d^2-3*ln(1/2*(2*((a*x+b)*x
)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*((a*d-b*c)*d/c^2)^(1/2)*x*b*c^3*d+2*a
^(3/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*b*c^3*d+a^(3/2)*ln((2*((a*
d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^2*c^2
*d^2+4*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*((a*d-b*c)
*d/c^2)^(1/2)*c*d^3-3*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))
*a^2*((a*d-b*c)*d/c^2)^(1/2)*b*c^2*d^2)*x*((a*x+b)/x)^(1/2)/c^4/((a*d-b*c)*
d/c^2)^(1/2)/a^(3/2)/(c*x+d)/d/((a*x+b)*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{\left(c+\frac{d}{x}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/(c+d/x)^2,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^(3/2)/(c + d/x)^2, x)
```

Fricas [A] time = 1.8083, size = 1682, normalized size = 10.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/(c+d/x)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((3*b*c*d - 4*a*d^2 + (3*b*c^2 - 4*a*c*d)*x)*sqrt(a)*log(2*a*x - 2*sq
rt(a)*x*sqrt((a*x + b)/x) + b) + (b*c*d - 4*a*d^2 + (b*c^2 - 4*a*c*d)*x)*sq
rt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) + b*d
- (b*c - 2*a*d)*x)/(c*x + d)) - 2*(a*c^2*x^2 - (b*c^2 - 2*a*c*d)*x)*sqrt((a
*x + b)/x))/(c^4*x + c^3*d), 1/2*(2*(b*c*d - 4*a*d^2 + (b*c^2 - 4*a*c*d)*x)
*sqrt((b*c - a*d)/d)*arctan(-d*sqrt((b*c - a*d)/d)*sqrt((a*x + b)/x)/(b*c -
 a*d)) - (3*b*c*d - 4*a*d^2 + (3*b*c^2 - 4*a*c*d)*x)*sqrt(a)*log(2*a*x - 2*
sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(a*c^2*x^2 - (b*c^2 - 2*a*c*d)*x)*sqrt
((a*x + b)/x))/(c^4*x + c^3*d), -1/2*(2*(3*b*c*d - 4*a*d^2 + (3*b*c^2 - 4*a
*c*d)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (b*c*d - 4*a*d^2 +
 (b*c^2 - 4*a*c*d)*x)*sqrt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c - a*d)/d)*
sqrt((a*x + b)/x) + b*d - (b*c - 2*a*d)*x)/(c*x + d)) - 2*(a*c^2*x^2 - (b*c
^2 - 2*a*c*d)*x)*sqrt((a*x + b)/x))/(c^4*x + c^3*d), ((b*c*d - 4*a*d^2 + (b
*c^2 - 4*a*c*d)*x)*sqrt((b*c - a*d)/d)*arctan(-d*sqrt((b*c - a*d)/d)*sqrt((
a*x + b)/x)/(b*c - a*d)) - (3*b*c*d - 4*a*d^2 + (3*b*c^2 - 4*a*c*d)*x)*sqrt
(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (a*c^2*x^2 - (b*c^2 - 2*a*c*d)*
```

```
x)*sqrt((a*x + b)/x))/(c^4*x + c^3*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2)/(c+d/x)**2,x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/(c+d/x)^2,x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.237 $\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^3} dx$

Optimal. Leaf size=209

$$-\frac{3\left(8a^2d^2-8abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4\sqrt{d}\sqrt{bc-ad}}-\frac{3\sqrt{a+\frac{b}{x}}(bc-4ad)}{4c^3\left(c+\frac{d}{x}\right)}-\frac{\sqrt{a+\frac{b}{x}}(bc-3ad)}{2c^2\left(c+\frac{d}{x}\right)^2}+\frac{3\sqrt{a}(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^4}$$

```
[Out] -((b*c - 3*a*d)*Sqrt[a + b/x])/(2*c^2*(c + d/x)^2) - (3*(b*c - 4*a*d)*Sqrt[
a + b/x])/(4*c^3*(c + d/x)) + (a*Sqrt[a + b/x]*x)/(c*(c + d/x)^2) - (3*(b^2
*c^2 - 8*a*b*c*d + 8*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d
]])/(4*c^4*Sqrt[d]*Sqrt[b*c - a*d]) + (3*Sqrt[a]*(b*c - 2*a*d)*ArcTanh[Sqrt
[a + b/x]/Sqrt[a]])/c^4
```

Rubi [A] time = 0.344981, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {375, 98, 151, 156, 63, 208, 205}

$$-\frac{3\left(8a^2d^2-8abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4\sqrt{d}\sqrt{bc-ad}}-\frac{3\sqrt{a+\frac{b}{x}}(bc-4ad)}{4c^3\left(c+\frac{d}{x}\right)}-\frac{\sqrt{a+\frac{b}{x}}(bc-3ad)}{2c^2\left(c+\frac{d}{x}\right)^2}+\frac{3\sqrt{a}(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(3/2)/(c + d/x)^3,x]
```

```
[Out] -((b*c - 3*a*d)*Sqrt[a + b/x])/(2*c^2*(c + d/x)^2) - (3*(b*c - 4*a*d)*Sqrt[
a + b/x])/(4*c^3*(c + d/x)) + (a*Sqrt[a + b/x]*x)/(c*(c + d/x)^2) - (3*(b^2
*c^2 - 8*a*b*c*d + 8*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d
]])/(4*c^4*Sqrt[d]*Sqrt[b*c - a*d]) + (3*Sqrt[a]*(b*c - 2*a*d)*ArcTanh[Sqrt
[a + b/x]/Sqrt[a]])/c^4
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{\left(c+\frac{d}{x}\right)^3} dx = -\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2(c+dx)^3} dx, x, \frac{1}{x}\right)$$

$$= \frac{a\sqrt{a+\frac{b}{x}}x}{c\left(c+\frac{d}{x}\right)^2} + \frac{\text{Subst}\left(\int \frac{-\frac{3}{2}a(bc-2ad)-\frac{1}{2}b(2bc-5ad)x}{x\sqrt{a+bx}(c+dx)^3} dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} + \frac{a\sqrt{a+\frac{b}{x}}x}{c\left(c+\frac{d}{x}\right)^2} - \frac{\text{Subst}\left(\int \frac{3a(bc-2ad)(bc-ad)+\frac{3}{2}b(bc-3ad)(bc-ad)x}{x\sqrt{a+bx}(c+dx)^2} dx, x, \frac{1}{x}\right)}{2c^2(bc-ad)}$$

$$= -\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} - \frac{3(bc-4ad)\sqrt{a+\frac{b}{x}}}{4c^3\left(c+\frac{d}{x}\right)} + \frac{a\sqrt{a+\frac{b}{x}}x}{c\left(c+\frac{d}{x}\right)^2} + \frac{\text{Subst}\left(\int \frac{-3a(bc-2ad)(bc-ad)^2-\frac{3}{4}b(bc-4ad)(bc-a}{x\sqrt{a+bx}(c+dx)}\right.}{2c^3(bc-ad)^2}$$

$$= -\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} - \frac{3(bc-4ad)\sqrt{a+\frac{b}{x}}}{4c^3\left(c+\frac{d}{x}\right)} + \frac{a\sqrt{a+\frac{b}{x}}x}{c\left(c+\frac{d}{x}\right)^2} - \frac{(3a(bc-2ad))\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, \frac{1}{x}\right)}{2c^4}$$

$$= -\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} - \frac{3(bc-4ad)\sqrt{a+\frac{b}{x}}}{4c^3\left(c+\frac{d}{x}\right)} + \frac{a\sqrt{a+\frac{b}{x}}x}{c\left(c+\frac{d}{x}\right)^2} - \frac{(3a(bc-2ad))\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}} dx, x, \sqrt{a\right.}{bc^4}$$

$$= -\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} - \frac{3(bc-4ad)\sqrt{a+\frac{b}{x}}}{4c^3\left(c+\frac{d}{x}\right)} + \frac{a\sqrt{a+\frac{b}{x}}x}{c\left(c+\frac{d}{x}\right)^2} - \frac{3\left(b^2c^2-8abcd+8a^2d^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right.}{4c^4\sqrt{d}\sqrt{bc-ad}}$$

Mathematica [A] time = 0.448489, size = 168, normalized size = 0.8

$$\frac{-\frac{3\left(8a^2d^2-8abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{\sqrt{d}\sqrt{bc-ad}} + \frac{cx\sqrt{a+\frac{b}{x}}\left(2a\left(2c^2x^2+9cdx+6d^2\right)-bc(5cx+3d)\right)}{(cx+d)^2} + 12\sqrt{a}(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{4c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(3/2)/(c + d/x)^3,x]
```

```
[Out] ((c*Sqrt[a + b/x]*x*(-(b*c*(3*d + 5*c*x)) + 2*a*(6*d^2 + 9*c*d*x + 2*c^2*x^
2)))/(d + c*x)^2 - (3*(b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*ArcTan[(Sqrt[d]*Sqr
t[a + b/x])/Sqrt[b*c - a*d]])/(Sqrt[d]*Sqrt[b*c - a*d]) + 12*Sqrt[a]*(b*c -
 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(4*c^4)
```

Maple [B] time = 0.011, size = 1817, normalized size = 8.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(3/2)/(c+d/x)^3,x)
```

```
[Out] -1/8*(24*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*((a*d-b*
c)*d/c^2)^(1/2)*x*b^2*c^4*d^2+54*a^(5/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x
)^(1/2)*x*b*c^4*d^2+12*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2)
)*a^2*((a*d-b*c)*d/c^2)^(1/2)*x^2*b^2*c^5*d+18*a^(5/2)*((a*d-b*c)*d/c^2)^(1
/2)*((a*x+b)*x)^(1/2)*x^2*b*c^5*d-72*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*
a*x+b)/a^(1/2))*a^3*((a*d-b*c)*d/c^2)^(1/2)*x*b*c^3*d^3-36*ln(1/2*(2*((a*x+
b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*((a*d-b*c)*d/c^2)^(1/2)*x^2*b*c^4
*d^2-36*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*((a*d-b*c
)*d/c^2)^(1/2)*b*c^2*d^4+12*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^
(1/2))*a^2*((a*d-b*c)*d/c^2)^(1/2)*b^2*c^3*d^3+30*a^(5/2)*((a*d-b*c)*d/c^2)
^(1/2)*((a*x+b)*x)^(1/2)*b*c^3*d^3+24*a^(9/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)
*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*d^6-6*a^(3/2)*((a*d-b*c)*d
/c^2)^(1/2)*((a*x+b)*x)^(1/2)*b^2*c^4*d^2+12*a^(7/2)*((a*d-b*c)*d/c^2)^(1/2
)*((a*x+b)*x)^(1/2)*x^3*c^5*d-6*a^(5/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)
^(1/2)*x^3*b*c^6+24*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a
^4*((a*d-b*c)*d/c^2)^(1/2)*x^2*c^3*d^3-12*a^(5/2)*((a*d-b*c)*d/c^2)^(1/2)*(
(a*x+b)*x)^(3/2)*x*c^5*d-48*a^(7/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*
x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x^2*b*c^3*d^3+27*a^(5/2)*ln((2*((a*d
-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x^2*b^2*
c^4*d^2+48*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^4*((a*d-
b*c)*d/c^2)^(1/2)*x*c^2*d^4-36*a^(7/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^
(1/2)*x*c^3*d^3-96*a^(7/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*
c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x*b*c^2*d^4+54*a^(5/2)*ln((2*((a*d-b*c)*d/c^2
)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x*b^2*c^3*d^3-6*a^(
3/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*x^2*b^2*c^6-3*a^(3/2)*ln((2*
((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x^2
*b^3*c^5*d-6*a^(3/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*
d*x+b*c*x-b*d)/(c*x+d))*x*b^3*c^4*d^2+2*a^(3/2)*((a*d-b*c)*d/c^2)^(1/2)*((a
*x+b)*x)^(3/2)*b*c^5*d-12*a^(3/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)
*x*b^2*c^5*d+24*a^(9/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2
*a*d*x+b*c*x-b*d)/(c*x+d))*x^2*c^2*d^4-8*a^(5/2)*((a*d-b*c)*d/c^2)^(1/2)*((
a*x+b)*x)^(3/2)*c^4*d^2+24*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(
1/2))*a^4*((a*d-b*c)*d/c^2)^(1/2)*c*d^5-24*a^(7/2)*((a*d-b*c)*d/c^2)^(1/2)*
((a*x+b)*x)^(1/2)*c^2*d^4-48*a^(7/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)
*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b*c*d^5+27*a^(5/2)*ln((2*((a*d-b*c)
*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^2*c^2*d^4-3
*a^(3/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*
d)/(c*x+d))*b^3*c^3*d^3+48*a^(9/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x
)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x*c*d^5+6*a^(3/2)*((a*d-b*c)*d/c^2)^(
1/2)*((a*x+b)*x)^(3/2)*x*b*c^6)*x*((a*x+b)/x)^(1/2)/d/c^5/((a*d-b*c)*d/c^2)
^(1/2)/a^(3/2)/(c*x+d)^2/(a*d-b*c)/((a*x+b)*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}{\left(c+\frac{d}{x}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/(c+d/x)^3,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^(3/2)/(c + d/x)^3, x)
```

Fricas [B] time = 1.81602, size = 3646, normalized size = 17.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/(c+d/x)^3,x, algorithm="fricas")
```

```
[Out] [-1/8*(12*(b^2*c^2*d^3 - 3*a*b*c*d^4 + 2*a^2*d^5 + (b^2*c^4*d - 3*a*b*c^3*d
^2 + 2*a^2*c^2*d^3)*x^2 + 2*(b^2*c^3*d^2 - 3*a*b*c^2*d^3 + 2*a^2*c*d^4)*x)*
sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 3*(b^2*c^2*d^2 - 8
*a*b*c*d^3 + 8*a^2*d^4 + (b^2*c^4 - 8*a*b*c^3*d + 8*a^2*c^2*d^2)*x^2 + 2*(b
^2*c^3*d - 8*a*b*c^2*d^2 + 8*a^2*c*d^3)*x)*sqrt(-b*c*d + a*d^2)*log((b*d -
(b*c - 2*a*d)*x + 2*sqrt(-b*c*d + a*d^2)*x*sqrt((a*x + b)/x))/(c*x + d)) -
2*(4*(a*b*c^4*d - a^2*c^3*d^2)*x^3 - (5*b^2*c^4*d - 23*a*b*c^3*d^2 + 18*a^2
*c^2*d^3)*x^2 - 3*(b^2*c^3*d^2 - 5*a*b*c^2*d^3 + 4*a^2*c*d^4)*x)*sqrt((a*x
+ b)/x))/(b*c^5*d^3 - a*c^4*d^4 + (b*c^7*d - a*c^6*d^2)*x^2 + 2*(b*c^6*d^2
- a*c^5*d^3)*x), -1/8*(24*(b^2*c^2*d^3 - 3*a*b*c*d^4 + 2*a^2*d^5 + (b^2*c^4
*d - 3*a*b*c^3*d^2 + 2*a^2*c^2*d^3)*x^2 + 2*(b^2*c^3*d^2 - 3*a*b*c^2*d^3 +
2*a^2*c*d^4)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + 3*(b^2*c^2*
d^2 - 8*a*b*c*d^3 + 8*a^2*d^4 + (b^2*c^4 - 8*a*b*c^3*d + 8*a^2*c^2*d^2)*x^2
 + 2*(b^2*c^3*d - 8*a*b*c^2*d^2 + 8*a^2*c*d^3)*x)*sqrt(-b*c*d + a*d^2)*log(
(b*d - (b*c - 2*a*d)*x + 2*sqrt(-b*c*d + a*d^2)*x*sqrt((a*x + b)/x))/(c*x +
 d)) - 2*(4*(a*b*c^4*d - a^2*c^3*d^2)*x^3 - (5*b^2*c^4*d - 23*a*b*c^3*d^2 +
 18*a^2*c^2*d^3)*x^2 - 3*(b^2*c^3*d^2 - 5*a*b*c^2*d^3 + 4*a^2*c*d^4)*x)*sqr
t((a*x + b)/x))/(b*c^5*d^3 - a*c^4*d^4 + (b*c^7*d - a*c^6*d^2)*x^2 + 2*(b*c
^6*d^2 - a*c^5*d^3)*x), 1/4*(3*(b^2*c^2*d^2 - 8*a*b*c*d^3 + 8*a^2*d^4 + (b^
2*c^4 - 8*a*b*c^3*d + 8*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d - 8*a*b*c^2*d^2 + 8
*a^2*c*d^3)*x)*sqrt(b*c*d - a*d^2)*arctan(sqrt(b*c*d - a*d^2)*x*sqrt((a*x +
 b)/x)/(a*d*x + b*d)) - 6*(b^2*c^2*d^3 - 3*a*b*c*d^4 + 2*a^2*d^5 + (b^2*c^4
*d - 3*a*b*c^3*d^2 + 2*a^2*c^2*d^3)*x^2 + 2*(b^2*c^3*d^2 - 3*a*b*c^2*d^3 +
2*a^2*c*d^4)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (4
*(a*b*c^4*d - a^2*c^3*d^2)*x^3 - (5*b^2*c^4*d - 23*a*b*c^3*d^2 + 18*a^2*c^2
*d^3)*x^2 - 3*(b^2*c^3*d^2 - 5*a*b*c^2*d^3 + 4*a^2*c*d^4)*x)*sqrt((a*x + b)
/x))/(b*c^5*d^3 - a*c^4*d^4 + (b*c^7*d - a*c^6*d^2)*x^2 + 2*(b*c^6*d^2 - a*
c^5*d^3)*x), 1/4*(3*(b^2*c^2*d^2 - 8*a*b*c*d^3 + 8*a^2*d^4 + (b^2*c^4 - 8*a
*b*c^3*d + 8*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d - 8*a*b*c^2*d^2 + 8*a^2*c*d^3)
*x)*sqrt(b*c*d - a*d^2)*arctan(sqrt(b*c*d - a*d^2)*x*sqrt((a*x + b)/x)/(a*d
*x + b*d)) - 12*(b^2*c^2*d^3 - 3*a*b*c*d^4 + 2*a^2*d^5 + (b^2*c^4*d - 3*a*b
*c^3*d^2 + 2*a^2*c^2*d^3)*x^2 + 2*(b^2*c^3*d^2 - 3*a*b*c^2*d^3 + 2*a^2*c*d^
4)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (4*(a*b*c^4*d - a^2*c
^3*d^2)*x^3 - (5*b^2*c^4*d - 23*a*b*c^3*d^2 + 18*a^2*c^2*d^3)*x^2 - 3*(b^2*
c^3*d^2 - 5*a*b*c^2*d^3 + 4*a^2*c*d^4)*x)*sqrt((a*x + b)/x))/(b*c^5*d^3 - a
*c^4*d^4 + (b*c^7*d - a*c^6*d^2)*x^2 + 2*(b*c^6*d^2 - a*c^5*d^3)*x)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2)/(c+d/x)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.31728, size = 981, normalized size = 4.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/(c+d/x)^3,x, algorithm="giac")
```

```
[Out] sqrt(a*x^2 + b*x)*a*sgn(x)/c^3 + 3/4*(b^2*c^2*sgn(x) - 8*a*b*c*d*sgn(x) + 8
*a^2*d^2*sgn(x))*arctan(-((sqrt(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)/sq
rt(b*c*d - a*d^2))/(sqrt(b*c*d - a*d^2)*c^4) - 3/2*(a*b*c*sgn(x) - 2*a^2*d*
sgn(x))*log(abs(2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))/(sqrt(a)*c^
4) + 1/4*(3*sqrt(a)*b^2*c^2*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 24*a^(3
/2)*b*c*d*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 24*a^(5/2)*d^2*arctan(sqr
t(a)*d/sqrt(b*c*d - a*d^2)) + 6*sqrt(b*c*d - a*d^2)*a*b*c*log(abs(b)) - 12*
sqrt(b*c*d - a*d^2)*a^2*d*log(abs(b)) + 5*sqrt(b*c*d - a*d^2)*a*b*c - 10*sq
rt(b*c*d - a*d^2)*a^2*d)*sgn(x)/(sqrt(b*c*d - a*d^2)*sqrt(a)*c^4) + 1/4*(5*
(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*sqrt(a)*b^2*c^3*sgn(x) - 24*(sqrt(a)*x -
sqrt(a*x^2 + b*x))^3*a^(3/2)*b*c^2*d*sgn(x) + 24*(sqrt(a)*x - sqrt(a*x^2 +
b*x))^3*a^(5/2)*c*d^2*sgn(x) - (sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a*b^2*c^2*
d*sgn(x) - 24*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^2*b*c*d^2*sgn(x) + 40*(sq
rt(a)*x - sqrt(a*x^2 + b*x))^2*a^3*d^3*sgn(x) + 3*(sqrt(a)*x - sqrt(a*x^2 +
 b*x))*sqrt(a)*b^3*c^2*d*sgn(x) - 28*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^(3/2
)*b^2*c*d^2*sgn(x) + 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^(5/2)*b*d^3*sgn(x
) - 5*a*b^3*c*d^2*sgn(x) + 10*a^2*b^2*d^3*sgn(x))/(((sqrt(a)*x - sqrt(a*x^2
 + b*x))^2*c + 2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*d + b*d)^2*sqrt(a)
*c^4)
```

3.238 $\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right)^3 \, dx$

Optimal. Leaf size=198

$$-\frac{d\left(a+\frac{b}{x}\right)^{5/2}\left(2\left(-10a^2d^2+135abcd+469b^2c^2\right)+\frac{5bd(10ad+89bc)}{x}\right)}{315b^2}+a^{3/2}c^2(6ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-\frac{1}{3}c^2\left(a+\frac{b}{x}\right.$$

```
[Out] -(a*c^2*(5*b*c + 6*a*d)*Sqrt[a + b/x]) - (c^2*(5*b*c + 6*a*d)*(a + b/x)^(3/
2))/3 - (11*d*(a + b/x)^(5/2)*(c + d/x)^2)/9 - (d*(a + b/x)^(5/2)*(2*(469*b
^2*c^2 + 135*a*b*c*d - 10*a^2*d^2) + (5*b*d*(89*b*c + 10*a*d))/x))/(315*b^2
) + (a + b/x)^(5/2)*(c + d/x)^3*x + a^(3/2)*c^2*(5*b*c + 6*a*d)*ArcTanh[Sqr
t[a + b/x]/Sqrt[a]]
```

Rubi [A] time = 0.158298, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {375, 97, 153, 147, 50, 63, 208}

$$-\frac{d\left(a+\frac{b}{x}\right)^{5/2}\left(2\left(-10a^2d^2+135abcd+469b^2c^2\right)+\frac{5bd(10ad+89bc)}{x}\right)}{315b^2}+a^{3/2}c^2(6ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-\frac{1}{3}c^2\left(a+\frac{b}{x}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(5/2)*(c + d/x)^3,x]
```


```
[Out] -(a*c^2*(5*b*c + 6*a*d)*Sqrt[a + b/x]) - (c^2*(5*b*c + 6*a*d)*(a + b/x)^(3/
2))/3 - (11*d*(a + b/x)^(5/2)*(c + d/x)^2)/9 - (d*(a + b/x)^(5/2)*(2*(469*b
^2*c^2 + 135*a*b*c*d - 10*a^2*d^2) + (5*b*d*(89*b*c + 10*a*d))/x))/(315*b^2
) + (a + b/x)^(5/2)*(c + d/x)^3*x + a^(3/2)*c^2*(5*b*c + 6*a*d)*ArcTanh[Sqr
t[a + b/x]/Sqrt[a]]
```


Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
```

```
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3 dx = -\text{Subst}\left(\int \frac{(a+bx)^{5/2}(c+dx)^3}{x^2}dx, x, \frac{1}{x}\right)$$
$$= \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3 x - \text{Subst}\left(\int \frac{(a+bx)^{3/2}(c+dx)^2\left(\frac{1}{2}(5bc+6ad)+\frac{11bdx}{2}\right)}{x}dx, x, \frac{1}{x}\right)$$
$$= -\frac{11}{9}d\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2 + \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3 x - \frac{2\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}(c+dx)\left(\frac{9}{4}bc(5bc+6ad)+\right.}{x}\right.}{9b}$$
$$= -\frac{11}{9}d\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2 - \frac{d\left(a+\frac{b}{x}\right)^{5/2}\left(2\left(469b^2c^2+135abcd-10a^2d^2\right)+\frac{5bd(89bc+10ad)}{x}\right.}{315b^2}$$
$$= -\frac{1}{3}c^2(5bc+6ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{11}{9}d\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2 - \frac{d\left(a+\frac{b}{x}\right)^{5/2}\left(2\left(469b^2c^2+135a\right.\right.}{315}$$
$$= -ac^2(5bc+6ad)\sqrt{a+\frac{b}{x}} - \frac{1}{3}c^2(5bc+6ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{11}{9}d\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2 - \frac{d\left(a+\right.}{}$$
$$= -ac^2(5bc+6ad)\sqrt{a+\frac{b}{x}} - \frac{1}{3}c^2(5bc+6ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{11}{9}d\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2 - \frac{d\left(a+\right.}{}$$
$$= -ac^2(5bc+6ad)\sqrt{a+\frac{b}{x}} - \frac{1}{3}c^2(5bc+6ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{11}{9}d\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2 - \frac{d\left(a+\right.}{}$$

Mathematica [A] time = 0.212277, size = 201, normalized size = 1.02

$$\frac{\sqrt{a+\frac{b}{x}}\left(-3a^2b^2x^2\left(966c^2dx^2-105c^3x^3+270cd^2x+50d^3\right)-10a^3bd^2x^3(27cx+d)+20a^4d^3x^4-2ab^3x\left(693c^2dx^2+7\right.\right.}{315b^2x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(5/2)*(c + d/x)^3,x]
```

```
[Out] (Sqrt[a + b/x]*(20*a^4*d^3*x^4 - 10*a^3*b*d^2*x^3*(d + 27*c*x) - 3*a^2*b^2*
x^2*(50*d^3 + 270*c*d^2*x + 966*c^2*d*x^2 - 105*c^3*x^3) - 2*b^4*(35*d^3 +
135*c*d^2*x + 189*c^2*d*x^2 + 105*c^3*x^3) - 2*a*b^3*x*(95*d^3 + 405*c*d^2*
x + 693*c^2*d*x^2 + 735*c^3*x^3)))/(315*b^2*x^4) + a^(3/2)*c^2*(5*b*c + 6*a
*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Maple [B] time = 0.013, size = 457, normalized size = 2.3

$$\frac{1}{630\,b^2x^5}\sqrt{\frac{ax+b}{x}}\left(3780\sqrt{ax^2+bx}a^{7/2}x^6bc^2d+3150\sqrt{ax^2+bx}a^{5/2}x^6b^2c^3+40\left(ax^2+bx\right)^{3/2}a^{7/2}x^3d^3-3780\left(ax^2+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(5/2)*(c+d/x)^3,x)
```

```
[Out] 1/630*((a*x+b)/x)^(1/2)*(3780*(a*x^2+b*x)^(1/2)*a^(7/2)*x^6*b*c^2*d+3150*(a
*x^2+b*x)^(1/2)*a^(5/2)*x^6*b^2*c^3+40*(a*x^2+b*x)^(3/2)*a^(7/2)*x^3*d^3-37
80*(a*x^2+b*x)^(3/2)*a^(5/2)*x^4*b*c^2*d-540*(a*x^2+b*x)^(3/2)*a^(5/2)*x^3*
b*c*d^2-2520*(a*x^2+b*x)^(3/2)*a^(3/2)*x^4*b^2*c^3+1890*ln(1/2*(2*(a*x^2+b*
x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x^6*a^3*b^2*c^2*d+1575*ln(1/2*(2*(a*x^2+
b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x^6*a^2*b^3*c^3-60*(a*x^2+b*x)^(3/2)*a
^(5/2)*x^2*b*d^3-2016*(a*x^2+b*x)^(3/2)*a^(3/2)*x^3*b^2*c^2*d-1080*(a*x^2+b
*x)^(3/2)*a^(3/2)*x^2*b^2*c*d^2-420*(a*x^2+b*x)^(3/2)*a^(1/2)*x^3*b^3*c^3-2
40*(a*x^2+b*x)^(3/2)*a^(3/2)*x*b^2*d^3-756*(a*x^2+b*x)^(3/2)*a^(1/2)*x^2*b^
3*c^2*d-540*(a*x^2+b*x)^(3/2)*a^(1/2)*x*b^3*c*d^2-140*(a*x^2+b*x)^(3/2)*a^(
1/2)*b^3*d^3)/x^5/b^2/((a*x+b)*x)^(1/2)/a^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*(c+d/x)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.37091, size = 1115, normalized size = 5.63

$$\left[\frac{315\left(5\,ab^{3}c^{3}+6\,a^{2}b^{2}c^{2}d\right)\sqrt{a}x^{4}\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(315\,a^{2}b^{2}c^{3}x^{5}-70\,b^{4}d^{3}-2\left(735\,ab^{3}c^{3}+1449\,a^{2}b^{2}c^{2}d\cdots\right.\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*(c+d/x)^3,x, algorithm="fricas")
```

```
[Out] [1/630*(315*(5*a*b^3*c^3 + 6*a^2*b^2*c^2*d)*sqrt(a)*x^4*log(2*a*x + 2*sqrt(
a)*x*sqrt((a*x + b)/x) + b) + 2*(315*a^2*b^2*c^3*x^5 - 70*b^4*d^3 - 2*(735*
a*b^3*c^3 + 1449*a^2*b^2*c^2*d + 135*a^3*b*c*d^2 - 10*a^4*d^3)*x^4 - 2*(105
*b^4*c^3 + 693*a*b^3*c^2*d + 405*a^2*b^2*c*d^2 + 5*a^3*b*d^3)*x^3 - 6*(63*b
^4*c^2*d + 135*a*b^3*c*d^2 + 25*a^2*b^2*d^3)*x^2 - 10*(27*b^4*c*d^2 + 19*a*
b^3*d^3)*x)*sqrt((a*x + b)/x))/(b^2*x^4), -1/315*(315*(5*a*b^3*c^3 + 6*a^2*
b^2*c^2*d)*sqrt(-a)*x^4*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (315*a^2*b^2
*c^3*x^5 - 70*b^4*d^3 - 2*(735*a*b^3*c^3 + 1449*a^2*b^2*c^2*d + 135*a^3*b*c
*d^2 - 10*a^4*d^3)*x^4 - 2*(105*b^4*c^3 + 693*a*b^3*c^2*d + 405*a^2*b^2*c*d
^2 + 5*a^3*b*d^3)*x^3 - 6*(63*b^4*c^2*d + 135*a*b^3*c*d^2 + 25*a^2*b^2*d^3)
*x^2 - 10*(27*b^4*c*d^2 + 19*a*b^3*d^3)*x)*sqrt((a*x + b)/x))/(b^2*x^4)]
```

Sympy [A] time = 91.14, size = 5513, normalized size = 27.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2)*(c+d/x)**3,x)


[Out] 32*a**(29/2)*b**(27/2)*d**3*x**10*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**(
21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6
300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(1
1/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) + 176*a**(27/2)*b**(29/
2)*d**3*x**9*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2
)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x
**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2)
+ 315*a**(9/2)*b**21*x**(9/2)) + 396*a**(25/2)*b**(31/2)*d**3*x**8*sqrt(a*x
/b + 1)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4
725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(1
3/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*
x**(9/2)) + 462*a**(23/2)*b**(33/2)*d**3*x**7*sqrt(a*x/b + 1)/(315*a**(21/2
)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x
**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2)
+ 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) + 210*a**(2
1/2)*b**(35/2)*d**3*x**6*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21/2) + 1
890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(1
5/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**2
0*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) - 32*a**(21/2)*b**(11/2)*d**3*x*
*6*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(1
1/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 378*a**(
19/2)*b**(37/2)*d**3*x**5*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21/2) +
1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(
15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**
20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) - 48*a**(19/2)*b**(13/2)*c*d**2
*x**6*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x*
*(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 80*a*
*(19/2)*b**(13/2)*d**3*x**5*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) +
 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b
**10*x**(7/2)) - 1134*a**(17/2)*b**(39/2)*d**3*x**4*sqrt(a*x/b + 1)/(315*a*
*(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b
**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(
13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) - 120
*a**(17/2)*b**(15/2)*c*d**2*x**5*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13
/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7
/2)*b**10*x**(7/2)) - 60*a**(17/2)*b**(15/2)*d**3*x**4*sqrt(a*x/b + 1)/(105
*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**
9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 1494*a**(15/2)*b**(41/2)*d**3*x
**3*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x
**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2)
+ 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**
(9/2)*b**21*x**(9/2)) - 90*a**(15/2)*b**(17/2)*c*d**2*x**4*sqrt(a*x/b + 1)/
(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)
*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 80*a**(15/2)*b**(17/2)*d**3
*x**3*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x*
*(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 4*a**
(15/2)*b**(3/2)*d**3*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a
**(5/2)*b**4*x**(5/2)) - 1098*a**(13/2)*b**(43/2)*d**3*x**2*sqrt(a*x/b + 1)
/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**
(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b*
*19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2
)) - 120*a**(13/2)*b**(19/2)*c*d**2*x**3*sqrt(a*x/b + 1)/(105*a**(13/2)*b**
7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 1
05*a**(7/2)*b**10*x**(7/2)) - 200*a**(13/2)*b**(19/2)*d**3*x**2*sqrt(a*x/b
+ 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**
(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 24*a**(13/2)*b**(5/2)*
c*d**2*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x
```

```
**(5/2)) + 2*a**(13/2)*b**(5/2)*d**3*x**2*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3
*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 430*a**(11/2)*b**(45/2)*d**3*x*sqr
t(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2
) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*
a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b
**21*x**(9/2)) - 300*a**(11/2)*b**(21/2)*c*d**2*x**2*sqrt(a*x/b + 1)/(105*a
**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*
x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 192*a**(11/2)*b**(21/2)*d**3*x*sq
rt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2)
+ 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 12*a**(11/2)*
b**(7/2)*c**2*d*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/
2)*b**4*x**(5/2)) + 12*a**(11/2)*b**(7/2)*c*d**2*x**2*sqrt(a*x/b + 1)/(15*a
**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 8*a**(11/2)*b**(7/2)*d
**3*x*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2
)) - 70*a**(9/2)*b**(47/2)*d**3*sqrt(a*x/b + 1)/(315*a**(21/2)*b**15*x**(21
/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 630
0*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/
2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) - 288*a**(9/2)*b**(23/2)*
c*d**2*x*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8
*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 60
*a**(9/2)*b**(23/2)*d**3*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 31
5*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**1
0*x**(7/2)) + 6*a**(9/2)*b**(9/2)*c**2*d*x**2*sqrt(a*x/b + 1)/(15*a**(7/2)*
b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 48*a**(9/2)*b**(9/2)*c*d**2*x*
sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 6
*a**(9/2)*b**(9/2)*d**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**
(5/2)*b**4*x**(5/2)) - 90*a**(7/2)*b**(25/2)*c*d**2*sqrt(a*x/b + 1)/(105*a*
*(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x
**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 24*a**(7/2)*b**(11/2)*c**2*d*x*sqr
t(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 36*a
**(7/2)*b**(11/2)*c*d**2*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a*
*(5/2)*b**4*x**(5/2)) - 18*a**(5/2)*b**(13/2)*c**2*d*sqrt(a*x/b + 1)/(15*a*
*(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) + a**(3/2)*b*c**3*asinh(s
qrt(a)*sqrt(x)/sqrt(b)) - 32*a**15*b**13*d**3*x**(21/2)/(315*a**(21/2)*b**1
5*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/
2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890
*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) - 192*a**14*b**14
*d**3*x**(19/2)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(1
9/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 47
25*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2
)*b**21*x**(9/2)) - 480*a**13*b**15*d**3*x**(17/2)/(315*a**(21/2)*b**15*x**
(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) +
6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(
11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) - 640*a**12*b**16*d**3
*x**(15/2)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2)
+ 4725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a*
*(13/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**
21*x**(9/2)) - 480*a**11*b**17*d**3*x**(13/2)/(315*a**(21/2)*b**15*x**(21/2
) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*
a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)
*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) + 32*a**11*b**5*d**3*x**(13
/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(
9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 192*a**10*b**18*d**3*x*
*(11/2)/(315*a**(21/2)*b**15*x**(21/2) + 1890*a**(19/2)*b**16*x**(19/2) + 4
725*a**(17/2)*b**17*x**(17/2) + 6300*a**(15/2)*b**18*x**(15/2) + 4725*a**(1
3/2)*b**19*x**(13/2) + 1890*a**(11/2)*b**20*x**(11/2) + 315*a**(9/2)*b**21*
x**(9/2)) + 48*a**10*b**6*c*d**2*x**(13/2)/(105*a**(13/2)*b**7*x**(13/2) +
315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b*
*10*x**(7/2)) + 96*a**10*b**6*d**3*x**(11/2)/(105*a**(13/2)*b**7*x**(13/2)
```

```
+ 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*
b**10*x**(7/2)) - 32*a**9*b**19*d**3*x**(9/2)/(315*a**(21/2)*b**15*x**(21/2
) + 1890*a**(19/2)*b**16*x**(19/2) + 4725*a**(17/2)*b**17*x**(17/2) + 6300*
a**(15/2)*b**18*x**(15/2) + 4725*a**(13/2)*b**19*x**(13/2) + 1890*a**(11/2)
*b**20*x**(11/2) + 315*a**(9/2)*b**21*x**(9/2)) + 144*a**9*b**7*c*d**2*x**(
11/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a*
*(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 96*a**9*b**7*d**3*x**
(9/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a*
*(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 144*a**8*b**8*c*d**2*
x**(9/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315
*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 32*a**8*b**8*d**3*
x**(7/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315
*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 4*a**8*b*d**3*x**(
7/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) + 48*a**7*b**9
*c*d**2*x**(7/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/
2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 24*a**7*b*
*2*c*d**2*x**(7/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2))
- 4*a**7*b**2*d**3*x**(5/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x
**(5/2)) - 12*a**6*b**3*c**2*d*x**(7/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**
(5/2)*b**4*x**(5/2)) - 24*a**6*b**3*c*d**2*x**(5/2)/(15*a**(7/2)*b**3*x**(7
/2) + 15*a**(5/2)*b**4*x**(5/2)) - 12*a**5*b**4*c**2*d*x**(5/2)/(15*a**(7/2
)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 6*a**3*c**2*d*atan(sqrt(a +
b/x)/sqrt(-a))/sqrt(-a) + a**2*sqrt(b)*c**3*sqrt(x)*sqrt(a*x/b + 1) - 4*a**
2*b*c**3*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a) - 6*a**2*c**2*d*sqrt(a + b/x
) + 3*a**2*c*d**2*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3
*b), True)) - 4*a*b*c**3*sqrt(a + b/x) + 6*a*b*c**2*d*Piecewise((-sqrt(a)/x
, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True)) + b**2*c**3*Piecewise((-sqr
t(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*(c+d/x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.239 $\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2 dx$

Optimal. Leaf size=152

$$a^{3/2}c(4ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)+\frac{c^2x\left(a+\frac{b}{x}\right)^{7/2}}{a}-\frac{c\left(a+\frac{b}{x}\right)^{5/2}(4ad+5bc)}{5a}-\frac{1}{3}c\left(a+\frac{b}{x}\right)^{3/2}(4ad+5bc)-ac\sqrt{a+\frac{b}{x}}(4$$

```
[Out] -(a*c*(5*b*c + 4*a*d)*Sqrt[a + b/x]) - (c*(5*b*c + 4*a*d)*(a + b/x)^(3/2))/
3 - (c*(5*b*c + 4*a*d)*(a + b/x)^(5/2))/(5*a) - (2*d^2*(a + b/x)^(7/2))/(7*
b) + (c^2*(a + b/x)^(7/2)*x)/a + a^(3/2)*c*(5*b*c + 4*a*d)*ArcTanh[Sqrt[a +
 b/x]/Sqrt[a]]
```

Rubi [A] time = 0.102039, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {375, 89, 80, 50, 63, 208}

$$a^{3/2}c(4ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)+\frac{c^2x\left(a+\frac{b}{x}\right)^{7/2}}{a}-\frac{c\left(a+\frac{b}{x}\right)^{5/2}(4ad+5bc)}{5a}-\frac{1}{3}c\left(a+\frac{b}{x}\right)^{3/2}(4ad+5bc)-ac\sqrt{a+\frac{b}{x}}(4$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(5/2)*(c + d/x)^2,x]
```

```
[Out] -(a*c*(5*b*c + 4*a*d)*Sqrt[a + b/x]) - (c*(5*b*c + 4*a*d)*(a + b/x)^(3/2))/
3 - (c*(5*b*c + 4*a*d)*(a + b/x)^(5/2))/(5*a) - (2*d^2*(a + b/x)^(7/2))/(7*
b) + (c^2*(a + b/x)^(7/2)*x)/a + a^(3/2)*c*(5*b*c + 4*a*d)*ArcTanh[Sqrt[a +
 b/x]/Sqrt[a]]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2 dx &= -\text{Subst}\left(\int \frac{(a+bx)^{5/2}(c+dx)^2}{x^2}\,dx, x, \frac{1}{x}\right)\\
&= \frac{c^2\left(a+\frac{b}{x}\right)^{7/2}x}{a} - \frac{\text{Subst}\left(\int \frac{(a+bx)^{5/2}\left(\frac{1}{2}c(5bc+4ad)+ad^2x\right)}{x}\,dx, x, \frac{1}{x}\right)}{a}\\
&= -\frac{2d^2\left(a+\frac{b}{x}\right)^{7/2}}{7b} + \frac{c^2\left(a+\frac{b}{x}\right)^{7/2}x}{a} - \frac{(c(5bc+4ad))\,\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x}\,dx, x, \frac{1}{x}\right)}{2a}\\
&= -\frac{c(5bc+4ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{7/2}}{7b} + \frac{c^2\left(a+\frac{b}{x}\right)^{7/2}x}{a} - \frac{1}{2}(c(5bc+4ad))\,\text{Subst}\Bigg(\int\\
&= -\frac{1}{3}c(5bc+4ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{c(5bc+4ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a} - \frac{2d^2\left(a+\frac{b}{x}\right)^{7/2}}{7b} + \frac{c^2\left(a+\frac{b}{x}\right)^{7/2}x}{a} -\\
&= -ac(5bc+4ad)\sqrt{a+\frac{b}{x}} - \frac{1}{3}c(5bc+4ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{c(5bc+4ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a} - \frac{2d^2\left(a+\right.}{7b}\\
&= -ac(5bc+4ad)\sqrt{a+\frac{b}{x}} - \frac{1}{3}c(5bc+4ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{c(5bc+4ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a} - \frac{2d^2\left(a+\right.}{7b}\\
&= -ac(5bc+4ad)\sqrt{a+\frac{b}{x}} - \frac{1}{3}c(5bc+4ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{c(5bc+4ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a} - \frac{2d^2\left(a+\right.}{7b}
\end{aligned}$$

Mathematica [A] time = 0.135628, size = 121, normalized size = 0.8

$$-\frac{c(4ad+5bc)\left(\sqrt{a+\frac{b}{x}}\left(23a^2x^2+11abx+3b^2\right)-15a^{5/2}x^2\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)\right)}{15ax^2}+\frac{c^2x\left(a+\frac{b}{x}\right)^{7/2}}{a}-\frac{2d^2\left(a+\frac{b}{x}\right)^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(5/2)*(c + d/x)^2,x]
```

```
[Out] (-2*d^2*(a + b/x)^(7/2))/(7*b) + (c^2*(a + b/x)^(7/2)*x)/a - (c*(5*b*c + 4*
a*d)*(Sqrt[a + b/x]*(3*b^2 + 11*a*b*x + 23*a^2*x^2) - 15*a^(5/2)*x^2*ArcTan
h[Sqrt[a + b/x]/Sqrt[a]]))/(15*a*x^2)
```

Maple [B] time = 0.013, size = 336, normalized size = 2.2

$$-\frac{1}{210\,bx^4}\sqrt{\frac{ax+b}{x}}\left(-840\,\sqrt{ax^2+bx}a^{7/2}x^5cd-1050\,\sqrt{ax^2+bx}a^{5/2}x^5bc^2-420\,\ln\left(1/2\,\frac{2\,\sqrt{ax^2+bx}\sqrt{a}+2\,ax+b}{\sqrt{a}}\right)x^5a^3bc$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(5/2)*(c+d/x)^2,x)
```

```
[Out] -1/210*((a*x+b)/x)^(1/2)/x^4/b*(-840*(a*x^2+b*x)^(1/2)*a^(7/2)*x^5*c*d-1050
*(a*x^2+b*x)^(1/2)*a^(5/2)*x^5*b*c^2-420*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2
)+2*a*x+b)/a^(1/2))*x^5*a^3*b*c*d-525*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2
*a*x+b)/a^(1/2))*x^5*a^2*b^2*c^2+840*(a*x^2+b*x)^(3/2)*a^(5/2)*x^3*c*d+840*
(a*x^2+b*x)^(3/2)*a^(3/2)*x^3*b*c^2+60*(a*x^2+b*x)^(3/2)*a^(5/2)*x^2*d^2+44
8*(a*x^2+b*x)^(3/2)*a^(3/2)*x^2*b*c*d+140*(a*x^2+b*x)^(3/2)*a^(1/2)*x^2*b^2
*c^2+120*(a*x^2+b*x)^(3/2)*a^(3/2)*x*b*d^2+168*(a*x^2+b*x)^(3/2)*a^(1/2)*x*
b^2*c*d+60*(a*x^2+b*x)^(3/2)*a^(1/2)*b^2*d^2)/((a*x+b)*x)^(1/2)/a^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*(c+d/x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.27295, size = 807, normalized size = 5.31

$$\left[\frac{105\left(5\,ab^2c^2+4\,a^2bcd\right)\sqrt{a}x^3\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(105\,a^2bc^2x^4-30\,b^3d^2-2\left(245\,ab^2c^2+322\,a^2bcd+15\,a^3\right.\right.}{210\,bx^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*(c+d/x)^2,x, algorithm="fricas")
```

```
[Out] [1/210*(105*(5*a*b^2*c^2 + 4*a^2*b*c*d)*sqrt(a)*x^3*log(2*a*x + 2*sqrt(a)*x
*sqrt((a*x + b)/x) + b) + 2*(105*a^2*b*c^2*x^4 - 30*b^3*d^2 - 2*(245*a*b^2*
c^2 + 322*a^2*b*c*d + 15*a^3*d^2)*x^3 - 2*(35*b^3*c^2 + 154*a*b^2*c*d + 45*
a^2*b*d^2)*x^2 - 6*(14*b^3*c*d + 15*a*b^2*d^2)*x)*sqrt((a*x + b)/x))/(b*x^3
), -1/105*(105*(5*a*b^2*c^2 + 4*a^2*b*c*d)*sqrt(-a)*x^3*arctan(sqrt(-a)*sqr
```

```
t((a*x + b)/x)/a) - (105*a^2*b*c^2*x^4 - 30*b^3*d^2 - 2*(245*a*b^2*c^2 + 32
2*a^2*b*c*d + 15*a^3*d^2)*x^3 - 2*(35*b^3*c^2 + 154*a*b^2*c*d + 45*a^2*b*d^
2)*x^2 - 6*(14*b^3*c*d + 15*a*b^2*d^2)*x)*sqrt((a*x + b)/x))/(b*x^3)]
```

Sympy [A] time = 57.4778, size = 1841, normalized size = 12.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2)*(c+d/x)**2,x)
```

```
[Out] -16*a**(19/2)*b**(13/2)*d**2*x**6*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(1
3/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(
7/2)*b**10*x**(7/2)) - 40*a**(17/2)*b**(15/2)*d**2*x**5*sqrt(a*x/b + 1)/(10
5*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b*
*9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 30*a**(15/2)*b**(17/2)*d**2*x*
*4*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(1
1/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 40*a**(1
3/2)*b**(19/2)*d**2*x**3*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 31
5*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**1
0*x**(7/2)) + 8*a**(13/2)*b**(5/2)*d**2*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b
**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 100*a**(11/2)*b**(21/2)*d**2*x*
*2*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(1
1/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 8*a**(11
/2)*b**(7/2)*c*d*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5
/2)*b**4*x**(5/2)) + 4*a**(11/2)*b**(7/2)*d**2*x**2*sqrt(a*x/b + 1)/(15*a**
(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 96*a**(9/2)*b**(23/2)*d*
*2*x*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**
(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 4*a**(
9/2)*b**(9/2)*c*d*x**2*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(
5/2)*b**4*x**(5/2)) - 16*a**(9/2)*b**(9/2)*d**2*x*sqrt(a*x/b + 1)/(15*a**(7
/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 30*a**(7/2)*b**(25/2)*d**2
*sqrt(a*x/b + 1)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/
2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 16*a**(7/2
)*b**(11/2)*c*d*x*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*
b**4*x**(5/2)) - 12*a**(7/2)*b**(11/2)*d**2*sqrt(a*x/b + 1)/(15*a**(7/2)*b*
*3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 12*a**(5/2)*b**(13/2)*c*d*sqrt(a
*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) + a**(3/2
)*b*c**2*asinh(sqrt(a)*sqrt(x)/sqrt(b)) + 16*a**10*b**6*d**2*x**(13/2)/(105
*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**
9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 48*a**9*b**7*d**2*x**(11/2)/(10
5*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b*
*9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 48*a**8*b**8*d**2*x**(9/2)/(10
5*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b*
*9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) + 16*a**7*b**9*d**2*x**(7/2)/(10
5*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b*
*9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 8*a**7*b**2*d**2*x**(7/2)/(15*
a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 8*a**6*b**3*c*d*x**(7
/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 8*a**6*b**3*d
**2*x**(5/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 8*a*
*5*b**4*c*d*x**(5/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)
) - 4*a**3*c*d*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a) + a**2*sqrt(b)*c**2*sq
rt(x)*sqrt(a*x/b + 1) - 4*a**2*b*c**2*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a)
 - 4*a**2*c*d*sqrt(a + b/x) + a**2*d**2*Piecewise((-sqrt(a)/x, Eq(b, 0)), (
-2*(a + b/x)**(3/2)/(3*b), True)) - 4*a*b*c**2*sqrt(a + b/x) + 4*a*b*c*d*Pi
ecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True)) + b**2*c
```

```
**2*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*(c+d/x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.240 $\int \left(a+\frac{b}{x}\right)^{5/2} \left(c+\frac{d}{x}\right) dx$

Optimal. Leaf size=125

$$a^{3/2}(2ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-\frac{\left(a+\frac{b}{x}\right)^{5/2}(2ad+5bc)}{5a}-\frac{1}{3}\left(a+\frac{b}{x}\right)^{3/2}(2ad+5bc)-a\sqrt{a+\frac{b}{x}}(2ad+5bc)+\frac{cx\left(a\right.}{}$$

```
[Out] -(a*(5*b*c + 2*a*d)*Sqrt[a + b/x]) - ((5*b*c + 2*a*d)*(a + b/x)^(3/2))/3 -
((5*b*c + 2*a*d)*(a + b/x)^(5/2))/(5*a) + (c*(a + b/x)^(7/2)*x)/a + a^(3/2)
*(5*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Rubi [A] time = 0.0766491, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {375, 78, 50, 63, 208}

$$a^{3/2}(2ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)-\frac{\left(a+\frac{b}{x}\right)^{5/2}(2ad+5bc)}{5a}-\frac{1}{3}\left(a+\frac{b}{x}\right)^{3/2}(2ad+5bc)-a\sqrt{a+\frac{b}{x}}(2ad+5bc)+\frac{cx\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(5/2)*(c + d/x),x]
```

```
[Out] -(a*(5*b*c + 2*a*d)*Sqrt[a + b/x]) - ((5*b*c + 2*a*d)*(a + b/x)^(3/2))/3 -
((5*b*c + 2*a*d)*(a + b/x)^(5/2))/(5*a) + (c*(a + b/x)^(7/2)*x)/a + a^(3/2)
*(5*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)dx = -\text{Subst}\left(\int \frac{(a+bx)^{5/2}(c+dx)}{x^2}dx, x, \frac{1}{x}\right)$$

$$= \frac{c\left(a+\frac{b}{x}\right)^{7/2}x}{a} - \frac{\left(\frac{5bc}{2}+ad\right)\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x}dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{(5bc+2ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a} + \frac{c\left(a+\frac{b}{x}\right)^{7/2}x}{a} - \frac{1}{2}(5bc+2ad)\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}dx, x, \frac{1}{x}\right)$$

$$= -\frac{1}{3}(5bc+2ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{(5bc+2ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a} + \frac{c\left(a+\frac{b}{x}\right)^{7/2}x}{a} - \frac{1}{2}(a(5bc+2ad))\,\text{Subst}$$

$$= -a(5bc+2ad)\sqrt{a+\frac{b}{x}} - \frac{1}{3}(5bc+2ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{(5bc+2ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a} + \frac{c\left(a+\frac{b}{x}\right)^{7/2}x}{a} -$$

$$= -a(5bc+2ad)\sqrt{a+\frac{b}{x}} - \frac{1}{3}(5bc+2ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{(5bc+2ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a} + \frac{c\left(a+\frac{b}{x}\right)^{7/2}x}{a} -$$

$$= -a(5bc+2ad)\sqrt{a+\frac{b}{x}} - \frac{1}{3}(5bc+2ad)\left(a+\frac{b}{x}\right)^{3/2} - \frac{(5bc+2ad)\left(a+\frac{b}{x}\right)^{5/2}}{5a} + \frac{c\left(a+\frac{b}{x}\right)^{7/2}x}{a} +$$

Mathematica [A] time = 0.0938498, size = 94, normalized size = 0.75

$$\frac{\sqrt{a+\frac{b}{x}}\left(a^2x^2(15cx-46d)-2abx(35cx+11d)-2b^2(5cx+3d)\right)}{15x^2} + a^{3/2}(2ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(5/2)*(c + d/x),x]
```

```
[Out] (Sqrt[a + b/x]*(-2*b^2*(3*d + 5*c*x) + a^2*x^2*(-46*d + 15*c*x) - 2*a*b*x*(
11*d + 35*c*x)))/(15*x^2) + a^(3/2)*(5*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/S
qrt[a]]
```

Maple [B] time = 0.01, size = 253, normalized size = 2.

$$-\frac{1}{30\,bx^3}\sqrt{\frac{ax+b}{x}}\left(-60\sqrt{ax^2+bx}a^{7/2}x^4d - 150\sqrt{ax^2+bx}a^{5/2}x^4bc - 30\ln\left(1/2\,\frac{2\sqrt{ax^2+bx}\sqrt{a}+2\,ax+b}{\sqrt{a}}\right)x^4a^3bd - 75\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(5/2)*(c+d/x),x)
```

```
[Out] -1/30*((a*x+b)/x)^(1/2)/x^3/b*(-60*(a*x^2+b*x)^(1/2)*a^(7/2)*x^4*d-150*(a*x
^2+b*x)^(1/2)*a^(5/2)*x^4*b*c-30*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+
b)/a^(1/2))*x^4*a^3*b*d-75*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(
1/2))*x^4*a^2*b^2*c+60*(a*x^2+b*x)^(3/2)*a^(5/2)*x^2*d+120*(a*x^2+b*x)^(3/2
)*a^(3/2)*x^2*b*c+32*(a*x^2+b*x)^(3/2)*a^(3/2)*x*b*d+20*(a*x^2+b*x)^(3/2)*a
^(1/2)*x*b^2*c+12*(a*x^2+b*x)^(3/2)*a^(1/2)*b^2*d)/((a*x+b)*x)^(1/2)/a^(1/2
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*(c+d/x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.23765, size = 527, normalized size = 4.22

$$\left[\frac{15\left(5\,abc+2\,a^2d\right)\sqrt{a}x^2\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(15\,a^2cx^3-6\,b^2d-2\left(35\,abc+23\,a^2d\right)x^2-2\left(5\,b^2c+11\,abc\right.\right.}{30\,x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*(c+d/x),x, algorithm="fricas")
```

```
[Out] [1/30*(15*(5*a*b*c + 2*a^2*d)*sqrt(a)*x^2*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x
 + b)/x) + b) + 2*(15*a^2*c*x^3 - 6*b^2*d - 2*(35*a*b*c + 23*a^2*d)*x^2 - 2
*(5*b^2*c + 11*a*b*d)*x)*sqrt((a*x + b)/x))/x^2, -1/15*(15*(5*a*b*c + 2*a^2
*d)*sqrt(-a)*x^2*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (15*a^2*c*x^3 - 6*b
^2*d - 2*(35*a*b*c + 23*a^2*d)*x^2 - 2*(5*b^2*c + 11*a*b*d)*x)*sqrt((a*x +
b)/x))/x^2]
```

Sympy [A] time = 34.9096, size = 520, normalized size = 4.16

$$\frac{4a^{\frac{11}{2}}b^{\frac{7}{2}}dx^3\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}+\frac{2a^{\frac{9}{2}}b^{\frac{9}{2}}dx^2\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{8a^{\frac{7}{2}}b^{\frac{11}{2}}dx\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}-\frac{6a^{\frac{5}{2}}b^{\frac{13}{2}}d\sqrt{\frac{ax}{b}+1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}}+15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}}+a^{\frac{3}{2}}bc\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{\ }}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2)*(c+d/x),x)
```

```
[Out] 4*a**(11/2)*b**(7/2)*d*x**3*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15
*a**(5/2)*b**4*x**(5/2)) + 2*a**(9/2)*b**(9/2)*d*x**2*sqrt(a*x/b + 1)/(15*a
**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 8*a**(7/2)*b**(11/2)*d
*x*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2))
- 6*a**(5/2)*b**(13/2)*d*sqrt(a*x/b + 1)/(15*a**(7/2)*b**3*x**(7/2) + 15*a*
*(5/2)*b**4*x**(5/2)) + a**(3/2)*b*c*asinh(sqrt(a)*sqrt(x)/sqrt(b)) - 4*a**
6*b**3*d*x**(7/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) -
 4*a**5*b**4*d*x**(5/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5
/2)) - 2*a**3*d*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a) + a**2*sqrt(b)*c*sqrt
(x)*sqrt(a*x/b + 1) - 4*a**2*b*c*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a) - 2*
a**2*d*sqrt(a + b/x) - 4*a*b*c*sqrt(a + b/x) + 2*a*b*d*Piecewise((-sqrt(a)/
x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True)) + b**2*c*Piecewise((-sqrt(
a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*(c+d/x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.241 $\int \left(a+\frac{b}{x}\right)^{5/2} dx$

Optimal. Leaf size=71

$$5a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)+x\left(a+\frac{b}{x}\right)^{5/2}-\frac{5}{3}b\left(a+\frac{b}{x}\right)^{3/2}-5ab\sqrt{a+\frac{b}{x}}$$

```
[Out] -5*a*b*Sqrt[a + b/x] - (5*b*(a + b/x)^(3/2))/3 + (a + b/x)^(5/2)*x + 5*a^(3
/2)*b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Rubi [A] time = 0.0343127, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {242, 47, 50, 63, 208}

$$5a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)+x\left(a+\frac{b}{x}\right)^{5/2}-\frac{5}{3}b\left(a+\frac{b}{x}\right)^{3/2}-5ab\sqrt{a+\frac{b}{x}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(5/2),x]
```

```
[Out] -5*a*b*Sqrt[a + b/x] - (5*b*(a + b/x)^(3/2))/3 + (a + b/x)^(5/2)*x + 5*a^(3
/2)*b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \left(a+\frac{b}{x}\right)^{5/2} dx &= -\operatorname{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^2} dx, x, \frac{1}{x}\right)\\
&= \left(a+\frac{b}{x}\right)^{5/2} x - \frac{1}{2}(5b)\operatorname{Subst}\left(\int \frac{(a+bx)^{3/2}}{x} dx, x, \frac{1}{x}\right)\\
&= -\frac{5}{3}b\left(a+\frac{b}{x}\right)^{3/2} + \left(a+\frac{b}{x}\right)^{5/2} x - \frac{1}{2}(5ab)\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x} dx, x, \frac{1}{x}\right)\\
&= -5ab\sqrt{a+\frac{b}{x}} - \frac{5}{3}b\left(a+\frac{b}{x}\right)^{3/2} + \left(a+\frac{b}{x}\right)^{5/2} x - \frac{1}{2}\left(5a^2b\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, \frac{1}{x}\right)\\
&= -5ab\sqrt{a+\frac{b}{x}} - \frac{5}{3}b\left(a+\frac{b}{x}\right)^{3/2} + \left(a+\frac{b}{x}\right)^{5/2} x - \left(5a^2\right)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}} dx, x, \sqrt{a+\frac{b}{x}}\right)\\
&= -5ab\sqrt{a+\frac{b}{x}} - \frac{5}{3}b\left(a+\frac{b}{x}\right)^{3/2} + \left(a+\frac{b}{x}\right)^{5/2} x + 5a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0442537, size = 64, normalized size = 0.9

$$\frac{\sqrt{a+\frac{b}{x}}\left(3a^2x^2-14abx-2b^2\right)}{3x}+5a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(5/2),x]
```

```
[Out] (Sqrt[a + b/x]*(-2*b^2 - 14*a*b*x + 3*a^2*x^2))/(3*x) + 5*a^(3/2)*b*ArcTanh
[Sqrt[a + b/x]/Sqrt[a]]
```

Maple [B] time = 0.007, size = 120, normalized size = 1.7

$$-\frac{1}{6x^2}\sqrt{\frac{ax+b}{x}}\left(-30\sqrt{ax^2+bx}a^{5/2}x^3-15\ln\left(1/2\frac{2\sqrt{ax^2+bx}\sqrt{a}+2ax+b}{\sqrt{a}}\right)x^3a^2b+24\left(ax^2+bx\right)^{3/2}a^{3/2}x+4b\left(ax^2+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(5/2),x)
```

```
[Out] -1/6*((a*x+b)/x)^(1/2)/x^2*(-30*(a*x^2+b*x)^(1/2)*a^(5/2)*x^3-15*ln(1/2*(2*
(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x^3*a^2*b+24*(a*x^2+b*x)^(3/2)*
a^(3/2)*x+4*b*(a*x^2+b*x)^(3/2)*a^(1/2))/((a*x+b)*x)^(1/2)/a^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.42284, size = 329, normalized size = 4.63

$$\left[\frac{15\,a^{\frac{3}{2}}bx\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(3\,a^{2}x^{2}-14\,abx-2\,b^{2}\right)\sqrt{\frac{ax+b}{x}}}{6\,x},-\frac{15\,\sqrt{-a}abx\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)-\left(3\,a^{2}x^{2}-\right.}{3\,x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(15*a^(3/2)*b*x*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(3*
a^2*x^2 - 14*a*b*x - 2*b^2)*sqrt((a*x + b)/x))/x, -1/3*(15*sqrt(-a)*a*b*x*a
rctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (3*a^2*x^2 - 14*a*b*x - 2*b^2)*sqrt((
a*x + b)/x))/x]
```

Sympy [A] time = 3.37097, size = 99, normalized size = 1.39

$$a^{\frac{5}{2}}x\sqrt{1+\frac{b}{ax}}-\frac{14a^{\frac{3}{2}}b\sqrt{1+\frac{b}{ax}}}{3}-\frac{5a^{\frac{3}{2}}b\log\left(\frac{b}{ax}\right)}{2}+5a^{\frac{3}{2}}b\log\left(\sqrt{1+\frac{b}{ax}}+1\right)-\frac{2\sqrt{a}b^{2}\sqrt{1+\frac{b}{ax}}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2),x)
```

```
[Out] a**(5/2)*x*sqrt(1 + b/(a*x)) - 14*a**(3/2)*b*sqrt(1 + b/(a*x))/3 - 5*a**(3/
2)*b*log(b/(a*x))/2 + 5*a**(3/2)*b*log(sqrt(1 + b/(a*x)) + 1) - 2*sqrt(a)*b
**2*sqrt(1 + b/(a*x))/(3*x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.242 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{c+\frac{d}{x}} \, dx$

Optimal. Leaf size=134

$$\frac{a^{3/2}(5bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^2}+\frac{2(bc-ad)^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2d^{3/2}}-\frac{b\sqrt{a+\frac{b}{x}}(ad+2bc)}{cd}+\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c}$$

```
[Out] -((b*(2*b*c + a*d)*Sqrt[a + b/x])/(c*d)) + (a*(a + b/x)^(3/2)*x)/c + (2*(b*
c - a*d)^(5/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c^2*d^(3/2
)) + (a^(3/2)*(5*b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/c^2
```

Rubi [A] time = 0.221343, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {375, 98, 154, 156, 63, 208, 205}

$$\frac{a^{3/2}(5bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^2}+\frac{2(bc-ad)^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2d^{3/2}}-\frac{b\sqrt{a+\frac{b}{x}}(ad+2bc)}{cd}+\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(5/2)/(c + d/x),x]
```

```
[Out] -((b*(2*b*c + a*d)*Sqrt[a + b/x])/(c*d)) + (a*(a + b/x)^(3/2)*x)/c + (2*(b*
c - a*d)^(5/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c^2*d^(3/2
)) + (a^(3/2)*(5*b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/c^2
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
```

```
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{c+\frac{d}{x}}\,dx &= -\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^2(c+dx)}\,dx, x, \frac{1}{x}\right)\\
&= \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}\left(-\frac{1}{2}a(5bc-2ad)-\frac{1}{2}b(2bc+ad)x\right)}{x(c+dx)}\,dx, x, \frac{1}{x}\right)}{c}\\
&= -\frac{b(2bc+ad)\sqrt{a+\frac{b}{x}}}{cd} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c} + \frac{2\,\text{Subst}\left(\int \frac{-\frac{1}{4}a^2d(5bc-2ad)+\frac{1}{4}b\left(2b^2c^2-6abcd+a^2d^2\right)x}{x\sqrt{a+bx}(c+dx)}\,dx, x, \frac{1}{x}\right)}{cd}\\
&= -\frac{b(2bc+ad)\sqrt{a+\frac{b}{x}}}{cd} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c} - \frac{\left(a^2(5bc-2ad)\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2c^2} + \frac{(bc-ad)^3\,\text{Su}}{}\\
&= -\frac{b(2bc+ad)\sqrt{a+\frac{b}{x}}}{cd} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c} - \frac{\left(a^2(5bc-2ad)\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^2} + \frac{(2(bc-}{}\\
&= -\frac{b(2bc+ad)\sqrt{a+\frac{b}{x}}}{cd} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c} + \frac{2(bc-ad)^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2d^{3/2}} + \frac{a^{3/2}(5bc-2ad)\tanh^{-1}\left(\right.}{c^2}
\end{aligned}
$$

Mathematica [A] time = 0.183027, size = 116, normalized size = 0.87

$$
\frac{\frac{c\sqrt{a+\frac{b}{x}}\left(a^2dx-2b^2c\right)}{d} + a^{3/2}(5bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right) + \frac{2(bc-ad)^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{d^{3/2}}}{c^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(5/2)/(c + d/x),x]
```

```
[Out] ((c*Sqrt[a + b/x]*(-2*b^2*c + a^2*d*x))/d + (2*(b*c - a*d)^(5/2)*ArcTan[(Sq
rt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/d^(3/2) + a^(3/2)*(5*b*c - 2*a*d)*Ar
cTanh[Sqrt[a + b/x]/Sqrt[a]])/c^2
```

Maple [B] time = 0.013, size = 859, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(5/2)/(c+d/x),x)
```

```
[Out] -1/2*((a*x+b)/x)^(1/2)/x*(2*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^
(1/2))*((a*d-b*c)*d/c^2)^(1/2)*x^2*a^3*c*d^3-5*ln(1/2*(2*((a*x+b)*x)^(1/2)*
a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*x^2*a^2*b*c^2*d^2+4*ln(1/
2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*x^
2*a*b^2*c^3*d-ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b
*c)*d/c^2)^(1/2)*x^2*b^3*c^4-8*((a*d-b*c)*d/c^2)^(1/2)*(a*x^2+b*x)^(1/2)*a^
(3/2)*x^2*b*c^3*d+2*((a*d-b*c)*d/c^2)^(1/2)*(a*x^2+b*x)^(1/2)*a^(1/2)*x^2*b
^2*c^4-4*((a*d-b*c)*d/c^2)^(1/2)*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+
b)/a^(1/2))*x^2*a*b^2*c^3*d+((a*d-b*c)*d/c^2)^(1/2)*ln(1/2*(2*(a*x^2+b*x)^(
1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x^2*b^3*c^4-2*((a*d-b*c)*d/c^2)^(1/2)*a^(5/2
)*((a*x+b)*x)^(1/2)*x^2*c^2*d^2+4*((a*d-b*c)*d/c^2)^(1/2)*a^(3/2)*((a*x+b)*
x)^(1/2)*x^2*b*c^3*d-2*((a*d-b*c)*d/c^2)^(1/2)*a^(1/2)*((a*x+b)*x)^(1/2)*x^
2*b^2*c^4+2*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x
-b*d)/(c*x+d))*a^(7/2)*x^2*d^4-6*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^
(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(5/2)*x^2*b*c*d^3+6*ln((2*((a*d-b*c)*
d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(3/2)*x^2*b^
2*c^2*d^2-2*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x
-b*d)/(c*x+d))*a^(1/2)*x^2*b^3*c^3*d+4*((a*d-b*c)*d/c^2)^(1/2)*(a*x^2+b*x)^
(3/2)*a^(1/2)*b*c^3*d)/((a*x+b)*x)^(1/2)/d^2/a^(1/2)/c^3/((a*d-b*c)*d/c^2)^
(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + \frac{b}{x}\right)^{\frac{5}{2}}}{c + \frac{d}{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/(c+d/x),x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^(5/2)/(c + d/x), x)
```

Fricas [A] time = 2.26667, size = 1431, normalized size = 10.68

$$\left[-\frac{\left(5\,abcd-2\,a^2d^2\right)\sqrt{a}\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)-2\left(b^2c^2-2\,abcd+a^2d^2\right)\sqrt{-\frac{bc-ad}{d}}\log\left(\frac{2\,dx\sqrt{-\frac{bc-ad}{d}}\sqrt{\frac{ax+b}{x}}+bd-(bc-2\,a}{cx+d}\right.}{2\,c^2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/(c+d/x),x, algorithm="fricas")
```

```
[Out] [-1/2*((5*a*b*c*d - 2*a^2*d^2)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x +
b)/x) + b) - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-(b*c - a*d)/d)*log((2*
d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) + b*d - (b*c - 2*a*d)*x)/(c*x +
d)) - 2*(a^2*c*d*x - 2*b^2*c^2)*sqrt((a*x + b)/x))/(c^2*d), -((5*a*b*c*d -
2*a^2*d^2)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (b^2*c^2 - 2*a*b
*c*d + a^2*d^2)*sqrt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((
a*x + b)/x) + b*d - (b*c - 2*a*d)*x)/(c*x + d)) - (a^2*c*d*x - 2*b^2*c^2)*s
qrt((a*x + b)/x))/(c^2*d), -1/2*(4*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt((b*
c - a*d)/d)*arctan(-d*sqrt((b*c - a*d)/d)*sqrt((a*x + b)/x)/(b*c - a*d)) +
(5*a*b*c*d - 2*a^2*d^2)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) +
 b) - 2*(a^2*c*d*x - 2*b^2*c^2)*sqrt((a*x + b)/x))/(c^2*d), -(2*(b^2*c^2 -
2*a*b*c*d + a^2*d^2)*sqrt((b*c - a*d)/d)*arctan(-d*sqrt((b*c - a*d)/d)*sqrt
((a*x + b)/x)/(b*c - a*d)) + (5*a*b*c*d - 2*a^2*d^2)*sqrt(-a)*arctan(sqrt(-
a)*sqrt((a*x + b)/x)/a) - (a^2*c*d*x - 2*b^2*c^2)*sqrt((a*x + b)/x))/(c^2*d
)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{cx+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2)/(c+d/x),x)
```

```
[Out] Integral(x*(a + b/x)**(5/2)/(c*x + d), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/(c+d/x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.243 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^2} dx$

Optimal. Leaf size=166

$$\frac{a^{3/2}(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^3}-\frac{(bc-ad)^{3/2}(4ad+bc)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3d^{3/2}}+\frac{\sqrt{a+\frac{b}{x}}(bc-2ad)(bc-ad)}{c^2d\left(c+\frac{d}{x}\right)}+\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c\left(c+\frac{d}{x}\right)}$$

```
[Out] ((b*c - 2*a*d)*(b*c - a*d)*Sqrt[a + b/x])/(c^2*d*(c + d/x)) + (a*(a + b/x)^
(3/2)*x)/(c*(c + d/x)) - ((b*c - a*d)^(3/2)*(b*c + 4*a*d)*ArcTan[(Sqrt[d]*S
qrt[a + b/x])/Sqrt[b*c - a*d]])/(c^3*d^(3/2)) + (a^(3/2)*(5*b*c - 4*a*d)*Ar
cTanh[Sqrt[a + b/x]/Sqrt[a]])/c^3
```

Rubi [A] time = 0.233858, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {375, 98, 149, 156, 63, 208, 205}

$$\frac{a^{3/2}(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^3}-\frac{(bc-ad)^{3/2}(4ad+bc)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3d^{3/2}}+\frac{\sqrt{a+\frac{b}{x}}(bc-2ad)(bc-ad)}{c^2d\left(c+\frac{d}{x}\right)}+\frac{ax\left(a+\frac{b}{x}\right)^{3/2}}{c\left(c+\frac{d}{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(5/2)/(c + d/x)^2,x]
```

```
[Out] ((b*c - 2*a*d)*(b*c - a*d)*Sqrt[a + b/x])/(c^2*d*(c + d/x)) + (a*(a + b/x)^
(3/2)*x)/(c*(c + d/x)) - ((b*c - a*d)^(3/2)*(b*c + 4*a*d)*ArcTan[(Sqrt[d]*S
qrt[a + b/x])/Sqrt[b*c - a*d]])/(c^3*d^(3/2)) + (a^(3/2)*(5*b*c - 4*a*d)*Ar
cTanh[Sqrt[a + b/x]/Sqrt[a]])/c^3
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
```

```
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^2}\,dx = -\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^2(c+dx)^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}\left(-\frac{1}{2}a(5bc-4ad)-\frac{1}{2}b(2bc-ad)x\right)}{x(c+dx)^2}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{(bc-2ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{c^2d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}a^2d(5bc-4ad)+\frac{1}{2}b\left(b^2c^2+2abcd-2a^2d^2\right)x}{x\sqrt{a+bx}(c+dx)}\,dx, x, \frac{1}{x}\right)}{c^2d}$$

$$= \frac{(bc-2ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{c^2d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)} - \frac{\left(a^2(5bc-4ad)\right)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2c^3} - \frac{\left((bc-ad)^2\right.}{}$$

$$= \frac{(bc-2ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{c^2d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)} - \frac{\left(a^2(5bc-4ad)\right)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^3} - \frac{\left((bc\right.}{}$$

$$= \frac{(bc-2ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{c^2d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)} - \frac{(bc-ad)^{3/2}(bc+4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3d^{3/2}} + \frac{a^{3/2}(5bc-4ad}{}$$

Mathematica [A] time = 0.340474, size = 145, normalized size = 0.87

$$\frac{\frac{cx\sqrt{a+\frac{b}{x}}\left(a^2d(cx+2d)-2abcd+b^2c^2\right)}{d(cx+d)} + a^{3/2}(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right) - \frac{(bc-ad)^{3/2}(4ad+bc)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{d^{3/2}}}{c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(5/2)/(c + d/x)^2,x]
```

```
[Out] ((c*Sqrt[a + b/x]*x*(b^2*c^2 - 2*a*b*c*d + a^2*d*(2*d + c*x)))/(d*(d + c*x)
) - ((b*c - a*d)^(3/2)*(b*c + 4*a*d)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*
c - a*d]])/d^(3/2) + a^(3/2)*(5*b*c - 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
)/c^3
```

Maple [B] time = 0.034, size = 1323, normalized size = 8.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(5/2)/(c+d/x)^2,x)
```

```
[Out] -1/2*(ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a*((a*d-b*c)*d/
c^2)^(1/2)*x*b^3*c^5+4*a^(9/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1
```

```
/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*d^5+2*a^(5/2)*((a*d-b*c)*d/c^2)^(1/2)*((a
*x+b)*x)^(1/2)*x*b*c^4*d-5*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(
1/2))*a^3*((a*d-b*c)*d/c^2)^(1/2)*x*b*c^3*d^2+2*a^(5/2)*ln((2*((a*d-b*c)*d/
c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^2*c^2*d^3+2*a^
(3/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(3/2)*b*c^5+a^(3/2)*ln((2*((a*d-b
*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^3*c^3*d^
2-2*a^(5/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(3/2)*c^4*d+4*a^(9/2)*ln((2
*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x*
c*d^4+4*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^4*((a*d-b*c
)*d/c^2)^(1/2)*c*d^4-4*a^(7/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c^
2*d^3-7*a^(7/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b
*c*x-b*d)/(c*x+d))*b*c*d^4-7*a^(7/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)
*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x*b*c^2*d^3+2*a^(5/2)*ln((2*((a*d-b
*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x*b^2*c^3*
d^2-5*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*((a*d-b*c)*
d/c^2)^(1/2)*b*c^2*d^3+2*a^(7/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*
x^2*c^4*d+4*a^(5/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*b*c^3*d^2-2*a
^(3/2)*((a*d-b*c)*d/c^2)^(1/2)*(a*x^2+b*x)^(1/2)*x*b^2*c^5+a^(3/2)*ln((2*((
a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x*b^3
*c^4*d-a*((a*d-b*c)*d/c^2)^(1/2)*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+
b)/a^(1/2))*x*b^3*c^5+ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))
*a*((a*d-b*c)*d/c^2)^(1/2)*b^3*c^4*d-2*a^(3/2)*((a*d-b*c)*d/c^2)^(1/2)*(a*x
^2+b*x)^(1/2)*b^2*c^4*d-a*((a*d-b*c)*d/c^2)^(1/2)*ln(1/2*(2*(a*x^2+b*x)^(1/
2)*a^(1/2)+2*a*x+b)/a^(1/2))*b^3*c^4*d-2*a^(5/2)*((a*d-b*c)*d/c^2)^(1/2)*((
a*x+b)*x)^(1/2)*x^2*b*c^5+4*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^
(1/2))*a^4*((a*d-b*c)*d/c^2)^(1/2)*x*c^2*d^3-2*a^(7/2)*((a*d-b*c)*d/c^2)^(1
/2)*((a*x+b)*x)^(1/2)*x*c^3*d^2)*x*((a*x+b)/x)^(1/2)/c^4/((a*d-b*c)*d/c^2)^
(1/2)/a^(3/2)/(c*x+d)/d^2/((a*x+b)*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{\left(c+\frac{d}{x}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/(c+d/x)^2,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^(5/2)/(c + d/x)^2, x)
```

Fricas [A] time = 1.95058, size = 2125, normalized size = 12.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/(c+d/x)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((5*a*b*c*d^2 - 4*a^2*d^3 + (5*a*b*c^2*d - 4*a^2*c*d^2)*x)*sqrt(a)*lo
g(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (b^2*c^2*d + 3*a*b*c*d^2 - 4
*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a^2*c*d^2)*x)*sqrt(-(b*c - a*d)/d)*lo
g((2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) + b*d - (b*c - 2*a*d)*x)/(c
*x + d)) - 2*(a^2*c^2*d*x^2 + (b^2*c^3 - 2*a*b*c^2*d + 2*a^2*c*d^2)*x)*sqrt
```

```
((a*x + b)/x))/(c^4*d*x + c^3*d^2), -1/2*(2*(5*a*b*c*d^2 - 4*a^2*d^3 + (5*a
*b*c^2*d - 4*a^2*c*d^2)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) +
(b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a^2*c*d^2
)*x)*sqrt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x)
 + b*d - (b*c - 2*a*d)*x)/(c*x + d)) - 2*(a^2*c^2*d*x^2 + (b^2*c^3 - 2*a*b*
c^2*d + 2*a^2*c*d^2)*x)*sqrt((a*x + b)/x))/(c^4*d*x + c^3*d^2), 1/2*(2*(b^2
*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a^2*c*d^2)*x)
*sqrt((b*c - a*d)/d)*arctan(-d*sqrt((b*c - a*d)/d)*sqrt((a*x + b)/x)/(b*c -
 a*d)) - (5*a*b*c*d^2 - 4*a^2*d^3 + (5*a*b*c^2*d - 4*a^2*c*d^2)*x)*sqrt(a)*
log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(a^2*c^2*d*x^2 + (b^2*c^
3 - 2*a*b*c^2*d + 2*a^2*c*d^2)*x)*sqrt((a*x + b)/x))/(c^4*d*x + c^3*d^2), (
(b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a^2*c*d^2
)*x)*sqrt((b*c - a*d)/d)*arctan(-d*sqrt((b*c - a*d)/d)*sqrt((a*x + b)/x)/(b
*c - a*d)) - (5*a*b*c*d^2 - 4*a^2*d^3 + (5*a*b*c^2*d - 4*a^2*c*d^2)*x)*sqrt
(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (a^2*c^2*d*x^2 + (b^2*c^3 - 2*a
*b*c^2*d + 2*a^2*c*d^2)*x)*sqrt((a*x + b)/x))/(c^4*d*x + c^3*d^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2)/(c+d/x)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/(c+d/x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.244 $\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^3} \, dx$

Optimal. Leaf size=237

$$-\frac{\sqrt{a+\frac{b}{x}}\left(-12a^2d^2+7abcd+b^2c^2\right)}{4c^3d\left(c+\frac{d}{x}\right)}-\frac{\sqrt{bc-ad}\left(-24a^2d^2+8abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4d^{3/2}}+\frac{a^{3/2}(5bc-6ad)\tanh^{-1}}{c^4}$$

```
[Out] ((b*c - 3*a*d)*(b*c - a*d)*Sqrt[a + b/x])/(2*c^2*d*(c + d/x)^2) - ((b^2*c^2
 + 7*a*b*c*d - 12*a^2*d^2)*Sqrt[a + b/x])/(4*c^3*d*(c + d/x)) + (a*(a + b/x
)^(3/2)*x)/(c*(c + d/x)^2) - (Sqrt[b*c - a*d]*(b^2*c^2 + 8*a*b*c*d - 24*a^2
*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(4*c^4*d^(3/2)) + (a
^(3/2)*(5*b*c - 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/c^4
```

Rubi [A] time = 0.372679, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {375, 98, 149, 151, 156, 63, 208, 205}

$$-\frac{\sqrt{a+\frac{b}{x}}\left(-12a^2d^2+7abcd+b^2c^2\right)}{4c^3d\left(c+\frac{d}{x}\right)}-\frac{\sqrt{bc-ad}\left(-24a^2d^2+8abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4d^{3/2}}+\frac{a^{3/2}(5bc-6ad)\tanh^{-1}}{c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(5/2)/(c + d/x)^3,x]
```

```
[Out] ((b*c - 3*a*d)*(b*c - a*d)*Sqrt[a + b/x])/(2*c^2*d*(c + d/x)^2) - ((b^2*c^2
 + 7*a*b*c*d - 12*a^2*d^2)*Sqrt[a + b/x])/(4*c^3*d*(c + d/x)) + (a*(a + b/x
)^(3/2)*x)/(c*(c + d/x)^2) - (Sqrt[b*c - a*d]*(b^2*c^2 + 8*a*b*c*d - 24*a^2
*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(4*c^4*d^(3/2)) + (a
^(3/2)*(5*b*c - 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/c^4
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\left(a+\frac{b}{x}\right)^{5/2}}{\left(c+\frac{d}{x}\right)^3}dx = -\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x^2(c+dx)^3}dx,x,\frac{1}{x}\right)$$
$$= \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)^2} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}\left(-\frac{1}{2}a(5bc-6ad)-\frac{1}{2}b(2bc-3ad)x\right)}{x(c+dx)^3}dx,x,\frac{1}{x}\right)}{c}$$
$$= \frac{(bc-3ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{2c^2d\left(c+\frac{d}{x}\right)^2} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)^2} - \frac{\text{Subst}\left(\int \frac{a^2d(5bc-6ad)+\frac{1}{2}b\left(b^2c^2+6abcd-9a^2d^2\right)x}{x\sqrt{a+bx}(c+dx)^2}dx,x,\frac{1}{x}\right)}{2c^2d}$$
$$= \frac{(bc-3ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{2c^2d\left(c+\frac{d}{x}\right)^2} - \frac{\left(b^2c^2+7abcd-12a^2d^2\right)\sqrt{a+\frac{b}{x}}}{4c^3d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)^2} + \frac{\text{Subst}\left(\int \frac{-a^2d(5b}{}\right.}{}$$
$$= \frac{(bc-3ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{2c^2d\left(c+\frac{d}{x}\right)^2} - \frac{\left(b^2c^2+7abcd-12a^2d^2\right)\sqrt{a+\frac{b}{x}}}{4c^3d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)^2} - \frac{\left(a^2(5bc-6ad)\right)}{}$$
$$= \frac{(bc-3ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{2c^2d\left(c+\frac{d}{x}\right)^2} - \frac{\left(b^2c^2+7abcd-12a^2d^2\right)\sqrt{a+\frac{b}{x}}}{4c^3d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)^2} - \frac{\left(a^2(5bc-6ad)\right)}{}$$
$$= \frac{(bc-3ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{2c^2d\left(c+\frac{d}{x}\right)^2} - \frac{\left(b^2c^2+7abcd-12a^2d^2\right)\sqrt{a+\frac{b}{x}}}{4c^3d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2}x}{c\left(c+\frac{d}{x}\right)^2} - \frac{\sqrt{bc-ad}\left(b^2c^2+\right.}{}$$

Mathematica [A] time = 0.546294, size = 191, normalized size = 0.81

$$\frac{\frac{cx\sqrt{a+\frac{b}{x}}\left(2a^2d\left(2c^2x^2+9cdx+6d^2\right)-abcd(11cx+7d)+b^2c^2(cx-d)\right)}{d(cx+d)^2} - \frac{\sqrt{bc-ad}\left(-24a^2d^2+8abcd+b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{d^{3/2}} - 4a^{3/2}(6ad-5bc)\tanh^{-1}\left(\right.}{4c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(5/2)/(c + d/x)^3,x]
```

```
[Out] ((c*Sqrt[a + b/x]*x*(b^2*c^2*(-d + c*x) - a*b*c*d*(7*d + 11*c*x) + 2*a^2*d*
(6*d^2 + 9*c*d*x + 2*c^2*x^2)))/(d*(d + c*x)^2) - (Sqrt[b*c - a*d]*(b^2*c^2
 + 8*a*b*c*d - 24*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])
/d^(3/2) - 4*a^(3/2)*(-5*b*c + 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(4*c^
4)
```

Maple [B] time = 0.011, size = 1638, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(5/2)/(c+d/x)^3,x)
```

```
[Out] -1/8*(30*a^(5/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*x*b*c^4*d^2+18*a
^(5/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*x^2*b*c^5*d-40*ln(1/2*(2*(
(a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*((a*d-b*c)*d/c^2)^(1/2)*x*b*
c^3*d^3-20*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*((a*d-
b*c)*d/c^2)^(1/2)*x^2*b*c^4*d^2-20*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*
x+b)/a^(1/2))*a^3*((a*d-b*c)*d/c^2)^(1/2)*b*c^2*d^4+14*a^(5/2)*((a*d-b*c)*d
/c^2)^(1/2)*((a*x+b)*x)^(1/2)*b*c^3*d^3+24*a^(9/2)*ln((2*((a*d-b*c)*d/c^2)^
(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*d^6+2*a^(3/2)*((a*d-b
*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*b^2*c^4*d^2+12*a^(7/2)*((a*d-b*c)*d/c^2)
^(1/2)*((a*x+b)*x)^(1/2)*x^3*c^5*d+2*a^(5/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+
b)*x)^(1/2)*x^3*b*c^6+24*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/
2))*a^4*((a*d-b*c)*d/c^2)^(1/2)*x^2*c^3*d^3-12*a^(5/2)*((a*d-b*c)*d/c^2)^(1
/2)*((a*x+b)*x)^(3/2)*x*c^5*d-32*a^(7/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*
x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x^2*b*c^3*d^3+7*a^(5/2)*ln((2*(
(a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x^2*
b^2*c^4*d^2+48*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^4*((
a*d-b*c)*d/c^2)^(1/2)*x*c^2*d^4-36*a^(7/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)
*x)^(1/2)*x*c^3*d^3-64*a^(7/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1
/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x*b*c^2*d^4+14*a^(5/2)*ln((2*((a*d-b*c)*d
/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x*b^2*c^3*d^3+2
*a^(3/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*x^2*b^2*c^6+a^(3/2)*ln((
2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x
^2*b^3*c^5*d+2*a^(3/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*
a*d*x+b*c*x-b*d)/(c*x+d))*x*b^3*c^4*d^2-6*a^(3/2)*((a*d-b*c)*d/c^2)^(1/2)*(
(a*x+b)*x)^(3/2)*b*c^5*d+4*a^(3/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2
)*x*b^2*c^5*d+24*a^(9/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-
2*a*d*x+b*c*x-b*d)/(c*x+d))*x^2*c^2*d^4-8*a^(5/2)*((a*d-b*c)*d/c^2)^(1/2)*(
(a*x+b)*x)^(3/2)*c^4*d^2+24*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^
(1/2))*a^4*((a*d-b*c)*d/c^2)^(1/2)*c*d^5-24*a^(7/2)*((a*d-b*c)*d/c^2)^(1/2)
*((a*x+b)*x)^(1/2)*c^2*d^4-32*a^(7/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b
)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b*c*d^5+7*a^(5/2)*ln((2*((a*d-b*c)
*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^2*c^2*d^4+a
^(3/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)
/(c*x+d))*b^3*c^3*d^3+48*a^(9/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^
(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x*c*d^5-2*a^(3/2)*((a*d-b*c)*d/c^2)^(1/
2)*((a*x+b)*x)^(3/2)*x*b*c^6)*x*((a*x+b)/x)^(1/2)/c^5/((a*d-b*c)*d/c^2)^(1/
2)/a^(3/2)/(c*x+d)^2/d^2/((a*x+b)*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}{\left(c+\frac{d}{x}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/(c+d/x)^3,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^(5/2)/(c + d/x)^3, x)
```

Fricas [A] time = 1.89564, size = 3040, normalized size = 12.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/(c+d/x)^3,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(5*a*b*c*d^3 - 6*a^2*d^4 + (5*a*b*c^3*d - 6*a^2*c^2*d^2)*x^2 + 2*(
5*a*b*c^2*d^2 - 6*a^2*c*d^3)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x +
 b)/x) + b) + (b^2*c^2*d^2 + 8*a*b*c*d^3 - 24*a^2*d^4 + (b^2*c^4 + 8*a*b*c^
3*d - 24*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d + 8*a*b*c^2*d^2 - 24*a^2*c*d^3)*x)
*sqrt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) + b
*d - (b*c - 2*a*d)*x)/(c*x + d)) - 2*(4*a^2*c^3*d*x^3 + (b^2*c^4 - 11*a*b*c
^3*d + 18*a^2*c^2*d^2)*x^2 - (b^2*c^3*d + 7*a*b*c^2*d^2 - 12*a^2*c*d^3)*x)*
sqrt((a*x + b)/x))/(c^6*d*x^2 + 2*c^5*d^2*x + c^4*d^3), 1/4*((b^2*c^2*d^2 +
 8*a*b*c*d^3 - 24*a^2*d^4 + (b^2*c^4 + 8*a*b*c^3*d - 24*a^2*c^2*d^2)*x^2 +
2*(b^2*c^3*d + 8*a*b*c^2*d^2 - 24*a^2*c*d^3)*x)*sqrt((b*c - a*d)/d)*arctan(
-d*sqrt((b*c - a*d)/d)*sqrt((a*x + b)/x)/(b*c - a*d)) - 2*(5*a*b*c*d^3 - 6*
a^2*d^4 + (5*a*b*c^3*d - 6*a^2*c^2*d^2)*x^2 + 2*(5*a*b*c^2*d^2 - 6*a^2*c*d^
3)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (4*a^2*c^3*d
*x^3 + (b^2*c^4 - 11*a*b*c^3*d + 18*a^2*c^2*d^2)*x^2 - (b^2*c^3*d + 7*a*b*c
^2*d^2 - 12*a^2*c*d^3)*x)*sqrt((a*x + b)/x))/(c^6*d*x^2 + 2*c^5*d^2*x + c^4
*d^3), -1/8*(8*(5*a*b*c*d^3 - 6*a^2*d^4 + (5*a*b*c^3*d - 6*a^2*c^2*d^2)*x^2
 + 2*(5*a*b*c^2*d^2 - 6*a^2*c*d^3)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x +
b)/x)/a) + (b^2*c^2*d^2 + 8*a*b*c*d^3 - 24*a^2*d^4 + (b^2*c^4 + 8*a*b*c^3*d
 - 24*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d + 8*a*b*c^2*d^2 - 24*a^2*c*d^3)*x)*sq
rt(-(b*c - a*d)/d)*log((2*d*x*sqrt(-(b*c - a*d)/d)*sqrt((a*x + b)/x) + b*d
- (b*c - 2*a*d)*x)/(c*x + d)) - 2*(4*a^2*c^3*d*x^3 + (b^2*c^4 - 11*a*b*c^3*
d + 18*a^2*c^2*d^2)*x^2 - (b^2*c^3*d + 7*a*b*c^2*d^2 - 12*a^2*c*d^3)*x)*sqr
t((a*x + b)/x))/(c^6*d*x^2 + 2*c^5*d^2*x + c^4*d^3), 1/4*((b^2*c^2*d^2 + 8*
a*b*c*d^3 - 24*a^2*d^4 + (b^2*c^4 + 8*a*b*c^3*d - 24*a^2*c^2*d^2)*x^2 + 2*(
b^2*c^3*d + 8*a*b*c^2*d^2 - 24*a^2*c*d^3)*x)*sqrt((b*c - a*d)/d)*arctan(-d*
sqrt((b*c - a*d)/d)*sqrt((a*x + b)/x)/(b*c - a*d)) - 4*(5*a*b*c*d^3 - 6*a^2
*d^4 + (5*a*b*c^3*d - 6*a^2*c^2*d^2)*x^2 + 2*(5*a*b*c^2*d^2 - 6*a^2*c*d^3)*
x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (4*a^2*c^3*d*x^3 + (b^2*
c^4 - 11*a*b*c^3*d + 18*a^2*c^2*d^2)*x^2 - (b^2*c^3*d + 7*a*b*c^2*d^2 - 12*
a^2*c*d^3)*x)*sqrt((a*x + b)/x))/(c^6*d*x^2 + 2*c^5*d^2*x + c^4*d^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2)/(c+d/x)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.37821, size = 1276, normalized size = 5.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)/(c+d/x)^3,x, algorithm="giac")
```

```
[Out] sqrt(a*x^2 + b*x)*a^2*sgn(x)/c^3 - 1/2*(5*a^2*b*c*sgn(x) - 6*a^3*d*sgn(x))*
log(abs(2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))/(sqrt(a)*c^4) + 1/4
```

```
*(b^3*c^3*sgn(x) + 7*a*b^2*c^2*d*sgn(x) - 32*a^2*b*c*d^2*sgn(x) + 24*a^3*d^
3*sgn(x))*arctan(-((sqrt(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)/sqrt(b*c*
d - a*d^2))/(sqrt(b*c*d - a*d^2)*c^4*d) + 1/4*(sqrt(a)*b^3*c^3*arctan(sqrt(
a)*d/sqrt(b*c*d - a*d^2)) + 7*a^(3/2)*b^2*c^2*d*arctan(sqrt(a)*d/sqrt(b*c*d
 - a*d^2)) - 32*a^(5/2)*b*c*d^2*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 24*
a^(7/2)*d^3*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 10*sqrt(b*c*d - a*d^2)*
a^2*b*c*d*log(abs(b)) - 12*sqrt(b*c*d - a*d^2)*a^3*d^2*log(abs(b)) - sqrt(b
*c*d - a*d^2)*a*b^2*c^2 + 11*sqrt(b*c*d - a*d^2)*a^2*b*c*d - 10*sqrt(b*c*d
- a*d^2)*a^3*d^2)*sgn(x)/(sqrt(b*c*d - a*d^2)*sqrt(a)*c^4*d) - 1/4*((sqrt(a
)*x - sqrt(a*x^2 + b*x))^3*sqrt(a)*b^3*c^4*sgn(x) - 17*(sqrt(a)*x - sqrt(a*
x^2 + b*x))^3*a^(3/2)*b^2*c^3*d*sgn(x) + 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))
^3*a^(5/2)*b*c^2*d^2*sgn(x) - 24*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a^(7/2)*
c*d^3*sgn(x) - 5*(sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a*b^3*c^3*d*sgn(x) - 3*(
sqrt(a)*x - sqrt(a*x^2 + b*x))^2*a^2*b^2*c^2*d^2*sgn(x) + 48*(sqrt(a)*x - s
qrt(a*x^2 + b*x))^2*a^3*b*c*d^3*sgn(x) - 40*(sqrt(a)*x - sqrt(a*x^2 + b*x))
^2*a^4*d^4*sgn(x) - (sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*b^4*c^3*d*sgn(x
) - 11*(sqrt(a)*x - sqrt(a*x^2 + b*x))*a^(3/2)*b^3*c^2*d^2*sgn(x) + 52*(sqr
t(a)*x - sqrt(a*x^2 + b*x))*a^(5/2)*b^2*c*d^3*sgn(x) - 40*(sqrt(a)*x - sqrt
(a*x^2 + b*x))*a^(7/2)*b*d^4*sgn(x) - a*b^4*c^2*d^2*sgn(x) + 11*a^2*b^3*c*d
^3*sgn(x) - 10*a^3*b^2*d^4*sgn(x))/(((sqrt(a)*x - sqrt(a*x^2 + b*x))^2*c +
2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a)*d + b*d)^2*sqrt(a)*c^4*d)
```

3.245 $\int \frac{\left(c+\frac{d}{x}\right)^3}{\sqrt{a+\frac{b}{x}}}\,dx$

Optimal. Leaf size=126

$$-\frac{d\sqrt{a+\frac{b}{x}}\left(2\left(-2a^2d^2+9abcd+3b^2c^2\right)+\frac{bd(2ad+3bc)}{x}\right)}{3ab^2}-\frac{c^2(bc-6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{cx\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}{a}$$

```
[Out] -(d*Sqrt[a + b/x]*(2*(3*b^2*c^2 + 9*a*b*c*d - 2*a^2*d^2) + (b*d*(3*b*c + 2*
a*d))/x))/(3*a*b^2) + (c*Sqrt[a + b/x]*(c + d/x)^2*x)/a - (c^2*(b*c - 6*a*d
)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)
```

Rubi [A] time = 0.0899013, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {375, 98, 147, 63, 208}

$$-\frac{d\sqrt{a+\frac{b}{x}}\left(2\left(-2a^2d^2+9abcd+3b^2c^2\right)+\frac{bd(2ad+3bc)}{x}\right)}{3ab^2}-\frac{c^2(bc-6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{cx\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d/x)^3/Sqrt[a + b/x],x]
```

```
[Out] -(d*Sqrt[a + b/x]*(2*(3*b^2*c^2 + 9*a*b*c*d - 2*a^2*d^2) + (b*d*(3*b*c + 2*
a*d))/x))/(3*a*b^2) + (c*Sqrt[a + b/x]*(c + d/x)^2*x)/a - (c^2*(b*c - 6*a*d
)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
```

```
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\sqrt{a+\frac{b}{x}}}\,dx = -\text{Subst}\left(\int \frac{(c+dx)^3}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 x}{a} + \frac{\text{Subst}\left(\int \frac{(c+dx)\left(\frac{1}{2}c(bc-6ad)-\frac{1}{2}d(3bc+2ad)x\right)}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{d\sqrt{a+\frac{b}{x}}\left(2\left(3b^2c^2+9abcd-2a^2d^2\right)+\frac{bd(3bc+2ad)}{x}\right)}{3ab^2} + \frac{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 x}{a} + \frac{\left(c^2(bc-6ad)\right)\text{Subst}\left(\int \ldots\right.}{2a}$$

$$= -\frac{d\sqrt{a+\frac{b}{x}}\left(2\left(3b^2c^2+9abcd-2a^2d^2\right)+\frac{bd(3bc+2ad)}{x}\right)}{3ab^2} + \frac{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 x}{a} + \frac{\left(c^2(bc-6ad)\right)\text{Subst}\left(\int \ldots\right.}{ab}$$

$$= -\frac{d\sqrt{a+\frac{b}{x}}\left(2\left(3b^2c^2+9abcd-2a^2d^2\right)+\frac{bd(3bc+2ad)}{x}\right)}{3ab^2} + \frac{c\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2 x}{a} - \frac{c^2(bc-6ad)\tanh^{-1}\left(\frac{\sqrt{a+\ldots}}{\sqrt{a}}\right.}{a^{3/2}}$$

Mathematica [A] time = 0.109219, size = 95, normalized size = 0.75

$$\frac{\sqrt{a+\frac{b}{x}}\left(4a^2d^3x-2abd^2(9cx+d)+3b^2c^3x^2\right)}{3ab^2x} + \frac{c^2(6ad-bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d/x)^3/Sqrt[a + b/x],x]
```

```
[Out] (Sqrt[a + b/x]*(4*a^2*d^3*x + 3*b^2*c^3*x^2 - 2*a*b*d^2*(d + 9*c*x)))/(3*a*
b^2*x) + (c^2*(-(b*c) + 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)
```

Maple [B] time = 0.014, size = 535, normalized size = 4.3

$$-\frac{1}{6b^3x^2}\sqrt{\frac{ax+b}{x}}\left(6a^{7/2}\sqrt{ax^2+bx}x^3d^3+6a^{7/2}\sqrt{(ax+b)x}x^3d^3-18a^{5/2}\sqrt{ax^2+bx}x^3bcd^2-18a^{5/2}\sqrt{(ax+b)x}x^3bcd^2-1\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)^3/(a+b/x)^(1/2),x)
```

```
[Out] -1/6*((a*x+b)/x)^(1/2)*(6*a^(7/2)*(a*x^2+b*x)^(1/2)*x^3*d^3+6*a^(7/2)*((a*x
+b)*x)^(1/2)*x^3*d^3-18*a^(5/2)*(a*x^2+b*x)^(1/2)*x^3*b*c*d^2-18*a^(5/2)*((
a*x+b)*x)^(1/2)*x^3*b*c*d^2-12*a^(5/2)*(a*x^2+b*x)^(3/2)*x*d^3-18*a^(3/2)*(
a*x^2+b*x)^(1/2)*x^3*b^2*c^2*d+18*a^(3/2)*((a*x+b)*x)^(1/2)*x^3*b^2*c^2*d+3
6*a^(3/2)*(a*x^2+b*x)^(3/2)*x*b*c*d^2-6*a^(1/2)*((a*x+b)*x)^(1/2)*x^3*b^3*c
^3+3*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x^3*a^3*b*d^3-9*
ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x^3*a^2*b^2*c*d^2-9*l
n(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x^3*a*b^3*c^2*d-3*ln(1
/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x^3*a^3*b*d^3+9*ln(1/2*(2
*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x^3*a^2*b^2*c*d^2-9*ln(1/2*(2*
((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x^3*a*b^3*c^2*d+3*ln(1/2*(2*((a
*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x^3*b^4*c^3+4*d^3*(a*x^2+b*x)^(3/2
)*b*a^(3/2))/x^2/((a*x+b)*x)^(1/2)/b^3/a^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^3/(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.37605, size = 510, normalized size = 4.05

$$\left[-\frac{3\left(b^3c^3-6\,ab^2c^2d\right)\sqrt{a}x\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)-2\left(3\,ab^2c^3x^2-2\,a^2bd^3-2\left(9\,a^2bcd^2-2\,a^3d^3\right)x\right)\sqrt{\frac{ax+b}{x}}}{6\,a^2b^2x}, \frac{3\,(b\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^3/(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(b^3*c^3 - 6*a*b^2*c^2*d)*sqrt(a)*x*log(2*a*x + 2*sqrt(a)*x*sqrt((
a*x + b)/x) + b) - 2*(3*a*b^2*c^3*x^2 - 2*a^2*b*d^3 - 2*(9*a^2*b*c*d^2 - 2*
a^3*d^3)*x)*sqrt((a*x + b)/x))/(a^2*b^2*x), 1/3*(3*(b^3*c^3 - 6*a*b^2*c^2*d
)*sqrt(-a)*x*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (3*a*b^2*c^3*x^2 - 2*a^
2*b*d^3 - 2*(9*a^2*b*c*d^2 - 2*a^3*d^3)*x)*sqrt((a*x + b)/x))/(a^2*b^2*x)]
```

Sympy [A] time = 40.8405, size = 386, normalized size = 3.06

$$\frac{4a^{\frac{7}{2}}b^{\frac{3}{2}}d^3x^2\sqrt{\frac{ax}{b}+1}}{3a^{\frac{5}{2}}b^3x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^4x^{\frac{3}{2}}}+\frac{2a^{\frac{5}{2}}b^{\frac{5}{2}}d^3x\sqrt{\frac{ax}{b}+1}}{3a^{\frac{5}{2}}b^3x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^4x^{\frac{3}{2}}}-\frac{2a^{\frac{3}{2}}b^{\frac{7}{2}}d^3\sqrt{\frac{ax}{b}+1}}{3a^{\frac{5}{2}}b^3x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^4x^{\frac{3}{2}}}-\frac{4a^4bd^3x^{\frac{5}{2}}}{3a^{\frac{5}{2}}b^3x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^4x^{\frac{3}{2}}}-\frac{4a^3b^2d^3x^{\frac{3}{2}}}{3a^{\frac{5}{2}}b^3x^{\frac{5}{2}}+3a^{\frac{3}{2}}b^4x^{\frac{3}{2}}}+3\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)**3/(a+b/x)**(1/2),x)
```

```
[Out] 4*a**(7/2)*b**(3/2)*d**3*x**2*sqrt(a*x/b + 1)/(3*a**(5/2)*b**3*x**(5/2) + 3
*a**(3/2)*b**4*x**(3/2)) + 2*a**(5/2)*b**(5/2)*d**3*x*sqrt(a*x/b + 1)/(3*a*
*(5/2)*b**3*x**(5/2) + 3*a**(3/2)*b**4*x**(3/2)) - 2*a**(3/2)*b**(7/2)*d**3
*sqrt(a*x/b + 1)/(3*a**(5/2)*b**3*x**(5/2) + 3*a**(3/2)*b**4*x**(3/2)) - 4*
a**4*b*d**3*x**(5/2)/(3*a**(5/2)*b**3*x**(5/2) + 3*a**(3/2)*b**4*x**(3/2))
- 4*a**3*b**2*d**3*x**(3/2)/(3*a**(5/2)*b**3*x**(5/2) + 3*a**(3/2)*b**4*x**
(3/2)) + 3*c*d**2*Piecewise((-1/(sqrt(a)*x), Eq(b, 0)), (-2*sqrt(a + b/x)/b
, True)) + sqrt(b)*c**3*sqrt(x)*sqrt(a*x/b + 1)/a - 6*c**2*d*atan(1/(sqrt(-
1/a)*sqrt(a + b/x)))/(a*sqrt(-1/a)) - b*c**3*asinh(sqrt(a)*sqrt(x)/sqrt(b))
/a**(3/2)
```

Giac [A] time = 1.2012, size = 207, normalized size = 1.64

$$-\frac{1}{3}\left(\frac{3\,c^3\sqrt{\frac{ax+b}{x}}}{\left(a-\frac{ax+b}{x}\right)a}-\frac{3\left(bc^3-6\,ac^2d\right)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}ab}+\frac{2\left(9\,b^7cd^2\sqrt{\frac{ax+b}{x}}-3\,ab^6d^3\sqrt{\frac{ax+b}{x}}+\frac{(ax+b)b^6d^3\sqrt{\frac{ax+b}{x}}}{x}\right)}{b^9}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^3/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*c^3*sqrt((a*x + b)/x)/((a - (a*x + b)/x)*a) - 3*(b*c^3 - 6*a*c^2*d)
*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a*b) + 2*(9*b^7*c*d^2*sqrt((a
*x + b)/x) - 3*a*b^6*d^3*sqrt((a*x + b)/x) + (a*x + b)*b^6*d^3*sqrt((a*x +
b)/x)/x)/b^9)*b
```

3.246 $\int \frac{\left(c+\frac{d}{x}\right)^2}{\sqrt{a+\frac{b}{x}}} \, dx$

Optimal. Leaf size=73

$$-\frac{c(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{c^2x\sqrt{a+\frac{b}{x}}}{a}-\frac{2d^2\sqrt{a+\frac{b}{x}}}{b}$$

```
[Out] (-2*d^2*Sqrt[a + b/x])/b + (c^2*Sqrt[a + b/x]*x)/a - (c*(b*c - 4*a*d)*ArcTa
nh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)
```

Rubi [A] time = 0.0540239, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {375, 89, 80, 63, 208}

$$-\frac{c(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{c^2x\sqrt{a+\frac{b}{x}}}{a}-\frac{2d^2\sqrt{a+\frac{b}{x}}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d/x)^2/Sqrt[a + b/x],x]
```

```
[Out] (-2*d^2*Sqrt[a + b/x])/b + (c^2*Sqrt[a + b/x]*x)/a - (c*(b*c - 4*a*d)*ArcTa
nh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(c+\frac{d}{x}\right)^2}{\sqrt{a+\frac{b}{x}}}\,dx &= -\operatorname{Subst}\left(\int \frac{(c+dx)^2}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)\\ &= \frac{c^2\sqrt{a+\frac{b}{x}}\,x}{a} - \frac{\operatorname{Subst}\left(\int \frac{-\frac{1}{2}c(bc-4ad)+ad^2x}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{a}\\ &= -\frac{2d^2\sqrt{a+\frac{b}{x}}}{b} + \frac{c^2\sqrt{a+\frac{b}{x}}\,x}{a} + \frac{(c(bc-4ad))\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a}\\ &= -\frac{2d^2\sqrt{a+\frac{b}{x}}}{b} + \frac{c^2\sqrt{a+\frac{b}{x}}\,x}{a} + \frac{(c(bc-4ad))\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{ab}\\ &= -\frac{2d^2\sqrt{a+\frac{b}{x}}}{b} + \frac{c^2\sqrt{a+\frac{b}{x}}\,x}{a} - \frac{c(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0714159, size = 66, normalized size = 0.9

$$\frac{c(4ad-bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}} + \frac{\sqrt{a+\frac{b}{x}}\left(bc^2x-2ad^2\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d/x)^2/Sqrt[a + b/x],x]
```

```
[Out] (Sqrt[a + b/x]*(-2*a*d^2 + b*c^2*x))/(a*b) + (c*(-(b*c) + 4*a*d)*ArcTanh[Sq
rt[a + b/x]/Sqrt[a]])/a^(3/2)
```

Maple [B] time = 0.013, size = 348, normalized size = 4.8

$$-\frac{1}{2b^2x}\sqrt{\frac{ax+b}{x}}\left(-2\sqrt{ax^2+bx}a^{5/2}x^2d^2-4\sqrt{ax^2+bx}a^{3/2}x^2bcd-\ln\left(\frac{1}{2}\left(2\sqrt{ax^2+bx}\sqrt{a}+2ax+b\right)\frac{1}{\sqrt{a}}\right)x^2a^2bd^2-2\ln\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)^2/(a+b/x)^(1/2),x)
```

```
[Out] -1/2*((a*x+b)/x)^(1/2)/x*(-2*(a*x^2+b*x)^(1/2)*a^(5/2)*x^2*d^2-4*(a*x^2+b*x
)^(1/2)*a^(3/2)*x^2*b*c*d-ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1
/2))*x^2*a^2*b*d^2-2*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*
x^2*a*b^2*c*d-2*a^(5/2)*((a*x+b)*x)^(1/2)*x^2*d^2+4*a^(3/2)*((a*x+b)*x)^(1/
2)*x^2*b*c*d-2*a^(1/2)*((a*x+b)*x)^(1/2)*x^2*b^2*c^2+ln(1/2*(2*((a*x+b)*x)^
(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x^2*a^2*b*d^2-2*ln(1/2*(2*((a*x+b)*x)^(1/2)
*a^(1/2)+2*a*x+b)/a^(1/2))*x^2*a*b^2*c*d+ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2
)+2*a*x+b)/a^(1/2))*x^2*b^3*c^2+4*(a*x^2+b*x)^(3/2)*a^(3/2)*d^2)/((a*x+b)*x
)^(1/2)/b^2/a^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2/(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.36408, size = 359, normalized size = 4.92

$$\left[-\frac{\left(b^2c^2-4\,abcd\right)\sqrt{a}\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)-2\left(abc^2x-2\,a^2d^2\right)\sqrt{\frac{ax+b}{x}}}{2\,a^2b},\ \frac{\left(b^2c^2-4\,abcd\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right.}{a^2b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2/(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2*c^2 - 4*a*b*c*d)*sqrt(a)*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)
/x) + b) - 2*(a*b*c^2*x - 2*a^2*d^2)*sqrt((a*x + b)/x))/(a^2*b), ((b^2*c^2
- 4*a*b*c*d)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (a*b*c^2*x - 2
*a^2*d^2)*sqrt((a*x + b)/x))/(a^2*b)]
```

Sympy [A] time = 25.404, size = 114, normalized size = 1.56

$$d^2\left(\begin{cases}-\frac{1}{\sqrt{a}x} & \text{for}\ b=0\\ -\frac{2\sqrt{a+\frac{b}{x}}}{b} & \text{otherwise}\end{cases}\right)+\frac{\sqrt{b}c^2\sqrt{x}\sqrt{\frac{ax}{b}+1}}{a}-\frac{4cd\,\mathrm{atan}\left(\frac{1}{\sqrt{-\frac{1}{a}}\sqrt{a+\frac{b}{x}}}\right)}{a\sqrt{-\frac{1}{a}}}-\frac{bc^2\,\mathrm{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)**2/(a+b/x)**(1/2),x)
```

```
[Out] d**2*Piecewise((-1/(sqrt(a)*x), Eq(b, 0)), (-2*sqrt(a + b/x)/b, True)) + sq
rt(b)*c**2*sqrt(x)*sqrt(a*x/b + 1)/a - 4*c*d*atan(1/(sqrt(-1/a)*sqrt(a + b/
x)))/(a*sqrt(-1/a)) - b*c**2*asinh(sqrt(a)*sqrt(x)/sqrt(b))/a**(3/2)
```

Giac [A] time = 1.20205, size = 131, normalized size = 1.79

$$-b\left(\frac{c^2\sqrt{\frac{ax+b}{x}}}{\left(a-\frac{ax+b}{x}\right)a}+\frac{2\,d^2\sqrt{\frac{ax+b}{x}}}{b^2}-\frac{\left(bc^2-4\,acd\right)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}ab}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] -b*(c^2*sqrt((a*x + b)/x)/((a - (a*x + b)/x)*a) + 2*d^2*sqrt((a*x + b)/x)/b
^2 - (b*c^2 - 4*a*c*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a*b))
```

3.247 $\int \frac{c+\frac{d}{x}}{\sqrt{a+\frac{b}{x}}} \, dx$

Optimal. Leaf size=51

$$\frac{cx\sqrt{a+\frac{b}{x}}}{a}-\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] (c*Sqrt[a + b/x]*x)/a - ((b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3
/2)
```

Rubi [A] time = 0.0331384, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {375, 78, 63, 208}

$$\frac{cx\sqrt{a+\frac{b}{x}}}{a}-\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d/x)/Sqrt[a + b/x],x]
```

```
[Out] (c*Sqrt[a + b/x]*x)/a - ((b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3
/2)
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{c+\frac{d}{x}}{\sqrt{a+\frac{b}{x}}}\,dx = -\operatorname{Subst}\left(\int \frac{c+dx}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{c\sqrt{a+\frac{b}{x}}x}{a} - \frac{\left(-\frac{bc}{2}+ad\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{c\sqrt{a+\frac{b}{x}}x}{a} - \frac{\left(2\left(-\frac{bc}{2}+ad\right)\right)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{ab}$$

$$= \frac{c\sqrt{a+\frac{b}{x}}x}{a} - \frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

Mathematica [A] time = 0.0339721, size = 53, normalized size = 1.04

$$\frac{2\left(ad-\frac{bc}{2}\right)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}} + \frac{cx\sqrt{a+\frac{b}{x}}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d/x)/Sqrt[a + b/x],x]
```

```
[Out] (c*Sqrt[a + b/x]*x)/a + (2*(-(b*c)/2 + a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])
/a^(3/2)
```

Maple [B] time = 0.01, size = 173, normalized size = 3.4

$$\frac{x}{2b}\sqrt{\frac{ax+b}{x}}\left(2\sqrt{ax^2+bx}a^{3/2}d+\ln\left(\frac{1}{2}\left(2\sqrt{ax^2+bx}\sqrt{a}+2ax+b\right)\frac{1}{\sqrt{a}}\right)abd-2a^{3/2}\sqrt{(ax+b)x}d+2\sqrt{a}\sqrt{(ax+b)x}bc+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)/(a+b/x)^(1/2),x)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x*(2*(a*x^2+b*x)^(1/2)*a^(3/2)*d+ln(1/2*(2*(a*x^2+b*x
)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a*b*d-2*a^(3/2)*((a*x+b)*x)^(1/2)*d+2*a^(
1/2)*((a*x+b)*x)^(1/2)*b*c+ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(
1/2))*a*b*d-ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*b^2*c)/((
a*x+b)*x)^(1/2)/b/a^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.29481, size = 277, normalized size = 5.43

$$\left[\frac{2\,acx\sqrt{\frac{ax+b}{x}}-(bc-2\,ad)\sqrt{a}\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)}{2\,a^{2}},\ \frac{acx\sqrt{\frac{ax+b}{x}}+(bc-2\,ad)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)}{a^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*c*x*sqrt((a*x + b)/x) - (b*c - 2*a*d)*sqrt(a)*log(2*a*x + 2*sqrt(
a)*x*sqrt((a*x + b)/x) + b))/a^2, (a*c*x*sqrt((a*x + b)/x) + (b*c - 2*a*d)*
sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a))/a^2]
```

Sympy [A] time = 24.4353, size = 82, normalized size = 1.61

$$\frac{\sqrt{b}c\sqrt{x}\sqrt{\frac{ax}{b}+1}}{a}-\frac{2d\,\mathrm{atan}\left(\frac{1}{\sqrt{-\frac{1}{a}}\sqrt{a+\frac{b}{x}}}\right)}{a\sqrt{-\frac{1}{a}}}-\frac{bc\,\mathrm{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)**(1/2),x)
```

```
[Out] sqrt(b)*c*sqrt(x)*sqrt(a*x/b + 1)/a - 2*d*atan(1/(sqrt(-1/a)*sqrt(a + b/x))
)/(a*sqrt(-1/a)) - b*c*asinh(sqrt(a)*sqrt(x)/sqrt(b))/a**(3/2)
```

Giac [A] time = 1.18868, size = 99, normalized size = 1.94

$$-b\left(\frac{c\sqrt{\frac{ax+b}{x}}}{\left(a-\frac{ax+b}{x}\right)a}-\frac{(bc-2\,ad)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}ab}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] -b*(c*sqrt((a*x + b)/x)/((a - (a*x + b)/x)*a) - (b*c - 2*a*d)*arctan(sqrt((
a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a*b))
```

3.248 $\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx$

Optimal. Leaf size=43

$$\frac{x\sqrt{a+\frac{b}{x}}}{a}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] (Sqrt[a + b/x]*x)/a - (b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)
```

Rubi [A] time = 0.0196423, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {242, 51, 63, 208}

$$\frac{x\sqrt{a+\frac{b}{x}}}{a}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b/x],x]
```

```
[Out] (Sqrt[a + b/x]*x)/a - (b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}}\,dx = -\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{\sqrt{a+\frac{b}{x}}x}{a} + \frac{b\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a}$$
$$= \frac{\sqrt{a+\frac{b}{x}}x}{a} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{a}$$
$$= \frac{\sqrt{a+\frac{b}{x}}x}{a} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

Mathematica [A] time = 0.0181801, size = 43, normalized size = 1.

$$\frac{x\sqrt{a+\frac{b}{x}}}{a} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b/x],x]
```

```
[Out] (Sqrt[a + b/x]*x)/a - (b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)
```

Maple [A] time = 0.006, size = 70, normalized size = 1.6

$$-\frac{x}{2}\sqrt{\frac{ax+b}{x}}\left(b\ln\left(\frac{1}{2}\left(2\sqrt{(ax+b)x}\sqrt{a}+2ax+b\right)\frac{1}{\sqrt{a}}\right)-2\sqrt{(ax+b)x}\sqrt{a}\right)\frac{1}{\sqrt{(ax+b)x}}a^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(1/2),x)
```

```
[Out] -1/2*((a*x+b)/x)^(1/2)*x*(b*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^
(1/2))-2*((a*x+b)*x)^(1/2)*a^(1/2))/((a*x+b)*x)^(1/2)/a^(3/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.23712, size = 239, normalized size = 5.56

$$\left[\frac{2\,ax\sqrt{\frac{ax+b}{x}}+\sqrt{a}b\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)}{2\,a^2},\,\frac{ax\sqrt{\frac{ax+b}{x}}+\sqrt{-a}b\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)}{a^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*x*sqrt((a*x + b)/x) + sqrt(a)*b*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x
 + b)/x) + b))/a^2, (a*x*sqrt((a*x + b)/x) + sqrt(-a)*b*arctan(sqrt(-a)*sqr
t((a*x + b)/x)/a))/a^2]
```

Sympy [A] time = 2.14315, size = 44, normalized size = 1.02

$$\frac{\sqrt{b}\sqrt{x}\sqrt{\frac{ax}{b}+1}}{a}-\frac{b\,\mathrm{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(1/2),x)
```

```
[Out] sqrt(b)*sqrt(x)*sqrt(a*x/b + 1)/a - b*asinh(sqrt(a)*sqrt(x)/sqrt(b))/a**(3/
2)
```

Giac [B] time = 1.15544, size = 96, normalized size = 2.23

$$-\frac{b\log\left(|b|\right)\mathrm{sgn}\left(x\right)}{2\,a^{\frac{3}{2}}}+\frac{b\log\left(\left|-2\left(\sqrt{a}x-\sqrt{ax^2+bx}\right)\sqrt{a}-b\right|\right)}{2\,a^{\frac{3}{2}}\mathrm{sgn}\left(x\right)}+\frac{\sqrt{ax^2+bx}}{a\mathrm{sgn}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*b*log(abs(b))*sgn(x)/a^(3/2) + 1/2*b*log(abs(-2*(sqrt(a)*x - sqrt(a*x^
2 + b*x))*sqrt(a) - b))/(a^(3/2)*sgn(x)) + sqrt(a*x^2 + b*x)/(a*sgn(x))
```

3.249 $\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}\,dx$

Optimal. Leaf size=108

$$-\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^2}-\frac{2d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2\sqrt{bc-ad}}+\frac{x\sqrt{a+\frac{b}{x}}}{ac}$$

```
[Out] (Sqrt[a + b/x]*x)/(a*c) - (2*d^(3/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*
c - a*d]])/(c^2*Sqrt[b*c - a*d]) - ((b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqr
t[a]])/(a^(3/2)*c^2)
```

Rubi [A] time = 0.0963475, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {375, 103, 156, 63, 208, 205}

$$-\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^2}-\frac{2d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2\sqrt{bc-ad}}+\frac{x\sqrt{a+\frac{b}{x}}}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x]*(c + d/x)),x]
```

```
[Out] (Sqrt[a + b/x]*x)/(a*c) - (2*d^(3/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*
c - a*d]])/(c^2*Sqrt[b*c - a*d]) - ((b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqr
t[a]])/(a^(3/2)*c^2)
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}\,dx &= -\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}(c+dx)}\,dx, x, \frac{1}{x}\right)\\
&= \frac{\sqrt{a+\frac{b}{x}}x}{ac} + \frac{\operatorname{Subst}\left(\int \frac{\frac{1}{2}(bc+2ad)+\frac{bdx}{2}}{x\sqrt{a+bx}(c+dx)}\,dx, x, \frac{1}{x}\right)}{ac}\\
&= \frac{\sqrt{a+\frac{b}{x}}x}{ac} - \frac{d^2\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx}(c+dx)}\,dx, x, \frac{1}{x}\right)}{c^2} + \frac{(bc+2ad)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2ac^2}\\
&= \frac{\sqrt{a+\frac{b}{x}}x}{ac} - \frac{\left(2d^2\right)\operatorname{Subst}\left(\int \frac{1}{c-\frac{ad}{b}+\frac{dx^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^2} + \frac{(bc+2ad)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{abc^2}\\
&= \frac{\sqrt{a+\frac{b}{x}}x}{ac} - \frac{2d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2\sqrt{bc-ad}} - \frac{(bc+2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^2}
\end{aligned}$$

Mathematica [A] time = 0.176637, size = 104, normalized size = 0.96

$$\frac{-\frac{(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{2d^{3/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}} + \frac{cx\sqrt{a+\frac{b}{x}}}{a}}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x]*(c + d/x)),x]
```

```
[Out] ((c*Sqrt[a + b/x]*x)/a - (2*d^(3/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c
 - a*d]])/Sqrt[b*c - a*d] - ((b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/
a^(3/2))/c^2
```

Maple [B] time = 0.011, size = 228, normalized size = 2.1

$$-\frac{x}{2c^3}\left(2\ln\left(1/2\,\frac{2\sqrt{(ax+b)x}\sqrt{a}+2ax+b}{\sqrt{a}}\right)\sqrt{\frac{(ad-bc)d}{c^2}}acd+\ln\left(\frac{1}{2}\left(2\sqrt{(ax+b)x}\sqrt{a}+2ax+b\right)\frac{1}{\sqrt{a}}\right)\sqrt{\frac{(ad-bc)d}{c^2}}b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d/x)/(a+b/x)^(1/2),x)
```

```
[Out] -1/2*(2*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/
c^2)^(1/2)*a*c*d+ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*
d-b*c)*d/c^2)^(1/2)*b*c^2-2*((a*x+b)*x)^(1/2)*c^2*a^(1/2)*((a*d-b*c)*d/c^2)
^(1/2)+2*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*
d)/(c*x+d))*a^(3/2)*d^2)*x*((a*x+b)/x)^(1/2)/((a*d-b*c)*d/c^2)^(1/2)/c^3/a^
(3/2)/((a*x+b)*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)/(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a + b/x)*(c + d/x)), x)
```

Fricas [A] time = 1.51129, size = 1215, normalized size = 11.25

$$\left[\frac{2\,a^2d\sqrt{-\frac{d}{bc-ad}}\log\left(-\frac{2\,(bc-ad)x\sqrt{-\frac{d}{bc-ad}}\sqrt{\frac{ax+b}{x}}-bd+(bc-2\,ad)x}{cx+d}\right)+2\,acx\sqrt{\frac{ax+b}{x}}+(bc+2\,ad)\sqrt{a}\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)}{2\,a^2c^2},\ \ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)/(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a^2*d*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d
))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*a*c*x*sqrt((a*
x + b)/x) + (b*c + 2*a*d)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x)
 + b))/(a^2*c^2), (a^2*d*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d
/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + a*c*x
*sqrt((a*x + b)/x) + (b*c + 2*a*d)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/
x)/a))/(a^2*c^2), -1/2*(4*a^2*d*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*s
qrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) - 2*a*c*x*sqrt((a*x + b
)/x) - (b*c + 2*a*d)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b)
)/(a^2*c^2), -(2*a^2*d*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*
c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) - a*c*x*sqrt((a*x + b)/x) - (b*c
 + 2*a*d)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a))/(a^2*c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a+\frac{b}{x}}\,(cx+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)/(a+b/x)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(a + b/x)*(c*x + d)), x)
```

Giac [A] time = 1.19339, size = 174, normalized size = 1.61

$$-b\left(\frac{2\,d^2\arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^2}}\right)}{\sqrt{bcd-ad^2}bc^2}+\frac{\sqrt{\frac{ax+b}{x}}}{\left(a-\frac{ax+b}{x}\right)ac}-\frac{(bc+2\,ad)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}abc^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] -b*(2*d^2*arctan(d*sqrt((a*x + b)/x)/sqrt(b*c*d - a*d^2))/(sqrt(b*c*d - a*d
^2)*b*c^2) + sqrt((a*x + b)/x)/((a - (a*x + b)/x)*a*c) - (b*c + 2*a*d)*arct
an(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a*b*c^2))
```

3.250 $\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}\,dx$

Optimal. Leaf size=172

$$-\frac{(4ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^3}-\frac{d^{3/2}(5bc-4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{3/2}}+\frac{d\sqrt{a+\frac{b}{x}}(bc-2ad)}{ac^2\left(c+\frac{d}{x}\right)(bc-ad)}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)}$$

```
[Out] (d*(b*c - 2*a*d)*Sqrt[a + b/x])/(a*c^2*(b*c - a*d)*(c + d/x)) + (Sqrt[a + b
/x]*x)/(a*c*(c + d/x)) - (d^(3/2)*(5*b*c - 4*a*d)*ArcTan[(Sqrt[d]*Sqrt[a +
b/x])/Sqrt[b*c - a*d]])/(c^3*(b*c - a*d)^(3/2)) - ((b*c + 4*a*d)*ArcTanh[Sq
rt[a + b/x]/Sqrt[a]])/(a^(3/2)*c^3)
```

Rubi [A] time = 0.218058, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {375, 103, 151, 156, 63, 208, 205}

$$-\frac{(4ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^3}-\frac{d^{3/2}(5bc-4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{3/2}}+\frac{d\sqrt{a+\frac{b}{x}}(bc-2ad)}{ac^2\left(c+\frac{d}{x}\right)(bc-ad)}+\frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x]*(c + d/x)^2),x]
```

```
[Out] (d*(b*c - 2*a*d)*Sqrt[a + b/x])/(a*c^2*(b*c - a*d)*(c + d/x)) + (Sqrt[a + b
/x]*x)/(a*c*(c + d/x)) - (d^(3/2)*(5*b*c - 4*a*d)*ArcTan[(Sqrt[d]*Sqrt[a +
b/x])/Sqrt[b*c - a*d]])/(c^3*(b*c - a*d)^(3/2)) - ((b*c + 4*a*d)*ArcTanh[Sq
rt[a + b/x]/Sqrt[a]])/(a^(3/2)*c^3)
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
```

```
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}\,dx = -\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}(c+dx)^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt{a+\frac{b}{x}}x}{ac\left(c+\frac{d}{x}\right)} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc+4ad)+\frac{3bdx}{2}}{x\sqrt{a+bx}(c+dx)^2}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{d(bc-2ad)\sqrt{a+\frac{b}{x}}}{ac^2(bc-ad)\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}x}{ac\left(c+\frac{d}{x}\right)} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}(bc-ad)(bc+4ad)-\frac{1}{2}bd(bc-2ad)x}{x\sqrt{a+bx}(c+dx)}\,dx, x, \frac{1}{x}\right)}{ac^2(bc-ad)}$$

$$= \frac{d(bc-2ad)\sqrt{a+\frac{b}{x}}}{ac^2(bc-ad)\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}x}{ac\left(c+\frac{d}{x}\right)} - \frac{\left(d^2(5bc-4ad)\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}(c+dx)}\,dx, x, \frac{1}{x}\right)}{2c^3(bc-ad)} + (bc+4ad$$

$$= \frac{d(bc-2ad)\sqrt{a+\frac{b}{x}}}{ac^2(bc-ad)\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}x}{ac\left(c+\frac{d}{x}\right)} - \frac{\left(d^2(5bc-4ad)\right)\text{Subst}\left(\int \frac{1}{c-\frac{ad}{b}+\frac{dx^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^3(bc-ad)} + (bc+$$

$$= \frac{d(bc-2ad)\sqrt{a+\frac{b}{x}}}{ac^2(bc-ad)\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}x}{ac\left(c+\frac{d}{x}\right)} - \frac{d^{3/2}(5bc-4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{3/2}} - \frac{(bc+4ad)\tanh^{-1}\left(\frac{\sqrt{a+}}{\sqrt{a}}\right.}{a^{3/2}c^3}$$

Mathematica [A] time = 0.609596, size = 150, normalized size = 0.87

$$\frac{\frac{ad^{3/2}(4ad-5bc)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{3/2}}+\frac{cx\sqrt{a+\frac{b}{x}}(bc(cx+d)-ad(cx+2d))}{(cx+d)(bc-ad)}-\frac{(4ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}}{ac^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x]*(c + d/x)^2),x]
```

```
[Out] ((c*Sqrt[a + b/x]*x*(b*c*(d + c*x) - a*d*(2*d + c*x)))/((b*c - a*d)*(d + c*
x)) + (a*d^(3/2)*(-5*b*c + 4*a*d)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c -
 a*d]])/(b*c - a*d)^(3/2) - ((b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/
Sqrt[a])/(a*c^3)
```

Maple [B] time = 0.017, size = 1135, normalized size = 6.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d/x)^2/(a+b/x)^(1/2),x)
```

```
[Out] -1/2*((a*x+b)/x)^(1/2)*x*(4*a^(9/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*
x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x*c*d^4+2*a^(7/2)*((a*d-b*c)*d/c^2)^
(1/2)*((a*x+b)*x)^(1/2)*x^2*c^4*d+4*a^(9/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*(
(a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*d^5-2*a^(7/2)*((a*d-b*c)*d/c
^2)^(1/2)*((a*x+b)*x)^(1/2)*x*c^3*d^2-9*a^(7/2)*ln((2*((a*d-b*c)*d/c^2)^(1/
2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x*b*c^2*d^3-4*a^(7/2)*((
a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c^2*d^3-9*a^(7/2)*ln((2*((a*d-b*c)*
d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b*c*d^4-2*a^(5
/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(3/2)*c^4*d+6*a^(5/2)*((a*d-b*c)*d/
c^2)^(1/2)*((a*x+b)*x)^(1/2)*x*b*c^4*d+5*a^(5/2)*ln((2*((a*d-b*c)*d/c^2)^(1
/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x*b^2*c^3*d^2+6*a^(5/2)
*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*b*c^3*d^2+5*a^(5/2)*ln((2*((a*d-
b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^2*c^2*d
^3-2*a^(3/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*x*b^2*c^5+4*ln(1/2*(
2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^4*((a*d-b*c)*d/c^2)^(1/2)*x
*c^2*d^3-7*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*((a*d-
b*c)*d/c^2)^(1/2)*x*b*c^3*d^2+2*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b
)/a^(1/2))*a^2*((a*d-b*c)*d/c^2)^(1/2)*x*b^2*c^4*d+ln(1/2*(2*((a*x+b)*x)^(1
/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a*((a*d-b*c)*d/c^2)^(1/2)*x*b^3*c^5-2*a^(3/2)
*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*b^2*c^4*d+4*ln(1/2*(2*((a*x+b)*x
)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^4*((a*d-b*c)*d/c^2)^(1/2)*c*d^4-7*ln(1/
2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*((a*d-b*c)*d/c^2)^(1/2
)*b*c^2*d^3+2*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*((a
*d-b*c)*d/c^2)^(1/2)*b^2*c^3*d^2+ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+
b)/a^(1/2))*a*((a*d-b*c)*d/c^2)^(1/2)*b^3*c^4*d)/c^4/((a*x+b)*x)^(1/2)/(a*d
-b*c)^2/(c*x+d)/a^(5/2)/((a*d-b*c)*d/c^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)^2/(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a + b/x)*(c + d/x)^2), x)
```

Fricas [A] time = 1.89441, size = 2408, normalized size = 14.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)^2/(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a^2
*c*d^2)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (5*a^2*
b*c*d^2 - 4*a^3*d^3 + (5*a^2*b*c^2*d - 4*a^3*c*d^2)*x)*sqrt(-d/(b*c - a*d))
*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c
- 2*a*d)*x)/(c*x + d)) + 2*((a*b*c^3 - a^2*c^2*d)*x^2 + (a*b*c^2*d - 2*a^2*
c*d^2)*x)*sqrt((a*x + b)/x))/(a^2*b*c^4*d - a^3*c^3*d^2 + (a^2*b*c^5 - a^3*
c^4*d)*x), -1/2*(2*(5*a^2*b*c*d^2 - 4*a^3*d^3 + (5*a^2*b*c^2*d - 4*a^3*c*d^
2)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a
*x + b)/x)/(a*d*x + b*d)) - (b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3
 + 3*a*b*c^2*d - 4*a^2*c*d^2)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x
+ b)/x) + b) - 2*((a*b*c^3 - a^2*c^2*d)*x^2 + (a*b*c^2*d - 2*a^2*c*d^2)*x)*
sqrt((a*x + b)/x))/(a^2*b*c^4*d - a^3*c^3*d^2 + (a^2*b*c^5 - a^3*c^4*d)*x),
 1/2*(2*(b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a
^2*c*d^2)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (5*a^2*b*c*d^2
 - 4*a^3*d^3 + (5*a^2*b*c^2*d - 4*a^3*c*d^2)*x)*sqrt(-d/(b*c - a*d))*log(-(
2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d
)*x)/(c*x + d)) + 2*((a*b*c^3 - a^2*c^2*d)*x^2 + (a*b*c^2*d - 2*a^2*c*d^2)*
x)*sqrt((a*x + b)/x))/(a^2*b*c^4*d - a^3*c^3*d^2 + (a^2*b*c^5 - a^3*c^4*d)*
x), -((5*a^2*b*c*d^2 - 4*a^3*d^3 + (5*a^2*b*c^2*d - 4*a^3*c*d^2)*x)*sqrt(d/
(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a
*d*x + b*d)) - (b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*
d - 4*a^2*c*d^2)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - ((a*b*c
^3 - a^2*c^2*d)*x^2 + (a*b*c^2*d - 2*a^2*c*d^2)*x)*sqrt((a*x + b)/x))/(a^2*
b*c^4*d - a^3*c^3*d^2 + (a^2*b*c^5 - a^3*c^4*d)*x)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)**2/(a+b/x)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [A] time = 1.21074, size = 393, normalized size = 2.28

$$-b\left(\frac{\left(5\,bcd^2-4\,ad^3\right)\arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^2}}\right)}{\left(b^2c^4-abc^3d\right)\sqrt{bcd-ad^2}}+\frac{b^2c^2\sqrt{\frac{ax+b}{x}}-2\,abcd\sqrt{\frac{ax+b}{x}}+2\,a^2d^2\sqrt{\frac{ax+b}{x}}+\frac{(ax+b)bcd\sqrt{\frac{ax+b}{x}}}{x}-\frac{2\,(ax+b)ad^2\sqrt{\frac{ax+b}{x}}}{x}}{\left(abc^3-a^2c^2d\right)\left(abc-a^2d-\frac{(ax+b)bc}{x}+\frac{2\,(ax+b)ad}{x}-\frac{(ax+b)^2d}{x^2}\right)}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)^2/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] -b*((5*b*c*d^2 - 4*a*d^3)*arctan(d*sqrt((a*x + b)/x)/sqrt(b*c*d - a*d^2))/(
(b^2*c^4 - a*b*c^3*d)*sqrt(b*c*d - a*d^2)) + (b^2*c^2*sqrt((a*x + b)/x) - 2
*a*b*c*d*sqrt((a*x + b)/x) + 2*a^2*d^2*sqrt((a*x + b)/x) + (a*x + b)*b*c*d*
sqrt((a*x + b)/x)/x - 2*(a*x + b)*a*d^2*sqrt((a*x + b)/x)/x)/((a*b*c^3 - a^
2*c^2*d)*(a*b*c - a^2*d - (a*x + b)*b*c/x + 2*(a*x + b)*a*d/x - (a*x + b)^2
*d/x^2)) - (b*c + 4*a*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a*b*c
^3))
```

3.251 $\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}\,dx$

Optimal. Leaf size=250

$$-\frac{d^{3/2}\left(24a^2d^2-56abcd+35b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{5/2}}-\frac{(6ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^4}+\frac{d\sqrt{a+\frac{b}{x}}(bc-4ad)(4bc-3ad)}{4ac^3\left(c+\frac{d}{x}\right)(bc-ad)^2}+\frac{d\sqrt{}}{2ac^2}$$

```
[Out] (d*(2*b*c - 3*a*d)*Sqrt[a + b/x])/(2*a*c^2*(b*c - a*d)*(c + d/x)^2) + (d*(b
*c - 4*a*d)*(4*b*c - 3*a*d)*Sqrt[a + b/x])/(4*a*c^3*(b*c - a*d)^2*(c + d/x)
) + (Sqrt[a + b/x]*x)/(a*c*(c + d/x)^2) - (d^(3/2)*(35*b^2*c^2 - 56*a*b*c*d
 + 24*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(4*c^4*(b*c
 - a*d)^(5/2)) - ((b*c + 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(a^(3/2)*c^
4)
```

Rubi [A] time = 0.400077, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {375, 103, 151, 156, 63, 208, 205}

$$-\frac{d^{3/2}\left(24a^2d^2-56abcd+35b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{5/2}}-\frac{(6ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^4}+\frac{d\sqrt{a+\frac{b}{x}}(bc-4ad)(4bc-3ad)}{4ac^3\left(c+\frac{d}{x}\right)(bc-ad)^2}+\frac{d\sqrt{}}{2ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b/x]*(c + d/x)^3),x]
```

```
[Out] (d*(2*b*c - 3*a*d)*Sqrt[a + b/x])/(2*a*c^2*(b*c - a*d)*(c + d/x)^2) + (d*(b
*c - 4*a*d)*(4*b*c - 3*a*d)*Sqrt[a + b/x])/(4*a*c^3*(b*c - a*d)^2*(c + d/x)
) + (Sqrt[a + b/x]*x)/(a*c*(c + d/x)^2) - (d^(3/2)*(35*b^2*c^2 - 56*a*b*c*d
 + 24*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(4*c^4*(b*c
 - a*d)^(5/2)) - ((b*c + 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(a^(3/2)*c^
4)
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}\,dx = -\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}(c+dx)^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt{a+\frac{b}{x}}\,x}{ac\left(c+\frac{d}{x}\right)^2} + \frac{\operatorname{Subst}\left(\int \frac{\frac{1}{2}(bc+6ad)+\frac{5bdx}{2}}{x\sqrt{a+bx}(c+dx)^3}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{d(2bc-3ad)\sqrt{a+\frac{b}{x}}}{2ac^2(bc-ad)\left(c+\frac{d}{x}\right)^2} + \frac{\sqrt{a+\frac{b}{x}}\,x}{ac\left(c+\frac{d}{x}\right)^2} - \frac{\operatorname{Subst}\left(\int \frac{-(bc-ad)(bc+6ad)-\frac{3}{2}bd(2bc-3ad)x}{x\sqrt{a+bx}(c+dx)^2}\,dx, x, \frac{1}{x}\right)}{2ac^2(bc-ad)}$$

$$= \frac{d(2bc-3ad)\sqrt{a+\frac{b}{x}}}{2ac^2(bc-ad)\left(c+\frac{d}{x}\right)^2} + \frac{d(bc-4ad)(4bc-3ad)\sqrt{a+\frac{b}{x}}}{4ac^3(bc-ad)^2\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{ac\left(c+\frac{d}{x}\right)^2} + \frac{\operatorname{Subst}\left(\int \frac{(bc-ad)^2(bc+6a\ldots}{\ldots}\right.}{2a\ldots}$$

$$= \frac{d(2bc-3ad)\sqrt{a+\frac{b}{x}}}{2ac^2(bc-ad)\left(c+\frac{d}{x}\right)^2} + \frac{d(bc-4ad)(4bc-3ad)\sqrt{a+\frac{b}{x}}}{4ac^3(bc-ad)^2\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{ac\left(c+\frac{d}{x}\right)^2} + \frac{(bc+6ad)\operatorname{Subst}\left(\int \ldots\right.}{2ac^4}$$

$$= \frac{d(2bc-3ad)\sqrt{a+\frac{b}{x}}}{2ac^2(bc-ad)\left(c+\frac{d}{x}\right)^2} + \frac{d(bc-4ad)(4bc-3ad)\sqrt{a+\frac{b}{x}}}{4ac^3(bc-ad)^2\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{ac\left(c+\frac{d}{x}\right)^2} + \frac{(bc+6ad)\operatorname{Subst}\left(\int \ldots\right.}{ab\ldots}$$

$$= \frac{d(2bc-3ad)\sqrt{a+\frac{b}{x}}}{2ac^2(bc-ad)\left(c+\frac{d}{x}\right)^2} + \frac{d(bc-4ad)(4bc-3ad)\sqrt{a+\frac{b}{x}}}{4ac^3(bc-ad)^2\left(c+\frac{d}{x}\right)} + \frac{\sqrt{a+\frac{b}{x}}\,x}{ac\left(c+\frac{d}{x}\right)^2} - \frac{d^{3/2}\left(35b^2c^2-56abcd\ldots\right.}{4c^4(\ldots}$$

Mathematica [A] time = 1.40395, size = 216, normalized size = 0.86

$$\frac{\frac{cx\sqrt{a+\frac{b}{x}}\left(2a^2d^2\left(2c^2x^2+9cdx+6d^2\right)-abcd\left(8c^2x^2+29cdx+19d^2\right)+4b^2c^2(cx+d)^2\right)}{(cx+d)^2(bc-ad)^2} - \frac{ad^{3/2}\left(24a^2d^2-56abcd+35b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{(bc-ad)^{5/2}} - \frac{4(6ad+bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}}{4ac^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x]*(c + d/x)^3),x]
```

```
[Out] ((c*Sqrt[a + b/x]*x*(4*b^2*c^2*(d + c*x)^2 + 2*a^2*d^2*(6*d^2 + 9*c*d*x + 2
*c^2*x^2) - a*b*c*d*(19*d^2 + 29*c*d*x + 8*c^2*x^2)))/((b*c - a*d)^2*(d + c
*x)^2) - (a*d^(3/2)*(35*b^2*c^2 - 56*a*b*c*d + 24*a^2*d^2)*ArcTan[(Sqrt[d]*
Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(b*c - a*d)^(5/2) - (4*(b*c + 6*a*d)*ArcTa
nh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a])/(4*a*c^4)
```

Maple [B] time = 0.014, size = 2269, normalized size = 9.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d/x)^3/(a+b/x)^(1/2),x)

[Out] -1/8*((a*x+b)/x)^(1/2)*x*(-70*a^(5/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b
)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x*b^3*c^4*d^3+60*ln(1/2*(2*((a*x+b
)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*((a*d-b*c)*d/c^2)^(1/2)*x^2*b^2*c^
5*d^2-12*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*((a*d-b*
c)*d/c^2)^(1/2)*x^2*b^3*c^6*d+22*a^(5/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x
)^(3/2)*x*b*c^6*d+18*a^(7/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*x^2*
b*c^5*d^2-8*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a*((a*d-b
*c)*d/c^2)^(1/2)*x*b^4*c^6*d+16*a^(3/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)
^(1/2)*x*b^3*c^6*d+24*a^(11/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1
/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*d^7-35*a^(5/2)*ln((2*((a*d-b*c)*d/c^2)^(1
/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x^2*b^3*c^5*d^2+48*ln(1
/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^5*((a*d-b*c)*d/c^2)^(1/
2)*x*c^2*d^5+18*a^(5/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(3/2)*b*c^5*d^2
-36*a^(9/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*x*c^3*d^4-160*a^(9/2)
*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+
d))*x*b*c^2*d^5+182*a^(7/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)
*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x*b^2*c^3*d^4+8*a^(3/2)*((a*d-b*c)*d/c^2)^(1
/2)*((a*x+b)*x)^(1/2)*x^2*b^3*c^7-4*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a
*x+b)/a^(1/2))*a*((a*d-b*c)*d/c^2)^(1/2)*b^4*c^5*d^2+8*a^(3/2)*((a*d-b*c)*d
/c^2)^(1/2)*((a*x+b)*x)^(1/2)*b^3*c^5*d^2-4*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(
1/2)+2*a*x+b)/a^(1/2))*a*((a*d-b*c)*d/c^2)^(1/2)*x^2*b^4*c^7+24*a^(11/2)*ln
((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))
*x^2*c^2*d^5-8*a^(7/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(3/2)*c^4*d^3+48
*a^(11/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b
*d)/(c*x+d))*x*c*d^6+24*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2
))*a^5*((a*d-b*c)*d/c^2)^(1/2)*c*d^6-24*a^(9/2)*((a*d-b*c)*d/c^2)^(1/2)*((a
*x+b)*x)^(1/2)*c^2*d^5-80*a^(9/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)
^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b*c*d^6+91*a^(7/2)*ln((2*((a*d-b*c)*d/
c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^2*c^2*d^5-35*a
^(5/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)
/(c*x+d))*b^3*c^3*d^4+120*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1
/2))*a^3*((a*d-b*c)*d/c^2)^(1/2)*x*b^2*c^4*d^3-24*ln(1/2*(2*((a*x+b)*x)^(1/
2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*((a*d-b*c)*d/c^2)^(1/2)*x*b^3*c^5*d^2+102*
a^(7/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*x*b*c^4*d^3-92*a^(5/2)*((
a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*x*b^2*c^5*d^2-46*a^(5/2)*((a*d-b*c)
*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*x^2*b^2*c^6*d-136*ln(1/2*(2*((a*x+b)*x)^(1/
2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^4*((a*d-b*c)*d/c^2)^(1/2)*x*b*c^3*d^4-22*a^(
7/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*x^3*b*c^6*d-68*ln(1/2*(2*((a
*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^4*((a*d-b*c)*d/c^2)^(1/2)*x^2*b*
c^4*d^3-68*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^4*((a*d-
b*c)*d/c^2)^(1/2)*b*c^2*d^5+60*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)
/a^(1/2))*a^3*((a*d-b*c)*d/c^2)^(1/2)*b^2*c^3*d^4-12*ln(1/2*(2*((a*x+b)*x)^
(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*((a*d-b*c)*d/c^2)^(1/2)*b^3*c^4*d^3+62*
a^(7/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*b*c^3*d^4-46*a^(5/2)*((a*
d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*b^2*c^4*d^3+12*a^(9/2)*((a*d-b*c)*d/c
^2)^(1/2)*((a*x+b)*x)^(1/2)*x^3*c^5*d^2+24*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1
/2)+2*a*x+b)/a^(1/2))*a^5*((a*d-b*c)*d/c^2)^(1/2)*x^2*c^3*d^4-12*a^(7/2)*((
a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(3/2)*x*c^5*d^2-80*a^(9/2)*ln((2*((a*d-b*
c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x^2*b*c^3*d
^4+91*a^(7/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c
*x-b*d)/(c*x+d))*x^2*b^2*c^4*d^3)/c^5/((a*x+b)*x)^(1/2)/(a*d-b*c)^3/(c*x+d)
^2/a^(5/2)/((a*d-b*c)*d/c^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)^3/(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a + b/x)*(c + d/x)^3), x)
```

Fricas [B] time = 3.7945, size = 4749, normalized size = 19.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)^3/(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*(b^3*c^3*d^2 + 4*a*b^2*c^2*d^3 - 11*a^2*b*c*d^4 + 6*a^3*d^5 + (b^3*
c^5 + 4*a*b^2*c^4*d - 11*a^2*b*c^3*d^2 + 6*a^3*c^2*d^3)*x^2 + 2*(b^3*c^4*d
+ 4*a*b^2*c^3*d^2 - 11*a^2*b*c^2*d^3 + 6*a^3*c*d^4)*x)*sqrt(a)*log(2*a*x -
2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (35*a^2*b^2*c^2*d^3 - 56*a^3*b*c*d^4 +
 24*a^4*d^5 + (35*a^2*b^2*c^4*d - 56*a^3*b*c^3*d^2 + 24*a^4*c^2*d^3)*x^2 +
2*(35*a^2*b^2*c^3*d^2 - 56*a^3*b*c^2*d^3 + 24*a^4*c*d^4)*x)*sqrt(-d/(b*c -
a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d +
(b*c - 2*a*d)*x)/(c*x + d)) + 2*(4*(a*b^2*c^5 - 2*a^2*b*c^4*d + a^3*c^3*d^2
)*x^3 + (8*a*b^2*c^4*d - 29*a^2*b*c^3*d^2 + 18*a^3*c^2*d^3)*x^2 + (4*a*b^2*
c^3*d^2 - 19*a^2*b*c^2*d^3 + 12*a^3*c*d^4)*x)*sqrt((a*x + b)/x))/(a^2*b^2*c
^6*d^2 - 2*a^3*b*c^5*d^3 + a^4*c^4*d^4 + (a^2*b^2*c^8 - 2*a^3*b*c^7*d + a^4
*c^6*d^2)*x^2 + 2*(a^2*b^2*c^7*d - 2*a^3*b*c^6*d^2 + a^4*c^5*d^3)*x), 1/8*(
8*(b^3*c^3*d^2 + 4*a*b^2*c^2*d^3 - 11*a^2*b*c*d^4 + 6*a^3*d^5 + (b^3*c^5 +
4*a*b^2*c^4*d - 11*a^2*b*c^3*d^2 + 6*a^3*c^2*d^3)*x^2 + 2*(b^3*c^4*d + 4*a*
b^2*c^3*d^2 - 11*a^2*b*c^2*d^3 + 6*a^3*c*d^4)*x)*sqrt(-a)*arctan(sqrt(-a)*s
qrt((a*x + b)/x)/a) + (35*a^2*b^2*c^2*d^3 - 56*a^3*b*c*d^4 + 24*a^4*d^5 + (
35*a^2*b^2*c^4*d - 56*a^3*b*c^3*d^2 + 24*a^4*c^2*d^3)*x^2 + 2*(35*a^2*b^2*c
^3*d^2 - 56*a^3*b*c^2*d^3 + 24*a^4*c*d^4)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(
b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x
)/(c*x + d)) + 2*(4*(a*b^2*c^5 - 2*a^2*b*c^4*d + a^3*c^3*d^2)*x^3 + (8*a*b^
2*c^4*d - 29*a^2*b*c^3*d^2 + 18*a^3*c^2*d^3)*x^2 + (4*a*b^2*c^3*d^2 - 19*a^
2*b*c^2*d^3 + 12*a^3*c*d^4)*x)*sqrt((a*x + b)/x))/(a^2*b^2*c^6*d^2 - 2*a^3*
b*c^5*d^3 + a^4*c^4*d^4 + (a^2*b^2*c^8 - 2*a^3*b*c^7*d + a^4*c^6*d^2)*x^2 +
 2*(a^2*b^2*c^7*d - 2*a^3*b*c^6*d^2 + a^4*c^5*d^3)*x), -1/4*((35*a^2*b^2*c^
2*d^3 - 56*a^3*b*c*d^4 + 24*a^4*d^5 + (35*a^2*b^2*c^4*d - 56*a^3*b*c^3*d^2
+ 24*a^4*c^2*d^3)*x^2 + 2*(35*a^2*b^2*c^3*d^2 - 56*a^3*b*c^2*d^3 + 24*a^4*c
*d^4)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt
((a*x + b)/x)/(a*d*x + b*d)) - 2*(b^3*c^3*d^2 + 4*a*b^2*c^2*d^3 - 11*a^2*b*
c*d^4 + 6*a^3*d^5 + (b^3*c^5 + 4*a*b^2*c^4*d - 11*a^2*b*c^3*d^2 + 6*a^3*c^2
*d^3)*x^2 + 2*(b^3*c^4*d + 4*a*b^2*c^3*d^2 - 11*a^2*b*c^2*d^3 + 6*a^3*c*d^4
)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - (4*(a*b^2*c^5
 - 2*a^2*b*c^4*d + a^3*c^3*d^2)*x^3 + (8*a*b^2*c^4*d - 29*a^2*b*c^3*d^2 + 1
8*a^3*c^2*d^3)*x^2 + (4*a*b^2*c^3*d^2 - 19*a^2*b*c^2*d^3 + 12*a^3*c*d^4)*x)
*sqrt((a*x + b)/x))/(a^2*b^2*c^6*d^2 - 2*a^3*b*c^5*d^3 + a^4*c^4*d^4 + (a^2
*b^2*c^8 - 2*a^3*b*c^7*d + a^4*c^6*d^2)*x^2 + 2*(a^2*b^2*c^7*d - 2*a^3*b*c^
6*d^2 + a^4*c^5*d^3)*x), -1/4*((35*a^2*b^2*c^2*d^3 - 56*a^3*b*c*d^4 + 24*a^
4*d^5 + (35*a^2*b^2*c^4*d - 56*a^3*b*c^3*d^2 + 24*a^4*c^2*d^3)*x^2 + 2*(35*
```

```
a^2*b^2*c^3*d^2 - 56*a^3*b*c^2*d^3 + 24*a^4*c*d^4)*x)*sqrt(d/(b*c - a*d))*a
rctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) -
 4*(b^3*c^3*d^2 + 4*a*b^2*c^2*d^3 - 11*a^2*b*c*d^4 + 6*a^3*d^5 + (b^3*c^5 +
 4*a*b^2*c^4*d - 11*a^2*b*c^3*d^2 + 6*a^3*c^2*d^3)*x^2 + 2*(b^3*c^4*d + 4*a
*b^2*c^3*d^2 - 11*a^2*b*c^2*d^3 + 6*a^3*c*d^4)*x)*sqrt(-a)*arctan(sqrt(-a)*
sqrt((a*x + b)/x)/a) - (4*(a*b^2*c^5 - 2*a^2*b*c^4*d + a^3*c^3*d^2)*x^3 + (
8*a*b^2*c^4*d - 29*a^2*b*c^3*d^2 + 18*a^3*c^2*d^3)*x^2 + (4*a*b^2*c^3*d^2 -
 19*a^2*b*c^2*d^3 + 12*a^3*c*d^4)*x)*sqrt((a*x + b)/x))/(a^2*b^2*c^6*d^2 -
2*a^3*b*c^5*d^3 + a^4*c^4*d^4 + (a^2*b^2*c^8 - 2*a^3*b*c^7*d + a^4*c^6*d^2)
*x^2 + 2*(a^2*b^2*c^7*d - 2*a^3*b*c^6*d^2 + a^4*c^5*d^3)*x)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)**3/(a+b/x)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [A] time = 1.22351, size = 459, normalized size = 1.84

$$-\frac{1}{4}\,b\left(\frac{\left(35\,b^2c^2d^2-56\,abcd^3+24\,a^2d^4\right)\arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^2}}\right)}{\left(b^3c^6-2\,ab^2c^5d+a^2bc^4d^2\right)\sqrt{bcd-ad^2}}+\frac{13\,b^2c^2d^2\sqrt{\frac{ax+b}{x}}-21\,abcd^3\sqrt{\frac{ax+b}{x}}+8\,a^2d^4\sqrt{\frac{ax+b}{x}}+\frac{11\,(ax+}{}}{\left(b^2c^5-2\,abc^4d+a^2c^3d^2\right)\left(bc-ad+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)^3/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*b*((35*b^2*c^2*d^2 - 56*a*b*c*d^3 + 24*a^2*d^4)*arctan(d*sqrt((a*x + b
)/x)/sqrt(b*c*d - a*d^2))/((b^3*c^6 - 2*a*b^2*c^5*d + a^2*b*c^4*d^2)*sqrt(b
*c*d - a*d^2)) + (13*b^2*c^2*d^2*sqrt((a*x + b)/x) - 21*a*b*c*d^3*sqrt((a*x
 + b)/x) + 8*a^2*d^4*sqrt((a*x + b)/x) + 11*(a*x + b)*b*c*d^3*sqrt((a*x + b
)/x)/x - 8*(a*x + b)*a*d^4*sqrt((a*x + b)/x)/x)/((b^2*c^5 - 2*a*b*c^4*d + a
^2*c^3*d^2)*(b*c - a*d + (a*x + b)*d/x)^2) + 4*sqrt((a*x + b)/x)/((a - (a*x
 + b)/x)*a*c^3) - 4*(b*c + 6*a*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(
-a)*a*b*c^4))
```

3.252 $\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{3/2}} dx$

Optimal. Leaf size=132

$$\frac{(bc-2ad)\left(2a^2d^2-2abcd+3b^2c^2\right)-\frac{abd^2(2ad+bc)}{x}}{a^2b^2\sqrt{a+\frac{b}{x}}}-\frac{3c^2(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{cx\left(c+\frac{d}{x}\right)^2}{a\sqrt{a+\frac{b}{x}}}$$

```
[Out] ((b*c - 2*a*d)*(3*b^2*c^2 - 2*a*b*c*d + 2*a^2*d^2) - (a*b*d^2*(b*c + 2*a*d)
)/x)/(a^2*b^2*Sqrt[a + b/x]) + (c*(c + d/x)^2*x)/(a*Sqrt[a + b/x]) - (3*c^2
*(b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(5/2)
```

Rubi [A] time = 0.10126, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {375, 98, 146, 63, 208}

$$\frac{(bc-2ad)\left(2a^2d^2-2abcd+3b^2c^2\right)-\frac{abd^2(2ad+bc)}{x}}{a^2b^2\sqrt{a+\frac{b}{x}}}-\frac{3c^2(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{cx\left(c+\frac{d}{x}\right)^2}{a\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d/x)^3/(a + b/x)^(3/2),x]
```

```
[Out] ((b*c - 2*a*d)*(3*b^2*c^2 - 2*a*b*c*d + 2*a^2*d^2) - (a*b*d^2*(b*c + 2*a*d)
)/x)/(a^2*b^2*Sqrt[a + b/x]) + (c*(c + d/x)^2*x)/(a*Sqrt[a + b/x]) - (3*c^2
*(b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(5/2)
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 146

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m
```

```
 + 1)*(m + n + 3)), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = -\operatorname{Subst}\left(\int \frac{(c+dx)^3}{x^2(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{c\left(c+\frac{d}{x}\right)^2 x}{a\sqrt{a+\frac{b}{x}}} + \frac{\operatorname{Subst}\left(\int \frac{(c+dx)\left(\frac{3}{2}c(bc-2ad)-\frac{1}{2}d(bc+2ad)x\right)}{x(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{(bc-2ad)\left(3b^2c^2-2abcd+2a^2d^2\right)-\frac{abd^2(bc+2ad)}{x}}{a^2b^2\sqrt{a+\frac{b}{x}}} + \frac{c\left(c+\frac{d}{x}\right)^2 x}{a\sqrt{a+\frac{b}{x}}} + \frac{\left(3c^2(bc-2ad)\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \ldots\right)}{2a^2}$$

$$= \frac{(bc-2ad)\left(3b^2c^2-2abcd+2a^2d^2\right)-\frac{abd^2(bc+2ad)}{x}}{a^2b^2\sqrt{a+\frac{b}{x}}} + \frac{c\left(c+\frac{d}{x}\right)^2 x}{a\sqrt{a+\frac{b}{x}}} + \frac{\left(3c^2(bc-2ad)\right)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\ldots\right)}{a^2b}$$

$$= \frac{(bc-2ad)\left(3b^2c^2-2abcd+2a^2d^2\right)-\frac{abd^2(bc+2ad)}{x}}{a^2b^2\sqrt{a+\frac{b}{x}}} + \frac{c\left(c+\frac{d}{x}\right)^2 x}{a\sqrt{a+\frac{b}{x}}} - \frac{3c^2(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

Mathematica [C] time = 0.0546606, size = 92, normalized size = 0.7

$$\frac{a\left(-4a^2d^3x-2abd^2(d-3cx)+b^2c^3x^2\right)+3b^2c^2x(bc-2ad)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b}{ax}+1\right)}{a^2b^2x\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d/x)^3/(a + b/x)^(3/2),x]
```

```
[Out] (a*(-4*a^2*d^3*x + b^2*c^3*x^2 - 2*a*b*d^2*(d - 3*c*x)) + 3*b^2*c^2*(b*c -
2*a*d)*x*Hypergeometric2F1[-1/2, 1, 1/2, 1 + b/(a*x)])/(a^2*b^2*Sqrt[a + b/
```

```
x]*x)
```

Maple [B] time = 0.016, size = 969, normalized size = 7.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)^3/(a+b/x)^(3/2),x)
```

```
[Out] -1/2*((a*x+b)/x)^(1/2)/x/a^(5/2)*(3*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a
*x+b)/a^(1/2))*x^4*a^4*b^2*c*d^2+3*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*
x+b)/a^(1/2))*x^2*b^6*c^3+4*(a*x^2+b*x)^(3/2)*a^(9/2)*x^2*d^3-4*a^(9/2)*((a
*x+b)*x)^(3/2)*x^2*d^3+4*(a*x^2+b*x)^(3/2)*a^(5/2)*b^2*d^3+4*a^(3/2)*((a*x+
b)*x)^(3/2)*x^2*b^3*c^3-12*a^(3/2)*((a*x+b)*x)^(1/2)*x^3*b^4*c^3+8*(a*x^2+b
*x)^(3/2)*a^(7/2)*x*b*d^3-6*a^(5/2)*((a*x+b)*x)^(1/2)*x^4*b^3*c^3+3*ln(1/2*
(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x^4*a^2*b^4*c^3+6*ln(1/2*(2*
((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x^3*a*b^5*c^3-6*a^(1/2)*((a*x+b
)*x)^(1/2)*x^2*b^5*c^3-3*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/
2))*x^2*a^2*b^4*c*d^2+3*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2
))*x^2*a^2*b^4*c*d^2-6*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2)
)*x^2*a*b^5*c^2*d-3*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x
^4*a^4*b^2*c*d^2-6*(a*x^2+b*x)^(1/2)*a^(9/2)*x^4*b*c*d^2-6*a^(9/2)*((a*x+b)
*x)^(1/2)*x^4*b*c*d^2+12*a^(7/2)*((a*x+b)*x)^(1/2)*x^4*b^2*c^2*d-12*(a*x^2+
b*x)^(1/2)*a^(7/2)*x^3*b^2*c*d^2+12*a^(7/2)*((a*x+b)*x)^(3/2)*x^2*b*c*d^2-1
2*a^(5/2)*((a*x+b)*x)^(3/2)*x^2*b^2*c^2*d-12*a^(7/2)*((a*x+b)*x)^(1/2)*x^3*
b^2*c*d^2+24*a^(5/2)*((a*x+b)*x)^(1/2)*x^3*b^3*c^2*d-6*(a*x^2+b*x)^(1/2)*a^
(5/2)*x^2*b^3*c*d^2-6*a^(5/2)*((a*x+b)*x)^(1/2)*x^2*b^3*c*d^2+12*a^(3/2)*((
a*x+b)*x)^(1/2)*x^2*b^4*c^2*d-6*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b
)/a^(1/2))*x^4*a^3*b^3*c^2*d-6*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)
/a^(1/2))*x^3*a^3*b^3*c*d^2+6*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/
a^(1/2))*x^3*a^3*b^3*c*d^2-12*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/
a^(1/2))*x^3*a^2*b^4*c^2*d)/((a*x+b)*x)^(1/2)/b^3/(a*x+b)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^3/(a+b/x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.32398, size = 699, normalized size = 5.3

$$\left[-\frac{3\left(b^4c^3 - 2\,ab^3c^2d + \left(ab^3c^3 - 2\,a^2b^2c^2d\right)x\right)\sqrt{a}\log\left(2\,ax + 2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}} + b\right) - 2\left(a^2b^2c^3x^2 - 2\,a^3bd^3 + \left(3\,ab^3c^3 - 6\,a^2b^2\right.\right.}{2\left(a^4b^2x + a^3b^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^3/(a+b/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(3*(b^4*c^3 - 2*a*b^3*c^2*d + (a*b^3*c^3 - 2*a^2*b^2*c^2*d)*x)*sqrt(a
)*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 2*(a^2*b^2*c^3*x^2 - 2*a
^3*b*d^3 + (3*a*b^3*c^3 - 6*a^2*b^2*c^2*d + 6*a^3*b*c*d^2 - 4*a^4*d^3)*x)*s
qrt((a*x + b)/x))/(a^4*b^2*x + a^3*b^3), (3*(b^4*c^3 - 2*a*b^3*c^2*d + (a*b
^3*c^3 - 2*a^2*b^2*c^2*d)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a)
+ (a^2*b^2*c^3*x^2 - 2*a^3*b*d^3 + (3*a*b^3*c^3 - 6*a^2*b^2*c^2*d + 6*a^3*b
*c*d^2 - 4*a^4*d^3)*x)*sqrt((a*x + b)/x))/(a^4*b^2*x + a^3*b^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx+d)^3}{x^3\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)**3/(a+b/x)**(3/2),x)
```

```
[Out] Integral((c*x + d)**3/(x**3*(a + b/x)**(3/2)), x)
```

Giac [A] time = 1.20549, size = 297, normalized size = 2.25

$$-b\left(\frac{2\,d^3\sqrt{\frac{ax+b}{x}}}{b^3}-\frac{3\left(bc^3-2\,ac^2d\right)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2b}-\frac{2\,ab^3c^3-6\,a^2b^2c^2d+6\,a^3bcd^2-2\,a^4d^3-\frac{3\,(ax+b)b^3c^3}{x}+\frac{6\,(ax+b)ab^2c^2d}{x}}{\left(a\sqrt{\frac{ax+b}{x}}-\frac{(ax+b)\sqrt{\frac{ax+b}{x}}}{x}\right)a^2b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^3/(a+b/x)^(3/2),x, algorithm="giac")
```

```
[Out] -b*(2*d^3*sqrt((a*x + b)/x)/b^3 - 3*(b*c^3 - 2*a*c^2*d)*arctan(sqrt((a*x +
b)/x)/sqrt(-a))/(sqrt(-a)*a^2*b) - (2*a*b^3*c^3 - 6*a^2*b^2*c^2*d + 6*a^3*b
*c*d^2 - 2*a^4*d^3 - 3*(a*x + b)*b^3*c^3/x + 6*(a*x + b)*a*b^2*c^2*d/x - 6*
(a*x + b)*a^2*b*c*d^2/x + 2*(a*x + b)*a^3*d^3/x)/((a*sqrt((a*x + b)/x) - (a
*x + b)*sqrt((a*x + b)/x)/x)*a^2*b^3))
```

3.253 $\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx$

Optimal. Leaf size=94

$$\frac{2a^2d^2+bc(3bc-4ad)}{a^2b\sqrt{a+\frac{b}{x}}}-\frac{c(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{c^2x}{a\sqrt{a+\frac{b}{x}}}$$

```
[Out] (2*a^2*d^2 + b*c*(3*b*c - 4*a*d))/(a^2*b*Sqrt[a + b/x]) + (c^2*x)/(a*Sqrt[a
 + b/x]) - (c*(3*b*c - 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(5/2)
```

Rubi [A] time = 0.0787857, antiderivative size = 90, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {375, 89, 78, 63, 208}

$$\frac{\frac{c(3bc-4ad)}{a^2}+\frac{2d^2}{b}}{\sqrt{a+\frac{b}{x}}}-\frac{c(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{c^2x}{a\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d/x)^2/(a + b/x)^(3/2),x]
```

```
[Out] ((2*d^2)/b + (c*(3*b*c - 4*a*d))/a^2)/Sqrt[a + b/x] + (c^2*x)/(a*Sqrt[a + b
/x]) - (c*(3*b*c - 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(5/2)
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx &= -\operatorname{Subst}\left(\int \frac{(c+dx)^2}{x^2(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{c^2x}{a\sqrt{a+\frac{b}{x}}} - \frac{\operatorname{Subst}\left(\int \frac{-\frac{1}{2}c(3bc-4ad)+ad^2x}{x(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)}{a}\\
&= \frac{\frac{2d^2}{b}+\frac{c(3bc-4ad)}{a^2}}{\sqrt{a+\frac{b}{x}}} + \frac{c^2x}{a\sqrt{a+\frac{b}{x}}} + \frac{(c(3bc-4ad))\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a^2}\\
&= \frac{\frac{2d^2}{b}+\frac{c(3bc-4ad)}{a^2}}{\sqrt{a+\frac{b}{x}}} + \frac{c^2x}{a\sqrt{a+\frac{b}{x}}} + \frac{(c(3bc-4ad))\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{a^2b}\\
&= \frac{\frac{2d^2}{b}+\frac{c(3bc-4ad)}{a^2}}{\sqrt{a+\frac{b}{x}}} + \frac{c^2x}{a\sqrt{a+\frac{b}{x}}} - \frac{c(3bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}\end{aligned}$$

Mathematica [A] time = 0.0857521, size = 81, normalized size = 0.86

$$\frac{2a^2d^2+abc(cx-4d)+3b^2c^2}{a^2b\sqrt{a+\frac{b}{x}}} + \frac{c(4ad-3bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d/x)^2/(a + b/x)^(3/2),x]
```

```
[Out] (3*b^2*c^2 + 2*a^2*d^2 + a*b*c*(-4*d + c*x))/(a^2*b*Sqrt[a + b/x]) + (c*(-3
*b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(5/2)
```

Maple [B] time = 0.012, size = 789, normalized size = 8.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)^2/(a+b/x)^(3/2),x)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x/a^(5/2)*(4*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*
x+b)/a^(1/2))*x^2*a^3*b^2*c*d+8*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b
)/a^(1/2))*x*a^2*b^3*c*d-8*a^(7/2)*((a*x+b)*x)^(1/2)*x^2*b*c*d-3*ln(1/2*(2*
((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*b^5*c^2-4*a^(7/2)*((a*x+b)*x)^(
3/2)*d^2-16*a^(5/2)*((a*x+b)*x)^(1/2)*x*b^2*c*d+2*(a*x^2+b*x)^(1/2)*a^(9/2)
*x^2*d^2+2*a^(9/2)*((a*x+b)*x)^(1/2)*x^2*d^2-4*a^(3/2)*((a*x+b)*x)^(3/2)*b^
2*c^2+2*(a*x^2+b*x)^(1/2)*a^(5/2)*b^2*d^2+2*a^(5/2)*((a*x+b)*x)^(1/2)*b^2*d
^2+ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*b^3*d^2+6*a^(1
/2)*((a*x+b)*x)^(1/2)*b^4*c^2-ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/
a^(1/2))*a^2*b^3*d^2+4*(a*x^2+b*x)^(1/2)*a^(7/2)*x*b*d^2+8*a^(5/2)*((a*x+b)
*x)^(3/2)*b*c*d+4*a^(7/2)*((a*x+b)*x)^(1/2)*x*b*d^2+12*a^(3/2)*((a*x+b)*x)^
(1/2)*x*b^3*c^2-8*a^(3/2)*((a*x+b)*x)^(1/2)*b^3*c*d+6*a^(5/2)*((a*x+b)*x)^(
1/2)*x^2*b^2*c^2+ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x^2*
a^4*b*d^2-ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x^2*a^4*b*d
^2-3*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x^2*a^2*b^3*c^2+
2*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x*a^3*b^2*d^2-2*ln(
1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x*a^3*b^2*d^2-6*ln(1/2*(
2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x*a*b^4*c^2+4*ln(1/2*(2*((a*x
+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a*b^4*c*d)/((a*x+b)*x)^(1/2)/b^2/(a*
x+b)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2/(a+b/x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.30421, size = 586, normalized size = 6.23

$$\left[-\frac{\left(3\,b^{3}c^{2}-4\,ab^{2}cd+\left(3\,ab^{2}c^{2}-4\,a^{2}bcd\right)x\right)\sqrt{a}\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)-2\left(a^{2}bc^{2}x^{2}+\left(3\,ab^{2}c^{2}-4\,a^{2}bcd+2\,a^{3}d^{2}\right)x\right.}{2\left(a^{4}bx+a^{3}b^{2}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2/(a+b/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((3*b^3*c^2 - 4*a*b^2*c*d + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x)*sqrt(a)*lo
g(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 2*(a^2*b*c^2*x^2 + (3*a*b^2*
c^2 - 4*a^2*b*c*d + 2*a^3*d^2)*x)*sqrt((a*x + b)/x))/(a^4*b*x + a^3*b^2), (
(3*b^3*c^2 - 4*a*b^2*c*d + (3*a*b^2*c^2 - 4*a^2*b*c*d)*x)*sqrt(-a)*arctan(s
qrt(-a)*sqrt((a*x + b)/x)/a) + (a^2*b*c^2*x^2 + (3*a*b^2*c^2 - 4*a^2*b*c*d
+ 2*a^3*d^2)*x)*sqrt((a*x + b)/x))/(a^4*b*x + a^3*b^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx+d)^2}{x^2\left(a+\frac{b}{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)**2/(a+b/x)**(3/2),x)
```

```
[Out] Integral((c*x + d)**2/(x**2*(a + b/x)**(3/2)), x)
```

Giac [A] time = 1.71262, size = 217, normalized size = 2.31

$$b\left(\frac{\left(3\,bc^2-4\,acd\right)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2b}+\frac{2\,ab^2c^2-4\,a^2bcd+2\,a^3d^2-\frac{3\,(ax+b)b^2c^2}{x}+\frac{4\,(ax+b)abcd}{x}-\frac{2\,(ax+b)a^2d^2}{x}}{\left(a\sqrt{\frac{ax+b}{x}}-\frac{(ax+b)\sqrt{\frac{ax+b}{x}}}{x}\right)a^2b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2/(a+b/x)^(3/2),x, algorithm="giac")
```

```
[Out] b*((3*b*c^2 - 4*a*c*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^2*b)
+ (2*a*b^2*c^2 - 4*a^2*b*c*d + 2*a^3*d^2 - 3*(a*x + b)*b^2*c^2/x + 4*(a*x +
 b)*a*b*c*d/x - 2*(a*x + b)*a^2*d^2/x)/((a*sqrt((a*x + b)/x) - (a*x + b)*sq
rt((a*x + b)/x)/x)*a^2*b^2))
```

3.254 $\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx$

Optimal. Leaf size=76

$$\frac{3bc-2ad}{a^2\sqrt{a+\frac{b}{x}}}-\frac{(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{cx}{a\sqrt{a+\frac{b}{x}}}$$

```
[Out] (3*b*c - 2*a*d)/(a^2*Sqrt[a + b/x]) + (c*x)/(a*Sqrt[a + b/x]) - ((3*b*c - 2
*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(5/2)
```

Rubi [A] time = 0.050474, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {375, 78, 51, 63, 208}

$$\frac{3bc-2ad}{a^2\sqrt{a+\frac{b}{x}}}-\frac{(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{cx}{a\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d/x)/(a + b/x)^(3/2),x]
```

```
[Out] (3*b*c - 2*a*d)/(a^2*Sqrt[a + b/x]) + (c*x)/(a*Sqrt[a + b/x]) - ((3*b*c - 2
*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(5/2)
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx &= -\text{Subst}\left(\int \frac{c+dx}{x^2(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{cx}{a\sqrt{a+\frac{b}{x}}} - \frac{\left(-\frac{3bc}{2}+ad\right)\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)}{a}\\
&= \frac{3bc-2ad}{a^2\sqrt{a+\frac{b}{x}}} + \frac{cx}{a\sqrt{a+\frac{b}{x}}} + \frac{(3bc-2ad)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a^2}\\
&= \frac{3bc-2ad}{a^2\sqrt{a+\frac{b}{x}}} + \frac{cx}{a\sqrt{a+\frac{b}{x}}} + \frac{(3bc-2ad)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{a^2b}\\
&= \frac{3bc-2ad}{a^2\sqrt{a+\frac{b}{x}}} + \frac{cx}{a\sqrt{a+\frac{b}{x}}} - \frac{(3bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}
\end{aligned}
$$

Mathematica [C] time = 0.0213857, size = 48, normalized size = 0.63

$$\frac{(3bc-2ad)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b}{ax}+1\right)+acx}{a^2\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d/x)/(a + b/x)^(3/2),x]
```

```
[Out] (a*c*x + (3*b*c - 2*a*d)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + b/(a*x)])/(a^2
*Sqrt[a + b/x])
```

Maple [B] time = 0.01, size = 387, normalized size = 5.1

$$\frac{x}{2\,b\,(ax+b)^2}\sqrt{\frac{ax+b}{x}}\left(2\,\ln\left(1/2\,\frac{2\,\sqrt{(ax+b)\,x}\sqrt{a}+2\,ax+b}{\sqrt{a}}\right)x^2a^3bd-3\,\ln\left(1/2\,\frac{2\,\sqrt{(ax+b)\,x}\sqrt{a}+2\,ax+b}{\sqrt{a}}\right)x^2a^2b^2c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)/(a+b/x)^(3/2),x)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x/a^(5/2)*(2*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*
x+b)/a^(1/2))*x^2*a^3*b*d-3*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^
(1/2))*x^2*a^2*b^2*c-4*a^(7/2)*((a*x+b)*x)^(1/2)*x^2*d+6*a^(5/2)*((a*x+b)*x
)^(1/2)*x^2*b*c+4*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x*a
^2*b^2*d-6*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x*a*b^3*c+
4*a^(5/2)*((a*x+b)*x)^(3/2)*d-4*a^(3/2)*((a*x+b)*x)^(3/2)*b*c-8*a^(5/2)*((a
*x+b)*x)^(1/2)*x*b*d+12*a^(3/2)*((a*x+b)*x)^(1/2)*x*b^2*c+2*ln(1/2*(2*((a*x
+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a*b^3*d-3*ln(1/2*(2*((a*x+b)*x)^(1/2
)*a^(1/2)+2*a*x+b)/a^(1/2))*b^4*c-4*a^(3/2)*((a*x+b)*x)^(1/2)*b^2*d+6*a^(1/
2)*((a*x+b)*x)^(1/2)*b^3*c)/((a*x+b)*x)^(1/2)/b/(a*x+b)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.35408, size = 473, normalized size = 6.22

$$\left[-\frac{\left(3\,b^2c - 2\,abd + \left(3\,abc - 2\,a^2d\right)x\right)\sqrt{a}\log\left(2\,ax + 2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}} + b\right) - 2\left(a^2cx^2 + \left(3\,abc - 2\,a^2d\right)x\right)\sqrt{\frac{ax+b}{x}}}{2\left(a^4x + a^3b\right)}, \frac{\left(3\,b^2c - 2\,abd\right.\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((3*b^2*c - 2*a*b*d + (3*a*b*c - 2*a^2*d)*x)*sqrt(a)*log(2*a*x + 2*sq
rt(a)*x*sqrt((a*x + b)/x) + b) - 2*(a^2*c*x^2 + (3*a*b*c - 2*a^2*d)*x)*sqrt
((a*x + b)/x))/(a^4*x + a^3*b), ((3*b^2*c - 2*a*b*d + (3*a*b*c - 2*a^2*d)*x
)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (a^2*c*x^2 + (3*a*b*c - 2
*a^2*d)*x)*sqrt((a*x + b)/x))/(a^4*x + a^3*b)]
```

Sympy [B] time = 21.5894, size = 224, normalized size = 2.95

$$c\left(\frac{x^{\frac{3}{2}}}{a\sqrt{b}\sqrt{\frac{ax}{b}+1}} + \frac{3\sqrt{b}\sqrt{x}}{a^2\sqrt{\frac{ax}{b}+1}} - \frac{3b\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{5}{2}}}\right) + d\left(-\frac{2a^3x\sqrt{1+\frac{b}{ax}}}{a^{\frac{9}{2}}x + a^{\frac{7}{2}}b} - \frac{a^3x\log\left(\frac{b}{ax}\right)}{a^{\frac{9}{2}}x + a^{\frac{7}{2}}b} + \frac{2a^3x\log\left(\sqrt{1+\frac{b}{ax}}+1\right)}{a^{\frac{9}{2}}x + a^{\frac{7}{2}}b} - \frac{a^2b\log\cdots}{a^{\frac{9}{2}}x\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)**(3/2),x)
```

```
[Out] c*(x**(3/2)/(a*sqrt(b)*sqrt(a*x/b + 1)) + 3*sqrt(b)*sqrt(x)/(a**2*sqrt(a*x/
b + 1)) - 3*b*asinh(sqrt(a)*sqrt(x)/sqrt(b))/a**(5/2)) + d*(-2*a**3*x*sqrt(
1 + b/(a*x))/(a**(9/2)*x + a**(7/2)*b) - a**3*x*log(b/(a*x))/(a**(9/2)*x +
a**(7/2)*b) + 2*a**3*x*log(sqrt(1 + b/(a*x)) + 1)/(a**(9/2)*x + a**(7/2)*b)
 - a**2*b*log(b/(a*x))/(a**(9/2)*x + a**(7/2)*b) + 2*a**2*b*log(sqrt(1 + b/
(a*x)) + 1)/(a**(9/2)*x + a**(7/2)*b))
```

Giac [A] time = 1.19867, size = 165, normalized size = 2.17

$$b\left(\frac{(3\,bc-2\,ad)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2b}+\frac{2\,abc-2\,a^2d-\frac{3\,(ax+b)bc}{x}+\frac{2\,(ax+b)ad}{x}}{\left(a\sqrt{\frac{ax+b}{x}}-\frac{(ax+b)\sqrt{\frac{ax+b}{x}}}{x}\right)a^2b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)^(3/2),x, algorithm="giac")
```

```
[Out] b*((3*b*c - 2*a*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^2*b) + (2
*a*b*c - 2*a^2*d - 3*(a*x + b)*b*c/x + 2*(a*x + b)*a*d/x)/((a*sqrt((a*x + b
)/x) - (a*x + b)*sqrt((a*x + b)/x)/x)*a^2*b))
```

3.255 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx$

Optimal. Leaf size=60

$$\frac{3b}{a^2\sqrt{a+\frac{b}{x}}}-\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{x}{a\sqrt{a+\frac{b}{x}}}$$

```
[Out] (3*b)/(a^2*Sqrt[a + b/x]) + x/(a*Sqrt[a + b/x]) - (3*b*ArcTanh[Sqrt[a + b/x
]/Sqrt[a]])/a^(5/2)
```

Rubi [A] time = 0.0306506, antiderivative size = 61, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {242, 51, 63, 208}

$$\frac{3x\sqrt{a+\frac{b}{x}}}{a^2}-\frac{3b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{2x}{a\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(-3/2),x]
```

```
[Out] (-2*x)/(a*Sqrt[a + b/x]) + (3*Sqrt[a + b/x]*x)/a^2 - (3*b*ArcTanh[Sqrt[a +
b/x]/Sqrt[a]])/a^(5/2)
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}}\,dx = -\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{2x}{a\sqrt{a+\frac{b}{x}}} - \frac{3\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{a}$$
$$= -\frac{2x}{a\sqrt{a+\frac{b}{x}}} + \frac{3\sqrt{a+\frac{b}{x}}\,x}{a^2} + \frac{(3b)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a^2}$$
$$= -\frac{2x}{a\sqrt{a+\frac{b}{x}}} + \frac{3\sqrt{a+\frac{b}{x}}\,x}{a^2} + \frac{3\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{a^2}$$
$$= -\frac{2x}{a\sqrt{a+\frac{b}{x}}} + \frac{3\sqrt{a+\frac{b}{x}}\,x}{a^2} - \frac{3b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

Mathematica [C] time = 0.0115065, size = 36, normalized size = 0.6

$$\frac{2b\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};\frac{a+\frac{b}{x}}{a}\right)}{a^2\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(-3/2),x]

[Out] (2*b*Hypergeometric2F1[-1/2, 2, 1/2, (a + b/x)/a])/(a^2*Sqrt[a + b/x])
```

Maple [B] time = 0.009, size = 198, normalized size = 3.3

$$-\frac{x}{2\,(ax+b)^2}\sqrt{\frac{ax+b}{x}}\left(3\,\ln\left(1/2\,\frac{2\,\sqrt{(ax+b)\,x}\sqrt{a}+2\,ax+b}{\sqrt{a}}\right)x^2a^2b-6\,a^{5/2}\sqrt{(ax+b)\,x}x^2+6\,\ln\left(1/2\,\frac{2\,\sqrt{(ax+b)\,x}\sqrt{a}}{\sqrt{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(3/2),x)

[Out] -1/2*((a*x+b)/x)^(1/2)*x/a^(5/2)*(3*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a
*x+b)/a^(1/2))*x^2*a^2*b-6*a^(5/2)*((a*x+b)*x)^(1/2)*x^2+6*ln(1/2*(2*((a*x+
b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x*a*b^2+4*a^(3/2)*((a*x+b)*x)^(3/2)-1
2*a^(3/2)*((a*x+b)*x)^(1/2)*x*b+3*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x
+b)/a^(1/2))*b^3-6*a^(1/2)*((a*x+b)*x)^(1/2)*b^2)/((a*x+b)*x)^(1/2)/(a*x+b)
^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.31091, size = 352, normalized size = 5.87

$$\left[\frac{3\left(abx+b^2\right)\sqrt{a}\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(a^2x^2+3\,abx\right)\sqrt{\frac{ax+b}{x}}}{2\left(a^4x+a^3b\right)},\ \frac{3\left(abx+b^2\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)+\left(a^2x^2+3\right.}{a^4x+a^3b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(3*(a*b*x + b^2)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b
) + 2*(a^2*x^2 + 3*a*b*x)*sqrt((a*x + b)/x))/(a^4*x + a^3*b), (3*(a*b*x + b
^2)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (a^2*x^2 + 3*a*b*x)*sqr
t((a*x + b)/x))/(a^4*x + a^3*b)]
```

Sympy [A] time = 3.06595, size = 71, normalized size = 1.18

$$\frac{x^{\frac{3}{2}}}{a\sqrt{b}\sqrt{\frac{ax}{b}+1}}+\frac{3\sqrt{b}\sqrt{x}}{a^2\sqrt{\frac{ax}{b}+1}}-\frac{3b\operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(3/2),x)
```

```
[Out] x**(3/2)/(a*sqrt(b)*sqrt(a*x/b + 1)) + 3*sqrt(b)*sqrt(x)/(a**2*sqrt(a*x/b +
 1)) - 3*b*asinh(sqrt(a)*sqrt(x)/sqrt(b))/a**(5/2)
```

Giac [A] time = 1.19081, size = 116, normalized size = 1.93

$$b\left(\frac{3\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}+\frac{2\,a-\frac{3\,(ax+b)}{x}}{\left(a\sqrt{\frac{ax+b}{x}}-\frac{(ax+b)\sqrt{\frac{ax+b}{x}}}{x}\right)a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2),x, algorithm="giac")
```

```
[Out] b*(3*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^2) + (2*a - 3*(a*x + b)
/x)/((a*sqrt((a*x + b)/x) - (a*x + b)*sqrt((a*x + b)/x)/x)*a^2))
```

3.256 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}\,dx$

Optimal. Leaf size=147

$$-\frac{(2ad+3bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^2}+\frac{b(3bc-ad)}{a^2c\sqrt{a+\frac{b}{x}}(bc-ad)}+\frac{2d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2(bc-ad)^{3/2}}+\frac{x}{ac\sqrt{a+\frac{b}{x}}}$$

```
[Out] (b*(3*b*c - a*d))/(a^2*c*(b*c - a*d)*Sqrt[a + b/x]) + x/(a*c*Sqrt[a + b/x])
 + (2*d^(5/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c^2*(b*c -
a*d)^(3/2)) - ((3*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(a^(5/2)*c^2
)
```

Rubi [A] time = 0.194021, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {375, 103, 152, 156, 63, 208, 205}

$$-\frac{(2ad+3bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^2}+\frac{b(3bc-ad)}{a^2c\sqrt{a+\frac{b}{x}}(bc-ad)}+\frac{2d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2(bc-ad)^{3/2}}+\frac{x}{ac\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(3/2)*(c + d/x)),x]
```

```
[Out] (b*(3*b*c - a*d))/(a^2*c*(b*c - a*d)*Sqrt[a + b/x]) + x/(a*c*Sqrt[a + b/x])
 + (2*d^(5/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c^2*(b*c -
a*d)^(3/2)) - ((3*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(a^(5/2)*c^2
)
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
```

```
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)}\,dx = -\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}(c+dx)}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{x}{ac\sqrt{a+\frac{b}{x}}} + \frac{\operatorname{Subst}\left(\int \frac{\frac{1}{2}(3bc+2ad)+\frac{3bdx}{2}}{x(a+bx)^{3/2}(c+dx)}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{b(3bc-ad)}{a^2c(bc-ad)\sqrt{a+\frac{b}{x}}} + \frac{x}{ac\sqrt{a+\frac{b}{x}}} + \frac{2\operatorname{Subst}\left(\int \frac{\frac{1}{4}(bc-ad)(3bc+2ad)+\frac{1}{4}bd(3bc-ad)x}{x\sqrt{a+bx}(c+dx)}\,dx, x, \frac{1}{x}\right)}{a^2c(bc-ad)}$$

$$= \frac{b(3bc-ad)}{a^2c(bc-ad)\sqrt{a+\frac{b}{x}}} + \frac{x}{ac\sqrt{a+\frac{b}{x}}} + \frac{d^3\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx}(c+dx)}\,dx, x, \frac{1}{x}\right)}{c^2(bc-ad)} + \frac{(3bc+2ad)\operatorname{Subst}}{2a}$$

$$= \frac{b(3bc-ad)}{a^2c(bc-ad)\sqrt{a+\frac{b}{x}}} + \frac{x}{ac\sqrt{a+\frac{b}{x}}} + \frac{\left(2d^3\right)\operatorname{Subst}\left(\int \frac{1}{c-\frac{ad}{b}+\frac{dx^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^2(bc-ad)} + \frac{(3bc+2ad)}{}$$

$$= \frac{b(3bc-ad)}{a^2c(bc-ad)\sqrt{a+\frac{b}{x}}} + \frac{x}{ac\sqrt{a+\frac{b}{x}}} + \frac{2d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2(bc-ad)^{3/2}} - \frac{(3bc+2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^2}$$

Mathematica [C] time = 0.0546908, size = 106, normalized size = 0.72

$$\frac{(ad-bc)\left((2ad+3bc)\,_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b}{ax}+1\right)+acx\right)-2a^2d^2\,_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{d\left(a+\frac{b}{x}\right)}{ad-bc}\right)}{a^2c^2\sqrt{a+\frac{b}{x}}(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(3/2)*(c + d/x)),x]
```

```
[Out] (-2*a^2*d^2*Hypergeometric2F1[-1/2, 1, 1/2, (d*(a + b/x))/(-(b*c) + a*d)] +
 (-(b*c) + a*d)*(a*c*x + (3*b*c + 2*a*d)*Hypergeometric2F1[-1/2, 1, 1/2, 1
+ b/(a*x)]))/(a^2*c^2*(-(b*c) + a*d)*Sqrt[a + b/x])
```

Maple [B] time = 0.016, size = 962, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(3/2)/(c+d/x),x)
```

```
[Out] -1/2*(2*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/
c^2)^(1/2)*x^2*a^4*c*d^2+ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/
2))*((a*d-b*c)*d/c^2)^(1/2)*x^2*a^3*b*c^2*d-3*ln(1/2*(2*((a*x+b)*x)^(1/2)*a
^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*x^2*a^2*b^2*c^3-2*a^(7/2)*
((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*x^2*c^2*d+6*a^(5/2)*((a*d-b*c)*d/
c^2)^(1/2)*((a*x+b)*x)^(1/2)*x^2*b*c^3+2*a^(9/2)*ln((2*((a*d-b*c)*d/c^2)^(1
/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x^2*d^3+4*ln(1/2*(2*((a
*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*x*a^3*b*c*
d^2+2*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^
2)^(1/2)*x*a^2*b^2*c^2*d-6*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(
1/2))*((a*d-b*c)*d/c^2)^(1/2)*x*a*b^3*c^3-4*a^(3/2)*((a*d-b*c)*d/c^2)^(1/2)
*((a*x+b)*x)^(3/2)*b*c^3-4*a^(5/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2
)*x*b*c^2*d+12*a^(3/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*x*b^2*c^3+
4*a^(7/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b
*d)/(c*x+d))*x*b*d^3+2*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2)
)*((a*d-b*c)*d/c^2)^(1/2)*a^2*b^2*c*d^2+ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)
+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*a*b^3*c^2*d-3*ln(1/2*(2*((a*x+b)
*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*b^4*c^3-2*a^(3/
2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*b^2*c^2*d+6*a^(1/2)*((a*d-b*c)
*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*b^3*c^3+2*a^(5/2)*ln((2*((a*d-b*c)*d/c^2)^(
1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^2*d^3)/a^(5/2)*x*((a
*x+b)/x)^(1/2)/((a*d-b*c)*d/c^2)^(1/2)/c^3/(a*x+b)^2/(a*d-b*c)/((a*x+b)*x)^
(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}\left(c+\frac{d}{x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/(c+d/x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a + b/x)^(3/2)*(c + d/x)), x)
```

Fricas [B] time = 2.03151, size = 2195, normalized size = 14.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/(c+d/x),x, algorithm="fricas")
```

```
[Out] [1/2*((3*b^3*c^2 - a*b^2*c*d - 2*a^2*b*d^2 + (3*a*b^2*c^2 - a^2*b*c*d - 2*a
^3*d^2)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 2*(a^4*
d^2*x + a^3*b*d^2)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c
- a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*((a^2*b*c
^2 - a^3*c*d)*x^2 + (3*a*b^2*c^2 - a^2*b*c*d)*x)*sqrt((a*x + b)/x))/(a^3*b^
2*c^3 - a^4*b*c^2*d + (a^4*b*c^3 - a^5*c^2*d)*x), ((3*b^3*c^2 - a*b^2*c*d -
 2*a^2*b*d^2 + (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x)*sqrt(-a)*arctan(sqr
t(-a)*sqrt((a*x + b)/x)/a) - (a^4*d^2*x + a^3*b*d^2)*sqrt(-d/(b*c - a*d))*l
og(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c -
2*a*d)*x)/(c*x + d)) + ((a^2*b*c^2 - a^3*c*d)*x^2 + (3*a*b^2*c^2 - a^2*b*c*
d)*x)*sqrt((a*x + b)/x))/(a^3*b^2*c^3 - a^4*b*c^2*d + (a^4*b*c^3 - a^5*c^2*
d)*x), 1/2*(4*(a^4*d^2*x + a^3*b*d^2)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*
d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) + (3*b^3*c^2 - a*
b^2*c*d - 2*a^2*b*d^2 + (3*a*b^2*c^2 - a^2*b*c*d - 2*a^3*d^2)*x)*sqrt(a)*lo
g(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*((a^2*b*c^2 - a^3*c*d)*x^2
 + (3*a*b^2*c^2 - a^2*b*c*d)*x)*sqrt((a*x + b)/x))/(a^3*b^2*c^3 - a^4*b*c^2
*d + (a^4*b*c^3 - a^5*c^2*d)*x), (2*(a^4*d^2*x + a^3*b*d^2)*sqrt(d/(b*c - a
*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b
*d)) + (3*b^3*c^2 - a*b^2*c*d - 2*a^2*b*d^2 + (3*a*b^2*c^2 - a^2*b*c*d - 2*
a^3*d^2)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + ((a^2*b*c^2 - a
^3*c*d)*x^2 + (3*a*b^2*c^2 - a^2*b*c*d)*x)*sqrt((a*x + b)/x))/(a^3*b^2*c^3
- a^4*b*c^2*d + (a^4*b*c^3 - a^5*c^2*d)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a + \frac{b}{x}\right)^{\frac{3}{2}} (cx + d)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(3/2)/(c+d/x),x)
```

```
[Out] Integral(x/((a + b/x)**(3/2)*(c*x + d)), x)
```

Giac [A] time = 1.18955, size = 261, normalized size = 1.78

$$\left(\frac{2\, d^3 \arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^2}}\right)}{\left(b^2c^3 - abc^2d\right)\sqrt{bcd-ad^2}} + \frac{2\, abc - \frac{3\,(ax+b)bc}{x} + \frac{(ax+b)ad}{x}}{\left(a^2bc^2 - a^3cd\right)\left(a\sqrt{\frac{ax+b}{x}} - \frac{(ax+b)\sqrt{\frac{ax+b}{x}}}{x}\right)} + \frac{(3\, bc + 2\, ad) \arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2bc^2}\right) b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/(c+d/x),x, algorithm="giac")
```

```
[Out] (2*d^3*arctan(d*sqrt((a*x + b)/x)/sqrt(b*c*d - a*d^2))/((b^2*c^3 - a*b*c^2*
d)*sqrt(b*c*d - a*d^2)) + (2*a*b*c - 3*(a*x + b)*b*c/x + (a*x + b)*a*d/x)/(
(a^2*b*c^2 - a^3*c*d)*(a*sqrt((a*x + b)/x) - (a*x + b)*sqrt((a*x + b)/x)/x)
) + (3*b*c + 2*a*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^2*b*c^2)
)*b
```

3.257 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}\,dx$

Optimal. Leaf size=224

$$\frac{b\left(2a^2d^2-2abcd+3b^2c^2\right)}{a^2c^2\sqrt{a+\frac{b}{x}}(bc-ad)^2}-\frac{(4ad+3bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^3}+\frac{d^{5/2}(7bc-4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{5/2}}+\frac{d(bc-2ad)}{ac^2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(bc$$

```
[Out] (b*(3*b^2*c^2 - 2*a*b*c*d + 2*a^2*d^2))/(a^2*c^2*(b*c - a*d)^2*Sqrt[a + b/x
]) + (d*(b*c - 2*a*d))/(a*c^2*(b*c - a*d)*Sqrt[a + b/x]*(c + d/x)) + x/(a*c
*Sqrt[a + b/x]*(c + d/x)) + (d^(5/2)*(7*b*c - 4*a*d)*ArcTan[(Sqrt[d]*Sqrt[a
 + b/x])/Sqrt[b*c - a*d]])/(c^3*(b*c - a*d)^(5/2)) - ((3*b*c + 4*a*d)*ArcTa
nh[Sqrt[a + b/x]/Sqrt[a]])/(a^(5/2)*c^3)
```

Rubi [A] time = 0.322693, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {375, 103, 151, 152, 156, 63, 208, 205}

$$\frac{b\left(2a^2d^2-2abcd+3b^2c^2\right)}{a^2c^2\sqrt{a+\frac{b}{x}}(bc-ad)^2}-\frac{(4ad+3bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^3}+\frac{d^{5/2}(7bc-4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{5/2}}+\frac{d(bc-2ad)}{ac^2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(bc$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(3/2)*(c + d/x)^2),x]
```


```
[Out] (b*(3*b^2*c^2 - 2*a*b*c*d + 2*a^2*d^2))/(a^2*c^2*(b*c - a*d)^2*Sqrt[a + b/x
]) + (d*(b*c - 2*a*d))/(a*c^2*(b*c - a*d)*Sqrt[a + b/x]*(c + d/x)) + x/(a*c
*Sqrt[a + b/x]*(c + d/x)) + (d^(5/2)*(7*b*c - 4*a*d)*ArcTan[(Sqrt[d]*Sqrt[a
 + b/x])/Sqrt[b*c - a*d]])/(c^3*(b*c - a*d)^(5/2)) - ((3*b*c + 4*a*d)*ArcTa
nh[Sqrt[a + b/x]/Sqrt[a]])/(a^(5/2)*c^3)
```


Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
```

```
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2}\,dx = -\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}(c+dx)^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(3bc+4ad)+\frac{5bdx}{2}}{x(a+bx)^{3/2}(c+dx)^2}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{d(bc-2ad)}{ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}(bc-ad)(3bc+4ad)-\frac{3}{2}bd(bc-2ad)x}{x(a+bx)^{3/2}(c+dx)}\right.}{ac^2(bc-ad)}$$

$$= \frac{b\left(3b^2c^2-2abcd+2a^2d^2\right)}{a^2c^2(bc-ad)^2\sqrt{a+\frac{b}{x}}} + \frac{d(bc-2ad)}{ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} - \frac{2\,\text{Subst}}{}$$

$$= \frac{b\left(3b^2c^2-2abcd+2a^2d^2\right)}{a^2c^2(bc-ad)^2\sqrt{a+\frac{b}{x}}} + \frac{d(bc-2ad)}{ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} + \frac{\left(d^3(7bc\right.}{}$$

$$= \frac{b\left(3b^2c^2-2abcd+2a^2d^2\right)}{a^2c^2(bc-ad)^2\sqrt{a+\frac{b}{x}}} + \frac{d(bc-2ad)}{ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} + \frac{\left(d^3(7bc\right.}{}$$

$$= \frac{b\left(3b^2c^2-2abcd+2a^2d^2\right)}{a^2c^2(bc-ad)^2\sqrt{a+\frac{b}{x}}} + \frac{d(bc-2ad)}{ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} + \frac{d^{5/2}(7bc}{}$$

Mathematica [C] time = 0.115698, size = 164, normalized size = 0.73

$$\frac{(bc-ad)\left((cx+d)\left(-4a^2d^2+abcd+3b^2c^2\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b}{ax}+1\right)+acx(bc(cx+d)-ad(cx+2d))\right)+a^2d^2(cx+d)(7bc}{a^2c^3\sqrt{a+\frac{b}{x}}(cx+d)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(3/2)*(c + d/x)^2),x]
```

```
[Out] (a^2*d^2*(7*b*c - 4*a*d)*(d + c*x)*Hypergeometric2F1[-1/2, 1, 1/2, (d*(a +
b/x))/(-(b*c) + a*d)] + (b*c - a*d)*(a*c*x*(b*c*(d + c*x) - a*d*(2*d + c*x)
) + (3*b^2*c^2 + a*b*c*d - 4*a^2*d^2)*(d + c*x)*Hypergeometric2F1[-1/2, 1,
1/2, 1 + b/(a*x)]))/(a^2*c^3*(b*c - a*d)^2*Sqrt[a + b/x]*(d + c*x))
```

Maple [B] time = 0.014, size = 3119, normalized size = 13.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(3/2)/(c+d/x)^2,x)
```

```
[Out] -1/2*((a*x+b)/x)^(1/2)*x*(7*a^(11/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)
*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x^3*b^2*c^3*d^3-3*a^(13/2)*ln((2*((
a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x^2*b
*c*d^5-15*a^(11/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*
x+b*c*x-b*d)/(c*x+d))*x^2*b^2*c^2*d^4+14*a^(9/2)*ln((2*((a*d-b*c)*d/c^2)^(1
/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x^2*b^3*c^3*d^3+2*a^(7/
2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(3/2)*b^2*c^4*d^2-4*a^(5/2)*((a*d-b*
c)*d/c^2)^(1/2)*((a*x+b)*x)^(3/2)*b^3*c^5*d-18*a^(11/2)*ln((2*((a*d-b*c)*d/
c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x*b^2*c*d^5+3*a^
(9/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/
(c*x+d))*x*b^3*c^2*d^4+2*a^(13/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)
*x^4*c^4*d^2+4*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^7*((
a*d-b*c)*d/c^2)^(1/2)*x^3*c^2*d^4-3*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a
*x+b)/a^(1/2))*a^3*((a*d-b*c)*d/c^2)^(1/2)*x^3*b^4*c^6-2*a^(11/2)*((a*d-b*c
)*d/c^2)^(1/2)*((a*x+b)*x)^(3/2)*x^2*c^4*d^2-2*a^(13/2)*((a*d-b*c)*d/c^2)^(
1/2)*((a*x+b)*x)^(1/2)*x^3*c^3*d^3+6*a^(7/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+
b)*x)^(1/2)*x^3*b^3*c^6-11*a^(13/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*
x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x^3*b*c^2*d^4-3*ln(1/2*(2*((a*x+b)*x
)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a*((a*d-b*c)*d/c^2)^(1/2)*b^6*c^5*d+6*a^(
3/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*b^5*c^5*d-3*ln(1/2*(2*((a*x+
b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a*((a*d-b*c)*d/c^2)^(1/2)*x*b^6*c^6+6
*a^(3/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*x*b^5*c^6+7*a^(7/2)*ln((
2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x
*b^4*c^3*d^3+4*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^5*((
a*d-b*c)*d/c^2)^(1/2)*b^2*c*d^5-9*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x
+b)/a^(1/2))*a^4*((a*d-b*c)*d/c^2)^(1/2)*b^3*c^2*d^4+3*ln(1/2*(2*((a*x+b)*x
)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*((a*d-b*c)*d/c^2)^(1/2)*b^4*c^3*d^3+5
*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*((a*d-b*c)*d/c^2
)^(1/2)*b^5*c^4*d^2-4*a^(9/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*b^2
*c^2*d^4+8*a^(7/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*b^3*c^3*d^3-10
*a^(5/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*b^4*c^4*d^2-3*ln(1/2*(2*
((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^4*((a*d-b*c)*d/c^2)^(1/2)*x*b
^3*c^3*d^3+13*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*((a
*d-b*c)*d/c^2)^(1/2)*x*b^4*c^4*d^2-ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*
x+b)/a^(1/2))*a^2*((a*d-b*c)*d/c^2)^(1/2)*x*b^5*c^5*d-8*a^(11/2)*((a*d-b*c)
*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*x*b*c^2*d^4+14*a^(9/2)*((a*d-b*c)*d/c^2)^(1
/2)*((a*x+b)*x)^(1/2)*x*b^2*c^3*d^3-12*a^(7/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*
x+b)*x)^(1/2)*x*b^3*c^4*d^2+8*a^(9/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(
1/2)*x^2*b^2*c^4*d^2-14*a^(7/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*x
^2*b^3*c^5*d+8*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^6*((
a*d-b*c)*d/c^2)^(1/2)*x*b*c*d^5-14*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*
x+b)/a^(1/2))*a^5*((a*d-b*c)*d/c^2)^(1/2)*x*b^2*c^2*d^4-4*a^(9/2)*((a*d-b*c
)*d/c^2)^(1/2)*((a*x+b)*x)^(3/2)*x*b*c^4*d^2+4*a^(7/2)*((a*d-b*c)*d/c^2)^(1
/2)*((a*x+b)*x)^(3/2)*x*b^2*c^5*d+4*a^(11/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+
b)*x)^(1/2)*x^2*b*c^3*d^3+11*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a
^(1/2))*a^4*((a*d-b*c)*d/c^2)^(1/2)*x^2*b^3*c^4*d^2+7*ln(1/2*(2*((a*x+b)*x)
^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*((a*d-b*c)*d/c^2)^(1/2)*x^2*b^4*c^5*d-
10*a^(9/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*x^3*b^2*c^5*d-ln(1/2*(
2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^6*((a*d-b*c)*d/c^2)^(1/2)*x
^2*b*c^2*d^4-15*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^5*(
(a*d-b*c)*d/c^2)^(1/2)*x^2*b^2*c^3*d^3+5*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2
)+2*a*x+b)/a^(1/2))*a^4*((a*d-b*c)*d/c^2)^(1/2)*x^3*b^3*c^5*d+12*a^(11/2)*(
(a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*x^3*b*c^4*d^2-9*ln(1/2*(2*((a*x+b)
*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^6*((a*d-b*c)*d/c^2)^(1/2)*x^3*b*c^3*d
^3+3*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^5*((a*d-b*c)*d
/c^2)^(1/2)*x^3*b^2*c^4*d^2+2*a^(5/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(
1/2)*x*b^4*c^5*d+4*a^(15/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)
*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x^2*d^6+4*a^(11/2)*ln((2*((a*d-b*c)*d/c^2)^(
1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^2*d^6+8*a^(13/2)*ln(
```

```
(2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*
x*b*d^6-11*a^(9/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*
x+b*c*x-b*d)/(c*x+d))*b^3*c*d^5+7*a^(7/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a
*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^4*c^2*d^4+4*a^(15/2)*ln((2*(
(a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x^3*
c*d^5+4*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^7*((a*d-b*c
)*d/c^2)^(1/2)*x^2*c*d^5-4*a^(5/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(3/2
)*x*b^3*c^6-4*a^(13/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*x^2*c^2*d^
4-6*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^2*((a*d-b*c)*d/
c^2)^(1/2)*x^2*b^5*c^6+12*a^(5/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)
*x^2*b^4*c^6)/c^4/a^(7/2)/((a*x+b)*x)^(1/2)/(a*d-b*c)^3/(c*x+d)/((a*d-b*c)*
d/c^2)^(1/2)/(a*x+b)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}\left(c+\frac{d}{x}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/(c+d/x)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((a + b/x)^(3/2)*(c + d/x)^2), x)
```

Fricas [B] time = 4.01772, size = 4641, normalized size = 20.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/(c+d/x)^2,x, algorithm="fricas")
```

```
[Out] [1/2*((3*b^4*c^3*d - 2*a*b^3*c^2*d^2 - 5*a^2*b^2*c*d^3 + 4*a^3*b*d^4 + (3*a
*b^3*c^4 - 2*a^2*b^2*c^3*d - 5*a^3*b*c^2*d^2 + 4*a^4*c*d^3)*x^2 + (3*b^4*c^
4 + a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 - a^3*b*c*d^3 + 4*a^4*d^4)*x)*sqrt(a)*l
og(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - (7*a^3*b^2*c*d^3 - 4*a^4*b*
d^4 + (7*a^4*b*c^2*d^2 - 4*a^5*c*d^3)*x^2 + (7*a^3*b^2*c^2*d^2 + 3*a^4*b*c*
d^3 - 4*a^5*d^4)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c
 - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*((a^2*b^
2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*x^3 + (3*a*b^3*c^4 - a^2*b^2*c^3*d - a
^3*b*c^2*d^2 + 2*a^4*c*d^3)*x^2 + (3*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + 2*a^
3*b*c*d^3)*x)*sqrt((a*x + b)/x))/(a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a^5*b
*c^3*d^3 + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^2 + (a^3*b^3*c^6 -
 a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x), 1/2*(2*(7*a^3*b^2*c*d^3 -
 4*a^4*b*d^4 + (7*a^4*b*c^2*d^2 - 4*a^5*c*d^3)*x^2 + (7*a^3*b^2*c^2*d^2 + 3
*a^4*b*c*d^3 - 4*a^5*d^4)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt
(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) + (3*b^4*c^3*d - 2*a*b^3*c
^2*d^2 - 5*a^2*b^2*c*d^3 + 4*a^3*b*d^4 + (3*a*b^3*c^4 - 2*a^2*b^2*c^3*d - 5
*a^3*b*c^2*d^2 + 4*a^4*c*d^3)*x^2 + (3*b^4*c^4 + a*b^3*c^3*d - 7*a^2*b^2*c^
2*d^2 - a^3*b*c*d^3 + 4*a^4*d^4)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a
*x + b)/x) + b) + 2*((a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*x^3 + (3*a
*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + 2*a^4*c*d^3)*x^2 + (3*a*b^3*c^3*
d - 2*a^2*b^2*c^2*d^2 + 2*a^3*b*c*d^3)*x)*sqrt((a*x + b)/x))/(a^3*b^3*c^5*d
 - 2*a^4*b^2*c^4*d^2 + a^5*b*c^3*d^3 + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c
```

```
^4*d^2)*x^2 + (a^3*b^3*c^6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x
), 1/2*(2*(3*b^4*c^3*d - 2*a*b^3*c^2*d^2 - 5*a^2*b^2*c*d^3 + 4*a^3*b*d^4 +
(3*a*b^3*c^4 - 2*a^2*b^2*c^3*d - 5*a^3*b*c^2*d^2 + 4*a^4*c*d^3)*x^2 + (3*b^
4*c^4 + a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 - a^3*b*c*d^3 + 4*a^4*d^4)*x)*sqrt(
-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (7*a^3*b^2*c*d^3 - 4*a^4*b*d^4 +
 (7*a^4*b*c^2*d^2 - 4*a^5*c*d^3)*x^2 + (7*a^3*b^2*c^2*d^2 + 3*a^4*b*c*d^3 -
 4*a^5*d^4)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*
d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*((a^2*b^2*c^4
 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*x^3 + (3*a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*
c^2*d^2 + 2*a^4*c*d^3)*x^2 + (3*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + 2*a^3*b*c
*d^3)*x)*sqrt((a*x + b)/x))/(a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a^5*b*c^3*
d^3 + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^2 + (a^3*b^3*c^6 - a^4*
b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x), ((7*a^3*b^2*c*d^3 - 4*a^4*b*d^
4 + (7*a^4*b*c^2*d^2 - 4*a^5*c*d^3)*x^2 + (7*a^3*b^2*c^2*d^2 + 3*a^4*b*c*d^
3 - 4*a^5*d^4)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a
*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) + (3*b^4*c^3*d - 2*a*b^3*c^2*d^2 - 5*
a^2*b^2*c*d^3 + 4*a^3*b*d^4 + (3*a*b^3*c^4 - 2*a^2*b^2*c^3*d - 5*a^3*b*c^2*
d^2 + 4*a^4*c*d^3)*x^2 + (3*b^4*c^4 + a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 - a^3
*b*c*d^3 + 4*a^4*d^4)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + ((
a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*x^3 + (3*a*b^3*c^4 - a^2*b^2*c^3
*d - a^3*b*c^2*d^2 + 2*a^4*c*d^3)*x^2 + (3*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2
+ 2*a^3*b*c*d^3)*x)*sqrt((a*x + b)/x))/(a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 +
 a^5*b*c^3*d^3 + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^2 + (a^3*b^3
*c^6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(3/2)/(c+d/x)**2,x)
```

```
[Out] Exception raised: ValueError
```

Giac [B] time = 1.18649, size = 560, normalized size = 2.5

$$b\left(\frac{\left(7\,bcd^3-4\,ad^4\right)\arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^2}}\right)}{\left(b^3c^5-2\,ab^2c^4d+a^2bc^3d^2\right)\sqrt{bcd-ad^2}}+\frac{2\,ab^3c^3-2\,a^2b^2c^2d-\frac{3\,(ax+b)b^3c^3}{x}+\frac{7\,(ax+b)ab^2c^2d}{x}-\frac{3\,(ax+b)a^2bcd^2}{x}+\frac{2\,(ax+b)a^3d^3}{x}-}{\left(a^2b^2c^4-2\,a^3bc^3d+a^4c^2d^2\right)\left(abc\sqrt{\frac{ax+b}{x}}-a^2d\sqrt{\frac{ax+b}{x}}-\frac{(ax+b)bc\sqrt{\frac{ax+b}{x}}}{x}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/(c+d/x)^2,x, algorithm="giac")
```

```
[Out] b*((7*b*c*d^3 - 4*a*d^4)*arctan(d*sqrt((a*x + b)/x)/sqrt(b*c*d - a*d^2))/((
b^3*c^5 - 2*a*b^2*c^4*d + a^2*b*c^3*d^2)*sqrt(b*c*d - a*d^2)) + (2*a*b^3*c^
3 - 2*a^2*b^2*c^2*d - 3*(a*x + b)*b^3*c^3/x + 7*(a*x + b)*a*b^2*c^2*d/x - 3
*(a*x + b)*a^2*b*c*d^2/x + 2*(a*x + b)*a^3*d^3/x - 3*(a*x + b)^2*b^2*c^2*d/
x^2 + 2*(a*x + b)^2*a*b*c*d^2/x^2 - 2*(a*x + b)^2*a^2*d^3/x^2)/((a^2*b^2*c^
4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*(a*b*c*sqrt((a*x + b)/x) - a^2*d*sqrt((a*x
 + b)/x) - (a*x + b)*b*c*sqrt((a*x + b)/x)/x + 2*(a*x + b)*a*d*sqrt((a*x +
```

```
b)/x)/x - (a*x + b)^2*d*sqrt((a*x + b)/x)/x^2)) + (3*b*c + 4*a*d)*arctan(sq
rt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^2*b*c^3))
```

3.258 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3} \, dx$

<u>**Optimal**</u>. Leaf size=320

$$\frac{d\left(12a^2d^2-21abcd+4b^2c^2\right)}{4ac^3\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(bc-ad)^2}+\frac{3b(2bc-ad)\left(4a^2d^2-abcd+2b^2c^2\right)}{4a^2c^3\sqrt{a+\frac{b}{x}}(bc-ad)^3}+\frac{3d^{5/2}\left(8a^2d^2-24abcd+21b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right.}{4c^4(bc-ad)^{7/2}}$$

```
[Out] (3*b*(2*b*c - a*d)*(2*b^2*c^2 - a*b*c*d + 4*a^2*d^2))/(4*a^2*c^3*(b*c - a*d
)^3*Sqrt[a + b/x]) + (d*(2*b*c - 3*a*d))/(2*a*c^2*(b*c - a*d)*Sqrt[a + b/x]
*(c + d/x)^2) + (d*(4*b^2*c^2 - 21*a*b*c*d + 12*a^2*d^2))/(4*a*c^3*(b*c - a
*d)^2*Sqrt[a + b/x]*(c + d/x)) + x/(a*c*Sqrt[a + b/x]*(c + d/x)^2) + (3*d^(
5/2)*(21*b^2*c^2 - 24*a*b*c*d + 8*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/S
qrt[b*c - a*d]])/(4*c^4*(b*c - a*d)^(7/2)) - (3*(b*c + 2*a*d)*ArcTanh[Sqrt[
a + b/x]/Sqrt[a]])/(a^(5/2)*c^4)
```

<u>**Rubi [A]**</u> time = 0.524976, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {375, 103, 151, 152, 156, 63, 208, 205}

$$\frac{d\left(12a^2d^2-21abcd+4b^2c^2\right)}{4ac^3\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)(bc-ad)^2}+\frac{3b(2bc-ad)\left(4a^2d^2-abcd+2b^2c^2\right)}{4a^2c^3\sqrt{a+\frac{b}{x}}(bc-ad)^3}+\frac{3d^{5/2}\left(8a^2d^2-24abcd+21b^2c^2\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right.}{4c^4(bc-ad)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(3/2)*(c + d/x)^3),x]
```

```
[Out] (3*b*(2*b*c - a*d)*(2*b^2*c^2 - a*b*c*d + 4*a^2*d^2))/(4*a^2*c^3*(b*c - a*d
)^3*Sqrt[a + b/x]) + (d*(2*b*c - 3*a*d))/(2*a*c^2*(b*c - a*d)*Sqrt[a + b/x]
*(c + d/x)^2) + (d*(4*b^2*c^2 - 21*a*b*c*d + 12*a^2*d^2))/(4*a*c^3*(b*c - a
*d)^2*Sqrt[a + b/x]*(c + d/x)) + x/(a*c*Sqrt[a + b/x]*(c + d/x)^2) + (3*d^(
5/2)*(21*b^2*c^2 - 24*a*b*c*d + 8*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/S
qrt[b*c - a*d]])/(4*c^4*(b*c - a*d)^(7/2)) - (3*(b*c + 2*a*d)*ArcTanh[Sqrt[
a + b/x]/Sqrt[a]])/(a^(5/2)*c^4)
```

<u>Rule 375</u>

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

<u>Rule 103</u>

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^3}\,dx = -\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}(c+dx)^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} + \frac{\text{Subst}\left(\int \frac{\frac{3}{2}(bc+2ad)+\frac{7bdx}{2}}{x(a+bx)^{3/2}(c+dx)^3}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{d(2bc-3ad)}{2ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} + \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} - \frac{\text{Subst}\left(\int \frac{-3(bc-ad)(bc+2ad)-\frac{5}{2}bd(2bc-3ad)x}{x(a+bx)^{3/2}(c+dx)^2}\right.}{2ac^2(bc-ad)}$$

$$= \frac{d(2bc-3ad)}{2ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} + \frac{d\left(4b^2c^2-21abcd+12a^2d^2\right)}{4ac^3(bc-ad)^2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} + \text{Sub}$$

$$= \frac{3b(2bc-ad)\left(2b^2c^2-abcd+4a^2d^2\right)}{4a^2c^3(bc-ad)^3\sqrt{a+\frac{b}{x}}} + \frac{d(2bc-3ad)}{2ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} + \frac{d\left(4b^2c^2-21abcd+\right.}{4ac^3(bc-ad)^2\sqrt{a+}}$$

$$= \frac{3b(2bc-ad)\left(2b^2c^2-abcd+4a^2d^2\right)}{4a^2c^3(bc-ad)^3\sqrt{a+\frac{b}{x}}} + \frac{d(2bc-3ad)}{2ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} + \frac{d\left(4b^2c^2-21abcd+\right.}{4ac^3(bc-ad)^2\sqrt{a+}}$$

$$= \frac{3b(2bc-ad)\left(2b^2c^2-abcd+4a^2d^2\right)}{4a^2c^3(bc-ad)^3\sqrt{a+\frac{b}{x}}} + \frac{d(2bc-3ad)}{2ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} + \frac{d\left(4b^2c^2-21abcd+\right.}{4ac^3(bc-ad)^2\sqrt{a+}}$$

$$= \frac{3b(2bc-ad)\left(2b^2c^2-abcd+4a^2d^2\right)}{4a^2c^3(bc-ad)^3\sqrt{a+\frac{b}{x}}} + \frac{d(2bc-3ad)}{2ac^2(bc-ad)\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)^2} + \frac{d\left(4b^2c^2-21abcd+\right.}{4ac^3(bc-ad)^2\sqrt{a+}}$$

Mathematica [C] time = 0.212408, size = 239, normalized size = 0.75

$$\frac{(cx+d)\left(2(cx+d)\left(\frac{3}{4}a^2d^2\left(8a^2d^2-24abcd+21b^2c^2\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{d\left(a+\frac{b}{x}\right)}{ad-bc}\right)+3(2ad+bc)(bc-ad)^3\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b}{ax}+1\right)\right.\right.}{2a^2c^4\sqrt{a+\frac{b}{x}}(cx+d)^2(bc-}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(3/2)*(c + d/x)^3),x]
```

```
[Out] (-(a*c^2*d*(b*c - a*d)^2*(-2*b*c + 3*a*d)*x^2) + 2*a*c^3*(b*c - a*d)^3*x^3
+ (d + c*x)*(-(a*c*d*(-(b*c) + a*d)*(4*b^2*c^2 - 21*a*b*c*d + 12*a^2*d^2)*x
)/2 + 2*(d + c*x)*((3*a^2*d^2*(21*b^2*c^2 - 24*a*b*c*d + 8*a^2*d^2)*Hyperge
ometric2F1[-1/2, 1, 1/2, (d*(a + b/x))/(-(b*c) + a*d)])/4 + 3*(b*c - a*d)^3
*(b*c + 2*a*d)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + b/(a*x)])))/(2*a^2*c^4*(
b*c - a*d)^3*Sqrt[a + b/x]*(d + c*x)^2)
```

Maple [B] time = 0.016, size = 5158, normalized size = 16.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(3/2)/(c+d/x)^3,x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{\frac{3}{2}}\left(c+\frac{d}{x}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/(c+d/x)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((a + b/x)^(3/2)*(c + d/x)^3), x)
```

Fricas [B] time = 10.4814, size = 8142, normalized size = 25.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/(c+d/x)^3,x, algorithm="fricas")
```

```
[Out] [1/8*(12*(b^5*c^4*d^2 - a*b^4*c^3*d^3 - 3*a^2*b^3*c^2*d^4 + 5*a^3*b^2*c*d^5
 - 2*a^4*b*d^6 + (a*b^4*c^6 - a^2*b^3*c^5*d - 3*a^3*b^2*c^4*d^2 + 5*a^4*b*c
^3*d^3 - 2*a^5*c^2*d^4)*x^3 + (b^5*c^6 + a*b^4*c^5*d - 5*a^2*b^3*c^4*d^2 -
a^3*b^2*c^3*d^3 + 8*a^4*b*c^2*d^4 - 4*a^5*c*d^5)*x^2 + (2*b^5*c^5*d - a*b^4
*c^4*d^2 - 7*a^2*b^3*c^3*d^3 + 7*a^3*b^2*c^2*d^4 + a^4*b*c*d^5 - 2*a^5*d^6)
*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 3*(21*a^3*b^3*
c^2*d^4 - 24*a^4*b^2*c*d^5 + 8*a^5*b*d^6 + (21*a^4*b^2*c^4*d^2 - 24*a^5*b*c
^3*d^3 + 8*a^6*c^2*d^4)*x^3 + (21*a^3*b^3*c^4*d^2 + 18*a^4*b^2*c^3*d^3 - 40
*a^5*b*c^2*d^4 + 16*a^6*c*d^5)*x^2 + (42*a^3*b^3*c^3*d^3 - 27*a^4*b^2*c^2*d
^4 - 8*a^5*b*c*d^5 + 8*a^6*d^6)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)
*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d
)) + 2*(4*(a^2*b^3*c^6 - 3*a^3*b^2*c^5*d + 3*a^4*b*c^4*d^2 - a^5*c^3*d^3)*x
^4 + (12*a*b^4*c^6 - 4*a^2*b^3*c^5*d - 12*a^3*b^2*c^4*d^2 + 37*a^4*b*c^3*d^
3 - 18*a^5*c^2*d^4)*x^3 + (24*a*b^4*c^5*d - 20*a^2*b^3*c^4*d^2 + 29*a^3*b^2
*c^3*d^3 + 9*a^4*b*c^2*d^4 - 12*a^5*c*d^5)*x^2 + 3*(4*a*b^4*c^4*d^2 - 4*a^2
*b^3*c^3*d^3 + 9*a^3*b^2*c^2*d^4 - 4*a^4*b*c*d^5)*x)*sqrt((a*x + b)/x))/(a^
3*b^4*c^7*d^2 - 3*a^4*b^3*c^6*d^3 + 3*a^5*b^2*c^5*d^4 - a^6*b*c^4*d^5 + (a^
4*b^3*c^9 - 3*a^5*b^2*c^8*d + 3*a^6*b*c^7*d^2 - a^7*c^6*d^3)*x^3 + (a^3*b^4
*c^9 - a^4*b^3*c^8*d - 3*a^5*b^2*c^7*d^2 + 5*a^6*b*c^6*d^3 - 2*a^7*c^5*d^4)
*x^2 + (2*a^3*b^4*c^8*d - 5*a^4*b^3*c^7*d^2 + 3*a^5*b^2*c^6*d^3 + a^6*b*c^5
*d^4 - a^7*c^4*d^5)*x), 1/8*(24*(b^5*c^4*d^2 - a*b^4*c^3*d^3 - 3*a^2*b^3*c^
2*d^4 + 5*a^3*b^2*c*d^5 - 2*a^4*b*d^6 + (a*b^4*c^6 - a^2*b^3*c^5*d - 3*a^3*
b^2*c^4*d^2 + 5*a^4*b*c^3*d^3 - 2*a^5*c^2*d^4)*x^3 + (b^5*c^6 + a*b^4*c^5*d
 - 5*a^2*b^3*c^4*d^2 - a^3*b^2*c^3*d^3 + 8*a^4*b*c^2*d^4 - 4*a^5*c*d^5)*x^2
 + (2*b^5*c^5*d - a*b^4*c^4*d^2 - 7*a^2*b^3*c^3*d^3 + 7*a^3*b^2*c^2*d^4 + a
^4*b*c*d^5 - 2*a^5*d^6)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) -
3*(21*a^3*b^3*c^2*d^4 - 24*a^4*b^2*c*d^5 + 8*a^5*b*d^6 + (21*a^4*b^2*c^4*d^
2 - 24*a^5*b*c^3*d^3 + 8*a^6*c^2*d^4)*x^3 + (21*a^3*b^3*c^4*d^2 + 18*a^4*b^
```

```
2*c^3*d^3 - 40*a^5*b*c^2*d^4 + 16*a^6*c*d^5)*x^2 + (42*a^3*b^3*c^3*d^3 - 27
*a^4*b^2*c^2*d^4 - 8*a^5*b*c*d^5 + 8*a^6*d^6)*x)*sqrt(-d/(b*c - a*d))*log(-
(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*
d)*x)/(c*x + d)) + 2*(4*(a^2*b^3*c^6 - 3*a^3*b^2*c^5*d + 3*a^4*b*c^4*d^2 -
a^5*c^3*d^3)*x^4 + (12*a*b^4*c^6 - 4*a^2*b^3*c^5*d - 12*a^3*b^2*c^4*d^2 + 3
7*a^4*b*c^3*d^3 - 18*a^5*c^2*d^4)*x^3 + (24*a*b^4*c^5*d - 20*a^2*b^3*c^4*d^
2 + 29*a^3*b^2*c^3*d^3 + 9*a^4*b*c^2*d^4 - 12*a^5*c*d^5)*x^2 + 3*(4*a*b^4*c
^4*d^2 - 4*a^2*b^3*c^3*d^3 + 9*a^3*b^2*c^2*d^4 - 4*a^4*b*c*d^5)*x)*sqrt((a*
x + b)/x))/(a^3*b^4*c^7*d^2 - 3*a^4*b^3*c^6*d^3 + 3*a^5*b^2*c^5*d^4 - a^6*b
*c^4*d^5 + (a^4*b^3*c^9 - 3*a^5*b^2*c^8*d + 3*a^6*b*c^7*d^2 - a^7*c^6*d^3)*
x^3 + (a^3*b^4*c^9 - a^4*b^3*c^8*d - 3*a^5*b^2*c^7*d^2 + 5*a^6*b*c^6*d^3 -
2*a^7*c^5*d^4)*x^2 + (2*a^3*b^4*c^8*d - 5*a^4*b^3*c^7*d^2 + 3*a^5*b^2*c^6*d
^3 + a^6*b*c^5*d^4 - a^7*c^4*d^5)*x), 1/4*(3*(21*a^3*b^3*c^2*d^4 - 24*a^4*b
^2*c*d^5 + 8*a^5*b*d^6 + (21*a^4*b^2*c^4*d^2 - 24*a^5*b*c^3*d^3 + 8*a^6*c^2
*d^4)*x^3 + (21*a^3*b^3*c^4*d^2 + 18*a^4*b^2*c^3*d^3 - 40*a^5*b*c^2*d^4 + 1
6*a^6*c*d^5)*x^2 + (42*a^3*b^3*c^3*d^3 - 27*a^4*b^2*c^2*d^4 - 8*a^5*b*c*d^5
 + 8*a^6*d^6)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*
d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) + 6*(b^5*c^4*d^2 - a*b^4*c^3*d^3 - 3*a
^2*b^3*c^2*d^4 + 5*a^3*b^2*c*d^5 - 2*a^4*b*d^6 + (a*b^4*c^6 - a^2*b^3*c^5*d
 - 3*a^3*b^2*c^4*d^2 + 5*a^4*b*c^3*d^3 - 2*a^5*c^2*d^4)*x^3 + (b^5*c^6 + a*
b^4*c^5*d - 5*a^2*b^3*c^4*d^2 - a^3*b^2*c^3*d^3 + 8*a^4*b*c^2*d^4 - 4*a^5*c
*d^5)*x^2 + (2*b^5*c^5*d - a*b^4*c^4*d^2 - 7*a^2*b^3*c^3*d^3 + 7*a^3*b^2*c^
2*d^4 + a^4*b*c*d^5 - 2*a^5*d^6)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a
*x + b)/x) + b) + (4*(a^2*b^3*c^6 - 3*a^3*b^2*c^5*d + 3*a^4*b*c^4*d^2 - a^5
*c^3*d^3)*x^4 + (12*a*b^4*c^6 - 4*a^2*b^3*c^5*d - 12*a^3*b^2*c^4*d^2 + 37*a
^4*b*c^3*d^3 - 18*a^5*c^2*d^4)*x^3 + (24*a*b^4*c^5*d - 20*a^2*b^3*c^4*d^2 +
 29*a^3*b^2*c^3*d^3 + 9*a^4*b*c^2*d^4 - 12*a^5*c*d^5)*x^2 + 3*(4*a*b^4*c^4*
d^2 - 4*a^2*b^3*c^3*d^3 + 9*a^3*b^2*c^2*d^4 - 4*a^4*b*c*d^5)*x)*sqrt((a*x +
 b)/x))/(a^3*b^4*c^7*d^2 - 3*a^4*b^3*c^6*d^3 + 3*a^5*b^2*c^5*d^4 - a^6*b*c^
4*d^5 + (a^4*b^3*c^9 - 3*a^5*b^2*c^8*d + 3*a^6*b*c^7*d^2 - a^7*c^6*d^3)*x^3
 + (a^3*b^4*c^9 - a^4*b^3*c^8*d - 3*a^5*b^2*c^7*d^2 + 5*a^6*b*c^6*d^3 - 2*a
^7*c^5*d^4)*x^2 + (2*a^3*b^4*c^8*d - 5*a^4*b^3*c^7*d^2 + 3*a^5*b^2*c^6*d^3
+ a^6*b*c^5*d^4 - a^7*c^4*d^5)*x), 1/4*(3*(21*a^3*b^3*c^2*d^4 - 24*a^4*b^2*
c*d^5 + 8*a^5*b*d^6 + (21*a^4*b^2*c^4*d^2 - 24*a^5*b*c^3*d^3 + 8*a^6*c^2*d^
4)*x^3 + (21*a^3*b^3*c^4*d^2 + 18*a^4*b^2*c^3*d^3 - 40*a^5*b*c^2*d^4 + 16*a
^6*c*d^5)*x^2 + (42*a^3*b^3*c^3*d^3 - 27*a^4*b^2*c^2*d^4 - 8*a^5*b*c*d^5 +
8*a^6*d^6)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))
*sqrt((a*x + b)/x)/(a*d*x + b*d)) + 12*(b^5*c^4*d^2 - a*b^4*c^3*d^3 - 3*a^2
*b^3*c^2*d^4 + 5*a^3*b^2*c*d^5 - 2*a^4*b*d^6 + (a*b^4*c^6 - a^2*b^3*c^5*d -
 3*a^3*b^2*c^4*d^2 + 5*a^4*b*c^3*d^3 - 2*a^5*c^2*d^4)*x^3 + (b^5*c^6 + a*b^
4*c^5*d - 5*a^2*b^3*c^4*d^2 - a^3*b^2*c^3*d^3 + 8*a^4*b*c^2*d^4 - 4*a^5*c*d
^5)*x^2 + (2*b^5*c^5*d - a*b^4*c^4*d^2 - 7*a^2*b^3*c^3*d^3 + 7*a^3*b^2*c^2*
d^4 + a^4*b*c*d^5 - 2*a^5*d^6)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x
)/a) + (4*(a^2*b^3*c^6 - 3*a^3*b^2*c^5*d + 3*a^4*b*c^4*d^2 - a^5*c^3*d^3)*x
^4 + (12*a*b^4*c^6 - 4*a^2*b^3*c^5*d - 12*a^3*b^2*c^4*d^2 + 37*a^4*b*c^3*d^
3 - 18*a^5*c^2*d^4)*x^3 + (24*a*b^4*c^5*d - 20*a^2*b^3*c^4*d^2 + 29*a^3*b^2
*c^3*d^3 + 9*a^4*b*c^2*d^4 - 12*a^5*c*d^5)*x^2 + 3*(4*a*b^4*c^4*d^2 - 4*a^2
*b^3*c^3*d^3 + 9*a^3*b^2*c^2*d^4 - 4*a^4*b*c*d^5)*x)*sqrt((a*x + b)/x))/(a^
3*b^4*c^7*d^2 - 3*a^4*b^3*c^6*d^3 + 3*a^5*b^2*c^5*d^4 - a^6*b*c^4*d^5 + (a^
4*b^3*c^9 - 3*a^5*b^2*c^8*d + 3*a^6*b*c^7*d^2 - a^7*c^6*d^3)*x^3 + (a^3*b^4
*c^9 - a^4*b^3*c^8*d - 3*a^5*b^2*c^7*d^2 + 5*a^6*b*c^6*d^3 - 2*a^7*c^5*d^4)
*x^2 + (2*a^3*b^4*c^8*d - 5*a^4*b^3*c^7*d^2 + 3*a^5*b^2*c^6*d^3 + a^6*b*c^5
*d^4 - a^7*c^4*d^5)*x)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(3/2)/(c+d/x)**3,x)
```

```
[Out] Exception raised: ValueError
```

Giac [A] time = 1.20721, size = 678, normalized size = 2.12

$$\frac{1}{4}\,b\left(\frac{3\left(21\,b^2c^2d^3-24\,abcd^4+8\,a^2d^5\right)\arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^2}}\right)}{\left(b^4c^7-3\,ab^3c^6d+3\,a^2b^2c^5d^2-a^3bc^4d^3\right)\sqrt{bcd-ad^2}}+\frac{4\left(2\,ab^3c^3-\frac{3\,(ax+b)b^3c^3}{x}+\frac{3\,(ax+b)ab^2c^2d}{x}-\frac{3\,(ax+b)a^2bcd^2}{x}+\frac{(ax+}{}\right.}{\left(a^2b^3c^6-3\,a^3b^2c^5d+3\,a^4bc^4d^2-a^5c^3d^3\right)\left(a\sqrt{\frac{ax+b}{x}}-\frac{(ax+b)}{}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/(c+d/x)^3,x, algorithm="giac")
```

```
[Out] 1/4*b*(3*(21*b^2*c^2*d^3 - 24*a*b*c*d^4 + 8*a^2*d^5)*arctan(d*sqrt((a*x + b
)/x)/sqrt(b*c*d - a*d^2))/((b^4*c^7 - 3*a*b^3*c^6*d + 3*a^2*b^2*c^5*d^2 - a
^3*b*c^4*d^3)*sqrt(b*c*d - a*d^2)) + 4*(2*a*b^3*c^3 - 3*(a*x + b)*b^3*c^3/x
 + 3*(a*x + b)*a*b^2*c^2*d/x - 3*(a*x + b)*a^2*b*c*d^2/x + (a*x + b)*a^3*d^
3/x)/((a^2*b^3*c^6 - 3*a^3*b^2*c^5*d + 3*a^4*b*c^4*d^2 - a^5*c^3*d^3)*(a*sq
rt((a*x + b)/x) - (a*x + b)*sqrt((a*x + b)/x)/x)) + (17*b^2*c^2*d^3*sqrt((a
*x + b)/x) - 25*a*b*c*d^4*sqrt((a*x + b)/x) + 8*a^2*d^5*sqrt((a*x + b)/x) +
 15*(a*x + b)*b*c*d^4*sqrt((a*x + b)/x)/x - 8*(a*x + b)*a*d^5*sqrt((a*x + b
)/x)/x)/((b^3*c^6 - 3*a*b^2*c^5*d + 3*a^2*b*c^4*d^2 - a^3*c^3*d^3)*(b*c - a
*d + (a*x + b)*d/x)^2) + 12*(b*c + 2*a*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a)
)/(sqrt(-a)*a^2*b*c^4))
```

3.259 $\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{5/2}} \, dx$

Optimal. Leaf size=143

$$\frac{(bc-ad)\left(-2a^2bd(5cx+3d)-4a^3d^2x+ab^2c(20cx-3d)+15b^3c^2\right)}{3a^3b^2x\left(a+\frac{b}{x}\right)^{3/2}}-\frac{c^2(5bc-6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{cx\left(c+\frac{d}{x}\right)^2}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

```
[Out] (c*(c + d/x)^2*x)/(a*(a + b/x)^(3/2)) + ((b*c - a*d)*(15*b^3*c^2 - 4*a^3*d^
2*x - 2*a^2*b*d*(3*d + 5*c*x) + a*b^2*c*(-3*d + 20*c*x)))/(3*a^3*b^2*(a + b
/x)^(3/2)*x) - (c^2*(5*b*c - 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(7/2)
```

Rubi [A] time = 0.151227, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {375, 98, 145, 63, 208}

$$\frac{(bc-ad)\left(-2a^2bd(5cx+3d)-4a^3d^2x+ab^2c(20cx-3d)+15b^3c^2\right)}{3a^3b^2x\left(a+\frac{b}{x}\right)^{3/2}}-\frac{c^2(5bc-6ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{cx\left(c+\frac{d}{x}\right)^2}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d/x)^3/(a + b/x)^(5/2),x]
```

```
[Out] (c*(c + d/x)^2*x)/(a*(a + b/x)^(3/2)) + ((b*c - a*d)*(15*b^3*c^2 - 4*a^3*d^
2*x - 2*a^2*b*d*(3*d + 5*c*x) + a*b^2*c*(-3*d + 20*c*x)))/(3*a^3*b^2*(a + b
/x)^(3/2)*x) - (c^2*(5*b*c - 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(7/2)
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 145

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(
n + 2) - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h)
 + d*e*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(
```

```
f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)*(
a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), x]
 + Dist[(f*h)/b^2 - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1
) - d*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)
)))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), Int[(a + b*x)^(m + 2)*(c + d*x)^n,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[
m + n + 3, 0] &&  !LtQ[n, -2]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(c+\frac{d}{x}\right)^3}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx &= -\text{Subst}\left(\int \frac{(c+dx)^3}{x^2(a+bx)^{5/2}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{c\left(c+\frac{d}{x}\right)^2 x}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{(c+dx)\left(\frac{1}{2}c(5bc-6ad)+\frac{1}{2}d(bc-2ad)x\right)}{x(a+bx)^{5/2}}\,dx, x, \frac{1}{x}\right)}{a}\\
&= \frac{c\left(c+\frac{d}{x}\right)^2 x}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{(bc-ad)\left(15b^3c^2-4a^3d^2x-ab^2c(3d-20cx)-2a^2bd(3d+5cx)\right)}{3a^3b^2\left(a+\frac{b}{x}\right)^{3/2}x} + \frac{\left(c^2(5bc-6ad)\right)\cdots}{\cdots}\\
&= \frac{c\left(c+\frac{d}{x}\right)^2 x}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{(bc-ad)\left(15b^3c^2-4a^3d^2x-ab^2c(3d-20cx)-2a^2bd(3d+5cx)\right)}{3a^3b^2\left(a+\frac{b}{x}\right)^{3/2}x} + \frac{\left(c^2(5bc-6ad)\right)\cdots}{\cdots}\\
&= \frac{c\left(c+\frac{d}{x}\right)^2 x}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{(bc-ad)\left(15b^3c^2-4a^3d^2x-ab^2c(3d-20cx)-2a^2bd(3d+5cx)\right)}{3a^3b^2\left(a+\frac{b}{x}\right)^{3/2}x} - \frac{c^2(5bc-6ad)\,\text{t}\cdots}{a^{7/}\cdots}
\end{aligned}$$

Mathematica [A] time = 0.294236, size = 145, normalized size = 1.01

$$\frac{\frac{4a^5d^3x}{b^2}+2a^2bc^2(10cx-9d)+\frac{6a^4d^2(cx+d)}{b}+3a^3c^2x(cx-8d)+15ab^2c^3+3ac^2\sqrt{\frac{b}{ax}+1}(ax+b)(6ad-5bc)\tanh^{-1}\left(\sqrt{\frac{b}{ax}\cdots}\right.}{3a^4\sqrt{a+\frac{b}{x}}(ax+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d/x)^3/(a + b/x)^(5/2),x]
```

```
[Out] (15*a*b^2*c^3 + (4*a^5*d^3*x)/b^2 + 3*a^3*c^2*x*(-8*d + c*x) + (6*a^4*d^2*(
d + c*x))/b + 2*a^2*b*c^2*(-9*d + 10*c*x) + 3*a*c^2*(-5*b*c + 6*a*d)*Sqrt[1
 + b/(a*x)]*(b + a*x)*ArcTanh[Sqrt[1 + b/(a*x)]])/(3*a^4*Sqrt[a + b/x]*(b +
 a*x))
```

Maple [B] time = 0.014, size = 1150, normalized size = 8.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)^3/(a+b/x)^(5/2),x)
```

```
[Out] 1/6*((a*x+b)/x)^(1/2)*x/a^(7/2)*(3*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*
x+b)/a^(1/2))*a^3*b^4*d^3+30*a^(1/2)*((a*x+b)*x)^(1/2)*b^6*c^3-3*ln(1/2*(2*
((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*b^4*d^3-16*a^(9/2)*((a*x+b)
*x)^(3/2)*b*d^3-20*a^(3/2)*((a*x+b)*x)^(3/2)*b^4*c^3+6*(a*x^2+b*x)^(1/2)*a^
(7/2)*b^3*d^3+6*a^(7/2)*((a*x+b)*x)^(1/2)*b^3*d^3+6*(a*x^2+b*x)^(1/2)*a^(13
/2)*x^3*d^3+6*a^(13/2)*((a*x+b)*x)^(1/2)*x^3*d^3-12*a^(11/2)*((a*x+b)*x)^(3
/2)*x*d^3-15*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*b^7*c^3+
18*a^(9/2)*((a*x+b)*x)^(1/2)*x*b^2*d^3+90*a^(3/2)*((a*x+b)*x)^(1/2)*x*b^5*c
^3-36*a^(3/2)*((a*x+b)*x)^(1/2)*b^5*c^2*d+18*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^
(1/2)+2*a*x+b)/a^(1/2))*a*b^6*c^2*d+3*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2
*a*x+b)/a^(1/2))*x^3*a^6*b*d^3-3*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+
b)/a^(1/2))*x^3*a^6*b*d^3-15*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a
^(1/2))*x^3*a^3*b^4*c^3+9*ln(1/2*(2*(a*x^2+b*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1
/2))*x^2*a^5*b^2*d^3-9*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2)
)*x^2*a^5*b^2*d^3-45*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*
x^2*a^2*b^5*c^3+90*a^(5/2)*((a*x+b)*x)^(1/2)*x^2*b^4*c^3+18*(a*x^2+b*x)^(1/
2)*a^(9/2)*x*b^2*d^3+12*a^(7/2)*((a*x+b)*x)^(3/2)*b^2*c*d^2+24*a^(5/2)*((a*
x+b)*x)^(3/2)*b^3*c^2*d-9*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1
/2))*x*a^4*b^3*d^3-45*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))
*x*a*b^6*c^3+30*a^(7/2)*((a*x+b)*x)^(1/2)*x^3*b^3*c^3+9*ln(1/2*(2*(a*x^2+b*
x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x*a^4*b^3*d^3+18*(a*x^2+b*x)^(1/2)*a^(11
/2)*x^2*b*d^3-24*a^(5/2)*((a*x+b)*x)^(3/2)*x*b^3*c^3+18*a^(11/2)*((a*x+b)*x
)^(1/2)*x^2*b*d^3+18*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*
x^3*a^4*b^3*c^2*d-108*a^(7/2)*((a*x+b)*x)^(1/2)*x^2*b^3*c^2*d-108*a^(5/2)*(
(a*x+b)*x)^(1/2)*x*b^4*c^2*d-36*a^(9/2)*((a*x+b)*x)^(1/2)*x^3*b^2*c^2*d+36*
a^(7/2)*((a*x+b)*x)^(3/2)*x*b^2*c^2*d+54*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2
)+2*a*x+b)/a^(1/2))*x^2*a^3*b^4*c^2*d+54*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2
)+2*a*x+b)/a^(1/2))*x*a^2*b^5*c^2*d)/((a*x+b)*x)^(1/2)/b^3/(a*x+b)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^3/(a+b/x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.32662, size = 1002, normalized size = 7.01

$$\left[-\frac{3\left(5\,b^{5}c^{3}-6\,ab^{4}c^{2}d+\left(5\,a^{2}b^{3}c^{3}-6\,a^{3}b^{2}c^{2}d\right)x^{2}+2\left(5\,ab^{4}c^{3}-6\,a^{2}b^{3}c^{2}d\right)x\right)\sqrt{a}\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)-2\left(3\,a\right.}{6\left(a^{6}b^{2}x^{2}+2\,a^{5}b^{3}x\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^3/(a+b/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(5*b^5*c^3 - 6*a*b^4*c^2*d + (5*a^2*b^3*c^3 - 6*a^3*b^2*c^2*d)*x^2
 + 2*(5*a*b^4*c^3 - 6*a^2*b^3*c^2*d)*x)*sqrt(a)*log(2*a*x + 2*sqrt(a)*x*sqr
t((a*x + b)/x) + b) - 2*(3*a^3*b^2*c^3*x^3 + 2*(10*a^2*b^3*c^3 - 12*a^3*b^2
*c^2*d + 3*a^4*b*c*d^2 + 2*a^5*d^3)*x^2 + 3*(5*a*b^4*c^3 - 6*a^2*b^3*c^2*d
+ 2*a^4*b*d^3)*x)*sqrt((a*x + b)/x))/(a^6*b^2*x^2 + 2*a^5*b^3*x + a^4*b^4),
 1/3*(3*(5*b^5*c^3 - 6*a*b^4*c^2*d + (5*a^2*b^3*c^3 - 6*a^3*b^2*c^2*d)*x^2
+ 2*(5*a*b^4*c^3 - 6*a^2*b^3*c^2*d)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x +
 b)/x)/a) + (3*a^3*b^2*c^3*x^3 + 2*(10*a^2*b^3*c^3 - 12*a^3*b^2*c^2*d + 3*a
^4*b*c*d^2 + 2*a^5*d^3)*x^2 + 3*(5*a*b^4*c^3 - 6*a^2*b^3*c^2*d + 2*a^4*b*d^
3)*x)*sqrt((a*x + b)/x))/(a^6*b^2*x^2 + 2*a^5*b^3*x + a^4*b^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx+d)^{3}}{x^{3}\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)**3/(a+b/x)**(5/2),x)
```

```
[Out] Integral((c*x + d)**3/(x**3*(a + b/x)**(5/2)), x)
```

Giac [A] time = 1.19536, size = 267, normalized size = 1.87

$$-\frac{1}{3}\,b\left(\frac{3\,c^{3}\sqrt{\frac{ax+b}{x}}}{\left(a-\frac{ax+b}{x}\right)a^{3}}-\frac{3\left(5\,bc^{3}-6\,ac^{2}d\right)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}a^{3}b}-\frac{2\left(ab^{3}c^{3}-3\,a^{2}b^{2}c^{2}d+3\,a^{3}bcd^{2}-a^{4}d^{3}+\frac{6\,(ax+b)b^{3}c^{3}}{x}-\frac{9\,(ax+b\right.}{}}{(ax+b)a^{3}b^{3}\sqrt{\frac{ax+b}{x}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^3/(a+b/x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*b*(3*c^3*sqrt((a*x + b)/x)/((a - (a*x + b)/x)*a^3) - 3*(5*b*c^3 - 6*a*
c^2*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^3*b) - 2*(a*b^3*c^3 -
 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3 + 6*(a*x + b)*b^3*c^3/x - 9*(a*x
 + b)*a*b^2*c^2*d/x + 3*(a*x + b)*a^3*d^3/x)*x/((a*x + b)*a^3*b^3*sqrt((a*x
 + b)/x)))
```

3.260 $\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx$

Optimal. Leaf size=122

$$\frac{2a^2d^2+bc(5bc-4ad)}{3a^2b\left(a+\frac{b}{x}\right)^{3/2}}+\frac{c(5bc-4ad)}{a^3\sqrt{a+\frac{b}{x}}}-\frac{c(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{c^2x}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

```
[Out] (2*a^2*d^2 + b*c*(5*b*c - 4*a*d))/(3*a^2*b*(a + b/x)^(3/2)) + (c*(5*b*c - 4
*a*d))/(a^3*Sqrt[a + b/x]) + (c^2*x)/(a*(a + b/x)^(3/2)) - (c*(5*b*c - 4*a*
d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(7/2)
```

Rubi [A] time = 0.0951476, antiderivative size = 118, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {375, 89, 78, 51, 63, 208}

$$\frac{\frac{c(5bc-4ad)}{a^2}+\frac{2d^2}{b}}{3\left(a+\frac{b}{x}\right)^{3/2}}+\frac{c(5bc-4ad)}{a^3\sqrt{a+\frac{b}{x}}}-\frac{c(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{c^2x}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d/x)^2/(a + b/x)^(5/2),x]
```

```
[Out] ((2*d^2)/b + (c*(5*b*c - 4*a*d))/a^2)/(3*(a + b/x)^(3/2)) + (c*(5*b*c - 4*a
*d))/(a^3*Sqrt[a + b/x]) + (c^2*x)/(a*(a + b/x)^(3/2)) - (c*(5*b*c - 4*a*d)
*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(7/2)
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
```

```
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(c+\frac{d}{x}\right)^2}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx &= -\text{Subst}\left(\int \frac{(c+dx)^2}{x^2(a+bx)^{5/2}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{c^2x}{a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}c(5bc-4ad)+ad^2x}{x(a+bx)^{5/2}}\,dx, x, \frac{1}{x}\right)}{a}\\
&= \frac{\frac{2d^2}{b}+\frac{c(5bc-4ad)}{a^2}}{3\left(a+\frac{b}{x}\right)^{3/2}} + \frac{c^2x}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{(c(5bc-4ad))\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)}{2a^2}\\
&= \frac{\frac{2d^2}{b}+\frac{c(5bc-4ad)}{a^2}}{3\left(a+\frac{b}{x}\right)^{3/2}} + \frac{c(5bc-4ad)}{a^3\sqrt{a+\frac{b}{x}}} + \frac{c^2x}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{(c(5bc-4ad))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a^3}\\
&= \frac{\frac{2d^2}{b}+\frac{c(5bc-4ad)}{a^2}}{3\left(a+\frac{b}{x}\right)^{3/2}} + \frac{c(5bc-4ad)}{a^3\sqrt{a+\frac{b}{x}}} + \frac{c^2x}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{(c(5bc-4ad))\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{a^3b}\\
&= \frac{\frac{2d^2}{b}+\frac{c(5bc-4ad)}{a^2}}{3\left(a+\frac{b}{x}\right)^{3/2}} + \frac{c(5bc-4ad)}{a^3\sqrt{a+\frac{b}{x}}} + \frac{c^2x}{a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{c(5bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}
\end{aligned}$$

Mathematica [C] time = 0.0491668, size = 97, normalized size = 0.8

$$\frac{ax\left(2a^2d^2+abc(3cx-4d)+5b^2c^2\right)+3bc(ax+b)(5bc-4ad)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b}{ax}+1\right)}{3a^3b\sqrt{a+\frac{b}{x}}(ax+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d/x)^2/(a + b/x)^(5/2),x]
```

```
[Out] (a*x*(5*b^2*c^2 + 2*a^2*d^2 + a*b*c*(-4*d + 3*c*x)) + 3*b*c*(5*b*c - 4*a*d)
*(b + a*x)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + b/(a*x)])/(3*a^3*b*Sqrt[a +
b/x]*(b + a*x))
```

Maple [B] time = 0.011, size = 588, normalized size = 4.8

$$\frac{x}{6b(ax+b)^3}\sqrt{\frac{ax+b}{x}}\left(12\ln\left(1/2\,\frac{2\sqrt{(ax+b)x}\sqrt{a}+2ax+b}{\sqrt{a}}\right)x^3a^4bcd-15\ln\left(1/2\,\frac{2\sqrt{(ax+b)x}\sqrt{a}+2ax+b}{\sqrt{a}}\right)x^3a^3b^2c^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)^2/(a+b/x)^(5/2),x)
```

```
[Out] 1/6*((a*x+b)/x)^(1/2)*x/a^(7/2)*(12*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a
*x+b)/a^(1/2))*x^3*a^4*b*c*d-15*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b
)/a^(1/2))*x^3*a^3*b^2*c^2-24*a^(9/2)*((a*x+b)*x)^(1/2)*x^3*c*d+30*a^(7/2)*
((a*x+b)*x)^(1/2)*x^3*b*c^2+36*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)
/a^(1/2))*x^2*a^3*b^2*c*d-45*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a
^(1/2))*x^2*a^2*b^3*c^2+24*a^(7/2)*((a*x+b)*x)^(3/2)*x*c*d-24*a^(5/2)*((a*x
+b)*x)^(3/2)*x*b*c^2-72*a^(7/2)*((a*x+b)*x)^(1/2)*x^2*b*c*d+90*a^(5/2)*((a*
x+b)*x)^(1/2)*x^2*b^2*c^2+36*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a
^(1/2))*x*a^2*b^3*c*d-45*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/
2))*x*a*b^4*c^2+4*a^(7/2)*((a*x+b)*x)^(3/2)*d^2+16*a^(5/2)*((a*x+b)*x)^(3/2
)*b*c*d-20*a^(3/2)*((a*x+b)*x)^(3/2)*b^2*c^2-72*a^(5/2)*((a*x+b)*x)^(1/2)*x
*b^2*c*d+90*a^(3/2)*((a*x+b)*x)^(1/2)*x*b^3*c^2+12*ln(1/2*(2*((a*x+b)*x)^(1
/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a*b^4*c*d-15*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1
/2)+2*a*x+b)/a^(1/2))*b^5*c^2-24*a^(3/2)*((a*x+b)*x)^(1/2)*b^3*c*d+30*a^(1/
2)*((a*x+b)*x)^(1/2)*b^4*c^2)/((a*x+b)*x)^(1/2)/b/(a*x+b)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2/(a+b/x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.36801, size = 864, normalized size = 7.08

$$\left[-\frac{3\left(5\,b^4c^2-4\,ab^3cd+\left(5\,a^2b^2c^2-4\,a^3bcd\right)x^2+2\left(5\,ab^3c^2-4\,a^2b^2cd\right)x\right)\sqrt{a}\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)-2\left(3\,a^3bc^2\right.}{6\left(a^6bx^2+2\,a^5b^2x+a^4b^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2/(a+b/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(5*b^4*c^2 - 4*a*b^3*c*d + (5*a^2*b^2*c^2 - 4*a^3*b*c*d)*x^2 + 2*(
5*a*b^3*c^2 - 4*a^2*b^2*c*d)*x)*sqrt(a)*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x +
 b)/x) + b) - 2*(3*a^3*b*c^2*x^3 + 2*(10*a^2*b^2*c^2 - 8*a^3*b*c*d + a^4*d^
2)*x^2 + 3*(5*a*b^3*c^2 - 4*a^2*b^2*c*d)*x)*sqrt((a*x + b)/x))/(a^6*b*x^2 +
 2*a^5*b^2*x + a^4*b^3), 1/3*(3*(5*b^4*c^2 - 4*a*b^3*c*d + (5*a^2*b^2*c^2 -
 4*a^3*b*c*d)*x^2 + 2*(5*a*b^3*c^2 - 4*a^2*b^2*c*d)*x)*sqrt(-a)*arctan(sqrt
(-a)*sqrt((a*x + b)/x)/a) + (3*a^3*b*c^2*x^3 + 2*(10*a^2*b^2*c^2 - 8*a^3*b*
c*d + a^4*d^2)*x^2 + 3*(5*a*b^3*c^2 - 4*a^2*b^2*c*d)*x)*sqrt((a*x + b)/x))/
(a^6*b*x^2 + 2*a^5*b^2*x + a^4*b^3)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx+d)^2}{x^2\left(a+\frac{b}{x}\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)**2/(a+b/x)**(5/2),x)
```

```
[Out] Integral((c*x + d)**2/(x**2*(a + b/x)**(5/2)), x)
```

Giac [A] time = 1.20202, size = 217, normalized size = 1.78

$$-\frac{1}{3}\,b\left(\frac{3\,c^2\sqrt{\frac{ax+b}{x}}}{\left(a-\frac{ax+b}{x}\right)a^3}-\frac{3\left(5\,bc^2-4\,acd\right)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3b}-\frac{2\left(ab^2c^2-2\,a^2bcd+a^3d^2+\frac{6\,(ax+b)b^2c^2}{x}-\frac{6\,(ax+b)abcd}{x}\right)x}{(ax+b)a^3b^2\sqrt{\frac{ax+b}{x}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2/(a+b/x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*b*(3*c^2*sqrt((a*x + b)/x)/((a - (a*x + b)/x)*a^3) - 3*(5*b*c^2 - 4*a*
c*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^3*b) - 2*(a*b^2*c^2 - 2
*a^2*b*c*d + a^3*d^2 + 6*(a*x + b)*b^2*c^2/x - 6*(a*x + b)*a*b*c*d/x)*x/((a
*x + b)*a^3*b^2*sqrt((a*x + b)/x)))
```

3.261 $\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx$

Optimal. Leaf size=103

$$\frac{5bc-2ad}{a^3\sqrt{a+\frac{b}{x}}}+\frac{5bc-2ad}{3a^2\left(a+\frac{b}{x}\right)^{3/2}}-\frac{(5bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{cx}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

```
[Out] (5*b*c - 2*a*d)/(3*a^2*(a + b/x)^(3/2)) + (5*b*c - 2*a*d)/(a^3*Sqrt[a + b/x
]) + (c*x)/(a*(a + b/x)^(3/2)) - ((5*b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqr
t[a]])/a^(7/2)
```

Rubi [A] time = 0.0654625, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {375, 78, 51, 63, 208}

$$\frac{5bc-2ad}{a^3\sqrt{a+\frac{b}{x}}}+\frac{5bc-2ad}{3a^2\left(a+\frac{b}{x}\right)^{3/2}}-\frac{(5bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{cx}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d/x)/(a + b/x)^(5/2),x]
```

```
[Out] (5*b*c - 2*a*d)/(3*a^2*(a + b/x)^(3/2)) + (5*b*c - 2*a*d)/(a^3*Sqrt[a + b/x
]) + (c*x)/(a*(a + b/x)^(3/2)) - ((5*b*c - 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqr
t[a]])/a^(7/2)
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+\frac{d}{x}}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx &= -\operatorname{Subst}\left(\int \frac{c+dx}{x^2(a+bx)^{5/2}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{cx}{a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{\left(-\frac{5bc}{2}+ad\right)\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx, x, \frac{1}{x}\right)}{a}\\
&= \frac{5bc-2ad}{3a^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{cx}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{(5bc-2ad)\operatorname{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)}{2a^2}\\
&= \frac{5bc-2ad}{3a^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{5bc-2ad}{a^3\sqrt{a+\frac{b}{x}}} + \frac{cx}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{(5bc-2ad)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a^3}\\
&= \frac{5bc-2ad}{3a^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{5bc-2ad}{a^3\sqrt{a+\frac{b}{x}}} + \frac{cx}{a\left(a+\frac{b}{x}\right)^{3/2}} + \frac{(5bc-2ad)\operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{a^3b}\\
&= \frac{5bc-2ad}{3a^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{5bc-2ad}{a^3\sqrt{a+\frac{b}{x}}} + \frac{cx}{a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{(5bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}
\end{aligned}
$$

Mathematica [C] time = 0.0231468, size = 60, normalized size = 0.58

$$
\frac{x\left((5bc-2ad)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b}{ax}+1\right)+3acx\right)}{3a^2\sqrt{a+\frac{b}{x}}(ax+b)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d/x)/(a + b/x)^(5/2),x]
```

```
[Out] (x*(3*a*c*x + (5*b*c - 2*a*d)*Hypergeometric2F1[-3/2, 1, -1/2, 1 + b/(a*x)]
))/(3*a^2*Sqrt[a + b/x]*(b + a*x))
```

Maple [B] time = 0.011, size = 541, normalized size = 5.3

$$\frac{x}{6\,b\,(ax+b)^3}\sqrt{\frac{ax+b}{x}}\left(6\,\ln\left(1/2\,\frac{2\,\sqrt{(ax+b)\,x}\sqrt{a}+2\,ax+b}{\sqrt{a}}\right)x^3a^4bd-15\,\ln\left(1/2\,\frac{2\,\sqrt{(ax+b)\,x}\sqrt{a}+2\,ax+b}{\sqrt{a}}\right)x^3a^3b^2c-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)/(a+b/x)^(5/2),x)
```

```
[Out] 1/6*((a*x+b)/x)^(1/2)*x/a^(7/2)*(6*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*
x+b)/a^(1/2))*x^3*a^4*b*d-15*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a
^(1/2))*x^3*a^3*b^2*c-12*a^(9/2)*((a*x+b)*x)^(1/2)*x^3*d+30*a^(7/2)*((a*x+b
)*x)^(1/2)*x^3*b*c+18*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))
*x^2*a^3*b^2*d-45*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x^2
*a^2*b^3*c+12*a^(7/2)*((a*x+b)*x)^(3/2)*x*d-24*a^(5/2)*((a*x+b)*x)^(3/2)*x*
b*c-36*a^(7/2)*((a*x+b)*x)^(1/2)*x^2*b*d+90*a^(5/2)*((a*x+b)*x)^(1/2)*x^2*b
^2*c+18*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x*a^2*b^3*d-4
5*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x*a*b^4*c+8*a^(5/2)
*((a*x+b)*x)^(3/2)*b*d-20*a^(3/2)*((a*x+b)*x)^(3/2)*b^2*c-36*a^(5/2)*((a*x+
b)*x)^(1/2)*x*b^2*d+90*a^(3/2)*((a*x+b)*x)^(1/2)*x*b^3*c+6*ln(1/2*(2*((a*x+
b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a*b^4*d-15*ln(1/2*(2*((a*x+b)*x)^(1/2
)*a^(1/2)+2*a*x+b)/a^(1/2))*b^5*c-12*a^(3/2)*((a*x+b)*x)^(1/2)*b^3*d+30*a^(
1/2)*((a*x+b)*x)^(1/2)*b^4*c)/((a*x+b)*x)^(1/2)/b/(a*x+b)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.39597, size = 726, normalized size = 7.05

$$\left[-\frac{3\left(5\,b^3c-2\,ab^2d+\left(5\,a^2bc-2\,a^3d\right)x^2+2\left(5\,ab^2c-2\,a^2bd\right)x\right)\sqrt{a}\log\left(2\,ax+2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)-2\left(3\,a^3cx^3+4\left(5\,a^2bc\right.\right.}{6\left(a^6x^2+2\,a^5bx+a^4b^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(5*b^3*c - 2*a*b^2*d + (5*a^2*b*c - 2*a^3*d)*x^2 + 2*(5*a*b^2*c -
2*a^2*b*d)*x)*sqrt(a)*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 2*(3
*a^3*c*x^3 + 4*(5*a^2*b*c - 2*a^3*d)*x^2 + 3*(5*a*b^2*c - 2*a^2*b*d)*x)*sqr
t((a*x + b)/x))/(a^6*x^2 + 2*a^5*b*x + a^4*b^2), 1/3*(3*(5*b^3*c - 2*a*b^2*
d + (5*a^2*b*c - 2*a^3*d)*x^2 + 2*(5*a*b^2*c - 2*a^2*b*d)*x)*sqrt(-a)*arcta
n(sqrt(-a)*sqrt((a*x + b)/x)/a) + (3*a^3*c*x^3 + 4*(5*a^2*b*c - 2*a^3*d)*x^
2 + 3*(5*a*b^2*c - 2*a^2*b*d)*x)*sqrt((a*x + b)/x))/(a^6*x^2 + 2*a^5*b*x +
```

```
a^4*b^2)]
```

Sympy [B] time = 57.6507, size = 1479, normalized size = 14.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)**(5/2),x)
```

```
[Out] c*(6*a**17*x**4*sqrt(1 + b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 +
 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 46*a**16*b*x**3*sqrt(1 + b/(a*x)
)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/
2)*b**3) + 15*a**16*b*x**3*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*
x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) - 30*a**16*b*x**3*log(sqrt(1
 + b/(a*x)) + 1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**
2*x + 6*a**(33/2)*b**3) + 70*a**15*b**2*x**2*sqrt(1 + b/(a*x))/(6*a**(39/2)
*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 45*
a**15*b**2*x**2*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a
**(35/2)*b**2*x + 6*a**(33/2)*b**3) - 90*a**15*b**2*x**2*log(sqrt(1 + b/(a*
x)) + 1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*
a**(33/2)*b**3) + 30*a**14*b**3*x*sqrt(1 + b/(a*x))/(6*a**(39/2)*x**3 + 18*
a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 45*a**14*b**3*
x*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*
x + 6*a**(33/2)*b**3) - 90*a**14*b**3*x*log(sqrt(1 + b/(a*x)) + 1)/(6*a**(3
9/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) +
 15*a**13*b**4*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a*
*(35/2)*b**2*x + 6*a**(33/2)*b**3) - 30*a**13*b**4*log(sqrt(1 + b/(a*x)) +
1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33
/2)*b**3)) + d*(-8*a**7*x**3*sqrt(1 + b/(a*x))/(3*a**(19/2)*x**3 + 9*a**(17
/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b**3) - 3*a**7*x**3*log(b/(a*
x))/(3*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/
2)*b**3) + 6*a**7*x**3*log(sqrt(1 + b/(a*x)) + 1)/(3*a**(19/2)*x**3 + 9*a**
(17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b**3) - 14*a**6*b*x**2*sqr
t(1 + b/(a*x))/(3*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b**2*x
+ 3*a**(13/2)*b**3) - 9*a**6*b*x**2*log(b/(a*x))/(3*a**(19/2)*x**3 + 9*a**(
17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b**3) + 18*a**6*b*x**2*log(
sqrt(1 + b/(a*x)) + 1)/(3*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)
*b**2*x + 3*a**(13/2)*b**3) - 6*a**5*b**2*x*sqrt(1 + b/(a*x))/(3*a**(19/2)*
x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b**3) - 9*a**5
*b**2*x*log(b/(a*x))/(3*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b
**2*x + 3*a**(13/2)*b**3) + 18*a**5*b**2*x*log(sqrt(1 + b/(a*x)) + 1)/(3*a*
*(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b**3)
- 3*a**4*b**3*log(b/(a*x))/(3*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(1
5/2)*b**2*x + 3*a**(13/2)*b**3) + 6*a**4*b**3*log(sqrt(1 + b/(a*x)) + 1)/(3
*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b**
3))
```

Giac [A] time = 1.19383, size = 185, normalized size = 1.8

$$-\frac{1}{3}\,b\left(\frac{3\,c\sqrt{\frac{ax+b}{x}}}{\left(a-\frac{ax+b}{x}\right)a^3}-\frac{3\,(5\,bc-2\,ad)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3b}-\frac{2\left(abc-a^2d+\frac{6\,(ax+b)bc}{x}-\frac{3\,(ax+b)ad}{x}\right)x}{(ax+b)a^3b\sqrt{\frac{ax+b}{x}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*b*(3*c*sqrt((a*x + b)/x)/((a - (a*x + b)/x)*a^3) - 3*(5*b*c - 2*a*d)*a
rctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^3*b) - 2*(a*b*c - a^2*d + 6*(
a*x + b)*b*c/x - 3*(a*x + b)*a*d/x)*x/((a*x + b)*a^3*b*sqrt((a*x + b)/x)))
```

3.262 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx$

Optimal. Leaf size=79

$$\frac{5b}{a^3\sqrt{a+\frac{b}{x}}}+\frac{5b}{3a^2\left(a+\frac{b}{x}\right)^{3/2}}-\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}+\frac{x}{a\left(a+\frac{b}{x}\right)^{3/2}}$$

```
[Out] (5*b)/(3*a^2*(a + b/x)^(3/2)) + (5*b)/(a^3*Sqrt[a + b/x]) + x/(a*(a + b/x)^
(3/2)) - (5*b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(7/2)
```

Rubi [A] time = 0.0377736, antiderivative size = 82, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {242, 51, 63, 208}

$$\frac{5x\sqrt{a+\frac{b}{x}}}{a^3}-\frac{10x}{3a^2\sqrt{a+\frac{b}{x}}}-\frac{5b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}-\frac{2x}{3a\left(a+\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^(-5/2),x]
```

```
[Out] (-2*x)/(3*a*(a + b/x)^(3/2)) - (10*x)/(3*a^2*Sqrt[a + b/x]) + (5*Sqrt[a + b
/x]*x)/a^3 - (5*b*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(7/2)
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}}\,dx = -\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{5/2}}\,dx, x, \frac{1}{x}\right)$$
$$= -\frac{2x}{3a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{5\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}}\,dx, x, \frac{1}{x}\right)}{3a}$$
$$= -\frac{2x}{3a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{10x}{3a^2\sqrt{a+\frac{b}{x}}} - \frac{5\,\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{a^2}$$
$$= -\frac{2x}{3a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{10x}{3a^2\sqrt{a+\frac{b}{x}}} + \frac{5\sqrt{a+\frac{b}{x}}\,x}{a^3} + \frac{(5b)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \frac{1}{x}\right)}{2a^3}$$
$$= -\frac{2x}{3a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{10x}{3a^2\sqrt{a+\frac{b}{x}}} + \frac{5\sqrt{a+\frac{b}{x}}\,x}{a^3} + \frac{5\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+\frac{b}{x}}\right)}{a^3}$$
$$= -\frac{2x}{3a\left(a+\frac{b}{x}\right)^{3/2}} - \frac{10x}{3a^2\sqrt{a+\frac{b}{x}}} + \frac{5\sqrt{a+\frac{b}{x}}\,x}{a^3} - \frac{5b\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}$$

Mathematica [C] time = 0.0157974, size = 38, normalized size = 0.48

$$\frac{2b\,{}_2F_1\left(-\frac{3}{2},2;-\frac{1}{2};\frac{a+\frac{b}{x}}{a}\right)}{3a^2\left(a+\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(-5/2),x]
```

```
[Out] (2*b*Hypergeometric2F1[-3/2, 2, -1/2, (a + b/x)/a])/(3*a^2*(a + b/x)^(3/2))
```

Maple [B] time = 0.008, size = 271, normalized size = 3.4

$$-\frac{x}{6\,(ax+b)^3}\sqrt{\frac{ax+b}{x}}\left(15\,\ln\left(1/2\,\frac{2\,\sqrt{(ax+b)\,x}\sqrt{a}+2\,ax+b}{\sqrt{a}}\right)x^3a^3b-30\,a^{7/2}\sqrt{(ax+b)\,x}x^3+45\,\ln\left(1/2\,\frac{2\,\sqrt{(ax+b)\,x}\sqrt{}}{\sqrt{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(5/2),x)
```

```
[Out] -1/6*((a*x+b)/x)^(1/2)*x/a^(7/2)*(15*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*
a*x+b)/a^(1/2))*x^3*a^3*b-30*a^(7/2)*((a*x+b)*x)^(1/2)*x^3+45*ln(1/2*(2*((a
*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*x^2*a^2*b^2+24*a^(5/2)*((a*x+b)*x)
^(3/2)*x-90*a^(5/2)*((a*x+b)*x)^(1/2)*x^2*b+45*ln(1/2*(2*((a*x+b)*x)^(1/2)*
a^(1/2)+2*a*x+b)/a^(1/2))*x*a*b^3+20*b*a^(3/2)*((a*x+b)*x)^(3/2)-90*a^(3/2)
```

```
*((a*x+b)*x)^(1/2)*x*b^2+15*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^
(1/2))*b^4-30*a^(1/2)*((a*x+b)*x)^(1/2)*b^3)/((a*x+b)*x)^(1/2)/(a*x+b)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.27383, size = 501, normalized size = 6.34

$$\left[\frac{15\left(a^2bx^2+2\,ab^2x+b^3\right)\sqrt{a}\log\left(2\,ax-2\,\sqrt{a}x\sqrt{\frac{ax+b}{x}}+b\right)+2\left(3\,a^3x^3+20\,a^2bx^2+15\,ab^2x\right)\sqrt{\frac{ax+b}{x}}}{6\left(a^6x^2+2\,a^5bx+a^4b^2\right)},\ \frac{15\left(a^2bx^2+2\,ab^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(15*(a^2*b*x^2 + 2*a*b^2*x + b^3)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt
((a*x + b)/x) + b) + 2*(3*a^3*x^3 + 20*a^2*b*x^2 + 15*a*b^2*x)*sqrt((a*x +
b)/x))/(a^6*x^2 + 2*a^5*b*x + a^4*b^2), 1/3*(15*(a^2*b*x^2 + 2*a*b^2*x + b^
3)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (3*a^3*x^3 + 20*a^2*b*x^
2 + 15*a*b^2*x)*sqrt((a*x + b)/x))/(a^6*x^2 + 2*a^5*b*x + a^4*b^2)]
```

Sympy [B] time = 4.93673, size = 774, normalized size = 9.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(5/2),x)
```

```
[Out] 6*a**17*x**4*sqrt(1 + b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18
*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 46*a**16*b*x**3*sqrt(1 + b/(a*x))/(
6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*
b**3) + 15*a**16*b*x**3*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**
2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) - 30*a**16*b*x**3*log(sqrt(1 +
b/(a*x)) + 1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x
 + 6*a**(33/2)*b**3) + 70*a**15*b**2*x**2*sqrt(1 + b/(a*x))/(6*a**(39/2)*x*
*3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 45*a**
15*b**2*x**2*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(
35/2)*b**2*x + 6*a**(33/2)*b**3) - 90*a**15*b**2*x**2*log(sqrt(1 + b/(a*x))
 + 1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**
(33/2)*b**3) + 30*a**14*b**3*x*sqrt(1 + b/(a*x))/(6*a**(39/2)*x**3 + 18*a**
(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 45*a**14*b**3*x*l
```

```
og(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x +
 6*a**(33/2)*b**3) - 90*a**14*b**3*x*log(sqrt(1 + b/(a*x)) + 1)/(6*a**(39/2
)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 15
*a**13*b**4*log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(3
5/2)*b**2*x + 6*a**(33/2)*b**3) - 30*a**13*b**4*log(sqrt(1 + b/(a*x)) + 1)/
(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)
*b**3)
```

Giac [A] time = 1.22499, size = 132, normalized size = 1.67

$$\frac{1}{3}\,b\left(\frac{2\left(a+\frac{6\,(ax+b)}{x}\right)x}{(ax+b)a^3\sqrt{\frac{ax+b}{x}}}+\frac{15\,\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3}-\frac{3\,\sqrt{\frac{ax+b}{x}}}{\left(a-\frac{ax+b}{x}\right)a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*b*(2*(a + 6*(a*x + b)/x)*x/((a*x + b)*a^3*sqrt((a*x + b)/x)) + 15*arcta
n(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^3) - 3*sqrt((a*x + b)/x)/((a - (a
*x + b)/x)*a^3))
```

3.263 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}\,dx$

Optimal. Leaf size=201

$$\frac{b\left(a^2d^2-8abcd+5b^2c^2\right)}{a^3c\sqrt{a+\frac{b}{x}}(bc-ad)^2}-\frac{(2ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^2}+\frac{b(5bc-3ad)}{3a^2c\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)}-\frac{2d^{7/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2(bc-ad)^{5/2}}+\frac{x}{ac\left(a+\right.}$$

```
[Out] (b*(5*b*c - 3*a*d))/(3*a^2*c*(b*c - a*d)*(a + b/x)^(3/2)) + (b*(5*b^2*c^2 -
 8*a*b*c*d + a^2*d^2))/(a^3*c*(b*c - a*d)^2*Sqrt[a + b/x]) + x/(a*c*(a + b/
x)^(3/2)) - (2*d^(7/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c^
2*(b*c - a*d)^(5/2)) - ((5*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(a^
(7/2)*c^2)
```

Rubi [A] time = 0.315132, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {375, 103, 152, 156, 63, 208, 205}

$$\frac{b\left(a^2d^2-8abcd+5b^2c^2\right)}{a^3c\sqrt{a+\frac{b}{x}}(bc-ad)^2}-\frac{(2ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^2}+\frac{b(5bc-3ad)}{3a^2c\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)}-\frac{2d^{7/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2(bc-ad)^{5/2}}+\frac{x}{ac\left(a+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(5/2)*(c + d/x)),x]
```

```
[Out] (b*(5*b*c - 3*a*d))/(3*a^2*c*(b*c - a*d)*(a + b/x)^(3/2)) + (b*(5*b^2*c^2 -
 8*a*b*c*d + a^2*d^2))/(a^3*c*(b*c - a*d)^2*Sqrt[a + b/x]) + x/(a*c*(a + b/
x)^(3/2)) - (2*d^(7/2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c^
2*(b*c - a*d)^(5/2)) - ((5*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(a^
(7/2)*c^2)
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
```

```
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)}\,dx = -\,\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{5/2}(c+dx)}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(5bc+2ad)+\frac{5bdx}{2}}{x(a+bx)^{5/2}(c+dx)}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{b(5bc-3ad)}{3a^2c(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}} + \frac{2\,\text{Subst}\left(\int \frac{\frac{3}{4}(bc-ad)(5bc+2ad)+\frac{3}{4}bd(5bc-3ad)x}{x(a+bx)^{3/2}(c+dx)}\,dx, x, \frac{1}{x}\right.}{3a^2c(bc-ad)}$$

$$= \frac{b(5bc-3ad)}{3a^2c(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b\left(5b^2c^2-8abcd+a^2d^2\right)}{a^3c(bc-ad)^2\sqrt{a+\frac{b}{x}}} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}} + \frac{4\,\text{Subst}\left(\int \frac{\frac{3}{8}(bc-ad)}{}\right.}{}$$

$$= \frac{b(5bc-3ad)}{3a^2c(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b\left(5b^2c^2-8abcd+a^2d^2\right)}{a^3c(bc-ad)^2\sqrt{a+\frac{b}{x}}} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}} - \frac{d^4\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}(c}\right.}{c^2(bc-a}$$

$$= \frac{b(5bc-3ad)}{3a^2c(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b\left(5b^2c^2-8abcd+a^2d^2\right)}{a^3c(bc-ad)^2\sqrt{a+\frac{b}{x}}} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}} - \frac{\left(2d^4\right)\text{Subst}\left(\int \frac{}{c-\frac{a}{b}}\right.}{bc^2(bc}$$

$$= \frac{b(5bc-3ad)}{3a^2c(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b\left(5b^2c^2-8abcd+a^2d^2\right)}{a^3c(bc-ad)^2\sqrt{a+\frac{b}{x}}} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}} - \frac{2d^{7/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+}}{\sqrt{bc-ad}}\right.}{c^2(bc-ad)^{5/2}}$$

Mathematica [C] time = 0.0585514, size = 118, normalized size = 0.59

$$\frac{x\left((ad-bc)\left((2ad+5bc)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b}{ax}+1\right)+3acx\right)-2a^2d^2\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{d\left(a+\frac{b}{x}\right)}{ad-bc}\right)\right)}{3a^2c^2\sqrt{a+\frac{b}{x}}(ax+b)(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(5/2)*(c + d/x)),x]
```

```
[Out] (x*(-2*a^2*d^2*Hypergeometric2F1[-3/2, 1, -1/2, (d*(a + b/x))/(-(b*c) + a*d
)] + (-(b*c) + a*d)*(3*a*c*x + (5*b*c + 2*a*d)*Hypergeometric2F1[-3/2, 1, -
1/2, 1 + b/(a*x)])))/(3*a^2*c^2*(-(b*c) + a*d)*Sqrt[a + b/x]*(b + a*x))
```

Maple [B] time = 0.013, size = 1767, normalized size = 8.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(5/2)/(c+d/x),x)
```

```
[Out] -1/6*(-32*a^(5/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(3/2)*b^2*c^3*d+18*ln
(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)
*x*a^4*b^2*c*d^3+9*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((
a*d-b*c)*d/c^2)^(1/2)*x*a^3*b^3*c^2*d^2-72*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1
/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*x*a^2*b^4*c^3*d+3*ln(1/2*(2*(
(a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*x^3*a^5*
b*c^2*d^2-24*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*
c)*d/c^2)^(1/2)*x^3*a^4*b^2*c^3*d+18*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*
a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*x^2*a^5*b*c*d^3+9*ln(1/2*(2*((a*x+b
)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*x^2*a^4*b^2*c^
2*d^2-72*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d
/c^2)^(1/2)*x^2*a^3*b^3*c^3*d+144*a^(5/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*
x)^(1/2)*x*b^3*c^3*d-18*a^(9/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*x
^2*b*c^2*d^2+144*a^(7/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*x^2*b^2*
c^3*d-18*a^(7/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*x*b^2*c^2*d^2+48
*a^(9/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*x^3*b*c^3*d-36*a^(7/2)*(
(a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(3/2)*x*b*c^3*d-30*a^(7/2)*((a*d-b*c)*d/
c^2)^(1/2)*((a*x+b)*x)^(1/2)*x^3*b^2*c^4+45*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(
1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*x^2*a^2*b^4*c^4+45*ln(1/2*(2
*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*x*a*b^
5*c^4+6*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/
c^2)^(1/2)*a^3*b^3*c*d^3+3*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(
1/2))*((a*d-b*c)*d/c^2)^(1/2)*a^2*b^4*c^2*d^2-24*ln(1/2*(2*((a*x+b)*x)^(1/2
)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*a*b^5*c^3*d+6*ln(1/2*(2
*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*x^3*a^
6*c*d^3+15*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*((a*d-b*c)
*d/c^2)^(1/2)*x^3*a^3*b^3*c^4-90*a^(3/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x
)^(1/2)*x*b^4*c^4-6*a^(5/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*b^3*c
^2*d^2+48*a^(3/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*b^4*c^3*d+24*a^
(5/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(3/2)*x*b^2*c^4-90*a^(5/2)*((a*d-
b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*x^2*b^3*c^4-6*a^(11/2)*((a*d-b*c)*d/c^2
)^(1/2)*((a*x+b)*x)^(1/2)*x^3*c^2*d^2+15*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2
)+2*a*x+b)/a^(1/2))*((a*d-b*c)*d/c^2)^(1/2)*b^6*c^4+6*a^(13/2)*ln((2*((a*d-
b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x^3*d^4+6
*a^(7/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*
d)/(c*x+d))*b^3*d^4+18*a^(11/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(
1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x^2*b*d^4+20*a^(3/2)*((a*d-b*c)*d/c^2)^(
1/2)*((a*x+b)*x)^(3/2)*b^3*c^4+18*a^(9/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a
*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x*b^2*d^4-30*a^(1/2)*((a*d-b*c
)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*b^5*c^4)/a^(7/2)*x*((a*x+b)/x)^(1/2)/((a*d
-b*c)*d/c^2)^(1/2)/c^3/(a*x+b)^3/(a*d-b*c)^2/((a*x+b)*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}\left(c+\frac{d}{x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/(c+d/x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a + b/x)^(5/2)*(c + d/x)), x)
```

Fricas [B] time = 4.55797, size = 4049, normalized size = 20.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/(c+d/x),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(5*b^5*c^3 - 8*a*b^4*c^2*d + a^2*b^3*c*d^2 + 2*a^3*b^2*d^3 + (5*a^2
*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^2 + 2*(5*a*b^4*c^3
- 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2 + 2*a^4*b*d^3)*x)*sqrt(a)*log(2*a*x - 2*s
qrt(a)*x*sqrt((a*x + b)/x) + b) + 6*(a^6*d^3*x^2 + 2*a^5*b*d^3*x + a^4*b^2*
d^3)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((
a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*(3*(a^3*b^2*c^3 - 2*a^4
*b*c^2*d + a^5*c*d^2)*x^3 + 2*(10*a^2*b^3*c^3 - 16*a^3*b^2*c^2*d + 3*a^4*b*
c*d^2)*x^2 + 3*(5*a*b^4*c^3 - 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2)*x)*sqrt((a*x
 + b)/x))/(a^4*b^4*c^4 - 2*a^5*b^3*c^3*d + a^6*b^2*c^2*d^2 + (a^6*b^2*c^4 -
 2*a^7*b*c^3*d + a^8*c^2*d^2)*x^2 + 2*(a^5*b^3*c^4 - 2*a^6*b^2*c^3*d + a^7*
b*c^2*d^2)*x), 1/3*(3*(5*b^5*c^3 - 8*a*b^4*c^2*d + a^2*b^3*c*d^2 + 2*a^3*b^
2*d^3 + (5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^2 + 2
*(5*a*b^4*c^3 - 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2 + 2*a^4*b*d^3)*x)*sqrt(-a)*
arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + 3*(a^6*d^3*x^2 + 2*a^5*b*d^3*x + a^4
*b^2*d^3)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*s
qrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + (3*(a^3*b^2*c^3 - 2*
a^4*b*c^2*d + a^5*c*d^2)*x^3 + 2*(10*a^2*b^3*c^3 - 16*a^3*b^2*c^2*d + 3*a^4
*b*c*d^2)*x^2 + 3*(5*a*b^4*c^3 - 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2)*x)*sqrt((
a*x + b)/x))/(a^4*b^4*c^4 - 2*a^5*b^3*c^3*d + a^6*b^2*c^2*d^2 + (a^6*b^2*c^
4 - 2*a^7*b*c^3*d + a^8*c^2*d^2)*x^2 + 2*(a^5*b^3*c^4 - 2*a^6*b^2*c^3*d + a
^7*b*c^2*d^2)*x), -1/6*(12*(a^6*d^3*x^2 + 2*a^5*b*d^3*x + a^4*b^2*d^3)*sqrt
(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)
/(a*d*x + b*d)) - 3*(5*b^5*c^3 - 8*a*b^4*c^2*d + a^2*b^3*c*d^2 + 2*a^3*b^2*
d^3 + (5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^2 + 2*(
5*a*b^4*c^3 - 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2 + 2*a^4*b*d^3)*x)*sqrt(a)*log
(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 2*(3*(a^3*b^2*c^3 - 2*a^4*b*c
^2*d + a^5*c*d^2)*x^3 + 2*(10*a^2*b^3*c^3 - 16*a^3*b^2*c^2*d + 3*a^4*b*c*d^
2)*x^2 + 3*(5*a*b^4*c^3 - 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2)*x)*sqrt((a*x + b
)/x))/(a^4*b^4*c^4 - 2*a^5*b^3*c^3*d + a^6*b^2*c^2*d^2 + (a^6*b^2*c^4 - 2*a
^7*b*c^3*d + a^8*c^2*d^2)*x^2 + 2*(a^5*b^3*c^4 - 2*a^6*b^2*c^3*d + a^7*b*c^
2*d^2)*x), -1/3*(6*(a^6*d^3*x^2 + 2*a^5*b*d^3*x + a^4*b^2*d^3)*sqrt(d/(b*c
- a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x
+ b*d)) - 3*(5*b^5*c^3 - 8*a*b^4*c^2*d + a^2*b^3*c*d^2 + 2*a^3*b^2*d^3 + (5
*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^2 + 2*(5*a*b^4*
c^3 - 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2 + 2*a^4*b*d^3)*x)*sqrt(-a)*arctan(sqr
t(-a)*sqrt((a*x + b)/x)/a) - (3*(a^3*b^2*c^3 - 2*a^4*b*c^2*d + a^5*c*d^2)*x
^3 + 2*(10*a^2*b^3*c^3 - 16*a^3*b^2*c^2*d + 3*a^4*b*c*d^2)*x^2 + 3*(5*a*b^4
*c^3 - 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2)*x)*sqrt((a*x + b)/x))/(a^4*b^4*c^4
- 2*a^5*b^3*c^3*d + a^6*b^2*c^2*d^2 + (a^6*b^2*c^4 - 2*a^7*b*c^3*d + a^8*c^
2*d^2)*x^2 + 2*(a^5*b^3*c^4 - 2*a^6*b^2*c^3*d + a^7*b*c^2*d^2)*x)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(a + \frac{b}{x}\right)^{\frac{5}{2}} (cx + d)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(5/2)/(c+d/x),x)
```

```
[Out] Integral(x/((a + b/x)**(5/2)*(c*x + d)), x)
```

Giac [A] time = 1.18169, size = 332, normalized size = 1.65

$$-\frac{1}{3}\left(\frac{6\,d^4\arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^2}}\right)}{\left(b^3c^4-2\,ab^2c^3d+a^2bc^2d^2\right)\sqrt{bcd-ad^2}}-\frac{2\left(ab^2c-a^2bd+\frac{6\,(ax+b)b^2c}{x}-\frac{9\,(ax+b)abd}{x}\right)x}{\left(a^3b^2c^2-2\,a^4bcd+a^5d^2\right)(ax+b)\sqrt{\frac{ax+b}{x}}}+\frac{3\sqrt{\frac{ax+b}{x}}}{\left(a-\frac{ax+b}{x}\right)a^3c}-\frac{3\,(5\,bc+2\,ad}{\sqrt{-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/(c+d/x),x, algorithm="giac")
```

```
[Out] -1/3*(6*d^4*arctan(d*sqrt((a*x + b)/x)/sqrt(b*c*d - a*d^2))/((b^3*c^4 - 2*a
*b^2*c^3*d + a^2*b*c^2*d^2)*sqrt(b*c*d - a*d^2)) - 2*(a*b^2*c - a^2*b*d + 6
*(a*x + b)*b^2*c/x - 9*(a*x + b)*a*b*d/x)*x/((a^3*b^2*c^2 - 2*a^4*b*c*d + a
^5*d^2)*(a*x + b)*sqrt((a*x + b)/x)) + 3*sqrt((a*x + b)/x)/((a - (a*x + b)/
x)*a^3*c) - 3*(5*b*c + 2*a*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*
a^3*b*c^2))*b
```

3.264 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}\,dx$

Optimal. Leaf size=287

$$\frac{b(bc-2ad)\left(a^2d^2-abcd+5b^2c^2\right)}{a^3c^2\sqrt{a+\frac{b}{x}}(bc-ad)^3}+\frac{b\left(6a^2d^2-6abcd+5b^2c^2\right)}{3a^2c^2\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)^2}-\frac{(4ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^3}-\frac{d^{7/2}(9bc-4ad)\tan^{-}}{c^3(bc-ad)^{7}}$$

```
[Out] (b*(5*b^2*c^2 - 6*a*b*c*d + 6*a^2*d^2))/(3*a^2*c^2*(b*c - a*d)^2*(a + b/x)^
(3/2)) + (b*(b*c - 2*a*d)*(5*b^2*c^2 - a*b*c*d + a^2*d^2))/(a^3*c^2*(b*c -
a*d)^3*Sqrt[a + b/x]) + (d*(b*c - 2*a*d))/(a*c^2*(b*c - a*d)*(a + b/x)^(3/2
)*(c + d/x)) + x/(a*c*(a + b/x)^(3/2)*(c + d/x)) - (d^(7/2)*(9*b*c - 4*a*d)
*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c^3*(b*c - a*d)^(7/2)) -
 ((5*b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(a^(7/2)*c^3)
```

Rubi [A] time = 0.447713, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {375, 103, 151, 152, 156, 63, 208, 205}

$$\frac{b(bc-2ad)\left(a^2d^2-abcd+5b^2c^2\right)}{a^3c^2\sqrt{a+\frac{b}{x}}(bc-ad)^3}+\frac{b\left(6a^2d^2-6abcd+5b^2c^2\right)}{3a^2c^2\left(a+\frac{b}{x}\right)^{3/2}(bc-ad)^2}-\frac{(4ad+5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^3}-\frac{d^{7/2}(9bc-4ad)\tan^{-}}{c^3(bc-ad)^{7}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(5/2)*(c + d/x)^2),x]
```


```
[Out] (b*(5*b^2*c^2 - 6*a*b*c*d + 6*a^2*d^2))/(3*a^2*c^2*(b*c - a*d)^2*(a + b/x)^
(3/2)) + (b*(b*c - 2*a*d)*(5*b^2*c^2 - a*b*c*d + a^2*d^2))/(a^3*c^2*(b*c -
a*d)^3*Sqrt[a + b/x]) + (d*(b*c - 2*a*d))/(a*c^2*(b*c - a*d)*(a + b/x)^(3/2
)*(c + d/x)) + x/(a*c*(a + b/x)^(3/2)*(c + d/x)) - (d^(7/2)*(9*b*c - 4*a*d)
*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(c^3*(b*c - a*d)^(7/2)) -
 ((5*b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/(a^(7/2)*c^3)
```


Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^2}\,dx = -\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{5/2}(c+dx)^2}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(5bc+4ad)+\frac{7bdx}{2}}{x(a+bx)^{5/2}(c+dx)^2}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{d(bc-2ad)}{ac^2(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}(bc-ad)(5bc+4ad)-\frac{5}{2}bd(bc}{x(a+bx)^{5/2}(c+dx)}\right.}{ac^2(bc-ad)}$$

$$= \frac{b\left(5b^2c^2-6abcd+6a^2d^2\right)}{3a^2c^2(bc-ad)^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{d(bc-2ad)}{ac^2(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} - \frac{2\,\text{S}}{}$$

$$= \frac{b\left(5b^2c^2-6abcd+6a^2d^2\right)}{3a^2c^2(bc-ad)^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b(bc-2ad)\left(5b^2c^2-abcd+a^2d^2\right)}{a^3c^2(bc-ad)^3\sqrt{a+\frac{b}{x}}} + \frac{d(bc-2ad)}{ac^2(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}}$$

$$= \frac{b\left(5b^2c^2-6abcd+6a^2d^2\right)}{3a^2c^2(bc-ad)^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b(bc-2ad)\left(5b^2c^2-abcd+a^2d^2\right)}{a^3c^2(bc-ad)^3\sqrt{a+\frac{b}{x}}} + \frac{d(bc-2ad)}{ac^2(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}}$$

$$= \frac{b\left(5b^2c^2-6abcd+6a^2d^2\right)}{3a^2c^2(bc-ad)^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b(bc-2ad)\left(5b^2c^2-abcd+a^2d^2\right)}{a^3c^2(bc-ad)^3\sqrt{a+\frac{b}{x}}} + \frac{d(bc-2ad)}{ac^2(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}}$$

$$= \frac{b\left(5b^2c^2-6abcd+6a^2d^2\right)}{3a^2c^2(bc-ad)^2\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b(bc-2ad)\left(5b^2c^2-abcd+a^2d^2\right)}{a^3c^2(bc-ad)^3\sqrt{a+\frac{b}{x}}} + \frac{d(bc-2ad)}{ac^2(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}}$$

Mathematica [C] time = 0.13204, size = 178, normalized size = 0.62

$$\frac{x\left((ad-bc)\left(3acx(ad(cx+2d)-bc(cx+d))-(cx+d)\left(-4a^2d^2-abcd+5b^2c^2\right)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b}{ax}+1\right)\right)+a^2d^2(cx+c\right.}{3a^2c^3\sqrt{a+\frac{b}{x}}(ax+b)(cx+d)(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(5/2)*(c + d/x)^2),x]
```

```
[Out] (x*(a^2*d^2*(9*b*c - 4*a*d)*(d + c*x)*Hypergeometric2F1[-3/2, 1, -1/2, (d*(
a + b/x))/(-(b*c) + a*d)] + (-(b*c) + a*d)*(3*a*c*x*(-(b*c*(d + c*x)) + a*d
*(2*d + c*x)) - (5*b^2*c^2 - a*b*c*d - 4*a^2*d^2)*(d + c*x)*Hypergeometric2
F1[-3/2, 1, -1/2, 1 + b/(a*x)])))/(3*a^2*c^3*(b*c - a*d)^2*Sqrt[a + b/x]*(b
 + a*x)*(d + c*x))
```

Maple [B] time = 0.017, size = 4644, normalized size = 16.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(5/2)/(c+d/x)^2,x)
```

```
[Out] -1/6*((a*x+b)/x)^(1/2)*x*(12*a^(19/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b
)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x^3*d^7+12*a^(13/2)*ln((2*((a*d-b*
c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^3*d^7+81*
a^(13/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*
d)/(c*x+d))*x^3*b^3*c^3*d^4+45*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)
/a^(1/2))*a^2*((a*d-b*c)*d/c^2)^(1/2)*x^2*b^7*c^7+20*a^(5/2)*((a*d-b*c)*d/c
^2)^(1/2)*((a*x+b)*x)^(3/2)*x*b^5*c^7-90*a^(5/2)*((a*d-b*c)*d/c^2)^(1/2)*((
a*x+b)*x)^(1/2)*x^2*b^6*c^7-81*a^(15/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x
+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x^2*b^2*c*d^6-36*a^(13/2)*ln((2*
((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x^2
*b^3*c^2*d^5+81*a^(11/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-
2*a*d*x+b*c*x-b*d)/(c*x+d))*x^2*b^4*c^3*d^4+38*a^(9/2)*((a*d-b*c)*d/c^2)^(1
/2)*((a*x+b)*x)^(3/2)*b^3*c^4*d^3-64*a^(7/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+
b)*x)^(3/2)*b^4*c^5*d^2+20*a^(5/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(3/2
)*b^5*c^6*d-105*a^(13/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-
2*a*d*x+b*c*x-b*d)/(c*x+d))*x*b^3*c*d^6+42*a^(11/2)*ln((2*((a*d-b*c)*d/c^2)
^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x*b^4*c^2*d^5+27*a^(
9/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(
c*x+d))*x*b^5*c^3*d^4+15*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/
2))*a*((a*d-b*c)*d/c^2)^(1/2)*b^8*c^6*d-30*a^(3/2)*((a*d-b*c)*d/c^2)^(1/2)*
((a*x+b)*x)^(1/2)*b^7*c^6*d-30*a^(3/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^
(1/2)*x*b^7*c^7+6*a^(17/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*x^5*c^
4*d^3+12*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^9*((a*d-b*
c)*d/c^2)^(1/2)*x^4*c^2*d^5+15*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)
/a^(1/2))*a^4*((a*d-b*c)*d/c^2)^(1/2)*x^4*b^5*c^7+15*ln(1/2*(2*((a*x+b)*x)^
(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a*((a*d-b*c)*d/c^2)^(1/2)*x*b^8*c^7-6*a^(15
/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(3/2)*x^3*c^4*d^3+36*a^(15/2)*ln((2
*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x*
b^2*d^7-39*a^(11/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d
*x+b*c*x-b*d)/(c*x+d))*b^4*c*d^6+27*a^(9/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*(
(a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*b^5*c^2*d^5+12*a^(19/2)*ln((
2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x
^4*c*d^6+36*a^(17/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*
d*x+b*c*x-b*d)/(c*x+d))*x^2*b*d^7+36*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*
a*x+b)/a^(1/2))*a^7*((a*d-b*c)*d/c^2)^(1/2)*x*b^2*c*d^6+30*a^(11/2)*((a*d-b
*c)*d/c^2)^(1/2)*((a*x+b)*x)^(3/2)*x*b^2*c^4*d^3-28*a^(9/2)*((a*d-b*c)*d/c^
2)^(1/2)*((a*x+b)*x)^(3/2)*x*b^3*c^5*d^2-6*a^(17/2)*((a*d-b*c)*d/c^2)^(1/2)
*((a*x+b)*x)^(1/2)*x^4*c^3*d^4-30*a^(9/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*
x)^(1/2)*x^4*b^4*c^7-39*a^(17/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^
(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x^4*b*c^2*d^5+27*a^(15/2)*ln((2*((a*d-b
*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x^4*b^2*c^
3*d^4+12*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^9*((a*d-b*
c)*d/c^2)^(1/2)*x^3*c*d^6+45*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a
^(1/2))*a^3*((a*d-b*c)*d/c^2)^(1/2)*x^3*b^6*c^7+24*a^(7/2)*((a*d-b*c)*d/c^2
)^(1/2)*((a*x+b)*x)^(3/2)*x^2*b^4*c^7-12*a^(17/2)*((a*d-b*c)*d/c^2)^(1/2)*(
(a*x+b)*x)^(1/2)*x^3*c^2*d^5-90*a^(7/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)
^(1/2)*x^3*b^5*c^7+12*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))
*a^6*((a*d-b*c)*d/c^2)^(1/2)*b^3*c*d^6-33*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/
2)+2*a*x+b)/a^(1/2))*a^5*((a*d-b*c)*d/c^2)^(1/2)*b^4*c^2*d^5+12*ln(1/2*(2*(
(a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^4*((a*d-b*c)*d/c^2)^(1/2)*b^5*
c^3*d^4+42*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*((a*d-
b*c)*d/c^2)^(1/2)*b^6*c^4*d^3-48*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+
b)/a^(1/2))*a^2*((a*d-b*c)*d/c^2)^(1/2)*b^7*c^5*d^2-12*a^(11/2)*((a*d-b*c)*
d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*b^3*c^2*d^5+30*a^(9/2)*((a*d-b*c)*d/c^2)^(1/
2)*((a*x+b)*x)^(1/2)*b^4*c^3*d^4-84*a^(7/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b
```

```
)*x)^(1/2)*b^5*c^4*d^3+96*a^(5/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)
*b^6*c^5*d^2-3*a^(17/2)*ln((2*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2
*a*d*x+b*c*x-b*d)/(c*x+d))*x^3*b*c*d^6-90*a^(15/2)*ln((2*((a*d-b*c)*d/c^2)^
(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*x^3*b^2*c^2*d^5-99*ln
(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*((a*d-b*c)*d/c^2)^(
1/2)*x^2*b^6*c^6*d+162*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2)
)*a^5*((a*d-b*c)*d/c^2)^(1/2)*x^2*b^4*c^4*d^3-18*ln(1/2*(2*((a*x+b)*x)^(1/2
)*a^(1/2)+2*a*x+b)/a^(1/2))*a^4*((a*d-b*c)*d/c^2)^(1/2)*x^2*b^5*c^5*d^2-222
*a^(9/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*x*b^4*c^4*d^3+204*a^(7/2
)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*x*b^5*c^5*d^2+6*a^(5/2)*((a*d-b
*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*x*b^6*c^6*d-63*ln(1/2*(2*((a*x+b)*x)^(1/
2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^6*((a*d-b*c)*d/c^2)^(1/2)*x^2*b^3*c^3*d^4+36
*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^8*((a*d-b*c)*d/c^2
)^(1/2)*x^2*b*c*d^6-63*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2)
)*a^7*((a*d-b*c)*d/c^2)^(1/2)*x^2*b^2*c^2*d^5+12*a^(15/2)*((a*d-b*c)*d/c^2)
^(1/2)*((a*x+b)*x)^(1/2)*x^3*b*c^3*d^4+24*a^(13/2)*((a*d-b*c)*d/c^2)^(1/2)*
((a*x+b)*x)^(1/2)*x^3*b^2*c^4*d^3-156*a^(11/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*
x+b)*x)^(1/2)*x^3*b^3*c^5*d^2+258*a^(9/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*
x)^(1/2)*x^3*b^4*c^6*d+48*a^(11/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(3/2
)*x^2*b^2*c^5*d^2-72*a^(9/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(3/2)*x^2*
b^3*c^6*d+78*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^5*((a*
d-b*c)*d/c^2)^(1/2)*x^3*b^4*c^5*d^2-129*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)
+2*a*x+b)/a^(1/2))*a^4*((a*d-b*c)*d/c^2)^(1/2)*x^3*b^5*c^6*d-18*a^(13/2)*((
a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(3/2)*x^2*b*c^4*d^3-84*a^(13/2)*((a*d-b*c
)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*x^4*b^2*c^5*d^2+96*a^(11/2)*((a*d-b*c)*d/c
^2)^(1/2)*((a*x+b)*x)^(1/2)*x^4*b^3*c^6*d+3*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(
1/2)+2*a*x+b)/a^(1/2))*a^8*((a*d-b*c)*d/c^2)^(1/2)*x^3*b*c^2*d^5-87*ln(1/2*
(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^7*((a*d-b*c)*d/c^2)^(1/2)*
x^3*b^2*c^3*d^4+78*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^
6*((a*d-b*c)*d/c^2)^(1/2)*x^3*b^3*c^4*d^3+48*a^(15/2)*((a*d-b*c)*d/c^2)^(1/
2)*((a*x+b)*x)^(1/2)*x^4*b*c^4*d^3+12*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2
*a*x+b)/a^(1/2))*a^7*((a*d-b*c)*d/c^2)^(1/2)*x^4*b^2*c^4*d^3+42*ln(1/2*(2*(
(a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^6*((a*d-b*c)*d/c^2)^(1/2)*x^4*
b^3*c^5*d^2-48*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^5*((
a*d-b*c)*d/c^2)^(1/2)*x^4*b^4*c^6*d-33*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+
2*a*x+b)/a^(1/2))*a^8*((a*d-b*c)*d/c^2)^(1/2)*x^4*b*c^3*d^4+138*ln(1/2*(2*(
(a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^4*((a*d-b*c)*d/c^2)^(1/2)*x*b^
5*c^4*d^3-102*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^3*((a
*d-b*c)*d/c^2)^(1/2)*x*b^6*c^5*d^2-3*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*
a*x+b)/a^(1/2))*a^2*((a*d-b*c)*d/c^2)^(1/2)*x*b^7*c^6*d-36*a^(13/2)*((a*d-b
*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*x*b^2*c^2*d^5+84*a^(11/2)*((a*d-b*c)*d/c
^2)^(1/2)*((a*x+b)*x)^(1/2)*x*b^3*c^3*d^4-87*ln(1/2*(2*((a*x+b)*x)^(1/2)*a^
(1/2)+2*a*x+b)/a^(1/2))*a^6*((a*d-b*c)*d/c^2)^(1/2)*x*b^3*c^2*d^5+3*ln(1/2*
(2*((a*x+b)*x)^(1/2)*a^(1/2)+2*a*x+b)/a^(1/2))*a^5*((a*d-b*c)*d/c^2)^(1/2)*
x*b^4*c^3*d^4-40*a^(7/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(3/2)*x*b^4*c^
6*d-36*a^(15/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*x^2*b*c^2*d^5+72*
a^(13/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*x^2*b^2*c^3*d^4-156*a^(1
1/2)*((a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*x^2*b^3*c^4*d^3+36*a^(9/2)*(
(a*d-b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*x^2*b^4*c^5*d^2+198*a^(7/2)*((a*d-
b*c)*d/c^2)^(1/2)*((a*x+b)*x)^(1/2)*x^2*b^5*c^6*d)/a^(9/2)/c^4/((a*x+b)*x)^
(1/2)/(a*d-b*c)^4/(c*x+d)/((a*d-b*c)*d/c^2)^(1/2)/(a*x+b)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}\left(c+\frac{d}{x}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/(c+d/x)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((a + b/x)^(5/2)*(c + d/x)^2), x)
```

Fricas [B] time = 11.7287, size = 7776, normalized size = 27.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/(c+d/x)^2,x, algorithm="fricas")
```

```
[Out] [1/6*(3*(5*b^6*c^4*d - 11*a*b^5*c^3*d^2 + 3*a^2*b^4*c^2*d^3 + 7*a^3*b^3*c*d
^4 - 4*a^4*b^2*d^5 + (5*a^2*b^4*c^5 - 11*a^3*b^3*c^4*d + 3*a^4*b^2*c^3*d^2
+ 7*a^5*b*c^2*d^3 - 4*a^6*c*d^4)*x^3 + (10*a*b^5*c^5 - 17*a^2*b^4*c^4*d - 5
*a^3*b^3*c^3*d^2 + 17*a^4*b^2*c^2*d^3 - a^5*b*c*d^4 - 4*a^6*d^5)*x^2 + (5*b
^6*c^5 - a*b^5*c^4*d - 19*a^2*b^4*c^3*d^2 + 13*a^3*b^3*c^2*d^3 + 10*a^4*b^2
*c*d^4 - 8*a^5*b*d^5)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x)
+ b) + 3*(9*a^4*b^3*c*d^4 - 4*a^5*b^2*d^5 + (9*a^6*b*c^2*d^3 - 4*a^7*c*d^4)
*x^3 + (18*a^5*b^2*c^2*d^3 + a^6*b*c*d^4 - 4*a^7*d^5)*x^2 + (9*a^4*b^3*c^2*
d^3 + 14*a^5*b^2*c*d^4 - 8*a^6*b*d^5)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c
- a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c
*x + d)) + 2*(3*(a^3*b^3*c^5 - 3*a^4*b^2*c^4*d + 3*a^5*b*c^3*d^2 - a^6*c^2*
d^3)*x^4 + (20*a^2*b^4*c^5 - 41*a^3*b^3*c^4*d + 9*a^4*b^2*c^3*d^2 + 3*a^5*b
*c^2*d^3 - 6*a^6*c*d^4)*x^3 + (15*a*b^5*c^5 - 13*a^2*b^4*c^4*d - 35*a^3*b^3
*c^3*d^2 + 15*a^4*b^2*c^2*d^3 - 12*a^5*b*c*d^4)*x^2 + 3*(5*a*b^5*c^4*d - 11
*a^2*b^4*c^3*d^2 + 3*a^3*b^3*c^2*d^3 - 2*a^4*b^2*c*d^4)*x)*sqrt((a*x + b)/x
))/(a^4*b^5*c^6*d - 3*a^5*b^4*c^5*d^2 + 3*a^6*b^3*c^4*d^3 - a^7*b^2*c^3*d^4
 + (a^6*b^3*c^7 - 3*a^7*b^2*c^6*d + 3*a^8*b*c^5*d^2 - a^9*c^4*d^3)*x^3 + (2
*a^5*b^4*c^7 - 5*a^6*b^3*c^6*d + 3*a^7*b^2*c^5*d^2 + a^8*b*c^4*d^3 - a^9*c^
3*d^4)*x^2 + (a^4*b^5*c^7 - a^5*b^4*c^6*d - 3*a^6*b^3*c^5*d^2 + 5*a^7*b^2*c
^4*d^3 - 2*a^8*b*c^3*d^4)*x), -1/6*(6*(9*a^4*b^3*c*d^4 - 4*a^5*b^2*d^5 + (9
*a^6*b*c^2*d^3 - 4*a^7*c*d^4)*x^3 + (18*a^5*b^2*c^2*d^3 + a^6*b*c*d^4 - 4*a
^7*d^5)*x^2 + (9*a^4*b^3*c^2*d^3 + 14*a^5*b^2*c*d^4 - 8*a^6*b*d^5)*x)*sqrt(
d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/
(a*d*x + b*d)) - 3*(5*b^6*c^4*d - 11*a*b^5*c^3*d^2 + 3*a^2*b^4*c^2*d^3 + 7*
a^3*b^3*c*d^4 - 4*a^4*b^2*d^5 + (5*a^2*b^4*c^5 - 11*a^3*b^3*c^4*d + 3*a^4*b
^2*c^3*d^2 + 7*a^5*b*c^2*d^3 - 4*a^6*c*d^4)*x^3 + (10*a*b^5*c^5 - 17*a^2*b^
4*c^4*d - 5*a^3*b^3*c^3*d^2 + 17*a^4*b^2*c^2*d^3 - a^5*b*c*d^4 - 4*a^6*d^5)
*x^2 + (5*b^6*c^5 - a*b^5*c^4*d - 19*a^2*b^4*c^3*d^2 + 13*a^3*b^3*c^2*d^3 +
 10*a^4*b^2*c*d^4 - 8*a^5*b*d^5)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a
*x + b)/x) + b) - 2*(3*(a^3*b^3*c^5 - 3*a^4*b^2*c^4*d + 3*a^5*b*c^3*d^2 - a
^6*c^2*d^3)*x^4 + (20*a^2*b^4*c^5 - 41*a^3*b^3*c^4*d + 9*a^4*b^2*c^3*d^2 +
3*a^5*b*c^2*d^3 - 6*a^6*c*d^4)*x^3 + (15*a*b^5*c^5 - 13*a^2*b^4*c^4*d - 35*
a^3*b^3*c^3*d^2 + 15*a^4*b^2*c^2*d^3 - 12*a^5*b*c*d^4)*x^2 + 3*(5*a*b^5*c^4
*d - 11*a^2*b^4*c^3*d^2 + 3*a^3*b^3*c^2*d^3 - 2*a^4*b^2*c*d^4)*x)*sqrt((a*x
 + b)/x))/(a^4*b^5*c^6*d - 3*a^5*b^4*c^5*d^2 + 3*a^6*b^3*c^4*d^3 - a^7*b^2*
c^3*d^4 + (a^6*b^3*c^7 - 3*a^7*b^2*c^6*d + 3*a^8*b*c^5*d^2 - a^9*c^4*d^3)*x
^3 + (2*a^5*b^4*c^7 - 5*a^6*b^3*c^6*d + 3*a^7*b^2*c^5*d^2 + a^8*b*c^4*d^3 -
 a^9*c^3*d^4)*x^2 + (a^4*b^5*c^7 - a^5*b^4*c^6*d - 3*a^6*b^3*c^5*d^2 + 5*a^
7*b^2*c^4*d^3 - 2*a^8*b*c^3*d^4)*x), 1/6*(6*(5*b^6*c^4*d - 11*a*b^5*c^3*d^2
 + 3*a^2*b^4*c^2*d^3 + 7*a^3*b^3*c*d^4 - 4*a^4*b^2*d^5 + (5*a^2*b^4*c^5 - 1
1*a^3*b^3*c^4*d + 3*a^4*b^2*c^3*d^2 + 7*a^5*b*c^2*d^3 - 4*a^6*c*d^4)*x^3 +
(10*a*b^5*c^5 - 17*a^2*b^4*c^4*d - 5*a^3*b^3*c^3*d^2 + 17*a^4*b^2*c^2*d^3 -
 a^5*b*c*d^4 - 4*a^6*d^5)*x^2 + (5*b^6*c^5 - a*b^5*c^4*d - 19*a^2*b^4*c^3*d
```

```
^2 + 13*a^3*b^3*c^2*d^3 + 10*a^4*b^2*c*d^4 - 8*a^5*b*d^5)*x)*sqrt(-a)*arcta
n(sqrt(-a)*sqrt((a*x + b)/x)/a) + 3*(9*a^4*b^3*c*d^4 - 4*a^5*b^2*d^5 + (9*a
^6*b*c^2*d^3 - 4*a^7*c*d^4)*x^3 + (18*a^5*b^2*c^2*d^3 + a^6*b*c*d^4 - 4*a^7
*d^5)*x^2 + (9*a^4*b^3*c^2*d^3 + 14*a^5*b^2*c*d^4 - 8*a^6*b*d^5)*x)*sqrt(-d
/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x)
- b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*(3*(a^3*b^3*c^5 - 3*a^4*b^2*c^4*d +
 3*a^5*b*c^3*d^2 - a^6*c^2*d^3)*x^4 + (20*a^2*b^4*c^5 - 41*a^3*b^3*c^4*d +
9*a^4*b^2*c^3*d^2 + 3*a^5*b*c^2*d^3 - 6*a^6*c*d^4)*x^3 + (15*a*b^5*c^5 - 13
*a^2*b^4*c^4*d - 35*a^3*b^3*c^3*d^2 + 15*a^4*b^2*c^2*d^3 - 12*a^5*b*c*d^4)*
x^2 + 3*(5*a*b^5*c^4*d - 11*a^2*b^4*c^3*d^2 + 3*a^3*b^3*c^2*d^3 - 2*a^4*b^2
*c*d^4)*x)*sqrt((a*x + b)/x))/(a^4*b^5*c^6*d - 3*a^5*b^4*c^5*d^2 + 3*a^6*b^
3*c^4*d^3 - a^7*b^2*c^3*d^4 + (a^6*b^3*c^7 - 3*a^7*b^2*c^6*d + 3*a^8*b*c^5*
d^2 - a^9*c^4*d^3)*x^3 + (2*a^5*b^4*c^7 - 5*a^6*b^3*c^6*d + 3*a^7*b^2*c^5*d
^2 + a^8*b*c^4*d^3 - a^9*c^3*d^4)*x^2 + (a^4*b^5*c^7 - a^5*b^4*c^6*d - 3*a^
6*b^3*c^5*d^2 + 5*a^7*b^2*c^4*d^3 - 2*a^8*b*c^3*d^4)*x), -1/3*(3*(9*a^4*b^3
*c*d^4 - 4*a^5*b^2*d^5 + (9*a^6*b*c^2*d^3 - 4*a^7*c*d^4)*x^3 + (18*a^5*b^2*
c^2*d^3 + a^6*b*c*d^4 - 4*a^7*d^5)*x^2 + (9*a^4*b^3*c^2*d^3 + 14*a^5*b^2*c*
d^4 - 8*a^6*b*d^5)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c
 - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) - 3*(5*b^6*c^4*d - 11*a*b^5*c^3*d
^2 + 3*a^2*b^4*c^2*d^3 + 7*a^3*b^3*c*d^4 - 4*a^4*b^2*d^5 + (5*a^2*b^4*c^5 -
 11*a^3*b^3*c^4*d + 3*a^4*b^2*c^3*d^2 + 7*a^5*b*c^2*d^3 - 4*a^6*c*d^4)*x^3
+ (10*a*b^5*c^5 - 17*a^2*b^4*c^4*d - 5*a^3*b^3*c^3*d^2 + 17*a^4*b^2*c^2*d^3
 - a^5*b*c*d^4 - 4*a^6*d^5)*x^2 + (5*b^6*c^5 - a*b^5*c^4*d - 19*a^2*b^4*c^3
*d^2 + 13*a^3*b^3*c^2*d^3 + 10*a^4*b^2*c*d^4 - 8*a^5*b*d^5)*x)*sqrt(-a)*arc
tan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (3*(a^3*b^3*c^5 - 3*a^4*b^2*c^4*d + 3*a
^5*b*c^3*d^2 - a^6*c^2*d^3)*x^4 + (20*a^2*b^4*c^5 - 41*a^3*b^3*c^4*d + 9*a^
4*b^2*c^3*d^2 + 3*a^5*b*c^2*d^3 - 6*a^6*c*d^4)*x^3 + (15*a*b^5*c^5 - 13*a^2
*b^4*c^4*d - 35*a^3*b^3*c^3*d^2 + 15*a^4*b^2*c^2*d^3 - 12*a^5*b*c*d^4)*x^2
+ 3*(5*a*b^5*c^4*d - 11*a^2*b^4*c^3*d^2 + 3*a^3*b^3*c^2*d^3 - 2*a^4*b^2*c*d
^4)*x)*sqrt((a*x + b)/x))/(a^4*b^5*c^6*d - 3*a^5*b^4*c^5*d^2 + 3*a^6*b^3*c^
4*d^3 - a^7*b^2*c^3*d^4 + (a^6*b^3*c^7 - 3*a^7*b^2*c^6*d + 3*a^8*b*c^5*d^2
- a^9*c^4*d^3)*x^3 + (2*a^5*b^4*c^7 - 5*a^6*b^3*c^6*d + 3*a^7*b^2*c^5*d^2 +
 a^8*b*c^4*d^3 - a^9*c^3*d^4)*x^2 + (a^4*b^5*c^7 - a^5*b^4*c^6*d - 3*a^6*b^
3*c^5*d^2 + 5*a^7*b^2*c^4*d^3 - 2*a^8*b*c^3*d^4)*x)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(5/2)/(c+d/x)**2,x)
```

```
[Out] Exception raised: ValueError
```

Giac [B] time = 1.23121, size = 779, normalized size = 2.71

$$-\frac{1}{3}\,b\left(\frac{3\left(9\,bcd^4-4\,ad^5\right)\arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^2}}\right)}{\left(b^4c^6-3\,ab^3c^5d+3\,a^2b^2c^4d^2-a^3bc^3d^3\right)\sqrt{bcd-ad^2}}-\frac{2\left(ab^3c-a^2b^2d+\frac{6\,(ax+b)b^3c}{x}-\frac{12\,(ax+b)ab^2d}{x}\right)x}{\left(a^3b^3c^3-3\,a^4b^2c^2d+3\,a^5bcd^2-a^6d^3\right)(ax+b)\sqrt{\frac{ax+b}{x}}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/(c+d/x)^2,x, algorithm="giac")
```

```
[Out] -1/3*b*(3*(9*b*c*d^4 - 4*a*d^5)*arctan(d*sqrt((a*x + b)/x)/sqrt(b*c*d - a*d
^2))/((b^4*c^6 - 3*a*b^3*c^5*d + 3*a^2*b^2*c^4*d^2 - a^3*b*c^3*d^3)*sqrt(b*
c*d - a*d^2)) - 2*(a*b^3*c - a^2*b^2*d + 6*(a*x + b)*b^3*c/x - 12*(a*x + b)
*a*b^2*d/x)*x/((a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3)*(a
*x + b)*sqrt((a*x + b)/x)) + 3*(b^4*c^4*sqrt((a*x + b)/x) - 4*a*b^3*c^3*d*s
qrt((a*x + b)/x) + 6*a^2*b^2*c^2*d^2*sqrt((a*x + b)/x) - 4*a^3*b*c*d^3*sqrt
((a*x + b)/x) + 2*a^4*d^4*sqrt((a*x + b)/x) + (a*x + b)*b^3*c^3*d*sqrt((a*x
 + b)/x)/x - 3*(a*x + b)*a*b^2*c^2*d^2*sqrt((a*x + b)/x)/x + 3*(a*x + b)*a^
2*b*c*d^3*sqrt((a*x + b)/x)/x - 2*(a*x + b)*a^3*d^4*sqrt((a*x + b)/x)/x)/((
a^3*b^3*c^5 - 3*a^4*b^2*c^4*d + 3*a^5*b*c^3*d^2 - a^6*c^2*d^3)*(a*b*c - a^2
*d - (a*x + b)*b*c/x + 2*(a*x + b)*a*d/x - (a*x + b)^2*d/x^2)) - 3*(5*b*c +
 4*a*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^3*b*c^3))
```

3.265 $\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3}\,dx$

Optimal. Leaf size=409

$$\frac{d\left(12a^2d^2-23abcd+4b^2c^2\right)}{4ac^3\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)(bc-ad)^2}+\frac{b\left(24a^2b^2c^2d^2-35a^3bcd^3+12a^4d^4-56ab^3c^3d+20b^4c^4\right)}{4a^3c^3\sqrt{a+\frac{b}{x}}(bc-ad)^4}+\frac{b\left(87a^2bcd^2-36a^3d\right.}{12a^2c^3\left(a+\right.}$$

```
[Out] (b*(20*b^3*c^3 - 36*a*b^2*c^2*d + 87*a^2*b*c*d^2 - 36*a^3*d^3))/(12*a^2*c^3
*(b*c - a*d)^3*(a + b/x)^(3/2)) + (b*(20*b^4*c^4 - 56*a*b^3*c^3*d + 24*a^2*
b^2*c^2*d^2 - 35*a^3*b*c*d^3 + 12*a^4*d^4))/(4*a^3*c^3*(b*c - a*d)^4*Sqrt[a
 + b/x]) + (d*(2*b*c - 3*a*d))/(2*a*c^2*(b*c - a*d)*(a + b/x)^(3/2)*(c + d/
x)^2) + (d*(4*b^2*c^2 - 23*a*b*c*d + 12*a^2*d^2))/(4*a*c^3*(b*c - a*d)^2*(a
 + b/x)^(3/2)*(c + d/x)) + x/(a*c*(a + b/x)^(3/2)*(c + d/x)^2) - (d^(7/2)*(
99*b^2*c^2 - 88*a*b*c*d + 24*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b
*c - a*d]])/(4*c^4*(b*c - a*d)^(9/2)) - ((5*b*c + 6*a*d)*ArcTanh[Sqrt[a + b
/x]/Sqrt[a]])/(a^(7/2)*c^4)
```

Rubi [A] time = 0.702414, antiderivative size = 409, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {375, 103, 151, 152, 156, 63, 208, 205}

$$\frac{d\left(12a^2d^2-23abcd+4b^2c^2\right)}{4ac^3\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)(bc-ad)^2}+\frac{b\left(24a^2b^2c^2d^2-35a^3bcd^3+12a^4d^4-56ab^3c^3d+20b^4c^4\right)}{4a^3c^3\sqrt{a+\frac{b}{x}}(bc-ad)^4}+\frac{b\left(87a^2bcd^2-36a^3d\right.}{12a^2c^3\left(a+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b/x)^(5/2)*(c + d/x)^3),x]
```

```
[Out] (b*(20*b^3*c^3 - 36*a*b^2*c^2*d + 87*a^2*b*c*d^2 - 36*a^3*d^3))/(12*a^2*c^3
*(b*c - a*d)^3*(a + b/x)^(3/2)) + (b*(20*b^4*c^4 - 56*a*b^3*c^3*d + 24*a^2*
b^2*c^2*d^2 - 35*a^3*b*c*d^3 + 12*a^4*d^4))/(4*a^3*c^3*(b*c - a*d)^4*Sqrt[a
 + b/x]) + (d*(2*b*c - 3*a*d))/(2*a*c^2*(b*c - a*d)*(a + b/x)^(3/2)*(c + d/
x)^2) + (d*(4*b^2*c^2 - 23*a*b*c*d + 12*a^2*d^2))/(4*a*c^3*(b*c - a*d)^2*(a
 + b/x)^(3/2)*(c + d/x)) + x/(a*c*(a + b/x)^(3/2)*(c + d/x)^2) - (d^(7/2)*(
99*b^2*c^2 - 88*a*b*c*d + 24*a^2*d^2)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b
*c - a*d]])/(4*c^4*(b*c - a*d)^(9/2)) - ((5*b*c + 6*a*d)*ArcTanh[Sqrt[a + b
/x]/Sqrt[a]])/(a^(7/2)*c^4)
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
```

```
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{5/2}\left(c+\frac{d}{x}\right)^3}\,dx = -\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx)^{5/2}(c+dx)^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2} + \frac{\operatorname{Subst}\left(\int \frac{\frac{1}{2}(5bc+6ad)+\frac{9bdx}{2}}{x(a+bx)^{5/2}(c+dx)^3}\,dx, x, \frac{1}{x}\right)}{ac}$$

$$= \frac{d(2bc-3ad)}{2ac^2(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2} - \frac{\operatorname{Subst}\left(\int \frac{-(bc-ad)(5bc+6ad)-\frac{7}{2}bd}{x(a+bx)^{5/2}(c+d}\right.}{2ac^2(bc-ad}$$

$$= \frac{d(2bc-3ad)}{2ac^2(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2} + \frac{d\left(4b^2c^2-23abcd+12a^2d^2\right)}{4ac^3(bc-ad)^2\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)} + \frac{x}{ac\left(a+\frac{b}{x}\right)^{3/2}\left(c+\right.}$$

$$= \frac{b\left(20b^3c^3-36ab^2c^2d+87a^2bcd^2-36a^3d^3\right)}{12a^2c^3(bc-ad)^3\left(a+\frac{b}{x}\right)^{3/2}} + \frac{d(2bc-3ad)}{2ac^2(bc-ad)\left(a+\frac{b}{x}\right)^{3/2}\left(c+\frac{d}{x}\right)^2} + \frac{d\left(4b\right.}{4ac^3(b}$$

$$= \frac{b\left(20b^3c^3-36ab^2c^2d+87a^2bcd^2-36a^3d^3\right)}{12a^2c^3(bc-ad)^3\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b\left(20b^4c^4-56ab^3c^3d+24a^2b^2c^2d^2-35a^3\right.}{4a^3c^3(bc-ad)^4\sqrt{a+\frac{b}{x}}}$$

$$= \frac{b\left(20b^3c^3-36ab^2c^2d+87a^2bcd^2-36a^3d^3\right)}{12a^2c^3(bc-ad)^3\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b\left(20b^4c^4-56ab^3c^3d+24a^2b^2c^2d^2-35a^3\right.}{4a^3c^3(bc-ad)^4\sqrt{a+\frac{b}{x}}}$$

$$= \frac{b\left(20b^3c^3-36ab^2c^2d+87a^2bcd^2-36a^3d^3\right)}{12a^2c^3(bc-ad)^3\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b\left(20b^4c^4-56ab^3c^3d+24a^2b^2c^2d^2-35a^3\right.}{4a^3c^3(bc-ad)^4\sqrt{a+\frac{b}{x}}}$$

$$= \frac{b\left(20b^3c^3-36ab^2c^2d+87a^2bcd^2-36a^3d^3\right)}{12a^2c^3(bc-ad)^3\left(a+\frac{b}{x}\right)^{3/2}} + \frac{b\left(20b^4c^4-56ab^3c^3d+24a^2b^2c^2d^2-35a^3\right.}{4a^3c^3(bc-ad)^4\sqrt{a+\frac{b}{x}}}$$

Mathematica [C] time = 0.214204, size = 239, normalized size = 0.58

$$\frac{(cx+d)\left(2(cx+d)\left(\frac{1}{4}a^2d^2\left(24a^2d^2-88abcd+99b^2c^2\right)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{d\left(a+\frac{b}{x}\right)}{ad-bc}\right)+(6ad+5bc)(bc-ad)^3\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\right.\right.\right.}{6a^2c^4\left(a+\frac{b}{x}\right)^{3/2}(cx+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b/x)^(5/2)*(c + d/x)^3),x]
```

```
[Out] (-3*a*c^2*d*(b*c - a*d)^2*(-2*b*c + 3*a*d)*x^2 + 6*a*c^3*(b*c - a*d)^3*x^3
+ (d + c*x)*((-3*a*c*d*(-(b*c) + a*d)*(4*b^2*c^2 - 23*a*b*c*d + 12*a^2*d^2)
*x)/2 + 2*(d + c*x)*((a^2*d^2*(99*b^2*c^2 - 88*a*b*c*d + 24*a^2*d^2)*Hyperg
eometric2F1[-3/2, 1, -1/2, (d*(a + b/x))/(-(b*c) + a*d)])/4 + (b*c - a*d)^3
*(5*b*c + 6*a*d)*Hypergeometric2F1[-3/2, 1, -1/2, 1 + b/(a*x)])))/(6*a^2*c^
```

```
4*(b*c - a*d)^3*(a + b/x)^(3/2)*(d + c*x)^2)
```

Maple [B] time = 0.019, size = 7300, normalized size = 17.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(5/2)/(c+d/x)^3,x)

[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+\frac{b}{x}\right)^{\frac{5}{2}}\left(c+\frac{d}{x}\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/(c+d/x)^3,x, algorithm="maxima")

[Out] integrate(1/((a + b/x)^(5/2)*(c + d/x)^3), x)
```

Fricas [B] time = 31.4349, size = 12556, normalized size = 30.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/(c+d/x)^3,x, algorithm="fricas")

[Out] [1/24*(12*(5*b^7*c^5*d^2 - 14*a*b^6*c^4*d^3 + 6*a^2*b^5*c^3*d^4 + 16*a^3*b^
4*c^2*d^5 - 19*a^4*b^3*c*d^6 + 6*a^5*b^2*d^7 + (5*a^2*b^5*c^7 - 14*a^3*b^4*
c^6*d + 6*a^4*b^3*c^5*d^2 + 16*a^5*b^2*c^4*d^3 - 19*a^6*b*c^3*d^4 + 6*a^7*c
^2*d^5)*x^4 + 2*(5*a*b^6*c^7 - 9*a^2*b^5*c^6*d - 8*a^3*b^4*c^5*d^2 + 22*a^4
*b^3*c^4*d^3 - 3*a^5*b^2*c^3*d^4 - 13*a^6*b*c^2*d^5 + 6*a^7*c*d^6)*x^3 + (5
*b^7*c^7 + 6*a*b^6*c^6*d - 45*a^2*b^5*c^5*d^2 + 26*a^3*b^4*c^4*d^3 + 51*a^4
*b^3*c^3*d^4 - 54*a^5*b^2*c^2*d^5 + 5*a^6*b*c*d^6 + 6*a^7*d^7)*x^2 + 2*(5*b
^7*c^6*d - 9*a*b^6*c^5*d^2 - 8*a^2*b^5*c^4*d^3 + 22*a^3*b^4*c^3*d^4 - 3*a^4
*b^3*c^2*d^5 - 13*a^5*b^2*c*d^6 + 6*a^6*b*d^7)*x)*sqrt(a)*log(2*a*x - 2*sqr
t(a)*x*sqrt((a*x + b)/x) + b) + 3*(99*a^4*b^4*c^2*d^5 - 88*a^5*b^3*c*d^6 +
24*a^6*b^2*d^7 + (99*a^6*b^2*c^4*d^3 - 88*a^7*b*c^3*d^4 + 24*a^8*c^2*d^5)*x
^4 + 2*(99*a^5*b^3*c^4*d^3 + 11*a^6*b^2*c^3*d^4 - 64*a^7*b*c^2*d^5 + 24*a^8
*c*d^6)*x^3 + (99*a^4*b^4*c^4*d^3 + 308*a^5*b^3*c^3*d^4 - 229*a^6*b^2*c^2*d
^5 + 8*a^7*b*c*d^6 + 24*a^8*d^7)*x^2 + 2*(99*a^4*b^4*c^3*d^4 + 11*a^5*b^3*c
^2*d^5 - 64*a^6*b^2*c*d^6 + 24*a^7*b*d^7)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(
b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x
)/(c*x + d)) + 2*(12*(a^3*b^4*c^7 - 4*a^4*b^3*c^6*d + 6*a^5*b^2*c^5*d^2 - 4
*a^6*b*c^4*d^3 + a^7*c^3*d^4)*x^5 + (80*a^2*b^5*c^7 - 200*a^3*b^4*c^6*d + 4
8*a^4*b^3*c^5*d^2 + 48*a^5*b^2*c^4*d^3 - 135*a^6*b*c^3*d^4 + 54*a^7*c^2*d^5
)*x^4 + (60*a*b^6*c^7 - 8*a^2*b^5*c^6*d - 364*a^3*b^4*c^5*d^2 + 192*a^4*b^3
```

```
*c^4*d^3 - 234*a^5*b^2*c^3*d^4 + 3*a^6*b*c^2*d^5 + 36*a^7*c*d^6)*x^3 + (120
*a*b^6*c^6*d - 256*a^2*b^5*c^5*d^2 - 80*a^3*b^4*c^4*d^3 - 15*a^4*b^3*c^3*d^
4 - 156*a^5*b^2*c^2*d^5 + 72*a^6*b*c*d^6)*x^2 + 3*(20*a*b^6*c^5*d^2 - 56*a^
2*b^5*c^4*d^3 + 24*a^3*b^4*c^3*d^4 - 35*a^4*b^3*c^2*d^5 + 12*a^5*b^2*c*d^6)
*x)*sqrt((a*x + b)/x))/(a^4*b^6*c^8*d^2 - 4*a^5*b^5*c^7*d^3 + 6*a^6*b^4*c^6
*d^4 - 4*a^7*b^3*c^5*d^5 + a^8*b^2*c^4*d^6 + (a^6*b^4*c^10 - 4*a^7*b^3*c^9*
d + 6*a^8*b^2*c^8*d^2 - 4*a^9*b*c^7*d^3 + a^10*c^6*d^4)*x^4 + 2*(a^5*b^5*c^
10 - 3*a^6*b^4*c^9*d + 2*a^7*b^3*c^8*d^2 + 2*a^8*b^2*c^7*d^3 - 3*a^9*b*c^6*
d^4 + a^10*c^5*d^5)*x^3 + (a^4*b^6*c^10 - 9*a^6*b^4*c^8*d^2 + 16*a^7*b^3*c^
7*d^3 - 9*a^8*b^2*c^6*d^4 + a^10*c^4*d^6)*x^2 + 2*(a^4*b^6*c^9*d - 3*a^5*b^
5*c^8*d^2 + 2*a^6*b^4*c^7*d^3 + 2*a^7*b^3*c^6*d^4 - 3*a^8*b^2*c^5*d^5 + a^9
*b*c^4*d^6)*x), 1/24*(24*(5*b^7*c^5*d^2 - 14*a*b^6*c^4*d^3 + 6*a^2*b^5*c^3*
d^4 + 16*a^3*b^4*c^2*d^5 - 19*a^4*b^3*c*d^6 + 6*a^5*b^2*d^7 + (5*a^2*b^5*c^
7 - 14*a^3*b^4*c^6*d + 6*a^4*b^3*c^5*d^2 + 16*a^5*b^2*c^4*d^3 - 19*a^6*b*c^
3*d^4 + 6*a^7*c^2*d^5)*x^4 + 2*(5*a*b^6*c^7 - 9*a^2*b^5*c^6*d - 8*a^3*b^4*c
^5*d^2 + 22*a^4*b^3*c^4*d^3 - 3*a^5*b^2*c^3*d^4 - 13*a^6*b*c^2*d^5 + 6*a^7*
c*d^6)*x^3 + (5*b^7*c^7 + 6*a*b^6*c^6*d - 45*a^2*b^5*c^5*d^2 + 26*a^3*b^4*c
^4*d^3 + 51*a^4*b^3*c^3*d^4 - 54*a^5*b^2*c^2*d^5 + 5*a^6*b*c*d^6 + 6*a^7*d^
7)*x^2 + 2*(5*b^7*c^6*d - 9*a*b^6*c^5*d^2 - 8*a^2*b^5*c^4*d^3 + 22*a^3*b^4*
c^3*d^4 - 3*a^4*b^3*c^2*d^5 - 13*a^5*b^2*c*d^6 + 6*a^6*b*d^7)*x)*sqrt(-a)*a
rctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + 3*(99*a^4*b^4*c^2*d^5 - 88*a^5*b^3*c*
d^6 + 24*a^6*b^2*d^7 + (99*a^6*b^2*c^4*d^3 - 88*a^7*b*c^3*d^4 + 24*a^8*c^2*
d^5)*x^4 + 2*(99*a^5*b^3*c^4*d^3 + 11*a^6*b^2*c^3*d^4 - 64*a^7*b*c^2*d^5 +
24*a^8*c*d^6)*x^3 + (99*a^4*b^4*c^4*d^3 + 308*a^5*b^3*c^3*d^4 - 229*a^6*b^2
*c^2*d^5 + 8*a^7*b*c*d^6 + 24*a^8*d^7)*x^2 + 2*(99*a^4*b^4*c^3*d^4 + 11*a^5
*b^3*c^2*d^5 - 64*a^6*b^2*c*d^6 + 24*a^7*b*d^7)*x)*sqrt(-d/(b*c - a*d))*log
(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*
a*d)*x)/(c*x + d)) + 2*(12*(a^3*b^4*c^7 - 4*a^4*b^3*c^6*d + 6*a^5*b^2*c^5*d
^2 - 4*a^6*b*c^4*d^3 + a^7*c^3*d^4)*x^5 + (80*a^2*b^5*c^7 - 200*a^3*b^4*c^6
*d + 48*a^4*b^3*c^5*d^2 + 48*a^5*b^2*c^4*d^3 - 135*a^6*b*c^3*d^4 + 54*a^7*c
^2*d^5)*x^4 + (60*a*b^6*c^7 - 8*a^2*b^5*c^6*d - 364*a^3*b^4*c^5*d^2 + 192*a
^4*b^3*c^4*d^3 - 234*a^5*b^2*c^3*d^4 + 3*a^6*b*c^2*d^5 + 36*a^7*c*d^6)*x^3
+ (120*a*b^6*c^6*d - 256*a^2*b^5*c^5*d^2 - 80*a^3*b^4*c^4*d^3 - 15*a^4*b^3*
c^3*d^4 - 156*a^5*b^2*c^2*d^5 + 72*a^6*b*c*d^6)*x^2 + 3*(20*a*b^6*c^5*d^2 -
 56*a^2*b^5*c^4*d^3 + 24*a^3*b^4*c^3*d^4 - 35*a^4*b^3*c^2*d^5 + 12*a^5*b^2*
c*d^6)*x)*sqrt((a*x + b)/x))/(a^4*b^6*c^8*d^2 - 4*a^5*b^5*c^7*d^3 + 6*a^6*b
^4*c^6*d^4 - 4*a^7*b^3*c^5*d^5 + a^8*b^2*c^4*d^6 + (a^6*b^4*c^10 - 4*a^7*b^
3*c^9*d + 6*a^8*b^2*c^8*d^2 - 4*a^9*b*c^7*d^3 + a^10*c^6*d^4)*x^4 + 2*(a^5*
b^5*c^10 - 3*a^6*b^4*c^9*d + 2*a^7*b^3*c^8*d^2 + 2*a^8*b^2*c^7*d^3 - 3*a^9*
b*c^6*d^4 + a^10*c^5*d^5)*x^3 + (a^4*b^6*c^10 - 9*a^6*b^4*c^8*d^2 + 16*a^7*
b^3*c^7*d^3 - 9*a^8*b^2*c^6*d^4 + a^10*c^4*d^6)*x^2 + 2*(a^4*b^6*c^9*d - 3*
a^5*b^5*c^8*d^2 + 2*a^6*b^4*c^7*d^3 + 2*a^7*b^3*c^6*d^4 - 3*a^8*b^2*c^5*d^5
 + a^9*b*c^4*d^6)*x), -1/12*(3*(99*a^4*b^4*c^2*d^5 - 88*a^5*b^3*c*d^6 + 24*
a^6*b^2*d^7 + (99*a^6*b^2*c^4*d^3 - 88*a^7*b*c^3*d^4 + 24*a^8*c^2*d^5)*x^4
+ 2*(99*a^5*b^3*c^4*d^3 + 11*a^6*b^2*c^3*d^4 - 64*a^7*b*c^2*d^5 + 24*a^8*c*
d^6)*x^3 + (99*a^4*b^4*c^4*d^3 + 308*a^5*b^3*c^3*d^4 - 229*a^6*b^2*c^2*d^5
+ 8*a^7*b*c*d^6 + 24*a^8*d^7)*x^2 + 2*(99*a^4*b^4*c^3*d^4 + 11*a^5*b^3*c^2*
d^5 - 64*a^6*b^2*c*d^6 + 24*a^7*b*d^7)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c
- a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) - 6*(5*b^7*c^
5*d^2 - 14*a*b^6*c^4*d^3 + 6*a^2*b^5*c^3*d^4 + 16*a^3*b^4*c^2*d^5 - 19*a^4*
b^3*c*d^6 + 6*a^5*b^2*d^7 + (5*a^2*b^5*c^7 - 14*a^3*b^4*c^6*d + 6*a^4*b^3*c
^5*d^2 + 16*a^5*b^2*c^4*d^3 - 19*a^6*b*c^3*d^4 + 6*a^7*c^2*d^5)*x^4 + 2*(5*
a*b^6*c^7 - 9*a^2*b^5*c^6*d - 8*a^3*b^4*c^5*d^2 + 22*a^4*b^3*c^4*d^3 - 3*a^
5*b^2*c^3*d^4 - 13*a^6*b*c^2*d^5 + 6*a^7*c*d^6)*x^3 + (5*b^7*c^7 + 6*a*b^6*
c^6*d - 45*a^2*b^5*c^5*d^2 + 26*a^3*b^4*c^4*d^3 + 51*a^4*b^3*c^3*d^4 - 54*a
^5*b^2*c^2*d^5 + 5*a^6*b*c*d^6 + 6*a^7*d^7)*x^2 + 2*(5*b^7*c^6*d - 9*a*b^6*
c^5*d^2 - 8*a^2*b^5*c^4*d^3 + 22*a^3*b^4*c^3*d^4 - 3*a^4*b^3*c^2*d^5 - 13*a
^5*b^2*c*d^6 + 6*a^6*b*d^7)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x +
b)/x) + b) - (12*(a^3*b^4*c^7 - 4*a^4*b^3*c^6*d + 6*a^5*b^2*c^5*d^2 - 4*a^6
```

```
*b*c^4*d^3 + a^7*c^3*d^4)*x^5 + (80*a^2*b^5*c^7 - 200*a^3*b^4*c^6*d + 48*a^
4*b^3*c^5*d^2 + 48*a^5*b^2*c^4*d^3 - 135*a^6*b*c^3*d^4 + 54*a^7*c^2*d^5)*x^
4 + (60*a*b^6*c^7 - 8*a^2*b^5*c^6*d - 364*a^3*b^4*c^5*d^2 + 192*a^4*b^3*c^4
*d^3 - 234*a^5*b^2*c^3*d^4 + 3*a^6*b*c^2*d^5 + 36*a^7*c*d^6)*x^3 + (120*a*b
^6*c^6*d - 256*a^2*b^5*c^5*d^2 - 80*a^3*b^4*c^4*d^3 - 15*a^4*b^3*c^3*d^4 -
156*a^5*b^2*c^2*d^5 + 72*a^6*b*c*d^6)*x^2 + 3*(20*a*b^6*c^5*d^2 - 56*a^2*b^
5*c^4*d^3 + 24*a^3*b^4*c^3*d^4 - 35*a^4*b^3*c^2*d^5 + 12*a^5*b^2*c*d^6)*x)*
sqrt((a*x + b)/x))/(a^4*b^6*c^8*d^2 - 4*a^5*b^5*c^7*d^3 + 6*a^6*b^4*c^6*d^4
 - 4*a^7*b^3*c^5*d^5 + a^8*b^2*c^4*d^6 + (a^6*b^4*c^10 - 4*a^7*b^3*c^9*d +
6*a^8*b^2*c^8*d^2 - 4*a^9*b*c^7*d^3 + a^10*c^6*d^4)*x^4 + 2*(a^5*b^5*c^10 -
 3*a^6*b^4*c^9*d + 2*a^7*b^3*c^8*d^2 + 2*a^8*b^2*c^7*d^3 - 3*a^9*b*c^6*d^4
+ a^10*c^5*d^5)*x^3 + (a^4*b^6*c^10 - 9*a^6*b^4*c^8*d^2 + 16*a^7*b^3*c^7*d^
3 - 9*a^8*b^2*c^6*d^4 + a^10*c^4*d^6)*x^2 + 2*(a^4*b^6*c^9*d - 3*a^5*b^5*c^
8*d^2 + 2*a^6*b^4*c^7*d^3 + 2*a^7*b^3*c^6*d^4 - 3*a^8*b^2*c^5*d^5 + a^9*b*c
^4*d^6)*x), -1/12*(3*(99*a^4*b^4*c^2*d^5 - 88*a^5*b^3*c*d^6 + 24*a^6*b^2*d^
7 + (99*a^6*b^2*c^4*d^3 - 88*a^7*b*c^3*d^4 + 24*a^8*c^2*d^5)*x^4 + 2*(99*a^
5*b^3*c^4*d^3 + 11*a^6*b^2*c^3*d^4 - 64*a^7*b*c^2*d^5 + 24*a^8*c*d^6)*x^3 +
 (99*a^4*b^4*c^4*d^3 + 308*a^5*b^3*c^3*d^4 - 229*a^6*b^2*c^2*d^5 + 8*a^7*b*
c*d^6 + 24*a^8*d^7)*x^2 + 2*(99*a^4*b^4*c^3*d^4 + 11*a^5*b^3*c^2*d^5 - 64*a
^6*b^2*c*d^6 + 24*a^7*b*d^7)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*s
qrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) - 12*(5*b^7*c^5*d^2 - 1
4*a*b^6*c^4*d^3 + 6*a^2*b^5*c^3*d^4 + 16*a^3*b^4*c^2*d^5 - 19*a^4*b^3*c*d^6
 + 6*a^5*b^2*d^7 + (5*a^2*b^5*c^7 - 14*a^3*b^4*c^6*d + 6*a^4*b^3*c^5*d^2 +
16*a^5*b^2*c^4*d^3 - 19*a^6*b*c^3*d^4 + 6*a^7*c^2*d^5)*x^4 + 2*(5*a*b^6*c^7
 - 9*a^2*b^5*c^6*d - 8*a^3*b^4*c^5*d^2 + 22*a^4*b^3*c^4*d^3 - 3*a^5*b^2*c^3
*d^4 - 13*a^6*b*c^2*d^5 + 6*a^7*c*d^6)*x^3 + (5*b^7*c^7 + 6*a*b^6*c^6*d - 4
5*a^2*b^5*c^5*d^2 + 26*a^3*b^4*c^4*d^3 + 51*a^4*b^3*c^3*d^4 - 54*a^5*b^2*c^
2*d^5 + 5*a^6*b*c*d^6 + 6*a^7*d^7)*x^2 + 2*(5*b^7*c^6*d - 9*a*b^6*c^5*d^2 -
 8*a^2*b^5*c^4*d^3 + 22*a^3*b^4*c^3*d^4 - 3*a^4*b^3*c^2*d^5 - 13*a^5*b^2*c*
d^6 + 6*a^6*b*d^7)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (12*(
a^3*b^4*c^7 - 4*a^4*b^3*c^6*d + 6*a^5*b^2*c^5*d^2 - 4*a^6*b*c^4*d^3 + a^7*c
^3*d^4)*x^5 + (80*a^2*b^5*c^7 - 200*a^3*b^4*c^6*d + 48*a^4*b^3*c^5*d^2 + 48
*a^5*b^2*c^4*d^3 - 135*a^6*b*c^3*d^4 + 54*a^7*c^2*d^5)*x^4 + (60*a*b^6*c^7
- 8*a^2*b^5*c^6*d - 364*a^3*b^4*c^5*d^2 + 192*a^4*b^3*c^4*d^3 - 234*a^5*b^2
*c^3*d^4 + 3*a^6*b*c^2*d^5 + 36*a^7*c*d^6)*x^3 + (120*a*b^6*c^6*d - 256*a^2
*b^5*c^5*d^2 - 80*a^3*b^4*c^4*d^3 - 15*a^4*b^3*c^3*d^4 - 156*a^5*b^2*c^2*d^
5 + 72*a^6*b*c*d^6)*x^2 + 3*(20*a*b^6*c^5*d^2 - 56*a^2*b^5*c^4*d^3 + 24*a^3
*b^4*c^3*d^4 - 35*a^4*b^3*c^2*d^5 + 12*a^5*b^2*c*d^6)*x)*sqrt((a*x + b)/x))
/(a^4*b^6*c^8*d^2 - 4*a^5*b^5*c^7*d^3 + 6*a^6*b^4*c^6*d^4 - 4*a^7*b^3*c^5*d
^5 + a^8*b^2*c^4*d^6 + (a^6*b^4*c^10 - 4*a^7*b^3*c^9*d + 6*a^8*b^2*c^8*d^2
- 4*a^9*b*c^7*d^3 + a^10*c^6*d^4)*x^4 + 2*(a^5*b^5*c^10 - 3*a^6*b^4*c^9*d +
 2*a^7*b^3*c^8*d^2 + 2*a^8*b^2*c^7*d^3 - 3*a^9*b*c^6*d^4 + a^10*c^5*d^5)*x^
3 + (a^4*b^6*c^10 - 9*a^6*b^4*c^8*d^2 + 16*a^7*b^3*c^7*d^3 - 9*a^8*b^2*c^6*
d^4 + a^10*c^4*d^6)*x^2 + 2*(a^4*b^6*c^9*d - 3*a^5*b^5*c^8*d^2 + 2*a^6*b^4*
c^7*d^3 + 2*a^7*b^3*c^6*d^4 - 3*a^8*b^2*c^5*d^5 + a^9*b*c^4*d^6)*x)]
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(5/2)/(c+d/x)**3,x)
```

```
[Out] Exception raised: ValueError
```

Giac [A] time = 1.22993, size = 703, normalized size = 1.72

$$-\frac{1}{12}b\left(\frac{3\left(99b^2c^2d^4-88abcd^5+24a^2d^6\right)\arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^2}}\right)}{\left(b^5c^8-4ab^4c^7d+6a^2b^3c^6d^2-4a^3b^2c^5d^3+a^4bc^4d^4\right)\sqrt{bcd-ad^2}}-\frac{8\left(ab^4c-a^2b^3d+\frac{6(ax+b)b^4c}{x}-\right.}{\left(a^3b^4c^4-4a^4b^3c^3d+6a^5b^2c^2d^2-4a^6bcd\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(5/2)/(c+d/x)^3,x, algorithm="giac")
```

```
[Out] -1/12*b*(3*(99*b^2*c^2*d^4 - 88*a*b*c*d^5 + 24*a^2*d^6)*arctan(d*sqrt((a*x
+ b)/x)/sqrt(b*c*d - a*d^2))/((b^5*c^8 - 4*a*b^4*c^7*d + 6*a^2*b^3*c^6*d^2
- 4*a^3*b^2*c^5*d^3 + a^4*b*c^4*d^4)*sqrt(b*c*d - a*d^2)) - 8*(a*b^4*c - a^
2*b^3*d + 6*(a*x + b)*b^4*c/x - 15*(a*x + b)*a*b^3*d/x)*x/((a^3*b^4*c^4 - 4
*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4)*(a*x + b)*sqr
t((a*x + b)/x)) + 3*(21*b^2*c^2*d^4*sqrt((a*x + b)/x) - 29*a*b*c*d^5*sqrt((
a*x + b)/x) + 8*a^2*d^6*sqrt((a*x + b)/x) + 19*(a*x + b)*b*c*d^5*sqrt((a*x
+ b)/x)/x - 8*(a*x + b)*a*d^6*sqrt((a*x + b)/x)/x)/((b^4*c^7 - 4*a*b^3*c^6*
d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4)*(b*c - a*d + (a*x +
b)*d/x)^2) + 12*sqrt((a*x + b)/x)/((a - (a*x + b)/x)*a^3*c^3) - 12*(5*b*c +
 6*a*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^3*b*c^4))
```

3.266 $\int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx$

Optimal. Leaf size=123

$$x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}+\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\sqrt{c}}-2\sqrt{b}\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{b}\sqrt{c+\frac{d}{x}}}\right)$$

```
[Out] Sqrt[a + b/x]*Sqrt[c + d/x]*x + ((b*c + a*d)*ArcTanh[(Sqrt[c]*Sqrt[a + b/x]
)/(Sqrt[a]*Sqrt[c + d/x])])/(Sqrt[a]*Sqrt[c]) - 2*Sqrt[b]*Sqrt[d]*ArcTanh[(
Sqrt[d]*Sqrt[a + b/x])/(Sqrt[b]*Sqrt[c + d/x])]
```

Rubi [A] time = 0.0938878, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {375, 97, 157, 63, 217, 206, 93, 208}

$$x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}+\frac{(ad+bc)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\sqrt{c}}-2\sqrt{b}\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{b}\sqrt{c+\frac{d}{x}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]*Sqrt[c + d/x],x]
```

```
[Out] Sqrt[a + b/x]*Sqrt[c + d/x]*x + ((b*c + a*d)*ArcTanh[(Sqrt[c]*Sqrt[a + b/x]
)/(Sqrt[a]*Sqrt[c + d/x])])/(Sqrt[a]*Sqrt[c]) - 2*Sqrt[b]*Sqrt[d]*ArcTanh[(
Sqrt[d]*Sqrt[a + b/x])/(Sqrt[b]*Sqrt[c + d/x])]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx &= -\text{Subst}\left(\int \frac{\sqrt{a+bx}\sqrt{c+dx}}{x^2}\,dx,x,\frac{1}{x}\right)\\
&= \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}x-\text{Subst}\left(\int \frac{\frac{1}{2}(bc+ad)+bdx}{x\sqrt{a+bx}\sqrt{c+dx}}\,dx,x,\frac{1}{x}\right)\\
&= \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}x-(bd)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx,x,\frac{1}{x}\right)-\frac{1}{2}(bc+ad)\,\text{Subst}\left(\int \frac{}{x\sqrt{a+\cdots}}\right.\\
&= \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}x-(2d)\,\text{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx,x,\sqrt{a+\frac{b}{x}}\right)-(bc+ad)\,\text{Subst}\left(\int \frac{}{-a\cdots}\right.\\
&= \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}x+\frac{(bc+ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\sqrt{c}}-(2d)\,\text{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx,x,\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\right)\\
&= \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}x+\frac{(bc+ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\sqrt{c}}-2\sqrt{b}\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{b}\sqrt{c+\frac{d}{x}}}\right)
\end{aligned}
$$

Mathematica [A] time = 1.00956, size = 167, normalized size = 1.36

$$\frac{\sqrt{a+\frac{b}{x}}(cx+d)-2\sqrt{d}\sqrt{bc-ad}\sqrt{\frac{bcx+bd}{bcx-adx}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)+\frac{\sqrt{c+\frac{d}{x}}(ad+bc)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}\sqrt{c}}}{\sqrt{c+\frac{d}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]*Sqrt[c + d/x],x]
```

```
[Out] (Sqrt[a + b/x]*(d + c*x) - 2*Sqrt[d]*Sqrt[b*c - a*d]*Sqrt[(b*d + b*c*x)/(b*
c*x - a*d*x)]*ArcSinh[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]] + ((b*c + a*
d)*Sqrt[c + d/x]*ArcTanh[(Sqrt[c]*Sqrt[a + b/x])/(Sqrt[a]*Sqrt[c + d/x])])/
(Sqrt[a]*Sqrt[c]))/Sqrt[c + d/x]
```

Maple [B] time = 0.046, size = 253, normalized size = 2.1

$$\frac{x}{2}\sqrt{\frac{cx+d}{x}}\sqrt{\frac{ax+b}{x}}\left(-2\,bd\ln\left(\frac{adx+bcx+2\sqrt{bd}\sqrt{acx^2+adx+bcx+bd}+2\,bd}{x}\right)\sqrt{ac}+\sqrt{bd}\ln\left(\frac{1}{2}\left(2\,acx+2\sqrt{acx^2+a\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)^(1/2)*(a+b/x)^(1/2),x)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x*((c*x+d)/x)^(1/2)*(-2*b*d*ln((a*d*x+b*c*x+2*(b*d)^(
1/2)*(a*c*x^2+a*d*x+b*c*x+b*d)^(1/2)+2*b*d)/x)*(a*c)^(1/2)+(b*d)^(1/2)*ln(1
/2*(2*a*c*x+2*(a*c*x^2+a*d*x+b*c*x+b*d)^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1
/2))*a*d+(b*d)^(1/2)*ln(1/2*(2*a*c*x+2*(a*c*x^2+a*d*x+b*c*x+b*d)^(1/2)*(a*c
)^(1/2)+a*d+b*c)/(a*c)^(1/2))*b*c+2*(a*c*x^2+a*d*x+b*c*x+b*d)^(1/2)*(a*c)^(
1/2)*(b*d)^(1/2))/(a*c*x^2+a*d*x+b*c*x+b*d)^(1/2)/(a*c)^(1/2)/(b*d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^(1/2)*(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x)*sqrt(c + d/x), x)
```

Fricas [A] time = 5.40341, size = 1995, normalized size = 16.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^(1/2)*(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*a*c*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) + 2*sqrt(b*d)*a*c*log(-(8
*b^2*d^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^2 - 4*(2*b*d*x + (b*c + a*d)*x
^2)*sqrt(b*d)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) + 8*(b^2*c*d + a*b*d^2)*x
)/x^2) + sqrt(a*c)*(b*c + a*d)*log(-8*a^2*c^2*x^2 - b^2*c^2 - 6*a*b*c*d - a
^2*d^2 - 4*(2*a*c*x^2 + (b*c + a*d)*x)*sqrt(a*c)*sqrt((a*x + b)/x)*sqrt((c*
x + d)/x) - 8*(a*b*c^2 + a^2*c*d)*x))/(a*c), 1/4*(4*a*c*x*sqrt((a*x + b)/x)
*sqrt((c*x + d)/x) + 4*sqrt(-b*d)*a*c*arctan(1/2*(2*b*d*x + (b*c + a*d)*x^2
)*sqrt(-b*d)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(a*b*c*d*x^2 + b^2*d^2 + (
b^2*c*d + a*b*d^2)*x)) + sqrt(a*c)*(b*c + a*d)*log(-8*a^2*c^2*x^2 - b^2*c^2
 - 6*a*b*c*d - a^2*d^2 - 4*(2*a*c*x^2 + (b*c + a*d)*x)*sqrt(a*c)*sqrt((a*x
+ b)/x)*sqrt((c*x + d)/x) - 8*(a*b*c^2 + a^2*c*d)*x))/(a*c), 1/2*(2*a*c*x*s
qrt((a*x + b)/x)*sqrt((c*x + d)/x) + sqrt(b*d)*a*c*log(-(8*b^2*d^2 + (b^2*c
^2 + 6*a*b*c*d + a^2*d^2)*x^2 - 4*(2*b*d*x + (b*c + a*d)*x^2)*sqrt(b*d)*sqr
t((a*x + b)/x)*sqrt((c*x + d)/x) + 8*(b^2*c*d + a*b*d^2)*x)/x^2) - sqrt(-a*
c)*(b*c + a*d)*arctan(2*sqrt(-a*c)*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(2
*a*c*x + b*c + a*d)))/(a*c), 1/2*(2*a*c*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/
x) + 2*sqrt(-b*d)*a*c*arctan(1/2*(2*b*d*x + (b*c + a*d)*x^2)*sqrt(-b*d)*sqr
t((a*x + b)/x)*sqrt((c*x + d)/x)/(a*b*c*d*x^2 + b^2*d^2 + (b^2*c*d + a*b*d^
2)*x)) - sqrt(-a*c)*(b*c + a*d)*arctan(2*sqrt(-a*c)*x*sqrt((a*x + b)/x)*sqr
t((c*x + d)/x)/(2*a*c*x + b*c + a*d)))/(a*c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)**(1/2)*(a+b/x)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b/x)*sqrt(c + d/x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^(1/2)*(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/x)*sqrt(c + d/x), x)
```

3.267 $\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\,dx$

Optimal. Leaf size=81

$$\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}c^{3/2}}+\frac{x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}}{c}$$

```
[Out] (Sqrt[a + b/x]*Sqrt[c + d/x]*x)/c + ((b*c - a*d)*ArcTanh[(Sqrt[c]*Sqrt[a +
b/x])/(Sqrt[a]*Sqrt[c + d/x])])/(Sqrt[a]*c^(3/2))
```

Rubi [A] time = 0.0520053, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {375, 94, 93, 208}

$$\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}c^{3/2}}+\frac{x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]/Sqrt[c + d/x],x]
```

```
[Out] (Sqrt[a + b/x]*Sqrt[c + d/x]*x)/c + ((b*c - a*d)*ArcTanh[(Sqrt[c]*Sqrt[a +
b/x])/(Sqrt[a]*Sqrt[c + d/x])])/(Sqrt[a]*c^(3/2))
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\,dx = -\operatorname{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2\sqrt{c+dx}}\,dx, x, \frac{1}{x}\right)$$
$$= \frac{\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}x}{c} - \frac{(bc-ad)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, \frac{1}{x}\right)}{2c}$$
$$= \frac{\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}x}{c} - \frac{(bc-ad)\operatorname{Subst}\left(\int \frac{1}{-a+cx^2}\,dx, x, \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\right)}{c}$$
$$= \frac{\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}x}{c} + \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}c^{3/2}}$$

Mathematica [A] time = 0.0607665, size = 81, normalized size = 1.

$$\frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}c^{3/2}} + \frac{x\sqrt{a+\frac{b}{x}}\sqrt{c+\frac{d}{x}}}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]/Sqrt[c + d/x],x]
```

```
[Out] (Sqrt[a + b/x]*Sqrt[c + d/x]*x)/c + ((b*c - a*d)*ArcTanh[(Sqrt[c]*Sqrt[a +
b/x])/(Sqrt[a]*Sqrt[c + d/x])])/(Sqrt[a]*c^(3/2))
```

Maple [B] time = 0.024, size = 155, normalized size = 1.9

$$-\frac{x}{2c}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}\left(\ln\left(\frac{1}{2}\left(2\,acx+2\,\sqrt{(cx+d)\,(ax+b)}\sqrt{ac}+ad+bc\right)\frac{1}{\sqrt{ac}}\right)ad-\ln\left(\frac{1}{2}\left(2\,acx+2\,\sqrt{(cx+d)\,(ax+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(1/2)/(c+d/x)^(1/2),x)
```

```
[Out] -1/2*((a*x+b)/x)^(1/2)*x*((c*x+d)/x)^(1/2)*(ln(1/2*(2*a*c*x+2*((c*x+d)*(a*x
+b))^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*a*d-ln(1/2*(2*a*c*x+2*((c*x+d)
*(a*x+b))^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*b*c-2*((c*x+d)*(a*x+b))^(
1/2)*(a*c)^(1/2))/((c*x+d)*(a*x+b))^(1/2)/c/(a*c)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x)/sqrt(c + d/x), x)
```

Fricas [A] time = 2.21108, size = 551, normalized size = 6.8

$$\Bigg[\frac{4\,acx\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}} - \sqrt{ac}(bc-ad)\log\Big(-8\,a^2c^2x^2 - b^2c^2 - 6\,abcd - a^2d^2 + 4\big(2\,acx^2 + (bc+ad)x\big)\sqrt{ac}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}} - 8\big(\cdots}{4\,ac^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*a*c*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) - sqrt(a*c)*(b*c - a*d)*l
og(-8*a^2*c^2*x^2 - b^2*c^2 - 6*a*b*c*d - a^2*d^2 + 4*(2*a*c*x^2 + (b*c + a
*d)*x)*sqrt(a*c)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) - 8*(a*b*c^2 + a^2*c*d
)*x))/(a*c^2), 1/2*(2*a*c*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) - sqrt(-a*c
)*(b*c - a*d)*arctan(2*sqrt(-a*c)*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(2*
a*c*x + b*c + a*d)))/(a*c^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{c + \frac{d}{x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(1/2)/(c+d/x)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b/x)/sqrt(c + d/x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{c + \frac{d}{x}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/x)/sqrt(c + d/x), x)
```

3.268 $\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{3/2}}\,dx$

Optimal. Leaf size=122

$$-\frac{\sqrt{a+\frac{b}{x}}(bc-3ad)}{ac^2\sqrt{c+\frac{d}{x}}}+\frac{(bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}c^{5/2}}+\frac{x\left(a+\frac{b}{x}\right)^{3/2}}{ac\sqrt{c+\frac{d}{x}}}$$

```
[Out] -(((b*c - 3*a*d)*Sqrt[a + b/x])/(a*c^2*Sqrt[c + d/x])) + ((a + b/x)^(3/2)*x
)/(a*c*Sqrt[c + d/x]) + ((b*c - 3*a*d)*ArcTanh[(Sqrt[c]*Sqrt[a + b/x])/(Sqr
t[a]*Sqrt[c + d/x])])/(Sqrt[a]*c^(5/2))
```

Rubi [A] time = 0.0803529, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {375, 96, 94, 93, 208}

$$-\frac{\sqrt{a+\frac{b}{x}}(bc-3ad)}{ac^2\sqrt{c+\frac{d}{x}}}+\frac{(bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}c^{5/2}}+\frac{x\left(a+\frac{b}{x}\right)^{3/2}}{ac\sqrt{c+\frac{d}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x]/(c + d/x)^(3/2),x]
```

```
[Out] -(((b*c - 3*a*d)*Sqrt[a + b/x])/(a*c^2*Sqrt[c + d/x])) + ((a + b/x)^(3/2)*x
)/(a*c*Sqrt[c + d/x]) + ((b*c - 3*a*d)*ArcTanh[(Sqrt[c]*Sqrt[a + b/x])/(Sqr
t[a]*Sqrt[c + d/x])])/(Sqrt[a]*c^(5/2))
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
```

```
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{3/2}}\,dx &= -\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2(c+dx)^{3/2}}\,dx, x, \frac{1}{x}\right)\\
&= \frac{\left(a+\frac{b}{x}\right)^{3/2} x}{ac\sqrt{c+\frac{d}{x}}} + \frac{\left(-\frac{bc}{2}+\frac{3ad}{2}\right)\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x(c+dx)^{3/2}}\,dx, x, \frac{1}{x}\right)}{ac}\\
&= -\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{ac^2\sqrt{c+\frac{d}{x}}} + \frac{\left(a+\frac{b}{x}\right)^{3/2} x}{ac\sqrt{c+\frac{d}{x}}} - \frac{(bc-3ad)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}\sqrt{c+dx}}\,dx, x, \frac{1}{x}\right)}{2c^2}\\
&= -\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{ac^2\sqrt{c+\frac{d}{x}}} + \frac{\left(a+\frac{b}{x}\right)^{3/2} x}{ac\sqrt{c+\frac{d}{x}}} - \frac{(bc-3ad)\,\text{Subst}\left(\int \frac{1}{-a+cx^2}\,dx, x, \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\right)}{c^2}\\
&= -\frac{(bc-3ad)\sqrt{a+\frac{b}{x}}}{ac^2\sqrt{c+\frac{d}{x}}} + \frac{\left(a+\frac{b}{x}\right)^{3/2} x}{ac\sqrt{c+\frac{d}{x}}} + \frac{(bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}c^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.0948413, size = 87, normalized size = 0.71

$$\frac{\sqrt{a+\frac{b}{x}}(cx+3d)}{c^2\sqrt{c+\frac{d}{x}}} + \frac{(bc-3ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]/(c + d/x)^(3/2),x]
```

```
[Out] (Sqrt[a + b/x]*(3*d + c*x))/(c^2*Sqrt[c + d/x]) + ((b*c - 3*a*d)*ArcTanh[(S
qrt[c]*Sqrt[a + b/x])/(Sqrt[a]*Sqrt[c + d/x])])/(Sqrt[a]*c^(5/2))
```

Maple [B] time = 0.032, size = 280, normalized size = 2.3

$$\frac{x}{(2cx+2d)c^2}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}\left(-3\ln\left(1/2\,\frac{2\,acx+2\sqrt{(cx+d)(ax+b)}\sqrt{ac}+ad+bc}{\sqrt{ac}}\right)xacd+\ln\left(\frac{1}{2}\left(2\,acx+2\sqrt{(cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(1/2)/(c+d/x)^(3/2),x)
```

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x*((c*x+d)/x)^(1/2)*(-3*ln(1/2*(2*a*c*x+2*((c*x+d)*(a
*x+b))^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*x*a*c*d+ln(1/2*(2*a*c*x+2*((
c*x+d)*(a*x+b))^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*x*b*c^2+2*x*c*((c*x
+d)*(a*x+b))^(1/2)*(a*c)^(1/2)-3*ln(1/2*(2*a*c*x+2*((c*x+d)*(a*x+b))^(1/2)*
(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*a*d^2+ln(1/2*(2*a*c*x+2*((c*x+d)*(a*x+b))
^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*b*c*d+6*d*((c*x+d)*(a*x+b))^(1/2)*
(a*c)^(1/2))/(a*c)^(1/2)/(c*x+d)/((c*x+d)*(a*x+b))^(1/2)/c^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x)/(c + d/x)^(3/2), x)
```

Fricas [A] time = 2.08185, size = 710, normalized size = 5.82

$$\left[-\frac{\left(bcd-3\,ad^2+\left(bc^2-3\,acd\right)x\right)\sqrt{ac}\log\left(-8\,a^2c^2x^2-b^2c^2-6\,abcd-a^2d^2+4\left(2\,acx^2+(bc+ad)x\right)\sqrt{ac}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}\right.}{4\left(ac^4x+ac^3d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((b*c*d - 3*a*d^2 + (b*c^2 - 3*a*c*d)*x)*sqrt(a*c)*log(-8*a^2*c^2*x^2
 - b^2*c^2 - 6*a*b*c*d - a^2*d^2 + 4*(2*a*c*x^2 + (b*c + a*d)*x)*sqrt(a*c)*
sqrt((a*x + b)/x)*sqrt((c*x + d)/x) - 8*(a*b*c^2 + a^2*c*d)*x) - 4*(a*c^2*x
^2 + 3*a*c*d*x)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x))/(a*c^4*x + a*c^3*d), -
1/2*((b*c*d - 3*a*d^2 + (b*c^2 - 3*a*c*d)*x)*sqrt(-a*c)*arctan(2*sqrt(-a*c)
*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(2*a*c*x + b*c + a*d)) - 2*(a*c^2*x^
2 + 3*a*c*d*x)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x))/(a*c^4*x + a*c^3*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(1/2)/(c+d/x)**(3/2),x)
```

```
[Out] Integral(sqrt(a + b/x)/(c + d/x)**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.269 $\int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q \, dx$

Optimal. Leaf size=96

$$-\frac{b\left(a+\frac{b}{x}\right)^{p+1}\left(c+\frac{d}{x}\right)^q\left(\frac{b\left(c+\frac{d}{x}\right)}{bc-ad}\right)^{-q}F_1\left(p+1;-q,2;p+2;-\frac{d\left(a+\frac{b}{x}\right)}{bc-ad},\frac{a+\frac{b}{x}}{a}\right)}{a^2(p+1)}$$

```
[Out] -((b*(a + b/x)^(1 + p)*(c + d/x)^q*AppellF1[1 + p, -q, 2, 2 + p, -((d*(a +
b/x))/(b*c - a*d)), (a + b/x)/a])/(a^2*(1 + p)*((b*(c + d/x))/(b*c - a*d))^
q))
```

Rubi [A] time = 0.0605418, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {375, 137, 136}

$$-\frac{b\left(a+\frac{b}{x}\right)^{p+1}\left(c+\frac{d}{x}\right)^q\left(\frac{b\left(c+\frac{d}{x}\right)}{bc-ad}\right)^{-q}F_1\left(p+1;-q,2;p+2;-\frac{d\left(a+\frac{b}{x}\right)}{bc-ad},\frac{a+\frac{b}{x}}{a}\right)}{a^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x)^p*(c + d/x)^q,x]
```

```
[Out] -((b*(a + b/x)^(1 + p)*(c + d/x)^q*AppellF1[1 + p, -q, 2, 2 + p, -((d*(a +
b/x))/(b*c - a*d)), (a + b/x)/a])/(a^2*(1 + p)*((b*(c + d/x))/(b*c - a*d))^
q))
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 137

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(c + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*
((b*(c + d*x))/(b*c - a*d))^FracPart[n]), Int[(a + b*x)^m*((b*c)/(b*c - a*d
) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !GtQ[b/(b*c
- a*d), 0] &&  !SimplerQ[c + d*x, a + b*x]
```

Rule 136

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -
n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/
(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rubi steps

$$\int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q dx = -\text{Subst}\left(\int \frac{(a+bx)^p(c+dx)^q}{x^2}\,dx, x, \frac{1}{x}\right)$$
$$= -\left(\left(\left(c+\frac{d}{x}\right)^q \left(\frac{b\left(c+\frac{d}{x}\right)}{bc-ad}\right)^{-q}\right)\text{Subst}\left(\int \frac{(a+bx)^p\left(\frac{bc}{bc-ad}+\frac{bdx}{bc-ad}\right)^q}{x^2}\,dx, x, \frac{1}{x}\right)\right)$$
$$= -\frac{b\left(a+\frac{b}{x}\right)^{1+p}\left(c+\frac{d}{x}\right)^q\left(\frac{b\left(c+\frac{d}{x}\right)}{bc-ad}\right)^{-q}F_1\left(1+p;-q,2;2+p;-\frac{d\left(a+\frac{b}{x}\right)}{bc-ad},\frac{a+\frac{b}{x}}{a}\right)}{a^2(1+p)}$$

Mathematica [B] time = 0.302972, size = 206, normalized size = 2.15

$$\frac{bdx(p+q-2)\left(a+\frac{b}{x}\right)^p\left(c+\frac{d}{x}\right)^q F_1\left(-p-q+1;-p,-q;-p-q+2;-\frac{a}{b}\right.}{(p+q-1)\left(x\left(adpF_1\left(-p-q+2;1-p,-q;-p-q+3;-\frac{ax}{b},-\frac{cx}{d}\right)+bcqF_1\left(-p-q+2;-p,1-q;-p-q+3;-\frac{ax}{b},-\frac{cx}{d}\right)\right)-\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b/x)^p*(c + d/x)^q,x]
```

```
[Out] (b*d*(-2 + p + q)*(a + b/x)^p*(c + d/x)^q*x*AppellF1[1 - p - q, -p, -q, 2 -
 p - q, -((a*x)/b), -((c*x)/d)])/((-1 + p + q)*(-(b*d*(-2 + p + q)*AppellF1
[1 - p - q, -p, -q, 2 - p - q, -((a*x)/b), -((c*x)/d)]) + x*(a*d*p*AppellF1
[2 - p - q, 1 - p, -q, 3 - p - q, -((a*x)/b), -((c*x)/d)] + b*c*q*AppellF1[
2 - p - q, -p, 1 - q, 3 - p - q, -((a*x)/b), -((c*x)/d)])))
```

Maple [F] time = 0.121, size = 0, normalized size = 0.

$$\int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^p*(c+d/x)^q,x)
```

```
[Out] int((a+b/x)^p*(c+d/x)^q,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+\frac{b}{x}\right)^p \left(c+\frac{d}{x}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^p*(c+d/x)^q,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x)^p*(c + d/x)^q, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\frac{ax+b}{x}\right)^p\left(\frac{cx+d}{x}\right)^q,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^p*(c+d/x)^q,x, algorithm="fricas")
```

```
[Out] integral(((a*x + b)/x)^p*((c*x + d)/x)^q, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+\frac{b}{x}\right)^p\left(c+\frac{d}{x}\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**p*(c+d/x)**q,x)
```

```
[Out] Integral((a + b/x)**p*(c + d/x)**q, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+\frac{b}{x}\right)^p\left(c+\frac{d}{x}\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^p*(c+d/x)^q,x, algorithm="giac")
```

```
[Out] integrate((a + b/x)^p*(c + d/x)^q, x)
```

3.270 $\int \frac{a+\frac{b}{x^2}}{c+\frac{d}{x^2}} dx$

Optimal. Leaf size=39

$$\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{c^{3/2}\sqrt{d}}+\frac{ax}{c}$$

```
[Out] (a*x)/c + ((b*c - a*d)*ArcTan[(Sqrt[c]*x)/Sqrt[d]])/(c^(3/2)*Sqrt[d])
```

Rubi [A] time = 0.0200836, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {374, 388, 205}

$$\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{c^{3/2}\sqrt{d}}+\frac{ax}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)/(c + d/x^2),x]
```

```
[Out] (a*x)/c + ((b*c - a*d)*ArcTan[(Sqrt[c]*x)/Sqrt[d]])/(c^(3/2)*Sqrt[d])
```

Rule 374

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[x^(n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[n]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+\frac{b}{x^2}}{c+\frac{d}{x^2}}\,dx &= \int \frac{b+ax^2}{d+cx^2}\,dx\\
&= \frac{ax}{c}-\frac{(-bc+ad)\int \frac{1}{d+cx^2}\,dx}{c}\\
&= \frac{ax}{c}+\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{c^{3/2}\sqrt{d}}
\end{aligned}$$

Mathematica [A] time = 0.0256489, size = 40, normalized size = 1.03

$$\frac{ax}{c}-\frac{(ad-bc)\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)}{c^{3/2}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^2)/(c + d/x^2),x]
```

```
[Out] (a*x)/c - ((-(b*c) + a*d)*ArcTan[(Sqrt[c]*x)/Sqrt[d]])/(c^(3/2)*Sqrt[d])
```

Maple [A] time = 0.004, size = 45, normalized size = 1.2

$$\frac{ax}{c}-\frac{ad}{c}\arctan\left(cx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}+b\arctan\left(cx\frac{1}{\sqrt{cd}}\right)\frac{1}{\sqrt{cd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+1/x^2*b)/(c+d/x^2),x)
```

```
[Out] a*x/c-1/c/(c*d)^(1/2)*arctan(x*c/(c*d)^(1/2))*a*d+1/(c*d)^(1/2)*arctan(x*c/
(c*d)^(1/2))*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.23323, size = 223, normalized size = 5.72

$$\left[\frac{2\,acdx+(bc-ad)\sqrt{-cd}\log\left(\frac{cx^2+2\sqrt{-cd}x-d}{cx^2+d}\right)}{2\,c^2d},\,\frac{acdx+(bc-ad)\sqrt{cd}\arctan\left(\frac{\sqrt{cd}x}{d}\right)}{c^2d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*c*d*x + (b*c - a*d)*sqrt(-c*d)*log((c*x^2 + 2*sqrt(-c*d)*x - d)/(
c*x^2 + d)))/(c^2*d), (a*c*d*x + (b*c - a*d)*sqrt(c*d)*arctan(sqrt(c*d)*x/d
))/(c^2*d)]
```

Sympy [B] time = 0.40391, size = 82, normalized size = 2.1

$$\frac{ax}{c} + \frac{\sqrt{-\frac{1}{c^3d}}\,(ad-bc)\log\left(-cd\sqrt{-\frac{1}{c^3d}}+x\right)}{2} - \frac{\sqrt{-\frac{1}{c^3d}}\,(ad-bc)\log\left(cd\sqrt{-\frac{1}{c^3d}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)/(c+d/x**2),x)
```

```
[Out] a*x/c + sqrt(-1/(c**3*d))*(a*d - b*c)*log(-c*d*sqrt(-1/(c**3*d)) + x)/2 - s
qrt(-1/(c**3*d))*(a*d - b*c)*log(c*d*sqrt(-1/(c**3*d)) + x)/2
```

Giac [A] time = 1.13505, size = 45, normalized size = 1.15

$$\frac{ax}{c} + \frac{(bc-ad)\arctan\left(\frac{cx}{\sqrt{cd}}\right)}{\sqrt{cd}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)/(c+d/x^2),x, algorithm="giac")
```

```
[Out] a*x/c + (b*c - a*d)*arctan(c*x/sqrt(c*d))/(sqrt(c*d)*c)
```

3.271 $\int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx$

Optimal. Leaf size=233

$$-\frac{\sqrt{c}\sqrt{a+\frac{b}{x^2}}(ad+bc)\text{EllipticF}\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}-\frac{2d\sqrt{a+\frac{b}{x^2}}}{x\sqrt{c+\frac{d}{x^2}}}+x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}+\frac{2\sqrt{c}\sqrt{d}\sqrt{a+\frac{b}{x^2}}E\left(\cot^{-1}\right.}{\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c(a+}{a(c+}}}$$

```
[Out] (-2*d*Sqrt[a + b/x^2])/(Sqrt[c + d/x^2]*x) + Sqrt[a + b/x^2]*Sqrt[c + d/x^2
]*x + (2*Sqrt[c]*Sqrt[d]*Sqrt[a + b/x^2]*EllipticE[ArcCot[(Sqrt[c]*x)/Sqrt[
d]], 1 - (b*c)/(a*d)])/(Sqrt[(c*(a + b/x^2))/(a*(c + d/x^2))]*Sqrt[c + d/x^
2]) - (Sqrt[c]*(b*c + a*d)*Sqrt[a + b/x^2]*EllipticF[ArcCot[(Sqrt[c]*x)/Sqr
t[d]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b/x^2))/(a*(c + d/x^2))]*S
qrt[c + d/x^2])
```

Rubi [A] time = 0.217579, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {375, 473, 531, 418, 492, 411}

$$-\frac{2d\sqrt{a+\frac{b}{x^2}}}{x\sqrt{c+\frac{d}{x^2}}}+x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}-\frac{\sqrt{c}\sqrt{a+\frac{b}{x^2}}(ad+bc)F\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}+\frac{2\sqrt{c}\sqrt{d}\sqrt{a+\frac{b}{x^2}}E\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1\right.}{\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x^2]*Sqrt[c + d/x^2],x]
```

```
[Out] (-2*d*Sqrt[a + b/x^2])/(Sqrt[c + d/x^2]*x) + Sqrt[a + b/x^2]*Sqrt[c + d/x^2
]*x + (2*Sqrt[c]*Sqrt[d]*Sqrt[a + b/x^2]*EllipticE[ArcCot[(Sqrt[c]*x)/Sqrt[
d]], 1 - (b*c)/(a*d)])/(Sqrt[(c*(a + b/x^2))/(a*(c + d/x^2))]*Sqrt[c + d/x^
2]) - (Sqrt[c]*(b*c + a*d)*Sqrt[a + b/x^2]*EllipticF[ArcCot[(Sqrt[c]*x)/Sqr
t[d]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b/x^2))/(a*(c + d/x^2))]*S
qrt[c + d/x^2])
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^p*(c + d*x^n)^q)/(e*(m
 + 1)), x] - Dist[n/(e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^(p - 1)*(c
 + d*x^n)^(q - 1)*Simp[b*c*p + a*d*q + b*d*(p + q)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && LtQ
[m, -1] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
```

```
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx = -\text{Subst}\left(\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{x^2}\,dx, x, \frac{1}{x}\right)$$

$$= \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}x - 2\,\text{Subst}\left(\int \frac{\frac{1}{2}(bc+ad)+bdx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)$$

$$= \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}x - (2bd)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right) - (bc+ad)\,\text{Subst}\left(\int \frac{}{\sqrt{a}}\right.$$

$$= -\frac{2d\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}x} + \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}x - \frac{\sqrt{c}(bc+ad)\sqrt{a+\frac{b}{x^2}}F\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\sqrt{c+\frac{d}{x^2}}} + (2cd)\,\text{Sub}$$

$$= -\frac{2d\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}x} + \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}x + \frac{2\sqrt{c}\sqrt{d}\sqrt{a+\frac{b}{x^2}}E\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\sqrt{c+\frac{d}{x^2}}} - \frac{\sqrt{c}(bc+ad)\sqrt{}}{a\sqrt{}}$$

Mathematica [C] time = 0.296365, size = 205, normalized size = 0.88

$$-\frac{x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\left(ix\sqrt{\frac{ax^2}{b}+1}\sqrt{\frac{cx^2}{d}+1}(bc-ad)\text{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{a}{b}}\right),\frac{bc}{ad}\right)+\sqrt{\frac{a}{b}}\left(ax^2+b\right)\left(cx^2+d\right)+2iadx\sqrt{\frac{ax^2}{b}}\right.}{\sqrt{\frac{a}{b}}\left(ax^2+b\right)\left(cx^2+d\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x^2]*Sqrt[c + d/x^2],x]
```

```
[Out] -((Sqrt[a + b/x^2]*Sqrt[c + d/x^2]*x*(Sqrt[a/b]*(b + a*x^2)*(d + c*x^2) + (
2*I)*a*d*x*Sqrt[1 + (a*x^2)/b]*Sqrt[1 + (c*x^2)/d]*EllipticE[I*ArcSinh[Sqrt
```

```
[a/b]*x], (b*c)/(a*d)] + I*(b*c - a*d)*x*Sqrt[1 + (a*x^2)/b]*Sqrt[1 + (c*x^
2)/d]*EllipticF[I*ArcSinh[Sqrt[a/b]*x], (b*c)/(a*d)]))/(Sqrt[a/b]*(b + a*x^
2)*(d + c*x^2)))
```

Maple [A] time = 0.038, size = 277, normalized size = 1.2

$$\frac{x}{acx^4+adx^2+bcx^2+bd}\sqrt{\frac{cx^2+d}{x^2}}\sqrt{\frac{ax^2+b}{x^2}}\left(-\sqrt{-\frac{c}{d}}x^4ac+\sqrt{\frac{cx^2+d}{d}}\sqrt{\frac{ax^2+b}{b}}\mathrm{EllipticF}\left(x\sqrt{-\frac{c}{d}},\sqrt{\frac{ad}{bc}}\right)xad-cb\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x^2)^(1/2)*(a+1/x^2*b)^(1/2),x)
```

```
[Out] ((a*x^2+b)/x^2)^(1/2)*x*((c*x^2+d)/x^2)^(1/2)*(-(-c/d)^(1/2)*x^4*a*c+((c*x^
2+d)/d)^(1/2)*((a*x^2+b)/b)^(1/2)*EllipticF(x*(-c/d)^(1/2),(a*d/b/c)^(1/2))
*x*a*d-c*b*((c*x^2+d)/d)^(1/2)*((a*x^2+b)/b)^(1/2)*x*EllipticF(x*(-c/d)^(1/
2),(a*d/b/c)^(1/2))+2*c*b*((c*x^2+d)/d)^(1/2)*((a*x^2+b)/b)^(1/2)*x*Ellipti
cE(x*(-c/d)^(1/2),(a*d/b/c)^(1/2))-(-c/d)^(1/2)*x^2*a*d-(-c/d)^(1/2)*x^2*b*
c-(-c/d)^(1/2)*b*d)/(a*c*x^4+a*d*x^2+b*c*x^2+b*d)/(-c/d)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x^2)^(1/2)*(a+b/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x^2)*sqrt(c + d/x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{\frac{ax^2+b}{x^2}}\sqrt{\frac{cx^2+d}{x^2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x^2)^(1/2)*(a+b/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt((a*x^2 + b)/x^2)*sqrt((c*x^2 + d)/x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x**2)**(1/2)*(a+b/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b/x**2)*sqrt(c + d/x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x^2)^(1/2)*(a+b/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/x^2)*sqrt(c + d/x^2), x)
```

3.272 $\int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx$

Optimal. Leaf size=232

$$-\frac{b\sqrt{c}\sqrt{a+\frac{b}{x^2}}\,\text{EllipticF}\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}-\frac{d\sqrt{a+\frac{b}{x^2}}}{cx\sqrt{c+\frac{d}{x^2}}}+\frac{x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}{c}+\frac{\sqrt{d}\sqrt{a+\frac{b}{x^2}}E\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}$$

```
[Out] -((d*Sqrt[a + b/x^2])/(c*Sqrt[c + d/x^2]*x)) + (Sqrt[a + b/x^2]*Sqrt[c + d/
x^2]*x)/c + (Sqrt[d]*Sqrt[a + b/x^2]*EllipticE[ArcCot[(Sqrt[c]*x)/Sqrt[d]],
 1 - (b*c)/(a*d)])/(Sqrt[c]*Sqrt[(c*(a + b/x^2))/(a*(c + d/x^2))]*Sqrt[c +
d/x^2]) - (b*Sqrt[c]*Sqrt[a + b/x^2]*EllipticF[ArcCot[(Sqrt[c]*x)/Sqrt[d]],
 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b/x^2))/(a*(c + d/x^2))]*Sqrt[c
+ d/x^2])
```

Rubi [A] time = 0.204726, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {375, 475, 21, 422, 418, 492, 411}

$$-\frac{d\sqrt{a+\frac{b}{x^2}}}{cx\sqrt{c+\frac{d}{x^2}}}+\frac{x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}{c}-\frac{b\sqrt{c}\sqrt{a+\frac{b}{x^2}}F\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}+\frac{\sqrt{d}\sqrt{a+\frac{b}{x^2}}E\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x^2]/Sqrt[c + d/x^2],x]
```


```
[Out] -((d*Sqrt[a + b/x^2])/(c*Sqrt[c + d/x^2]*x)) + (Sqrt[a + b/x^2]*Sqrt[c + d/
x^2]*x)/c + (Sqrt[d]*Sqrt[a + b/x^2]*EllipticE[ArcCot[(Sqrt[c]*x)/Sqrt[d]],
 1 - (b*c)/(a*d)])/(Sqrt[c]*Sqrt[(c*(a + b/x^2))/(a*(c + d/x^2))]*Sqrt[c +
d/x^2]) - (b*Sqrt[c]*Sqrt[a + b/x^2]*EllipticF[ArcCot[(Sqrt[c]*x)/Sqrt[d]],
 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b/x^2))/(a*(c + d/x^2))]*Sqrt[c
+ d/x^2])
```


Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\sqrt{c+\frac{d}{x^2}}}\,dx = -\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,x}{c} - \frac{\text{Subst}\left(\int \frac{bc+bdx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,x}{c} - \frac{b\,\text{Subst}\left(\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,x}{c} - b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right) - \frac{(bd)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{d\sqrt{a+\frac{b}{x^2}}}{c\sqrt{c+\frac{d}{x^2}}\,x} + \frac{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,x}{c} - \frac{b\sqrt{c}\sqrt{a+\frac{b}{x^2}}\,F\left(\cot^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\sqrt{c+\frac{d}{x^2}}} + d\,\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{\left(c+dx^2\right)^{3/2}}\right.$$

$$= -\frac{d\sqrt{a+\frac{b}{x^2}}}{c\sqrt{c+\frac{d}{x^2}}\,x} + \frac{\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,x}{c} + \frac{\sqrt{d}\sqrt{a+\frac{b}{x^2}}\,E\left(\cot^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\sqrt{c+\frac{d}{x^2}}} - \frac{b\sqrt{c}\sqrt{a+\frac{b}{x^2}}\,F\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right.\right.}{a\sqrt{d}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\sqrt{c+}}$$

Mathematica [A] time = 0.0609942, size = 86, normalized size = 0.37

$$\frac{\sqrt{a+\frac{b}{x^2}}\sqrt{\frac{cx^2+d}{d}}\,E\left(\sin^{-1}\left(\sqrt{-\frac{c}{d}}\,x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{c}{d}}\sqrt{\frac{ax^2+b}{b}}\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x^2]/Sqrt[c + d/x^2],x]
```

```
[Out] (Sqrt[a + b/x^2]*Sqrt[(d + c*x^2)/d]*EllipticE[ArcSin[Sqrt[-(c/d)]*x], (a*d
)/(b*c)])/(Sqrt[-(c/d)]*Sqrt[c + d/x^2]*Sqrt[(b + a*x^2)/b])
```

Maple [A] time = 0.018, size = 94, normalized size = 0.4

$$\frac{b}{ax^2+b}\sqrt{\frac{ax^2+b}{x^2}}\,\text{EllipticE}\left(x\sqrt{-\frac{c}{d}},\sqrt{\frac{ad}{bc}}\right)\sqrt{\frac{ax^2+b}{b}}\sqrt{\frac{cx^2+d}{d}}\,\frac{1}{\sqrt{-\frac{c}{d}}}\,\frac{1}{\sqrt{\frac{cx^2+d}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+1/x^2*b)^(1/2)/(c+d/x^2)^(1/2),x)
```

```
[Out] ((a*x^2+b)/x^2)^(1/2)/(a*x^2+b)*EllipticE(x*(-c/d)^(1/2),(a*d/b/c)^(1/2))*(
(a*x^2+b)/b)^(1/2)*((c*x^2+d)/d)^(1/2)*b/(-c/d)^(1/2)/((c*x^2+d)/x^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + \frac{b}{x^2}}}{\sqrt{c + \frac{d}{x^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(1/2)/(c+d/x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x^2)/sqrt(c + d/x^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^2\sqrt{\frac{ax^2+b}{x^2}}\sqrt{\frac{cx^2+d}{x^2}}}{cx^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(1/2)/(c+d/x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^2*sqrt((a*x^2 + b)/x^2)*sqrt((c*x^2 + d)/x^2)/(c*x^2 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + \frac{b}{x^2}}}{\sqrt{c + \frac{d}{x^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**(1/2)/(c+d/x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b/x**2)/sqrt(c + d/x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + \frac{b}{x^2}}}{\sqrt{c + \frac{d}{x^2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(1/2)/(c+d/x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/x^2)/sqrt(c + d/x^2), x)
```

3.273 $\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx$

Optimal. Leaf size=262

$$-\frac{b\sqrt{a+\frac{b}{x^2}}\,\text{EllipticF}\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{c}\sqrt{d}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}-\frac{2d\sqrt{a+\frac{b}{x^2}}}{c^2x\sqrt{c+\frac{d}{x^2}}}+\frac{2x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}{c^2}+\frac{2\sqrt{d}\sqrt{a+\frac{b}{x^2}}E\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right.}{c^{3/2}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}$$

```
[Out] (-2*d*Sqrt[a + b/x^2])/(c^2*Sqrt[c + d/x^2]*x) - (Sqrt[a + b/x^2]*x)/(c*Sqr
t[c + d/x^2]) + (2*Sqrt[a + b/x^2]*Sqrt[c + d/x^2]*x)/c^2 + (2*Sqrt[d]*Sqrt
[a + b/x^2]*EllipticE[ArcCot[(Sqrt[c]*x)/Sqrt[d]], 1 - (b*c)/(a*d)])/(c^(3/
2)*Sqrt[(c*(a + b/x^2))/(a*(c + d/x^2))]*Sqrt[c + d/x^2]) - (b*Sqrt[a + b/x
^2]*EllipticF[ArcCot[(Sqrt[c]*x)/Sqrt[d]], 1 - (b*c)/(a*d)])/(a*Sqrt[c]*Sqr
t[d]*Sqrt[(c*(a + b/x^2))/(a*(c + d/x^2))]*Sqrt[c + d/x^2])
```

Rubi [A] time = 0.280129, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {375, 469, 583, 531, 418, 492, 411}

$$-\frac{2d\sqrt{a+\frac{b}{x^2}}}{c^2x\sqrt{c+\frac{d}{x^2}}}+\frac{2x\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}}{c^2}+\frac{2\sqrt{d}\sqrt{a+\frac{b}{x^2}}E\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{c^{3/2}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}-\frac{x\sqrt{a+\frac{b}{x^2}}}{c\sqrt{c+\frac{d}{x^2}}}-\frac{b\sqrt{a+\frac{b}{x^2}}F\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1\right.}{a\sqrt{c}\sqrt{d}\sqrt{c+\frac{d}{x^2}}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b/x^2]/(c + d/x^2)^(3/2),x]
```


```
[Out] (-2*d*Sqrt[a + b/x^2])/(c^2*Sqrt[c + d/x^2]*x) - (Sqrt[a + b/x^2]*x)/(c*Sqr
t[c + d/x^2]) + (2*Sqrt[a + b/x^2]*Sqrt[c + d/x^2]*x)/c^2 + (2*Sqrt[d]*Sqrt
[a + b/x^2]*EllipticE[ArcCot[(Sqrt[c]*x)/Sqrt[d]], 1 - (b*c)/(a*d)])/(c^(3/
2)*Sqrt[(c*(a + b/x^2))/(a*(c + d/x^2))]*Sqrt[c + d/x^2]) - (b*Sqrt[a + b/x
^2]*EllipticF[ArcCot[(Sqrt[c]*x)/Sqrt[d]], 1 - (b*c)/(a*d)])/(a*Sqrt[c]*Sqr
t[d]*Sqrt[(c*(a + b/x^2))/(a*(c + d/x^2))]*Sqrt[c + d/x^2])
```


Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 469

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
q)/(a*e*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p +
 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1) +
 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e, m,
 n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{3/2}}\,dx = -\mathrm{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^2\left(c+dx^2\right)^{3/2}}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\sqrt{a+\frac{b}{x^2}}\,x}{c\sqrt{c+\frac{d}{x^2}}} + \frac{\mathrm{Subst}\left(\int \frac{-2a-bx^2}{x^2\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{\sqrt{a+\frac{b}{x^2}}\,x}{c\sqrt{c+\frac{d}{x^2}}} + \frac{2\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,x}{c^2} - \frac{\mathrm{Subst}\left(\int \frac{abc+2abdx^2}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)}{ac^2}$$

$$= -\frac{\sqrt{a+\frac{b}{x^2}}\,x}{c\sqrt{c+\frac{d}{x^2}}} + \frac{2\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,x}{c^2} - \frac{b\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx, x, \frac{1}{x}\right)}{c} - \frac{(2bd)\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{a+bx^2}\sqrt{c\cdots}}\right.}{c^2}$$

$$= -\frac{2d\sqrt{a+\frac{b}{x^2}}}{c^2\sqrt{c+\frac{d}{x^2}}\,x} - \frac{\sqrt{a+\frac{b}{x^2}}\,x}{c\sqrt{c+\frac{d}{x^2}}} + \frac{2\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,x}{c^2} - \frac{b\sqrt{a+\frac{b}{x^2}}\,F\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{c}\sqrt{d}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\sqrt{c+\frac{d}{x^2}}} + \frac{(2d)\,\mathrm{Subst}\cdots}{\cdots}$$

$$= -\frac{2d\sqrt{a+\frac{b}{x^2}}}{c^2\sqrt{c+\frac{d}{x^2}}\,x} - \frac{\sqrt{a+\frac{b}{x^2}}\,x}{c\sqrt{c+\frac{d}{x^2}}} + \frac{2\sqrt{a+\frac{b}{x^2}}\sqrt{c+\frac{d}{x^2}}\,x}{c^2} + \frac{2\sqrt{d}\sqrt{a+\frac{b}{x^2}}\,E\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)|1-\frac{bc}{ad}\right)}{c^{3/2}\sqrt{\frac{c\left(a+\frac{b}{x^2}\right)}{a\left(c+\frac{d}{x^2}\right)}}\sqrt{c+\frac{d}{x^2}}} - \frac{b\sqrt{a+\cdots}}{a\sqrt{\cdots}}$$

Mathematica [C] time = 0.224402, size = 191, normalized size = 0.73

$$-\frac{\sqrt{a+\frac{b}{x^2}}\left(i\sqrt{\frac{ax^2}{b}+1}\sqrt{\frac{cx^2}{d}+1}(bc-2ad)\mathrm{EllipticF}\left(i\sinh^{-1}\left(x\sqrt{\frac{a}{b}}\right),\frac{bc}{ad}\right)+2iad\sqrt{\frac{ax^2}{b}+1}\sqrt{\frac{cx^2}{d}+1}E\left(i\sinh^{-1}\left(\sqrt{\frac{a}{b}}x\right)|\frac{b}{a}\cdots\right.\right.}{c^2\sqrt{\frac{a}{b}}\left(ax^2+b\right)\sqrt{c+\frac{d}{x^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x^2]/(c + d/x^2)^(3/2),x]
```

```
[Out] -((Sqrt[a + b/x^2]*(Sqrt[a/b]*c*x*(b + a*x^2) + (2*I)*a*d*Sqrt[1 + (a*x^2)/
b]*Sqrt[1 + (c*x^2)/d]*EllipticE[I*ArcSinh[Sqrt[a/b]*x], (b*c)/(a*d)] + I*(
b*c - 2*a*d)*Sqrt[1 + (a*x^2)/b]*Sqrt[1 + (c*x^2)/d]*EllipticF[I*ArcSinh[Sq
rt[a/b]*x], (b*c)/(a*d)]))/(Sqrt[a/b]*c^2*Sqrt[c + d/x^2]*(b + a*x^2)))
```

Maple [A] time = 0.071, size = 187, normalized size = 0.7

$$\frac{cx^2+d}{x^2c\left(ax^2+b\right)}\sqrt{\frac{ax^2+b}{x^2}}\left(-x^3a\sqrt{-\frac{c}{d}}+2\,\mathrm{EllipticE}\left(x\sqrt{-\frac{c}{d}},\sqrt{\frac{ad}{bc}}\right)b\sqrt{\frac{cx^2+d}{d}}\sqrt{\frac{ax^2+b}{b}}-\mathrm{EllipticF}\left(x\sqrt{-\frac{c}{d}},\sqrt{\frac{ad}{bc}}\right)\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+1/x^2*b)^(1/2)/(c+d/x^2)^(3/2),x)
```

```
[Out] ((a*x^2+b)/x^2)^(1/2)/x^2/(a*x^2+b)*(-x^3*a*(-c/d)^(1/2)+2*EllipticE(x*(-c/
d)^(1/2),(a*d/b/c)^(1/2))*b*((c*x^2+d)/d)^(1/2)*((a*x^2+b)/b)^(1/2)-Ellipti
```

```
cF(x*(-c/d)^(1/2),(a*d/b/c)^(1/2))*b*((c*x^2+d)/d)^(1/2)*((a*x^2+b)/b)^(1/2
)-x*b*(-c/d)^(1/2))*(c*x^2+d)/(-c/d)^(1/2)/c/((c*x^2+d)/x^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(1/2)/(c+d/x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x^2)/(c + d/x^2)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^4\sqrt{\frac{ax^2+b}{x^2}}\sqrt{\frac{cx^2+d}{x^2}}}{c^2x^4+2\,cdx^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(1/2)/(c+d/x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(x^4*sqrt((a*x^2 + b)/x^2)*sqrt((c*x^2 + d)/x^2)/(c^2*x^4 + 2*c*d*x
^2 + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**(1/2)/(c+d/x**2)**(3/2),x)
```

```
[Out] Integral(sqrt(a + b/x**2)/(c + d/x**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+\frac{b}{x^2}}}{\left(c+\frac{d}{x^2}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^(1/2)/(c+d/x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a + b/x^2)/(c + d/x^2)^(3/2), x)
```

3.274 $\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$

Optimal. Leaf size=79

$$x\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(-\frac{1}{2};-p,-q;\frac{1}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

```
[Out] ((a + b/x^2)^p*(c + d/x^2)^q*x*AppellF1[-1/2, -p, -q, 1/2, -(b/(a*x^2)), -(
d/(c*x^2))])/((1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

Rubi [A] time = 0.0883045, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {375, 511, 510}

$$x\left(a+\frac{b}{x^2}\right)^p\left(\frac{b}{ax^2}+1\right)^{-p}\left(c+\frac{d}{x^2}\right)^q\left(\frac{d}{cx^2}+1\right)^{-q}F_1\left(-\frac{1}{2};-p,-q;\frac{1}{2};-\frac{b}{ax^2},-\frac{d}{cx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^2)^p*(c + d/x^2)^q,x]
```

```
[Out] ((a + b/x^2)^p*(c + d/x^2)^q*x*AppellF1[-1/2, -p, -q, 1/2, -(b/(a*x^2)), -(
d/(c*x^2))])/((1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx = -\text{Subst}\left(\int \frac{\left(a+bx^2\right)^p \left(c+dx^2\right)^q}{x^2} dx, x, \frac{1}{x}\right)$$
$$= -\left(\left(\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p}\right) \text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p \left(c+dx^2\right)^q}{x^2} dx, x, \frac{1}{x}\right)\right)$$
$$= -\left(\left(\left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q}\right) \text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p \left(1+\frac{dx^2}{c}\right)^q}{x^2} dx, x,\right.\right.$$
$$= \left(a+\frac{b}{x^2}\right)^p \left(1+\frac{b}{ax^2}\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(1+\frac{d}{cx^2}\right)^{-q} x F_1\left(-\frac{1}{2}; -p, -q; \frac{1}{2}; -\frac{b}{ax^2}, -\frac{d}{cx^2}\right)$$

Mathematica [A] time = 0.110181, size = 104, normalized size = 1.32

$$-\frac{x \left(a+\frac{b}{x^2}\right)^p \left(\frac{ax^2}{b}+1\right)^{-p} \left(c+\frac{d}{x^2}\right)^q \left(\frac{cx^2}{d}+1\right)^{-q} F_1\left(-p-q+\frac{1}{2}; -p, -q; -p-q+\frac{3}{2}; -\frac{ax^2}{b}, -\frac{cx^2}{d}\right)}{2p+2q-1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b/x^2)^p*(c + d/x^2)^q,x]
```

```
[Out] -(((a + b/x^2)^p*(c + d/x^2)^q*x*AppellF1[1/2 - p - q, -p, -q, 3/2 - p - q,
 -((a*x^2)/b), -((c*x^2)/d)])/((-1 + 2*p + 2*q)*(1 + (a*x^2)/b)^p*(1 + (c*x
^2)/d)^q))
```

Maple [F] time = 0.093, size = 0, normalized size = 0.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+1/x^2*b)^p*(c+d/x^2)^q,x)
```

```
[Out] int((a+1/x^2*b)^p*(c+d/x^2)^q,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+\frac{b}{x^2}\right)^p \left(c+\frac{d}{x^2}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q,x, algorithm="maxima")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\frac{ax^2+b}{x^2}\right)^p\left(\frac{cx^2+d}{x^2}\right)^q,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q,x, algorithm="fricas")
```

```
[Out] integral(((a*x^2 + b)/x^2)^p*((c*x^2 + d)/x^2)^q, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**2)**p*(c+d/x**2)**q,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+\frac{b}{x^2}\right)^p\left(c+\frac{d}{x^2}\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^2)^p*(c+d/x^2)^q,x, algorithm="giac")
```

```
[Out] integrate((a + b/x^2)^p*(c + d/x^2)^q, x)
```

3.275 $\int \frac{a+\frac{b}{x^3}}{c+\frac{d}{x^3}}\,dx$

Optimal. Leaf size=145

$$-\frac{(bc-ad)\log\left(c^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}\right)}{6c^{4/3}d^{2/3}}+\frac{(bc-ad)\log\left(\sqrt[3]{c}x+\sqrt[3]{d}\right)}{3c^{4/3}d^{2/3}}-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{c}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}c^{4/3}d^{2/3}}+\frac{ax}{c}$$

```
[Out] (a*x)/c - ((b*c - a*d)*ArcTan[(d^(1/3) - 2*c^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(
Sqrt[3]*c^(4/3)*d^(2/3)) + ((b*c - a*d)*Log[d^(1/3) + c^(1/3)*x])/(3*c^(4/3
)*d^(2/3)) - ((b*c - a*d)*Log[d^(2/3) - c^(1/3)*d^(1/3)*x + c^(2/3)*x^2])/(
6*c^(4/3)*d^(2/3))
```

Rubi [A] time = 0.105609, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.471, Rules used = {374, 388, 200, 31, 634, 617, 204, 628}

$$-\frac{(bc-ad)\log\left(c^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}\right)}{6c^{4/3}d^{2/3}}+\frac{(bc-ad)\log\left(\sqrt[3]{c}x+\sqrt[3]{d}\right)}{3c^{4/3}d^{2/3}}-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{c}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}c^{4/3}d^{2/3}}+\frac{ax}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b/x^3)/(c + d/x^3),x]
```

```
[Out] (a*x)/c - ((b*c - a*d)*ArcTan[(d^(1/3) - 2*c^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(
Sqrt[3]*c^(4/3)*d^(2/3)) + ((b*c - a*d)*Log[d^(1/3) + c^(1/3)*x])/(3*c^(4/3
)*d^(2/3)) - ((b*c - a*d)*Log[d^(2/3) - c^(1/3)*d^(1/3)*x + c^(2/3)*x^2])/(
6*c^(4/3)*d^(2/3))
```

Rule 374

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[x^(n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[n]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+\frac{b}{x^3}}{c+\frac{d}{x^3}}\,dx &= \int \frac{b+ax^3}{d+cx^3}\,dx\\
&= \frac{ax}{c}-\frac{(-bc+ad)\int \frac{1}{d+cx^3}\,dx}{c}\\
&= \frac{ax}{c}+\frac{(bc-ad)\int \frac{1}{\sqrt[3]{d}+\sqrt[3]{c}x}\,dx}{3cd^{2/3}}+\frac{(bc-ad)\int \frac{2\sqrt[3]{d}-\sqrt[3]{c}x}{d^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}x^2}\,dx}{3cd^{2/3}}\\
&= \frac{ax}{c}+\frac{(bc-ad)\log\left(\sqrt[3]{d}+\sqrt[3]{c}x\right)}{3c^{4/3}d^{2/3}}-\frac{(bc-ad)\int \frac{-\sqrt[3]{c}\sqrt[3]{d}+2c^{2/3}x}{d^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}x^2}\,dx}{6c^{4/3}d^{2/3}}+\frac{(bc-ad)\int \frac{1}{d^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}x^2}\,dx}{2c\sqrt[3]{d}}\\
&= \frac{ax}{c}+\frac{(bc-ad)\log\left(\sqrt[3]{d}+\sqrt[3]{c}x\right)}{3c^{4/3}d^{2/3}}-\frac{(bc-ad)\log\left(d^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}x^2\right)}{6c^{4/3}d^{2/3}}+\frac{(bc-ad)\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,\right.}{c^{4/3}d^{2/3}}\\
&= \frac{ax}{c}-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{c}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}c^{4/3}d^{2/3}}+\frac{(bc-ad)\log\left(\sqrt[3]{d}+\sqrt[3]{c}x\right)}{3c^{4/3}d^{2/3}}-\frac{(bc-ad)\log\left(d^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+c^{2/3}x^2\right)}{6c^{4/3}d^{2/3}}
\end{aligned}$$

Mathematica [A] time = 0.0835389, size = 129, normalized size = 0.89

$$\frac{-(bc-ad)\log\left(c^{2/3}x^2-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}\right)+2(bc-ad)\log\left(\sqrt[3]{c}x+\sqrt[3]{d}\right)-2\sqrt{3}(bc-ad)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{c}x}{\sqrt[3]{d}}}{\sqrt{3}}\right)+6a\sqrt[3]{c}d^{2/3}x}{6c^{4/3}d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x^3)/(c + d/x^3),x]
```

```
[Out] (6*a*c^(1/3)*d^(2/3)*x - 2*Sqrt[3]*(b*c - a*d)*ArcTan[(1 - (2*c^(1/3)*x)/d^
(1/3))/Sqrt[3]] + 2*(b*c - a*d)*Log[d^(1/3) + c^(1/3)*x] - (b*c - a*d)*Log[
d^(2/3) - c^(1/3)*d^(1/3)*x + c^(2/3)*x^2])/(6*c^(4/3)*d^(2/3))
```

Maple [A] time = 0.004, size = 195, normalized size = 1.3

$$\frac{ax}{c}-\frac{ad}{3c^{2}}\ln\left(x+\sqrt[3]{\frac{d}{c}}\right)\left(\frac{d}{c}\right)^{-\frac{2}{3}}+\frac{b}{3c}\ln\left(x+\sqrt[3]{\frac{d}{c}}\right)\left(\frac{d}{c}\right)^{-\frac{2}{3}}+\frac{ad}{6c^{2}}\ln\left(x^{2}-\sqrt[3]{\frac{d}{c}}x+\left(\frac{d}{c}\right)^{\frac{2}{3}}\right)\left(\frac{d}{c}\right)^{-\frac{2}{3}}-\frac{b}{6c}\ln\left(x^{2}-\sqrt[3]{\frac{d}{c}}x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x^3)/(c+d/x^3),x)
```

```
[Out] a*x/c-1/3/c^2/(d/c)^(2/3)*ln(x+(d/c)^(1/3))*a*d+1/3/c/(d/c)^(2/3)*ln(x+(d/c
)^(1/3))*b+1/6/c^2/(d/c)^(2/3)*ln(x^2-(d/c)^(1/3)*x+(d/c)^(2/3))*a*d-1/6/c/
(d/c)^(2/3)*ln(x^2-(d/c)^(1/3)*x+(d/c)^(2/3))*b-1/3/c^2/(d/c)^(2/3)*3^(1/2)
*arctan(1/3*3^(1/2)*(2/(d/c)^(1/3)*x-1))*a*d+1/3/c/(d/c)^(2/3)*3^(1/2)*arct
an(1/3*3^(1/2)*(2/(d/c)^(1/3)*x-1))*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)/(c+d/x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.34319, size = 926, normalized size = 6.39

$$\left[\frac{6\,acd^{2}x-3\sqrt{\frac{1}{3}}\left(bc^{2}d-acd^{2}\right)\sqrt{\frac{\left(-cd^{2}\right)^{\frac{1}{3}}}{c}}\log\left(\frac{2\,cdx^{3}+3\left(-cd^{2}\right)^{\frac{1}{3}}dx-d^{2}-3\sqrt{\frac{1}{3}}\left(2\,cdx^{2}+\left(-cd^{2}\right)^{\frac{2}{3}}x+\left(-cd^{2}\right)^{\frac{1}{3}}d\right)\sqrt{\frac{\left(-cd^{2}\right)^{\frac{1}{3}}}{c}}}{cx^{3}+d}\right)-\left(-cd^{2}\right)^{\frac{2}{3}}\left(bc-\right.}{6\,c^{2}d^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)/(c+d/x^3),x, algorithm="fricas")
```

```
[Out] [1/6*(6*a*c*d^2*x - 3*sqrt(1/3)*(b*c^2*d - a*c*d^2)*sqrt((-c*d^2)^(1/3)/c)*
log((2*c*d*x^3 + 3*(-c*d^2)^(1/3)*d*x - d^2 - 3*sqrt(1/3)*(2*c*d*x^2 + (-c*
d^2)^(2/3)*x + (-c*d^2)^(1/3)*d)*sqrt((-c*d^2)^(1/3)/c))/(c*x^3 + d)) - (-c
*d^2)^(2/3)*(b*c - a*d)*log(c*d*x^2 - (-c*d^2)^(2/3)*x - (-c*d^2)^(1/3)*d)
+ 2*(-c*d^2)^(2/3)*(b*c - a*d)*log(c*d*x + (-c*d^2)^(2/3)))/(c^2*d^2), 1/6*
```

```
(6*a*c*d^2*x + 6*sqrt(1/3)*(b*c^2*d - a*c*d^2)*sqrt(-(-c*d^2)^(1/3)/c)*arct
an(sqrt(1/3)*(2*(-c*d^2)^(2/3)*x + (-c*d^2)^(1/3)*d)*sqrt(-(-c*d^2)^(1/3)/c
)/d^2) - (-c*d^2)^(2/3)*(b*c - a*d)*log(c*d*x^2 - (-c*d^2)^(2/3)*x - (-c*d^
2)^(1/3)*d) + 2*(-c*d^2)^(2/3)*(b*c - a*d)*log(c*d*x + (-c*d^2)^(2/3)))/(c^
2*d^2)]
```

Sympy [A] time = 0.51837, size = 71, normalized size = 0.49

$$\frac{ax}{c} + \mathrm{RootSum}\left(27t^3c^4d^2 + a^3d^3 - 3a^2bcd^2 + 3ab^2c^2d - b^3c^3, \left(t \mapsto t\log\left(-\frac{3tcd}{ad-bc} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x**3)/(c+d/x**3),x)
```

```
[Out] a*x/c + RootSum(27*_t**3*c**4*d**2 + a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2
*c**2*d - b**3*c**3, Lambda(_t, _t*log(-3*_t*c*d/(a*d - b*c) + x)))
```

Giac [A] time = 1.13226, size = 217, normalized size = 1.5

$$\frac{ax}{c} - \frac{(bc-ad)\left(-\frac{d}{c}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{d}{c}\right)^{\frac{1}{3}}\right|\right)}{3\,cd} + \frac{\sqrt{3}\left(\left(-c^2d\right)^{\frac{1}{3}}bc - \left(-c^2d\right)^{\frac{1}{3}}ad\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{d}{c}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{d}{c}\right)^{\frac{1}{3}}}\right)}{3\,c^2d} + \frac{\left(\left(-c^2d\right)^{\frac{1}{3}}bc - \left(-c^2d\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x^3)/(c+d/x^3),x, algorithm="giac")
```

```
[Out] a*x/c - 1/3*(b*c - a*d)*(-d/c)^(1/3)*log(abs(x - (-d/c)^(1/3)))/(c*d) + 1/3
*sqrt(3)*((-c^2*d)^(1/3)*b*c - (-c^2*d)^(1/3)*a*d)*arctan(1/3*sqrt(3)*(2*x
+ (-d/c)^(1/3))/(-d/c)^(1/3))/(c^2*d) + 1/6*((-c^2*d)^(1/3)*b*c - (-c^2*d)^
(1/3)*a*d)*log(x^2 + x*(-d/c)^(1/3) + (-d/c)^(2/3))/(c^2*d)
```

3.276 $\int \frac{a+b\sqrt{x}}{c+d\sqrt{x}}\,dx$

Optimal. Leaf size=49

$$-\frac{2\sqrt{x}(bc-ad)}{d^2}+\frac{2c(bc-ad)\log\left(c+d\sqrt{x}\right)}{d^3}+\frac{bx}{d}$$

```
[Out] (-2*(b*c - a*d)*Sqrt[x])/d^2 + (b*x)/d + (2*c*(b*c - a*d)*Log[c + d*Sqrt[x]
])/d^3
```

Rubi [A] time = 0.0487507, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {376, 77}

$$-\frac{2\sqrt{x}(bc-ad)}{d^2}+\frac{2c(bc-ad)\log\left(c+d\sqrt{x}\right)}{d^3}+\frac{bx}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sqrt[x])/(c + d*Sqrt[x]),x]
```

```
[Out] (-2*(b*c - a*d)*Sqrt[x])/d^2 + (b*x)/d + (2*c*(b*c - a*d)*Log[c + d*Sqrt[x]
])/d^3
```

Rule 376

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sqrt{x}}{c+d\sqrt{x}}\,dx &= 2\,\text{Subst}\left(\int \frac{x(a+bx)}{c+dx}\,dx, x, \sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(\frac{-bc+ad}{d^2}+\frac{bx}{d}+\frac{c(bc-ad)}{d^2(c+dx)}\right)dx, x, \sqrt{x}\right)\\ &= -\frac{2(bc-ad)\sqrt{x}}{d^2}+\frac{bx}{d}+\frac{2c(bc-ad)\log\left(c+d\sqrt{x}\right)}{d^3}\end{aligned}$$

Mathematica [A] time = 0.0406529, size = 41, normalized size = 0.84

$$\frac{2(ad-bc)\left(d\sqrt{x}-c\log\left(c+d\sqrt{x}\right)\right)}{d^3}+\frac{bx}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[x])/(c + d*Sqrt[x]),x]
```

```
[Out] (b*x)/d + (2*(-(b*c) + a*d)*(d*Sqrt[x] - c*Log[c + d*Sqrt[x]]))/d^3
```

Maple [A] time = 0.004, size = 59, normalized size = 1.2

$$\frac{bx}{d}+2\frac{a\sqrt{x}}{d}-2\frac{b\sqrt{x}c}{d^2}-2\frac{c\ln\left(c+d\sqrt{x}\right)a}{d^2}+2\frac{c^2\ln\left(c+d\sqrt{x}\right)b}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^(1/2))/(c+d*x^(1/2)),x)
```

```
[Out] b*x/d+2/d*a*x^(1/2)-2/d^2*x^(1/2)*b*c-2*c/d^2*ln(c+d*x^(1/2))*a+2*c^2/d^3*l
n(c+d*x^(1/2))*b
```

Maxima [A] time = 0.966631, size = 63, normalized size = 1.29

$$\frac{bdx-2\,(bc-ad)\sqrt{x}}{d^2}+\frac{2\left(bc^2-acd\right)\log\left(d\sqrt{x}+c\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))/(c+d*x^(1/2)),x, algorithm="maxima")
```

```
[Out] (b*d*x - 2*(b*c - a*d)*sqrt(x))/d^2 + 2*(b*c^2 - a*c*d)*log(d*sqrt(x) + c)/
d^3
```

Fricas [A] time = 1.30843, size = 111, normalized size = 2.27

$$\frac{bd^2x+2\left(bc^2-acd\right)\log\left(d\sqrt{x}+c\right)-2\left(bcd-ad^2\right)\sqrt{x}}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))/(c+d*x^(1/2)),x, algorithm="fricas")
```

```
[Out] (b*d^2*x + 2*(b*c^2 - a*c*d)*log(d*sqrt(x) + c) - 2*(b*c*d - a*d^2)*sqrt(x)
)/d^3
```

Sympy [A] time = 0.271116, size = 82, normalized size = 1.67

$$\begin{cases}-\frac{2ac\log\left(\frac{c}{d}+\sqrt{x}\right)}{d^2}+\frac{2a\sqrt{x}}{d}+\frac{2bc^2\log\left(\frac{c}{d}+\sqrt{x}\right)}{d^3}-\frac{2bc\sqrt{x}}{d^2}+\frac{bx}{d} & \text{for}\ d\neq 0\\ \frac{ax+\frac{2bx^{\frac{3}{2}}}{3}}{c} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**(1/2))/(c+d*x**(1/2)),x)
```

```
[Out] Piecewise((-2*a*c*log(c/d + sqrt(x))/d**2 + 2*a*sqrt(x)/d + 2*b*c**2*log(c/
d + sqrt(x))/d**3 - 2*b*c*sqrt(x)/d**2 + b*x/d, Ne(d, 0)), ((a*x + 2*b*x**(
3/2)/3)/c, True))
```

Giac [A] time = 1.0981, size = 66, normalized size = 1.35

$$\frac{bdx - 2\,bc\sqrt{x} + 2\,ad\sqrt{x}}{d^2} + \frac{2\left(bc^2 - acd\right)\log\left(\left|d\sqrt{x} + c\right|\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^(1/2))/(c+d*x^(1/2)),x, algorithm="giac")
```

```
[Out] (b*d*x - 2*b*c*sqrt(x) + 2*a*d*sqrt(x))/d^2 + 2*(b*c^2 - a*c*d)*log(abs(d*s
qrt(x) + c))/d^3
```

3.277 $\int \frac{-1+\sqrt[3]{x}}{1+\sqrt[3]{x}} \, dx$

Optimal. Leaf size=26

$$-3x^{2/3}+x+6\sqrt[3]{x}-6\log\left(\sqrt[3]{x}+1\right)$$

```
[Out] 6*x^(1/3) - 3*x^(2/3) + x - 6*Log[1 + x^(1/3)]
```

Rubi [A] time = 0.0159524, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {376, 77}

$$-3x^{2/3}+x+6\sqrt[3]{x}-6\log\left(\sqrt[3]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^(1/3))/(1 + x^(1/3)),x]
```

```
[Out] 6*x^(1/3) - 3*x^(2/3) + x - 6*Log[1 + x^(1/3)]
```

Rule 376

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+\sqrt[3]{x}}{1+\sqrt[3]{x}} \, dx &= 3\,\text{Subst}\left(\int \frac{(-1+x)x^2}{1+x} \, dx, x, \sqrt[3]{x}\right)\\
&= 3\,\text{Subst}\left(\int \left(2-2x+x^2-\frac{2}{1+x}\right) dx, x, \sqrt[3]{x}\right)\\
&= 6\sqrt[3]{x}-3x^{2/3}+x-6\log\left(1+\sqrt[3]{x}\right)
\end{aligned}$$

Mathematica [A] time = 0.0103988, size = 26, normalized size = 1.

$$-3x^{2/3}+x+6\sqrt[3]{x}-6\log\left(\sqrt[3]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^(1/3))/(1 + x^(1/3)),x]
```

```
[Out] 6*x^(1/3) - 3*x^(2/3) + x - 6*Log[1 + x^(1/3)]
```

Maple [A] time = 0.003, size = 21, normalized size = 0.8

$$6\sqrt[3]{x}-3x^{2/3}+x-6\ln\left(\sqrt[3]{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x^(1/3))/(x^(1/3)+1),x)
```

```
[Out] 6*x^(1/3)-3*x^(2/3)+x-6*ln(x^(1/3)+1)
```

Maxima [A] time = 0.940811, size = 27, normalized size = 1.04

$$x-3x^{\frac{2}{3}}+6x^{\frac{1}{3}}-6\log\left(x^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/3))/(1+x^(1/3)),x, algorithm="maxima")
```

```
[Out] x - 3*x^(2/3) + 6*x^(1/3) - 6*log(x^(1/3) + 1)
```

Fricas [A] time = 1.3296, size = 65, normalized size = 2.5

$$x-3x^{\frac{2}{3}}+6x^{\frac{1}{3}}-6\log\left(x^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/3))/(1+x^(1/3)),x, algorithm="fricas")
```

```
[Out] x - 3*x^(2/3) + 6*x^(1/3) - 6*log(x^(1/3) + 1)
```

Sympy [A] time = 0.166887, size = 24, normalized size = 0.92

$$-3x^{\frac{2}{3}}+6\sqrt[3]{x}+x-6\log\left(\sqrt[3]{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x**(1/3))/(1+x**(1/3)),x)
```

```
[Out] -3*x**(2/3) + 6*x**(1/3) + x - 6*log(x**(1/3) + 1)
```

Giac [A] time = 1.10426, size = 27, normalized size = 1.04

$$x-3x^{\frac{2}{3}}+6x^{\frac{1}{3}}-6\log\left(x^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(1/3))/(1+x^(1/3)),x, algorithm="giac")
```

```
[Out] x - 3*x^(2/3) + 6*x^(1/3) - 6*log(x^(1/3) + 1)
```

3.278 $\int \frac{1+x^{2/3}}{-1+x^{2/3}}\,dx$

Optimal. Leaf size=17

$$x+6\sqrt[3]{x}-6\tanh^{-1}\left(\sqrt[3]{x}\right)$$

```
[Out] 6*x^(1/3) + x - 6*ArcTanh[x^(1/3)]
```

Rubi [A] time = 0.0146946, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {376, 459, 321, 207}

$$x+6\sqrt[3]{x}-6\tanh^{-1}\left(\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^(2/3))/(-1 + x^(2/3)),x]
```

```
[Out] 6*x^(1/3) + x - 6*ArcTanh[x^(1/3)]
```

Rule 376

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1+x^{2/3}}{-1+x^{2/3}}\,dx = 3\,\text{Subst}\left(\int \frac{x^2(1+x^2)}{-1+x^2}\,dx, x, \sqrt[3]{x}\right)$$
$$= x + 6\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \sqrt[3]{x}\right)$$
$$= 6\sqrt[3]{x} + x + 6\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt[3]{x}\right)$$
$$= 6\sqrt[3]{x} + x - 6\tanh^{-1}\left(\sqrt[3]{x}\right)$$

Mathematica [A] time = 0.0036745, size = 17, normalized size = 1.

$$x + 6\sqrt[3]{x} - 6\tanh^{-1}\left(\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^(2/3))/(-1 + x^(2/3)),x]
```

```
[Out] 6*x^(1/3) + x - 6*ArcTanh[x^(1/3)]
```

Maple [A] time = 0.004, size = 24, normalized size = 1.4

$$x + 6\sqrt[3]{x} + 3\ln\left(-1 + \sqrt[3]{x}\right) - 3\ln\left(\sqrt[3]{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x^(2/3))/(-1+x^(2/3)),x)
```

```
[Out] x+6*x^(1/3)+3*ln(-1+x^(1/3))-3*ln(x^(1/3)+1)
```

Maxima [A] time = 0.973211, size = 31, normalized size = 1.82

$$x + 6x^{\frac{1}{3}} - 3\log\left(x^{\frac{1}{3}} + 1\right) + 3\log\left(x^{\frac{1}{3}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(2/3))/(-1+x^(2/3)),x, algorithm="maxima")
```

```
[Out] x + 6*x^(1/3) - 3*log(x^(1/3) + 1) + 3*log(x^(1/3) - 1)
```

Fricas [A] time = 1.42153, size = 77, normalized size = 4.53

$$x + 6x^{\frac{1}{3}} - 3\log\left(x^{\frac{1}{3}} + 1\right) + 3\log\left(x^{\frac{1}{3}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(2/3))/(-1+x^(2/3)),x, algorithm="fricas")
```

```
[Out] x + 6*x^(1/3) - 3*log(x^(1/3) + 1) + 3*log(x^(1/3) - 1)
```

Sympy [A] time = 0.223739, size = 27, normalized size = 1.59

$$6\sqrt[3]{x}+x+3\log\left(\sqrt[3]{x}-1\right)-3\log\left(\sqrt[3]{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**(2/3))/(-1+x**(2/3)),x)
```

```
[Out] 6*x**(1/3) + x + 3*log(x**(1/3) - 1) - 3*log(x**(1/3) + 1)
```

Giac [A] time = 1.36138, size = 32, normalized size = 1.88

$$x+6\,x^{\frac{1}{3}}-3\,\log\left(x^{\frac{1}{3}}+1\right)+3\,\log\left(\left|x^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(2/3))/(-1+x^(2/3)),x, algorithm="giac")
```

```
[Out] x + 6*x^(1/3) - 3*log(x^(1/3) + 1) + 3*log(abs(x^(1/3) - 1))
```

3.279 $\int \frac{-16+x^{3/4}}{16+x^{3/4}}\,dx$

Optimal. Leaf size=104

$$x-128\sqrt[4]{x}+\frac{256}{3}\sqrt[3]{2}\log\left(\sqrt[4]{x}+2\sqrt[3]{2}\right)-\frac{128}{3}\sqrt[3]{2}\log\left(\sqrt{x}-2\sqrt[3]{2}\sqrt[4]{x}+4\ 2^{2/3}\right)-\frac{256\sqrt[3]{2}\tan^{-1}\left(\frac{\sqrt[3]{2}-\sqrt[4]{x}}{\sqrt[3]{2}\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -128*x^(1/4) + x - (256*2^(1/3)*ArcTan[(2^(1/3) - x^(1/4))/(2^(1/3)*Sqrt[3]
)])/Sqrt[3] + (256*2^(1/3)*Log[2*2^(1/3) + x^(1/4)])/3 - (128*2^(1/3)*Log[4
*2^(2/3) - 2*2^(1/3)*x^(1/4) + Sqrt[x]])/3
```

Rubi [A] time = 0.0861164, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {376, 459, 321, 200, 31, 634, 617, 204, 628}

$$x-128\sqrt[4]{x}+\frac{256}{3}\sqrt[3]{2}\log\left(\sqrt[4]{x}+2\sqrt[3]{2}\right)-\frac{128}{3}\sqrt[3]{2}\log\left(\sqrt{x}-2\sqrt[3]{2}\sqrt[4]{x}+4\ 2^{2/3}\right)-\frac{256\sqrt[3]{2}\tan^{-1}\left(\frac{\sqrt[3]{2}-\sqrt[4]{x}}{\sqrt[3]{2}\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-16 + x^(3/4))/(16 + x^(3/4)),x]
```

```
[Out] -128*x^(1/4) + x - (256*2^(1/3)*ArcTan[(2^(1/3) - x^(1/4))/(2^(1/3)*Sqrt[3]
)])/Sqrt[3] + (256*2^(1/3)*Log[2*2^(1/3) + x^(1/4)])/3 - (128*2^(1/3)*Log[4
*2^(2/3) - 2*2^(1/3)*x^(1/4) + Sqrt[x]])/3
```

Rule 376

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

```
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-16+x^{3/4}}{16+x^{3/4}}\,dx &= 4\,\text{Subst}\left(\int \frac{x^3\left(-16+x^3\right)}{16+x^3}\,dx, x, \sqrt[4]{x}\right)\\
&= x-128\,\text{Subst}\left(\int \frac{x^3}{16+x^3}\,dx, x, \sqrt[4]{x}\right)\\
&= -128\sqrt[4]{x}+x+2048\,\text{Subst}\left(\int \frac{1}{16+x^3}\,dx, x, \sqrt[4]{x}\right)\\
&= -128\sqrt[4]{x}+x+\frac{1}{3}\left(256\sqrt[3]{2}\right)\text{Subst}\left(\int \frac{1}{2\sqrt[3]{2}+x}\,dx, x, \sqrt[4]{x}\right)+\frac{1}{3}\left(256\sqrt[3]{2}\right)\text{Subst}\left(\int \frac{4\sqrt[3]{2}-x}{4\,2^{2/3}-2\sqrt[3]{2}x\ldots}\right.\\
&= -128\sqrt[4]{x}+x+\frac{256}{3}\sqrt[3]{2}\log\left(2\sqrt[3]{2}+\sqrt[4]{x}\right)-\frac{1}{3}\left(128\sqrt[3]{2}\right)\text{Subst}\left(\int \frac{-2\sqrt[3]{2}+2x}{4\,2^{2/3}-2\sqrt[3]{2}x+x^2}\,dx, x, \sqrt[4]{x}\right)+\ldots\\
&= -128\sqrt[4]{x}+x+\frac{256}{3}\sqrt[3]{2}\log\left(2\sqrt[3]{2}+\sqrt[4]{x}\right)-\frac{128}{3}\sqrt[3]{2}\log\left(4\,2^{2/3}-2\sqrt[3]{2}\sqrt[4]{x}+\sqrt{x}\right)+\left(256\sqrt[3]{2}\right)\text{Subst}\left(\ldots\right.\\
&= -128\sqrt[4]{x}+x-\frac{256\sqrt[3]{2}\tan^{-1}\left(\frac{2-2^{2/3}\sqrt[4]{x}}{2\sqrt{3}}\right)}{\sqrt{3}}+\frac{256}{3}\sqrt[3]{2}\log\left(2\sqrt[3]{2}+\sqrt[4]{x}\right)-\frac{128}{3}\sqrt[3]{2}\log\left(4\,2^{2/3}-2\sqrt[3]{2}\sqrt[4]{x}\ldots\right.
\end{aligned}
$$

Mathematica [C] time = 0.0031688, size = 22, normalized size = 0.21

$$x-2x\,{}_2F_1\left(1,\frac{4}{3};\frac{7}{3};-\frac{x^{3/4}}{16}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-16 + x^(3/4))/(16 + x^(3/4)),x]
```

```
[Out] x - 2*x*Hypergeometric2F1[1, 4/3, 7/3, -x^(3/4)/16]
```

Maple [A] time = 0.006, size = 66, normalized size = 0.6

$$x-128\,\sqrt[4]{x}+\frac{128\,\sqrt[3]{16}}{3}\ln\left(\sqrt[4]{x}+\sqrt[3]{16}\right)-\frac{64\,\sqrt[3]{16}}{3}\ln\left(\sqrt{x}-\sqrt[3]{16}\sqrt[4]{x}+16^{\frac{2}{3}}\right)+\frac{128\,\sqrt[3]{16}\sqrt{3}}{3}\arctan\left(\frac{\sqrt{3}}{3}\left(\frac{16^{\frac{2}{3}}}{8}\sqrt[4]{x}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-16+x^(3/4))/(16+x^(3/4)),x)
```

```
[Out] x-128*x^(1/4)+128/3*16^(1/3)*ln(x^(1/4)+16^(1/3))-64/3*16^(1/3)*ln(x^(1/2)-
16^(1/3)*x^(1/4)+16^(2/3))+128/3*16^(1/3)*3^(1/2)*arctan(1/3*3^(1/2)*(1/8*1
6^(2/3)*x^(1/4)-1))
```

Maxima [A] time = 1.45643, size = 96, normalized size = 0.92

$$\frac{256}{3}\sqrt{3}2^{\frac{1}{3}}\arctan\left(-\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2^{\frac{1}{3}}-x^{\frac{1}{4}}\right)\right)-\frac{128}{3}\cdot 2^{\frac{1}{3}}\log\left(4\cdot 2^{\frac{2}{3}}-2\cdot 2^{\frac{1}{3}}x^{\frac{1}{4}}+\sqrt{x}\right)+\frac{256}{3}\cdot 2^{\frac{1}{3}}\log\left(2\cdot 2^{\frac{1}{3}}+x^{\frac{1}{4}}\right)+x-128\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16+x^(3/4))/(16+x^(3/4)),x, algorithm="maxima")
```

```
[Out] 256/3*sqrt(3)*2^(1/3)*arctan(-1/6*sqrt(3)*2^(2/3)*(2^(1/3) - x^(1/4))) - 12
8/3*2^(1/3)*log(4*2^(2/3) - 2*2^(1/3)*x^(1/4) + sqrt(x)) + 256/3*2^(1/3)*lo
g(2*2^(1/3) + x^(1/4)) + x - 128*x^(1/4)
```

Fricas [A] time = 1.57667, size = 261, normalized size = 2.51

$$\frac{256}{3}\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}x^{\frac{1}{4}}-\frac{1}{3}\sqrt{3}\right)-\frac{128}{3}\cdot 2^{\frac{1}{3}}\log\left(4\cdot 2^{\frac{2}{3}}-2\cdot 2^{\frac{1}{3}}x^{\frac{1}{4}}+\sqrt{x}\right)+\frac{256}{3}\cdot 2^{\frac{1}{3}}\log\left(2\cdot 2^{\frac{1}{3}}+x^{\frac{1}{4}}\right)+x-128\,x^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16+x^(3/4))/(16+x^(3/4)),x, algorithm="fricas")
```

```
[Out] 256/3*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*x^(1/4) - 1/3*sqrt(3)) - 1
28/3*2^(1/3)*log(4*2^(2/3) - 2*2^(1/3)*x^(1/4) + sqrt(x)) + 256/3*2^(1/3)*l
og(2*2^(1/3) + x^(1/4)) + x - 128*x^(1/4)
```

Sympy [A] time = 8.5829, size = 102, normalized size = 0.98

$$-128\sqrt[4]{x}+x+\frac{256\sqrt[3]{2}\log\left(\sqrt[4]{x}+2\sqrt[3]{2}\right)}{3}-\frac{128\sqrt[3]{2}\log\left(-2\sqrt[3]{2}\sqrt[4]{x}+\sqrt{x}+4\cdot 2^{\frac{2}{3}}\right)}{3}+\frac{256\sqrt[3]{2}\sqrt{3}\operatorname{atan}\left(\frac{2^{\frac{2}{3}}\sqrt{3}\sqrt[4]{x}}{6}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16+x**(3/4))/(16+x**(3/4)),x)
```

```
[Out] -128*x**(1/4) + x + 256*2**(1/3)*log(x**(1/4) + 2*2**(1/3))/3 - 128*2**(1/3
)*log(-2*2**(1/3)*x**(1/4) + sqrt(x) + 4*2**(2/3))/3 + 256*2**(1/3)*sqrt(3)
*atan(2**(2/3)*sqrt(3)*x**(1/4)/6 - sqrt(3)/3)/3
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-16+x^(3/4))/(16+x^(3/4)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.280 $\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}} \, dx$

Optimal. Leaf size=30

$$-3x^{2/3} - x - 6\sqrt[3]{x} - 6\log\left(1 - \sqrt[3]{x}\right)$$

```
[Out] -6*x^(1/3) - 3*x^(2/3) - x - 6*Log[1 - x^(1/3)]
```

Rubi [A] time = 0.0185796, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {374, 376, 77}

$$-3x^{2/3} - x - 6\sqrt[3]{x} - 6\log\left(1 - \sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^(-1/3))/(-1 + x^(-1/3)),x]

[Out] -6*x^(1/3) - 3*x^(2/3) - x - 6*Log[1 - x^(1/3)]
```

Rule 374

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[x^(n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[n]
```

Rule 376

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}}\,dx &= \int \frac{1+\sqrt[3]{x}}{1-\sqrt[3]{x}}\,dx \\
&= 3\,\text{Subst}\left(\int \frac{x^2(1+x)}{1-x}\,dx, x, \sqrt[3]{x}\right) \\
&= 3\,\text{Subst}\left(\int \left(-2-\frac{2}{-1+x}-2x-x^2\right)dx, x, \sqrt[3]{x}\right) \\
&= -6\sqrt[3]{x}-3x^{2/3}-x-6\log\left(1-\sqrt[3]{x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0128408, size = 30, normalized size = 1.

$$-3x^{2/3}-x-6\sqrt[3]{x}-6\log\left(1-\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^(-1/3))/(-1 + x^(-1/3)),x]
```

```
[Out] -6*x^(1/3) - 3*x^(2/3) - x - 6*Log[1 - x^(1/3)]
```

Maple [A] time = 0.003, size = 23, normalized size = 0.8

$$-x-3\,x^{2/3}-6\,\sqrt[3]{x}-6\,\ln\left(-1+\sqrt[3]{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+1/x^(1/3))/(-1+1/x^(1/3)),x)
```

```
[Out] -x-3*x^(2/3)-6*x^(1/3)-6*ln(-1+x^(1/3))
```

Maxima [A] time = 0.932322, size = 30, normalized size = 1.

$$-x-3\,x^{\frac{2}{3}}-6\,x^{\frac{1}{3}}-6\,\log\left(x^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x^(1/3))/(-1+1/x^(1/3)),x, algorithm="maxima")
```

```
[Out] -x - 3*x^(2/3) - 6*x^(1/3) - 6*log(x^(1/3) - 1)
```

Fricas [A] time = 1.44237, size = 66, normalized size = 2.2

$$-x-3\,x^{\frac{2}{3}}-6\,x^{\frac{1}{3}}-6\,\log\left(x^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x^(1/3))/(-1+1/x^(1/3)),x, algorithm="fricas")
```

```
[Out] -x - 3*x^(2/3) - 6*x^(1/3) - 6*log(x^(1/3) - 1)
```

Sympy [A] time = 0.159724, size = 26, normalized size = 0.87

$$-3x^{\frac{2}{3}} - 6\sqrt[3]{x} - x - 6\log\left(\sqrt[3]{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x**(1/3))/(-1+1/x**(1/3)),x)
```

```
[Out] -3*x**(2/3) - 6*x**(1/3) - x - 6*log(x**(1/3) - 1)
```

Giac [A] time = 1.13532, size = 31, normalized size = 1.03

$$-x - 3\,x^{\frac{2}{3}} - 6\,x^{\frac{1}{3}} - 6\,\log\left(\left|x^{\frac{1}{3}} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x^(1/3))/(-1+1/x^(1/3)),x, algorithm="giac")
```

```
[Out] -x - 3*x^(2/3) - 6*x^(1/3) - 6*log(abs(x^(1/3) - 1))
```

3.281 $\int (a-bx^n)^{3/2} (a+bx^n)^{3/2} \, dx$

Optimal. Leaf size=79

$$\frac{a^2 x \sqrt{a-bx^n} \sqrt{a+bx^n} \, {}_2F_1\left(-\frac{3}{2},\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$

```
[Out] (a^2*x*Sqrt[a - b*x^n]*Sqrt[a + b*x^n]*Hypergeometric2F1[-3/2, 1/(2*n), (2
+ n^(-1))/2, (b^2*x^(2*n))/a^2])/Sqrt[1 - (b^2*x^(2*n))/a^2]
```

Rubi [A] time = 0.0331956, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {253, 246, 245}

$$\frac{a^2 x \sqrt{a-bx^n} \sqrt{a+bx^n} \, {}_2F_1\left(-\frac{3}{2},\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^n)^(3/2)*(a + b*x^n)^(3/2),x]
```

```
[Out] (a^2*x*Sqrt[a - b*x^n]*Sqrt[a + b*x^n]*Hypergeometric2F1[-3/2, 1/(2*n), (2
+ n^(-1))/2, (b^2*x^(2*n))/a^2])/Sqrt[1 - (b^2*x^(2*n))/a^2]
```

Rule 253

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Dist[((a1 + b1*x^n)^FracPart[p]*(a2 + b2*x^n)^FracPart[p])/(a1*a2
+ b1*b2*x^(2*n))^FracPart[p], Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Free
Q[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && !IntegerQ[p]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int (a-bx^n)^{3/2}(a+bx^n)^{3/2}\,dx = \frac{\left(\sqrt{a-bx^n}\sqrt{a+bx^n}\right)\int \left(a^2-b^2x^{2n}\right)^{3/2}dx}{\sqrt{a^2-b^2x^{2n}}}$$
$$= \frac{\left(a^2\sqrt{a-bx^n}\sqrt{a+bx^n}\right)\int \left(1-\frac{b^2x^{2n}}{a^2}\right)^{3/2}dx}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$
$$= \frac{a^2x\sqrt{a-bx^n}\sqrt{a+bx^n}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$

Mathematica [A] time = 0.0336167, size = 79, normalized size = 1.

$$\frac{a^2x\sqrt{a-bx^n}\sqrt{a+bx^n}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2n};1+\frac{1}{2n};\frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^n)^(3/2)*(a + b*x^n)^(3/2),x]
```

```
[Out] (a^2*x*Sqrt[a - b*x^n]*Sqrt[a + b*x^n]*Hypergeometric2F1[-3/2, 1/(2*n), 1 +
 1/(2*n), (b^2*x^(2*n))/a^2])/Sqrt[1 - (b^2*x^(2*n))/a^2]
```

Maple [F] time = 0.641, size = 0, normalized size = 0.

$$\int (a-bx^n)^{\frac{3}{2}}(a+bx^n)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-b*x^n)^(3/2)*(a+b*x^n)^(3/2),x)
```

```
[Out] int((a-b*x^n)^(3/2)*(a+b*x^n)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^n+a)^{\frac{3}{2}}(-bx^n+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^n)^(3/2)*(a+b*x^n)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^(3/2)*(-b*x^n + a)^(3/2), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^n)^(3/2)*(a+b*x^n)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x**n)**(3/2)*(a+b*x**n)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^n + a)^{\frac{3}{2}}(-bx^n + a)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^n)^(3/2)*(a+b*x^n)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^(3/2)*(-b*x^n + a)^(3/2), x)
```

3.282 $\int \sqrt{a-bx^n}\sqrt{a+bx^n}\,dx$

Optimal. Leaf size=76

$$\frac{x\sqrt{a-bx^n}\sqrt{a+bx^n}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$

```
[Out] (x*Sqrt[a - b*x^n]*Sqrt[a + b*x^n]*Hypergeometric2F1[-1/2, 1/(2*n), (2 + n^
(-1))/2, (b^2*x^(2*n))/a^2])/Sqrt[1 - (b^2*x^(2*n))/a^2]
```

Rubi [A] time = 0.0272434, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {253, 246, 245}

$$\frac{x\sqrt{a-bx^n}\sqrt{a+bx^n}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - b*x^n]*Sqrt[a + b*x^n],x]
```

```
[Out] (x*Sqrt[a - b*x^n]*Sqrt[a + b*x^n]*Hypergeometric2F1[-1/2, 1/(2*n), (2 + n^
(-1))/2, (b^2*x^(2*n))/a^2])/Sqrt[1 - (b^2*x^(2*n))/a^2]
```

Rule 253

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Dist[((a1 + b1*x^n)^FracPart[p]*(a2 + b2*x^n)^FracPart[p])/(a1*a2
+ b1*b2*x^(2*n))^FracPart[p], Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Free
Q[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && !IntegerQ[p]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \sqrt{a-bx^n}\sqrt{a+bx^n}\,dx = \frac{\left(\sqrt{a-bx^n}\sqrt{a+bx^n}\right)\int \sqrt{a^2-b^2x^{2n}}\,dx}{\sqrt{a^2-b^2x^{2n}}}$$
$$= \frac{\left(\sqrt{a-bx^n}\sqrt{a+bx^n}\right)\int \sqrt{1-\frac{b^2x^{2n}}{a^2}}\,dx}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$
$$= \frac{x\sqrt{a-bx^n}\sqrt{a+bx^n}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2n};\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$

Mathematica [A] time = 0.0164294, size = 76, normalized size = 1.

$$\frac{x\sqrt{a-bx^n}\sqrt{a+bx^n}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2n};1+\frac{1}{2n};\frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1-\frac{b^2x^{2n}}{a^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - b*x^n]*Sqrt[a + b*x^n],x]
```

```
[Out] (x*Sqrt[a - b*x^n]*Sqrt[a + b*x^n]*Hypergeometric2F1[-1/2, 1/(2*n), 1 + 1/(
2*n), (b^2*x^(2*n))/a^2])/Sqrt[1 - (b^2*x^(2*n))/a^2]
```

Maple [F] time = 0.376, size = 0, normalized size = 0.

$$\int \sqrt{a-bx^n}\sqrt{a+bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-b*x^n)^(1/2)*(a+b*x^n)^(1/2),x)
```

```
[Out] int((a-b*x^n)^(1/2)*(a+b*x^n)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^n+a}\sqrt{-bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^n)^(1/2)*(a+b*x^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^n + a)*sqrt(-b*x^n + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^n)^(1/2)*(a+b*x^n)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a-bx^n}\sqrt{a+bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x**n)**(1/2)*(a+b*x**n)**(1/2),x)
```

```
[Out] Integral(sqrt(a - b*x**n)*sqrt(a + b*x**n), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{bx^n+a}\sqrt{-bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^n)^(1/2)*(a+b*x^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x^n + a)*sqrt(-b*x^n + a), x)
```

3.283 $\int (a-bx^n)^p (a+bx^n)^p \, dx$

Optimal. Leaf size=72

$$x(a-bx^n)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}{}_2F_1\left(\frac{1}{2n},-p;\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2}\right)$$

```
[Out] (x*(a - b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[1/(2*n), -p, (2 + n^(-1))/
2, (b^2*x^(2*n))/a^2])/(1 - (b^2*x^(2*n))/a^2)^p
```

Rubi [A] time = 0.0285352, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {253, 246, 245}

$$x(a-bx^n)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}{}_2F_1\left(\frac{1}{2n},-p;\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^n)^p*(a + b*x^n)^p,x]
```


```
[Out] (x*(a - b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[1/(2*n), -p, (2 + n^(-1))/
2, (b^2*x^(2*n))/a^2])/(1 - (b^2*x^(2*n))/a^2)^p
```


Rule 253

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Dist[((a1 + b1*x^n)^FracPart[p]*(a2 + b2*x^n)^FracPart[p])/(a1*a2
+ b1*b2*x^(2*n))^FracPart[p], Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Free
Q[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && !IntegerQ[p]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int (a-bx^n)^p(a+bx^n)^p\,dx &= \left((a-bx^n)^p(a+bx^n)^p\left(a^2-b^2x^{2n}\right)^{-p}\right)\int \left(a^2-b^2x^{2n}\right)^p\,dx\\
&= \left((a-bx^n)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\right)\int \left(1-\frac{b^2x^{2n}}{a^2}\right)^p\,dx\\
&= x(a-bx^n)^p(a+bx^n)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}{}_2F_1\left(\frac{1}{2n},-p;\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0188087, size = 72, normalized size = 1.

$$x\left(a-bx^n\right)^p\left(a+bx^n\right)^p\left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}{}_2F_1\left(\frac{1}{2n},-p;1+\frac{1}{2n};\frac{b^2x^{2n}}{a^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^n)^p*(a + b*x^n)^p,x]
```

```
[Out] (x*(a - b*x^n)^p*(a + b*x^n)^p*Hypergeometric2F1[1/(2*n), -p, 1 + 1/(2*n),
(b^2*x^(2*n))/a^2])/(1 - (b^2*x^(2*n))/a^2)^p
```

Maple [F] time = 0.856, size = 0, normalized size = 0.

$$\int\left(a-bx^n\right)^p\left(a+bx^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-b*x^n)^p*(a+b*x^n)^p,x)
```

```
[Out] int((a-b*x^n)^p*(a+b*x^n)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int(bx^n+a)^p(-bx^n+a)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^n)^p*(a+b*x^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p*(-b*x^n + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^n+a)^p(-bx^n+a)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^n)^p*(a+b*x^n)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^p*(-b*x^n + a)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a-bx^n\right)^p\left(a+bx^n\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x**n)**p*(a+b*x**n)**p,x)
```

```
[Out] Integral((a - b*x**n)**p*(a + b*x**n)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^n + a)^p (-bx^n + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^n)^p*(a+b*x^n)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^p*(-b*x^n + a)^p, x)
```

3.284 $\int (a+bx^n)(c+dx^n)^4\,dx$

Optimal. Leaf size=132

$$\frac{c^3x^{n+1}(4ad+bc)}{n+1}+\frac{2c^2dx^{2n+1}(3ad+2bc)}{2n+1}+\frac{2cd^2x^{3n+1}(2ad+3bc)}{3n+1}+\frac{d^3x^{4n+1}(ad+4bc)}{4n+1}+ac^4x+\frac{bd^4x^{5n+1}}{5n+1}$$

```
[Out] a*c^4*x + (c^3*(b*c + 4*a*d)*x^(1 + n))/(1 + n) + (2*c^2*d*(2*b*c + 3*a*d)*
x^(1 + 2*n))/(1 + 2*n) + (2*c*d^2*(3*b*c + 2*a*d)*x^(1 + 3*n))/(1 + 3*n) +
(d^3*(4*b*c + a*d)*x^(1 + 4*n))/(1 + 4*n) + (b*d^4*x^(1 + 5*n))/(1 + 5*n)
```

Rubi [A] time = 0.111141, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$\frac{c^3x^{n+1}(4ad+bc)}{n+1}+\frac{2c^2dx^{2n+1}(3ad+2bc)}{2n+1}+\frac{2cd^2x^{3n+1}(2ad+3bc)}{3n+1}+\frac{d^3x^{4n+1}(ad+4bc)}{4n+1}+ac^4x+\frac{bd^4x^{5n+1}}{5n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)*(c + d*x^n)^4,x]
```

```
[Out] a*c^4*x + (c^3*(b*c + 4*a*d)*x^(1 + n))/(1 + n) + (2*c^2*d*(2*b*c + 3*a*d)*
x^(1 + 2*n))/(1 + 2*n) + (2*c*d^2*(3*b*c + 2*a*d)*x^(1 + 3*n))/(1 + 3*n) +
(d^3*(4*b*c + a*d)*x^(1 + 4*n))/(1 + 4*n) + (b*d^4*x^(1 + 5*n))/(1 + 5*n)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int (a+bx^n)(c+dx^n)^4\,dx = \int \left(ac^4+c^3(bc+4ad)x^n+2c^2d(2bc+3ad)x^{2n}+2cd^2(3bc+2ad)x^{3n}+d^3(4bc+ad)x^{4n}+bc\right.$$
$$= ac^4x+\frac{c^3(bc+4ad)x^{1+n}}{1+n}+\frac{2c^2d(2bc+3ad)x^{1+2n}}{1+2n}+\frac{2cd^2(3bc+2ad)x^{1+3n}}{1+3n}+\frac{d^3(4bc+ad)x^{1}}{1+4n}$$

Mathematica [A] time = 0.147514, size = 110, normalized size = 0.83

$$\frac{bx(c+dx^n)^5-x\left(\frac{6c^2d^2x^{2n}}{2n+1}+\frac{4c^3dx^n}{n+1}+c^4+\frac{4cd^3x^{3n}}{3n+1}+\frac{d^4x^{4n}}{4n+1}\right)(bc-ad(5n+1))}{5dn+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)*(c + d*x^n)^4,x]
```

```
[Out] (b*x*(c + d*x^n)^5 - (b*c - a*d*(1 + 5*n))*x*(c^4 + (4*c^3*d*x^n)/(1 + n) +
 (6*c^2*d^2*x^(2*n))/(1 + 2*n) + (4*c*d^3*x^(3*n))/(1 + 3*n) + (d^4*x^(4*n)
)/(1 + 4*n)))/(d + 5*d*n)
```

Maple [A] time = 0.059, size = 138, normalized size = 1.1

$$ac^4x+\frac{bd^4x\left(e^{n\ln(x)}\right)^5}{1+5n}+\frac{c^3\left(4ad+bc\right)xe^{n\ln(x)}}{1+n}+\frac{d^3\left(ad+4bc\right)x\left(e^{n\ln(x)}\right)^4}{1+4n}+2\frac{cd^2\left(2ad+3bc\right)x\left(e^{n\ln(x)}\right)^3}{1+3n}+2\frac{c^2d\,(3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)*(c+d*x^n)^4,x)
```

```
[Out] a*c^4*x+b*d^4/(1+5*n)*x*exp(n*ln(x))^5+c^3*(4*a*d+b*c)/(1+n)*x*exp(n*ln(x))
+d^3*(a*d+4*b*c)/(1+4*n)*x*exp(n*ln(x))^4+2*c*d^2*(2*a*d+3*b*c)/(1+3*n)*x*e
xp(n*ln(x))^3+2*c^2*d*(3*a*d+2*b*c)/(1+2*n)*x*exp(n*ln(x))^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.64474, size = 1166, normalized size = 8.83

$$\left(24\,bd^4n^4+50\,bd^4n^3+35\,bd^4n^2+10\,bd^4n+bd^4\right)xx^{5\,n}+\left(4\,bcd^3+ad^4+30\left(4\,bcd^3+ad^4\right)n^4+61\left(4\,bcd^3+ad^4\right)n^3+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^4,x, algorithm="fricas")
```

```
[Out] ((24*b*d^4*n^4 + 50*b*d^4*n^3 + 35*b*d^4*n^2 + 10*b*d^4*n + b*d^4)*x*x^(5*n
) + (4*b*c*d^3 + a*d^4 + 30*(4*b*c*d^3 + a*d^4)*n^4 + 61*(4*b*c*d^3 + a*d^4
)*n^3 + 41*(4*b*c*d^3 + a*d^4)*n^2 + 11*(4*b*c*d^3 + a*d^4)*n)*x*x^(4*n) +
2*(3*b*c^2*d^2 + 2*a*c*d^3 + 40*(3*b*c^2*d^2 + 2*a*c*d^3)*n^4 + 78*(3*b*c^2
*d^2 + 2*a*c*d^3)*n^3 + 49*(3*b*c^2*d^2 + 2*a*c*d^3)*n^2 + 12*(3*b*c^2*d^2
+ 2*a*c*d^3)*n)*x*x^(3*n) + 2*(2*b*c^3*d + 3*a*c^2*d^2 + 60*(2*b*c^3*d + 3*
a*c^2*d^2)*n^4 + 107*(2*b*c^3*d + 3*a*c^2*d^2)*n^3 + 59*(2*b*c^3*d + 3*a*c^
2*d^2)*n^2 + 13*(2*b*c^3*d + 3*a*c^2*d^2)*n)*x*x^(2*n) + (b*c^4 + 4*a*c^3*d
 + 120*(b*c^4 + 4*a*c^3*d)*n^4 + 154*(b*c^4 + 4*a*c^3*d)*n^3 + 71*(b*c^4 +
4*a*c^3*d)*n^2 + 14*(b*c^4 + 4*a*c^3*d)*n)*x*x^n + (120*a*c^4*n^5 + 274*a*c
^4*n^4 + 225*a*c^4*n^3 + 85*a*c^4*n^2 + 15*a*c^4*n + a*c^4)*x)/(120*n^5 + 2
74*n^4 + 225*n^3 + 85*n^2 + 15*n + 1)
```

Sympy [A] time = 3.27936, size = 2744, normalized size = 20.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)*(c+d*x**n)**4,x)
```

```
[Out] Piecewise((a*c**4*x + 4*a*c**3*d*log(x) - 6*a*c**2*d**2/x - 2*a*c*d**3/x**2
 - a*d**4/(3*x**3) + b*c**4*log(x) - 4*b*c**3*d/x - 3*b*c**2*d**2/x**2 - 4*
b*c*d**3/(3*x**3) - b*d**4/(4*x**4), Eq(n, -1)), (a*c**4*x + 8*a*c**3*d*sqr
t(x) + 6*a*c**2*d**2*log(x) - 8*a*c*d**3/sqrt(x) - a*d**4/x + 2*b*c**4*sqrt
(x) + 4*b*c**3*d*log(x) - 12*b*c**2*d**2/sqrt(x) - 4*b*c*d**3/x - 2*b*d**4/
(3*x**(3/2)), Eq(n, -1/2)), (a*c**4*x + 6*a*c**3*d*x**(2/3) + 18*a*c**2*d**
2*x**(1/3) + 4*a*c*d**3*log(x) - 3*a*d**4/x**(1/3) + 3*b*c**4*x**(2/3)/2 +
12*b*c**3*d*x**(1/3) + 6*b*c**2*d**2*log(x) - 12*b*c*d**3/x**(1/3) - 3*b*d*
*4/(2*x**(2/3)), Eq(n, -1/3)), (a*c**4*x + 16*a*c**3*d*x**(3/4)/3 + 12*a*c*
*2*d**2*sqrt(x) + 16*a*c*d**3*x**(1/4) + a*d**4*log(x) + 4*b*c**4*x**(3/4)/
3 + 8*b*c**3*d*sqrt(x) + 24*b*c**2*d**2*x**(1/4) + 4*b*c*d**3*log(x) - 4*b*
d**4/x**(1/4), Eq(n, -1/4)), (a*c**4*x + 5*a*c**3*d*x**(4/5) + 10*a*c**2*d*
*2*x**(3/5) + 10*a*c*d**3*x**(2/5) + 5*a*d**4*x**(1/5) + 5*b*c**4*x**(4/5)/
4 + 20*b*c**3*d*x**(3/5)/3 + 15*b*c**2*d**2*x**(2/5) + 20*b*c*d**3*x**(1/5)
 + b*d**4*log(x), Eq(n, -1/5)), (120*a*c**4*n**5*x/(120*n**5 + 274*n**4 + 2
25*n**3 + 85*n**2 + 15*n + 1) + 274*a*c**4*n**4*x/(120*n**5 + 274*n**4 + 22
5*n**3 + 85*n**2 + 15*n + 1) + 225*a*c**4*n**3*x/(120*n**5 + 274*n**4 + 225
*n**3 + 85*n**2 + 15*n + 1) + 85*a*c**4*n**2*x/(120*n**5 + 274*n**4 + 225*n
**3 + 85*n**2 + 15*n + 1) + 15*a*c**4*n*x/(120*n**5 + 274*n**4 + 225*n**3 +
 85*n**2 + 15*n + 1) + a*c**4*x/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 +
 15*n + 1) + 480*a*c**3*d*n**4*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*
n**2 + 15*n + 1) + 616*a*c**3*d*n**3*x*x**n/(120*n**5 + 274*n**4 + 225*n**3
 + 85*n**2 + 15*n + 1) + 284*a*c**3*d*n**2*x*x**n/(120*n**5 + 274*n**4 + 22
5*n**3 + 85*n**2 + 15*n + 1) + 56*a*c**3*d*n*x*x**n/(120*n**5 + 274*n**4 +
225*n**3 + 85*n**2 + 15*n + 1) + 4*a*c**3*d*x*x**n/(120*n**5 + 274*n**4 + 2
25*n**3 + 85*n**2 + 15*n + 1) + 360*a*c**2*d**2*n**4*x*x**(2*n)/(120*n**5 +
 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 642*a*c**2*d**2*n**3*x*x**(2*n
)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 354*a*c**2*d**2*n
**2*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 78*a
*c**2*d**2*n*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n +
1) + 6*a*c**2*d**2*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 1
5*n + 1) + 160*a*c*d**3*n**4*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 8
5*n**2 + 15*n + 1) + 312*a*c*d**3*n**3*x*x**(3*n)/(120*n**5 + 274*n**4 + 22
5*n**3 + 85*n**2 + 15*n + 1) + 196*a*c*d**3*n**2*x*x**(3*n)/(120*n**5 + 274
*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 48*a*c*d**3*n*x*x**(3*n)/(120*n**5
 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 4*a*c*d**3*x*x**(3*n)/(120*n
**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 30*a*d**4*n**4*x*x**(4*n)
/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 61*a*d**4*n**3*x*x
**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 41*a*d**4*n
**2*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 11*a
*d**4*n*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) +
a*d**4*x*x**(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 1
20*b*c**4*n**4*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1)
 + 154*b*c**4*n**3*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n
+ 1) + 71*b*c**4*n**2*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15
*n + 1) + 14*b*c**4*n*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15
*n + 1) + b*c**4*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n +
1) + 240*b*c**3*d*n**4*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2
 + 15*n + 1) + 428*b*c**3*d*n**3*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3
 + 85*n**2 + 15*n + 1) + 236*b*c**3*d*n**2*x*x**(2*n)/(120*n**5 + 274*n**4
+ 225*n**3 + 85*n**2 + 15*n + 1) + 52*b*c**3*d*n*x*x**(2*n)/(120*n**5 + 274
*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 4*b*c**3*d*x*x**(2*n)/(120*n**5 +
274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 240*b*c**2*d**2*n**4*x*x**(3*n)
/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 468*b*c**2*d**2*n*
*3*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 294*b
*c**2*d**2*n**2*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n
 + 1) + 72*b*c**2*d**2*n*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n*
```

```
*2 + 15*n + 1) + 6*b*c**2*d**2*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 +
 85*n**2 + 15*n + 1) + 120*b*c*d**3*n**4*x*x**(4*n)/(120*n**5 + 274*n**4 +
225*n**3 + 85*n**2 + 15*n + 1) + 244*b*c*d**3*n**3*x*x**(4*n)/(120*n**5 + 2
74*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 164*b*c*d**3*n**2*x*x**(4*n)/(12
0*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 44*b*c*d**3*n*x*x**(4*
n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 4*b*c*d**3*x*x**
(4*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 24*b*d**4*n**
4*x*x**(5*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 50*b*d
**4*n**3*x*x**(5*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) +
 35*b*d**4*n**2*x*x**(5*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n
 + 1) + 10*b*d**4*n*x*x**(5*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 +
15*n + 1) + b*d**4*x*x**(5*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 1
5*n + 1), True))
```

Giac [B] time = 1.15031, size = 999, normalized size = 7.57

$$\frac{120\,ac^4n^5x + 24\,bd^4n^4xx^{5n} + 120\,bcd^3n^4xx^{4n} + 30\,ad^4n^4xx^{4n} + 240\,bc^2d^2n^4xx^{3n} + 160\,acd^3n^4xx^{3n} + 240\,bc^3dn^4xx^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^4,x, algorithm="giac")
```

```
[Out] (120*a*c^4*n^5*x + 24*b*d^4*n^4*x*x^(5*n) + 120*b*c*d^3*n^4*x*x^(4*n) + 30*
a*d^4*n^4*x*x^(4*n) + 240*b*c^2*d^2*n^4*x*x^(3*n) + 160*a*c*d^3*n^4*x*x^(3*
n) + 240*b*c^3*d*n^4*x*x^(2*n) + 360*a*c^2*d^2*n^4*x*x^(2*n) + 120*b*c^4*n^
4*x*x^n + 480*a*c^3*d*n^4*x*x^n + 274*a*c^4*n^4*x + 50*b*d^4*n^3*x*x^(5*n)
+ 244*b*c*d^3*n^3*x*x^(4*n) + 61*a*d^4*n^3*x*x^(4*n) + 468*b*c^2*d^2*n^3*x*
x^(3*n) + 312*a*c*d^3*n^3*x*x^(3*n) + 428*b*c^3*d*n^3*x*x^(2*n) + 642*a*c^2
*d^2*n^3*x*x^(2*n) + 154*b*c^4*n^3*x*x^n + 616*a*c^3*d*n^3*x*x^n + 225*a*c^
4*n^3*x + 35*b*d^4*n^2*x*x^(5*n) + 164*b*c*d^3*n^2*x*x^(4*n) + 41*a*d^4*n^2
*x*x^(4*n) + 294*b*c^2*d^2*n^2*x*x^(3*n) + 196*a*c*d^3*n^2*x*x^(3*n) + 236*
b*c^3*d*n^2*x*x^(2*n) + 354*a*c^2*d^2*n^2*x*x^(2*n) + 71*b*c^4*n^2*x*x^n +
284*a*c^3*d*n^2*x*x^n + 85*a*c^4*n^2*x + 10*b*d^4*n*x*x^(5*n) + 44*b*c*d^3*
n*x*x^(4*n) + 11*a*d^4*n*x*x^(4*n) + 72*b*c^2*d^2*n*x*x^(3*n) + 48*a*c*d^3*
n*x*x^(3*n) + 52*b*c^3*d*n*x*x^(2*n) + 78*a*c^2*d^2*n*x*x^(2*n) + 14*b*c^4*
n*x*x^n + 56*a*c^3*d*n*x*x^n + 15*a*c^4*n*x + b*d^4*x*x^(5*n) + 4*b*c*d^3*x
*x^(4*n) + a*d^4*x*x^(4*n) + 6*b*c^2*d^2*x*x^(3*n) + 4*a*c*d^3*x*x^(3*n) +
4*b*c^3*d*x*x^(2*n) + 6*a*c^2*d^2*x*x^(2*n) + b*c^4*x*x^n + 4*a*c^3*d*x*x^n
 + a*c^4*x)/(120*n^5 + 274*n^4 + 225*n^3 + 85*n^2 + 15*n + 1)
```

3.285 $\int (a+bx^n)(c+dx^n)^3\,dx$

Optimal. Leaf size=99

$$\frac{c^2x^{n+1}(3ad+bc)}{n+1}+\frac{d^2x^{3n+1}(ad+3bc)}{3n+1}+\frac{3cdx^{2n+1}(ad+bc)}{2n+1}+ac^3x+\frac{bd^3x^{4n+1}}{4n+1}$$

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^(1 + n))/(1 + n) + (3*c*d*(b*c + a*d)*x^(1 + 2*n))/(1 + 2*n) + (d^2*(3*b*c + a*d)*x^(1 + 3*n))/(1 + 3*n) + (b*d^3*x^(1 + 4*n))/(1 + 4*n)
```

Rubi [A] time = 0.0727884, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$\frac{c^2x^{n+1}(3ad+bc)}{n+1}+\frac{d^2x^{3n+1}(ad+3bc)}{3n+1}+\frac{3cdx^{2n+1}(ad+bc)}{2n+1}+ac^3x+\frac{bd^3x^{4n+1}}{4n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)*(c + d*x^n)^3,x]
```

```
[Out] a*c^3*x + (c^2*(b*c + 3*a*d)*x^(1 + n))/(1 + n) + (3*c*d*(b*c + a*d)*x^(1 + 2*n))/(1 + 2*n) + (d^2*(3*b*c + a*d)*x^(1 + 3*n))/(1 + 3*n) + (b*d^3*x^(1 + 4*n))/(1 + 4*n)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^n)(c+dx^n)^3\,dx &= \int \left(ac^3+c^2(bc+3ad)x^n+3cd(bc+ad)x^{2n}+d^2(3bc+ad)x^{3n}+bd^3x^{4n}\right)dx\\ &= ac^3x+\frac{c^2(bc+3ad)x^{1+n}}{1+n}+\frac{3cd(bc+ad)x^{1+2n}}{1+2n}+\frac{d^2(3bc+ad)x^{1+3n}}{1+3n}+\frac{bd^3x^{1+4n}}{1+4n}\end{aligned}$$

Mathematica [A] time = 0.105617, size = 90, normalized size = 0.91

$$\frac{bx(c+dx^n)^4-x\left(\frac{3c^2dx^n}{n+1}+c^3+\frac{3cd^2x^{2n}}{2n+1}+\frac{d^3x^{3n}}{3n+1}\right)(bc-ad(4n+1))}{4dn+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)*(c + d*x^n)^3,x]
```

```
[Out] (b*x*(c + d*x^n)^4 - (b*c - a*d*(1 + 4*n))*x*(c^3 + (3*c^2*d*x^n)/(1 + n) + (3*c*d^2*x^(2*n))/(1 + 2*n) + (d^3*x^(3*n))/(1 + 3*n)))/(d + 4*d*n)
```

Maple [A] time = 0.009, size = 104, normalized size = 1.1

$$ac^3x+\frac{bd^3x\left(e^{n\ln(x)}\right)^4}{1+4n}+\frac{c^2\,(3\,ad+bc)\,xe^{n\ln(x)}}{1+n}+\frac{d^2\,(ad+3\,bc)\,x\left(e^{n\ln(x)}\right)^3}{1+3\,n}+3\,\frac{cd\,(ad+bc)\,x\left(e^{n\ln(x)}\right)^2}{1+2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)*(c+d*x^n)^3,x)
```

```
[Out] a*c^3*x+b*d^3/(1+4*n)*x*exp(n*ln(x))^4+c^2*(3*a*d+b*c)/(1+n)*x*exp(n*ln(x))
+d^2*(a*d+3*b*c)/(1+3*n)*x*exp(n*ln(x))^3+3*c*d*(a*d+b*c)/(1+2*n)*x*exp(n*l
n(x))^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.59923, size = 695, normalized size = 7.02

$$\frac{\left(6\,bd^3n^3+11\,bd^3n^2+6\,bd^3n+bd^3\right)xx^{4\,n}+\left(3\,bcd^2+ad^3+8\left(3\,bcd^2+ad^3\right)n^3+14\left(3\,bcd^2+ad^3\right)n^2+7\left(3\,bcd^2+ad\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^3,x, algorithm="fricas")
```

```
[Out] ((6*b*d^3*n^3 + 11*b*d^3*n^2 + 6*b*d^3*n + b*d^3)*x*x^(4*n) + (3*b*c*d^2 +
a*d^3 + 8*(3*b*c*d^2 + a*d^3)*n^3 + 14*(3*b*c*d^2 + a*d^3)*n^2 + 7*(3*b*c*d
^2 + a*d^3)*n)*x*x^(3*n) + 3*(b*c^2*d + a*c*d^2 + 12*(b*c^2*d + a*c*d^2)*n^
3 + 19*(b*c^2*d + a*c*d^2)*n^2 + 8*(b*c^2*d + a*c*d^2)*n)*x*x^(2*n) + (b*c^
3 + 3*a*c^2*d + 24*(b*c^3 + 3*a*c^2*d)*n^3 + 26*(b*c^3 + 3*a*c^2*d)*n^2 + 9
*(b*c^3 + 3*a*c^2*d)*n)*x*x^n + (24*a*c^3*n^4 + 50*a*c^3*n^3 + 35*a*c^3*n^2
 + 10*a*c^3*n + a*c^3)*x)/(24*n^4 + 50*n^3 + 35*n^2 + 10*n + 1)
```

Sympy [A] time = 1.7615, size = 1540, normalized size = 15.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)*(c+d*x**n)**3,x)
```

```
[Out] Piecewise((a*c**3*x + 3*a*c**2*d*log(x) - 3*a*c*d**2/x - a*d**3/(2*x**2) +
b*c**3*log(x) - 3*b*c**2*d/x - 3*b*c*d**2/(2*x**2) - b*d**3/(3*x**3), Eq(n,
 -1)), (a*c**3*x + 6*a*c**2*d*sqrt(x) + 3*a*c*d**2*log(x) - 2*a*d**3/sqrt(x
```

```
) + 2*b*c**3*sqrt(x) + 3*b*c**2*d*log(x) - 6*b*c*d**2/sqrt(x) - b*d**3/x, E
q(n, -1/2)), (a*c**3*x + 9*a*c**2*d*x**(2/3)/2 + 9*a*c*d**2*x**(1/3) + a*d*
*3*log(x) + 3*b*c**3*x**(2/3)/2 + 9*b*c**2*d*x**(1/3) + 3*b*c*d**2*log(x) -
 3*b*d**3/x**(1/3), Eq(n, -1/3)), (a*c**3*x + 4*a*c**2*d*x**(3/4) + 6*a*c*d
**2*sqrt(x) + 4*a*d**3*x**(1/4) + 4*b*c**3*x**(3/4)/3 + 6*b*c**2*d*sqrt(x)
+ 12*b*c*d**2*x**(1/4) + b*d**3*log(x), Eq(n, -1/4)), (24*a*c**3*n**4*x/(24
*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 50*a*c**3*n**3*x/(24*n**4 + 50*n**3
 + 35*n**2 + 10*n + 1) + 35*a*c**3*n**2*x/(24*n**4 + 50*n**3 + 35*n**2 + 10
*n + 1) + 10*a*c**3*n*x/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + a*c**3*x
/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 72*a*c**2*d*n**3*x*x**n/(24*n**
4 + 50*n**3 + 35*n**2 + 10*n + 1) + 78*a*c**2*d*n**2*x*x**n/(24*n**4 + 50*n
**3 + 35*n**2 + 10*n + 1) + 27*a*c**2*d*n*x*x**n/(24*n**4 + 50*n**3 + 35*n*
*2 + 10*n + 1) + 3*a*c**2*d*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1)
 + 36*a*c*d**2*n**3*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 5
7*a*c*d**2*n**2*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 24*a*
c*d**2*n*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 3*a*c*d**2*x
*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 8*a*d**3*n**3*x*x**(3*
n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 14*a*d**3*n**2*x*x**(3*n)/(24
*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 7*a*d**3*n*x*x**(3*n)/(24*n**4 + 50
*n**3 + 35*n**2 + 10*n + 1) + a*d**3*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**
2 + 10*n + 1) + 24*b*c**3*n**3*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n +
 1) + 26*b*c**3*n**2*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 9*b*
c**3*n*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + b*c**3*x*x**n/(24*
n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 36*b*c**2*d*n**3*x*x**(2*n)/(24*n**4
 + 50*n**3 + 35*n**2 + 10*n + 1) + 57*b*c**2*d*n**2*x*x**(2*n)/(24*n**4 + 5
0*n**3 + 35*n**2 + 10*n + 1) + 24*b*c**2*d*n*x*x**(2*n)/(24*n**4 + 50*n**3
+ 35*n**2 + 10*n + 1) + 3*b*c**2*d*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2
+ 10*n + 1) + 24*b*c*d**2*n**3*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10
*n + 1) + 42*b*c*d**2*n**2*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n +
 1) + 21*b*c*d**2*n*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 3
*b*c*d**2*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 6*b*d**3*n*
*3*x*x**(4*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 11*b*d**3*n**2*x*x
**(4*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 6*b*d**3*n*x*x**(4*n)/(2
4*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + b*d**3*x*x**(4*n)/(24*n**4 + 50*n*
*3 + 35*n**2 + 10*n + 1), True))
```

Giac [B] time = 1.11209, size = 608, normalized size = 6.14

$$\frac{24\,ac^3n^4x + 6\,bd^3n^3xx^{4\,n} + 24\,bcd^2n^3xx^{3\,n} + 8\,ad^3n^3xx^{3\,n} + 36\,bc^2dn^3xx^{2\,n} + 36\,acd^2n^3xx^{2\,n} + 24\,bc^3n^3xx^{n} + 72\,ac^2dn^3x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^3,x, algorithm="giac")
```

```
[Out] (24*a*c^3*n^4*x + 6*b*d^3*n^3*x*x^(4*n) + 24*b*c*d^2*n^3*x*x^(3*n) + 8*a*d^
3*n^3*x*x^(3*n) + 36*b*c^2*d*n^3*x*x^(2*n) + 36*a*c*d^2*n^3*x*x^(2*n) + 24*
b*c^3*n^3*x*x^n + 72*a*c^2*d*n^3*x*x^n + 50*a*c^3*n^3*x + 11*b*d^3*n^2*x*x^
(4*n) + 42*b*c*d^2*n^2*x*x^(3*n) + 14*a*d^3*n^2*x*x^(3*n) + 57*b*c^2*d*n^2*
x*x^(2*n) + 57*a*c*d^2*n^2*x*x^(2*n) + 26*b*c^3*n^2*x*x^n + 78*a*c^2*d*n^2*
x*x^n + 35*a*c^3*n^2*x + 6*b*d^3*n*x*x^(4*n) + 21*b*c*d^2*n*x*x^(3*n) + 7*a
*d^3*n*x*x^(3*n) + 24*b*c^2*d*n*x*x^(2*n) + 24*a*c*d^2*n*x*x^(2*n) + 9*b*c^
3*n*x*x^n + 27*a*c^2*d*n*x*x^n + 10*a*c^3*n*x + b*d^3*x*x^(4*n) + 3*b*c*d^2
*x*x^(3*n) + a*d^3*x*x^(3*n) + 3*b*c^2*d*x*x^(2*n) + 3*a*c*d^2*x*x^(2*n) +
b*c^3*x*x^n + 3*a*c^2*d*x*x^n + a*c^3*x)/(24*n^4 + 50*n^3 + 35*n^2 + 10*n +
 1)
```

3.286 $\int (a+bx^n)(c+dx^n)^2\,dx$

Optimal. Leaf size=70

$$\frac{cx^{n+1}(2ad+bc)}{n+1}+\frac{dx^{2n+1}(ad+2bc)}{2n+1}+ac^2x+\frac{bd^2x^{3n+1}}{3n+1}$$

```
[Out] a*c^2*x + (c*(b*c + 2*a*d)*x^(1 + n))/(1 + n) + (d*(2*b*c + a*d)*x^(1 + 2*n
))/(1 + 2*n) + (b*d^2*x^(1 + 3*n))/(1 + 3*n)
```

Rubi [A] time = 0.0451552, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$\frac{cx^{n+1}(2ad+bc)}{n+1}+\frac{dx^{2n+1}(ad+2bc)}{2n+1}+ac^2x+\frac{bd^2x^{3n+1}}{3n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)*(c + d*x^n)^2,x]
```

```
[Out] a*c^2*x + (c*(b*c + 2*a*d)*x^(1 + n))/(1 + n) + (d*(2*b*c + a*d)*x^(1 + 2*n
))/(1 + 2*n) + (b*d^2*x^(1 + 3*n))/(1 + 3*n)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^n)(c+dx^n)^2\,dx &= \int \left(ac^2+c(bc+2ad)x^n+d(2bc+ad)x^{2n}+bd^2x^{3n}\right)dx\\ &= ac^2x+\frac{c(bc+2ad)x^{1+n}}{1+n}+\frac{d(2bc+ad)x^{1+2n}}{1+2n}+\frac{bd^2x^{1+3n}}{1+3n}\end{aligned}$$

Mathematica [A] time = 0.0812045, size = 70, normalized size = 1.

$$\frac{bx(c+dx^n)^3-x\left(c^2+\frac{2cdx^n}{n+1}+\frac{d^2x^{2n}}{2n+1}\right)(bc-ad(3n+1))}{3dn+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)*(c + d*x^n)^2,x]
```

```
[Out] (b*x*(c + d*x^n)^3 - (b*c - a*d*(1 + 3*n))*x*(c^2 + (2*c*d*x^n)/(1 + n) + (
d^2*x^(2*n))/(1 + 2*n)))/(d + 3*d*n)
```

Maple [A] time = 0.009, size = 74, normalized size = 1.1

$$ac^2x+\frac{bd^2x\left(e^{n\ln(x)}\right)^3}{1+3n}+\frac{c(2ad+bc)xe^{n\ln(x)}}{1+n}+\frac{d(ad+2bc)x\left(e^{n\ln(x)}\right)^2}{1+2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)*(c+d*x^n)^2,x)
```

```
[Out] a*c^2*x+b*d^2/(1+3*n)*x*exp(n*ln(x))^3+c*(2*a*d+b*c)/(1+n)*x*exp(n*ln(x))+d
*(a*d+2*b*c)/(1+2*n)*x*exp(n*ln(x))^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.65614, size = 382, normalized size = 5.46

$$\frac{\left(2\,bd^2n^2+3\,bd^2n+bd^2\right)xx^{3\,n}+\left(2\,bcd+ad^2+3\left(2\,bcd+ad^2\right)n^2+4\left(2\,bcd+ad^2\right)n\right)xx^{2\,n}+\left(bc^2+2\,acd+6\left(bc^2+2\,acd\right.\right.}{6\,n^3+11\,n^2+6\,n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^2,x, algorithm="fricas")
```

```
[Out] ((2*b*d^2*n^2 + 3*b*d^2*n + b*d^2)*x*x^(3*n) + (2*b*c*d + a*d^2 + 3*(2*b*c*
d + a*d^2)*n^2 + 4*(2*b*c*d + a*d^2)*n)*x*x^(2*n) + (b*c^2 + 2*a*c*d + 6*(b
*c^2 + 2*a*c*d)*n^2 + 5*(b*c^2 + 2*a*c*d)*n)*x*x^n + (6*a*c^2*n^3 + 11*a*c^
2*n^2 + 6*a*c^2*n + a*c^2)*x)/(6*n^3 + 11*n^2 + 6*n + 1)
```

Sympy [A] time = 0.927817, size = 726, normalized size = 10.37

$$\begin{cases} ac^2x+2acd\log(x)-\frac{ad^2}{x}+bc^2\log(x)-\frac{2bcd}{x}-\frac{bd^2}{2x^2} \\ ac^2x+4acd\sqrt{x}+ad^2\log(x)+2bc^2\sqrt{x}+2bcd\log(x)-\frac{2bd^2}{\sqrt{x}} \\ ac^2x+3acdx^{\frac{2}{3}}+3ad^2\sqrt[3]{x}+\frac{3bc^2x^{\frac{2}{3}}}{2}+6bcd\sqrt[3]{x}+bd^2\log(x) \\ \frac{6ac^2n^3x}{6n^3+11n^2+6n+1}+\frac{11ac^2n^2x}{6n^3+11n^2+6n+1}+\frac{6ac^2nx}{6n^3+11n^2+6n+1}+\frac{ac^2x}{6n^3+11n^2+6n+1}+\frac{12acdn^2xx^n}{6n^3+11n^2+6n+1}+\frac{10acdnxx^n}{6n^3+11n^2+6n+1}+\frac{2acdxx^n}{6n^3+11n^2+6n+1}+\frac{3ad^2}{6n^3+11} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)*(c+d*x**n)**2,x)
```

```
[Out] Piecewise((a*c**2*x + 2*a*c*d*log(x) - a*d**2/x + b*c**2*log(x) - 2*b*c*d/x
 - b*d**2/(2*x**2), Eq(n, -1)), (a*c**2*x + 4*a*c*d*sqrt(x) + a*d**2*log(x)
 + 2*b*c**2*sqrt(x) + 2*b*c*d*log(x) - 2*b*d**2/sqrt(x), Eq(n, -1/2)), (a*c
**2*x + 3*a*c*d*x**(2/3) + 3*a*d**2*x**(1/3) + 3*b*c**2*x**(2/3)/2 + 6*b*c*
d*x**(1/3) + b*d**2*log(x), Eq(n, -1/3)), (6*a*c**2*n**3*x/(6*n**3 + 11*n**
2 + 6*n + 1) + 11*a*c**2*n**2*x/(6*n**3 + 11*n**2 + 6*n + 1) + 6*a*c**2*n*x
/(6*n**3 + 11*n**2 + 6*n + 1) + a*c**2*x/(6*n**3 + 11*n**2 + 6*n + 1) + 12*
```

```
a*c*d*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 10*a*c*d*n*x*x**n/(6*n**3
+ 11*n**2 + 6*n + 1) + 2*a*c*d*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 3*a*d*
*2*n**2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 4*a*d**2*n*x*x**(2*n)/(6*
n**3 + 11*n**2 + 6*n + 1) + a*d**2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1)
+ 6*b*c**2*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 5*b*c**2*n*x*x**n/(6*
n**3 + 11*n**2 + 6*n + 1) + b*c**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 6*
b*c*d*n**2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 8*b*c*d*n*x*x**(2*n)/(
6*n**3 + 11*n**2 + 6*n + 1) + 2*b*c*d*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n +
1) + 2*b*d**2*n**2*x*x**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 3*b*d**2*n*x*x
**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1) + b*d**2*x*x**(3*n)/(6*n**3 + 11*n**2
+ 6*n + 1), True))
```

Giac [B] time = 1.12922, size = 313, normalized size = 4.47

$$\frac{6\,ac^2n^3x + 2\,bd^2n^2xx^{3\,n} + 6\,bcdn^2xx^{2\,n} + 3\,ad^2n^2xx^{2\,n} + 6\,bc^2n^2xx^n + 12\,acdn^2xx^n + 11\,ac^2n^2x + 3\,bd^2nxx^{3\,n} + 8\,bcdn}{6\,n^3 + 11\,}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^2,x, algorithm="giac")
```

```
[Out] (6*a*c^2*n^3*x + 2*b*d^2*n^2*x*x^(3*n) + 6*b*c*d*n^2*x*x^(2*n) + 3*a*d^2*n^
2*x*x^(2*n) + 6*b*c^2*n^2*x*x^n + 12*a*c*d*n^2*x*x^n + 11*a*c^2*n^2*x + 3*b
*d^2*n*x*x^(3*n) + 8*b*c*d*n*x*x^(2*n) + 4*a*d^2*n*x*x^(2*n) + 5*b*c^2*n*x*
x^n + 10*a*c*d*n*x*x^n + 6*a*c^2*n*x + b*d^2*x*x^(3*n) + 2*b*c*d*x*x^(2*n)
+ a*d^2*x*x^(2*n) + b*c^2*x*x^n + 2*a*c*d*x*x^n + a*c^2*x)/(6*n^3 + 11*n^2
+ 6*n + 1)
```

3.287 $\int (a+bx^n)(c+dx^n)\,dx$

Optimal. Leaf size=40

$$\frac{x^{n+1}(ad+bc)}{n+1}+acx+\frac{bdx^{2n+1}}{2n+1}$$

```
[Out] a*c*x + ((b*c + a*d)*x^(1 + n))/(1 + n) + (b*d*x^(1 + 2*n))/(1 + 2*n)
```

Rubi [A] time = 0.0209865, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {373}

$$\frac{x^{n+1}(ad+bc)}{n+1}+acx+\frac{bdx^{2n+1}}{2n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)*(c + d*x^n),x]
```

```
[Out] a*c*x + ((b*c + a*d)*x^(1 + n))/(1 + n) + (b*d*x^(1 + 2*n))/(1 + 2*n)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^n)(c+dx^n)\,dx &= \int \left(ac+(bc+ad)x^n+bdx^{2n}\right)dx\\ &= acx+\frac{(bc+ad)x^{1+n}}{1+n}+\frac{bdx^{1+2n}}{1+2n}\end{aligned}$$

Mathematica [A] time = 0.0618715, size = 37, normalized size = 0.92

$$x\left(\frac{x^n(ad+bc)}{n+1}+ac+\frac{bdx^{2n}}{2n+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)*(c + d*x^n),x]
```

```
[Out] x*(a*c + ((b*c + a*d)*x^n)/(1 + n) + (b*d*x^(2*n))/(1 + 2*n))
```

Maple [A] time = 0.006, size = 43, normalized size = 1.1

$$acx+\frac{(ad+bc)\,x\mathrm{e}^{n\ln(x)}}{1+n}+\frac{bdx\left(\mathrm{e}^{n\ln(x)}\right)^2}{1+2\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)*(c+d*x^n),x)
```

```
[Out] a*c*x+(a*d+b*c)/(1+n)*x*exp(n*ln(x))+b*d/(1+2*n)*x*exp(n*ln(x))^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.60407, size = 155, normalized size = 3.88

$$\frac{(bdn + bd)xx^{2n} + (bc + ad + 2(bc + ad)n)xx^{n} + \left(2\,acn^{2} + 3\,acn + ac\right)x}{2\,n^{2} + 3\,n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n),x, algorithm="fricas")
```

```
[Out] ((b*d*n + b*d)*x*x^(2*n) + (b*c + a*d + 2*(b*c + a*d)*n)*x*x^n + (2*a*c*n^2
 + 3*a*c*n + a*c)*x)/(2*n^2 + 3*n + 1)
```

Sympy [A] time = 0.439729, size = 236, normalized size = 5.9

$$\begin{cases} acx + ad\log{(x)} + bc\log{(x)} - \frac{bd}{x} & \text{for}\: n = -1 \\ acx + 2ad\sqrt{x} + 2bc\sqrt{x} + bd\log{(x)} & \text{for}\: n = -\frac{1}{2} \\ \frac{2acn^{2}x}{2n^{2}+3n+1} + \frac{3acnx}{2n^{2}+3n+1} + \frac{acx}{2n^{2}+3n+1} + \frac{2adnxx^{n}}{2n^{2}+3n+1} + \frac{adxx^{n}}{2n^{2}+3n+1} + \frac{2bcnxx^{n}}{2n^{2}+3n+1} + \frac{bcxx^{n}}{2n^{2}+3n+1} + \frac{bdnxx^{2n}}{2n^{2}+3n+1} + \frac{bdxx^{2n}}{2n^{2}+3n+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)*(c+d*x**n),x)
```

```
[Out] Piecewise((a*c*x + a*d*log(x) + b*c*log(x) - b*d/x, Eq(n, -1)), (a*c*x + 2*
a*d*sqrt(x) + 2*b*c*sqrt(x) + b*d*log(x), Eq(n, -1/2)), (2*a*c*n**2*x/(2*n*
*2 + 3*n + 1) + 3*a*c*n*x/(2*n**2 + 3*n + 1) + a*c*x/(2*n**2 + 3*n + 1) + 2
*a*d*n*x*x**n/(2*n**2 + 3*n + 1) + a*d*x*x**n/(2*n**2 + 3*n + 1) + 2*b*c*n*
x*x**n/(2*n**2 + 3*n + 1) + b*c*x*x**n/(2*n**2 + 3*n + 1) + b*d*n*x*x**(2*n
)/(2*n**2 + 3*n + 1) + b*d*x*x**(2*n)/(2*n**2 + 3*n + 1), True))
```

Giac [B] time = 1.09448, size = 112, normalized size = 2.8

$$\frac{2\,acn^{2}x + bdnxx^{2n} + 2\,bcnxx^{n} + 2\,adnxx^{n} + 3\,acnx + bdxx^{2n} + bcxx^{n} + adxx^{n} + acx}{2\,n^{2} + 3\,n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n),x, algorithm="giac")
```

```
[Out] (2*a*c*n^2*x + b*d*n*x*x^(2*n) + 2*b*c*n*x*x^n + 2*a*d*n*x*x^n + 3*a*c*n*x
+ b*d*x*x^(2*n) + b*c*x*x^n + a*d*x*x^n + a*c*x)/(2*n^2 + 3*n + 1)
```

3.288 $\int \frac{a+bx^n}{c+dx^n}\,dx$

Optimal. Leaf size=43

$$\frac{bx}{d}-\frac{x(bc-ad)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{cd}$$

```
[Out] (b*x)/d - ((b*c - a*d)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)
/c)])/(c*d)
```

Rubi [A] time = 0.0170337, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {388, 245}

$$\frac{bx}{d}-\frac{x(bc-ad)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{cd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)/(c + d*x^n),x]
```

```
[Out] (b*x)/d - ((b*c - a*d)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)
/c)])/(c*d)
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^n}{c+dx^n}\,dx&=\frac{bx}{d}-\frac{(bc-ad)\int \frac{1}{c+dx^n}\,dx}{d}\\&=\frac{bx}{d}-\frac{(bc-ad)x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{cd}\end{aligned}$$

Mathematica [A] time = 0.0117517, size = 40, normalized size = 0.93

$$\frac{x\left((ad-bc)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)+bc\right)}{cd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)/(c + d*x^n),x]
```

```
[Out] (x*(b*c + (-(b*c) + a*d)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)
/c)]))/(c*d)
```

Maple [F] time = 0.407, size = 0, normalized size = 0.

$$\int \frac{a+bx^n}{c+dx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)/(c+d*x^n),x)
```

```
[Out] int((a+b*x^n)/(c+d*x^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-(bc-ad)\int \frac{1}{d^2x^n+cd}\,dx+\frac{bx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] -(b*c - a*d)*integrate(1/(d^2*x^n + c*d), x) + b*x/d
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{bx^n+a}{dx^n+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)/(d*x^n + c), x)
```

Sympy [C] time = 1.74438, size = 73, normalized size = 1.7

$$\frac{ax\Phi\left(\frac{dx^ne^{i\pi}}{c},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{cn^2\Gamma\left(1+\frac{1}{n}\right)}-\frac{bx\Phi\left(\frac{cx^{-n}e^{i\pi}}{d},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{dn^2\Gamma\left(1+\frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)/(c+d*x**n),x)
```

```
[Out] a*x*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1/n)*gamma(1/n)/(c*n**2*gamma(1 +
 1/n)) - b*x*lerchphi(c*x**(-n)*exp_polar(I*pi)/d, 1, exp_polar(I*pi)/n)*ga
```

```
mma(1/n)/(d*n**2*gamma(1 + 1/n))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^n + a}{dx^n + c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)/(d*x^n + c), x)
```

3.289 $\int \frac{a+bx^n}{(c+dx^n)^2} \, dx$

Optimal. Leaf size=73

$$\frac{x(bc-ad(1-n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2dn}-\frac{x(bc-ad)}{cdn\,(c+dx^n)}$$

```
[Out] -(((b*c - a*d)*x)/(c*d*n*(c + d*x^n))) + ((b*c - a*d*(1 - n))*x*Hypergeomet
ric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c^2*d*n)
```

Rubi [A] time = 0.0308483, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {385, 245}

$$\frac{x(bc-ad(1-n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2dn}-\frac{x(bc-ad)}{cdn\,(c+dx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)/(c + d*x^n)^2,x]
```

```
[Out] -(((b*c - a*d)*x)/(c*d*n*(c + d*x^n))) + ((b*c - a*d*(1 - n))*x*Hypergeomet
ric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c^2*d*n)
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^n}{(c+dx^n)^2}\,dx &= -\frac{(bc-ad)x}{cdn\,(c+dx^n)}+\frac{(bc-ad(1-n))\int \frac{1}{c+dx^n}\,dx}{cdn}\\ &= -\frac{(bc-ad)x}{cdn\,(c+dx^n)}+\frac{(bc-ad(1-n))x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2dn}\end{aligned}$$

Mathematica [A] time = 0.0366805, size = 56, normalized size = 0.77

$$\frac{x\left(\frac{b}{c+dx^n}-\frac{(ad(n-1)+bc)\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2}\right)}{d-dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)/(c + d*x^n)^2,x]
```

```
[Out] (x*(b/(c + d*x^n) - ((b*c + a*d*(-1 + n))*Hypergeometric2F1[2, n^(-1), 1 +
n^(-1), -((d*x^n)/c)])/c^2))/(d - d*n)
```

Maple [F] time = 0.36, size = 0, normalized size = 0.

$$\int \frac{a + bx^n}{(c + dx^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)/(c+d*x^n)^2,x)
```

```
[Out] int((a+b*x^n)/(c+d*x^n)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$(ad(n-1) + bc)\int \frac{1}{cd^2nx^n + c^2dn}\,dx - \frac{(bc - ad)x}{cd^2nx^n + c^2dn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)/(c+d*x^n)^2,x, algorithm="maxima")
```

```
[Out] (a*d*(n - 1) + b*c)*integrate(1/(c*d^2*n*x^n + c^2*d*n), x) - (b*c - a*d)*x
/(c*d^2*n*x^n + c^2*d*n)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^n + a}{d^2x^{2n} + 2\,cdx^n + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)/(c+d*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)/(d^2*x^(2*n) + 2*c*d*x^n + c^2), x)
```

Sympy [C] time = 4.6078, size = 592, normalized size = 8.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)/(c+d*x**n)**2,x)
```

```
[Out] a*(n*x*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1/n)*gamma(1/n)/(c*(c*n**3*gam
ma(1 + 1/n) + d*n**3*x**n*gamma(1 + 1/n))) + n*x*gamma(1/n)/(c*(c*n**3*gamm
a(1 + 1/n) + d*n**3*x**n*gamma(1 + 1/n))) - x*lerchphi(d*x**n*exp_polar(I*p
i)/c, 1, 1/n)*gamma(1/n)/(c*(c*n**3*gamma(1 + 1/n) + d*n**3*x**n*gamma(1 +
1/n))) + d*n*x*x**n*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1/n)*gamma(1/n)/(
c**2*(c*n**3*gamma(1 + 1/n) + d*n**3*x**n*gamma(1 + 1/n))) - d*x*x**n*lerch
phi(d*x**n*exp_polar(I*pi)/c, 1, 1/n)*gamma(1/n)/(c**2*(c*n**3*gamma(1 + 1/
n) + d*n**3*x**n*gamma(1 + 1/n)))) + b*(n**2*x*x**n*gamma(1 + 1/n)/(c*(c*n*
*3*gamma(2 + 1/n) + d*n**3*x**n*gamma(2 + 1/n))) - n*x*x**n*lerchphi(d*x**n
*exp_polar(I*pi)/c, 1, 1 + 1/n)*gamma(1 + 1/n)/(c*(c*n**3*gamma(2 + 1/n) +
d*n**3*x**n*gamma(2 + 1/n))) + n*x*x**n*gamma(1 + 1/n)/(c*(c*n**3*gamma(2 +
 1/n) + d*n**3*x**n*gamma(2 + 1/n))) - x*x**n*lerchphi(d*x**n*exp_polar(I*p
i)/c, 1, 1 + 1/n)*gamma(1 + 1/n)/(c*(c*n**3*gamma(2 + 1/n) + d*n**3*x**n*ga
mma(2 + 1/n))) - d*n*x*x**(2*n)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1 + 1
/n)*gamma(1 + 1/n)/(c**2*(c*n**3*gamma(2 + 1/n) + d*n**3*x**n*gamma(2 + 1/n
))) - d*x*x**(2*n)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1 + 1/n)*gamma(1 +
 1/n)/(c**2*(c*n**3*gamma(2 + 1/n) + d*n**3*x**n*gamma(2 + 1/n))))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^n + a}{(dx^n + c)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)/(c+d*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)/(d*x^n + c)^2, x)
```

3.290 $\int \frac{a+bx^n}{(c+dx^n)^3} \, dx$

Optimal. Leaf size=78

$$\frac{x(bc-ad(1-2n))\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{2c^3dn}-\frac{x(bc-ad)}{2cdn\,(c+dx^n)^2}$$

```
[Out] -((b*c - a*d)*x)/(2*c*d*n*(c + d*x^n)^2) + ((b*c - a*d*(1 - 2*n))*x*Hyperge
ometric2F1[2, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(2*c^3*d*n)
```

Rubi [A] time = 0.0310969, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {385, 245}

$$\frac{x(bc-ad(1-2n))\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{2c^3dn}-\frac{x(bc-ad)}{2cdn\,(c+dx^n)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)/(c + d*x^n)^3,x]
```

```
[Out] -((b*c - a*d)*x)/(2*c*d*n*(c + d*x^n)^2) + ((b*c - a*d*(1 - 2*n))*x*Hyperge
ometric2F1[2, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(2*c^3*d*n)
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^n}{(c+dx^n)^3}\,dx &= -\frac{(bc-ad)x}{2cdn\,(c+dx^n)^2}+\frac{(bc-ad(1-2n))\int \frac{1}{(c+dx^n)^2}\,dx}{2cdn}\\ &= -\frac{(bc-ad)x}{2cdn\,(c+dx^n)^2}+\frac{(bc-ad(1-2n))x\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{2c^3dn}\end{aligned}$$

Mathematica [A] time = 0.0421899, size = 58, normalized size = 0.74

$$\frac{x\left(\frac{b}{(c+dx^n)^2}-\frac{(ad(2n-1)+bc)\,{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^3}\right)}{d-2dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)/(c + d*x^n)^3,x]
```

```
[Out] (x*(b/(c + d*x^n)^2 - ((b*c + a*d*(-1 + 2*n))*Hypergeometric2F1[3, n^(-1),
1 + n^(-1), -((d*x^n)/c)])/c^3))/(d - 2*d*n)
```

Maple [F] time = 0.373, size = 0, normalized size = 0.

$$\int \frac{a + bx^n}{(c + dx^n)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)/(c+d*x^n)^3,x)
```

```
[Out] int((a+b*x^n)/(c+d*x^n)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\left(\left(2\,n^2 - 3\,n + 1\right)ad + bc(n-1)\right)\int \frac{1}{2\left(c^2d^2n^2x^n + c^3dn^2\right)}\,dx + \frac{\left(ad^2(2\,n-1) + bcd\right)xx^n + \left(acd(3\,n-1) - bc^2(n-1)\right)x}{2\left(c^2d^3n^2x^{2\,n} + 2\,c^3d^2n^2x^n + c^4dn^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)/(c+d*x^n)^3,x, algorithm="maxima")
```

```
[Out] ((2*n^2 - 3*n + 1)*a*d + b*c*(n - 1))*integrate(1/2/(c^2*d^2*n^2*x^n + c^3*
d*n^2), x) + 1/2*((a*d^2*(2*n - 1) + b*c*d)*x*x^n + (a*c*d*(3*n - 1) - b*c^
2*(n - 1))*x)/(c^2*d^3*n^2*x^(2*n) + 2*c^3*d^2*n^2*x^n + c^4*d*n^2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{bx^n + a}{d^3x^{3\,n} + 3\,cd^2x^{2\,n} + 3\,c^2dx^n + c^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)/(c+d*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)/(d^3*x^(3*n) + 3*c*d^2*x^(2*n) + 3*c^2*d*x^n + c^3), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)/(c+d*x**n)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^n + a}{(dx^n + c)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)/(c+d*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)/(d*x^n + c)^3, x)
```

3.291 $\int \frac{a+bx^n}{(c+dx^n)^4}\,dx$

Optimal. Leaf size=78

$$\frac{x(bc-ad(1-3n))\,{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{3c^4dn}-\frac{x(bc-ad)}{3cdn\,(c+dx^n)^3}$$

```
[Out] -((b*c - a*d)*x)/(3*c*d*n*(c + d*x^n)^3) + ((b*c - a*d*(1 - 3*n))*x*Hyperge
ometric2F1[3, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(3*c^4*d*n)
```

Rubi [A] time = 0.0307568, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {385, 245}

$$\frac{x(bc-ad(1-3n))\,{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{3c^4dn}-\frac{x(bc-ad)}{3cdn\,(c+dx^n)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)/(c + d*x^n)^4,x]
```

```
[Out] -((b*c - a*d)*x)/(3*c*d*n*(c + d*x^n)^3) + ((b*c - a*d*(1 - 3*n))*x*Hyperge
ometric2F1[3, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(3*c^4*d*n)
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^n}{(c+dx^n)^4}\,dx &= -\frac{(bc-ad)x}{3cdn\,(c+dx^n)^3}+\frac{(bc-ad(1-3n))\int \frac{1}{(c+dx^n)^3}\,dx}{3cdn}\\ &= -\frac{(bc-ad)x}{3cdn\,(c+dx^n)^3}+\frac{(bc-ad(1-3n))x\,{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{3c^4dn}\end{aligned}$$

Mathematica [A] time = 0.0486125, size = 58, normalized size = 0.74

$$\frac{x\left(\frac{b}{(c+dx^n)^3}-\frac{(ad(3n-1)+bc)\,{}_2F_1\left(4,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^4}\right)}{d-3dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)/(c + d*x^n)^4,x]
```

```
[Out] (x*(b/(c + d*x^n)^3 - ((b*c + a*d*(-1 + 3*n))*Hypergeometric2F1[4, n^(-1), 
1 + n^(-1), -((d*x^n)/c)])/c^4))/(d - 3*d*n)
```

Maple [F] time = 0.373, size = 0, normalized size = 0.

$$\int \frac{a + bx^n}{(c + dx^n)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)/(c+d*x^n)^4,x)
```

```
[Out] int((a+b*x^n)/(c+d*x^n)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\left(\left(2\,n^2 - 3\,n + 1\right)bc + \left(6\,n^3 - 11\,n^2 + 6\,n - 1\right)ad\right)\int \frac{1}{6\left(c^3d^2n^3x^n + c^4dn^3\right)}\, dx + \frac{\left(\left(6\,n^2 - 5\,n + 1\right)ad^3 + bcd^2(2\,n - 1)\right)x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)/(c+d*x^n)^4,x, algorithm="maxima")
```

```
[Out] ((2*n^2 - 3*n + 1)*b*c + (6*n^3 - 11*n^2 + 6*n - 1)*a*d)*integrate(1/6/(c^3
*d^2*n^3*x^n + c^4*d*n^3), x) + 1/6*(((6*n^2 - 5*n + 1)*a*d^3 + b*c*d^2*(2*
n - 1))*x*x^(2*n) + ((15*n^2 - 11*n + 2)*a*c*d^2 + b*c^2*d*(5*n - 2))*x*x^n
 - ((2*n^2 - 3*n + 1)*b*c^3 - (11*n^2 - 6*n + 1)*a*c^2*d)*x)/(c^3*d^4*n^3*x
^(3*n) + 3*c^4*d^3*n^3*x^(2*n) + 3*c^5*d^2*n^3*x^n + c^6*d*n^3)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^n + a}{d^4x^{4\,n} + 4\,cd^3x^{3\,n} + 6\,c^2d^2x^{2\,n} + 4\,c^3dx^n + c^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)/(c+d*x^n)^4,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)/(d^4*x^(4*n) + 4*c*d^3*x^(3*n) + 6*c^2*d^2*x^(2*n) + 4
*c^3*d*x^n + c^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)/(c+d*x**n)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{bx^n + a}{(dx^n + c)^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)/(c+d*x^n)^4,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)/(d*x^n + c)^4, x)
```

3.292 $\int (a+bx^n)^2 (d+ex^n)^3 \, dx$

Optimal. Leaf size=158

$$\frac{dx^{2n+1}\left(3a^2e^2+6abde+b^2d^2\right)}{2n+1}+\frac{ex^{3n+1}\left(a^2e^2+6abde+3b^2d^2\right)}{3n+1}+a^2d^3x+\frac{ad^2x^{n+1}(3ae+2bd)}{n+1}+\frac{be^2x^{4n+1}(2ae+3bd}{4n+1}$$

```
[Out] a^2*d^3*x + (a*d^2*(2*b*d + 3*a*e)*x^(1 + n))/(1 + n) + (d*(b^2*d^2 + 6*a*b
*d*e + 3*a^2*e^2)*x^(1 + 2*n))/(1 + 2*n) + (e*(3*b^2*d^2 + 6*a*b*d*e + a^2*
e^2)*x^(1 + 3*n))/(1 + 3*n) + (b*e^2*(3*b*d + 2*a*e)*x^(1 + 4*n))/(1 + 4*n)
 + (b^2*e^3*x^(1 + 5*n))/(1 + 5*n)
```

Rubi [A] time = 0.12993, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {373}

$$\frac{dx^{2n+1}\left(3a^2e^2+6abde+b^2d^2\right)}{2n+1}+\frac{ex^{3n+1}\left(a^2e^2+6abde+3b^2d^2\right)}{3n+1}+a^2d^3x+\frac{ad^2x^{n+1}(3ae+2bd)}{n+1}+\frac{be^2x^{4n+1}(2ae+3bd}{4n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^2*(d + e*x^n)^3,x]
```

```
[Out] a^2*d^3*x + (a*d^2*(2*b*d + 3*a*e)*x^(1 + n))/(1 + n) + (d*(b^2*d^2 + 6*a*b
*d*e + 3*a^2*e^2)*x^(1 + 2*n))/(1 + 2*n) + (e*(3*b^2*d^2 + 6*a*b*d*e + a^2*
e^2)*x^(1 + 3*n))/(1 + 3*n) + (b*e^2*(3*b*d + 2*a*e)*x^(1 + 4*n))/(1 + 4*n)
 + (b^2*e^3*x^(1 + 5*n))/(1 + 5*n)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int (a+bx^n)^2 (d+ex^n)^3 \, dx = \int \left(a^2d^3+ad^2(2bd+3ae)x^n+d\left(b^2d^2+6abde+3a^2e^2\right)x^{2n}+e\left(3b^2d^2+6abde+a^2e^2\right)\right.$$

$$= a^2d^3x+\frac{ad^2(2bd+3ae)x^{1+n}}{1+n}+\frac{d\left(b^2d^2+6abde+3a^2e^2\right)x^{1+2n}}{1+2n}+\frac{e\left(3b^2d^2+6abde+a^2e\right.}{1+3n}$$

Mathematica [A] time = 0.158896, size = 149, normalized size = 0.94

$$x\left(\frac{dx^{2n}\left(3a^2e^2+6abde+b^2d^2\right)}{2n+1}+\frac{ex^{3n}\left(a^2e^2+6abde+3b^2d^2\right)}{3n+1}+a^2d^3+\frac{ad^2x^n(3ae+2bd)}{n+1}+\frac{be^2x^{4n}(2ae+3bd)}{4n+1}+\frac{b^2e}{5n}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^2*(d + e*x^n)^3,x]
```

```
[Out] x*(a^2*d^3 + (a*d^2*(2*b*d + 3*a*e)*x^n)/(1 + n) + (d*(b^2*d^2 + 6*a*b*d*e
+ 3*a^2*e^2)*x^(2*n))/(1 + 2*n) + (e*(3*b^2*d^2 + 6*a*b*d*e + a^2*e^2)*x^(3
*n))/(1 + 3*n) + (b*e^2*(3*b*d + 2*a*e)*x^(4*n))/(1 + 4*n) + (b^2*e^3*x^(5*
```

```
n))/(1 + 5*n))
```

Maple [A] time = 0.055, size = 164, normalized size = 1.

$$a^2d^3x + \frac{b^2e^3x\left(e^{n\ln(x)}\right)^5}{1+5n} + \frac{d\left(3a^2e^2+6abde+b^2d^2\right)x\left(e^{n\ln(x)}\right)^2}{1+2n} + \frac{e\left(a^2e^2+6abde+3b^2d^2\right)x\left(e^{n\ln(x)}\right)^3}{1+3n} + \frac{ad^2(3ae+2}{1+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^2*(d+e*x^n)^3,x)
```

```
[Out] a^2*d^3*x+b^2*e^3/(1+5*n)*x*exp(n*ln(x))^5+d*(3*a^2*e^2+6*a*b*d*e+b^2*d^2)/
(1+2*n)*x*exp(n*ln(x))^2+e*(a^2*e^2+6*a*b*d*e+3*b^2*d^2)/(1+3*n)*x*exp(n*ln
(x))^3+a*d^2*(3*a*e+2*b*d)/(1+n)*x*exp(n*ln(x))+b*e^2*(2*a*e+3*b*d)/(1+4*n)
*x*exp(n*ln(x))^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(d+e*x^n)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.70087, size = 1434, normalized size = 9.08

$$\frac{\left(24b^2e^3n^4+50b^2e^3n^3+35b^2e^3n^2+10b^2e^3n+b^2e^3\right)xx^{5n}+\left(3b^2de^2+2abe^3+30\left(3b^2de^2+2abe^3\right)n^4+61\left(3b^2de^2+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(d+e*x^n)^3,x, algorithm="fricas")
```

```
[Out] ((24*b^2*e^3*n^4 + 50*b^2*e^3*n^3 + 35*b^2*e^3*n^2 + 10*b^2*e^3*n + b^2*e^3
)*x*x^(5*n) + (3*b^2*d*e^2 + 2*a*b*e^3 + 30*(3*b^2*d*e^2 + 2*a*b*e^3)*n^4 +
 61*(3*b^2*d*e^2 + 2*a*b*e^3)*n^3 + 41*(3*b^2*d*e^2 + 2*a*b*e^3)*n^2 + 11*(
3*b^2*d*e^2 + 2*a*b*e^3)*n)*x*x^(4*n) + (3*b^2*d^2*e + 6*a*b*d*e^2 + a^2*e^
3 + 40*(3*b^2*d^2*e + 6*a*b*d*e^2 + a^2*e^3)*n^4 + 78*(3*b^2*d^2*e + 6*a*b*
d*e^2 + a^2*e^3)*n^3 + 49*(3*b^2*d^2*e + 6*a*b*d*e^2 + a^2*e^3)*n^2 + 12*(3
*b^2*d^2*e + 6*a*b*d*e^2 + a^2*e^3)*n)*x*x^(3*n) + (b^2*d^3 + 6*a*b*d^2*e +
 3*a^2*d*e^2 + 60*(b^2*d^3 + 6*a*b*d^2*e + 3*a^2*d*e^2)*n^4 + 107*(b^2*d^3
+ 6*a*b*d^2*e + 3*a^2*d*e^2)*n^3 + 59*(b^2*d^3 + 6*a*b*d^2*e + 3*a^2*d*e^2)
*n^2 + 13*(b^2*d^3 + 6*a*b*d^2*e + 3*a^2*d*e^2)*n)*x*x^(2*n) + (2*a*b*d^3 +
 3*a^2*d^2*e + 120*(2*a*b*d^3 + 3*a^2*d^2*e)*n^4 + 154*(2*a*b*d^3 + 3*a^2*d
^2*e)*n^3 + 71*(2*a*b*d^3 + 3*a^2*d^2*e)*n^2 + 14*(2*a*b*d^3 + 3*a^2*d^2*e)
*n)*x*x^n + (120*a^2*d^3*n^5 + 274*a^2*d^3*n^4 + 225*a^2*d^3*n^3 + 85*a^2*d
^3*n^2 + 15*a^2*d^3*n + a^2*d^3)*x)/(120*n^5 + 274*n^4 + 225*n^3 + 85*n^2 +
 15*n + 1)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**2*(d+e*x**n)**3,x)
```

```
[Out] Exception raised: TypeError
```

Giac [B] time = 1.64841, size = 1278, normalized size = 8.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(d+e*x^n)^3,x, algorithm="giac")
```

```
[Out] (120*a^2*d^3*n^5*x + 60*b^2*d^3*n^4*x*x^(2*n) + 240*a*b*d^3*n^4*x*x^n + 120
*b^2*d^2*n^4*x*x^(3*n)*e + 360*a*b*d^2*n^4*x*x^(2*n)*e + 360*a^2*d^2*n^4*x*
x^n*e + 274*a^2*d^3*n^4*x + 107*b^2*d^3*n^3*x*x^(2*n) + 308*a*b*d^3*n^3*x*x
^n + 90*b^2*d*n^4*x*x^(4*n)*e^2 + 240*a*b*d*n^4*x*x^(3*n)*e^2 + 180*a^2*d*n
^4*x*x^(2*n)*e^2 + 234*b^2*d^2*n^3*x*x^(3*n)*e + 642*a*b*d^2*n^3*x*x^(2*n)*
e + 462*a^2*d^2*n^3*x*x^n*e + 225*a^2*d^3*n^3*x + 59*b^2*d^3*n^2*x*x^(2*n)
+ 142*a*b*d^3*n^2*x*x^n + 24*b^2*n^4*x*x^(5*n)*e^3 + 60*a*b*n^4*x*x^(4*n)*e
^3 + 40*a^2*n^4*x*x^(3*n)*e^3 + 183*b^2*d*n^3*x*x^(4*n)*e^2 + 468*a*b*d*n^3
*x*x^(3*n)*e^2 + 321*a^2*d*n^3*x*x^(2*n)*e^2 + 147*b^2*d^2*n^2*x*x^(3*n)*e
+ 354*a*b*d^2*n^2*x*x^(2*n)*e + 213*a^2*d^2*n^2*x*x^n*e + 85*a^2*d^3*n^2*x
+ 13*b^2*d^3*n*x*x^(2*n) + 28*a*b*d^3*n*x*x^n + 50*b^2*n^3*x*x^(5*n)*e^3 +
122*a*b*n^3*x*x^(4*n)*e^3 + 78*a^2*n^3*x*x^(3*n)*e^3 + 123*b^2*d*n^2*x*x^(4
*n)*e^2 + 294*a*b*d*n^2*x*x^(3*n)*e^2 + 177*a^2*d*n^2*x*x^(2*n)*e^2 + 36*b^
2*d^2*n*x*x^(3*n)*e + 78*a*b*d^2*n*x*x^(2*n)*e + 42*a^2*d^2*n*x*x^n*e + 15*
a^2*d^3*n*x + b^2*d^3*x*x^(2*n) + 2*a*b*d^3*x*x^n + 35*b^2*n^2*x*x^(5*n)*e^
3 + 82*a*b*n^2*x*x^(4*n)*e^3 + 49*a^2*n^2*x*x^(3*n)*e^3 + 33*b^2*d*n*x*x^(4
*n)*e^2 + 72*a*b*d*n*x*x^(3*n)*e^2 + 39*a^2*d*n*x*x^(2*n)*e^2 + 3*b^2*d^2*x
*x^(3*n)*e + 6*a*b*d^2*x*x^(2*n)*e + 3*a^2*d^2*x*x^n*e + a^2*d^3*x + 10*b^2
*n*x*x^(5*n)*e^3 + 22*a*b*n*x*x^(4*n)*e^3 + 12*a^2*n*x*x^(3*n)*e^3 + 3*b^2*
d*x*x^(4*n)*e^2 + 6*a*b*d*x*x^(3*n)*e^2 + 3*a^2*d*x*x^(2*n)*e^2 + b^2*x*x^(
5*n)*e^3 + 2*a*b*x*x^(4*n)*e^3 + a^2*x*x^(3*n)*e^3)/(120*n^5 + 274*n^4 + 22
5*n^3 + 85*n^2 + 15*n + 1)
```

3.293 $\int (a+bx^n)^2 (d+ex^n)^2 \, dx$

Optimal. Leaf size=112

$$\frac{x^{2n+1}\left(a^2e^2+4abde+b^2d^2\right)}{2n+1}+a^2d^2x+\frac{2adx^{n+1}(ae+bd)}{n+1}+\frac{2bex^{3n+1}(ae+bd)}{3n+1}+\frac{b^2e^2x^{4n+1}}{4n+1}$$

```
[Out] a^2*d^2*x + (2*a*d*(b*d + a*e)*x^(1 + n))/(1 + n) + ((b^2*d^2 + 4*a*b*d*e +
 a^2*e^2)*x^(1 + 2*n))/(1 + 2*n) + (2*b*e*(b*d + a*e)*x^(1 + 3*n))/(1 + 3*n
) + (b^2*e^2*x^(1 + 4*n))/(1 + 4*n)
```

Rubi [A] time = 0.0816809, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {373}

$$\frac{x^{2n+1}\left(a^2e^2+4abde+b^2d^2\right)}{2n+1}+a^2d^2x+\frac{2adx^{n+1}(ae+bd)}{n+1}+\frac{2bex^{3n+1}(ae+bd)}{3n+1}+\frac{b^2e^2x^{4n+1}}{4n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^2*(d + e*x^n)^2,x]
```

```
[Out] a^2*d^2*x + (2*a*d*(b*d + a*e)*x^(1 + n))/(1 + n) + ((b^2*d^2 + 4*a*b*d*e +
 a^2*e^2)*x^(1 + 2*n))/(1 + 2*n) + (2*b*e*(b*d + a*e)*x^(1 + 3*n))/(1 + 3*n
) + (b^2*e^2*x^(1 + 4*n))/(1 + 4*n)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^n)^2 (d+ex^n)^2 \, dx &= \int \left(a^2d^2+2ad(bd+ae)x^n+\left(b^2d^2+4abde+a^2e^2\right)x^{2n}+2be(bd+ae)x^{3n}+b^2e^2x^{4n}\right) dx\\ &= a^2d^2x+\frac{2ad(bd+ae)x^{1+n}}{1+n}+\frac{\left(b^2d^2+4abde+a^2e^2\right)x^{1+2n}}{1+2n}+\frac{2be(bd+ae)x^{1+3n}}{1+3n}+\frac{b^2e^2x^{1+4n}}{1+4n}\end{aligned}$$

Mathematica [A] time = 0.157799, size = 105, normalized size = 0.94

$$x\left(\frac{x^{2n}\left(a^2e^2+4abde+b^2d^2\right)}{2n+1}+a^2d^2+\frac{2adx^n(ae+bd)}{n+1}+\frac{2bex^{3n}(ae+bd)}{3n+1}+\frac{b^2e^2x^{4n}}{4n+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^2*(d + e*x^n)^2,x]
```

```
[Out] x*(a^2*d^2 + (2*a*d*(b*d + a*e)*x^n)/(1 + n) + ((b^2*d^2 + 4*a*b*d*e + a^2*
e^2)*x^(2*n))/(1 + 2*n) + (2*b*e*(b*d + a*e)*x^(3*n))/(1 + 3*n) + (b^2*e^2*
x^(4*n))/(1 + 4*n))
```

Maple [A] time = 0.01, size = 117, normalized size = 1.

$$a^2d^2x + \frac{(a^2e^2 + 4\,abde + b^2d^2)\,x\left(e^{n\ln(x)}\right)^2}{1 + 2\,n} + \frac{b^2e^2x\left(e^{n\ln(x)}\right)^4}{1 + 4\,n} + 2\,\frac{ad\,(ae + bd)\,xe^{n\ln(x)}}{1 + n} + 2\,\frac{be\,(ae + bd)\,x\left(e^{n\ln(x)}\right)^3}{1 + 3\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^2*(d+e*x^n)^2,x)
```

```
[Out] a^2*d^2*x+(a^2*e^2+4*a*b*d*e+b^2*d^2)/(1+2*n)*x*exp(n*ln(x))^2+b^2*e^2/(1+4
*n)*x*exp(n*ln(x))^4+2*a*d*(a*e+b*d)/(1+n)*x*exp(n*ln(x))+2*b*e*(a*e+b*d)/(
1+3*n)*x*exp(n*ln(x))^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(d+e*x^n)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.64533, size = 787, normalized size = 7.03

$$\frac{\left(6\,b^2e^2n^3 + 11\,b^2e^2n^2 + 6\,b^2e^2n + b^2e^2\right)xx^{4\,n} + 2\left(b^2de + abe^2 + 8\left(b^2de + abe^2\right)n^3 + 14\left(b^2de + abe^2\right)n^2 + 7\left(b^2de + ab\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(d+e*x^n)^2,x, algorithm="fricas")
```

```
[Out] ((6*b^2*e^2*n^3 + 11*b^2*e^2*n^2 + 6*b^2*e^2*n + b^2*e^2)*x*x^(4*n) + 2*(b^
2*d*e + a*b*e^2 + 8*(b^2*d*e + a*b*e^2)*n^3 + 14*(b^2*d*e + a*b*e^2)*n^2 +
7*(b^2*d*e + a*b*e^2)*n)*x*x^(3*n) + (b^2*d^2 + 4*a*b*d*e + a^2*e^2 + 12*(b
^2*d^2 + 4*a*b*d*e + a^2*e^2)*n^3 + 19*(b^2*d^2 + 4*a*b*d*e + a^2*e^2)*n^2
+ 8*(b^2*d^2 + 4*a*b*d*e + a^2*e^2)*n)*x*x^(2*n) + 2*(a*b*d^2 + a^2*d*e + 2
4*(a*b*d^2 + a^2*d*e)*n^3 + 26*(a*b*d^2 + a^2*d*e)*n^2 + 9*(a*b*d^2 + a^2*d
*e)*n)*x*x^n + (24*a^2*d^2*n^4 + 50*a^2*d^2*n^3 + 35*a^2*d^2*n^2 + 10*a^2*d
^2*n + a^2*d^2)*x)/(24*n^4 + 50*n^3 + 35*n^2 + 10*n + 1)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**2*(d+e*x**n)**2,x)
```

```
[Out] Exception raised: TypeError
```

Giac [B] time = 1.13428, size = 728, normalized size = 6.5

$$\frac{24\,a^2d^2n^4x + 12\,b^2d^2n^3xx^{2\,n} + 48\,abd^2n^3xx^n + 16\,b^2dn^3xx^{3\,n}e + 48\,abdn^3xx^{2\,n}e + 48\,a^2dn^3xx^ne + 50\,a^2d^2n^3x + 19\,b^2d^2n^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(d+e*x^n)^2,x, algorithm="giac")
```

```
[Out] (24*a^2*d^2*n^4*x + 12*b^2*d^2*n^3*x*x^(2*n) + 48*a*b*d^2*n^3*x*x^n + 16*b^
2*d*n^3*x*x^(3*n)*e + 48*a*b*d*n^3*x*x^(2*n)*e + 48*a^2*d*n^3*x*x^n*e + 50*
a^2*d^2*n^3*x + 19*b^2*d^2*n^2*x*x^(2*n) + 52*a*b*d^2*n^2*x*x^n + 6*b^2*n^3
*x*x^(4*n)*e^2 + 16*a*b*n^3*x*x^(3*n)*e^2 + 12*a^2*n^3*x*x^(2*n)*e^2 + 28*b
^2*d*n^2*x*x^(3*n)*e + 76*a*b*d*n^2*x*x^(2*n)*e + 52*a^2*d*n^2*x*x^n*e + 35
*a^2*d^2*n^2*x + 8*b^2*d^2*n*x*x^(2*n) + 18*a*b*d^2*n*x*x^n + 11*b^2*n^2*x*
x^(4*n)*e^2 + 28*a*b*n^2*x*x^(3*n)*e^2 + 19*a^2*n^2*x*x^(2*n)*e^2 + 14*b^2*
d*n*x*x^(3*n)*e + 32*a*b*d*n*x*x^(2*n)*e + 18*a^2*d*n*x*x^n*e + 10*a^2*d^2*
n*x + b^2*d^2*x*x^(2*n) + 2*a*b*d^2*x*x^n + 6*b^2*n*x*x^(4*n)*e^2 + 14*a*b*
n*x*x^(3*n)*e^2 + 8*a^2*n*x*x^(2*n)*e^2 + 2*b^2*d*x*x^(3*n)*e + 4*a*b*d*x*x
^(2*n)*e + 2*a^2*d*x*x^n*e + a^2*d^2*x + b^2*x*x^(4*n)*e^2 + 2*a*b*x*x^(3*n
)*e^2 + a^2*x*x^(2*n)*e^2)/(24*n^4 + 50*n^3 + 35*n^2 + 10*n + 1)
```

3.294 $\int (a+bx^n)^2 (c+dx^n)\, dx$

Optimal. Leaf size=70

$$a^2cx+\frac{ax^{n+1}(ad+2bc)}{n+1}+\frac{bx^{2n+1}(2ad+bc)}{2n+1}+\frac{b^2dx^{3n+1}}{3n+1}$$

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^(1 + n))/(1 + n) + (b*(b*c + 2*a*d)*x^(1 + 2*n
))/(1 + 2*n) + (b^2*d*x^(1 + 3*n))/(1 + 3*n)
```

Rubi [A] time = 0.0471064, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {373}

$$a^2cx+\frac{ax^{n+1}(ad+2bc)}{n+1}+\frac{bx^{2n+1}(2ad+bc)}{2n+1}+\frac{b^2dx^{3n+1}}{3n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^2*(c + d*x^n),x]
```

```
[Out] a^2*c*x + (a*(2*b*c + a*d)*x^(1 + n))/(1 + n) + (b*(b*c + 2*a*d)*x^(1 + 2*n
))/(1 + 2*n) + (b^2*d*x^(1 + 3*n))/(1 + 3*n)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^n)^2(c+dx^n)\, dx &= \int \left(a^2c+a(2bc+ad)x^n+b(bc+2ad)x^{2n}+b^2dx^{3n}\right) dx\\ &= a^2cx+\frac{a(2bc+ad)x^{1+n}}{1+n}+\frac{b(bc+2ad)x^{1+2n}}{1+2n}+\frac{b^2dx^{1+3n}}{1+3n}\end{aligned}$$

Mathematica [A] time = 0.0860143, size = 70, normalized size = 1.

$$\frac{dx(a+bx^n)^3-x\left(a^2+\frac{2abx^n}{n+1}+\frac{b^2x^{2n}}{2n+1}\right)(ad-b(3cn+c))}{3bn+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^2*(c + d*x^n),x]
```

```
[Out] (d*x*(a + b*x^n)^3 - (a*d - b*(c + 3*c*n))*x*(a^2 + (2*a*b*x^n)/(1 + n) + (
b^2*x^(2*n))/(1 + 2*n)))/(b + 3*b*n)
```

Maple [A] time = 0.008, size = 74, normalized size = 1.1

$$a^2cx+\frac{a(ad+2bc)x{e}^{n\ln(x)}}{1+n}+\frac{b(2ad+bc)x\left({e}^{n\ln(x)}\right)^2}{1+2n}+\frac{b^2dx\left({e}^{n\ln(x)}\right)^3}{1+3n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^2*(c+d*x^n),x)
```

```
[Out] a^2*c*x+a*(a*d+2*b*c)/(1+n)*x*exp(n*ln(x))+b*(2*a*d+b*c)/(1+2*n)*x*exp(n*ln
(x))^2+b^2*d/(1+3*n)*x*exp(n*ln(x))^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(c+d*x^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.63465, size = 382, normalized size = 5.46

$$\frac{\left(2\,b^2dn^2 + 3\,b^2dn + b^2d\right)xx^{3\,n} + \left(b^2c + 2\,abd + 3\left(b^2c + 2\,abd\right)n^2 + 4\left(b^2c + 2\,abd\right)n\right)xx^{2\,n} + \left(2\,abc + a^2d + 6\left(2\,abc + a^2c\right.\right.}{6\,n^3 + 11\,n^2 + 6\,n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(c+d*x^n),x, algorithm="fricas")
```

```
[Out] ((2*b^2*d*n^2 + 3*b^2*d*n + b^2*d)*x*x^(3*n) + (b^2*c + 2*a*b*d + 3*(b^2*c
+ 2*a*b*d)*n^2 + 4*(b^2*c + 2*a*b*d)*n)*x*x^(2*n) + (2*a*b*c + a^2*d + 6*(2
*a*b*c + a^2*d)*n^2 + 5*(2*a*b*c + a^2*d)*n)*x*x^n + (6*a^2*c*n^3 + 11*a^2*
c*n^2 + 6*a^2*c*n + a^2*c)*x)/(6*n^3 + 11*n^2 + 6*n + 1)
```

Sympy [A] time = 0.970842, size = 726, normalized size = 10.37

$$\begin{cases} a^2cx + a^2d\log(x) + 2abc\log(x) - \frac{2abd}{x} - \frac{b^2c}{x} - \frac{b^2d}{2x^2} \\ a^2cx + 2a^2d\sqrt{x} + 4abc\sqrt{x} + 2abd\log(x) + b^2c\log(x) - \frac{2b^2d}{\sqrt{x}} \\ a^2cx + \frac{3a^2dx^{\frac{2}{3}}}{2} + 3abcx^{\frac{2}{3}} + 6abd\sqrt[3]{x} + 3b^2c\sqrt[3]{x} + b^2d\log(x) \\ \frac{6a^2cn^3x}{6n^3+11n^2+6n+1} + \frac{11a^2cn^2x}{6n^3+11n^2+6n+1} + \frac{6a^2cnx}{6n^3+11n^2+6n+1} + \frac{a^2cx}{6n^3+11n^2+6n+1} + \frac{6a^2dn^2xx^n}{6n^3+11n^2+6n+1} + \frac{5a^2dnxx^n}{6n^3+11n^2+6n+1} + \frac{a^2dxx^n}{6n^3+11n^2+6n+1} + \frac{12ab\ldots}{6n^3+11\ldots} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**2*(c+d*x**n),x)
```

```
[Out] Piecewise((a**2*c*x + a**2*d*log(x) + 2*a*b*c*log(x) - 2*a*b*d/x - b**2*c/x
 - b**2*d/(2*x**2), Eq(n, -1)), (a**2*c*x + 2*a**2*d*sqrt(x) + 4*a*b*c*sqrt
(x) + 2*a*b*d*log(x) + b**2*c*log(x) - 2*b**2*d/sqrt(x), Eq(n, -1/2)), (a**
2*c*x + 3*a**2*d*x**(2/3)/2 + 3*a*b*c*x**(2/3) + 6*a*b*d*x**(1/3) + 3*b**2*
c*x**(1/3) + b**2*d*log(x), Eq(n, -1/3)), (6*a**2*c*n**3*x/(6*n**3 + 11*n**
2 + 6*n + 1) + 11*a**2*c*n**2*x/(6*n**3 + 11*n**2 + 6*n + 1) + 6*a**2*c*n*x
/(6*n**3 + 11*n**2 + 6*n + 1) + a**2*c*x/(6*n**3 + 11*n**2 + 6*n + 1) + 6*a
```

```
**2*d*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 5*a**2*d*n*x*x**n/(6*n**3
+ 11*n**2 + 6*n + 1) + a**2*d*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 12*a*b*
c*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 10*a*b*c*n*x*x**n/(6*n**3 + 11
*n**2 + 6*n + 1) + 2*a*b*c*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 6*a*b*d*n*
*2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 8*a*b*d*n*x*x**(2*n)/(6*n**3 +
 11*n**2 + 6*n + 1) + 2*a*b*d*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 3*b
**2*c*n**2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 4*b**2*c*n*x*x**(2*n)/
(6*n**3 + 11*n**2 + 6*n + 1) + b**2*c*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n +
1) + 2*b**2*d*n**2*x*x**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 3*b**2*d*n*x*x
**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1) + b**2*d*x*x**(3*n)/(6*n**3 + 11*n**2
+ 6*n + 1), True))
```

Giac [B] time = 1.11375, size = 313, normalized size = 4.47

$$\frac{6\,a^2cn^3x + 2\,b^2dn^2xx^{3\,n} + 3\,b^2cn^2xx^{2\,n} + 6\,abdn^2xx^{2\,n} + 12\,abcn^2xx^n + 6\,a^2dn^2xx^n + 11\,a^2cn^2x + 3\,b^2dnxx^{3\,n} + 4\,b^2cn}{6\,n^3 + 11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(c+d*x^n),x, algorithm="giac")
```

```
[Out] (6*a^2*c*n^3*x + 2*b^2*d*n^2*x*x^(3*n) + 3*b^2*c*n^2*x*x^(2*n) + 6*a*b*d*n^
2*x*x^(2*n) + 12*a*b*c*n^2*x*x^n + 6*a^2*d*n^2*x*x^n + 11*a^2*c*n^2*x + 3*b
^2*d*n*x*x^(3*n) + 4*b^2*c*n*x*x^(2*n) + 8*a*b*d*n*x*x^(2*n) + 10*a*b*c*n*x
*x^n + 5*a^2*d*n*x*x^n + 6*a^2*c*n*x + b^2*d*x*x^(3*n) + b^2*c*x*x^(2*n) +
2*a*b*d*x*x^(2*n) + 2*a*b*c*x*x^n + a^2*d*x*x^n + a^2*c*x)/(6*n^3 + 11*n^2
+ 6*n + 1)
```

3.295 $\int \frac{(a+bx^n)^2}{c+dx^n}\,dx$

Optimal. Leaf size=84

$$\frac{x(bc-ad)^2\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{cd^2}-\frac{bx(bc(n+1)-ad(2n+1))}{d^2(n+1)}+\frac{bx\,(a+bx^n)}{d(n+1)}$$

```
[Out] -((b*(b*c*(1 + n) - a*d*(1 + 2*n))*x)/(d^2*(1 + n))) + (b*x*(a + b*x^n))/(d
*(1 + n)) + ((b*c - a*d)^2*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*
x^n)/c)])/(c*d^2)
```

Rubi [A] time = 0.0940034, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {416, 388, 245}

$$\frac{x(bc-ad)^2\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{cd^2}-\frac{bx(bc(n+1)-ad(2n+1))}{d^2(n+1)}+\frac{bx\,(a+bx^n)}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^2/(c + d*x^n),x]
```

```
[Out] -((b*(b*c*(1 + n) - a*d*(1 + 2*n))*x)/(d^2*(1 + n))) + (b*x*(a + b*x^n))/(d
*(1 + n)) + ((b*c - a*d)^2*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*
x^n)/c)])/(c*d^2)
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \frac{(a+bx^n)^2}{c+dx^n}\,dx = \frac{bx\,(a+bx^n)}{d(1+n)} + \frac{\int \frac{-a(bc-ad(1+n))-b(bc(1+n)-ad(1+2n))x^n}{c+dx^n}\,dx}{d(1+n)}$$
$$= -\frac{b(bc(1+n)-ad(1+2n))x}{d^2(1+n)} + \frac{bx\,(a+bx^n)}{d(1+n)} + \frac{(bc-ad)^2 \int \frac{1}{c+dx^n}\,dx}{d^2}$$
$$= -\frac{b(bc(1+n)-ad(1+2n))x}{d^2(1+n)} + \frac{bx\,(a+bx^n)}{d(1+n)} + \frac{(bc-ad)^2 x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{cd^2}$$

Mathematica [A] time = 0.0444925, size = 82, normalized size = 0.98

$$\frac{a^2x}{c} + \frac{x(ad-bc)^2\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{cd^2} - \frac{x(bc-ad)^2}{cd^2} + \frac{b^2x^{n+1}}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^2/(c + d*x^n),x]
```

```
[Out] (a^2*x)/c - ((b*c - a*d)^2*x)/(c*d^2) + (b^2*x^(1 + n))/(d*(1 + n)) + ((-(b
*c) + a*d)^2*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c*d
^2)
```

Maple [F] time = 0.353, size = 0, normalized size = 0.

$$\int \frac{(a+bx^n)^2}{c+dx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^2/(c+d*x^n),x)
```

```
[Out] int((a+b*x^n)^2/(c+d*x^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\left(b^2c^2 - 2\,abcd + a^2d^2\right) \int \frac{1}{d^3x^n + cd^2}\,dx + \frac{b^2dxx^n - \left(b^2c(n+1) - 2\,abd(n+1)\right)x}{d^2(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*integrate(1/(d^3*x^n + c*d^2), x) + (b^2*d*
x*x^n - (b^2*c*(n + 1) - 2*a*b*d*(n + 1))*x)/(d^2*(n + 1))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2x^{2\,n} + 2\,abx^n + a^2}{dx^n + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral((b^2*x^(2*n) + 2*a*b*x^n + a^2)/(d*x^n + c), x)
```

Sympy [C] time = 3.31903, size = 170, normalized size = 2.02

$$\frac{a^2x\Phi\left(\frac{dx^ne^{i\pi}}{c},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{cn^2\Gamma\left(1+\frac{1}{n}\right)}-\frac{2abx\Phi\left(\frac{cx^{-n}e^{i\pi}}{d},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{dn^2\Gamma\left(1+\frac{1}{n}\right)}+\frac{2b^2xx^{2n}\Phi\left(\frac{dx^ne^{i\pi}}{c},1,2+\frac{1}{n}\right)\Gamma\left(2+\frac{1}{n}\right)}{cn\Gamma\left(3+\frac{1}{n}\right)}+\frac{b^2xx^{2n}\Phi\left(\frac{dx^ne^{i\pi}}{c},1,2+\right.}{cn^2\Gamma\left(3+\frac{1}{n}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] a**2*x*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 1/n)*gamma(1/n)/(c*n**2*gamma(
1 + 1/n)) - 2*a*b*x*lerchphi(c*x**(-n)*exp_polar(I*pi)/d, 1, exp_polar(I*pi
)/n)*gamma(1/n)/(d*n**2*gamma(1 + 1/n)) + 2*b**2*x*x**(2*n)*lerchphi(d*x**n
*exp_polar(I*pi)/c, 1, 2 + 1/n)*gamma(2 + 1/n)/(c*n*gamma(3 + 1/n)) + b**2*
x*x**(2*n)*lerchphi(d*x**n*exp_polar(I*pi)/c, 1, 2 + 1/n)*gamma(2 + 1/n)/(c
*n**2*gamma(3 + 1/n))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^n+a)^2}{dx^n+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^2/(d*x^n + c), x)
```

3.296 $\int \frac{(a+bx^n)^2}{(c+dx^n)^2} dx$

Optimal. Leaf size=115

$$\frac{x(bc-ad)(ad(1-n)-bc(n+1))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2d^2n}-\frac{bx(ad-bc(n+1))}{cd^2n}-\frac{x(bc-ad)\,(a+bx^n)}{cdn\,(c+dx^n)}$$

```
[Out] -((b*(a*d - b*c*(1 + n))*x)/(c*d^2*n)) - ((b*c - a*d)*x*(a + b*x^n))/(c*d*n
*(c + d*x^n)) + ((b*c - a*d)*(a*d*(1 - n) - b*c*(1 + n))*x*Hypergeometric2F
1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c^2*d^2*n)
```

Rubi [A] time = 0.092344, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {413, 388, 245}

$$\frac{x(bc-ad)(ad(1-n)-bc(n+1))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2d^2n}-\frac{bx(ad-bc(n+1))}{cd^2n}-\frac{x(bc-ad)\,(a+bx^n)}{cdn\,(c+dx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^2/(c + d*x^n)^2,x]
```


```
[Out] -((b*(a*d - b*c*(1 + n))*x)/(c*d^2*n)) - ((b*c - a*d)*x*(a + b*x^n))/(c*d*n
*(c + d*x^n)) + ((b*c - a*d)*(a*d*(1 - n) - b*c*(1 + n))*x*Hypergeometric2F
1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c^2*d^2*n)
```


Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^2}\,dx = -\frac{(bc-ad)x(a+bx^n)}{cdn(c+dx^n)} + \frac{\int \frac{a(bc-ad(1-n))-b(ad-bc(1+n))x^n}{c+dx^n}\,dx}{cdn}$$

$$= -\frac{b(ad-bc(1+n))x}{cd^2n} - \frac{(bc-ad)x(a+bx^n)}{cdn(c+dx^n)} + \frac{((bc-ad)(ad(1-n)-bc(1+n)))\int \frac{1}{c+dx^n}\,dx}{cd^2n}$$

$$= -\frac{b(ad-bc(1+n))x}{cd^2n} - \frac{(bc-ad)x(a+bx^n)}{cdn(c+dx^n)} + \frac{(bc-ad)(ad(1-n)-bc(1+n))x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2d^2n}$$

Mathematica [A] time = 0.113189, size = 95, normalized size = 0.83

$$\frac{x\left(\frac{c\left(a^2d^2-2abcd+b^2c(cn+c+dnx^n)\right)}{c+dx^n} - (bc-ad)(ad(n-1)+bc(n+1))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)\right)}{c^2d^2n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^2/(c + d*x^n)^2,x]
```

```
[Out] (x*((c*(-2*a*b*c*d + a^2*d^2 + b^2*c*(c + c*n + d*n*x^n)))/(c + d*x^n) - (b
*c - a*d)*(a*d*(-1 + n) + b*c*(1 + n))*Hypergeometric2F1[1, n^(-1), 1 + n^(
-1), -((d*x^n)/c)]))/(c^2*d^2*n)
```

Maple [F] time = 0.385, size = 0, normalized size = 0.

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^2/(c+d*x^n)^2,x)
```

```
[Out] int((a+b*x^n)^2/(c+d*x^n)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\left(b^2c^2(n+1)-a^2d^2(n-1)-2\,abcd\right)\int \frac{1}{cd^3nx^n+c^2d^2n}\,dx + \frac{b^2cdnxx^n + \left(b^2c^2(n+1)-2\,abcd+a^2d^2\right)x}{cd^3nx^n+c^2d^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2/(c+d*x^n)^2,x, algorithm="maxima")
```

```
[Out] -(b^2*c^2*(n + 1) - a^2*d^2*(n - 1) - 2*a*b*c*d)*integrate(1/(c*d^3*n*x^n +
 c^2*d^2*n), x) + (b^2*c*d*n*x*x^n + (b^2*c^2*(n + 1) - 2*a*b*c*d + a^2*d^2
)*x)/(c*d^3*n*x^n + c^2*d^2*n)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2x^{2n}+2\,abx^n+a^2}{d^2x^{2n}+2\,cdx^n+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2/(c+d*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^(2*n) + 2*a*b*x^n + a^2)/(d^2*x^(2*n) + 2*c*d*x^n + c^2), x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**2/(c+d*x**n)**2,x)
```

```
[Out] Integral((a + b*x**n)**2/(c + d*x**n)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^n+a)^2}{(dx^n+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2/(c+d*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^2/(d*x^n + c)^2, x)
```

3.297 $\int \frac{(a+bx^n)^2}{(c+dx^n)^3} dx$

Optimal. Leaf size=160

$$-\frac{x\left(-a^2d^2\left(2n^2-3n+1\right)+2abcd(1-n)-b^2c^2(n+1)\right){}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{2c^3d^2n^2}+\frac{x(bc-ad)(ad(1-2n)-bc(n+1))}{2c^2d^2n^2\left(c+dx^n\right)}-\frac{x(\cdots}{}$$

```
[Out] -((b*c - a*d)*x*(a + b*x^n))/(2*c*d*n*(c + d*x^n)^2) + ((b*c - a*d)*(a*d*(1
 - 2*n) - b*c*(1 + n))*x)/(2*c^2*d^2*n^2*(c + d*x^n)) - ((2*a*b*c*d*(1 - n)
 - b^2*c^2*(1 + n) - a^2*d^2*(1 - 3*n + 2*n^2))*x*Hypergeometric2F1[1, n^(-
1), 1 + n^(-1), -((d*x^n)/c)])/(2*c^3*d^2*n^2)
```

Rubi [A] time = 0.159603, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {413, 385, 245}

$$-\frac{x\left(-a^2d^2\left(2n^2-3n+1\right)+2abcd(1-n)-b^2c^2(n+1)\right){}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{2c^3d^2n^2}+\frac{x(bc-ad)(ad(1-2n)-bc(n+1))}{2c^2d^2n^2\left(c+dx^n\right)}-\frac{x(\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^2/(c + d*x^n)^3,x]
```

```
[Out] -((b*c - a*d)*x*(a + b*x^n))/(2*c*d*n*(c + d*x^n)^2) + ((b*c - a*d)*(a*d*(1
 - 2*n) - b*c*(1 + n))*x)/(2*c^2*d^2*n^2*(c + d*x^n)) - ((2*a*b*c*d*(1 - n)
 - b^2*c^2*(1 + n) - a^2*d^2*(1 - 3*n + 2*n^2))*x*Hypergeometric2F1[1, n^(-
1), 1 + n^(-1), -((d*x^n)/c)])/(2*c^3*d^2*n^2)
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^3}\,dx = -\frac{(bc-ad)x\,(a+bx^n)}{2cdn\,(c+dx^n)^2} + \frac{\int \frac{a(bc-ad(1-2n))-b(ad(1-n)-bc(1+n))x^n}{(c+dx^n)^2}\,dx}{2cdn}$$

$$= -\frac{(bc-ad)x\,(a+bx^n)}{2cdn\,(c+dx^n)^2} + \frac{(bc-ad)(ad(1-2n)-bc(1+n))x}{2c^2d^2n^2\,(c+dx^n)} - \frac{\left(2abcd(1-n)-b^2c^2(1+n)-a^2d^2\left(\right.\right.}{2c^2d^2n^2}$$

$$= -\frac{(bc-ad)x\,(a+bx^n)}{2cdn\,(c+dx^n)^2} + \frac{(bc-ad)(ad(1-2n)-bc(1+n))x}{2c^2d^2n^2\,(c+dx^n)} - \frac{\left(2abcd(1-n)-b^2c^2(1+n)-a^2d^2\left(\right.\right.}{2c^3}$$

Mathematica [A] time = 0.106566, size = 133, normalized size = 0.83

$$\frac{x\left(\left(a^2d^2\left(2n^2-3n+1\right)+2abcd(n-1)+b^2c^2(n+1)\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)+\frac{c^2n(bc-ad)^2}{(c+dx^n)^2}-\frac{c(bc-ad)(ad(2n-1)+b(2cn+c))}{c+dx^n}\right)}{2c^3d^2n^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^2/(c + d*x^n)^3,x]
```

```
[Out] (x*((c^2*(b*c - a*d)^2*n)/(c + d*x^n)^2 - (c*(b*c - a*d)*(a*d*(-1 + 2*n) +
b*(c + 2*c*n)))/(c + d*x^n) + (2*a*b*c*d*(-1 + n) + b^2*c^2*(1 + n) + a^2*d
^2*(1 - 3*n + 2*n^2))*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)
]))/(2*c^3*d^2*n^2)
```

Maple [F] time = 0.353, size = 0, normalized size = 0.

$$\int \frac{(a+bx^n)^2}{(c+dx^n)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^2/(c+d*x^n)^3,x)
```

```
[Out] int((a+b*x^n)^2/(c+d*x^n)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\left(\left(2\,n^2-3\,n+1\right)a^2d^2+b^2c^2(n+1)+2\,abcd(n-1)\right)\int \frac{1}{2\left(c^2d^3n^2x^n+c^3d^2n^2\right)}\,dx - \frac{\left(b^2c^2d(2\,n+1)-a^2d^3(2\,n-1)-2\right.}{2\left(c^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2/(c+d*x^n)^3,x, algorithm="maxima")
```

```
[Out] ((2*n^2 - 3*n + 1)*a^2*d^2 + b^2*c^2*(n + 1) + 2*a*b*c*d*(n - 1))*integrate
(1/2/(c^2*d^3*n^2*x^n + c^3*d^2*n^2), x) - 1/2*((b^2*c^2*d*(2*n + 1) - a^2*
d^3*(2*n - 1) - 2*a*b*c*d^2)*x*x^n - (a^2*c*d^2*(3*n - 1) - b^2*c^3*(n + 1)
 - 2*a*b*c^2*d*(n - 1))*x)/(c^2*d^4*n^2*x^(2*n) + 2*c^3*d^3*n^2*x^n + c^4*d
^2*n^2)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^{2}x^{2\,n} + 2\,abx^{n} + a^{2}}{d^{3}x^{3\,n} + 3\,cd^{2}x^{2\,n} + 3\,c^{2}dx^{n} + c^{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2/(c+d*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral((b^2*x^(2*n) + 2*a*b*x^n + a^2)/(d^3*x^(3*n) + 3*c*d^2*x^(2*n) + 3
*c^2*d*x^n + c^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**2/(c+d*x**n)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^{n} + a)^{2}}{(dx^{n} + c)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2/(c+d*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^2/(d*x^n + c)^3, x)
```

3.298 $\int \frac{(c+dx^n)^4}{a+bx^n} dx$

Optimal. Leaf size=310

$$-\frac{dx\,(c+dx^n)\left(-a^2d^2\left(6n^2+5n+1\right)+2abcd(3n+1)^2-b^2c^2\left(18n^2+7n+1\right)\right)}{b^3(n+1)(2n+1)(3n+1)}-\frac{dx\left(-a^2bcd^2\left(24n^3+38n^2+19n+3\right.\right.}{}$$

```
[Out] -((d*(a^3*d^3*(1 + 6*n + 11*n^2 + 6*n^3) - b^3*c^3*(1 + 7*n + 18*n^2 + 24*n
^3) - a^2*b*c*d^2*(3 + 19*n + 38*n^2 + 24*n^3) + a*b^2*c^2*d*(3 + 20*n + 45
*n^2 + 36*n^3))*x)/(b^4*(1 + n)*(1 + 2*n)*(1 + 3*n))) - (d*(2*a*b*c*d*(1 +
3*n)^2 - a^2*d^2*(1 + 5*n + 6*n^2) - b^2*c^2*(1 + 7*n + 18*n^2))*x*(c + d*x
^n))/(b^3*(1 + n)*(1 + 2*n)*(1 + 3*n)) - (d*(a*d*(1 + 3*n) - b*(c + 6*c*n))
*x*(c + d*x^n)^2)/(b^2*(1 + 5*n + 6*n^2)) + (d*x*(c + d*x^n)^3)/(b*(1 + 3*n
)) + ((b*c - a*d)^4*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)
])/(a*b^4)
```

Rubi [A] time = 0.501013, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {416, 528, 388, 245}

$$-\frac{dx\,(c+dx^n)\left(-a^2d^2\left(6n^2+5n+1\right)+2abcd(3n+1)^2-b^2c^2\left(18n^2+7n+1\right)\right)}{b^3(n+1)(2n+1)(3n+1)}-\frac{dx\left(-a^2bcd^2\left(24n^3+38n^2+19n+3\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^n)^4/(a + b*x^n),x]
```

```
[Out] -((d*(a^3*d^3*(1 + 6*n + 11*n^2 + 6*n^3) - b^3*c^3*(1 + 7*n + 18*n^2 + 24*n
^3) - a^2*b*c*d^2*(3 + 19*n + 38*n^2 + 24*n^3) + a*b^2*c^2*d*(3 + 20*n + 45
*n^2 + 36*n^3))*x)/(b^4*(1 + n)*(1 + 2*n)*(1 + 3*n))) - (d*(2*a*b*c*d*(1 +
3*n)^2 - a^2*d^2*(1 + 5*n + 6*n^2) - b^2*c^2*(1 + 7*n + 18*n^2))*x*(c + d*x
^n))/(b^3*(1 + n)*(1 + 2*n)*(1 + 3*n)) - (d*(a*d*(1 + 3*n) - b*(c + 6*c*n))
*x*(c + d*x^n)^2)/(b^2*(1 + 5*n + 6*n^2)) + (d*x*(c + d*x^n)^3)/(b*(1 + 3*n
)) + ((b*c - a*d)^4*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)
])/(a*b^4)
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \frac{(c+dx^n)^4}{a+bx^n}\,dx = \frac{dx\,(c+dx^n)^3}{b(1+3n)} + \frac{\int \frac{(c+dx^n)^2(-c(ad-b(c+3cn))-d(ad(1+3n)-b(c+6cn))x^n)}{a+bx^n}\,dx}{b(1+3n)}$$

$$= -\frac{d(ad(1+3n)-b(c+6cn))x\,(c+dx^n)^2}{b^2\left(1+5n+6n^2\right)} + \frac{dx\,(c+dx^n)^3}{b(1+3n)} + \frac{\int \frac{(c+dx^n)\left(c\left(a^2d^2(1+3n)-2abcd(1+4n)+b^2c^2\left(1+5n+6n\right.\right.\right.}{\cdots}}{b^2\left(1\cdots\right.}$$

$$= -\frac{d\left(2abcd(1+3n)^2-a^2d^2\left(1+5n+6n^2\right)-b^2c^2\left(1+7n+18n^2\right)\right)x\,(c+dx^n)}{b^3(1+n)\left(1+5n+6n^2\right)} - \frac{d(ad(1+3n)-b(c+\cdots}{b^2\left(1+5n\cdots\right.}$$

$$= -\frac{d\left(a^3d^3\left(1+6n+11n^2+6n^3\right)-b^3c^3\left(1+7n+18n^2+24n^3\right)-a^2bcd^2\left(3+19n+38n^2+24n^3\right)+a\cdots\right.}{b^4(1+n)\left(1+5n+6n^2\right)}$$

$$= -\frac{d\left(a^3d^3\left(1+6n+11n^2+6n^3\right)-b^3c^3\left(1+7n+18n^2+24n^3\right)-a^2bcd^2\left(3+19n+38n^2+24n^3\right)+a\cdots\right.}{b^4(1+n)\left(1+5n+6n^2\right)}$$

Mathematica [C] time = 2.47083, size = 133, normalized size = 0.43

$$\frac{x\left(6c^2d^2x^{2n}\Phi\left(-\frac{bx^n}{a},1,2+\frac{1}{n}\right)+4c^3dx^n\Phi\left(-\frac{bx^n}{a},1,1+\frac{1}{n}\right)+c^4\Phi\left(-\frac{bx^n}{a},1,\frac{1}{n}\right)+4cd^3x^{3n}\Phi\left(-\frac{bx^n}{a},1,3+\frac{1}{n}\right)+d^4x^{4n}\Phi\left(-\frac{bx^n}{a},\cdots\right.\right.}{an}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^n)^4/(a + b*x^n),x]
```

```
[Out] (x*(4*c^3*d*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, 1 + n^(-1)] + 6*c^2*d^2*x^
(2*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, 2 + n^(-1)] + 4*c*d^3*x^(3*n)*Hurwit
zLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + d^4*x^(4*n)*HurwitzLerchPhi[-((b*x
^n)/a), 1, 4 + n^(-1)] + c^4*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)]))/(a*
n)
```

Maple [F] time = 0.364, size = 0, normalized size = 0.

$$\int \frac{(c+dx^n)^4}{a+bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^n)^4/(a+b*x^n),x)
```

```
[Out] int((c+d*x^n)^4/(a+b*x^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\left(b^4c^4 - 4\,ab^3c^3d + 6\,a^2b^2c^2d^2 - 4\,a^3bcd^3 + a^4d^4\right)\int \frac{1}{b^5x^n + ab^4}\,dx + \frac{\left(2\,n^2+3\,n+1\right)b^3d^4xx^{3\,n} + \left(4\left(3\,n^2+4\,n+1\right)b^3c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^4/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*int
egrate(1/(b^5*x^n + a*b^4), x) + ((2*n^2 + 3*n + 1)*b^3*d^4*x*x^(3*n) + (4*
(3*n^2 + 4*n + 1)*b^3*c*d^3 - (3*n^2 + 4*n + 1)*a*b^2*d^4)*x*x^(2*n) + (6*(
6*n^2 + 5*n + 1)*b^3*c^2*d^2 - 4*(6*n^2 + 5*n + 1)*a*b^2*c*d^3 + (6*n^2 + 5
*n + 1)*a^2*b*d^4)*x*x^n + (4*(6*n^3 + 11*n^2 + 6*n + 1)*b^3*c^3*d - 6*(6*n
^3 + 11*n^2 + 6*n + 1)*a*b^2*c^2*d^2 + 4*(6*n^3 + 11*n^2 + 6*n + 1)*a^2*b*c
*d^3 - (6*n^3 + 11*n^2 + 6*n + 1)*a^3*d^4)*x)/((6*n^3 + 11*n^2 + 6*n + 1)*b
^4)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{d^4x^{4\,n} + 4\,cd^3x^{3\,n} + 6\,c^2d^2x^{2\,n} + 4\,c^3dx^n + c^4}{bx^n + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^4/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] integral((d^4*x^(4*n) + 4*c*d^3*x^(3*n) + 6*c^2*d^2*x^(2*n) + 4*c^3*d*x^n +
 c^4)/(b*x^n + a), x)
```

Sympy [C] time = 6.64649, size = 369, normalized size = 1.19

$$-\frac{4c^3dx\Phi\left(\frac{ax^{-n}e^{i\pi}}{b},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{bn^2\Gamma\left(1+\frac{1}{n}\right)} + \frac{c^4x\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^2\Gamma\left(1+\frac{1}{n}\right)} + \frac{12c^2d^2xx^{2n}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,2+\frac{1}{n}\right)\Gamma\left(2+\frac{1}{n}\right)}{an\Gamma\left(3+\frac{1}{n}\right)} + \frac{6c^2d^2xx^{2n}\Phi\left(\right.}{an}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)**4/(a+b*x**n),x)
```

```
[Out] -4*c**3*d*x*lerchphi(a*x**(-n)*exp_polar(I*pi)/b, 1, exp_polar(I*pi)/n)*gam
ma(1/n)/(b*n**2*gamma(1 + 1/n)) + c**4*x*lerchphi(b*x**n*exp_polar(I*pi)/a,
 1, 1/n)*gamma(1/n)/(a*n**2*gamma(1 + 1/n)) + 12*c**2*d**2*x*x**(2*n)*lerch
phi(b*x**n*exp_polar(I*pi)/a, 1, 2 + 1/n)*gamma(2 + 1/n)/(a*n*gamma(3 + 1/n
)) + 6*c**2*d**2*x*x**(2*n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2 + 1/n)*
gamma(2 + 1/n)/(a*n**2*gamma(3 + 1/n)) + 12*c*d**3*x*x**(3*n)*lerchphi(b*x*
```

```
*n*exp_polar(I*pi)/a, 1, 3 + 1/n)*gamma(3 + 1/n)/(a*n*gamma(4 + 1/n)) + 4*c
*d**3*x*x**(3*n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 3 + 1/n)*gamma(3 + 1
/n)/(a*n**2*gamma(4 + 1/n)) + 4*d**4*x*x**(4*n)*lerchphi(b*x**n*exp_polar(I
*pi)/a, 1, 4 + 1/n)*gamma(4 + 1/n)/(a*n*gamma(5 + 1/n)) + d**4*x*x**(4*n)*l
erchphi(b*x**n*exp_polar(I*pi)/a, 1, 4 + 1/n)*gamma(4 + 1/n)/(a*n**2*gamma(
5 + 1/n))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^n + c)^4}{bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^4/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^4/(b*x^n + a), x)
```

3.299 $\int \frac{(c+dx^n)^3}{a+bx^n} dx$

Optimal. Leaf size=173

$$\frac{dx\left(a^2d^2\left(2n^2+3n+1\right)-abcd\left(6n^2+7n+2\right)+b^2c^2\left(6n^2+4n+1\right)\right)}{b^3(n+1)(2n+1)}+\frac{x(bc-ad)^3\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{ab^3}-\frac{dx\,(c+}{}$$

```
[Out] (d*(a^2*d^2*(1 + 3*n + 2*n^2) + b^2*c^2*(1 + 4*n + 6*n^2) - a*b*c*d*(2 + 7*
n + 6*n^2))*x)/(b^3*(1 + n)*(1 + 2*n)) - (d*(a*d*(1 + 2*n) - b*(c + 4*c*n))
*x*(c + d*x^n))/(b^2*(1 + n)*(1 + 2*n)) + (d*x*(c + d*x^n)^2)/(b*(1 + 2*n))
 + ((b*c - a*d)^3*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])
/(a*b^3)
```

Rubi [A] time = 0.267249, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {416, 528, 388, 245}

$$\frac{dx\left(a^2d^2\left(2n^2+3n+1\right)-abcd\left(6n^2+7n+2\right)+b^2c^2\left(6n^2+4n+1\right)\right)}{b^3(n+1)(2n+1)}+\frac{x(bc-ad)^3\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{ab^3}-\frac{dx\,(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^n)^3/(a + b*x^n),x]
```

```
[Out] (d*(a^2*d^2*(1 + 3*n + 2*n^2) + b^2*c^2*(1 + 4*n + 6*n^2) - a*b*c*d*(2 + 7*
n + 6*n^2))*x)/(b^3*(1 + n)*(1 + 2*n)) - (d*(a*d*(1 + 2*n) - b*(c + 4*c*n))
*x*(c + d*x^n))/(b^2*(1 + n)*(1 + 2*n)) + (d*x*(c + d*x^n)^2)/(b*(1 + 2*n))
 + ((b*c - a*d)^3*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])
/(a*b^3)
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \frac{(c+dx^n)^3}{a+bx^n}\,dx = \frac{dx\,(c+dx^n)^2}{b(1+2n)} + \frac{\int \frac{(c+dx^n)(-c(ad-b(c+2cn))-d(ad(1+2n)-b(c+4cn))x^n)}{a+bx^n}\,dx}{b(1+2n)}$$

$$= -\frac{d(ad(1+2n)-b(c+4cn))x\,(c+dx^n)}{b^2(1+n)(1+2n)} + \frac{dx\,(c+dx^n)^2}{b(1+2n)} + \frac{\int \frac{c\left(a^2d^2(1+2n)-abcd(2+5n)+b^2c^2\left(1+3n+2n^2\right)\right)+d\left(a^2d^2\right.}{a+b}}{b^2(1+n}$$

$$= \frac{d\left(a^2d^2\left(1+3n+2n^2\right)+b^2c^2\left(1+4n+6n^2\right)-abcd\left(2+7n+6n^2\right)\right)x}{b^3(1+n)(1+2n)} - \frac{d(ad(1+2n)-b(c+4cn))x}{b^2(1+n)(1+2n)}$$

$$= \frac{d\left(a^2d^2\left(1+3n+2n^2\right)+b^2c^2\left(1+4n+6n^2\right)-abcd\left(2+7n+6n^2\right)\right)x}{b^3(1+n)(1+2n)} - \frac{d(ad(1+2n)-b(c+4cn))x}{b^2(1+n)(1+2n)}$$

Mathematica [C] time = 0.982742, size = 104, normalized size = 0.6

$$\frac{x\left(3c^2dx^n\Phi\left(-\frac{bx^n}{a},1,1+\frac{1}{n}\right)+c^3\Phi\left(-\frac{bx^n}{a},1,\frac{1}{n}\right)+3cd^2x^{2n}\Phi\left(-\frac{bx^n}{a},1,2+\frac{1}{n}\right)+d^3x^{3n}\Phi\left(-\frac{bx^n}{a},1,3+\frac{1}{n}\right)\right)}{an}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^n)^3/(a + b*x^n),x]
```

```
[Out] (x*(3*c^2*d*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, 1 + n^(-1)] + 3*c*d^2*x^(2
*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, 2 + n^(-1)] + d^3*x^(3*n)*HurwitzLerch
Phi[-((b*x^n)/a), 1, 3 + n^(-1)] + c^3*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(
-1)]))/(a*n)
```

Maple [F] time = 0.382, size = 0, normalized size = 0.

$$\int \frac{(c+dx^n)^3}{a+bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^n)^3/(a+b*x^n),x)
```

```
[Out] int((c+d*x^n)^3/(a+b*x^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\left(b^3c^3-3\,ab^2c^2d+3\,a^2bcd^2-a^3d^3\right)\int \frac{1}{b^4x^n+ab^3}\,dx + \frac{b^2d^3(n+1)xx^{2\,n}+\left(3\,b^2cd^2(2\,n+1)-abd^3(2\,n+1)\right)xx^n+\left(3\left(2\,n\right.\right.}{\left(2\,n^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^3/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*integrate(1/(b^4*x^n +
a*b^3), x) + (b^2*d^3*(n + 1)*x*x^(2*n) + (3*b^2*c*d^2*(2*n + 1) - a*b*d^3*
(2*n + 1))*x*x^n + (3*(2*n^2 + 3*n + 1)*b^2*c^2*d - 3*(2*n^2 + 3*n + 1)*a*b
*c*d^2 + (2*n^2 + 3*n + 1)*a^2*d^3)*x)/((2*n^2 + 3*n + 1)*b^3)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{d^3x^{3\,n} + 3\,cd^2x^{2\,n} + 3\,c^2dx^n + c^3}{bx^n + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^3/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] integral((d^3*x^(3*n) + 3*c*d^2*x^(2*n) + 3*c^2*d*x^n + c^3)/(b*x^n + a), x
)
```

Sympy [C] time = 4.7349, size = 269, normalized size = 1.55

$$-\frac{3c^2dx\Phi\left(\frac{ax^{-n}e^{i\pi}}{b},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{bn^2\Gamma\left(1+\frac{1}{n}\right)} + \frac{c^3x\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^2\Gamma\left(1+\frac{1}{n}\right)} + \frac{6cd^2xx^{2n}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,2+\frac{1}{n}\right)\Gamma\left(2+\frac{1}{n}\right)}{an\Gamma\left(3+\frac{1}{n}\right)} + \frac{3cd^2xx^{2n}\Phi\left(\frac{bx^ne^{i}}{a}\right.}{an^2\Gamma}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)**3/(a+b*x**n),x)
```

```
[Out] -3*c**2*d*x*lerchphi(a*x**(-n)*exp_polar(I*pi)/b, 1, exp_polar(I*pi)/n)*gam
ma(1/n)/(b*n**2*gamma(1 + 1/n)) + c**3*x*lerchphi(b*x**n*exp_polar(I*pi)/a,
 1, 1/n)*gamma(1/n)/(a*n**2*gamma(1 + 1/n)) + 6*c*d**2*x*x**(2*n)*lerchphi(
b*x**n*exp_polar(I*pi)/a, 1, 2 + 1/n)*gamma(2 + 1/n)/(a*n*gamma(3 + 1/n)) +
 3*c*d**2*x*x**(2*n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2 + 1/n)*gamma(2
 + 1/n)/(a*n**2*gamma(3 + 1/n)) + 3*d**3*x*x**(3*n)*lerchphi(b*x**n*exp_pol
ar(I*pi)/a, 1, 3 + 1/n)*gamma(3 + 1/n)/(a*n*gamma(4 + 1/n)) + d**3*x*x**(3*
n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 3 + 1/n)*gamma(3 + 1/n)/(a*n**2*ga
mma(4 + 1/n))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^n + c)^3}{bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^3/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^3/(b*x^n + a), x)
```

3.300 $\int \frac{(c+dx^n)^2}{a+bx^n} dx$

Optimal. Leaf size=84

$$\frac{x(bc-ad)^2 {}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{ab^2}-\frac{dx(ad(n+1)-b(2cn+c))}{b^2(n+1)}+\frac{dx\left(c+dx^n\right)}{b(n+1)}$$

```
[Out] -((d*(a*d*(1 + n) - b*(c + 2*c*n))*x)/(b^2*(1 + n))) + (d*x*(c + d*x^n))/(b
*(1 + n)) + ((b*c - a*d)^2*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*
x^n)/a)])/(a*b^2)
```

Rubi [A] time = 0.0974269, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {416, 388, 245}

$$\frac{x(bc-ad)^2 {}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{ab^2}-\frac{dx(ad(n+1)-b(2cn+c))}{b^2(n+1)}+\frac{dx\left(c+dx^n\right)}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^n)^2/(a + b*x^n),x]
```

```
[Out] -((d*(a*d*(1 + n) - b*(c + 2*c*n))*x)/(b^2*(1 + n))) + (d*x*(c + d*x^n))/(b
*(1 + n)) + ((b*c - a*d)^2*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*
x^n)/a)])/(a*b^2)
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \frac{(c+dx^n)^2}{a+bx^n}\,dx = \frac{dx\,(c+dx^n)}{b(1+n)} + \frac{\int \frac{-c(ad-bc(1+n))-d(ad(1+n)-b(c+2cn))x^n}{a+bx^n}\,dx}{b(1+n)}$$
$$= -\frac{d(ad(1+n)-b(c+2cn))x}{b^2(1+n)} + \frac{dx\,(c+dx^n)}{b(1+n)} + \frac{(bc-ad)^2 \int \frac{1}{a+bx^n}\,dx}{b^2}$$
$$= -\frac{d(ad(1+n)-b(c+2cn))x}{b^2(1+n)} + \frac{dx\,(c+dx^n)}{b(1+n)} + \frac{(bc-ad)^2 x\, {}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{ab^2}$$

Mathematica [C] time = 0.295013, size = 75, normalized size = 0.89

$$\frac{x\left(c^2\Phi\left(-\frac{bx^n}{a},1,\frac{1}{n}\right)+2cdx^n\Phi\left(-\frac{bx^n}{a},1,1+\frac{1}{n}\right)+d^2x^{2n}\Phi\left(-\frac{bx^n}{a},1,2+\frac{1}{n}\right)\right)}{an}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^n)^2/(a + b*x^n),x]
```

```
[Out] (x*(2*c*d*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, 1 + n^(-1)] + d^2*x^(2*n)*Hu
rwitzLerchPhi[-((b*x^n)/a), 1, 2 + n^(-1)] + c^2*HurwitzLerchPhi[-((b*x^n)/
a), 1, n^(-1)]))/(a*n)
```

Maple [F] time = 0.429, size = 0, normalized size = 0.

$$\int \frac{(c+dx^n)^2}{a+bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^n)^2/(a+b*x^n),x)
```

```
[Out] int((c+d*x^n)^2/(a+b*x^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\left(b^2c^2 - 2\,abcd + a^2d^2\right)\int \frac{1}{b^3x^n + ab^2}\,dx + \frac{bd^2xx^n + \left(2\,bcd(n+1) - ad^2(n+1)\right)x}{b^2(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^2/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*integrate(1/(b^3*x^n + a*b^2), x) + (b*d^2*
x*x^n + (2*b*c*d*(n + 1) - a*d^2*(n + 1))*x)/(b^2*(n + 1))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{d^2x^{2\,n} + 2\,cdx^n + c^2}{bx^n + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^2/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] integral((d^2*x^(2*n) + 2*c*d*x^n + c^2)/(b*x^n + a), x)
```

Sympy [C] time = 3.18218, size = 170, normalized size = 2.02

$$-\frac{2cdx\Phi\left(\frac{ax^{-n}e^{i\pi}}{b},1,\frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{bn^2\Gamma\left(1+\frac{1}{n}\right)}+\frac{c^2x\Phi\left(\frac{bx^ne^{i\pi}}{a},1,\frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^2\Gamma\left(1+\frac{1}{n}\right)}+\frac{2d^2xx^{2n}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,2+\frac{1}{n}\right)\Gamma\left(2+\frac{1}{n}\right)}{an\Gamma\left(3+\frac{1}{n}\right)}+\frac{d^2xx^{2n}\Phi\left(\frac{bx^ne^{i\pi}}{a},1,2+\right.}{an^2\Gamma\left(3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)**2/(a+b*x**n),x)
```

```
[Out] -2*c*d*x*lerchphi(a*x**(-n)*exp_polar(I*pi)/b, 1, exp_polar(I*pi)/n)*gamma(
1/n)/(b*n**2*gamma(1 + 1/n)) + c**2*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1,
 1/n)*gamma(1/n)/(a*n**2*gamma(1 + 1/n)) + 2*d**2*x*x**(2*n)*lerchphi(b*x**
n*exp_polar(I*pi)/a, 1, 2 + 1/n)*gamma(2 + 1/n)/(a*n*gamma(3 + 1/n)) + d**2
*x*x**(2*n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2 + 1/n)*gamma(2 + 1/n)/(
a*n**2*gamma(3 + 1/n))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^n+c)^2}{bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^2/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^2/(b*x^n + a), x)
```

3.301 $\int \frac{c+dx^n}{a+bx^n}\,dx$

Optimal. Leaf size=42

$$\frac{x(bc-ad)\,_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{ab}+\frac{dx}{b}$$

```
[Out] (d*x)/b + ((b*c - a*d)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)
/a)])/(a*b)
```

Rubi [A] time = 0.0159626, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {388, 245}

$$\frac{x(bc-ad)\,_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{ab}+\frac{dx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^n)/(a + b*x^n),x]
```

```
[Out] (d*x)/b + ((b*c - a*d)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)
/a)])/(a*b)
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^n}{a+bx^n}\,dx &= \frac{dx}{b}-\frac{(-bc+ad)\int \frac{1}{a+bx^n}\,dx}{b}\\ &= \frac{dx}{b}+\frac{(bc-ad)x\,_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{ab}\end{aligned}$$

Mathematica [A] time = 0.0102025, size = 40, normalized size = 0.95

$$\frac{x\left((bc-ad)\,_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)+ad\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^n)/(a + b*x^n),x]
```

```
[Out] (x*(a*d + (b*c - a*d)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)
]))/(a*b)
```

Maple [F] time = 0.416, size = 0, normalized size = 0.

$$\int \frac{c + dx^n}{a + bx^n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^n)/(a+b*x^n),x)
```

```
[Out] int((c+d*x^n)/(a+b*x^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$(bc - ad) \int \frac{1}{b^2 x^n + ab}\, dx + \frac{dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] (b*c - a*d)*integrate(1/(b^2*x^n + a*b), x) + d*x/b
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dx^n + c}{bx^n + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] integral((d*x^n + c)/(b*x^n + a), x)
```

Sympy [C] time = 1.69671, size = 73, normalized size = 1.74

$$-\frac{dx\Phi\left(\frac{ax^{-n}e^{i\pi}}{b}, 1, \frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{bn^2\Gamma\left(1 + \frac{1}{n}\right)} + \frac{cx\Phi\left(\frac{bx^n e^{i\pi}}{a}, 1, \frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^2\Gamma\left(1 + \frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)/(a+b*x**n),x)
```

```
[Out] -d*x*lerchphi(a*x**(-n)*exp_polar(I*pi)/b, 1, exp_polar(I*pi)/n)*gamma(1/n)
/(b*n**2*gamma(1 + 1/n)) + c*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*g
```

```
amma(1/n)/(a*n**2*gamma(1 + 1/n))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^n + c}{bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)/(b*x^n + a), x)
```

3.302 $\int \frac{1}{(a+bx^n)(c+dx^n)} \, dx$

Optimal. Leaf size=72

$$\frac{bx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)}-\frac{dx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)}$$

```
[Out] (b*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*(b*c - a*d)
) - (d*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c*(b*c -
a*d))
```

Rubi [A] time = 0.0258087, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {391, 245}

$$\frac{bx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)}-\frac{dx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (b*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*(b*c - a*d)
) - (d*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c*(b*c -
a*d))
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx^n)(c+dx^n)} \, dx &= \frac{b\int \frac{1}{a+bx^n} \, dx}{bc-ad}-\frac{d\int \frac{1}{c+dx^n} \, dx}{bc-ad}\\
&= \frac{bx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)}-\frac{dx\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)}
\end{aligned}$$

Mathematica [A] time = 0.0340111, size = 64, normalized size = 0.89

$$\frac{x\left(ad\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)-bc\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)\right)}{ac(ad-bc)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^n)*(c + d*x^n)),x]
```

```
[Out] (x*(-(b*c*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)]) + a*d*Hyp
ergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)]))/(a*c*(-(b*c) + a*d))
```

Maple [F] time = 0.711, size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*x^n)/(c+d*x^n),x)
```

```
[Out] int(1/(a+b*x^n)/(c+d*x^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^n+a)(dx^n+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{bdx^{2n}+ac+(bc+ad)x^n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral(1/(b*d*x^(2*n) + a*c + (b*c + a*d)*x^n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^n)(c+dx^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**n)/(c+d*x**n),x)
```

```
[Out] Integral(1/((a + b*x**n)*(c + d*x**n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^n + a)(dx^n + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)), x)
```

3.303 $\int \frac{1}{(a+bx^n)(c+dx^n)^2}\,dx$

Optimal. Leaf size=123

$$\frac{b^2x\,_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)^2}+\frac{dx(bc(1-2n)-ad(1-n))\,_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2n(bc-ad)^2}-\frac{dx}{cn(bc-ad)\,(c+dx^n)}$$

```
[Out] -((d*x)/(c*(b*c - a*d)*n*(c + d*x^n))) + (b^2*x*Hypergeometric2F1[1, n^(-1)
, 1 + n^(-1), -((b*x^n)/a)])/(a*(b*c - a*d)^2) + (d*(b*c*(1 - 2*n) - a*d*(1
 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c^2*(b*c
- a*d)^2*n)
```

Rubi [A] time = 0.14596, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {414, 522, 245}

$$\frac{b^2x\,_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)^2}+\frac{dx(bc(1-2n)-ad(1-n))\,_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2n(bc-ad)^2}-\frac{dx}{cn(bc-ad)\,(c+dx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^n)*(c + d*x^n)^2),x]
```

```
[Out] -((d*x)/(c*(b*c - a*d)*n*(c + d*x^n))) + (b^2*x*Hypergeometric2F1[1, n^(-1)
, 1 + n^(-1), -((b*x^n)/a)])/(a*(b*c - a*d)^2) + (d*(b*c*(1 - 2*n) - a*d*(1
 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c^2*(b*c
- a*d)^2*n)
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx^n)(c+dx^n)^2}\,dx = -\frac{dx}{c(bc-ad)n(c+dx^n)} + \frac{\int \frac{bcn+a(d-dn)+bd(1-n)x^n}{(a+bx^n)(c+dx^n)}\,dx}{c(bc-ad)n}$$
$$= -\frac{dx}{c(bc-ad)n(c+dx^n)} + \frac{b^2\int \frac{1}{a+bx^n}\,dx}{(bc-ad)^2} - \frac{(d(ad(1-n)-b(c-2cn)))\int \frac{1}{c+dx^n}\,dx}{c(bc-ad)^2n}$$
$$= -\frac{dx}{c(bc-ad)n(c+dx^n)} + \frac{b^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)^2} + \frac{d(bc(1-2n)-ad(1-n))x\,{}_2F_1\left(1,\frac{1}{n};1\right.}{c^2(bc-ad)^2n}$$

Mathematica [A] time = 0.132505, size = 121, normalized size = 0.98

$$\frac{x\left(b^2c^2n(c+dx^n)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)+ad\left((c+dx^n)(ad(n-1)+b(c-2cn))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)+c(ad-bc)\right)\right)}{ac^2n(bc-ad)^2(c+dx^n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^n)*(c + d*x^n)^2),x]
```

```
[Out] (x*(b^2*c^2*n*(c + d*x^n)*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n
)/a)] + a*d*(c*(-(b*c) + a*d) + (a*d*(-1 + n) + b*(c - 2*c*n))*(c + d*x^n)*
Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])))/(a*c^2*(b*c - a*d
)^2*n*(c + d*x^n))
```

Maple [F] time = 0.721, size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^n)(c+dx^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*x^n)/(c+d*x^n)^2,x)
```

```
[Out] int(1/(a+b*x^n)/(c+d*x^n)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b^2\int \frac{1}{ab^2c^2-2\,a^2bcd+a^3d^2+\left(b^3c^2-2\,ab^2cd+a^2bd^2\right)x^n}\,dx-\left(bcd(2\,n-1)-ad^2(n-1)\right)\int \frac{}{b^2c^4n-2\,abc^3dn+a^2c^2d^2r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)/(c+d*x^n)^2,x, algorithm="maxima")
```

```
[Out] b^2*integrate(1/(a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2 + (b^3*c^2 - 2*a*b^2*c*d
 + a^2*b*d^2)*x^n), x) - (b*c*d*(2*n - 1) - a*d^2*(n - 1))*integrate(1/(b^2
*c^4*n - 2*a*b*c^3*d*n + a^2*c^2*d^2*n + (b^2*c^3*d*n - 2*a*b*c^2*d^2*n + a
^2*c*d^3*n)*x^n), x) - d*x/(b*c^3*n - a*c^2*d*n + (b*c^2*d*n - a*c*d^2*n)*x
^n)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{bd^2x^{3n}+ac^2+\left(2\,bcd+ad^2\right)x^{2n}+\left(bc^2+2\,acd\right)x^n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)/(c+d*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral(1/(b*d^2*x^(3*n) + a*c^2 + (2*b*c*d + a*d^2)*x^(2*n) + (b*c^2 + 2*
a*c*d)*x^n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(a+bx^n)\,(c+dx^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**n)/(c+d*x**n)**2,x)
```

```
[Out] Integral(1/((a + b*x**n)*(c + d*x**n)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(bx^n+a)(dx^n+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)/(c+d*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)^2), x)
```

3.304 $\int \frac{1}{(a+bx^n)(c+dx^n)^3} dx$

Optimal. Leaf size=210

$$-\frac{dx\left(a^2d^2\left(2n^2-3n+1\right)-2abcd\left(3n^2-4n+1\right)+b^2c^2\left(6n^2-5n+1\right)\right){}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{2c^3n^2(bc-ad)^3}+\frac{b^3x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{b\ldots}{}\right)}{a(bc-ad)^3}$$

```
[Out] -(d*x)/(2*c*(b*c - a*d)*n*(c + d*x^n)^2) - (d*(a*d*(1 - 2*n) - b*(c - 4*c*n
))*x)/(2*c^2*(b*c - a*d)^2*n^2*(c + d*x^n)) + (b^3*x*Hypergeometric2F1[1, n
^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*(b*c - a*d)^3) - (d*(a^2*d^2*(1 - 3*n
+ 2*n^2) - 2*a*b*c*d*(1 - 4*n + 3*n^2) + b^2*c^2*(1 - 5*n + 6*n^2))*x*Hyper
geometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(2*c^3*(b*c - a*d)^3*n^2
)
```

Rubi [A] time = 0.328671, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {414, 527, 522, 245}

$$-\frac{dx\left(a^2d^2\left(2n^2-3n+1\right)-2abcd\left(3n^2-4n+1\right)+b^2c^2\left(6n^2-5n+1\right)\right){}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{2c^3n^2(bc-ad)^3}+\frac{b^3x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{b\ldots}{}\right)}{a(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^n)*(c + d*x^n)^3),x]
```

```
[Out] -(d*x)/(2*c*(b*c - a*d)*n*(c + d*x^n)^2) - (d*(a*d*(1 - 2*n) - b*(c - 4*c*n
))*x)/(2*c^2*(b*c - a*d)^2*n^2*(c + d*x^n)) + (b^3*x*Hypergeometric2F1[1, n
^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*(b*c - a*d)^3) - (d*(a^2*d^2*(1 - 3*n
+ 2*n^2) - 2*a*b*c*d*(1 - 4*n + 3*n^2) + b^2*c^2*(1 - 5*n + 6*n^2))*x*Hyper
geometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(2*c^3*(b*c - a*d)^3*n^2
)
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
```

```
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx^n)(c+dx^n)^3}dx = -\frac{dx}{2c(bc-ad)n(c+dx^n)^2} + \frac{\int \frac{2bcn+a(d-2dn)+bd(1-2n)x^n}{(a+bx^n)(c+dx^n)^2}dx}{2c(bc-ad)n}$$

$$= -\frac{dx}{2c(bc-ad)n(c+dx^n)^2} + \frac{d(bc(1-4n)-ad(1-2n))x}{2c^2(bc-ad)^2n^2(c+dx^n)} + \frac{\int \frac{2b^2c^2n^2+a^2d^2(1-3n+2n^2)-abcd(1-}{(a+b}}{2c^2(b}$$

$$= -\frac{dx}{2c(bc-ad)n(c+dx^n)^2} + \frac{d(bc(1-4n)-ad(1-2n))x}{2c^2(bc-ad)^2n^2(c+dx^n)} + \frac{b^3\int \frac{1}{a+bx^n}dx}{(bc-ad)^3} - \frac{(d(a^2d^2(1-3}{}$$

$$= -\frac{dx}{2c(bc-ad)n(c+dx^n)^2} + \frac{d(bc(1-4n)-ad(1-2n))x}{2c^2(bc-ad)^2n^2(c+dx^n)} + \frac{b^3x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a(bc-ad)^3} - \ldots$$

Mathematica [A] time = 0.197811, size = 210, normalized size = 1.

$$\frac{x\left(-ad(c+dx^n)^2\left(a^2d^2\left(2n^2-3n+1\right)-2abcd\left(3n^2-4n+1\right)+b^2c^2\left(6n^2-5n+1\right)\right){}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)+2b^3c^3n\ldots}{2ac^3n^2(bc-ad)^3(c+\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^n)*(c + d*x^n)^3),x]
```

```
[Out] (x*(-(a*c^2*d*(b*c - a*d)^2*n) + a*c*d*(b*c - a*d)*(a*d*(-1 + 2*n) + b*(c -
 4*c*n))*(c + d*x^n) + 2*b^3*c^3*n^2*(c + d*x^n)^2*Hypergeometric2F1[1, n^(
-1), 1 + n^(-1), -((b*x^n)/a)] - a*d*(a^2*d^2*(1 - 3*n + 2*n^2) - 2*a*b*c*d
*(1 - 4*n + 3*n^2) + b^2*c^2*(1 - 5*n + 6*n^2))*(c + d*x^n)^2*Hypergeometri
c2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)]))/(2*a*c^3*(b*c - a*d)^3*n^2*(c +
 d*x^n)^2)
```

Maple [F] time = 0.757, size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^n)(c+dx^n)^3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*x^n)/(c+d*x^n)^3,x)
```

```
[Out] int(1/(a+b*x^n)/(c+d*x^n)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-b^3 \int -\frac{1}{ab^3c^3 - 3\,a^2b^2c^2d + 3\,a^3bcd^2 - a^4d^3 + \left(b^4c^3 - 3\,ab^3c^2d + 3\,a^2b^2cd^2 - a^3bd^3\right)x^n}\,dx + \left(\left(6\,n^2 - 5\,n + 1\right)b^2c^2d - 2\,(3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)/(c+d*x^n)^3,x, algorithm="maxima")
```

```
[Out] -b^3*integrate(-1/(a*b^3*c^3 - 3*a^2*b^2*c^2*d + 3*a^3*b*c*d^2 - a^4*d^3 +
(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*x^n), x) + ((6*n^2
- 5*n + 1)*b^2*c^2*d - 2*(3*n^2 - 4*n + 1)*a*b*c*d^2 + (2*n^2 - 3*n + 1)*a^
2*d^3)*integrate(-1/2/(b^3*c^6*n^2 - 3*a*b^2*c^5*d*n^2 + 3*a^2*b*c^4*d^2*n^
2 - a^3*c^3*d^3*n^2 + (b^3*c^5*d*n^2 - 3*a*b^2*c^4*d^2*n^2 + 3*a^2*b*c^3*d^
3*n^2 - a^3*c^2*d^4*n^2)*x^n), x) - 1/2*((b*c*d^2*(4*n - 1) - a*d^3*(2*n -
1))*x*x^n + (b*c^2*d*(5*n - 1) - a*c*d^2*(3*n - 1))*x)/(b^2*c^6*n^2 - 2*a*b
*c^5*d*n^2 + a^2*c^4*d^2*n^2 + (b^2*c^4*d^2*n^2 - 2*a*b*c^3*d^3*n^2 + a^2*c
^2*d^4*n^2)*x^(2*n) + 2*(b^2*c^5*d*n^2 - 2*a*b*c^4*d^2*n^2 + a^2*c^3*d^3*n^
2)*x^n)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{bd^3x^{4\,n} + ac^3 + \left(3\,bcd^2 + ad^3\right)x^{3\,n} + 3\,\left(bc^2d + acd^2\right)x^{2\,n} + \left(bc^3 + 3\,ac^2d\right)x^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)/(c+d*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral(1/(b*d^3*x^(4*n) + a*c^3 + (3*b*c*d^2 + a*d^3)*x^(3*n) + 3*(b*c^2*
d + a*c*d^2)*x^(2*n) + (b*c^3 + 3*a*c^2*d)*x^n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + bx^n)\,(c + dx^n)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**n)/(c+d*x**n)**3,x)
```

```
[Out] Integral(1/((a + b*x**n)*(c + d*x**n)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^n + a)(dx^n + c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)/(c+d*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)^3), x)
```

3.305 $\int \frac{(c+dx^n)^4}{(a+bx^n)^2}\,dx$

Optimal. Leaf size=341

$$-\frac{dx\,(c+dx^n)\left(a^2d^2\left(6n^2+5n+1\right)-2abcd\left(5n^2+4n+1\right)+b^2c^2\left(2n^2+3n+1\right)\right)}{ab^3n(n+1)(2n+1)}-\frac{dx\left(a^2bcd^2\left(16n^3+26n^2+15n+\right.\right.}{}$$

```
[Out] -((d*(b^3*c^3*(1 + 3*n + 2*n^2) - a^3*d^3*(1 + 6*n + 11*n^2 + 6*n^3) - a*b^
2*c^2*d*(3 + 12*n + 17*n^2 + 12*n^3) + a^2*b*c*d^2*(3 + 15*n + 26*n^2 + 16*
n^3))*x)/(a*b^4*n*(1 + n)*(1 + 2*n))) - (d*(b^2*c^2*(1 + 3*n + 2*n^2) - 2*a
*b*c*d*(1 + 4*n + 5*n^2) + a^2*d^2*(1 + 5*n + 6*n^2))*x*(c + d*x^n))/(a*b^3
*n*(1 + n)*(1 + 2*n)) + (d*(a*d*(1 + 3*n) - b*(c + 2*c*n))*x*(c + d*x^n)^2)
/(a*b^2*n*(1 + 2*n)) + ((b*c - a*d)*x*(c + d*x^n)^3)/(a*b*n*(a + b*x^n)) -
((b*c - a*d)^3*(b*c*(1 - n) - a*d*(1 + 3*n))*x*Hypergeometric2F1[1, n^(-1),
 1 + n^(-1), -((b*x^n)/a)])/(a^2*b^4*n)
```

Rubi [A] time = 0.547905, antiderivative size = 341, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {413, 528, 388, 245}

$$-\frac{dx\,(c+dx^n)\left(a^2d^2\left(6n^2+5n+1\right)-2abcd\left(5n^2+4n+1\right)+b^2c^2\left(2n^2+3n+1\right)\right)}{ab^3n(n+1)(2n+1)}-\frac{dx\left(a^2bcd^2\left(16n^3+26n^2+15n+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^n)^4/(a + b*x^n)^2,x]
```

```
[Out] -((d*(b^3*c^3*(1 + 3*n + 2*n^2) - a^3*d^3*(1 + 6*n + 11*n^2 + 6*n^3) - a*b^
2*c^2*d*(3 + 12*n + 17*n^2 + 12*n^3) + a^2*b*c*d^2*(3 + 15*n + 26*n^2 + 16*
n^3))*x)/(a*b^4*n*(1 + n)*(1 + 2*n))) - (d*(b^2*c^2*(1 + 3*n + 2*n^2) - 2*a
*b*c*d*(1 + 4*n + 5*n^2) + a^2*d^2*(1 + 5*n + 6*n^2))*x*(c + d*x^n))/(a*b^3
*n*(1 + n)*(1 + 2*n)) + (d*(a*d*(1 + 3*n) - b*(c + 2*c*n))*x*(c + d*x^n)^2)
/(a*b^2*n*(1 + 2*n)) + ((b*c - a*d)*x*(c + d*x^n)^3)/(a*b*n*(a + b*x^n)) -
((b*c - a*d)^3*(b*c*(1 - n) - a*d*(1 + 3*n))*x*Hypergeometric2F1[1, n^(-1),
 1 + n^(-1), -((b*x^n)/a)])/(a^2*b^4*n)
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \frac{(c+dx^n)^4}{(a+bx^n)^2}\,dx = \frac{(bc-ad)x\,(c+dx^n)^3}{abn\,(a+bx^n)} + \frac{\int \frac{(c+dx^n)^2(c(ad-bc(1-n))+d(ad(1+3n)-b(c+2cn))x^n)}{a+bx^n}\,dx}{abn}$$

$$= \frac{d(ad(1+3n)-b(c+2cn))x\,(c+dx^n)^2}{ab^2n(1+2n)} + \frac{(bc-ad)x\,(c+dx^n)^3}{abn\,(a+bx^n)} + \frac{\int \frac{(c+dx^n)\left(c\left(2abcd(1+2n)-a^2d^2(1+3n)-b^2c^2(1\right.\right.}{\ldots}}{\ldots}$$

$$= -\frac{d\left(b^2c^2\left(1+3n+2n^2\right)-2abcd\left(1+4n+5n^2\right)+a^2d^2\left(1+5n+6n^2\right)\right)x\,(c+dx^n)}{ab^3n(1+n)(1+2n)} + \frac{d(ad(1+3n)-\ldots}{ab^2\ldots}$$

$$= -\frac{d\left(b^3c^3\left(1+3n+2n^2\right)-a^3d^3\left(1+6n+11n^2+6n^3\right)-ab^2c^2d\left(3+12n+17n^2+12n^3\right)+a^2bcd^2\left(3\ldots\right.\right.}{ab^4n(1+n)(1+2n)}$$

$$= -\frac{d\left(b^3c^3\left(1+3n+2n^2\right)-a^3d^3\left(1+6n+11n^2+6n^3\right)-ab^2c^2d\left(3+12n+17n^2+12n^3\right)+a^2bcd^2\left(3\ldots\right.\right.}{ab^4n(1+n)(1+2n)}$$

Mathematica [A] time = 5.18974, size = 217, normalized size = 0.64

$$\frac{x\left(\frac{-6a^2b^2c^2d^2+4a^3bcd^3-a^4d^4+4ab^3c^3d+b^4c^4(n-1)}{a^2n} + \frac{(bc-ad)^3(ad(3n+1)+bc(n-1))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2n} + \frac{(ad-bc)^3(ad(3n+1)+bc(n-1))}{a^2n} + \frac{2bd^3x^n(2bc-ad)}{n+1}\ldots\right.}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^n)^4/(a + b*x^n)^2,x]
```

```
[Out] (x*((4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4 + b^4*c^4*
(-1 + n))/(a^2*n) + ((-(b*c) + a*d)^3*(b*c*(-1 + n) + a*d*(1 + 3*n)))/(a^2*
n) + (2*b*d^3*(2*b*c - a*d)*x^n)/(1 + n) + (b^2*d^4*x^(2*n))/(1 + 2*n) + (b
*c - a*d)^4/(a*n*(a + b*x^n)) + ((b*c - a*d)^3*(b*c*(-1 + n) + a*d*(1 + 3*n
))*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*n)))/b^4
```

Maple [F] time = 0.379, size = 0, normalized size = 0.

$$\int \frac{(c+dx^n)^4}{(a+bx^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^n)^4/(a+b*x^n)^2,x)
```

```
[Out] int((c+d*x^n)^4/(a+b*x^n)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\left(a^4d^4(3n+1)-4a^3bcd^3(2n+1)+6a^2b^2c^2d^2(n+1)-b^4c^4(n-1)-4ab^3c^3d\right)\int\frac{1}{ab^5nx^n+a^2b^4n}\,dx+\frac{(n^2+n)ab^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^4/(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] -(a^4*d^4*(3*n + 1) - 4*a^3*b*c*d^3*(2*n + 1) + 6*a^2*b^2*c^2*d^2*(n + 1) -
 b^4*c^4*(n - 1) - 4*a*b^3*c^3*d)*integrate(1/(a*b^5*n*x^n + a^2*b^4*n), x)
 + ((n^2 + n)*a*b^3*d^4*x*x^(3*n) + (4*(2*n^2 + n)*a*b^3*c*d^3 - (3*n^2 + n
)*a^2*b^2*d^4)*x*x^(2*n) + (6*(2*n^3 + 3*n^2 + n)*a*b^3*c^2*d^2 - 4*(4*n^3
+ 4*n^2 + n)*a^2*b^2*c*d^3 + (6*n^3 + 5*n^2 + n)*a^3*b*d^4)*x*x^n + ((2*n^2
 + 3*n + 1)*b^4*c^4 - 4*(2*n^2 + 3*n + 1)*a*b^3*c^3*d + 6*(2*n^3 + 5*n^2 +
4*n + 1)*a^2*b^2*c^2*d^2 - 4*(4*n^3 + 8*n^2 + 5*n + 1)*a^3*b*c*d^3 + (6*n^3
 + 11*n^2 + 6*n + 1)*a^4*d^4)*x)/((2*n^3 + 3*n^2 + n)*a*b^5*x^n + (2*n^3 +
3*n^2 + n)*a^2*b^4)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{d^4x^{4n}+4cd^3x^{3n}+6c^2d^2x^{2n}+4c^3dx^n+c^4}{b^2x^{2n}+2abx^n+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^4/(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral((d^4*x^(4*n) + 4*c*d^3*x^(3*n) + 6*c^2*d^2*x^(2*n) + 4*c^3*d*x^n +
 c^4)/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)**4/(a+b*x**n)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(dx^n+c)^4}{(bx^n+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^4/(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^4/(b*x^n + a)^2, x)
```

3.306 $\int \frac{(c+dx^n)^3}{(a+bx^n)^2} \, dx$

Optimal. Leaf size=200

$$-\frac{dx\left(a^2d^2\left(2n^2+3n+1\right)-abcd\left(3n^2+4n+2\right)+b^2c^2(n+1)\right)}{ab^3n(n+1)}-\frac{x(bc-ad)^2(bc(1-n)-ad(2n+1))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};\right.}{a^2b^3n}$$

```
[Out] -((d*(b^2*c^2*(1 + n) + a^2*d^2*(1 + 3*n + 2*n^2) - a*b*c*d*(2 + 4*n + 3*n^
2))*x)/(a*b^3*n*(1 + n))) - (d*(b*c*(1 + n) - a*d*(1 + 2*n))*x*(c + d*x^n))
/(a*b^2*n*(1 + n)) + ((b*c - a*d)*x*(c + d*x^n)^2)/(a*b*n*(a + b*x^n)) - ((
b*c - a*d)^2*(b*c*(1 - n) - a*d*(1 + 2*n))*x*Hypergeometric2F1[1, n^(-1), 1
 + n^(-1), -((b*x^n)/a)])/(a^2*b^3*n)
```

Rubi [A] time = 0.259678, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {413, 528, 388, 245}

$$-\frac{dx\left(a^2d^2\left(2n^2+3n+1\right)-abcd\left(3n^2+4n+2\right)+b^2c^2(n+1)\right)}{ab^3n(n+1)}-\frac{x(bc-ad)^2(bc(1-n)-ad(2n+1))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};\right.}{a^2b^3n}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^n)^3/(a + b*x^n)^2,x]
```

```
[Out] -((d*(b^2*c^2*(1 + n) + a^2*d^2*(1 + 3*n + 2*n^2) - a*b*c*d*(2 + 4*n + 3*n^
2))*x)/(a*b^3*n*(1 + n))) - (d*(b*c*(1 + n) - a*d*(1 + 2*n))*x*(c + d*x^n))
/(a*b^2*n*(1 + n)) + ((b*c - a*d)*x*(c + d*x^n)^2)/(a*b*n*(a + b*x^n)) - ((
b*c - a*d)^2*(b*c*(1 - n) - a*d*(1 + 2*n))*x*Hypergeometric2F1[1, n^(-1), 1
 + n^(-1), -((b*x^n)/a)])/(a^2*b^3*n)
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \frac{(c+dx^n)^3}{(a+bx^n)^2}\,dx = \frac{(bc-ad)x\,(c+dx^n)^2}{abn\,(a+bx^n)} + \frac{\int \frac{(c+dx^n)(c(ad-bc(1-n))-d(bc(1+n)-ad(1+2n))x^n)}{a+bx^n}\,dx}{abn}$$

$$= -\frac{d(bc(1+n)-ad(1+2n))x\,(c+dx^n)}{ab^2n(1+n)} + \frac{(bc-ad)x\,(c+dx^n)^2}{abn\,(a+bx^n)} + \frac{\int \frac{c\left(2abcd(1+n)-a^2d^2(1+2n)-b^2c^2\left(1-n^2\right)\right)-d\left(b\right.}{a}}{ab^2 n}$$

$$= -\frac{d\left(b^2c^2(1+n)+a^2d^2\left(1+3n+2n^2\right)-abcd\left(2+4n+3n^2\right)\right)x}{ab^3n(1+n)} - \frac{d(bc(1+n)-ad(1+2n))x\,(c+dx^n}{ab^2n(1+n)}$$

$$= -\frac{d\left(b^2c^2(1+n)+a^2d^2\left(1+3n+2n^2\right)-abcd\left(2+4n+3n^2\right)\right)x}{ab^3n(1+n)} - \frac{d(bc(1+n)-ad(1+2n))x\,(c+dx^n}{ab^2n(1+n)}$$

Mathematica [C] time = 4.2095, size = 2050, normalized size = 10.25

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^n)^3/(a + b*x^n)^2,x]
```

```
[Out] (x*(3*a*(1 + 10*n + 35*n^2 + 50*n^3 + 24*n^4)*(c^3*(1 + n)^4 + 3*c^2*d*(1 +
 4*n + 6*n^2 + 2*n^3 + n^4)*x^n + 3*c*d^2*(1 + n)^4*x^(2*n) + d^3*(1 + n)^4
*x^(3*n))*HurwitzLerchPhi[-((b*x^n)/a), 1, 1 + n^(-1)] - 3*a*(1 + 10*n + 35
*n^2 + 50*n^3 + 24*n^4)*(c^3*(1 + 2*n)^4 + 3*c^2*d*(1 + 2*n)^4*x^n + 3*c*d^
2*(1 + 8*n + 24*n^2 + 34*n^3 + 18*n^4)*x^(2*n) + d^3*(1 + 2*n)^4*x^(3*n))*H
urwitzLerchPhi[-((b*x^n)/a), 1, 2 + n^(-1)] + a*c^3*HurwitzLerchPhi[-((b*x^
n)/a), 1, 3 + n^(-1)] + 22*a*c^3*n*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(
-1)] + 209*a*c^3*n^2*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 1118*a*
c^3*n^3*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 3675*a*c^3*n^4*Hurwi
tzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 7578*a*c^3*n^5*HurwitzLerchPhi[-(
(b*x^n)/a), 1, 3 + n^(-1)] + 9531*a*c^3*n^6*HurwitzLerchPhi[-((b*x^n)/a), 1
, 3 + n^(-1)] + 6642*a*c^3*n^7*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)]
 + 1944*a*c^3*n^8*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 3*a*c^2*d*
x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 66*a*c^2*d*n*x^n*Hurwitz
LerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 627*a*c^2*d*n^2*x^n*HurwitzLerchPhi
[-((b*x^n)/a), 1, 3 + n^(-1)] + 3354*a*c^2*d*n^3*x^n*HurwitzLerchPhi[-((b*x
^n)/a), 1, 3 + n^(-1)] + 11025*a*c^2*d*n^4*x^n*HurwitzLerchPhi[-((b*x^n)/a)
, 1, 3 + n^(-1)] + 22734*a*c^2*d*n^5*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, 3
 + n^(-1)] + 28593*a*c^2*d*n^6*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(
-1)] + 19926*a*c^2*d*n^7*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] +
 5832*a*c^2*d*n^8*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 3*a*c*
d^2*x^(2*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 66*a*c*d^2*n*x^(
2*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 627*a*c*d^2*n^2*x^(2*n)
*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 3354*a*c*d^2*n^3*x^(2*n)*Hu
rwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 11025*a*c*d^2*n^4*x^(2*n)*Hurw
itzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 22734*a*c*d^2*n^5*x^(2*n)*Hurwit
zLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 28593*a*c*d^2*n^6*x^(2*n)*HurwitzL
erchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 19926*a*c*d^2*n^7*x^(2*n)*HurwitzLer
```

```
chPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 5832*a*c*d^2*n^8*x^(2*n)*HurwitzLerchP
hi[-((b*x^n)/a), 1, 3 + n^(-1)] + a*d^3*x^(3*n)*HurwitzLerchPhi[-((b*x^n)/a
), 1, 3 + n^(-1)] + 22*a*d^3*n*x^(3*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 +
 n^(-1)] + 209*a*d^3*n^2*x^(3*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1
)] + 1112*a*d^3*n^3*x^(3*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] +
3603*a*d^3*n^4*x^(3*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 7248*
a*d^3*n^5*x^(3*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 8811*a*d^3
*n^6*x^(3*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 5898*a*d^3*n^7*
x^(3*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] + 1656*a*d^3*n^8*x^(3*
n)*HurwitzLerchPhi[-((b*x^n)/a), 1, 3 + n^(-1)] - a*c^3*HurwitzLerchPhi[-((
b*x^n)/a), 1, n^(-1)] - 10*a*c^3*n*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)]
 - 35*a*c^3*n^2*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] - 56*a*c^3*n^3*Hur
witzLerchPhi[-((b*x^n)/a), 1, n^(-1)] - 78*a*c^3*n^4*HurwitzLerchPhi[-((b*x
^n)/a), 1, n^(-1)] - 150*a*c^3*n^5*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)]
 - 90*a*c^3*n^6*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 156*a*c^3*n^7*Hu
rwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 144*a*c^3*n^8*HurwitzLerchPhi[-((b
*x^n)/a), 1, n^(-1)] - 3*a*c^2*d*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1
)] - 30*a*c^2*d*n*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] - 105*a*c^2*
d*n^2*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] - 150*a*c^2*d*n^3*x^n*Hu
rwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] - 72*a*c^2*d*n^4*x^n*HurwitzLerchPhi
[-((b*x^n)/a), 1, n^(-1)] - 3*a*c*d^2*x^(2*n)*HurwitzLerchPhi[-((b*x^n)/a),
 1, n^(-1)] - 30*a*c*d^2*n*x^(2*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)]
 - 105*a*c*d^2*n^2*x^(2*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] - 150*a
*c*d^2*n^3*x^(2*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] - 72*a*c*d^2*n^
4*x^(2*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] - a*d^3*x^(3*n)*HurwitzL
erchPhi[-((b*x^n)/a), 1, n^(-1)] - 10*a*d^3*n*x^(3*n)*HurwitzLerchPhi[-((b*
x^n)/a), 1, n^(-1)] - 35*a*d^3*n^2*x^(3*n)*HurwitzLerchPhi[-((b*x^n)/a), 1,
 n^(-1)] - 50*a*d^3*n^3*x^(3*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] -
24*a*d^3*n^4*x^(3*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] - 6*b*c^3*n^8
*x^n*HypergeometricPFQ[{2, 2, 2, 2, 1 + n^(-1)}, {1, 1, 1, 5 + n^(-1)}, -((
b*x^n)/a)] - 18*b*c^2*d*n^8*x^(2*n)*HypergeometricPFQ[{2, 2, 2, 2, 1 + n^(-
1)}, {1, 1, 1, 5 + n^(-1)}, -((b*x^n)/a)] - 18*b*c*d^2*n^8*x^(3*n)*Hypergeo
metricPFQ[{2, 2, 2, 2, 1 + n^(-1)}, {1, 1, 1, 5 + n^(-1)}, -((b*x^n)/a)] -
6*b*d^3*n^8*x^(4*n)*HypergeometricPFQ[{2, 2, 2, 2, 1 + n^(-1)}, {1, 1, 1, 5
 + n^(-1)}, -((b*x^n)/a)]))/(6*a^3*n^5*(1 + n)*(1 + 2*n)*(1 + 3*n)*(1 + 4*n
))
```

Maple [F] time = 0.368, size = 0, normalized size = 0.

$$\int \frac{(c+dx^n)^3}{(a+bx^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^n)^3/(a+b*x^n)^2,x)

[Out] int((c+d*x^n)^3/(a+b*x^n)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\left(a^3d^3(2n+1)-3a^2bcd^2(n+1)+b^3c^3(n-1)+3ab^2c^2d\right)\int \frac{1}{ab^4nx^n+a^2b^3n}\,dx+\frac{ab^2d^3nxx^{2n}+\left(3\left(n^2+n\right)ab^2cd^2-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^3/(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] (a^3*d^3*(2*n + 1) - 3*a^2*b*c*d^2*(n + 1) + b^3*c^3*(n - 1) + 3*a*b^2*c^2*
d)*integrate(1/(a*b^4*n*x^n + a^2*b^3*n), x) + (a*b^2*d^3*n*x*x^(2*n) + (3*
(n^2 + n)*a*b^2*c*d^2 - (2*n^2 + n)*a^2*b*d^3)*x*x^n + (3*(n^2 + 2*n + 1)*a
^2*b*c*d^2 - (2*n^2 + 3*n + 1)*a^3*d^3 + b^3*c^3*(n + 1) - 3*a*b^2*c^2*d*(n
 + 1))*x)/((n^2 + n)*a*b^4*x^n + (n^2 + n)*a^2*b^3)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{d^3x^{3\,n} + 3\,cd^2x^{2\,n} + 3\,c^2dx^n + c^3}{b^2x^{2\,n} + 2\,abx^n + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^3/(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral((d^3*x^(3*n) + 3*c*d^2*x^(2*n) + 3*c^2*d*x^n + c^3)/(b^2*x^(2*n) +
 2*a*b*x^n + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)**3/(a+b*x**n)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^n + c)^3}{(bx^n + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^3/(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^3/(b*x^n + a)^2, x)
```

3.307 $\int \frac{(c+dx^n)^2}{(a+bx^n)^2}\,dx$

Optimal. Leaf size=115

$$-\frac{x(bc-ad)(bc(1-n)-ad(n+1))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2b^2n}-\frac{dx(bc-ad(n+1))}{ab^2n}+\frac{x(bc-ad)\left(c+dx^n\right)}{abn\left(a+bx^n\right)}$$

```
[Out] -((d*(b*c - a*d*(1 + n))*x)/(a*b^2*n)) + ((b*c - a*d)*x*(c + d*x^n))/(a*b*n
*(a + b*x^n)) - ((b*c - a*d)*(b*c*(1 - n) - a*d*(1 + n))*x*Hypergeometric2F
1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*b^2*n)
```

Rubi [A] time = 0.0939848, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {413, 388, 245}

$$-\frac{x(bc-ad)(bc(1-n)-ad(n+1))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2b^2n}-\frac{dx(bc-ad(n+1))}{ab^2n}+\frac{x(bc-ad)\left(c+dx^n\right)}{abn\left(a+bx^n\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^n)^2/(a + b*x^n)^2,x]
```

```
[Out] -((d*(b*c - a*d*(1 + n))*x)/(a*b^2*n)) + ((b*c - a*d)*x*(c + d*x^n))/(a*b*n
*(a + b*x^n)) - ((b*c - a*d)*(b*c*(1 - n) - a*d*(1 + n))*x*Hypergeometric2F
1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*b^2*n)
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \frac{(c+dx^n)^2}{(a+bx^n)^2}\,dx = \frac{(bc-ad)x\,(c+dx^n)}{abn\,(a+bx^n)} + \frac{\int \frac{c(ad-bc(1-n))-d(bc-ad(1+n))x^n}{a+bx^n}\,dx}{abn}$$
$$= -\frac{d(bc-ad(1+n))x}{ab^2n} + \frac{(bc-ad)x\,(c+dx^n)}{abn\,(a+bx^n)} - \frac{((bc-ad)(bc(1-n)-ad(1+n)))\int \frac{1}{a+bx^n}\,dx}{ab^2n}$$
$$= -\frac{d(bc-ad(1+n))x}{ab^2n} + \frac{(bc-ad)x\,(c+dx^n)}{abn\,(a+bx^n)} - \frac{(bc-ad)(bc(1-n)-ad(1+n))x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2b^2n}$$

Mathematica [C] time = 1.57463, size = 666, normalized size = 5.79

$$\frac{x\left(-2bc^2n^6x^n\text{HypergeometricPFQ}\left(\left\{2,2,2,\frac{1}{n}+1\right\},\left\{1,1,\frac{1}{n}+4\right\},-\frac{bx^n}{a}\right)-4bcdn^6x^{2n}\text{HypergeometricPFQ}\left(\left\{2,2,2,\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^n)^2/(a + b*x^n)^2,x]
```

```
[Out] (x*(-2*a*(1 + 6*n + 11*n^2 + 6*n^3)*(c^2*(1 + n)^3 + 2*c*d*(1 + 3*n + 4*n^2
 + n^3)*x^n + d^2*(1 + n)^3*x^(2*n))*HurwitzLerchPhi[-((b*x^n)/a), 1, 1 + n
^(-1)] + a*(1 + 6*n + 11*n^2 + 6*n^3)*(c^2*(1 + 2*n)^3 + 2*c*d*(1 + 2*n)^3*
x^n + d^2*(1 + 6*n + 10*n^2 + 6*n^3)*x^(2*n))*HurwitzLerchPhi[-((b*x^n)/a),
 1, 2 + n^(-1)] + a*c^2*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 6*a*c^2*
n*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 9*a*c^2*n^2*HurwitzLerchPhi[-(
(b*x^n)/a), 1, n^(-1)] - 4*a*c^2*n^3*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1
)] - 10*a*c^2*n^4*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 10*a*c^2*n^5*H
urwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 12*a*c^2*n^6*HurwitzLerchPhi[-((b
*x^n)/a), 1, n^(-1)] + 2*a*c*d*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)]
 + 12*a*c*d*n*x^n*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 22*a*c*d*n^2*x
^n*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 12*a*c*d*n^3*x^n*HurwitzLerch
Phi[-((b*x^n)/a), 1, n^(-1)] + a*d^2*x^(2*n)*HurwitzLerchPhi[-((b*x^n)/a),
1, n^(-1)] + 6*a*d^2*n*x^(2*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 1
1*a*d^2*n^2*x^(2*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] + 6*a*d^2*n^3*
x^(2*n)*HurwitzLerchPhi[-((b*x^n)/a), 1, n^(-1)] - 2*b*c^2*n^6*x^n*Hypergeo
metricPFQ[{2, 2, 2, 1 + n^(-1)}, {1, 1, 4 + n^(-1)}, -((b*x^n)/a)] - 4*b*c*
d*n^6*x^(2*n)*HypergeometricPFQ[{2, 2, 2, 1 + n^(-1)}, {1, 1, 4 + n^(-1)},
-((b*x^n)/a)] - 2*b*d^2*n^6*x^(3*n)*HypergeometricPFQ[{2, 2, 2, 1 + n^(-1)}
, {1, 1, 4 + n^(-1)}, -((b*x^n)/a)]))/(2*a^3*n^4*(1 + 6*n + 11*n^2 + 6*n^3)
)
```

Maple [F] time = 0.369, size = 0, normalized size = 0.

$$\int \frac{(c+dx^n)^2}{(a+bx^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^n)^2/(a+b*x^n)^2,x)
```

```
[Out] int((c+d*x^n)^2/(a+b*x^n)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\left(a^2d^2(n+1) - b^2c^2(n-1) - 2\,abcd\right)\int \frac{1}{ab^3nx^n + a^2b^2n}\,dx + \frac{abd^2nxx^n + \left(a^2d^2(n+1) + b^2c^2 - 2\,abcd\right)x}{ab^3nx^n + a^2b^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^2/(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] -(a^2*d^2*(n + 1) - b^2*c^2*(n - 1) - 2*a*b*c*d)*integrate(1/(a*b^3*n*x^n +
 a^2*b^2*n), x) + (a*b*d^2*n*x*x^n + (a^2*d^2*(n + 1) + b^2*c^2 - 2*a*b*c*d
)*x)/(a*b^3*n*x^n + a^2*b^2*n)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{d^2x^{2\,n} + 2\,cdx^n + c^2}{b^2x^{2\,n} + 2\,abx^n + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^2/(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral((d^2*x^(2*n) + 2*c*d*x^n + c^2)/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c + dx^n)^2}{(a + bx^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)**2/(a+b*x**n)**2,x)
```

```
[Out] Integral((c + d*x**n)**2/(a + b*x**n)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^n + c)^2}{(bx^n + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^2/(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^2/(b*x^n + a)^2, x)
```

3.308 $\int \frac{c+dx^n}{(a+bx^n)^2}\,dx$

Optimal. Leaf size=72

$$\frac{x(ad-bc(1-n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2bn}+\frac{x(bc-ad)}{abn\,(a+bx^n)}$$

```
[Out] ((b*c - a*d)*x)/(a*b*n*(a + b*x^n)) + ((a*d - b*c*(1 - n))*x*Hypergeometric
2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*b*n)
```

Rubi [A] time = 0.0307196, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {385, 245}

$$\frac{x(ad-bc(1-n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2bn}+\frac{x(bc-ad)}{abn\,(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^n)/(a + b*x^n)^2,x]
```

```
[Out] ((b*c - a*d)*x)/(a*b*n*(a + b*x^n)) + ((a*d - b*c*(1 - n))*x*Hypergeometric
2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*b*n)
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^n}{(a+bx^n)^2}\,dx &= \frac{(bc-ad)x}{abn\,(a+bx^n)}+\frac{(ad-bc(1-n))\int \frac{1}{a+bx^n}\,dx}{abn}\\ &= \frac{(bc-ad)x}{abn\,(a+bx^n)}+\frac{(ad-bc(1-n))x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2bn}\end{aligned}$$

Mathematica [A] time = 0.0389247, size = 56, normalized size = 0.78

$$\frac{x\left(\frac{d}{a+bx^n}-\frac{(ad+bc(n-1))\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2}\right)}{b-bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^n)/(a + b*x^n)^2,x]
```

```
[Out] (x*(d/(a + b*x^n) - ((a*d + b*c*(-1 + n))*Hypergeometric2F1[2, n^(-1), 1 +
n^(-1), -((b*x^n)/a)])/a^2))/(b - b*n)
```

Maple [F] time = 0.356, size = 0, normalized size = 0.

$$\int \frac{c + dx^n}{(a + bx^n)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^n)/(a+b*x^n)^2,x)
```

```
[Out] int((c+d*x^n)/(a+b*x^n)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$(bc(n-1) + ad) \int \frac{1}{ab^2nx^n + a^2bn} \, dx + \frac{(bc - ad)x}{ab^2nx^n + a^2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)/(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] (b*c*(n - 1) + a*d)*integrate(1/(a*b^2*n*x^n + a^2*b*n), x) + (b*c - a*d)*x
/(a*b^2*n*x^n + a^2*b*n)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{dx^n + c}{b^2x^{2n} + 2\,abx^n + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)/(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x^n + c)/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Sympy [C] time = 4.51472, size = 592, normalized size = 8.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)/(a+b*x**n)**2,x)
```

```
[Out] c*(n*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma(1/n)/(a*(a*n**3*gam
ma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n))) + n*x*gamma(1/n)/(a*(a*n**3*gamm
a(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n))) - x*lerchphi(b*x**n*exp_polar(I*p
i)/a, 1, 1/n)*gamma(1/n)/(a*(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 +
1/n))) + b*n*x*x**n*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma(1/n)/(
a**2*(a*n**3*gamma(1 + 1/n) + b*n**3*x**n*gamma(1 + 1/n))) - b*x*x**n*lerch
phi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma(1/n)/(a**2*(a*n**3*gamma(1 + 1/
n) + b*n**3*x**n*gamma(1 + 1/n)))) + d*(n**2*x*x**n*gamma(1 + 1/n)/(a*(a*n*
*3*gamma(2 + 1/n) + b*n**3*x**n*gamma(2 + 1/n))) - n*x*x**n*lerchphi(b*x**n
*exp_polar(I*pi)/a, 1, 1 + 1/n)*gamma(1 + 1/n)/(a*(a*n**3*gamma(2 + 1/n) +
b*n**3*x**n*gamma(2 + 1/n))) + n*x*x**n*gamma(1 + 1/n)/(a*(a*n**3*gamma(2 +
 1/n) + b*n**3*x**n*gamma(2 + 1/n))) - x*x**n*lerchphi(b*x**n*exp_polar(I*p
i)/a, 1, 1 + 1/n)*gamma(1 + 1/n)/(a*(a*n**3*gamma(2 + 1/n) + b*n**3*x**n*ga
mma(2 + 1/n))) - b*n*x*x**(2*n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1 + 1
/n)*gamma(1 + 1/n)/(a**2*(a*n**3*gamma(2 + 1/n) + b*n**3*x**n*gamma(2 + 1/n
))) - b*x*x**(2*n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1 + 1/n)*gamma(1 +
 1/n)/(a**2*(a*n**3*gamma(2 + 1/n) + b*n**3*x**n*gamma(2 + 1/n))))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^n + c}{(bx^n + a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)/(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)/(b*x^n + a)^2, x)
```

3.309 $\int \frac{1}{(a+bx^n)^2(c+dx^n)} dx$

Optimal. Leaf size=122

$$\frac{bx(ad(1-2n)-bc(1-n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2n(bc-ad)^2}+\frac{d^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)^2}+\frac{bx}{an(bc-ad)(a+bx^n)}$$

```
[Out] (b*x)/(a*(b*c - a*d)*n*(a + b*x^n)) + (b*(a*d*(1 - 2*n) - b*c*(1 - n))*x*Hy
pergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*(b*c - a*d)^2*n)
 + (d^2*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c*(b*c -
 a*d)^2)
```

Rubi [A] time = 0.142295, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {414, 522, 245}

$$\frac{bx(ad(1-2n)-bc(1-n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2n(bc-ad)^2}+\frac{d^2x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c(bc-ad)^2}+\frac{bx}{an(bc-ad)(a+bx^n)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (b*x)/(a*(b*c - a*d)*n*(a + b*x^n)) + (b*(a*d*(1 - 2*n) - b*c*(1 - n))*x*Hy
pergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*(b*c - a*d)^2*n)
 + (d^2*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c*(b*c -
 a*d)^2)
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)}\,dx = \frac{bx}{a(bc-ad)n(a+bx^n)} - \frac{\int \frac{adn+b(c-cn)+bd(1-n)x^n}{(a+bx^n)(c+dx^n)}\,dx}{a(bc-ad)n}$$
$$= \frac{bx}{a(bc-ad)n(a+bx^n)} + \frac{d^2\int \frac{1}{c+dx^n}\,dx}{(bc-ad)^2} + \frac{(b(ad(1-2n)-bc(1-n)))\int \frac{1}{a+bx^n}\,dx}{a(bc-ad)^2n}$$
$$= \frac{bx}{a(bc-ad)n(a+bx^n)} + \frac{b(ad(1-2n)-bc(1-n))x\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2(bc-ad)^2n} + \frac{d^2x\,{}_2F_1\left(1,\frac{1}{n};1\right.}{c(bc-a}$$

Mathematica [A] time = 0.150821, size = 108, normalized size = 0.89

$$\frac{x\left(\frac{b^2c-abd}{a^2n+abnx^n}+\frac{b(ad(1-2n)+bc(n-1))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2n}+\frac{d^2\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c}\right)}{(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^n)^2*(c + d*x^n)),x]
```

```
[Out] (x*((b^2*c - a*b*d)/(a^2*n + a*b*n*x^n) + (b*(a*d*(1 - 2*n) + b*c*(-1 + n))
*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*n) + (d^2*Hyp
ergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/c))/(b*c - a*d)^2
```

Maple [F] time = 0.719, size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*x^n)^2/(c+d*x^n),x)
```

```
[Out] int(1/(a+b*x^n)^2/(c+d*x^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$d^2\int \frac{1}{b^2c^3-2\,abc^2d+a^2cd^2+\left(b^2c^2d-2\,abcd^2+a^2d^3\right)x^n}\,dx - \left(abd(2\,n-1)-b^2c(n-1)\right)\int \frac{}{a^2b^2c^2n-2\,a^3bcdn+a^4d^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^2/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] d^2*integrate(1/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2 + (b^2*c^2*d - 2*a*b*c*d
^2 + a^2*d^3)*x^n), x) - (a*b*d*(2*n - 1) - b^2*c*(n - 1))*integrate(1/(a^2
*b^2*c^2*n - 2*a^3*b*c*d*n + a^4*d^2*n + (a*b^3*c^2*n - 2*a^2*b^2*c*d*n + a
^3*b*d^2*n)*x^n), x) + b*x/(a^2*b*c*n - a^3*d*n + (a*b^2*c*n - a^2*b*d*n)*x
^n)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b^2dx^{3n} + a^2c + \left(b^2c + 2\,abd\right)x^{2n} + \left(2\,abc + a^2d\right)x^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^2/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*d*x^(3*n) + a^2*c + (b^2*c + 2*a*b*d)*x^(2*n) + (2*a*b*c +
a^2*d)*x^n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + bx^n)^2 (c + dx^n)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**n)**2/(c+d*x**n),x)
```

```
[Out] Integral(1/((a + b*x**n)**2*(c + d*x**n)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^n + a)^2(dx^n + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^2/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^2*(d*x^n + c)), x)
```

3.310 $\int \frac{1}{(a+bx^n)^2(c+dx^n)^2}\,dx$

Optimal. Leaf size=193

$$\frac{b^2x(ad(1-3n)-b(c-cn))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2n(bc-ad)^3}-\frac{d^2x(bc(1-3n)-ad(1-n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2n(bc-ad)^3}+\frac{b\ldots}{an(bc-ad)\,(a+\ldots}$$

```
[Out] (d*(b*c + a*d)*x)/(a*c*(b*c - a*d)^2*n*(c + d*x^n)) + (b*x)/(a*(b*c - a*d)*
n*(a + b*x^n)*(c + d*x^n)) + (b^2*(a*d*(1 - 3*n) - b*(c - c*n))*x*Hypergeom
etric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*(b*c - a*d)^3*n) - (d^2
*(b*c*(1 - 3*n) - a*d*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -
((d*x^n)/c)])/(c^2*(b*c - a*d)^3*n)
```

Rubi [A] time = 0.271006, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {414, 527, 522, 245}

$$\frac{b^2x(ad(1-3n)-b(c-cn))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2n(bc-ad)^3}-\frac{d^2x(bc(1-3n)-ad(1-n))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2n(bc-ad)^3}+\frac{b\ldots}{an(bc-ad)\,(a+\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^n)^2*(c + d*x^n)^2),x]
```

```
[Out] (d*(b*c + a*d)*x)/(a*c*(b*c - a*d)^2*n*(c + d*x^n)) + (b*x)/(a*(b*c - a*d)*
n*(a + b*x^n)*(c + d*x^n)) + (b^2*(a*d*(1 - 3*n) - b*(c - c*n))*x*Hypergeom
etric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a^2*(b*c - a*d)^3*n) - (d^2
*(b*c*(1 - 3*n) - a*d*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -
((d*x^n)/c)])/(c^2*(b*c - a*d)^3*n)
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, n}, x]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)^2}\,dx = \frac{bx}{a(bc-ad)n\,(a+bx^n)\,(c+dx^n)} - \frac{\int \frac{adn+b(c-cn)+bd(1-2n)x^n}{(a+bx^n)(c+dx^n)^2}\,dx}{a(bc-ad)n}$$

$$= \frac{d(bc+ad)x}{ac(bc-ad)^2n\,(c+dx^n)} + \frac{bx}{a(bc-ad)n\,(a+bx^n)\,(c+dx^n)} - \frac{\int \frac{n\left(b^2c^2(1-n)+a^2d^2(1-n)+2abcdn\right)}{(a+bx^n)(c+dx}}{ac(bc-ad)}$$

$$= \frac{d(bc+ad)x}{ac(bc-ad)^2n\,(c+dx^n)} + \frac{bx}{a(bc-ad)n\,(a+bx^n)\,(c+dx^n)} + \frac{\left(d^2(ad(1-n)-b(c-3cn))\right)}{c(bc-ad)^3n}$$

$$= \frac{d(bc+ad)x}{ac(bc-ad)^2n\,(c+dx^n)} + \frac{bx}{a(bc-ad)n\,(a+bx^n)\,(c+dx^n)} + \frac{b^2(ad(1-3n)-b(c-cn))x}{a^2(bc-a}$$

Mathematica [A] time = 0.208677, size = 147, normalized size = 0.76

$$\frac{x\left(\frac{b^2(ad(1-3n)+bc(n-1))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2}+\frac{b^2(bc-ad)}{a(a+bx^n)}+\frac{d^2(bc(3n-1)-ad(n-1))\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2}+\frac{d^2(bc-ad)}{c(c+dx^n)}\right)}{n(bc-ad)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^n)^2*(c + d*x^n)^2),x]
```

```
[Out] (x*((b^2*(b*c - a*d))/(a*(a + b*x^n)) + (d^2*(b*c - a*d))/(c*(c + d*x^n)) +
 (b^2*(a*d*(1 - 3*n) + b*c*(-1 + n))*Hypergeometric2F1[1, n^(-1), 1 + n^(-1
), -((b*x^n)/a)])/a^2 + (d^2*(-(a*d*(-1 + n)) + b*c*(-1 + 3*n))*Hypergeomet
ric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/c^2))/((b*c - a*d)^3*n)
```

Maple [F] time = 0.734, size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*x^n)^2/(c+d*x^n)^2,x)
```

```
[Out] int(1/(a+b*x^n)^2/(c+d*x^n)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\left(ab^2d(3n-1)-b^3c(n-1)\right)\int -\frac{1}{a^2b^3c^3n-3a^3b^2c^2dn+3a^4bcd^2n-a^5d^3n+\left(ab^4c^3n-3a^2b^3c^2dn+3a^3b^2cd^2n-a^4bd^3n\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^2/(c+d*x^n)^2,x, algorithm="maxima")
```

```
[Out] (a*b^2*d*(3*n - 1) - b^3*c*(n - 1))*integrate(-1/(a^2*b^3*c^3*n - 3*a^3*b^2
*c^2*d*n + 3*a^4*b*c*d^2*n - a^5*d^3*n + (a*b^4*c^3*n - 3*a^2*b^3*c^2*d*n +
 3*a^3*b^2*c*d^2*n - a^4*b*d^3*n)*x^n), x) - (b*c*d^2*(3*n - 1) - a*d^3*(n
- 1))*integrate(-1/(b^3*c^5*n - 3*a*b^2*c^4*d*n + 3*a^2*b*c^3*d^2*n - a^3*c
^2*d^3*n + (b^3*c^4*d*n - 3*a*b^2*c^3*d^2*n + 3*a^2*b*c^2*d^3*n - a^3*c*d^4
*n)*x^n), x) + ((b^2*c*d + a*b*d^2)*x*x^n + (b^2*c^2 + a^2*d^2)*x)/(a^2*b^2
*c^4*n - 2*a^3*b*c^3*d*n + a^4*c^2*d^2*n + (a*b^3*c^3*d*n - 2*a^2*b^2*c^2*d
^2*n + a^3*b*c*d^3*n)*x^(2*n) + (a*b^3*c^4*n - a^2*b^2*c^3*d*n - a^3*b*c^2*
d^2*n + a^4*c*d^3*n)*x^n)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b^2d^2x^{4n}+a^2c^2+2\left(b^2cd+abd^2\right)x^{3n}+\left(b^2c^2+4abcd+a^2d^2\right)x^{2n}+2\left(abc^2+a^2cd\right)x^n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^2/(c+d*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*d^2*x^(4*n) + a^2*c^2 + 2*(b^2*c*d + a*b*d^2)*x^(3*n) + (b^
2*c^2 + 4*a*b*c*d + a^2*d^2)*x^(2*n) + 2*(a*b*c^2 + a^2*c*d)*x^n), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^n)^2\,(c+dx^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**n)**2/(c+d*x**n)**2,x)
```

```
[Out] Integral(1/((a + b*x**n)**2*(c + d*x**n)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^n+a)^2(dx^n+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^2/(c+d*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^2*(d*x^n + c)^2), x)
```

3.311 $\int \frac{1}{(a+bx^n)^2(c+dx^n)^3}\,dx$

Optimal. Leaf size=299

$$\frac{d^2x\left(a^2d^2\left(2n^2-3n+1\right)-2abcd\left(4n^2-5n+1\right)+b^2c^2\left(12n^2-7n+1\right)\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{2c^3n^2(bc-ad)^4}-\frac{dx\left(-a^2d^2(1-2n)\right.}{2ac^2n^2(bc}$$

```
[Out] (d*(2*b*c + a*d)*x)/(2*a*c*(b*c - a*d)^2*n*(c + d*x^n)^2) + (b*x)/(a*(b*c -
 a*d)*n*(a + b*x^n)*(c + d*x^n)^2) - (d*(a*b*c*d*(1 - 6*n) - a^2*d^2*(1 - 2
*n) - 2*b^2*c^2*n)*x)/(2*a*c^2*(b*c - a*d)^3*n^2*(c + d*x^n)) + (b^3*(a*d*(
1 - 4*n) - b*c*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n
)/a)])/(a^2*(b*c - a*d)^4*n) + (d^2*(a^2*d^2*(1 - 3*n + 2*n^2) - 2*a*b*c*d*
(1 - 5*n + 4*n^2) + b^2*c^2*(1 - 7*n + 12*n^2))*x*Hypergeometric2F1[1, n^(-
1), 1 + n^(-1), -((d*x^n)/c)])/(2*c^3*(b*c - a*d)^4*n^2)
```

Rubi [A] time = 0.546893, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {414, 527, 522, 245}

$$\frac{d^2x\left(a^2d^2\left(2n^2-3n+1\right)-2abcd\left(4n^2-5n+1\right)+b^2c^2\left(12n^2-7n+1\right)\right)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{2c^3n^2(bc-ad)^4}-\frac{dx\left(-a^2d^2(1-2n)\right.}{2ac^2n^2(bc}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^n)^2*(c + d*x^n)^3),x]
```

```
[Out] (d*(2*b*c + a*d)*x)/(2*a*c*(b*c - a*d)^2*n*(c + d*x^n)^2) + (b*x)/(a*(b*c -
 a*d)*n*(a + b*x^n)*(c + d*x^n)^2) - (d*(a*b*c*d*(1 - 6*n) - a^2*d^2*(1 - 2
*n) - 2*b^2*c^2*n)*x)/(2*a*c^2*(b*c - a*d)^3*n^2*(c + d*x^n)) + (b^3*(a*d*(
1 - 4*n) - b*c*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n
)/a)])/(a^2*(b*c - a*d)^4*n) + (d^2*(a^2*d^2*(1 - 3*n + 2*n^2) - 2*a*b*c*d*
(1 - 5*n + 4*n^2) + b^2*c^2*(1 - 7*n + 12*n^2))*x*Hypergeometric2F1[1, n^(-
1), 1 + n^(-1), -((d*x^n)/c)])/(2*c^3*(b*c - a*d)^4*n^2)
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)^3}\,dx = \frac{bx}{a(bc-ad)n(a+bx^n)(c+dx^n)^2} - \frac{\int \frac{adn+b(c-cn)+bd(1-3n)x^n}{(a+bx^n)(c+dx^n)^3}\,dx}{a(bc-ad)n}$$

$$= \frac{d(2bc+ad)x}{2ac(bc-ad)^2n(c+dx^n)^2} + \frac{bx}{a(bc-ad)n(a+bx^n)(c+dx^n)^2} - \frac{\int \frac{n\left(a^2d^2(1-2n)+2b^2c^2(1-n)+4abc\cdots\right.}{(a+bx^n)(c+\cdots}}{2ac(bc-a\cdots}$$

$$= \frac{d(2bc+ad)x}{2ac(bc-ad)^2n(c+dx^n)^2} + \frac{bx}{a(bc-ad)n(a+bx^n)(c+dx^n)^2} - \frac{d\left(abcd(1-6n)-a^2d^2(1-\cdots\right.}{2ac^2(bc-ad)^3n^2(c\cdots}$$

$$= \frac{d(2bc+ad)x}{2ac(bc-ad)^2n(c+dx^n)^2} + \frac{bx}{a(bc-ad)n(a+bx^n)(c+dx^n)^2} - \frac{d\left(abcd(1-6n)-a^2d^2(1-\cdots\right.}{2ac^2(bc-ad)^3n^2(c\cdots}$$

$$= \frac{d(2bc+ad)x}{2ac(bc-ad)^2n(c+dx^n)^2} + \frac{bx}{a(bc-ad)n(a+bx^n)(c+dx^n)^2} - \frac{d\left(abcd(1-6n)-a^2d^2(1-\cdots\right.}{2ac^2(bc-ad)^3n^2(c\cdots}$$

Mathematica [A] time = 0.324012, size = 233, normalized size = 0.78

$$\frac{x\left(\frac{d^2\left(a^2d^2(2n^2-3n+1)-2abcd(4n^2-5n+1)+b^2c^2(12n^2-7n+1)\right){}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^3} + \frac{2b^3n(ad(1-4n)+bc(n-1)){}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{bx^n}{a}\right)}{a^2} + \frac{2b^3n(bc-ad)}{a(a+bx^n)} + \frac{d^2(a\cdots}{\cdots}\right.}{2n^2(bc-ad)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^n)^2*(c + d*x^n)^3),x]
```

```
[Out] (x*((2*b^3*(b*c - a*d)*n)/(a*(a + b*x^n)) + (d^2*(b*c - a*d)^2*n)/(c*(c + d
*x^n)^2) + (d^2*(-(b*c) + a*d)*(a*d*(-1 + 2*n) + b*(c - 6*c*n)))/(c^2*(c +
d*x^n)) + (2*b^3*(a*d*(1 - 4*n) + b*c*(-1 + n))*n*Hypergeometric2F1[1, n^(-
1), 1 + n^(-1), -((b*x^n)/a)])/a^2 + (d^2*(a^2*d^2*(1 - 3*n + 2*n^2) - 2*a*
b*c*d*(1 - 5*n + 4*n^2) + b^2*c^2*(1 - 7*n + 12*n^2))*Hypergeometric2F1[1,
n^(-1), 1 + n^(-1), -((d*x^n)/c)])/c^3))/(2*(b*c - a*d)^4*n^2)
```

Maple [F] time = 0.734, size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^n)^2(c+dx^n)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*x^n)^2/(c+d*x^n)^3,x)
```

```
[Out] int(1/(a+b*x^n)^2/(c+d*x^n)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^2/(c+d*x^n)^3,x, algorithm="maxima")
```

```
[Out] ((12*n^2 - 7*n + 1)*b^2*c^2*d^2 - 2*(4*n^2 - 5*n + 1)*a*b*c*d^3 + (2*n^2 -
3*n + 1)*a^2*d^4)*integrate(1/2/(b^4*c^7*n^2 - 4*a*b^3*c^6*d*n^2 + 6*a^2*b^
2*c^5*d^2*n^2 - 4*a^3*b*c^4*d^3*n^2 + a^4*c^3*d^4*n^2 + (b^4*c^6*d*n^2 - 4*
a*b^3*c^5*d^2*n^2 + 6*a^2*b^2*c^4*d^3*n^2 - 4*a^3*b*c^3*d^4*n^2 + a^4*c^2*d
^5*n^2)*x^n), x) - (a*b^3*d*(4*n - 1) - b^4*c*(n - 1))*integrate(1/(a^2*b^4
*c^4*n - 4*a^3*b^3*c^3*d*n + 6*a^4*b^2*c^2*d^2*n - 4*a^5*b*c*d^3*n + a^6*d^
4*n + (a*b^5*c^4*n - 4*a^2*b^4*c^3*d*n + 6*a^3*b^3*c^2*d^2*n - 4*a^4*b^2*c*
d^3*n + a^5*b*d^4*n)*x^n), x) + 1/2*((a*b^2*c*d^3*(6*n - 1) - a^2*b*d^4*(2*
n - 1) + 2*b^3*c^2*d^2*n)*x*x^(2*n) + (a*b^2*c^2*d^2*(7*n - 1) - a^3*d^4*(2
*n - 1) + 4*b^3*c^3*d*n + 3*a^2*b*c*d^3*n)*x*x^n + (a^2*b*c^2*d^2*(7*n - 1)
 - a^3*c*d^3*(3*n - 1) + 2*b^3*c^4*n)*x)/(a^2*b^3*c^7*n^2 - 3*a^3*b^2*c^6*d
*n^2 + 3*a^4*b*c^5*d^2*n^2 - a^5*c^4*d^3*n^2 + (a*b^4*c^5*d^2*n^2 - 3*a^2*b
^3*c^4*d^3*n^2 + 3*a^3*b^2*c^3*d^4*n^2 - a^4*b*c^2*d^5*n^2)*x^(3*n) + (2*a*
b^4*c^6*d*n^2 - 5*a^2*b^3*c^5*d^2*n^2 + 3*a^3*b^2*c^4*d^3*n^2 + a^4*b*c^3*d
^4*n^2 - a^5*c^2*d^5*n^2)*x^(2*n) + (a*b^4*c^7*n^2 - a^2*b^3*c^6*d*n^2 - 3*
a^3*b^2*c^5*d^2*n^2 + 5*a^4*b*c^4*d^3*n^2 - 2*a^5*c^3*d^4*n^2)*x^n)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b^2d^3x^{5n} + a^2c^3 + \left(3\,b^2cd^2 + 2\,abd^3\right)x^{4n} + \left(3\,b^2c^2d + 6\,abcd^2 + a^2d^3\right)x^{3n} + \left(b^2c^3 + 6\,abc^2d + 3\,a^2cd^2\right)x^{2n} +}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^2/(c+d*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*d^3*x^(5*n) + a^2*c^3 + (3*b^2*c*d^2 + 2*a*b*d^3)*x^(4*n) +
 (3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^(3*n) + (b^2*c^3 + 6*a*b*c^2*d + 3
*a^2*c*d^2)*x^(2*n) + (2*a*b*c^3 + 3*a^2*c^2*d)*x^n), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**n)**2/(c+d*x**n)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^n + a)^2 (dx^n + c)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^2/(c+d*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^2*(d*x^n + c)^3), x)
```

3.312 $\int (a+bx^n)^p (c+dx^n)^q \, dx$

Optimal. Leaf size=81

$$x (a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} (c+dx^n)^q \left(\frac{dx^n}{c}+1\right)^{-q} F_1\left(\frac{1}{n};-p,-q;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)$$

```
[Out] (x*(a + b*x^n)^p*(c + d*x^n)^q*AppellF1[n^(-1), -p, -q, 1 + n^(-1), -((b*x^
n)/a), -((d*x^n)/c)])/((1 + (b*x^n)/a)^p*(1 + (d*x^n)/c)^q)
```

Rubi [A] time = 0.052638, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {430, 429}

$$x (a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} (c+dx^n)^q \left(\frac{dx^n}{c}+1\right)^{-q} F_1\left(\frac{1}{n};-p,-q;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^p*(c + d*x^n)^q,x]
```

```
[Out] (x*(a + b*x^n)^p*(c + d*x^n)^q*AppellF1[n^(-1), -p, -q, 1 + n^(-1), -((b*x^
n)/a), -((d*x^n)/c)])/((1 + (b*x^n)/a)^p*(1 + (d*x^n)/c)^q)
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int (a+bx^n)^p (c+dx^n)^q \, dx &= \left((a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p}\right) \int \left(1+\frac{bx^n}{a}\right)^p (c+dx^n)^q \, dx \\
&= \left((a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} (c+dx^n)^q \left(1+\frac{dx^n}{c}\right)^{-q}\right) \int \left(1+\frac{bx^n}{a}\right)^p \left(1+\frac{dx^n}{c}\right)^q \, dx \\
&= x (a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} (c+dx^n)^q \left(1+\frac{dx^n}{c}\right)^{-q} F_1\left(\frac{1}{n};-p,-q;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)
\end{aligned}$$

Mathematica [B] time = 0.393458, size = 190, normalized size = 2.35

$$\frac{ac(n+1)x (a+bx^n)^p (c+dx^n)^q F_1\left(\frac{1}{n};-p,-q;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{bcnpx^nF_1\left(1+\frac{1}{n};1-p,-q;2+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)+adnqx^nF_1\left(1+\frac{1}{n};-p,1-q;2+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)+ac(n+1)F_1\left(\frac{1}{n};-p,-\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^n)^p*(c + d*x^n)^q,x]
```

```
[Out] (a*c*(1 + n)*x*(a + b*x^n)^p*(c + d*x^n)^q*AppellF1[n^(-1), -p, -q, 1 + n^(
-1), -((b*x^n)/a), -((d*x^n)/c)])/(b*c*n*p*x^n*AppellF1[1 + n^(-1), 1 - p,
-q, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + a*d*n*q*x^n*AppellF1[1 + n^(-
1), -p, 1 - q, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + a*c*(1 + n)*Appell
F1[n^(-1), -p, -q, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)])
```

Maple [F] time = 0.816, size = 0, normalized size = 0.

$$\int (a + bx^n)^p (c + dx^n)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^p*(c+d*x^n)^q,x)
```

```
[Out] int((a+b*x^n)^p*(c+d*x^n)^q,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^n + a)^p (dx^n + c)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^q,x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p*(d*x^n + c)^q, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^n + a)^p (dx^n + c)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^q,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^p*(d*x^n + c)^q, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + bx^n)^p (c + dx^n)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**p*(c+d*x**n)**q,x)
```

```
[Out] Integral((a + b*x**n)**p*(c + d*x**n)**q, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^n + a)^p (dx^n + c)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^q,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^p*(d*x^n + c)^q, x)
```

3.313 $\int (a+bx^n)^p (c+dx^n)^3 \, dx$

Optimal. Leaf size=402

$$-\frac{x(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(-3a^2bcd^2(n+1)(n(p+3)+1)+a^3d^3\left(2n^2+3n+1\right)+3ab^2c^2d\left(n^2\left(p^2+5p+6\right)+n(2p+5)\right.\right. \cdots}{b^3(np+n+1)(n(p+2)+1)(n \cdots}$$

```
[Out] (d*(a^2*d^2*(1 + 3*n + 2*n^2) - a*b*c*d*(2 + n^2*(7 + p) + n*(9 + 2*p)) + b
^2*c^2*(1 + 2*n*(3 + p) + n^2*(11 + 6*p + p^2)))*x*(a + b*x^n)^(1 + p))/(b^
3*(1 + n + n*p)*(1 + n*(2 + p))*(1 + n*(3 + p))) - (d*(a*d*(1 + 2*n) - b*c*
(1 + n*(5 + p)))*x*(a + b*x^n)^(1 + p)*(c + d*x^n))/(b^2*(1 + n*(2 + p))*(1
 + n*(3 + p))) + (d*x*(a + b*x^n)^(1 + p)*(c + d*x^n)^2)/(b*(1 + 3*n + n*p)
) - ((a^3*d^3*(1 + 3*n + 2*n^2) - 3*a^2*b*c*d^2*(1 + n)*(1 + n*(3 + p)) + 3
*a*b^2*c^2*d*(1 + n*(5 + 2*p) + n^2*(6 + 5*p + p^2)) - b^3*c^3*(1 + 3*n*(2
+ p) + n^2*(11 + 12*p + 3*p^2) + n^3*(6 + 11*p + 6*p^2 + p^3)))*x*(a + b*x^
n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)])/(b^3*(1 + n +
 n*p)*(1 + n*(2 + p))*(1 + n*(3 + p))*(1 + (b*x^n)/a)^p)
```

Rubi [A] time = 0.578185, antiderivative size = 401, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {416, 528, 388, 246, 245}

$$-\frac{x(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(-3a^2bcd^2(n+1)(n(p+3)+1)+a^3d^3\left(2n^2+3n+1\right)+3ab^2c^2d\left(n^2\left(p^2+5p+6\right)+n(2p+5)\right.\right. \cdots}{b^3(np+n+1)(n(p+2)+1)(n \cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^p*(c + d*x^n)^3,x]
```

```
[Out] (d*(a^2*d^2*(1 + 3*n + 2*n^2) - a*b*c*d*(2 + n^2*(7 + p) + n*(9 + 2*p)) + b
^2*c^2*(1 + 2*n*(3 + p) + n^2*(11 + 6*p + p^2)))*x*(a + b*x^n)^(1 + p))/(b^
3*(1 + n + n*p)*(1 + n*(2 + p))*(1 + n*(3 + p))) - (d*(a*d*(1 + 2*n) - b*(c
 + c*n*(5 + p)))*x*(a + b*x^n)^(1 + p)*(c + d*x^n))/(b^2*(1 + n*(2 + p))*(1
 + n*(3 + p))) + (d*x*(a + b*x^n)^(1 + p)*(c + d*x^n)^2)/(b*(1 + n*(3 + p))
) - ((a^3*d^3*(1 + 3*n + 2*n^2) - 3*a^2*b*c*d^2*(1 + n)*(1 + n*(3 + p)) + 3
*a*b^2*c^2*d*(1 + n*(5 + 2*p) + n^2*(6 + 5*p + p^2)) - b^3*c^3*(1 + 3*n*(2
+ p) + n^2*(11 + 12*p + 3*p^2) + n^3*(6 + 11*p + 6*p^2 + p^3)))*x*(a + b*x^
n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)])/(b^3*(1 + n +
 n*p)*(1 + n*(2 + p))*(1 + n*(3 + p))*(1 + (b*x^n)/a)^p)
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
```

```
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int (a+bx^n)^p (c+dx^n)^3\, dx = \frac{dx(a+bx^n)^{1+p}(c+dx^n)^2}{b(1+n(3+p))} + \frac{\int (a+bx^n)^p (c+dx^n)\left(-c(ad-b(c+cn(3+p)))-d(ad(1\right.}{b(1+n(3+p))}$$

$$= -\frac{d(ad(1+2n)-b(c+cn(5+p)))x(a+bx^n)^{1+p}(c+dx^n)}{b^2(1+n(2+p))(1+n(3+p))} + \frac{dx(a+bx^n)^{1+p}(c+dx^n)^2}{b(1+n(3+p))}$$

$$= \frac{d\left(a^2d^2\left(1+3n+2n^2\right)-abcd\left(2+n^2(7+p)+n(9+2p)\right)+b^2c^2\left(1+2n(3+p)+n^2\left(1\right.\right.\right.}{b^3(1+n+np)(1+n(2+p))(1+n(3+p))}$$

$$= \frac{d\left(a^2d^2\left(1+3n+2n^2\right)-abcd\left(2+n^2(7+p)+n(9+2p)\right)+b^2c^2\left(1+2n(3+p)+n^2\left(1\right.\right.\right.}{b^3(1+n+np)(1+n(2+p))(1+n(3+p))}$$

$$= \frac{d\left(a^2d^2\left(1+3n+2n^2\right)-abcd\left(2+n^2(7+p)+n(9+2p)\right)+b^2c^2\left(1+2n(3+p)+n^2\left(1\right.\right.\right.}{b^3(1+n+np)(1+n(2+p))(1+n(3+p))}$$

Mathematica [A] time = 5.25378, size = 168, normalized size = 0.42

$$x(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(\frac{3c^2dx^n\,{}_2F_1\left(1+\frac{1}{n},-p;2+\frac{1}{n};-\frac{bx^n}{a}\right)}{n+1}+c^3\,{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{bx^n}{a}\right)+\frac{3cd^2x^{2n}\,{}_2F_1\left(2+\frac{1}{n},-p;\right.}{2n+1}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^p*(c + d*x^n)^3,x]
```

```
[Out] (x*(a + b*x^n)^p*((3*c^2*d*x^n*Hypergeometric2F1[1 + n^(-1), -p, 2 + n^(-1)
, -((b*x^n)/a)])/(1 + n) + (3*c*d^2*x^(2*n)*Hypergeometric2F1[2 + n^(-1), -
p, 3 + n^(-1), -((b*x^n)/a)])/(1 + 2*n) + (d^3*x^(3*n)*Hypergeometric2F1[3
+ n^(-1), -p, 4 + n^(-1), -((b*x^n)/a)])/(1 + 3*n) + c^3*Hypergeometric2F1[
```

```
n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)]))/(1 + (b*x^n)/a)^p
```

Maple [F] time = 0.623, size = 0, normalized size = 0.

$$\int (a+bx^n)^p (c+dx^n)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^p*(c+d*x^n)^3,x)
```

```
[Out] int((a+b*x^n)^p*(c+d*x^n)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx^n+c)^3 (bx^n+a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x^n + c)^3*(b*x^n + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(d^3x^{3\,n}+3\,cd^2x^{2\,n}+3\,c^2dx^n+c^3\right)(bx^n+a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral((d^3*x^(3*n) + 3*c*d^2*x^(2*n) + 3*c^2*d*x^n + c^3)*(b*x^n + a)^p,
 x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**p*(c+d*x**n)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.314 $\int (a+bx^n)^p (c+dx^n)^2\,dx$

Optimal. Leaf size=202

$$-\frac{x(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}(bc(np+n+1)(ad-bc(n(p+2)+1))-ad(ad(n+1)-bc(n(p+3)+1)))\,{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{bx}{a}\right.}{b^2(np+n+1)(n(p+2)+1)}$$

```
[Out] -((d*(a*d*(1 + n) - b*c*(1 + n*(3 + p)))*x*(a + b*x^n)^(1 + p))/(b^2*(1 + n
 + n*p)*(1 + n*(2 + p)))) + (d*x*(a + b*x^n)^(1 + p)*(c + d*x^n))/(b*(1 + 2
*n + n*p)) - ((b*c*(1 + n + n*p)*(a*d - b*c*(1 + n*(2 + p))) - a*d*(a*d*(1
+ n) - b*c*(1 + n*(3 + p))))*x*(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p,
1 + n^(-1), -((b*x^n)/a)])/(b^2*(1 + n + n*p)*(1 + n*(2 + p))*(1 + (b*x^n)/
a)^p)
```

Rubi [A] time = 0.26866, antiderivative size = 197, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {416, 388, 246, 245}

$$-\frac{dx(a+bx^n)^{p+1}(ad(n+1)-b(cn(p+3)+c))}{b^2(np+n+1)(n(p+2)+1)}-\frac{x(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(c(ad-b(cn(p+2)+c))-\frac{ad(ad(n+1)-b(cn(p+3)+c)}{b(np+n+1)}\right.}{b(n(p+2)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^p*(c + d*x^n)^2,x]
```

```
[Out] -((d*(a*d*(1 + n) - b*(c + c*n*(3 + p)))*x*(a + b*x^n)^(1 + p))/(b^2*(1 + n
 + n*p)*(1 + n*(2 + p)))) + (d*x*(a + b*x^n)^(1 + p)*(c + d*x^n))/(b*(1 + n
*(2 + p))) - ((c*(a*d - b*(c + c*n*(2 + p))) - (a*d*(a*d*(1 + n) - b*(c + c
*n*(3 + p))))/(b*(1 + n + n*p)))*x*(a + b*x^n)^p*Hypergeometric2F1[n^(-1),
-p, 1 + n^(-1), -((b*x^n)/a)])/(b*(1 + n*(2 + p))*(1 + (b*x^n)/a)^p)
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int (a+bx^n)^p (c+dx^n)^2\,dx = \frac{dx\,(a+bx^n)^{1+p}(c+dx^n)}{b(1+n(2+p))} + \frac{\int (a+bx^n)^p \left(-c(ad-b(c+cn(2+p)))-d(ad(1+n)-b(c+\right.}{b(1+n(2+p))}$$

$$= -\frac{d(ad(1+n)-b(c+cn(3+p)))x\,(a+bx^n)^{1+p}}{b^2(1+n+np)(1+n(2+p))} + \frac{dx\,(a+bx^n)^{1+p}(c+dx^n)}{b(1+n(2+p))} - \frac{\left(c(ad-b(\right.}{}$$

$$= -\frac{d(ad(1+n)-b(c+cn(3+p)))x\,(a+bx^n)^{1+p}}{b^2(1+n+np)(1+n(2+p))} + \frac{dx\,(a+bx^n)^{1+p}(c+dx^n)}{b(1+n(2+p))} - \frac{\left(\left(c(ad-b\right.\right.}{}$$

$$= -\frac{d(ad(1+n)-b(c+cn(3+p)))x\,(a+bx^n)^{1+p}}{b^2(1+n+np)(1+n(2+p))} + \frac{dx\,(a+bx^n)^{1+p}(c+dx^n)}{b(1+n(2+p))} - \frac{\left(c(ad-b(\right.}{}$$

Mathematica [A] time = 5.1585, size = 140, normalized size = 0.69

$$\frac{x\,(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} \left((n+1)\left(c^2(2n+1)\,{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{bx^n}{a}\right)+d^2x^{2n}\,{}_2F_1\left(2+\frac{1}{n},-p;3+\frac{1}{n};-\frac{bx^n}{a}\right)\right)+2cd(2n+\right.}{(n+1)(2n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^p*(c + d*x^n)^2,x]
```

```
[Out] (x*(a + b*x^n)^p*(2*c*d*(1 + 2*n)*x^n*Hypergeometric2F1[1 + n^(-1), -p, 2 +
 n^(-1), -((b*x^n)/a)] + (1 + n)*(d^2*x^(2*n)*Hypergeometric2F1[2 + n^(-1),
 -p, 3 + n^(-1), -((b*x^n)/a)] + c^2*(1 + 2*n)*Hypergeometric2F1[n^(-1), -p
, 1 + n^(-1), -((b*x^n)/a)])))/((1 + n)*(1 + 2*n)*(1 + (b*x^n)/a)^p)
```

Maple [F] time = 0.55, size = 0, normalized size = 0.

$$\int (a+bx^n)^p (c+dx^n)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^p*(c+d*x^n)^2,x)
```

```
[Out] int((a+b*x^n)^p*(c+d*x^n)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx^n+c)^2(bx^n+a)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^n + c)^2*(b*x^n + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(d^2x^{2n} + 2\,cdx^n + c^2\right)(bx^n + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral((d^2*x^(2*n) + 2*c*d*x^n + c^2)*(b*x^n + a)^p, x)
```

Sympy [C] time = 23.8943, size = 143, normalized size = 0.71

$$\frac{a^p c^2 x\Gamma\left(\frac{1}{n}\right) {}_2F_1\left(\begin{matrix}\frac{1}{n}, -p \\ 1+\frac{1}{n}\end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)} + \frac{2a^p cdxx^n\Gamma\left(1+\frac{1}{n}\right) {}_2F_1\left(\begin{matrix}-p, 1+\frac{1}{n} \\ 2+\frac{1}{n}\end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{1}{n}\right)} + \frac{a^p d^2 xx^{2n}\Gamma\left(2+\frac{1}{n}\right) {}_2F_1\left(\begin{matrix}-p, 2+\frac{1}{n} \\ 3+\frac{1}{n}\end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(3+\frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**p*(c+d*x**n)**2,x)
```

```
[Out] a**p*c**2*x*gamma(1/n)*hyper((1/n, -p), (1 + 1/n,), b*x**n*exp_polar(I*pi)/
a)/(n*gamma(1 + 1/n)) + 2*a**p*c*d*x*x**n*gamma(1 + 1/n)*hyper((-p, 1 + 1/n
), (2 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(2 + 1/n)) + a**p*d**2*x*x
**(2*n)*gamma(2 + 1/n)*hyper((-p, 2 + 1/n), (3 + 1/n,), b*x**n*exp_polar(I*
pi)/a)/(n*gamma(3 + 1/n))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.315 $\int (a+bx^n)^p (c+dx^n)\, dx$

Optimal. Leaf size=98

$$\frac{dx\,(a+bx^n)^{p+1}}{b(np+n+1)}-\frac{x\,(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}(ad-bc(np+n+1))\,{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{bx^n}{a}\right)}{b(np+n+1)}$$

```
[Out] (d*x*(a + b*x^n)^(1 + p))/(b*(1 + n + n*p)) - ((a*d - b*c*(1 + n + n*p))*x*
(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)])/(b*(
1 + n + n*p)*(1 + (b*x^n)/a)^p)
```

Rubi [A] time = 0.0473692, antiderivative size = 89, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {388, 246, 245}

$$x\,(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left(c-\frac{ad}{bnp+bn+b}\right){}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{bx^n}{a}\right)+\frac{dx\,(a+bx^n)^{p+1}}{b(np+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^p*(c + d*x^n),x]
```

```
[Out] (d*x*(a + b*x^n)^(1 + p))/(b*(1 + n + n*p)) + ((c - (a*d)/(b + b*n + b*n*p)
)*x*(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)])/
(1 + (b*x^n)/a)^p
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int (a+bx^n)^p (c+dx^n)\,dx = \frac{dx(a+bx^n)^{1+p}}{b(1+n+np)} - \left(-c+\frac{ad}{b+bn+bnp}\right)\int (a+bx^n)^p\,dx$$
$$= \frac{dx(a+bx^n)^{1+p}}{b(1+n+np)} - \left(\left(-c+\frac{ad}{b+bn+bnp}\right)(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\right)\int\left(1+\frac{bx^n}{a}\right)^p\,dx$$
$$= \frac{dx(a+bx^n)^{1+p}}{b(1+n+np)} + \left(c-\frac{ad}{b+bn+bnp}\right)x(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\,{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{bx^n}{a}\right)$$

Mathematica [A] time = 0.0429777, size = 94, normalized size = 0.96

$$\frac{x(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}\left((bc(np+n+1)-ad)\,{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{bx^n}{a}\right)+d(a+bx^n)\left(\frac{bx^n}{a}+1\right)^p\right)}{b(np+n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^p*(c + d*x^n),x]
```

```
[Out] (x*(a + b*x^n)^p*(d*(a + b*x^n)*(1 + (b*x^n)/a)^p + (-(a*d) + b*c*(1 + n +
n*p))*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)]))/(b*(1 + n +
 n*p)*(1 + (b*x^n)/a)^p)
```

Maple [F] time = 0.401, size = 0, normalized size = 0.

$$\int (a+bx^n)^p (c+dx^n)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^p*(c+d*x^n),x)
```

```
[Out] int((a+b*x^n)^p*(c+d*x^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx^n+c)(bx^n+a)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n),x, algorithm="maxima")
```

```
[Out] integrate((d*x^n + c)*(b*x^n + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((dx^n+c)(bx^n+a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral((d*x^n + c)*(b*x^n + a)^p, x)
```

Sympy [C] time = 4.2984, size = 87, normalized size = 0.89

$$\frac{a^{p}cx\Gamma\left(\frac{1}{n}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{n},-p\\1+\frac{1}{n}\end{matrix}\middle|\frac{bx^{n}e^{i\pi}}{a}\right)}{n\Gamma\left(1+\frac{1}{n}\right)}+\frac{a^{p}dxx^{n}\Gamma\left(1+\frac{1}{n}\right){}_{2}F_{1}\left(\begin{matrix}-p,1+\frac{1}{n}\\2+\frac{1}{n}\end{matrix}\middle|\frac{bx^{n}e^{i\pi}}{a}\right)}{n\Gamma\left(2+\frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**p*(c+d*x**n),x)
```

```
[Out] a**p*c*x*gamma(1/n)*hyper((1/n, -p), (1 + 1/n,), b*x**n*exp_polar(I*pi)/a)/
(n*gamma(1 + 1/n)) + a**p*d*x*x**n*gamma(1 + 1/n)*hyper((-p, 1 + 1/n), (2 +
 1/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(2 + 1/n))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.316 $\int (a+bx^n)^p\,dx$

Optimal. Leaf size=46

$$x\left(a+bx^n\right)^p\left(\frac{bx^n}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{bx^n}{a}\right)$$

```
[Out] (x*(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)])/(
1 + (b*x^n)/a)^p
```

Rubi [A] time = 0.0108094, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {246, 245}

$$x\left(a+bx^n\right)^p\left(\frac{bx^n}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{bx^n}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^p,x]
```

```
[Out] (x*(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)])/(
1 + (b*x^n)/a)^p
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^n\right)^p\,dx &= \left(\left(a+bx^n\right)^p\left(1+\frac{bx^n}{a}\right)^{-p}\right)\int \left(1+\frac{bx^n}{a}\right)^p\,dx\\ &= x\left(a+bx^n\right)^p\left(1+\frac{bx^n}{a}\right)^{-p}{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{bx^n}{a}\right)\end{aligned}$$

Mathematica [A] time = 0.0047375, size = 46, normalized size = 1.

$$x\left(a+bx^n\right)^p\left(\frac{bx^n}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{bx^n}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^p,x]
```

```
[Out] (x*(a + b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*x^n)/a)])/(
1 + (b*x^n)/a)^p
```

Maple [F] time = 0., size = 0, normalized size = 0.

$$\int (a + bx^n)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^p,x)
```

```
[Out] int((a+b*x^n)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^n + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((bx^n + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^p, x)
```

Sympy [C] time = 1.43392, size = 37, normalized size = 0.8

$$\frac{a^p x \Gamma\left(\frac{1}{n}\right) {}_2F_1\left(\begin{matrix} \frac{1}{n}, -p \\ 1 + \frac{1}{n} \end{matrix}\middle| \frac{bx^n e^{i\pi}}{a}\right)}{n\Gamma\left(1 + \frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**p,x)
```

```
[Out] a**p*x*gamma(1/n)*hyper((1/n, -p), (1 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(n
*gamma(1 + 1/n))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^n + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^p, x)
```

3.317 $\int \frac{(a+bx^n)^p}{c+dx^n} dx$

Optimal. Leaf size=59

$$\frac{x(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}F_1\left(\frac{1}{n};-p,1;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c}$$

```
[Out] (x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 1, 1 + n^(-1), -((b*x^n)/a), -((d*x^n
)/c)])/(c*(1 + (b*x^n)/a)^p)
```

Rubi [A] time = 0.0289819, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {430, 429}

$$\frac{x(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}F_1\left(\frac{1}{n};-p,1;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^p/(c + d*x^n),x]
```

```
[Out] (x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 1, 1 + n^(-1), -((b*x^n)/a), -((d*x^n
)/c)])/(c*(1 + (b*x^n)/a)^p)
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^n)^p}{c+dx^n}\,dx &= \left((a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^n}{a}\right)^p}{c+dx^n}\,dx\\ &= \frac{x(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}F_1\left(\frac{1}{n};-p,1;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c}\end{aligned}$$

Mathematica [B] time = 0.255761, size = 180, normalized size = 3.05

$$\frac{ac(n+1)x(a+bx^n)^p F_1\left(\frac{1}{n};-p,1;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{(c+dx^n)\left(bcnpx^nF_1\left(1+\frac{1}{n};1-p,1;2+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)-adnx^nF_1\left(1+\frac{1}{n};-p,2;2+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)+ac(n+1)F_1\left(\frac{1}{n};-\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^n)^p/(c + d*x^n),x]
```

```
[Out] (a*c*(1 + n)*x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 1, 1 + n^(-1), -((b*x^n)/
a), -((d*x^n)/c)])/((c + d*x^n)*(b*c*n*p*x^n*AppellF1[1 + n^(-1), 1 - p, 1,
 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] - a*d*n*x^n*AppellF1[1 + n^(-1), -
p, 2, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + a*c*(1 + n)*AppellF1[n^(-1)
, -p, 1, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)]))
```

Maple [F] time = 0.711, size = 0, normalized size = 0.

$$\int \frac{(a+bx^n)^p}{c+dx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^p/(c+d*x^n),x)
```

```
[Out] int((a+b*x^n)^p/(c+d*x^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^n+a)^p}{dx^n+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p/(c+d*x^n),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p/(d*x^n + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^n+a)^p}{dx^n+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p/(c+d*x^n),x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^p/(d*x^n + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+bx^n)^p}{c+dx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**p/(c+d*x**n),x)
```

```
[Out] Integral((a + b*x**n)**p/(c + d*x**n), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^n + a)^p}{dx^n + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p/(c+d*x^n),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^p/(d*x^n + c), x)
```

3.318 $\int \frac{(a+bx^n)^p}{(c+dx^n)^2} dx$

Optimal. Leaf size=59

$$\frac{x(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}F_1\left(\frac{1}{n};-p,2;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^2}$$

```
[Out] (x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 2, 1 + n^(-1), -((b*x^n)/a), -((d*x^n
)/c)])/(c^2*(1 + (b*x^n)/a)^p)
```

Rubi [A] time = 0.0295756, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {430, 429}

$$\frac{x(a+bx^n)^p\left(\frac{bx^n}{a}+1\right)^{-p}F_1\left(\frac{1}{n};-p,2;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^p/(c + d*x^n)^2,x]
```

```
[Out] (x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 2, 1 + n^(-1), -((b*x^n)/a), -((d*x^n
)/c)])/(c^2*(1 + (b*x^n)/a)^p)
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^n)^p}{(c+dx^n)^2}\,dx &= \left((a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}\right)\int \frac{\left(1+\frac{bx^n}{a}\right)^p}{(c+dx^n)^2}\,dx\\ &= \frac{x(a+bx^n)^p\left(1+\frac{bx^n}{a}\right)^{-p}F_1\left(\frac{1}{n};-p,2;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^2}\end{aligned}$$

Mathematica [B] time = 0.277856, size = 180, normalized size = 3.05

$$\frac{ac(n+1)x(a+bx^n)^pF_1\left(\frac{1}{n};-p,2;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{(c+dx^n)^2\left(bcnpx^nF_1\left(1+\frac{1}{n};1-p,2;2+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)-2adnx^nF_1\left(1+\frac{1}{n};-p,3;2+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)+ac(n+1)F_1\left(\frac{1}{n};-p\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^n)^p/(c + d*x^n)^2,x]
```

```
[Out] (a*c*(1 + n)*x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 2, 1 + n^(-1), -((b*x^n)/
a), -((d*x^n)/c)])/((c + d*x^n)^2*(b*c*n*p*x^n*AppellF1[1 + n^(-1), 1 - p,
2, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] - 2*a*d*n*x^n*AppellF1[1 + n^(-1
), -p, 3, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + a*c*(1 + n)*AppellF1[n^
(-1), -p, 2, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)]))
```

Maple [F] time = 0.676, size = 0, normalized size = 0.

$$\int \frac{(a + bx^n)^p}{(c + dx^n)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^p/(c+d*x^n)^2,x)
```

```
[Out] int((a+b*x^n)^p/(c+d*x^n)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^n + a)^p}{(dx^n + c)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p/(c+d*x^n)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p/(d*x^n + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^n + a)^p}{d^2x^{2n} + 2\,cdx^n + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p/(c+d*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^p/(d^2*x^(2*n) + 2*c*d*x^n + c^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx^n)^p}{(c + dx^n)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**p/(c+d*x**n)**2,x)
```

```
[Out] Integral((a + b*x**n)**p/(c + d*x**n)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^n + a)^p}{(dx^n + c)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p/(c+d*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^p/(d*x^n + c)^2, x)
```

3.319 $\int \frac{(a+bx^n)^p}{(c+dx^n)^3} \, dx$

Optimal. Leaf size=59

$$\frac{x(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} F_1\left(\frac{1}{n};-p,3;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^3}$$

```
[Out] (x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 3, 1 + n^(-1), -((b*x^n)/a), -((d*x^n
)/c)])/(c^3*(1 + (b*x^n)/a)^p)
```

Rubi [A] time = 0.0287971, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {430, 429}

$$\frac{x(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} F_1\left(\frac{1}{n};-p,3;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^p/(c + d*x^n)^3,x]
```

```
[Out] (x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 3, 1 + n^(-1), -((b*x^n)/a), -((d*x^n
)/c)])/(c^3*(1 + (b*x^n)/a)^p)
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^n)^p}{(c+dx^n)^3} \, dx &= \left((a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p}\right) \int \frac{\left(1+\frac{bx^n}{a}\right)^p}{(c+dx^n)^3} \, dx \\ &= \frac{x(a+bx^n)^p \left(1+\frac{bx^n}{a}\right)^{-p} F_1\left(\frac{1}{n};-p,3;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{c^3}\end{aligned}$$

Mathematica [B] time = 0.405605, size = 180, normalized size = 3.05

$$\frac{ac(n+1)x(a+bx^n)^p F_1\left(\frac{1}{n};-p,3;1+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right)}{(c+dx^n)^3 \left(bcnpx^n F_1\left(1+\frac{1}{n};1-p,3;2+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right) - 3adnx^n F_1\left(1+\frac{1}{n};-p,4;2+\frac{1}{n};-\frac{bx^n}{a},-\frac{dx^n}{c}\right) + ac(n+1)F_1\left(\frac{1}{n}\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^n)^p/(c + d*x^n)^3,x]
```

```
[Out] (a*c*(1 + n)*x*(a + b*x^n)^p*AppellF1[n^(-1), -p, 3, 1 + n^(-1), -((b*x^n)/
a), -((d*x^n)/c)])/((c + d*x^n)^3*(b*c*n*p*x^n*AppellF1[1 + n^(-1), 1 - p,
3, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] - 3*a*d*n*x^n*AppellF1[1 + n^(-1
), -p, 4, 2 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + a*c*(1 + n)*AppellF1[n^
(-1), -p, 3, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)]))
```

Maple [F] time = 0.708, size = 0, normalized size = 0.

$$\int \frac{(a + bx^n)^p}{(c + dx^n)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^p/(c+d*x^n)^3,x)
```

```
[Out] int((a+b*x^n)^p/(c+d*x^n)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^n + a)^p}{(dx^n + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p/(c+d*x^n)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p/(d*x^n + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^n + a)^p}{d^3x^{3n} + 3\,cd^2x^{2n} + 3\,c^2dx^n + c^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p/(c+d*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^p/(d^3*x^(3*n) + 3*c*d^2*x^(2*n) + 3*c^2*d*x^n + c^3),
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx^n)^p}{(c + dx^n)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**p/(c+d*x**n)**3,x)
```

```
[Out] Integral((a + b*x**n)**p/(c + d*x**n)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^n + a)^p}{(dx^n + c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p/(c+d*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^p/(d*x^n + c)^3, x)
```

3.320 $\int (a+bx^n)^p (c+dx^n)^{-1-\frac{1}{n}-p}\,dx$

Optimal. Leaf size=93

$$\frac{x(a+bx^n)^p (c+dx^n)^{-\frac{1}{n}-p} \left(\frac{c(a+bx^n)}{a(c+dx^n)}\right)^{-p} {}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{c}$$

```
[Out] (x*(a + b*x^n)^p*(c + d*x^n)^(-n^(-1) - p)*Hypergeometric2F1[n^(-1), -p, 1
+ n^(-1), -(((b*c - a*d)*x^n)/(a*(c + d*x^n)))])/(c*((c*(a + b*x^n))/(a*(c
+ d*x^n)))^p)
```

Rubi [A] time = 0.0219041, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {380}

$$\frac{x(a+bx^n)^p (c+dx^n)^{-\frac{1}{n}-p} \left(\frac{c(a+bx^n)}{a(c+dx^n)}\right)^{-p} {}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^p*(c + d*x^n)^(-1 - n^(-1) - p),x]
```

```
[Out] (x*(a + b*x^n)^p*(c + d*x^n)^(-n^(-1) - p)*Hypergeometric2F1[n^(-1), -p, 1
+ n^(-1), -(((b*c - a*d)*x^n)/(a*(c + d*x^n)))])/(c*((c*(a + b*x^n))/(a*(c
+ d*x^n)))^p)
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(x*(a + b*x^n)^p*Hypergeometric2F1[1/n, -p, 1 + 1/n, -(((b*c - a*d)
*x^n)/(a*(c + d*x^n)))])/(c*((c*(a + b*x^n))/(a*(c + d*x^n)))^p*(c + d*x^n)
^(1/n + p)), x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int (a+bx^n)^p (c+dx^n)^{-1-\frac{1}{n}-p}\,dx = \frac{x(a+bx^n)^p \left(\frac{c(a+bx^n)}{a(c+dx^n)}\right)^{-p} (c+dx^n)^{-\frac{1}{n}-p} {}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{c}$$

Mathematica [A] time = 0.067432, size = 94, normalized size = 1.01

$$\frac{x(a+bx^n)^p \left(\frac{bx^n}{a}+1\right)^{-p} (c+dx^n)^{-\frac{np+1}{n}} \left(\frac{dx^n}{c}+1\right)^p {}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};\frac{(ad-bc)x^n}{a(dx^n+c)}\right)}{c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^n)^p*(c + d*x^n)^(-1 - n^(-1) - p),x]
```

```
[Out] (x*(a + b*x^n)^p*(1 + (d*x^n)/c)^p*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1)
, ((-(b*c) + a*d)*x^n)/(a*(c + d*x^n))])/(c*(1 + (b*x^n)/a)^p*(c + d*x^n)^(
```

```
(1 + n*p)/n))
```

Maple [F] time = 0.773, size = 0, normalized size = 0.

$$\int (a+bx^n)^p (c+dx^n)^{-1-n^{-1}-p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^p*(c+d*x^n)^(-1-1/n-p),x)
```

```
[Out] int((a+b*x^n)^p*(c+d*x^n)^(-1-1/n-p),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^n+a)^p (dx^n+c)^{-p-\frac{1}{n}-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^(-1-1/n-p),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p*(d*x^n + c)^(-p - 1/n - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^n+a)^p}{(dx^n+c)^{\frac{np+n+1}{n}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^(-1-1/n-p),x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^p/(d*x^n + c)^((n*p + n + 1)/n), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**p*(c+d*x**n)**(-1-1/n-p),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^n+a)^p (dx^n+c)^{-p-\frac{1}{n}-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^(-1-1/n-p),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^p*(d*x^n + c)^(-p - 1/n - 1), x)
```

3.321 $\int (a+bx^n)^3 (c+dx^n)^{-4-\frac{1}{n}}\,dx$

Optimal. Leaf size=178

$$\frac{6a^2n^2x(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}}{c^3(n+1)(2n+1)(3n+1)}+\frac{6a^3n^3x(c+dx^n)^{-1/n}}{c^4(n+1)(2n+1)(3n+1)}+\frac{3anx(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}}{c^2\left(6n^2+5n+1\right)}+\frac{x(a+bx^n)^3(c+dx^n)}{c(3n+1)}$$

```
[Out] (x*(a + b*x^n)^3*(c + d*x^n)^(-3 - n^(-1)))/(c*(1 + 3*n)) + (3*a*n*x*(a + b
*x^n)^2*(c + d*x^n)^(-2 - n^(-1)))/(c^2*(1 + 5*n + 6*n^2)) + (6*a^2*n^2*x*(
a + b*x^n)*(c + d*x^n)^(-1 - n^(-1)))/(c^3*(1 + n)*(1 + 2*n)*(1 + 3*n)) + (
6*a^3*n^3*x)/(c^4*(1 + n)*(1 + 2*n)*(1 + 3*n)*(c + d*x^n)^n^(-1))
```

Rubi [A] time = 0.0859997, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {378, 191}

$$\frac{6a^2n^2x(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}}{c^3(n+1)(2n+1)(3n+1)}+\frac{6a^3n^3x(c+dx^n)^{-1/n}}{c^4(n+1)(2n+1)(3n+1)}+\frac{3anx(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}}{c^2\left(6n^2+5n+1\right)}+\frac{x(a+bx^n)^3(c+dx^n)}{c(3n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^3*(c + d*x^n)^(-4 - n^(-1)),x]
```

```
[Out] (x*(a + b*x^n)^3*(c + d*x^n)^(-3 - n^(-1)))/(c*(1 + 3*n)) + (3*a*n*x*(a + b
*x^n)^2*(c + d*x^n)^(-2 - n^(-1)))/(c^2*(1 + 5*n + 6*n^2)) + (6*a^2*n^2*x*(
a + b*x^n)*(c + d*x^n)^(-1 - n^(-1)))/(c^3*(1 + n)*(1 + 2*n)*(1 + 3*n)) + (
6*a^3*n^3*x)/(c^4*(1 + n)*(1 + 2*n)*(1 + 3*n)*(c + d*x^n)^n^(-1))
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int (a+bx^n)^3 (c+dx^n)^{-4-\frac{1}{n}}\,dx = \frac{x(a+bx^n)^3 (c+dx^n)^{-3-\frac{1}{n}}}{c(1+3n)} + \frac{(3an)\int (a+bx^n)^2 (c+dx^n)^{-3-\frac{1}{n}}\,dx}{c(1+3n)}$$

$$= \frac{x(a+bx^n)^3 (c+dx^n)^{-3-\frac{1}{n}}}{c(1+3n)} + \frac{3anx(a+bx^n)^2 (c+dx^n)^{-2-\frac{1}{n}}}{c^2\left(1+5n+6n^2\right)} + \frac{\left(6a^2n^2\right)\int (a+bx^n)(c+}{c^2\left(1+5n+6n\right.}$$

$$= \frac{x(a+bx^n)^3 (c+dx^n)^{-3-\frac{1}{n}}}{c(1+3n)} + \frac{3anx(a+bx^n)^2 (c+dx^n)^{-2-\frac{1}{n}}}{c^2\left(1+5n+6n^2\right)} + \frac{6a^2n^2x(a+bx^n)(c+dx^n}{c^3(1+n)\left(1+5n+6n\right.}$$

$$= \frac{x(a+bx^n)^3 (c+dx^n)^{-3-\frac{1}{n}}}{c(1+3n)} + \frac{3anx(a+bx^n)^2 (c+dx^n)^{-2-\frac{1}{n}}}{c^2\left(1+5n+6n^2\right)} + \frac{6a^2n^2x(a+bx^n)(c+dx^n}{c^3(1+n)\left(1+5n+6n\right.}$$

Mathematica [A] time = 0.128798, size = 218, normalized size = 1.22

$$\frac{x(c+dx^n)^{-\frac{1}{n}-3}\left(3a^2bcx^n\left(c^2\left(6n^2+5n+1\right)+2cdn(3n+1)x^n+2d^2n^2x^{2n}\right)+a^3\left(3c^2dn\left(6n^2+5n+1\right)x^n+c^3\left(6n^3+11n\right.\right.\right.}{c^4(n+1)(2n+1)(}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^3*(c + d*x^n)^(-4 - n^(-1)),x]
```

```
[Out] (x*(c + d*x^n)^(-3 - n^(-1))*(b^3*c^3*(1 + 3*n + 2*n^2)*x^(3*n) + 3*a*b^2*c
^2*(1 + n)*x^(2*n)*(c + 3*c*n + d*n*x^n) + 3*a^2*b*c*x^n*(c^2*(1 + 5*n + 6*
n^2) + 2*c*d*n*(1 + 3*n)*x^n + 2*d^2*n^2*x^(2*n)) + a^3*(c^3*(1 + 6*n + 11*
n^2 + 6*n^3) + 3*c^2*d*n*(1 + 5*n + 6*n^2)*x^n + 6*c*d^2*n^2*(1 + 3*n)*x^(2
*n) + 6*d^3*n^3*x^(3*n))))/(c^4*(1 + n)*(1 + 2*n)*(1 + 3*n))
```

Maple [F] time = 0.594, size = 0, normalized size = 0.

$$\int (a+bx^n)^3 (c+dx^n)^{-4-n^{-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^3*(c+d*x^n)^(-4-1/n),x)
```

```
[Out] int((a+b*x^n)^3*(c+d*x^n)^(-4-1/n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^n+a)^3(dx^n+c)^{-\frac{1}{n}-4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^3*(c+d*x^n)^(-4-1/n),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^3*(d*x^n + c)^(-1/n - 4), x)
```

Fricas [B] time = 1.66309, size = 971, normalized size = 5.46

$$\frac{\left(6\,a^3d^4n^3 + b^3c^3d + \left(2\,b^3c^3d + 3\,ab^2c^2d^2 + 6\,a^2bcd^3\right)n^2 + 3\left(b^3c^3d + ab^2c^2d^2\right)n\right)xx^{4\,n} + \left(24\,a^3cd^3n^3 + b^3c^4 + 3\,ab^2c^3d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^3*(c+d*x^n)^(-4-1/n),x, algorithm="fricas")
```

```
[Out] ((6*a^3*d^4*n^3 + b^3*c^3*d + (2*b^3*c^3*d + 3*a*b^2*c^2*d^2 + 6*a^2*b*c*d^
3)*n^2 + 3*(b^3*c^3*d + a*b^2*c^2*d^2)*n)*x*x^(4*n) + (24*a^3*c*d^3*n^3 + b
^3*c^4 + 3*a*b^2*c^3*d + 2*(b^3*c^4 + 6*a*b^2*c^3*d + 12*a^2*b*c^2*d^2 + 3*
a^3*c*d^3)*n^2 + 3*(b^3*c^4 + 5*a*b^2*c^3*d + 2*a^2*b*c^2*d^2)*n)*x*x^(3*n)
 + 3*(12*a^3*c^2*d^2*n^3 + a*b^2*c^4 + a^2*b*c^3*d + (3*a*b^2*c^4 + 12*a^2*
b*c^3*d + 7*a^3*c^2*d^2)*n^2 + (4*a*b^2*c^4 + 7*a^2*b*c^3*d + a^3*c^2*d^2)*
n)*x*x^(2*n) + (24*a^3*c^3*d*n^3 + 3*a^2*b*c^4 + a^3*c^3*d + 2*(9*a^2*b*c^4
 + 13*a^3*c^3*d)*n^2 + 3*(5*a^2*b*c^4 + 3*a^3*c^3*d)*n)*x*x^n + (6*a^3*c^4*
n^3 + 11*a^3*c^4*n^2 + 6*a^3*c^4*n + a^3*c^4)*x)/((6*c^4*n^3 + 11*c^4*n^2 +
 6*c^4*n + c^4)*(d*x^n + c)^((4*n + 1)/n))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**3*(c+d*x**n)**(-4-1/n),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^3*(c+d*x^n)^(-4-1/n),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.322 $\int (a+bx^n)^2 (c+dx^n)^{-3-\frac{1}{n}}\, dx$

Optimal. Leaf size=116

$$\frac{2a^2n^2x(c+dx^n)^{-1/n}}{c^3(n+1)(2n+1)}+\frac{2anx(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}}{c^2(n+1)(2n+1)}+\frac{x(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}}{c(2n+1)}$$

```
[Out] (x*(a + b*x^n)^2*(c + d*x^n)^(-2 - n^(-1)))/(c*(1 + 2*n)) + (2*a*n*x*(a + b
*x^n)*(c + d*x^n)^(-1 - n^(-1)))/(c^2*(1 + n)*(1 + 2*n)) + (2*a^2*n^2*x)/(c
^3*(1 + n)*(1 + 2*n)*(c + d*x^n)^n^(-1))
```

Rubi [A] time = 0.0360545, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {378, 191}

$$\frac{2a^2n^2x(c+dx^n)^{-1/n}}{c^3(n+1)(2n+1)}+\frac{2anx(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}}{c^2(n+1)(2n+1)}+\frac{x(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}}{c(2n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^2*(c + d*x^n)^(-3 - n^(-1)),x]
```

```
[Out] (x*(a + b*x^n)^2*(c + d*x^n)^(-2 - n^(-1)))/(c*(1 + 2*n)) + (2*a*n*x*(a + b
*x^n)*(c + d*x^n)^(-1 - n^(-1)))/(c^2*(1 + n)*(1 + 2*n)) + (2*a^2*n^2*x)/(c
^3*(1 + n)*(1 + 2*n)*(c + d*x^n)^n^(-1))
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^n)^2(c+dx^n)^{-3-\frac{1}{n}}\,dx &= \frac{x(a+bx^n)^2(c+dx^n)^{-2-\frac{1}{n}}}{c(1+2n)}+\frac{(2an)\int (a+bx^n)(c+dx^n)^{-2-\frac{1}{n}}\,dx}{c(1+2n)}\\ &= \frac{x(a+bx^n)^2(c+dx^n)^{-2-\frac{1}{n}}}{c(1+2n)}+\frac{2anx(a+bx^n)(c+dx^n)^{-1-\frac{1}{n}}}{c^2(1+n)(1+2n)}+\frac{\left(2a^2n^2\right)\int (c+dx^n)^{-1-\frac{1}{n}}\,dx}{c^2(1+n)(1+2n)}\\ &= \frac{x(a+bx^n)^2(c+dx^n)^{-2-\frac{1}{n}}}{c(1+2n)}+\frac{2anx(a+bx^n)(c+dx^n)^{-1-\frac{1}{n}}}{c^2(1+n)(1+2n)}+\frac{2a^2n^2x(c+dx^n)^{-1/n}}{c^3(1+n)(1+2n)}\end{aligned}$$

Mathematica [A] time = 0.073117, size = 113, normalized size = 0.97

$$\frac{x(c+dx^n)^{-\frac{1}{n}-2}\left(a^2\left(c^2\left(2n^2+3n+1\right)+2cdn(2n+1)x^n+2d^2n^2x^{2n}\right)+2abcx^n(2cn+c+dnx^n)+b^2c^2(n+1)x^{2n}\right)}{c^3(n+1)(2n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^2*(c + d*x^n)^(-3 - n^(-1)),x]
```

```
[Out] (x*(c + d*x^n)^(-2 - n^(-1))*(b^2*c^2*(1 + n)*x^(2*n) + 2*a*b*c*x^n*(c + 2*
c*n + d*n*x^n) + a^2*(c^2*(1 + 3*n + 2*n^2) + 2*c*d*n*(1 + 2*n)*x^n + 2*d^2
*n^2*x^(2*n))))/(c^3*(1 + n)*(1 + 2*n))
```

Maple [F] time = 0.577, size = 0, normalized size = 0.

$$\int (a+bx^n)^2 (c+dx^n)^{-3-n^{-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^2*(c+d*x^n)^(-3-1/n),x)
```

```
[Out] int((a+b*x^n)^2*(c+d*x^n)^(-3-1/n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^n+a)^2 (dx^n+c)^{-\frac{1}{n}-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(c+d*x^n)^(-3-1/n),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^2*(d*x^n + c)^(-1/n - 3), x)
```

Fricas [A] time = 1.58541, size = 468, normalized size = 4.03

$$\frac{\left(2\,a^2d^3n^2+b^2c^2d+\left(b^2c^2d+2\,abcd^2\right)n\right)xx^{3\,n}+\left(6\,a^2cd^2n^2+b^2c^3+2\,abc^2d+\left(b^2c^3+6\,abc^2d+2\,a^2cd^2\right)n\right)xx^{2\,n}+\left(6\cdots\right.}{\left(2\,c^3n^2+3\,c^3n+c^3\right)(dx^n+c)^{\frac{3n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(c+d*x^n)^(-3-1/n),x, algorithm="fricas")
```

```
[Out] ((2*a^2*d^3*n^2 + b^2*c^2*d + (b^2*c^2*d + 2*a*b*c*d^2)*n)*x*x^(3*n) + (6*a
^2*c*d^2*n^2 + b^2*c^3 + 2*a*b*c^2*d + (b^2*c^3 + 6*a*b*c^2*d + 2*a^2*c*d^2
)*n)*x*x^(2*n) + (6*a^2*c^2*d*n^2 + 2*a*b*c^3 + a^2*c^2*d + (4*a*b*c^3 + 5*
a^2*c^2*d)*n)*x*x^n + (2*a^2*c^3*n^2 + 3*a^2*c^3*n + a^2*c^3)*x)/((2*c^3*n^
2 + 3*c^3*n + c^3)*(d*x^n + c)^((3*n + 1)/n))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**2*(c+d*x**n)**(-3-1/n),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(c+d*x^n)^(-3-1/n),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.323 $\int (a+bx^n)(c+dx^n)^{-2-\frac{1}{n}}\,dx$

Optimal. Leaf size=58

$$\frac{x(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}}{c(n+1)}+\frac{anx(c+dx^n)^{-1/n}}{c^2(n+1)}$$

```
[Out] (x*(a + b*x^n)*(c + d*x^n)^(-1 - n^(-1)))/(c*(1 + n)) + (a*n*x)/(c^2*(1 + n
)*(c + d*x^n)^n^(-1))
```

Rubi [A] time = 0.0136548, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {378, 191}

$$\frac{x(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}}{c(n+1)}+\frac{anx(c+dx^n)^{-1/n}}{c^2(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)*(c + d*x^n)^(-2 - n^(-1)),x]
```

```
[Out] (x*(a + b*x^n)*(c + d*x^n)^(-1 - n^(-1)))/(c*(1 + n)) + (a*n*x)/(c^2*(1 + n
)*(c + d*x^n)^n^(-1))
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx^n)(c+dx^n)^{-2-\frac{1}{n}}\,dx &= \frac{x(a+bx^n)(c+dx^n)^{-1-\frac{1}{n}}}{c(1+n)}+\frac{(an)\int (c+dx^n)^{-1-\frac{1}{n}}\,dx}{c(1+n)}\\ &= \frac{x(a+bx^n)(c+dx^n)^{-1-\frac{1}{n}}}{c(1+n)}+\frac{anx(c+dx^n)^{-1/n}}{c^2(1+n)}\end{aligned}$$

Mathematica [C] time = 0.107201, size = 82, normalized size = 1.41

$$\frac{x(c+dx^n)^{-\frac{n+1}{n}}\left(a(n+1)(c+dx^n)\left(\frac{dx^n}{c}+1\right)^{\frac{1}{n}}{}_2F_1\left(2+\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)+bcx^n\right)}{c^2(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)*(c + d*x^n)^(-2 - n^(-1)),x]
```

```
[Out] (x*(b*c*x^n + a*(1 + n)*(c + d*x^n)*(1 + (d*x^n)/c)^n^(-1)*Hypergeometric2F
1[2 + n^(-1), n^(-1), 1 + n^(-1), -((d*x^n)/c)]))/(c^2*(1 + n)*(c + d*x^n)^
((1 + n)/n))
```

Maple [F] time = 0.421, size = 0, normalized size = 0.

$$\int (a+bx^n)(c+dx^n)^{-2-n^{-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)*(c+d*x^n)^(-2-1/n),x)
```

```
[Out] int((a+b*x^n)*(c+d*x^n)^(-2-1/n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^n+a)(dx^n+c)^{-\frac{1}{n}-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^(-2-1/n),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)*(d*x^n + c)^(-1/n - 2), x)
```

Fricas [A] time = 1.61014, size = 177, normalized size = 3.05

$$\frac{\left(ad^2n+bcd\right)xx^{2n}+\left(2\,acdn+bc^2+acd\right)xx^n+\left(ac^2n+ac^2\right)x}{\left(c^2n+c^2\right)(dx^n+c)^{\frac{2n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^(-2-1/n),x, algorithm="fricas")
```

```
[Out] ((a*d^2*n + b*c*d)*x*x^(2*n) + (2*a*c*d*n + b*c^2 + a*c*d)*x*x^n + (a*c^2*n
 + a*c^2)*x)/((c^2*n + c^2)*(d*x^n + c)^((2*n + 1)/n))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)*(c+d*x**n)**(-2-1/n),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^(-2-1/n),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.324 $\int (c+dx^n)^{-1-\frac{1}{n}}\,dx$

Optimal. Leaf size=18

$$\frac{x(c+dx^n)^{-1/n}}{c}$$

```
[Out] x/(c*(c + d*x^n)^n^(-1))
```

Rubi [A] time = 0.0026634, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {191}

$$\frac{x(c+dx^n)^{-1/n}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^n)^(-1 - n^(-1)),x]
```

```
[Out] x/(c*(c + d*x^n)^n^(-1))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int (c+dx^n)^{-1-\frac{1}{n}}\,dx=\frac{x(c+dx^n)^{-1/n}}{c}$$

Mathematica [A] time = 0.0267099, size = 18, normalized size = 1.

$$\frac{x(c+dx^n)^{-1/n}}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^n)^(-1 - n^(-1)),x]
```

```
[Out] x/(c*(c + d*x^n)^n^(-1))
```

Maple [B] time = 0.061, size = 53, normalized size = 2.9

$$x\mathrm{e}^{\left(-1-n^{-1}\right)\ln\left(c+d\mathrm{e}^{n\ln(x)}\right)}+\frac{dx\mathrm{e}^{n\ln(x)}}{c}\mathrm{e}^{\left(-1-n^{-1}\right)\ln\left(c+d\mathrm{e}^{n\ln(x)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^n)^(-1-1/n),x)
```

```
[Out] x*exp((-1-1/n)*ln(c+d*exp(n*ln(x))))+d/c*x*exp(n*ln(x))*exp((-1-1/n)*ln(c+d
*exp(n*ln(x))))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx^n + c)^{-\frac{1}{n}-1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(-1-1/n),x, algorithm="maxima")
```

```
[Out] integrate((d*x^n + c)^(-1/n - 1), x)
```

Fricas [A] time = 1.58121, size = 61, normalized size = 3.39

$$\frac{dxx^n + cx}{(dx^n + c)^{\frac{n+1}{n}} c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(-1-1/n),x, algorithm="fricas")
```

```
[Out] (d*x*x^n + c*x)/((d*x^n + c)^((n + 1)/n)*c)
```

Sympy [A] time = 35.6476, size = 211, normalized size = 11.72

$$\begin{cases} -\frac{d^{-\frac{1}{n}} x x^{-n} (x^n)^{-\frac{1}{n}}}{dn} & \text{for } c = 0 \\ 0^{-1-\frac{1}{n}} x & \text{for } c = -dx^n \\ x (0^n)^{-1-\frac{1}{n}} & \text{for } c = 0^n - \\ \frac{c^2 x}{c^3 (c+dx^n)^{\frac{1}{n}} + 2c^2 dx^n (c+dx^n)^{\frac{1}{n}} + cd^2 x^{2n} (c+dx^n)^{\frac{1}{n}}} + \frac{cdxx^n}{c^3 (c+dx^n)^{\frac{1}{n}} + 2c^2 dx^n (c+dx^n)^{\frac{1}{n}} + cd^2 x^{2n} (c+dx^n)^{\frac{1}{n}}} + \frac{dxx^n}{c^2 (c+dx^n)^{\frac{1}{n}} + cdx^n (c+dx^n)^{\frac{1}{n}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)**(-1-1/n),x)
```

```
[Out] Piecewise((-d**(-1/n)*x*x**(-n)*(x**n)**(-1/n)/(d*n), Eq(c, 0)), (0**(-1 -
1/n)*x, Eq(c, -d*x**n)), (x*(0**n)**(-1 - 1/n), Eq(c, 0**n - d*x**n)), (c**
2*x/(c**3*(c + d*x**n)**(1/n) + 2*c**2*d*x**n*(c + d*x**n)**(1/n) + c*d**2*
x**(2*n)*(c + d*x**n)**(1/n)) + c*d*x*x**n/(c**3*(c + d*x**n)**(1/n) + 2*c*
*2*d*x**n*(c + d*x**n)**(1/n) + c*d**2*x**(2*n)*(c + d*x**n)**(1/n)) + d*x*
x**n/(c**2*(c + d*x**n)**(1/n) + c*d*x**n*(c + d*x**n)**(1/n)), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (dx^n + c)^{-\frac{1}{n}-1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(-1-1/n),x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^(-1/n - 1), x)
```

3.325 $\int \frac{(c+dx^n)^{-1/n}}{a+bx^n}\,dx$

Optimal. Leaf size=53

$$\frac{x\left(c+dx^n\right)^{-1/n}\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a}$$

```
[Out] (x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(((b*c - a*d)*x^n)/(a*(c + d*x
^n)))])/(a*(c + d*x^n)^n^(-1))
```

Rubi [A] time = 0.0139553, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {379}

$$\frac{x\left(c+dx^n\right)^{-1/n}\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^n)*(c + d*x^n)^n^(-1)),x]
```

```
[Out] (x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(((b*c - a*d)*x^n)/(a*(c + d*x
^n)))])/(a*(c + d*x^n)^n^(-1))
```

Rule 379

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a^p*x*Hypergeometric2F1[1/n, -p, 1 + 1/n, -(((b*c - a*d)*x^n)/(a*(
c + d*x^n)))])/(c^(p + 1)*(c + d*x^n)^(1/n)), x] /; FreeQ[{a, b, c, d, n, q
}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && ILtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(c+dx^n\right)^{-1/n}}{a+bx^n}\,dx=\frac{x\left(c+dx^n\right)^{-1/n}\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{a}$$

Mathematica [A] time = 0.0129253, size = 52, normalized size = 0.98

$$\frac{x\left(c+dx^n\right)^{-1/n}\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};\frac{(ad-bc)x^n}{a(dx^n+c)}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^n)*(c + d*x^n)^n^(-1)),x]
```

```
[Out] (x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), ((-(b*c) + a*d)*x^n)/(a*(c + d*
x^n))])/(a*(c + d*x^n)^n^(-1))
```

Maple [F] time = 0.714, size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^n)\sqrt[n]{c+dx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*x^n)/((c+d*x^n)^(1/n)),x)
```

```
[Out] int(1/(a+b*x^n)/((c+d*x^n)^(1/n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^n+a)(dx^n+c)^{\left(\frac{1}{n}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)/((c+d*x^n)^(1/n)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)^(1/n)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(bx^n+a)(dx^n+c)^{\left(\frac{1}{n}\right)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)/((c+d*x^n)^(1/n)),x, algorithm="fricas")
```

```
[Out] integral(1/((b*x^n + a)*(d*x^n + c)^(1/n)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(c+dx^n)^{-\frac{1}{n}}}{a+bx^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**n)/((c+d*x**n)**(1/n)),x)
```

```
[Out] Integral((c + d*x**n)**(-1/n)/(a + b*x**n), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^n+a)(dx^n+c)^{\left(\frac{1}{n}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)/((c+d*x^n)^(1/n)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)*(d*x^n + c)^(1/n)), x)
```

3.326 $\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^2}\,dx$

Optimal. Leaf size=54

$$\frac{cx\,(c+dx^n)^{-1/n}\;{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a^2}$$

```
[Out] (c*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -(((b*c - a*d)*x^n)/(a*(c + d
*x^n)))])/(a^2*(c + d*x^n)^n^(-1))
```

Rubi [A] time = 0.0136726, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {379}

$$\frac{cx\,(c+dx^n)^{-1/n}\;{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^n)^(1 - n^(-1))/(a + b*x^n)^2,x]
```

```
[Out] (c*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -(((b*c - a*d)*x^n)/(a*(c + d
*x^n)))])/(a^2*(c + d*x^n)^n^(-1))
```

Rule 379

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a^p*x*Hypergeometric2F1[1/n, -p, 1 + 1/n, -(((b*c - a*d)*x^n)/(a*(
c + d*x^n)))])/(c^(p + 1)*(c + d*x^n)^(1/n)), x] /; FreeQ[{a, b, c, d, n, q
}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && ILtQ[p, 0]
```

Rubi steps

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^2}\,dx = \frac{cx\,(c+dx^n)^{-1/n}\;{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{a^2}$$

Mathematica [A] time = 0.0132257, size = 53, normalized size = 0.98

$$\frac{cx\,(c+dx^n)^{-1/n}\;{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};\frac{(ad-bc)x^n}{a(dx^n+c)}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^n)^(1 - n^(-1))/(a + b*x^n)^2,x]
```

```
[Out] (c*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), ((-(b*c) + a*d)*x^n)/(a*(c +
d*x^n))])/(a^2*(c + d*x^n)^n^(-1))
```

Maple [F] time = 0.7, size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b x^{n}\right)^{2}} \left(c+d x^{n}\right)^{1-n^{-1}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^n)^(1-1/n)/(a+b*x^n)^2,x)
```

```
[Out] int((c+d*x^n)^(1-1/n)/(a+b*x^n)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^{n}+c\right)^{-\frac{1}{n}+1}}{\left(bx^{n}+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(1-1/n)/(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x^n + c)^(-1/n + 1)/(b*x^n + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(dx^{n}+c\right)^{\frac{n-1}{n}}}{b^{2}x^{2\,n}+2\,abx^{n}+a^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(1-1/n)/(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x^n + c)^((n - 1)/n)/(b^2*x^(2*n) + 2*a*b*x^n + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)**(1-1/n)/(a+b*x**n)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^{n}+c\right)^{-\frac{1}{n}+1}}{\left(bx^{n}+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(1-1/n)/(a+b*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^(-1/n + 1)/(b*x^n + a)^2, x)
```

3.327 $\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^3}\,dx$

Optimal. Leaf size=56

$$\frac{c^2x\left(c+dx^n\right)^{-1/n}{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a^3}$$

```
[Out] (c^2*x*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -(((b*c - a*d)*x^n)/(a*(c +
 d*x^n)))])/(a^3*(c + d*x^n)^n^(-1))
```

Rubi [A] time = 0.0151126, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {379}

$$\frac{c^2x\left(c+dx^n\right)^{-1/n}{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^n)^(2 - n^(-1))/(a + b*x^n)^3,x]
```

```
[Out] (c^2*x*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -(((b*c - a*d)*x^n)/(a*(c +
 d*x^n)))])/(a^3*(c + d*x^n)^n^(-1))
```

Rule 379

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a^p*x*Hypergeometric2F1[1/n, -p, 1 + 1/n, -(((b*c - a*d)*x^n)/(a*(
c + d*x^n)))])/(c^(p + 1)*(c + d*x^n)^(1/n)), x] /; FreeQ[{a, b, c, d, n, q
}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && ILtQ[p, 0]
```

Rubi steps

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^3}\,dx = \frac{c^2x\left(c+dx^n\right)^{-1/n}{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{a^3}$$

Mathematica [A] time = 0.011421, size = 55, normalized size = 0.98

$$\frac{c^2x\left(c+dx^n\right)^{-1/n}{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};\frac{(ad-bc)x^n}{a(dx^n+c)}\right)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^n)^(2 - n^(-1))/(a + b*x^n)^3,x]
```

```
[Out] (c^2*x*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), ((-(b*c) + a*d)*x^n)/(a*(c
+ d*x^n))])/(a^3*(c + d*x^n)^n^(-1))
```

Maple [F] time = 0.724, size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^n)^3} (c+dx^n)^{2-n^{-1}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^n)^(2-1/n)/(a+b*x^n)^3,x)
```

```
[Out] int((c+d*x^n)^(2-1/n)/(a+b*x^n)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^n+c)^{-\frac{1}{n}+2}}{(bx^n+a)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(2-1/n)/(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x^n + c)^(-1/n + 2)/(b*x^n + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx^n+c)^{\frac{2n-1}{n}}}{b^3x^{3n}+3ab^2x^{2n}+3a^2bx^n+a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(2-1/n)/(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral((d*x^n + c)^((2*n - 1)/n)/(b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*b
*x^n + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)**(2-1/n)/(a+b*x**n)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^n+c)^{-\frac{1}{n}+2}}{(bx^n+a)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(2-1/n)/(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^(-1/n + 2)/(b*x^n + a)^3, x)
```

3.328 $\int (a+bx^n)^p (c+dx^n)^{-2-\frac{1}{n}-p} \, dx$

Optimal. Leaf size=193

$$\frac{x(a+bx^n)^{p+1}(c+dx^n)^{-\frac{1}{n}-p-1}(n(p+1)(bc-ad)+bc)\left(\frac{c(a+bx^n)}{a(c+dx^n)}\right)^{-p-1} {}_2F_1\left(\frac{1}{n},-p-1;1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{acn(p+1)(bc-ad)} - \frac{bx(a+bx^n)^{p+1}(}{an(p+1)(}$$

```
[Out] -((b*x*(a + b*x^n)^(1 + p)*(c + d*x^n)^(-1 - n^(-1) - p))/(a*(b*c - a*d)*n*
(1 + p))) + ((b*c + (b*c - a*d)*n*(1 + p))*x*(a + b*x^n)^(1 + p)*((c*(a + b
*x^n))/(a*(c + d*x^n)))^(-1 - p)*(c + d*x^n)^(-1 - n^(-1) - p)*Hypergeometr
ic2F1[n^(-1), -1 - p, 1 + n^(-1), -(((b*c - a*d)*x^n)/(a*(c + d*x^n)))])/(a
*c*(b*c - a*d)*n*(1 + p))
```

Rubi [A] time = 0.0812199, antiderivative size = 179, normalized size of antiderivative = 0.93, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {382, 380}

$$\frac{x(a+bx^n)^{p+1}(c+dx^n)^{-\frac{1}{n}-p-1}\left(\frac{b}{n(p+1)(bc-ad)}+\frac{1}{c}\right)\left(\frac{c(a+bx^n)}{a(c+dx^n)}\right)^{-p-1} {}_2F_1\left(\frac{1}{n},-p-1;1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a} - \frac{bx(a+bx^n)^{p+1}(c+dx}{an(p+1)(bc-a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^p*(c + d*x^n)^(-2 - n^(-1) - p),x]
```

```
[Out] -((b*x*(a + b*x^n)^(1 + p)*(c + d*x^n)^(-1 - n^(-1) - p))/(a*(b*c - a*d)*n*
(1 + p))) + ((c^(-1) + b/((b*c - a*d)*n*(1 + p)))*x*(a + b*x^n)^(1 + p)*((c
*(a + b*x^n))/(a*(c + d*x^n)))^(-1 - p)*(c + d*x^n)^(-1 - n^(-1) - p)*Hyper
geometric2F1[n^(-1), -1 - p, 1 + n^(-1), -(((b*c - a*d)*x^n)/(a*(c + d*x^n)
))])/a
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(x*(a + b*x^n)^p*Hypergeometric2F1[1/n, -p, 1 + 1/n, -(((b*c - a*d)
*x^n)/(a*(c + d*x^n)))])/(c*((c*(a + b*x^n))/(a*(c + d*x^n)))^p*(c + d*x^n)
^(1/n + p)), x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] &&
EqQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\int (a+bx^n)^p (c+dx^n)^{-2-\frac{1}{n}-p}\,dx = -\frac{bx(a+bx^n)^{1+p}(c+dx^n)^{-1-\frac{1}{n}-p}}{a(bc-ad)n(1+p)} + \frac{\left(1+\frac{bc}{(bc-ad)n(1+p)}\right)\int (a+bx^n)^{1+p}(c+dx^n)^{-2-\frac{1}{n}}}{a}$$

$$= -\frac{bx(a+bx^n)^{1+p}(c+dx^n)^{-1-\frac{1}{n}-p}}{a(bc-ad)n(1+p)} + \frac{\left(1+\frac{bc}{(bc-ad)n(1+p)}\right)x(a+bx^n)^{1+p}\left(\frac{c(a+bx^n)}{a(c+dx^n)}\right)^{-1-p}}{}$$

Mathematica [B] time = 45.8204, size = 1414, normalized size = 7.33

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^n)^p*(c + d*x^n)^(-2 - n^(-1) - p),x]
```

```
[Out] (c^4*(1 + n)*(1 + 2*n)*(1 + 3*n)*x*(a + b*x^n)^(3 + p)*(c + d*x^n)^(-2 - n^
(-1) - p)*(1 + (d*x^n)/c)*Gamma[2 + n^(-1)]*Gamma[-p]*(Hypergeometric2F1[1,
 -p, 1 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + (d*n*x^n*((c*Hypergeo
metric2F1[1, -p, 2 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))])/(1 + n) +
((b*c - a*d)*x^n*Gamma[1 + n^(-1)]*Gamma[1 - p]*Hypergeometric2F1[2, 1 - p,
 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))])/((1 + 2*n)*(a + b*x^n)*Gam
ma[2 + n^(-1)]*Gamma[-p])))/c^2))/(-(c*d*(1 + 3*n)*(1 + n + n*p)*x^n*(a + b
*x^n)^2*(c^2*(1 + n)*(1 + 2*n)*(a + b*x^n)*Gamma[2 + n^(-1)]*Gamma[-p]*Hype
rgeometric2F1[1, -p, 1 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + d*n*x
^n*(c*(1 + 2*n)*(a + b*x^n)*Gamma[2 + n^(-1)]*Gamma[-p]*Hypergeometric2F1[1
, -p, 2 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + (b*c - a*d)*(1 + n)*
x^n*Gamma[1 + n^(-1)]*Gamma[1 - p]*Hypergeometric2F1[2, 1 - p, 3 + n^(-1),
((b*c - a*d)*x^n)/(c*(a + b*x^n))]))) + b*c*n*(1 + 3*n)*p*x^n*(a + b*x^n)*(
c + d*x^n)*(c^2*(1 + n)*(1 + 2*n)*(a + b*x^n)*Gamma[2 + n^(-1)]*Gamma[-p]*H
ypergeometric2F1[1, -p, 1 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + d*
n*x^n*(c*(1 + 2*n)*(a + b*x^n)*Gamma[2 + n^(-1)]*Gamma[-p]*Hypergeometric2F
1[1, -p, 2 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + (b*c - a*d)*(1 +
n)*x^n*Gamma[1 + n^(-1)]*Gamma[1 - p]*Hypergeometric2F1[2, 1 - p, 3 + n^(-1
), ((b*c - a*d)*x^n)/(c*(a + b*x^n))])) + c*(1 + 3*n)*(a + b*x^n)^2*(c + d*
x^n)*(c^2*(1 + n)*(1 + 2*n)*(a + b*x^n)*Gamma[2 + n^(-1)]*Gamma[-p]*Hyperge
ometric2F1[1, -p, 1 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + d*n*x^n*
(c*(1 + 2*n)*(a + b*x^n)*Gamma[2 + n^(-1)]*Gamma[-p]*Hypergeometric2F1[1, -
p, 2 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + (b*c - a*d)*(1 + n)*x^n
*Gamma[1 + n^(-1)]*Gamma[1 - p]*Hypergeometric2F1[2, 1 - p, 3 + n^(-1), ((b
*c - a*d)*x^n)/(c*(a + b*x^n))])) + n^2*x^n*(c + d*x^n)*(a*c^2*(-(b*c) + a*
d)*(1 + 2*n)*(1 + 3*n)*p*(a + b*x^n)*Gamma[2 + n^(-1)]*Gamma[-p]*Hypergeome
tric2F1[2, 1 - p, 2 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + c*d*(1 +
 3*n)*(a + b*x^n)^2*(c*(1 + 2*n)*(a + b*x^n)*Gamma[2 + n^(-1)]*Gamma[-p]*Hy
pergeometric2F1[1, -p, 2 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + (b*
c - a*d)*(1 + n)*x^n*Gamma[1 + n^(-1)]*Gamma[1 - p]*Hypergeometric2F1[2, 1
- p, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))]) - d*(b*c - a*d)*x^n*(b
*c*(1 + n)*(1 + 3*n)*x^n*(a + b*x^n)*Gamma[1 + n^(-1)]*Gamma[1 - p]*Hyperge
ometric2F1[2, 1 - p, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] - c*(1
+ n)*(1 + 3*n)*(a + b*x^n)^2*Gamma[1 + n^(-1)]*Gamma[1 - p]*Hypergeometric2
F1[2, 1 - p, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + a*c*n*(1 + 3*
n)*p*(a + b*x^n)*Gamma[2 + n^(-1)]*Gamma[-p]*Hypergeometric2F1[2, 1 - p, 3
+ n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] - 2*a*(-(b*c) + a*d)*n*(1 + n)
*(-1 + p)*x^n*Gamma[1 + n^(-1)]*Gamma[1 - p]*Hypergeometric2F1[3, 2 - p, 4
+ n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))])))
```

Maple [F] time = 0.793, size = 0, normalized size = 0.

$$\int (a+bx^n)^p (c+dx^n)^{-2-n^{-1}-p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^p*(c+d*x^n)^(-2-1/n-p),x)
```

```
[Out] int((a+b*x^n)^p*(c+d*x^n)^(-2-1/n-p),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^n+a)^p (dx^n+c)^{-p-\frac{1}{n}-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^(-2-1/n-p),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p*(d*x^n + c)^(-p - 1/n - 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^n+a)^p}{(dx^n+c)^{\frac{np+2n+1}{n}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^(-2-1/n-p),x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^p/(d*x^n + c)^((n*p + 2*n + 1)/n), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**p*(c+d*x**n)**(-2-1/n-p),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^n+a)^p (dx^n+c)^{-p-\frac{1}{n}-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p*(c+d*x^n)^(-2-1/n-p),x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^p*(d*x^n + c)^(-p - 1/n - 2), x)
```

3.329 $\int (a+bx^n)^{\frac{adn-bc(1+n)}{(bc-ad)n}} (c+dx^n)^{\frac{ad-bcn+adn}{bcn-adn}}\,dx$

Optimal. Leaf size=57

$$\frac{x(a+bx^n)^{-\frac{bc}{n(bc-ad)}}(c+dx^n)^{\frac{ad}{n(bc-ad)}}}{ac}$$

```
[Out] (x*(c + d*x^n)^((a*d)/((b*c - a*d)*n)))/(a*c*(a + b*x^n)^((b*c)/((b*c - a*d
)*n)))
```

Rubi [A] time = 0.0271107, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 69, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.014, Rules used = {381}

$$\frac{x(a+bx^n)^{-\frac{bc}{n(bc-ad)}}(c+dx^n)^{\frac{ad}{n(bc-ad)}}}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^((a*d*n - b*c*(1 + n))/((b*c - a*d)*n))*(c + d*x^n)^((a*d -
 b*c*n + a*d*n)/(b*c*n - a*d*n)),x]
```

```
[Out] (x*(c + d*x^n)^((a*d)/((b*c - a*d)*n)))/(a*c*(a + b*x^n)^((b*c)/((b*c - a*d
)*n)))
```

Rule 381

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c), x] /; FreeQ[{a,
b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] &&
EqQ[a*d*(p + 1) + b*c*(q + 1), 0]
```

Rubi steps

$$\int (a+bx^n)^{\frac{adn-bc(1+n)}{(bc-ad)n}} (c+dx^n)^{\frac{ad-bcn+adn}{bcn-adn}}\,dx=\frac{x(a+bx^n)^{-\frac{bc}{(bc-ad)n}}(c+dx^n)^{\frac{ad}{(bc-ad)n}}}{ac}$$

Mathematica [A] time = 0.0492145, size = 55, normalized size = 0.96

$$\frac{x(a+bx^n)^{-\frac{bc}{bcn-adn}}(c+dx^n)^{\frac{ad}{bcn-adn}}}{ac}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^((a*d*n - b*c*(1 + n))/((b*c - a*d)*n))*(c + d*x^n)^(
(a*d - b*c*n + a*d*n)/(b*c*n - a*d*n)),x]
```

```
[Out] (x*(c + d*x^n)^((a*d)/(b*c*n - a*d*n)))/(a*c*(a + b*x^n)^((b*c)/(b*c*n - a*
d*n)))
```

Maple [F] time = 0.817, size = 0, normalized size = 0.

$$\int \left(a+b x^{n}\right)^{\frac{a d n -b c \left(1+n \right)}{\left(-a d +b c \right) n}} \left(c +d x^{n}\right)^{\frac{a d n -b c n +a d}{-a d n +b c n}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^((a*d*n-b*c*(1+n))/(-a*d+b*c)/n)*(c+d*x^n)^((a*d*n-b*c*n+a*d)
/(-a*d*n+b*c*n)),x)
```

```
[Out] int((a+b*x^n)^((a*d*n-b*c*(1+n))/(-a*d+b*c)/n)*(c+d*x^n)^((a*d*n-b*c*n+a*d)
/(-a*d*n+b*c*n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^n + a)^{\frac{bc(n+1)-adn}{(bc-ad)n}} (dx^n + c)^{\frac{bcn-adn-ad}{bcn-adn}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^((a*d*n-b*c*(1+n))/(-a*d+b*c)/n)*(c+d*x^n)^((a*d*n-b*c*
n+a*d)/(-a*d*n+b*c*n)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^n + a)^((b*c*(n + 1) - a*d*n)/((b*c - a*d)*n))*(d*x^n + c
)^((b*c*n - a*d*n - a*d)/(b*c*n - a*d*n))), x)
```

Fricas [A] time = 1.71863, size = 209, normalized size = 3.67

$$\frac{\left(bdxx^{2n} + acx + (bc + ad)xx^n\right)(dx^n + c)^{\frac{ad-(bc-ad)n}{(bc-ad)n}}}{(bx^n + a)^{\frac{bc+(bc-ad)n}{(bc-ad)n}}\, ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^((a*d*n-b*c*(1+n))/(-a*d+b*c)/n)*(c+d*x^n)^((a*d*n-b*c*
n+a*d)/(-a*d*n+b*c*n)),x, algorithm="fricas")
```

```
[Out] (b*d*x*x^(2*n) + a*c*x + (b*c + a*d)*x*x^n)*(d*x^n + c)^((a*d - (b*c - a*d)
*n)/((b*c - a*d)*n))/((b*x^n + a)^((b*c + (b*c - a*d)*n)/((b*c - a*d)*n))*a
*c)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**((a*d*n-b*c*(1+n))/(-a*d+b*c)/n)*(c+d*x**n)**((a*d*n-
b*c*n+a*d)/(-a*d*n+b*c*n)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^n+a\right)^{\frac{bc(n+1)-adn}{(bc-ad)n}}\left(dx^n+c\right)^{\frac{bcn-adn-ad}{bcn-adn}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^((a*d*n-b*c*(1+n))/(-a*d+b*c)/n)*(c+d*x^n)^((a*d*n-b*c*
n+a*d)/(-a*d*n+b*c*n)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^((b*c*(n + 1) - a*d*n)/((b*c - a*d)*n))*(d*x^n + c
)^((b*c*n - a*d*n - a*d)/(b*c*n - a*d*n))), x)
```

3.330 $\int (a+bx^n)^2 (c+dx^n)^{-4-\frac{1}{n}}\, dx$

Optimal. Leaf size=327

$$-\frac{2a^2n^2x(c+dx^n)^{-1/n}(3adn-b(3cn+c))}{c^4(n+1)(2n+1)(3n+1)(bc-ad)}-\frac{x(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}(3adn-b(3cn+c))}{c^2\left(6n^2+5n+1\right)(bc-ad)}-\frac{2anx(a+bx^n)(c+dx^n}{c^3(n+1)(2n+1}$$

```
[Out] -(b*x*(a + b*x^n)^3*(c + d*x^n)^(-3 - n^(-1)))/(3*a*(b*c - a*d)*n) - ((3*a*
d*n - b*(c + 3*c*n))*x*(a + b*x^n)^3*(c + d*x^n)^(-3 - n^(-1)))/(3*a*c*(b*c
 - a*d)*n*(1 + 3*n)) - ((3*a*d*n - b*(c + 3*c*n))*x*(a + b*x^n)^2*(c + d*x^
n)^(-2 - n^(-1)))/(c^2*(b*c - a*d)*(1 + 5*n + 6*n^2)) - (2*a*n*(3*a*d*n - b
*(c + 3*c*n))*x*(a + b*x^n)*(c + d*x^n)^(-1 - n^(-1)))/(c^3*(b*c - a*d)*(1
+ n)*(1 + 2*n)*(1 + 3*n)) - (2*a^2*n^2*(3*a*d*n - b*(c + 3*c*n))*x)/(c^4*(b
*c - a*d)*(1 + n)*(1 + 2*n)*(1 + 3*n)*(c + d*x^n)^n^(-1))
```

Rubi [A] time = 0.184194, antiderivative size = 327, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {382, 378, 191}

$$-\frac{2a^2n^2x(c+dx^n)^{-1/n}(3adn-b(3cn+c))}{c^4(n+1)(2n+1)(3n+1)(bc-ad)}-\frac{x(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}(3adn-b(3cn+c))}{c^2\left(6n^2+5n+1\right)(bc-ad)}-\frac{2anx(a+bx^n)(c+dx^n}{c^3(n+1)(2n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)^2*(c + d*x^n)^(-4 - n^(-1)),x]
```

```
[Out] -(b*x*(a + b*x^n)^3*(c + d*x^n)^(-3 - n^(-1)))/(3*a*(b*c - a*d)*n) - ((3*a*
d*n - b*(c + 3*c*n))*x*(a + b*x^n)^3*(c + d*x^n)^(-3 - n^(-1)))/(3*a*c*(b*c
 - a*d)*n*(1 + 3*n)) - ((3*a*d*n - b*(c + 3*c*n))*x*(a + b*x^n)^2*(c + d*x^
n)^(-2 - n^(-1)))/(c^2*(b*c - a*d)*(1 + 5*n + 6*n^2)) - (2*a*n*(3*a*d*n - b
*(c + 3*c*n))*x*(a + b*x^n)*(c + d*x^n)^(-1 - n^(-1)))/(c^3*(b*c - a*d)*(1
+ n)*(1 + 2*n)*(1 + 3*n)) - (2*a^2*n^2*(3*a*d*n - b*(c + 3*c*n))*x)/(c^4*(b
*c - a*d)*(1 + n)*(1 + 2*n)*(1 + 3*n)*(c + d*x^n)^n^(-1))
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int (a+bx^n)^2 (c+dx^n)^{-4-\frac{1}{n}}\,dx = -\frac{bx(a+bx^n)^3(c+dx^n)^{-3-\frac{1}{n}}}{3a(bc-ad)n} + \frac{\left(3+\frac{bc}{bcn-adn}\right)\int (a+bx^n)^3(c+dx^n)^{-4-\frac{1}{n}}\,dx}{3a}$$

$$= -\frac{bx(a+bx^n)^3(c+dx^n)^{-3-\frac{1}{n}}}{3a(bc-ad)n} + \frac{\left(3+\frac{bc}{bcn-adn}\right)x(a+bx^n)^3(c+dx^n)^{-3-\frac{1}{n}}}{3ac(1+3n)} + \frac{\left(n\left(3+\frac{bc}{bcn-a\ldots}\right.\right.}{\ldots}$$

$$= -\frac{bx(a+bx^n)^3(c+dx^n)^{-3-\frac{1}{n}}}{3a(bc-ad)n} + \frac{\left(3+\frac{bc}{bcn-adn}\right)x(a+bx^n)^3(c+dx^n)^{-3-\frac{1}{n}}}{3ac(1+3n)} + \frac{n\left(3+\frac{bc}{bcn-ad\ldots}\right.}{c\ldots}$$

$$= -\frac{bx(a+bx^n)^3(c+dx^n)^{-3-\frac{1}{n}}}{3a(bc-ad)n} + \frac{\left(3+\frac{bc}{bcn-adn}\right)x(a+bx^n)^3(c+dx^n)^{-3-\frac{1}{n}}}{3ac(1+3n)} + \frac{n\left(3+\frac{bc}{bcn-ad\ldots}\right.}{c\ldots}$$

$$= -\frac{bx(a+bx^n)^3(c+dx^n)^{-3-\frac{1}{n}}}{3a(bc-ad)n} + \frac{\left(3+\frac{bc}{bcn-adn}\right)x(a+bx^n)^3(c+dx^n)^{-3-\frac{1}{n}}}{3ac(1+3n)} + \frac{n\left(3+\frac{bc}{bcn-ad\ldots}\right.}{c\ldots}$$

Mathematica [C] time = 0.198272, size = 153, normalized size = 0.47

$$\frac{x(c+dx^n)^{-1/n}\left(\frac{dx^n}{c}+1\right)^{\frac{1}{n}}\left((n+1)\left(a^2(2n+1)\,{}_2F_1\left(4+\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)+b^2x^{2n}\,{}_2F_1\left(2+\frac{1}{n},4+\frac{1}{n};3+\frac{1}{n};-\frac{dx^n}{c}\right)\right)+2ab(2\ldots\right.}{c^4(n+1)(2n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)^2*(c + d*x^n)^(-4 - n^(-1)),x]
```

```
[Out] (x*(1 + (d*x^n)/c)^n^(-1)*(2*a*b*(1 + 2*n)*x^n*Hypergeometric2F1[1 + n^(-1)
, 4 + n^(-1), 2 + n^(-1), -((d*x^n)/c)] + (1 + n)*(b^2*x^(2*n)*Hypergeometr
ic2F1[2 + n^(-1), 4 + n^(-1), 3 + n^(-1), -((d*x^n)/c)] + a^2*(1 + 2*n)*Hyp
ergeometric2F1[4 + n^(-1), n^(-1), 1 + n^(-1), -((d*x^n)/c)])))/(c^4*(1 + n
)*(1 + 2*n)*(c + d*x^n)^n^(-1))
```

Maple [F] time = 0.586, size = 0, normalized size = 0.

$$\int (a+bx^n)^2 (c+dx^n)^{-4-n^{-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)^2*(c+d*x^n)^(-4-1/n),x)
```

```
[Out] int((a+b*x^n)^2*(c+d*x^n)^(-4-1/n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^n+a)^2(dx^n+c)^{-\frac{1}{n}-4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(c+d*x^n)^(-4-1/n),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^2*(d*x^n + c)^(-1/n - 4), x)
```

Fricas [A] time = 1.65607, size = 819, normalized size = 2.5

$$\frac{\left(6 a^2 d^4 n^3 + b^2 c^2 d^2 n + \left(b^2 c^2 d^2 + 4 abcd^3\right) n^2\right) x x^{4n} + \left(24 a^2 c d^3 n^3 + b^2 c^3 d + 2 \left(2 b^2 c^3 d + 8 abc^2 d^2 + 3 a^2 c d^3\right) n^2 + \left(5 b^2 c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(c+d*x^n)^(-4-1/n),x, algorithm="fricas")
```

```
[Out] ((6*a^2*d^4*n^3 + b^2*c^2*d^2*n + (b^2*c^2*d^2 + 4*a*b*c*d^3)*n^2)*x*x^(4*n
) + (24*a^2*c*d^3*n^3 + b^2*c^3*d + 2*(2*b^2*c^3*d + 8*a*b*c^2*d^2 + 3*a^2*
c*d^3)*n^2 + (5*b^2*c^3*d + 4*a*b*c^2*d^2)*n)*x*x^(3*n) + (36*a^2*c^2*d^2*n
^3 + b^2*c^4 + 2*a*b*c^3*d + 3*(b^2*c^4 + 8*a*b*c^3*d + 7*a^2*c^2*d^2)*n^2
+ (4*b^2*c^4 + 14*a*b*c^3*d + 3*a^2*c^2*d^2)*n)*x*x^(2*n) + (24*a^2*c^3*d*n
^3 + 2*a*b*c^4 + a^2*c^3*d + 2*(6*a*b*c^4 + 13*a^2*c^3*d)*n^2 + (10*a*b*c^4
 + 9*a^2*c^3*d)*n)*x*x^n + (6*a^2*c^4*n^3 + 11*a^2*c^4*n^2 + 6*a^2*c^4*n +
a^2*c^4)*x)/((6*c^4*n^3 + 11*c^4*n^2 + 6*c^4*n + c^4)*(d*x^n + c)^((4*n + 1
)/n))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**2*(c+d*x**n)**(-4-1/n),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(c+d*x^n)^(-4-1/n),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.331 $\int (a+bx^n)(c+dx^n)^{-3-\frac{1}{n}}\,dx$

Optimal. Leaf size=127

$$\frac{x(c+dx^n)^{-\frac{1}{n}-1}(2adn+bc)}{c^2d(n+1)(2n+1)}+\frac{nx(c+dx^n)^{-1/n}(2adn+bc)}{c^3d(n+1)(2n+1)}-\frac{x(bc-ad)(c+dx^n)^{-\frac{1}{n}-2}}{cd(2n+1)}$$

```
[Out] -(((b*c - a*d)*x*(c + d*x^n)^(-2 - n^(-1)))/(c*d*(1 + 2*n))) + ((b*c + 2*a*
d*n)*x*(c + d*x^n)^(-1 - n^(-1)))/(c^2*d*(1 + n)*(1 + 2*n)) + (n*(b*c + 2*a
*d*n)*x)/(c^3*d*(1 + n)*(1 + 2*n)*(c + d*x^n)^n^(-1))
```

Rubi [A] time = 0.063562, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {385, 192, 191}

$$\frac{x(c+dx^n)^{-\frac{1}{n}-1}(2adn+bc)}{c^2d(n+1)(2n+1)}+\frac{nx(c+dx^n)^{-1/n}(2adn+bc)}{c^3d(n+1)(2n+1)}-\frac{x(bc-ad)(c+dx^n)^{-\frac{1}{n}-2}}{cd(2n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^n)*(c + d*x^n)^(-3 - n^(-1)),x]
```

```
[Out] -(((b*c - a*d)*x*(c + d*x^n)^(-2 - n^(-1)))/(c*d*(1 + 2*n))) + ((b*c + 2*a*
d*n)*x*(c + d*x^n)^(-1 - n^(-1)))/(c^2*d*(1 + n)*(1 + 2*n)) + (n*(b*c + 2*a
*d*n)*x)/(c^3*d*(1 + n)*(1 + 2*n)*(c + d*x^n)^n^(-1))
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int (a+bx^n)(c+dx^n)^{-3-\frac{1}{n}}\,dx = -\frac{(bc-ad)x(c+dx^n)^{-2-\frac{1}{n}}}{cd(1+2n)} + \frac{(bc+2adn)\int (c+dx^n)^{-2-\frac{1}{n}}\,dx}{cd(1+2n)}$$
$$= -\frac{(bc-ad)x(c+dx^n)^{-2-\frac{1}{n}}}{cd(1+2n)} + \frac{(bc+2adn)x(c+dx^n)^{-1-\frac{1}{n}}}{c^2d(1+n)(1+2n)} + \frac{(n(bc+2adn))\int (c+dx^n}{c^2d(1+n)(1+2n}$$
$$= -\frac{(bc-ad)x(c+dx^n)^{-2-\frac{1}{n}}}{cd(1+2n)} + \frac{(bc+2adn)x(c+dx^n)^{-1-\frac{1}{n}}}{c^2d(1+n)(1+2n)} + \frac{n(bc+2adn)x(c+dx^n)^{-1}}{c^3d(1+n)(1+2n)}$$

Mathematica [C] time = 0.0969818, size = 96, normalized size = 0.76

$$\frac{x(c+dx^n)^{-1/n}\left(\frac{dx^n}{c}+1\right)^{\frac{1}{n}}\left(a(n+1)\,{}_2F_1\left(3+\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)+bx^n\,{}_2F_1\left(1+\frac{1}{n},3+\frac{1}{n};2+\frac{1}{n};-\frac{dx^n}{c}\right)\right)}{c^3(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^n)*(c + d*x^n)^(-3 - n^(-1)),x]
```

```
[Out] (x*(1 + (d*x^n)/c)^n^(-1)*(b*x^n*Hypergeometric2F1[1 + n^(-1), 3 + n^(-1),
2 + n^(-1), -((d*x^n)/c)] + a*(1 + n)*Hypergeometric2F1[3 + n^(-1), n^(-1),
 1 + n^(-1), -((d*x^n)/c)]))/(c^3*(1 + n)*(c + d*x^n)^n^(-1))
```

Maple [F] time = 0.449, size = 0, normalized size = 0.

$$\int (a+bx^n)(c+dx^n)^{-3-n^{-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x^n)*(c+d*x^n)^(-3-1/n),x)
```

```
[Out] int((a+b*x^n)*(c+d*x^n)^(-3-1/n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (bx^n+a)(dx^n+c)^{-\frac{1}{n}-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^(-3-1/n),x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)*(d*x^n + c)^(-1/n - 3), x)
```

Fricas [A] time = 1.61052, size = 360, normalized size = 2.83

$$\frac{\left(2\,ad^3n^2+bcd^2n\right)xx^{3\,n}+\left(6\,acd^2n^2+bc^2d+\left(3\,bc^2d+2\,acd^2\right)n\right)xx^{2\,n}+\left(6\,ac^2dn^2+bc^3+ac^2d+\left(2\,bc^3+5\,ac^2d\right)n\right)x}{\left(2\,c^3n^2+3\,c^3n+c^3\right)(dx^n+c)^{\frac{3\,n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^(-3-1/n),x, algorithm="fricas")
```

```
[Out] ((2*a*d^3*n^2 + b*c*d^2*n)*x*x^(3*n) + (6*a*c*d^2*n^2 + b*c^2*d + (3*b*c^2*
d + 2*a*c*d^2)*n)*x*x^(2*n) + (6*a*c^2*d*n^2 + b*c^3 + a*c^2*d + (2*b*c^3 +
 5*a*c^2*d)*n)*x*x^n + (2*a*c^3*n^2 + 3*a*c^3*n + a*c^3)*x)/((2*c^3*n^2 + 3
*c^3*n + c^3)*(d*x^n + c)^((3*n + 1)/n))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)*(c+d*x**n)**(-3-1/n),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)*(c+d*x^n)^(-3-1/n),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.332 $\int (c+dx^n)^{-2-\frac{1}{n}}\,dx$

Optimal. Leaf size=50

$$\frac{nx(c+dx^n)^{-1/n}}{c^2(n+1)}+\frac{x(c+dx^n)^{-\frac{1}{n}-1}}{c(n+1)}$$

```
[Out] (x*(c + d*x^n)^(-1 - n^(-1)))/(c*(1 + n)) + (n*x)/(c^2*(1 + n)*(c + d*x^n)^
n^(-1))
```

Rubi [A] time = 0.0123664, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {192, 191}

$$\frac{nx(c+dx^n)^{-1/n}}{c^2(n+1)}+\frac{x(c+dx^n)^{-\frac{1}{n}-1}}{c(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^n)^(-2 - n^(-1)),x]
```

```
[Out] (x*(c + d*x^n)^(-1 - n^(-1)))/(c*(1 + n)) + (n*x)/(c^2*(1 + n)*(c + d*x^n)^
n^(-1))
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx^n)^{-2-\frac{1}{n}}\,dx &= \frac{x(c+dx^n)^{-1-\frac{1}{n}}}{c(1+n)}+\frac{n\int (c+dx^n)^{-1-\frac{1}{n}}\,dx}{c(1+n)}\\ &= \frac{x(c+dx^n)^{-1-\frac{1}{n}}}{c(1+n)}+\frac{nx(c+dx^n)^{-1/n}}{c^2(1+n)}\end{aligned}$$

Mathematica [C] time = 0.0262842, size = 55, normalized size = 1.1

$$\frac{x(c+dx^n)^{-1/n}\left(\frac{dx^n}{c}+1\right)^{\frac{1}{n}}{}_2F_1\left(2+\frac{1}{n},\frac{1}{n};1+\frac{1}{n};-\frac{dx^n}{c}\right)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^n)^(-2 - n^(-1)),x]
```

```
[Out] (x*(1 + (d*x^n)/c)^n^(-1)*Hypergeometric2F1[2 + n^(-1), n^(-1), 1 + n^(-1),
 -((d*x^n)/c)])/(c^2*(c + d*x^n)^n^(-1))
```

Maple [F] time = 0.398, size = 0, normalized size = 0.

$$\int (c + dx^n)^{-2-n^{-1}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^n)^(-2-1/n),x)
```

```
[Out] int((c+d*x^n)^(-2-1/n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (dx^n + c)^{-\frac{1}{n}-2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(-2-1/n),x, algorithm="maxima")
```

```
[Out] integrate((d*x^n + c)^(-1/n - 2), x)
```

Fricas [A] time = 1.58591, size = 139, normalized size = 2.78

$$\frac{d^2nxx^{2n} + (2\,cdn + cd)xx^n + \left(c^2n + c^2\right)x}{\left(c^2n + c^2\right)(dx^n + c)^{\frac{2n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(-2-1/n),x, algorithm="fricas")
```

```
[Out] (d^2*n*x*x^(2*n) + (2*c*d*n + c*d)*x*x^n + (c^2*n + c^2)*x)/((c^2*n + c^2)*
(d*x^n + c)^((2*n + 1)/n))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)**(-2-1/n),x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int (dx^n + c)^{-\frac{1}{n}-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(-2-1/n),x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^(-1/n - 2), x)
```

3.333 $\int \frac{(c+dx^n)^{-1-\frac{1}{n}}}{a+bx^n} dx$

Optimal. Leaf size=95

$$\frac{bx(c+dx^n)^{-1/n}\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a(bc-ad)}-\frac{dx(c+dx^n)^{-1/n}}{c(bc-ad)}$$

```
[Out] -((d*x)/(c*(b*c - a*d)*(c + d*x^n)^n^(-1))) + (b*x*Hypergeometric2F1[1, n^(
-1), 1 + n^(-1), -(((b*c - a*d)*x^n)/(a*(c + d*x^n)))])/(a*(b*c - a*d)*(c +
 d*x^n)^n^(-1))
```

Rubi [A] time = 0.0313336, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {382, 379}

$$\frac{bx(c+dx^n)^{-1/n}\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a(bc-ad)}-\frac{dx(c+dx^n)^{-1/n}}{c(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^n)^(-1 - n^(-1))/(a + b*x^n),x]
```

```
[Out] -((d*x)/(c*(b*c - a*d)*(c + d*x^n)^n^(-1))) + (b*x*Hypergeometric2F1[1, n^(
-1), 1 + n^(-1), -(((b*c - a*d)*x^n)/(a*(c + d*x^n)))])/(a*(b*c - a*d)*(c +
 d*x^n)^n^(-1))
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 379

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a^p*x*Hypergeometric2F1[1/n, -p, 1 + 1/n, -(((b*c - a*d)*x^n)/(a*(
c + d*x^n)))])/(c^(p + 1)*(c + d*x^n)^(1/n)), x] /; FreeQ[{a, b, c, d, n, q
}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx^n)^{-1-\frac{1}{n}}}{a+bx^n}\,dx &= -\frac{dx(c+dx^n)^{-1/n}}{c(bc-ad)}+\frac{b\int \frac{(c+dx^n)^{-1/n}}{a+bx^n}\,dx}{bc-ad}\\ &= -\frac{dx(c+dx^n)^{-1/n}}{c(bc-ad)}+\frac{bx(c+dx^n)^{-1/n}\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{a(bc-ad)}\end{aligned}$$

Mathematica [C] time = 6.44092, size = 153, normalized size = 1.61

$$\frac{x\left(c+dx^n\right)^{-\frac{n+1}{n}}\left(\frac{bnx^{2n}(ad-bc)\,{}_2F_1\left(2,2+\frac{1}{n};3+\frac{1}{n};\frac{(ad-bc)x^n}{a(dx^n+c)}\right)}{a^2(2n+1)(c+dx^n)}+\frac{bx^n\Phi\left(\frac{(ad-bc)x^n}{a(dx^n+c)},1,1+\frac{1}{n}\right)}{a}+\frac{a(c+dx^n)}{c(a+bx^n)}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^n)^(-1 - n^(-1))/(a + b*x^n),x]
```

```
[Out] (x*((a*(c + d*x^n))/(c*(a + b*x^n)) + (b*x^n*HurwitzLerchPhi[((-(b*c) + a*d
)*x^n)/(a*(c + d*x^n)), 1, 1 + n^(-1)])/a + (b*(-(b*c) + a*d)*n*x^(2*n)*Hyp
ergeometric2F1[2, 2 + n^(-1), 3 + n^(-1), ((-(b*c) + a*d)*x^n)/(a*(c + d*x^
n))])/(a^2*(1 + 2*n)*(c + d*x^n))))/(a*(c + d*x^n)^((1 + n)/n))
```

Maple [F] time = 0.696, size = 0, normalized size = 0.

$$\int \frac{1}{a+bx^n}\left(c+dx^n\right)^{-1-n^{-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^n)^(-1-1/n)/(a+b*x^n),x)
```

```
[Out] int((c+d*x^n)^(-1-1/n)/(a+b*x^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(dx^n+c\right)^{-\frac{1}{n}-1}}{bx^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(-1-1/n)/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] integrate((d*x^n + c)^(-1/n - 1)/(b*x^n + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{(bx^n+a)(dx^n+c)^{\frac{n+1}{n}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(-1-1/n)/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] integral(1/((b*x^n + a)*(d*x^n + c)^((n + 1)/n)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)**(-1-1/n)/(a+b*x**n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^n + c)^{-\frac{1}{n}-1}}{bx^n + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(-1-1/n)/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^(-1/n - 1)/(b*x^n + a), x)
```

3.334 $\int \frac{(c+dx^n)^{-1/n}}{(a+bx^n)^2} dx$

Optimal. Leaf size=127

$$\frac{bx\left(c+dx^n\right)^{-\frac{1-n}{n}}}{an(bc-ad)\left(a+bx^n\right)}-\frac{x\left(c+dx^n\right)^{-1/n}\left(adn+bc(1-n)\right){}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a^2n(bc-ad)}$$

```
[Out] (b*x)/(a*(b*c - a*d)*n*(a + b*x^n)*(c + d*x^n)^((1 - n)/n)) - ((b*c*(1 - n)
 + a*d*n)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(((b*c - a*d)*x^n)/(a
*(c + d*x^n)))])/(a^2*(b*c - a*d)*n*(c + d*x^n)^n^(-1))
```

Rubi [A] time = 0.0522504, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {382, 379}

$$\frac{bx\left(c+dx^n\right)^{-\frac{1-n}{n}}}{an(bc-ad)\left(a+bx^n\right)}-\frac{x\left(c+dx^n\right)^{-1/n}\left(adn+bc(1-n)\right){}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a^2n(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x^n)^2*(c + d*x^n)^n^(-1)),x]
```

```
[Out] (b*x)/(a*(b*c - a*d)*n*(a + b*x^n)*(c + d*x^n)^((1 - n)/n)) - ((b*c*(1 - n)
 + a*d*n)*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(((b*c - a*d)*x^n)/(a
*(c + d*x^n)))])/(a^2*(b*c - a*d)*n*(c + d*x^n)^n^(-1))
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 379

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a^p*x*Hypergeometric2F1[1/n, -p, 1 + 1/n, -(((b*c - a*d)*x^n)/(a*(
c + d*x^n)))])/(c^(p + 1)*(c + d*x^n)^(1/n)), x] /; FreeQ[{a, b, c, d, n, q
}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(c+dx^n\right)^{-1/n}}{\left(a+bx^n\right)^2}\,dx &= \frac{bx\left(c+dx^n\right)^{-\frac{1-n}{n}}}{a(bc-ad)n\left(a+bx^n\right)}-\frac{(bc-(bc-ad)n)\int \frac{\left(c+dx^n\right)^{-1/n}}{a+bx^n}\,dx}{a(bc-ad)n}\\ &= \frac{bx\left(c+dx^n\right)^{-\frac{1-n}{n}}}{a(bc-ad)n\left(a+bx^n\right)}-\frac{(bc(1-n)+adn)x\left(c+dx^n\right)^{-1/n}\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{a^2(bc-ad)n}\end{aligned}$$

Mathematica [B] time = 49.8391, size = 1070, normalized size = 8.43

$$-cd(1-n)(2n+1)(3n+1)\left(bx^n+a\right)^2\left(2(bc-ad)n\left(dx^n+c\right)\text{Gamma}\left(2+\frac{1}{n}\right){}_2F_1\left(2,3;3+\frac{1}{n};\frac{(bc-ad)x^n}{c(bx^n+a)}\right)x^n+c\left(bx^n+a\right)\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^n)^2*(c + d*x^n)^n^(-1)),x]
```

```
[Out] (c^2*(1 + 2*n)*(1 + 3*n)*x*(a + b*x^n)*(1 + (d*x^n)/c)*Gamma[2 + n^(-1)]*Ga
mma[3 + n^(-1)]*((c*(c + c*n + d*n*x^n)*Hypergeometric2F1[1, 2, 2 + n^(-1),
 ((b*c - a*d)*x^n)/(c*(a + b*x^n))])/Gamma[2 + n^(-1)] + (2*(b*c - a*d)*n*x
^n*(c + d*x^n)*Hypergeometric2F1[2, 3, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a
+ b*x^n))])/((a + b*x^n)*Gamma[3 + n^(-1)])))/((c + d*x^n)^n^(-1)*(-(c*d*(1
 - n)*(1 + 2*n)*(1 + 3*n)*x^n*(a + b*x^n)^2*(c*(a + b*x^n)*(c + c*n + d*n*x
^n)*Gamma[3 + n^(-1)]*Hypergeometric2F1[1, 2, 2 + n^(-1), ((b*c - a*d)*x^n)
/(c*(a + b*x^n))] + 2*(b*c - a*d)*n*x^n*(c + d*x^n)*Gamma[2 + n^(-1)]*Hyper
geometric2F1[2, 3, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))])) - 2*b*c
*n*(1 + 2*n)*(1 + 3*n)*x^n*(a + b*x^n)*(c + d*x^n)*(c*(a + b*x^n)*(c + c*n
+ d*n*x^n)*Gamma[3 + n^(-1)]*Hypergeometric2F1[1, 2, 2 + n^(-1), ((b*c - a*
d)*x^n)/(c*(a + b*x^n))] + 2*(b*c - a*d)*n*x^n*(c + d*x^n)*Gamma[2 + n^(-1)
]*Hypergeometric2F1[2, 3, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))]) +
 c*(1 + 2*n)*(1 + 3*n)*(a + b*x^n)^2*(c + d*x^n)*(c*(a + b*x^n)*(c + c*n +
d*n*x^n)*Gamma[3 + n^(-1)]*Hypergeometric2F1[1, 2, 2 + n^(-1), ((b*c - a*d)
*x^n)/(c*(a + b*x^n))] + 2*(b*c - a*d)*n*x^n*(c + d*x^n)*Gamma[2 + n^(-1)]*
Hypergeometric2F1[2, 3, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))]) + n
^2*x^n*(c + d*x^n)*(c^2*d*(1 + 2*n)*(1 + 3*n)*(a + b*x^n)^3*Gamma[3 + n^(-1
)]*Hypergeometric2F1[1, 2, 2 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] +
 2*c*d*(b*c - a*d)*(1 + 2*n)*(1 + 3*n)*x^n*(a + b*x^n)^2*Gamma[2 + n^(-1)]*
Hypergeometric2F1[2, 3, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] - 2*
b*c*(b*c - a*d)*(1 + 2*n)*(1 + 3*n)*x^n*(a + b*x^n)*(c + d*x^n)*Gamma[2 + n
^(-1)]*Hypergeometric2F1[2, 3, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n)
)] + 2*c*(b*c - a*d)*(1 + 2*n)*(1 + 3*n)*(a + b*x^n)^2*(c + d*x^n)*Gamma[2
+ n^(-1)]*Hypergeometric2F1[2, 3, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x
^n))] + 2*a*c*(b*c - a*d)*(1 + 3*n)*(a + b*x^n)*(c + c*n + d*n*x^n)*Gamma[3
 + n^(-1)]*Hypergeometric2F1[2, 3, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*
x^n))] + 12*a*(b*c - a*d)^2*n*(1 + 2*n)*x^n*(c + d*x^n)*Gamma[2 + n^(-1)]*H
ypergeometric2F1[3, 4, 4 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))])))
```

Maple [F] time = 0.682, size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx^n\right)^2\sqrt[n]{c+dx^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*x^n)^2/((c+d*x^n)^(1/n)),x)
```

```
[Out] int(1/(a+b*x^n)^2/((c+d*x^n)^(1/n)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bx^n+a\right)^2\left(dx^n+c\right)^{\left(\frac{1}{n}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^2/((c+d*x^n)^(1/n)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*x^n + a)^2*(d*x^n + c)^(1/n)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(b^2x^{2n} + 2\,abx^n + a^2\right)(dx^n + c)^{\left(\frac{1}{n}\right)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^2/((c+d*x^n)^(1/n)),x, algorithm="fricas")
```

```
[Out] integral(1/((b^2*x^(2*n) + 2*a*b*x^n + a^2)*(d*x^n + c)^(1/n)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**n)**2/((c+d*x**n)**(1/n)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx^n + a)^2(dx^n + c)^{\left(\frac{1}{n}\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x^n)^2/((c+d*x^n)^(1/n)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^n + a)^2*(d*x^n + c)^(1/n)), x)
```

3.335 $\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^3}\,dx$

Optimal. Leaf size=131

$$\frac{bx\,(c+dx^n)^{2-\frac{1}{n}}}{2an(bc-ad)\,(a+bx^n)^2}-\frac{cx\,(c+dx^n)^{-1/n}\,(2adn+bc(1-2n))\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{2a^3n(bc-ad)}$$

```
[Out] (b*x*(c + d*x^n)^(2 - n^(-1)))/(2*a*(b*c - a*d)*n*(a + b*x^n)^2) - (c*(b*c*
(1 - 2*n) + 2*a*d*n)*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -(((b*c - a
*d)*x^n)/(a*(c + d*x^n)))])/(2*a^3*(b*c - a*d)*n*(c + d*x^n)^n^(-1))
```

Rubi [A] time = 0.0557339, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {382, 379}

$$\frac{bx\,(c+dx^n)^{2-\frac{1}{n}}}{2an(bc-ad)\,(a+bx^n)^2}-\frac{cx\,(c+dx^n)^{-1/n}\,(2adn+bc(1-2n))\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{2a^3n(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^n)^(1 - n^(-1))/(a + b*x^n)^3,x]
```


```
[Out] (b*x*(c + d*x^n)^(2 - n^(-1)))/(2*a*(b*c - a*d)*n*(a + b*x^n)^2) - (c*(b*c*
(1 - 2*n) + 2*a*d*n)*x*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -(((b*c - a
*d)*x^n)/(a*(c + d*x^n)))])/(2*a^3*(b*c - a*d)*n*(c + d*x^n)^n^(-1))
```


Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 379

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a^p*x*Hypergeometric2F1[1/n, -p, 1 + 1/n, -(((b*c - a*d)*x^n)/(a*(
c + d*x^n)))])/(c^(p + 1)*(c + d*x^n)^(1/n)), x] /; FreeQ[{a, b, c, d, n, q
}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^3}\,dx &= \frac{bx\,(c+dx^n)^{2-\frac{1}{n}}}{2a(bc-ad)n\,(a+bx^n)^2}-\frac{(bc-2(bc-ad)n)\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^2}\,dx}{2a(bc-ad)n}\\ &= \frac{bx\,(c+dx^n)^{2-\frac{1}{n}}}{2a(bc-ad)n\,(a+bx^n)^2}-\frac{c(bc(1-2n)+2adn)x\,(c+dx^n)^{-1/n}\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{2a^3(bc-ad)n}\end{aligned}$$

Mathematica [B] time = 42.82, size = 1241, normalized size = 9.47

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^n)^(1 - n^(-1))/(a + b*x^n)^3,x]
```

```
[Out] -((c^3*(1 + n)*(1 + 2*n)*(1 + 3*n)*x*(c + d*x^n)^(2 - n^(-1))*Gamma[2 + n^(
-1)]*(Hypergeometric2F1[1, 3, 1 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))
] + (d*n*x^n*((c*Hypergeometric2F1[1, 3, 2 + n^(-1), ((b*c - a*d)*x^n)/(c*(
a + b*x^n))])/(1 + n) + (3*(b*c - a*d)*x^n*Gamma[1 + n^(-1)]*Hypergeometric
2F1[2, 4, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))])/((1 + 2*n)*(a + b
*x^n)*Gamma[2 + n^(-1)])))/c^2))/(c*d*(1 - 2*n)*(1 + 3*n)*x^n*(a + b*x^n)^2
*(c^2*(1 + n)*(1 + 2*n)*(a + b*x^n)*Gamma[2 + n^(-1)]*Hypergeometric2F1[1,
3, 1 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + d*n*x^n*(c*(1 + 2*n)*(a
 + b*x^n)*Gamma[2 + n^(-1)]*Hypergeometric2F1[1, 3, 2 + n^(-1), ((b*c - a*d
)*x^n)/(c*(a + b*x^n))] + 3*(b*c - a*d)*(1 + n)*x^n*Gamma[1 + n^(-1)]*Hyper
geometric2F1[2, 4, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))])) + 3*b*c
*n*(1 + 3*n)*x^n*(a + b*x^n)*(c + d*x^n)*(c^2*(1 + n)*(1 + 2*n)*(a + b*x^n)
*Gamma[2 + n^(-1)]*Hypergeometric2F1[1, 3, 1 + n^(-1), ((b*c - a*d)*x^n)/(c
*(a + b*x^n))] + d*n*x^n*(c*(1 + 2*n)*(a + b*x^n)*Gamma[2 + n^(-1)]*Hyperge
ometric2F1[1, 3, 2 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + 3*(b*c -
a*d)*(1 + n)*x^n*Gamma[1 + n^(-1)]*Hypergeometric2F1[2, 4, 3 + n^(-1), ((b*
c - a*d)*x^n)/(c*(a + b*x^n))])) - c*(1 + 3*n)*(a + b*x^n)^2*(c + d*x^n)*(c
^2*(1 + n)*(1 + 2*n)*(a + b*x^n)*Gamma[2 + n^(-1)]*Hypergeometric2F1[1, 3,
1 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + d*n*x^n*(c*(1 + 2*n)*(a +
b*x^n)*Gamma[2 + n^(-1)]*Hypergeometric2F1[1, 3, 2 + n^(-1), ((b*c - a*d)*x
^n)/(c*(a + b*x^n))] + 3*(b*c - a*d)*(1 + n)*x^n*Gamma[1 + n^(-1)]*Hypergeo
metric2F1[2, 4, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))])) + n^2*x^n*
(c + d*x^n)*(3*a*c^2*(-(b*c) + a*d)*(1 + 2*n)*(1 + 3*n)*(a + b*x^n)*Gamma[2
 + n^(-1)]*Hypergeometric2F1[2, 4, 2 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*
x^n))] - c*d*(1 + 3*n)*(a + b*x^n)^2*(c*(1 + 2*n)*(a + b*x^n)*Gamma[2 + n^(
-1)]*Hypergeometric2F1[1, 3, 2 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))]
 + 3*(b*c - a*d)*(1 + n)*x^n*Gamma[1 + n^(-1)]*Hypergeometric2F1[2, 4, 3 +
n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))]) + 3*d*(b*c - a*d)*x^n*(b*c*(1 +
 n)*(1 + 3*n)*x^n*(a + b*x^n)*Gamma[1 + n^(-1)]*Hypergeometric2F1[2, 4, 3 +
 n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] - c*(1 + n)*(1 + 3*n)*(a + b*x^
n)^2*Gamma[1 + n^(-1)]*Hypergeometric2F1[2, 4, 3 + n^(-1), ((b*c - a*d)*x^n
)/(c*(a + b*x^n))] - a*c*n*(1 + 3*n)*(a + b*x^n)*Gamma[2 + n^(-1)]*Hypergeo
metric2F1[2, 4, 3 + n^(-1), ((b*c - a*d)*x^n)/(c*(a + b*x^n))] + 8*a*(-(b*c
) + a*d)*n*(1 + n)*x^n*Gamma[1 + n^(-1)]*Hypergeometric2F1[3, 5, 4 + n^(-1)
, ((b*c - a*d)*x^n)/(c*(a + b*x^n))]))))
```

Maple [F] time = 0.702, size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^n)^3} (c+dx^n)^{1-n^{-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^n)^(1-1/n)/(a+b*x^n)^3,x)
```

```
[Out] int((c+d*x^n)^(1-1/n)/(a+b*x^n)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^n + c)^{-\frac{1}{n}+1}}{(bx^n + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(1-1/n)/(a+b*x^n)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x^n + c)^(-1/n + 1)/(b*x^n + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx^n + c)^{\frac{n-1}{n}}}{b^3x^{3n} + 3\,ab^2x^{2n} + 3\,a^2bx^n + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(1-1/n)/(a+b*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral((d*x^n + c)^((n - 1)/n)/(b^3*x^(3*n) + 3*a*b^2*x^(2*n) + 3*a^2*b*x
^n + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)**(1-1/n)/(a+b*x**n)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^n + c)^{-\frac{1}{n}+1}}{(bx^n + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(1-1/n)/(a+b*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^(-1/n + 1)/(b*x^n + a)^3, x)
```

3.336 $\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^4} dx$

Optimal. Leaf size=133

$$\frac{bx(c+dx^n)^{3-\frac{1}{n}}}{3an(bc-ad)(a+bx^n)^3}-\frac{c^2x(c+dx^n)^{-1/n}(3adn+bc(1-3n))\,{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{3a^4n(bc-ad)}$$

```
[Out] (b*x*(c + d*x^n)^(3 - n^(-1)))/(3*a*(b*c - a*d)*n*(a + b*x^n)^3) - (c^2*(b*
c*(1 - 3*n) + 3*a*d*n)*x*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -(((b*c -
 a*d)*x^n)/(a*(c + d*x^n)))])/(3*a^4*(b*c - a*d)*n*(c + d*x^n)^n^(-1))
```

Rubi [A] time = 0.0546743, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {382, 379}

$$\frac{bx(c+dx^n)^{3-\frac{1}{n}}}{3an(bc-ad)(a+bx^n)^3}-\frac{c^2x(c+dx^n)^{-1/n}(3adn+bc(1-3n))\,{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{3a^4n(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^n)^(2 - n^(-1))/(a + b*x^n)^4,x]
```

```
[Out] (b*x*(c + d*x^n)^(3 - n^(-1)))/(3*a*(b*c - a*d)*n*(a + b*x^n)^3) - (c^2*(b*
c*(1 - 3*n) + 3*a*d*n)*x*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -(((b*c -
 a*d)*x^n)/(a*(c + d*x^n)))])/(3*a^4*(b*c - a*d)*n*(c + d*x^n)^n^(-1))
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 379

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(a^p*x*Hypergeometric2F1[1/n, -p, 1 + 1/n, -(((b*c - a*d)*x^n)/(a*(
c + d*x^n)))])/(c^(p + 1)*(c + d*x^n)^(1/n)), x] /; FreeQ[{a, b, c, d, n, q
}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^4}dx &= \frac{bx(c+dx^n)^{3-\frac{1}{n}}}{3a(bc-ad)n(a+bx^n)^3}-\frac{(bc-3(bc-ad)n)\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^3}dx}{3a(bc-ad)n}\\ &= \frac{bx(c+dx^n)^{3-\frac{1}{n}}}{3a(bc-ad)n(a+bx^n)^3}-\frac{c^2(bc(1-3n)+3adn)x(c+dx^n)^{-1/n}\,{}_2F_1\left(3,\frac{1}{n};1+\frac{1}{n};-\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{3a^4(bc-ad)n}\end{aligned}$$

Mathematica [F] time = 180.006, size = 0, normalized size = 0.

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x^n)^(2 - n^(-1))/(a + b*x^n)^4,x]
```

```
[Out] $Aborted
```

Maple [F] time = 0.717, size = 0, normalized size = 0.

$$\int \frac{1}{(a+bx^n)^4}(c+dx^n)^{2-n^{-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^n)^(2-1/n)/(a+b*x^n)^4,x)
```

```
[Out] int((c+d*x^n)^(2-1/n)/(a+b*x^n)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^n+c)^{-\frac{1}{n}+2}}{(bx^n+a)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(2-1/n)/(a+b*x^n)^4,x, algorithm="maxima")
```

```
[Out] integrate((d*x^n + c)^(-1/n + 2)/(b*x^n + a)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx^n+c)^{\frac{2n-1}{n}}}{b^4x^{4n}+4ab^3x^{3n}+6a^2b^2x^{2n}+4a^3bx^n+a^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(2-1/n)/(a+b*x^n)^4,x, algorithm="fricas")
```

```
[Out] integral((d*x^n + c)^((2*n - 1)/n)/(b^4*x^(4*n) + 4*a*b^3*x^(3*n) + 6*a^2*b
^2*x^(2*n) + 4*a^3*b*x^n + a^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)**(2-1/n)/(a+b*x**n)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^n + c)^{-\frac{1}{n}+2}}{(bx^n + a)^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^(2-1/n)/(a+b*x^n)^4,x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^(-1/n + 2)/(b*x^n + a)^4, x)
```

3.337 $\int x^5\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)dx$

Optimal. Leaf size=152

$$\frac{(dx-c)^{7/2}(c+dx)^{7/2}\left(ad^2+3bc^2\right)}{7d^8}+\frac{c^2(dx-c)^{5/2}(c+dx)^{5/2}\left(2ad^2+3bc^2\right)}{5d^8}+\frac{c^4(dx-c)^{3/2}(c+dx)^{3/2}\left(ad^2+bc^2\right)}{3d^8}+\frac{b(dx-$$

```
[Out] (c^4*(b*c^2 + a*d^2)*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(3*d^8) + (c^2*(3*b*
c^2 + 2*a*d^2)*(-c + d*x)^(5/2)*(c + d*x)^(5/2))/(5*d^8) + ((3*b*c^2 + a*d^
2)*(-c + d*x)^(7/2)*(c + d*x)^(7/2))/(7*d^8) + (b*(-c + d*x)^(9/2)*(c + d*x
)^(9/2))/(9*d^8)
```

Rubi [A] time = 0.118074, antiderivative size = 164, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {460, 100, 12, 74}

$$\frac{x^4(dx-c)^{3/2}(c+dx)^{3/2}\left(3ad^2+2bc^2\right)}{21d^4}+\frac{4c^2x^2(dx-c)^{3/2}(c+dx)^{3/2}\left(3ad^2+2bc^2\right)}{105d^6}+\frac{8c^4(dx-c)^{3/2}(c+dx)^{3/2}\left(3ad^2+2bc\right.}{315d^8}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (8*c^4*(2*b*c^2 + 3*a*d^2)*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(315*d^8) + (4
*c^2*(2*b*c^2 + 3*a*d^2)*x^2*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(105*d^6) +
((2*b*c^2 + 3*a*d^2)*x^4*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(21*d^4) + (b*x^
6*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(9*d^2)
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
```

```
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int x^5\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)dx = \frac{bx^6(-c+dx)^{3/2}(c+dx)^{3/2}}{9d^2}+\frac{1}{3}\left(3a+\frac{2bc^2}{d^2}\right)\int x^5\sqrt{-c+dx}\sqrt{c+dx}\,dx$$

$$= \frac{\left(2bc^2+3ad^2\right)x^4(-c+dx)^{3/2}(c+dx)^{3/2}}{21d^4}+\frac{bx^6(-c+dx)^{3/2}(c+dx)^{3/2}}{9d^2}+\frac{\left(2bc^2+\right.}{}$$

$$= \frac{\left(2bc^2+3ad^2\right)x^4(-c+dx)^{3/2}(c+dx)^{3/2}}{21d^4}+\frac{bx^6(-c+dx)^{3/2}(c+dx)^{3/2}}{9d^2}+\frac{\left(4c^2\left(2\right.\right.}{}$$

$$= \frac{4c^2\left(2bc^2+3ad^2\right)x^2(-c+dx)^{3/2}(c+dx)^{3/2}}{105d^6}+\frac{\left(2bc^2+3ad^2\right)x^4(-c+dx)^{3/2}(c+}{21d^4}$$

$$= \frac{4c^2\left(2bc^2+3ad^2\right)x^2(-c+dx)^{3/2}(c+dx)^{3/2}}{105d^6}+\frac{\left(2bc^2+3ad^2\right)x^4(-c+dx)^{3/2}(c+}{21d^4}$$

$$= \frac{8c^4\left(2bc^2+3ad^2\right)(-c+dx)^{3/2}(c+dx)^{3/2}}{315d^8}+\frac{4c^2\left(2bc^2+3ad^2\right)x^2(-c+dx)^{3/2}(c}{105d^6}$$

Mathematica [A] time = 0.0729676, size = 110, normalized size = 0.72

$$\frac{\sqrt{dx-c}\sqrt{c+dx}\left(d^2x^2-c^2\right)\left(3ad^2\left(12c^2d^2x^2+8c^4+15d^4x^4\right)+b\left(24c^4d^2x^2+30c^2d^4x^4+16c^6+35d^6x^6\right)\right)}{315d^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (Sqrt[-c + d*x]*Sqrt[c + d*x]*(-c^2 + d^2*x^2)*(3*a*d^2*(8*c^4 + 12*c^2*d^2
*x^2 + 15*d^4*x^4) + b*(16*c^6 + 24*c^4*d^2*x^2 + 30*c^2*d^4*x^4 + 35*d^6*x
^6)))/(315*d^8)
```

Maple [A] time = 0.006, size = 92, normalized size = 0.6

$$\frac{35\,bx^6d^6+45\,ad^6x^4+30\,bc^2d^4x^4+36\,ac^2d^4x^2+24\,bc^4d^2x^2+24\,ac^4d^2+16\,bc^6}{315\,d^8}(dx+c)^{\frac{3}{2}}(dx-c)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x)
```

```
[Out] 1/315*(d*x+c)^(3/2)*(35*b*d^6*x^6+45*a*d^6*x^4+30*b*c^2*d^4*x^4+36*a*c^2*d^
4*x^2+24*b*c^4*d^2*x^2+24*a*c^4*d^2+16*b*c^6)*(d*x-c)^(3/2)/d^8
```

Maxima [A] time = 0.974225, size = 240, normalized size = 1.58

$$\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}bx^6}{9\,d^2}+\frac{2\left(d^2x^2-c^2\right)^{\frac{3}{2}}bc^2x^4}{21\,d^4}+\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}ax^4}{7\,d^2}+\frac{8\left(d^2x^2-c^2\right)^{\frac{3}{2}}bc^4x^2}{105\,d^6}+\frac{4\left(d^2x^2-c^2\right)^{\frac{3}{2}}ac^2x^2}{35\,d^4}+\frac{16\left(d^2x^2-\right.}{315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/9*(d^2*x^2 - c^2)^(3/2)*b*x^6/d^2 + 2/21*(d^2*x^2 - c^2)^(3/2)*b*c^2*x^4/
d^4 + 1/7*(d^2*x^2 - c^2)^(3/2)*a*x^4/d^2 + 8/105*(d^2*x^2 - c^2)^(3/2)*b*c
^4*x^2/d^6 + 4/35*(d^2*x^2 - c^2)^(3/2)*a*c^2*x^2/d^4 + 16/315*(d^2*x^2 - c
^2)^(3/2)*b*c^6/d^8 + 8/105*(d^2*x^2 - c^2)^(3/2)*a*c^4/d^6
```

Fricas [A] time = 1.93724, size = 246, normalized size = 1.62

$$\frac{\left(35\,bd^{8}x^{8}-16\,bc^{8}-24\,ac^{6}d^{2}-5\left(bc^{2}d^{6}-9\,ad^{8}\right)x^{6}-3\left(2\,bc^{4}d^{4}+3\,ac^{2}d^{6}\right)x^{4}-4\left(2\,bc^{6}d^{2}+3\,ac^{4}d^{4}\right)x^{2}\right)\sqrt{dx+c}\sqrt{dx-c}}{315\,d^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*b*d^8*x^8 - 16*b*c^8 - 24*a*c^6*d^2 - 5*(b*c^2*d^6 - 9*a*d^8)*x^6
 - 3*(2*b*c^4*d^4 + 3*a*c^2*d^6)*x^4 - 4*(2*b*c^6*d^2 + 3*a*c^4*d^4)*x^2)*s
qrt(d*x + c)*sqrt(d*x - c)/d^8
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{5}\left(a+bx^{2}\right)\sqrt{-c+dx}\sqrt{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)
```

```
[Out] Integral(x**5*(a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x), x)
```

Giac [A] time = 1.29829, size = 309, normalized size = 2.03

$$3\left(\left(3\left((dx+c)\left(5\,(dx+c)\left(\frac{dx+c}{d^{5}}-\frac{6\,c}{d^{5}}\right)+\frac{74\,c^{2}}{d^{5}}\right)-\frac{96\,c^{3}}{d^{5}}\right)(dx+c)+\frac{203\,c^{4}}{d^{5}}\right)(dx+c)-\frac{70\,c^{5}}{d^{5}}\right)(dx+c)^{\frac{3}{2}}\sqrt{dx-c}a+\left(\left(\left(\left(5\left((dx+c\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/315*(3*((3*((d*x + c)*(5*(d*x + c)*((d*x + c)/d^5 - 6*c/d^5) + 74*c^2/d^5
) - 96*c^3/d^5)*(d*x + c) + 203*c^4/d^5)*(d*x + c) - 70*c^5/d^5)*(d*x + c)^
(3/2)*sqrt(d*x - c)*a + ((((5*((d*x + c)*(7*(d*x + c)*((d*x + c)/d^7 - 8*c/
d^7) + 195*c^2/d^7) - 386*c^3/d^7)*(d*x + c) + 2369*c^4/d^7)*(d*x + c) - 18
36*c^5/d^7)*(d*x + c) + 861*c^6/d^7)*(d*x + c) - 210*c^7/d^7)*(d*x + c)^(3/
2)*sqrt(d*x - c)*b)/d
```

3.338 $\int x^3\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx$

Optimal. Leaf size=109

$$\frac{(dx-c)^{5/2}(c+dx)^{5/2}\left(ad^2+2bc^2\right)}{5d^6}+\frac{c^2(dx-c)^{3/2}(c+dx)^{3/2}\left(ad^2+bc^2\right)}{3d^6}+\frac{b(dx-c)^{7/2}(c+dx)^{7/2}}{7d^6}$$

```
[Out] (c^2*(b*c^2 + a*d^2)*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(3*d^6) + ((2*b*c^2
+ a*d^2)*(-c + d*x)^(5/2)*(c + d*x)^(5/2))/(5*d^6) + (b*(-c + d*x)^(7/2)*(c
 + d*x)^(7/2))/(7*d^6)
```

Rubi [A] time = 0.0867472, antiderivative size = 118, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {460, 100, 12, 74}

$$\frac{x^2(dx-c)^{3/2}(c+dx)^{3/2}\left(7ad^2+4bc^2\right)}{35d^4}+\frac{2c^2(dx-c)^{3/2}(c+dx)^{3/2}\left(7ad^2+4bc^2\right)}{105d^6}+\frac{bx^4(dx-c)^{3/2}(c+dx)^{3/2}}{7d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (2*c^2*(4*b*c^2 + 7*a*d^2)*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(105*d^6) + ((
4*b*c^2 + 7*a*d^2)*x^2*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(35*d^4) + (b*x^4*
(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(7*d^2)
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
```

```
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int x^3\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx = \frac{bx^4(-c+dx)^{3/2}(c+dx)^{3/2}}{7d^2}-\frac{1}{7}\left(-7a-\frac{4bc^2}{d^2}\right)\int x^3\sqrt{-c+dx}\sqrt{c+dx}\,dx$$
$$=\frac{\left(4bc^2+7ad^2\right)x^2(-c+dx)^{3/2}(c+dx)^{3/2}}{35d^4}+\frac{bx^4(-c+dx)^{3/2}(c+dx)^{3/2}}{7d^2}+\frac{\left(4bc^2+7a\right.}{}$$
$$=\frac{\left(4bc^2+7ad^2\right)x^2(-c+dx)^{3/2}(c+dx)^{3/2}}{35d^4}+\frac{bx^4(-c+dx)^{3/2}(c+dx)^{3/2}}{7d^2}+\frac{\left(2c^2\left(4bc^2\right.\right.}{}$$
$$=\frac{2c^2\left(4bc^2+7ad^2\right)(-c+dx)^{3/2}(c+dx)^{3/2}}{105d^6}+\frac{\left(4bc^2+7ad^2\right)x^2(-c+dx)^{3/2}(c+dx)^{3/}}{35d^4}$$

Mathematica [A] time = 0.0528235, size = 88, normalized size = 0.81

$$\frac{\sqrt{dx-c}\sqrt{c+dx}\left(d^2x^2-c^2\right)\left(7ad^2\left(2c^2+3d^2x^2\right)+b\left(12c^2d^2x^2+8c^4+15d^4x^4\right)\right)}{105d^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (Sqrt[-c + d*x]*Sqrt[c + d*x]*(-c^2 + d^2*x^2)*(7*a*d^2*(2*c^2 + 3*d^2*x^2)
 + b*(8*c^4 + 12*c^2*d^2*x^2 + 15*d^4*x^4)))/(105*d^6)
```

Maple [A] time = 0.006, size = 68, normalized size = 0.6

$$\frac{15\,bd^4x^4+21\,ad^4x^2+12\,bc^2d^2x^2+14\,ac^2d^2+8\,bc^4}{105\,d^6}(dx+c)^{\frac{3}{2}}(dx-c)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x)
```

```
[Out] 1/105*(d*x+c)^(3/2)*(15*b*d^4*x^4+21*a*d^4*x^2+12*b*c^2*d^2*x^2+14*a*c^2*d^
2+8*b*c^4)*(d*x-c)^(3/2)/d^6
```

Maxima [A] time = 0.984139, size = 167, normalized size = 1.53

$$\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}bx^4}{7\,d^2}+\frac{4\left(d^2x^2-c^2\right)^{\frac{3}{2}}bc^2x^2}{35\,d^4}+\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}ax^2}{5\,d^2}+\frac{8\left(d^2x^2-c^2\right)^{\frac{3}{2}}bc^4}{105\,d^6}+\frac{2\left(d^2x^2-c^2\right)^{\frac{3}{2}}ac^2}{15\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/7*(d^2*x^2 - c^2)^(3/2)*b*x^4/d^2 + 4/35*(d^2*x^2 - c^2)^(3/2)*b*c^2*x^2/
d^4 + 1/5*(d^2*x^2 - c^2)^(3/2)*a*x^2/d^2 + 8/105*(d^2*x^2 - c^2)^(3/2)*b*c
^4/d^6 + 2/15*(d^2*x^2 - c^2)^(3/2)*a*c^2/d^4
```

Fricas [A] time = 1.67887, size = 193, normalized size = 1.77

$$\frac{\left(15\,bd^6x^6 - 8\,bc^6 - 14\,ac^4d^2 - 3\left(bc^2d^4 - 7\,ad^6\right)x^4 - \left(4\,bc^4d^2 + 7\,ac^2d^4\right)x^2\right)\sqrt{dx+c}\sqrt{dx-c}}{105\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*b*d^6*x^6 - 8*b*c^6 - 14*a*c^4*d^2 - 3*(b*c^2*d^4 - 7*a*d^6)*x^4
- (4*b*c^4*d^2 + 7*a*c^2*d^4)*x^2)*sqrt(d*x + c)*sqrt(d*x - c)/d^6
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\left(a + bx^2\right)\sqrt{-c+dx}\sqrt{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)
```

```
[Out] Integral(x**3*(a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x), x)
```

Giac [A] time = 1.19113, size = 227, normalized size = 2.08

$$\frac{7\left((dx+c)\left(3\,(dx+c)\left(\frac{dx+c}{d^3} - \frac{4\,c}{d^3}\right) + \frac{17\,c^2}{d^3}\right) - \frac{10\,c^3}{d^3}\right)(dx+c)^{\frac{3}{2}}\sqrt{dx-c}a + \left(\left(3\left((dx+c)\left(5\,(dx+c)\left(\frac{dx+c}{d^5} - \frac{6\,c}{d^5}\right) + \frac{74\,c^2}{d^5}\right) - \cdots\right.\right.\right.}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/105*(7*((d*x + c)*(3*(d*x + c)*((d*x + c)/d^3 - 4*c/d^3) + 17*c^2/d^3) -
10*c^3/d^3)*(d*x + c)^(3/2)*sqrt(d*x - c)*a + ((3*((d*x + c)*(5*(d*x + c)*(
(d*x + c)/d^5 - 6*c/d^5) + 74*c^2/d^5) - 96*c^3/d^5)*(d*x + c) + 203*c^4/d^
5)*(d*x + c) - 70*c^5/d^5)*(d*x + c)^(3/2)*sqrt(d*x - c)*b)/d
```

3.339 $\int x\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx$

Optimal. Leaf size=67

$$\frac{(dx-c)^{3/2}(c+dx)^{3/2}\left(ad^2+bc^2\right)}{3d^4}+\frac{b(dx-c)^{5/2}(c+dx)^{5/2}}{5d^4}$$

```
[Out] ((b*c^2 + a*d^2)*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(3*d^4) + (b*(-c + d*x)^
(5/2)*(c + d*x)^(5/2))/(5*d^4)
```

Rubi [A] time = 0.0443008, antiderivative size = 72, normalized size of antiderivative = 1.07, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {460, 74}

$$\frac{(dx-c)^{3/2}(c+dx)^{3/2}\left(5ad^2+2bc^2\right)}{15d^4}+\frac{bx^2(dx-c)^{3/2}(c+dx)^{3/2}}{5d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] ((2*b*c^2 + 5*a*d^2)*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(15*d^4) + (b*x^2*(-
c + d*x)^(3/2)*(c + d*x)^(3/2))/(5*d^2)
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx &= \frac{bx^2(-c+dx)^{3/2}(c+dx)^{3/2}}{5d^2}-\frac{1}{5}\left(-5a-\frac{2bc^2}{d^2}\right)\int x\sqrt{-c+dx}\sqrt{c+dx}\,dx\\ &= \frac{\left(2bc^2+5ad^2\right)(-c+dx)^{3/2}(c+dx)^{3/2}}{15d^4}+\frac{bx^2(-c+dx)^{3/2}(c+dx)^{3/2}}{5d^2}\end{aligned}$$

Mathematica [A] time = 0.03065, size = 62, normalized size = 0.93

$$\frac{\sqrt{dx-c}\sqrt{c+dx}\left(d^2x^2-c^2\right)\left(5ad^2+2bc^2+3bd^2x^2\right)}{15d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (Sqrt[-c + d*x]*Sqrt[c + d*x]*(-c^2 + d^2*x^2)*(2*b*c^2 + 5*a*d^2 + 3*b*d^2
*x^2))/(15*d^4)
```

Maple [A] time = 0.002, size = 44, normalized size = 0.7

$$\frac{3\,bd^2x^2+5\,ad^2+2\,bc^2}{15\,d^4}\,(dx+c)^{\frac{3}{2}}\,(dx-c)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x)
```

```
[Out] 1/15*(d*x+c)^(3/2)*(3*b*d^2*x^2+5*a*d^2+2*b*c^2)*(d*x-c)^(3/2)/d^4
```

Maxima [A] time = 0.96054, size = 95, normalized size = 1.42

$$\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}bx^2}{5\,d^2}+\frac{2\left(d^2x^2-c^2\right)^{\frac{3}{2}}bc^2}{15\,d^4}+\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}a}{3\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*(d^2*x^2 - c^2)^(3/2)*b*x^2/d^2 + 2/15*(d^2*x^2 - c^2)^(3/2)*b*c^2/d^4
+ 1/3*(d^2*x^2 - c^2)^(3/2)*a/d^2
```

Fricas [A] time = 1.5493, size = 140, normalized size = 2.09

$$\frac{\left(3\,bd^4x^4-2\,bc^4-5\,ac^2d^2-\left(bc^2d^2-5\,ad^4\right)x^2\right)\sqrt{dx+c}\sqrt{dx-c}}{15\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*b*d^4*x^4 - 2*b*c^4 - 5*a*c^2*d^2 - (b*c^2*d^2 - 5*a*d^4)*x^2)*sqrt
(d*x + c)*sqrt(d*x - c)/d^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(a+bx^2\right)\sqrt{-c+dx}\sqrt{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)
```

```
[Out] Integral(x*(a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x), x)
```

Giac [A] time = 1.51827, size = 126, normalized size = 1.88

$$\frac{\left((dx+c)\left(3\,(dx+c)\left(\frac{dx+c}{d^3}-\frac{4\,c}{d^3}\right)+\frac{17\,c^2}{d^3}\right)-\frac{10\,c^3}{d^3}\right)(dx+c)^{\frac{3}{2}}\sqrt{dx-c}\,b+\frac{5\,(dx+c)^{\frac{3}{2}}(dx-c)^{\frac{3}{2}}a}{d}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/15*(((d*x + c)*(3*(d*x + c)*((d*x + c)/d^3 - 4*c/d^3) + 17*c^2/d^3) - 10*
c^3/d^3)*(d*x + c)^(3/2)*sqrt(d*x - c)*b + 5*(d*x + c)^(3/2)*(d*x - c)^(3/2
)*a/d)/d
```

3.340 $\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x}\,dx$

Optimal. Leaf size=80

$$a\sqrt{dx-c}\sqrt{c+dx}-ac\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)+\frac{b(dx-c)^{3/2}(c+dx)^{3/2}}{3d^2}$$

```
[Out] a*Sqrt[-c + d*x]*Sqrt[c + d*x] + (b*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(3*d^
2) - a*c*ArcTan[(Sqrt[-c + d*x]*Sqrt[c + d*x])/c]
```

Rubi [A] time = 0.0782181, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {460, 101, 12, 92, 205}

$$a\sqrt{dx-c}\sqrt{c+dx}-ac\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)+\frac{b(dx-c)^{3/2}(c+dx)^{3/2}}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x,x]
```

```
[Out] a*Sqrt[-c + d*x]*Sqrt[c + d*x] + (b*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(3*d^
2) - a*c*ArcTan[(Sqrt[-c + d*x]*Sqrt[c + d*x])/c]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f
*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)}{x}\,dx &= \frac{b(-c+dx)^{3/2}(c+dx)^{3/2}}{3d^2}+a\int \frac{\sqrt{-c+dx}\sqrt{c+dx}}{x}\,dx\\ &= a\sqrt{-c+dx}\sqrt{c+dx}+\frac{b(-c+dx)^{3/2}(c+dx)^{3/2}}{3d^2}-a\int \frac{c^2}{x\sqrt{-c+dx}\sqrt{c+dx}}\,dx\\ &= a\sqrt{-c+dx}\sqrt{c+dx}+\frac{b(-c+dx)^{3/2}(c+dx)^{3/2}}{3d^2}-\left(ac^2\right)\int \frac{1}{x\sqrt{-c+dx}\sqrt{c+dx}}\,dx\\ &= a\sqrt{-c+dx}\sqrt{c+dx}+\frac{b(-c+dx)^{3/2}(c+dx)^{3/2}}{3d^2}-\left(ac^2d\right)\text{Subst}\left(\int \frac{1}{c^2d+dx^2}\,dx,x,\sqrt{\right.\\ &= a\sqrt{-c+dx}\sqrt{c+dx}+\frac{b(-c+dx)^{3/2}(c+dx)^{3/2}}{3d^2}-ac\tan^{-1}\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)\end{aligned}$$

Mathematica [A] time = 0.161454, size = 85, normalized size = 1.06

$$\frac{1}{3}\sqrt{dx-c}\sqrt{c+dx}\left(-\frac{3ac\tan^{-1}\left(\frac{\sqrt{d^2x^2-c^2}}{c}\right)}{\sqrt{d^2x^2-c^2}}+3a+b\left(x^2-\frac{c^2}{d^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x,x]
```

```
[Out] (Sqrt[-c + d*x]*Sqrt[c + d*x]*(3*a + b*(-(c^2/d^2) + x^2) - (3*a*c*ArcTan[S
qrt[-c^2 + d^2*x^2]/c])/Sqrt[-c^2 + d^2*x^2]))/3
```

Maple [B] time = 0.036, size = 174, normalized size = 2.2

$$\frac{1}{3\,d^2}\sqrt{dx-c}\sqrt{dx+c}\left(x^2bd^2\sqrt{-c^2}\sqrt{d^2x^2-c^2}+3\,\ln\left(-2\,\frac{c^2-\sqrt{-c^2}\sqrt{d^2x^2-c^2}}{x}\right)ac^2d^2+3\,\sqrt{-c^2}\sqrt{d^2x^2-c^2}ad^2-bc^2\sqrt{-c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x,x)
```

```
[Out] 1/3*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(x^2*b*d^2*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2)
+3*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*a*c^2*d^2+3*(-c^2)^(1/2)
*(d^2*x^2-c^2)^(1/2)*a*d^2-b*c^2*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/(d^2*x^2
-c^2)^(1/2)/d^2/(-c^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.55539, size = 174, normalized size = 2.17

$$-\frac{6\,acd^2\arctan\left(-\frac{dx-\sqrt{dx+c}\sqrt{dx-c}}{c}\right)-\left(bd^2x^2-bc^2+3\,ad^2\right)\sqrt{dx+c}\sqrt{dx-c}}{3\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] -1/3*(6*a*c*d^2*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c))/c) - (b*d^2*x^2
 - b*c^2 + 3*a*d^2)*sqrt(d*x + c)*sqrt(d*x - c))/d^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+bx^2\right)\sqrt{-c+dx}\sqrt{c+dx}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2)/x,x)
```

```
[Out] Integral((a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x)/x, x)
```

Giac [A] time = 1.69464, size = 109, normalized size = 1.36

$$2\,ac\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2\,c}\right)+\frac{1}{1920}\left(3\,ad^6+\left((dx+c)bd^4-2\,bcd^4\right)(dx+c)\right)\sqrt{dx+c}\sqrt{dx-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a*c*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/c) + 1/1920*(3*a*d^6 + (
(d*x + c)*b*d^4 - 2*b*c*d^4)*(d*x + c))*sqrt(d*x + c)*sqrt(d*x - c)
```

3.341 $\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^3}\,dx$

Optimal. Leaf size=96

$$-\frac{(2bc^2-ad^2)\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{2c}-\frac{a\sqrt{dx-c}\sqrt{c+dx}}{2x^2}+b\sqrt{dx-c}\sqrt{c+dx}$$

```
[Out] b*Sqrt[-c + d*x]*Sqrt[c + d*x] - (a*Sqrt[-c + d*x]*Sqrt[c + d*x])/(2*x^2) -
 ((2*b*c^2 - a*d^2)*ArcTan[(Sqrt[-c + d*x]*Sqrt[c + d*x])/c])/(2*c)
```

Rubi [A] time = 0.0840813, antiderivative size = 114, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {454, 101, 12, 92, 205}

$$\frac{1}{2}\sqrt{dx-c}\sqrt{c+dx}\left(2b-\frac{ad^2}{c^2}\right)-\frac{(2bc^2-ad^2)\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{2c}+\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{2c^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^3,x]
```

```
[Out] ((2*b - (a*d^2)/c^2)*Sqrt[-c + d*x]*Sqrt[c + d*x])/2 + (a*(-c + d*x)^(3/2)*
(c + d*x)^(3/2))/(2*c^2*x^2) - ((2*b*c^2 - a*d^2)*ArcTan[(Sqrt[-c + d*x]*Sq
rt[c + d*x])/c])/(2*c)
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f
*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(
c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*
(b*e - a*f) + b*n*(d*e - c*f))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}
, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m,
2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
```

```
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)}{x^3}\,dx = \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{2c^2x^2} + \frac{1}{2}\left(2b-\frac{ad^2}{c^2}\right)\int \frac{\sqrt{-c+dx}\sqrt{c+dx}}{x}\,dx$$
$$= \frac{1}{2}\left(2b-\frac{ad^2}{c^2}\right)\sqrt{-c+dx}\sqrt{c+dx} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{2c^2x^2} + \frac{1}{2}\left(-2b+\frac{ad^2}{c^2}\right)\int$$
$$= \frac{1}{2}\left(2b-\frac{ad^2}{c^2}\right)\sqrt{-c+dx}\sqrt{c+dx} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{2c^2x^2} + \frac{1}{2}\left(-2bc^2+ad^2\right)\int$$
$$= \frac{1}{2}\left(2b-\frac{ad^2}{c^2}\right)\sqrt{-c+dx}\sqrt{c+dx} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{2c^2x^2} - \frac{1}{2}\left(d\left(2bc^2-ad^2\right)\right)$$
$$= \frac{1}{2}\left(2b-\frac{ad^2}{c^2}\right)\sqrt{-c+dx}\sqrt{c+dx} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{2c^2x^2} - \frac{\left(2bc^2-ad^2\right)\tan^{-1}}{2c}$$

Mathematica [A] time = 0.0490826, size = 114, normalized size = 1.19

$$-\frac{\sqrt{dx-c}\sqrt{c+dx}\left(c\left(a-2bx^2\right)\sqrt{d^2x^2-c^2}+x^2\left(2bc^2-ad^2\right)\tan^{-1}\left(\frac{\sqrt{d^2x^2-c^2}}{c}\right)\right)}{2cx^2\sqrt{d^2x^2-c^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^3,x]
```

```
[Out] -(Sqrt[-c + d*x]*Sqrt[c + d*x]*(c*(a - 2*b*x^2)*Sqrt[-c^2 + d^2*x^2] + (2*b
*c^2 - a*d^2)*x^2*ArcTan[Sqrt[-c^2 + d^2*x^2]/c]))/(2*c*x^2*Sqrt[-c^2 + d^2
*x^2])
```

Maple [B] time = 0.014, size = 182, normalized size = 1.9

$$-\frac{1}{2x^2}\sqrt{dx-c}\sqrt{dx+c}\left(\ln\left(-2\frac{c^2-\sqrt{-c^2}\sqrt{d^2x^2-c^2}}{x}\right)x^2ad^2-2\ln\left(-2\frac{c^2-\sqrt{-c^2}\sqrt{d^2x^2-c^2}}{x}\right)x^2bc^2-2x^2b\sqrt{-c^2}\sqrt{d^2x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^3,x)
```

```
[Out] -1/2*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/
2))/x)*x^2*a*d^2-2*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*x^2*b*c^
2-2*x^2*b*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2)+(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2)
*a)/(d^2*x^2-c^2)^(1/2)/x^2/(-c^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.54694, size = 182, normalized size = 1.9

$$-\frac{2\left(2bc^2-ad^2\right)x^2\arctan\left(-\frac{dx-\sqrt{dx+c}\sqrt{dx-c}}{c}\right)-\left(2bcx^2-ac\right)\sqrt{dx+c}\sqrt{dx-c}}{2cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*(2*b*c^2 - a*d^2)*x^2*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c))/c
) - (2*b*c*x^2 - a*c)*sqrt(d*x + c)*sqrt(d*x - c))/(c*x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+bx^2\right)\sqrt{-c+dx}\sqrt{c+dx}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2)/x**3,x)
```

```
[Out] Integral((a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x)/x**3, x)
```

Giac [A] time = 1.18163, size = 212, normalized size = 2.21

$$\frac{\sqrt{dx+c}\sqrt{dx-c}bd+\frac{\left(2bc^2d-ad^3\right)\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2c}\right)}{c}+\frac{2\left(ad^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^6-4ac^2d^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4c^2\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] (sqrt(d*x + c)*sqrt(d*x - c)*b*d + (2*b*c^2*d - a*d^3)*arctan(1/2*(sqrt(d*x
 + c) - sqrt(d*x - c))^2/c)/c + 2*(a*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^6
- 4*a*c^2*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^2)/((sqrt(d*x + c) - sqrt(d*x
 - c))^4 + 4*c^2)^2)/d
```

3.342 $\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^5}\,dx$

Optimal. Leaf size=121

$$-\frac{\sqrt{dx-c}\sqrt{c+dx}\left(ad^2+4bc^2\right)}{8c^2x^2}+\frac{d^2\left(ad^2+4bc^2\right)\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{8c^3}+\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{4c^2x^4}$$

```
[Out] -((4*b*c^2 + a*d^2)*Sqrt[-c + d*x]*Sqrt[c + d*x])/(8*c^2*x^2) + (a*(-c + d*
x)^(3/2)*(c + d*x)^(3/2))/(4*c^2*x^4) + (d^2*(4*b*c^2 + a*d^2)*ArcTan[(Sqrt
[-c + d*x]*Sqrt[c + d*x])/c])/(8*c^3)
```

Rubi [A] time = 0.103594, antiderivative size = 164, normalized size of antiderivative = 1.36, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {454, 94, 92, 205}

$$-\frac{\sqrt{dx-c}(c+dx)^{3/2}\left(ad^2+4bc^2\right)}{8c^3x^2}+\frac{d\sqrt{dx-c}\sqrt{c+dx}\left(ad^2+4bc^2\right)}{8c^3x}+\frac{d^2\left(ad^2+4bc^2\right)\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{8c^3}+\frac{a(dx-c)}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^5,x]
```

```
[Out] (d*(4*b*c^2 + a*d^2)*Sqrt[-c + d*x]*Sqrt[c + d*x])/(8*c^3*x) - ((4*b*c^2 +
a*d^2)*Sqrt[-c + d*x]*(c + d*x)^(3/2))/(8*c^3*x^2) + (a*(-c + d*x)^(3/2)*(c
 + d*x)^(3/2))/(4*c^2*x^4) + (d^2*(4*b*c^2 + a*d^2)*ArcTan[(Sqrt[-c + d*x]*
Sqrt[c + d*x])/c])/(8*c^3)
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)}{x^5}\,dx = \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{4c^2x^4} + \frac{1}{4}\left(4b+\frac{ad^2}{c^2}\right)\int \frac{\sqrt{-c+dx}\sqrt{c+dx}}{x^3}\,dx$$
$$= -\frac{\left(4bc^2+ad^2\right)\sqrt{-c+dx}(c+dx)^{3/2}}{8c^3x^2} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{4c^2x^4} + \frac{1}{8}\left(d\left(4b+\frac{ad^2}{c^2}\right)\right)\int$$
$$= \frac{d\left(4bc^2+ad^2\right)\sqrt{-c+dx}\sqrt{c+dx}}{8c^3x} - \frac{\left(4bc^2+ad^2\right)\sqrt{-c+dx}(c+dx)^{3/2}}{8c^3x^2} + \frac{a(-c+dx)^3}{4c}$$
$$= \frac{d\left(4bc^2+ad^2\right)\sqrt{-c+dx}\sqrt{c+dx}}{8c^3x} - \frac{\left(4bc^2+ad^2\right)\sqrt{-c+dx}(c+dx)^{3/2}}{8c^3x^2} + \frac{a(-c+dx)^3}{4c}$$
$$= \frac{d\left(4bc^2+ad^2\right)\sqrt{-c+dx}\sqrt{c+dx}}{8c^3x} - \frac{\left(4bc^2+ad^2\right)\sqrt{-c+dx}(c+dx)^{3/2}}{8c^3x^2} + \frac{a(-c+dx)^3}{4c}$$

Mathematica [A] time = 0.0843731, size = 137, normalized size = 1.13

$$\frac{\sqrt{dx-c}\sqrt{c+dx}\left(\left(c^2-d^2x^2\right)\left(2ac^2-ad^2x^2+4bc^2x^2\right)-d^2x^4\sqrt{1-\frac{d^2x^2}{c^2}}\left(ad^2+4bc^2\right)\tanh^{-1}\left(\sqrt{1-\frac{d^2x^2}{c^2}}\right)\right)}{8c^2d^2x^6-8c^4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^5,x]
```

```
[Out] (Sqrt[-c + d*x]*Sqrt[c + d*x]*((c^2 - d^2*x^2)*(2*a*c^2 + 4*b*c^2*x^2 - a*d
^2*x^2) - d^2*(4*b*c^2 + a*d^2)*x^4*Sqrt[1 - (d^2*x^2)/c^2]*ArcTanh[Sqrt[1
- (d^2*x^2)/c^2]]))/(-8*c^4*x^4 + 8*c^2*d^2*x^6)
```

Maple [B] time = 0.014, size = 226, normalized size = 1.9

$$-\frac{1}{8\,c^2x^4}\sqrt{dx-c}\sqrt{dx+c}\left(\ln\left(-2\,\frac{c^2-\sqrt{-c^2}\sqrt{d^2x^2-c^2}}{x}\right)x^4ad^4+4\,\ln\left(-2\,\frac{c^2-\sqrt{-c^2}\sqrt{d^2x^2-c^2}}{x}\right)x^4bc^2d^2-\sqrt{-c^2}\sqrt{d^2x^2-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^5,x)
```

```
[Out] -1/8*(d*x-c)^(1/2)*(d*x+c)^(1/2)/c^2*(ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)
^(1/2))/x)*x^4*a*d^4+4*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*x^4*
b*c^2*d^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2)*x^2*a*d^2+4*(-c^2)^(1/2)*(d^2*x^
2-c^2)^(1/2)*x^2*b*c^2+2*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2)*a*c^2)/(d^2*x^2-c
^2)^(1/2)/x^4/(-c^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.58017, size = 213, normalized size = 1.76

$$\frac{2\left(4\,bc^2d^2+ad^4\right)x^4\arctan\left(-\frac{dx-\sqrt{dx+c}\sqrt{dx-c}}{c}\right)-\left(2\,ac^3+\left(4\,bc^3-acd^2\right)x^2\right)\sqrt{dx+c}\sqrt{dx-c}}{8\,c^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/8*(2*(4*b*c^2*d^2 + a*d^4)*x^4*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c)
)/c) - (2*a*c^3 + (4*b*c^3 - a*c*d^2)*x^2)*sqrt(d*x + c)*sqrt(d*x - c))/(c^
3*x^4)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: MellinTransformStripError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2)/x**5,x)
```

```
[Out] Exception raised: MellinTransformStripError
```

Giac [B] time = 1.21363, size = 437, normalized size = 3.61

$$-\frac{\frac{\left(4\,bc^2d^3+ad^5\right)\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2\,c}\right)}{c^3}-\frac{2\left(4\,bc^2d^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{14}-ad^5\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{14}+16\,bc^4d^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{10}+28\,ac^2d^5\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{1\cdots}\right.}{\left(\left(\sqrt{dx+c}-\cdots\right.\right.}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] -1/4*((4*b*c^2*d^3 + a*d^5)*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/c)
/c^3 - 2*(4*b*c^2*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^14 - a*d^5*(sqrt(d*x
+ c) - sqrt(d*x - c))^14 + 16*b*c^4*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^10
+ 28*a*c^2*d^5*(sqrt(d*x + c) - sqrt(d*x - c))^10 - 64*b*c^6*d^3*(sqrt(d*x
+ c) - sqrt(d*x - c))^6 - 112*a*c^4*d^5*(sqrt(d*x + c) - sqrt(d*x - c))^6 -
 256*b*c^8*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^2 + 64*a*c^6*d^5*(sqrt(d*x +
 c) - sqrt(d*x - c))^2)/(((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2)^4*c^2)
)/d
```

3.343 $\int x^4\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)dx$

Optimal. Leaf size=208

$$\frac{x^3(dx-c)^{3/2}(c+dx)^{3/2}\left(8ad^2+5bc^2\right)}{48d^4}+\frac{c^4x\sqrt{dx-c}\sqrt{c+dx}\left(8ad^2+5bc^2\right)}{128d^6}+\frac{c^2x(dx-c)^{3/2}(c+dx)^{3/2}\left(8ad^2+5bc^2\right)}{64d^6}-\frac{c^6}{}$$

```
[Out] (c^4*(5*b*c^2 + 8*a*d^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(128*d^6) + (c^2*(
5*b*c^2 + 8*a*d^2)*x*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(64*d^6) + ((5*b*c^2
 + 8*a*d^2)*x^3*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(48*d^4) + (b*x^5*(-c + d
*x)^(3/2)*(c + d*x)^(3/2))/(8*d^2) - (c^6*(5*b*c^2 + 8*a*d^2)*ArcTanh[Sqrt[
-c + d*x]/Sqrt[c + d*x]])/(64*d^7)
```

Rubi [A] time = 0.148884, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {460, 100, 12, 90, 38, 63, 217, 206}

$$\frac{x^3(dx-c)^{3/2}(c+dx)^{3/2}\left(8ad^2+5bc^2\right)}{48d^4}+\frac{c^4x\sqrt{dx-c}\sqrt{c+dx}\left(8ad^2+5bc^2\right)}{128d^6}+\frac{c^2x(dx-c)^{3/2}(c+dx)^{3/2}\left(8ad^2+5bc^2\right)}{64d^6}-\frac{c^6}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```


```
[Out] (c^4*(5*b*c^2 + 8*a*d^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(128*d^6) + (c^2*(
5*b*c^2 + 8*a*d^2)*x*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(64*d^6) + ((5*b*c^2
 + 8*a*d^2)*x^3*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(48*d^4) + (b*x^5*(-c + d
*x)^(3/2)*(c + d*x)^(3/2))/(8*d^2) - (c^6*(5*b*c^2 + 8*a*d^2)*ArcTanh[Sqrt[
-c + d*x]/Sqrt[c + d*x]])/(64*d^7)
```


Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^4\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)dx = \frac{bx^5(-c+dx)^{3/2}(c+dx)^{3/2}}{8d^2} - \frac{1}{8}\left(-8a-\frac{5bc^2}{d^2}\right)\int x^4\sqrt{-c+dx}\sqrt{c+dx}\,dx$$
$$= \frac{\left(5bc^2+8ad^2\right)x^3(-c+dx)^{3/2}(c+dx)^{3/2}}{48d^4} + \frac{bx^5(-c+dx)^{3/2}(c+dx)^{3/2}}{8d^2} + \frac{\left(5bc^2+8a\right.}{}$$
$$= \frac{\left(5bc^2+8ad^2\right)x^3(-c+dx)^{3/2}(c+dx)^{3/2}}{48d^4} + \frac{bx^5(-c+dx)^{3/2}(c+dx)^{3/2}}{8d^2} + \frac{\left(c^2\left(5bc^2\right.\right.}{}$$
$$= \frac{c^2\left(5bc^2+8ad^2\right)x(-c+dx)^{3/2}(c+dx)^{3/2}}{64d^6} + \frac{\left(5bc^2+8ad^2\right)x^3(-c+dx)^{3/2}(c+dx)^{3/}}{48d^4}$$
$$= \frac{c^2\left(5bc^2+8ad^2\right)x(-c+dx)^{3/2}(c+dx)^{3/2}}{64d^6} + \frac{\left(5bc^2+8ad^2\right)x^3(-c+dx)^{3/2}(c+dx)^{3/}}{48d^4}$$
$$= \frac{c^4\left(5bc^2+8ad^2\right)x\sqrt{-c+dx}\sqrt{c+dx}}{128d^6} + \frac{c^2\left(5bc^2+8ad^2\right)x(-c+dx)^{3/2}(c+dx)^{3/2}}{64d^6} +$$
$$= \frac{c^4\left(5bc^2+8ad^2\right)x\sqrt{-c+dx}\sqrt{c+dx}}{128d^6} + \frac{c^2\left(5bc^2+8ad^2\right)x(-c+dx)^{3/2}(c+dx)^{3/2}}{64d^6} +$$
$$= \frac{c^4\left(5bc^2+8ad^2\right)x\sqrt{-c+dx}\sqrt{c+dx}}{128d^6} + \frac{c^2\left(5bc^2+8ad^2\right)x(-c+dx)^{3/2}(c+dx)^{3/2}}{64d^6} +$$
$$= \frac{c^4\left(5bc^2+8ad^2\right)x\sqrt{-c+dx}\sqrt{c+dx}}{128d^6} + \frac{c^2\left(5bc^2+8ad^2\right)x(-c+dx)^{3/2}(c+dx)^{3/2}}{64d^6} +$$

Mathematica [A] time = 0.214394, size = 161, normalized size = 0.77

$$\frac{\sqrt{dx-c}\sqrt{c+dx}\left(3\left(8ac^5d^2+5bc^7\right)\sin^{-1}\left(\frac{dx}{c}\right)-dx\sqrt{1-\frac{d^2x^2}{c^2}}\left(8ad^2\left(2c^2d^2x^2+3c^4-8d^4x^4\right)+b\left(10c^4d^2x^2+8c^2d^4x^4+1\right.\right.\right.}{384d^7\sqrt{1-\frac{d^2x^2}{c^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (Sqrt[-c + d*x]*Sqrt[c + d*x]*(-(d*x*Sqrt[1 - (d^2*x^2)/c^2]*(8*a*d^2*(3*c^
4 + 2*c^2*d^2*x^2 - 8*d^4*x^4) + b*(15*c^6 + 10*c^4*d^2*x^2 + 8*c^2*d^4*x^4
 - 48*d^6*x^6))) + 3*(5*b*c^7 + 8*a*c^5*d^2)*ArcSin[(d*x)/c]))/(384*d^7*Sqr
t[1 - (d^2*x^2)/c^2])
```

Maple [C] time = 0.022, size = 298, normalized size = 1.4

$$\frac{\operatorname{csgn}(d)}{384\,d^7}\sqrt{dx-c}\sqrt{dx+c}\left(48\operatorname{csgn}(d)\,x^7bd^7\sqrt{d^2x^2-c^2}+64\operatorname{csgn}(d)\,x^5ad^7\sqrt{d^2x^2-c^2}-8\operatorname{csgn}(d)\,x^5bc^2d^5\sqrt{d^2x^2-c^2}-1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x)
```

```
[Out] 1/384*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(48*csgn(d)*x^7*b*d^7*(d^2*x^2-c^2)^(1/2)
+64*csgn(d)*x^5*a*d^7*(d^2*x^2-c^2)^(1/2)-8*csgn(d)*x^5*b*c^2*d^5*(d^2*x^2-
c^2)^(1/2)-16*csgn(d)*x^3*a*c^2*d^5*(d^2*x^2-c^2)^(1/2)-10*csgn(d)*x^3*b*c^
4*d^3*(d^2*x^2-c^2)^(1/2)-24*csgn(d)*d^3*(d^2*x^2-c^2)^(1/2)*x*a*c^4-15*csg
n(d)*d*(d^2*x^2-c^2)^(1/2)*x*b*c^6-24*ln(((d^2*x^2-c^2)^(1/2)*csgn(d)+d*x)*
```

```
csgn(d))*a*c^6*d^2-15*ln(((d^2*x^2-c^2)^(1/2)*csgn(d)+d*x)*csgn(d))*b*c^8)*
csgn(d)/(d^2*x^2-c^2)^(1/2)/d^7
```

Maxima [A] time = 0.948575, size = 356, normalized size = 1.71

$$\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}bx^5}{8\,d^2}+\frac{5\left(d^2x^2-c^2\right)^{\frac{3}{2}}bc^2x^3}{48\,d^4}+\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}ax^3}{6\,d^2}-\frac{5\,bc^8\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\sqrt{d^2}\right)}{128\,\sqrt{d^2}d^6}-\frac{ac^6\log\left(2\,d^2x+2\right.}{16\,\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*(d^2*x^2 - c^2)^(3/2)*b*x^5/d^2 + 5/48*(d^2*x^2 - c^2)^(3/2)*b*c^2*x^3/
d^4 + 1/6*(d^2*x^2 - c^2)^(3/2)*a*x^3/d^2 - 5/128*b*c^8*log(2*d^2*x + 2*sqr
t(d^2*x^2 - c^2)*sqrt(d^2))/(sqrt(d^2)*d^6) - 1/16*a*c^6*log(2*d^2*x + 2*sq
rt(d^2*x^2 - c^2)*sqrt(d^2))/(sqrt(d^2)*d^4) + 5/128*sqrt(d^2*x^2 - c^2)*b*
c^6*x/d^6 + 1/16*sqrt(d^2*x^2 - c^2)*a*c^4*x/d^4 + 5/64*(d^2*x^2 - c^2)^(3/
2)*b*c^4*x/d^6 + 1/8*(d^2*x^2 - c^2)^(3/2)*a*c^2*x/d^4
```

Fricas [A] time = 1.82932, size = 300, normalized size = 1.44

$$\frac{\left(48\,bd^7x^7-8\left(bc^2d^5-8\,ad^7\right)x^5-2\left(5\,bc^4d^3+8\,ac^2d^5\right)x^3-3\left(5\,bc^6d+8\,ac^4d^3\right)x\right)\sqrt{dx+c}\sqrt{dx-c}+3\left(5\,bc^8+8\,ac^6d^2\right.}{384\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/384*((48*b*d^7*x^7 - 8*(b*c^2*d^5 - 8*a*d^7)*x^5 - 2*(5*b*c^4*d^3 + 8*a*c
^2*d^5)*x^3 - 3*(5*b*c^6*d + 8*a*c^4*d^3)*x)*sqrt(d*x + c)*sqrt(d*x - c) +
3*(5*b*c^8 + 8*a*c^6*d^2)*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c)))/d^7
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4\left(a+bx^2\right)\sqrt{-c+dx}\sqrt{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)
```

```
[Out] Integral(x**4*(a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x), x)
```

Giac [A] time = 1.35004, size = 394, normalized size = 1.89

$$8\left(\frac{6\,c^6\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^4}+\left(\left(2\left((dx+c)\left(4\,(dx+c)\left(\frac{dx+c}{d^4}-\frac{5\,c}{d^4}\right)+\frac{39\,c^2}{d^4}\right)-\frac{37\,c^3}{d^4}\right)(dx+c)+\frac{31\,c^4}{d^4}\right)(dx+c)-\frac{3\,c^5}{d^4}\right)\sqrt{dx+c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/384*(8*(6*c^6*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^4 + ((2*((d*x +
c)*(4*(d*x + c)*((d*x + c)/d^4 - 5*c/d^4) + 39*c^2/d^4) - 37*c^3/d^4)*(d*x
+ c) + 31*c^4/d^4)*(d*x + c) - 3*c^5/d^4)*sqrt(d*x + c)*sqrt(d*x - c))*a +
(30*c^8*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^6 + ((2*((4*((d*x + c)*(
6*(d*x + c)*((d*x + c)/d^6 - 7*c/d^6) + 125*c^2/d^6) - 205*c^3/d^6)*(d*x +
c) + 795*c^4/d^6)*(d*x + c) - 449*c^5/d^6)*(d*x + c) + 251*c^6/d^6)*(d*x +
c) - 15*c^7/d^6)*sqrt(d*x + c)*sqrt(d*x - c))*b)/d
```

3.344 $\int x^2\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx$

Optimal. Leaf size=159

$$\frac{c^2x\sqrt{dx-c}\sqrt{c+dx}\left(2ad^2+bc^2\right)}{16d^4}+\frac{x(dx-c)^{3/2}(c+dx)^{3/2}\left(2ad^2+bc^2\right)}{8d^4}-\frac{c^4\left(2ad^2+bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{8d^5}+\frac{bx^3(dx-$$

```
[Out] (c^2*(b*c^2 + 2*a*d^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(16*d^4) + ((b*c^2 +
 2*a*d^2)*x*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(8*d^4) + (b*x^3*(-c + d*x)^(
3/2)*(c + d*x)^(3/2))/(6*d^2) - (c^4*(b*c^2 + 2*a*d^2)*ArcTanh[Sqrt[-c + d*
x]/Sqrt[c + d*x]])/(8*d^5)
```

Rubi [A] time = 0.123454, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {460, 90, 12, 38, 63, 217, 206}

$$\frac{c^2x\sqrt{dx-c}\sqrt{c+dx}\left(2ad^2+bc^2\right)}{16d^4}+\frac{x(dx-c)^{3/2}(c+dx)^{3/2}\left(2ad^2+bc^2\right)}{8d^4}-\frac{c^4\left(2ad^2+bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{8d^5}+\frac{bx^3(dx-$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (c^2*(b*c^2 + 2*a*d^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(16*d^4) + ((b*c^2 +
 2*a*d^2)*x*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(8*d^4) + (b*x^3*(-c + d*x)^(
3/2)*(c + d*x)^(3/2))/(6*d^2) - (c^4*(b*c^2 + 2*a*d^2)*ArcTanh[Sqrt[-c + d*
x]/Sqrt[c + d*x]])/(8*d^5)
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
```

```
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int x^2\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)dx = \frac{bx^3(-c+dx)^{3/2}(c+dx)^{3/2}}{6d^2}+\frac{1}{2}\left(2a+\frac{bc^2}{d^2}\right)\int x^2\sqrt{-c+dx}\sqrt{c+dx}\,dx$$

$$= \frac{\left(bc^2+2ad^2\right)x(-c+dx)^{3/2}(c+dx)^{3/2}}{8d^4}+\frac{bx^3(-c+dx)^{3/2}(c+dx)^{3/2}}{6d^2}+\frac{\left(bc^2+2ad^2\right)}{}$$

$$= \frac{\left(bc^2+2ad^2\right)x(-c+dx)^{3/2}(c+dx)^{3/2}}{8d^4}+\frac{bx^3(-c+dx)^{3/2}(c+dx)^{3/2}}{6d^2}+\frac{\left(c^2\left(bc^2+2a\right.\right.}{}$$

$$= \frac{c^2\left(bc^2+2ad^2\right)x\sqrt{-c+dx}\sqrt{c+dx}}{16d^4}+\frac{\left(bc^2+2ad^2\right)x(-c+dx)^{3/2}(c+dx)^{3/2}}{8d^4}+\frac{bx^3}{}$$

$$= \frac{c^2\left(bc^2+2ad^2\right)x\sqrt{-c+dx}\sqrt{c+dx}}{16d^4}+\frac{\left(bc^2+2ad^2\right)x(-c+dx)^{3/2}(c+dx)^{3/2}}{8d^4}+\frac{bx^3}{}$$

$$= \frac{c^2\left(bc^2+2ad^2\right)x\sqrt{-c+dx}\sqrt{c+dx}}{16d^4}+\frac{\left(bc^2+2ad^2\right)x(-c+dx)^{3/2}(c+dx)^{3/2}}{8d^4}+\frac{bx^3}{}$$

$$= \frac{c^2\left(bc^2+2ad^2\right)x\sqrt{-c+dx}\sqrt{c+dx}}{16d^4}+\frac{\left(bc^2+2ad^2\right)x(-c+dx)^{3/2}(c+dx)^{3/2}}{8d^4}+\frac{bx^3}{}$$

Mathematica [A] time = 0.163397, size = 135, normalized size = 0.85

$$\frac{\sqrt{dx-c}\sqrt{c+dx}\left(dx\sqrt{1-\frac{d^2x^2}{c^2}}\left(b\left(-2c^2d^2x^2-3c^4+8d^4x^4\right)-6ad^2\left(c^2-2d^2x^2\right)\right)+3\left(2ac^3d^2+bc^5\right)\sin^{-1}\left(\frac{dx}{c}\right)\right)}{48d^5\sqrt{1-\frac{d^2x^2}{c^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (Sqrt[-c + d*x]*Sqrt[c + d*x]*(d*x*Sqrt[1 - (d^2*x^2)/c^2]*(-6*a*d^2*(c^2 -
 2*d^2*x^2) + b*(-3*c^4 - 2*c^2*d^2*x^2 + 8*d^4*x^4)) + 3*(b*c^5 + 2*a*c^3*
```

```
d^2)*ArcSin[(d*x)/c]))/(48*d^5*Sqrt[1 - (d^2*x^2)/c^2])
```

Maple [C] time = 0.013, size = 240, normalized size = 1.5

$$\frac{\operatorname{csgn}(d)}{48\,d^5}\sqrt{dx-c}\sqrt{dx+c}\left(8\operatorname{csgn}(d)\,x^5bd^5\sqrt{d^2x^2-c^2}+12\operatorname{csgn}(d)\,x^3ad^5\sqrt{d^2x^2-c^2}-2\operatorname{csgn}(d)\,x^3bc^2d^3\sqrt{d^2x^2-c^2}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x)
```

```
[Out] 1/48*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(8*csgn(d)*x^5*b*d^5*(d^2*x^2-c^2)^(1/2)+1
2*csgn(d)*x^3*a*d^5*(d^2*x^2-c^2)^(1/2)-2*csgn(d)*x^3*b*c^2*d^3*(d^2*x^2-c^
2)^(1/2)-6*csgn(d)*d^3*(d^2*x^2-c^2)^(1/2)*x*a*c^2-3*csgn(d)*d*(d^2*x^2-c^2
)^(1/2)*x*b*c^4-6*ln(((d^2*x^2-c^2)^(1/2)*csgn(d)+d*x)*csgn(d))*a*c^4*d^2-3
*ln(((d^2*x^2-c^2)^(1/2)*csgn(d)+d*x)*csgn(d))*b*c^6)*csgn(d)/(d^2*x^2-c^2)
^(1/2)/d^5
```

Maxima [A] time = 0.969889, size = 284, normalized size = 1.79

$$\frac{\left(d^2x^2-c^2\right)^{\frac{3}{2}}bx^3}{6\,d^2}-\frac{bc^6\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\sqrt{d^2}\right)}{16\,\sqrt{d^2}d^4}-\frac{ac^4\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\sqrt{d^2}\right)}{8\,\sqrt{d^2}d^2}+\frac{\sqrt{d^2x^2-c^2}bc^4x}{16\,d^4}+\frac{\sqrt{d^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(d^2*x^2 - c^2)^(3/2)*b*x^3/d^2 - 1/16*b*c^6*log(2*d^2*x + 2*sqrt(d^2*x
^2 - c^2)*sqrt(d^2))/(sqrt(d^2)*d^4) - 1/8*a*c^4*log(2*d^2*x + 2*sqrt(d^2*x
^2 - c^2)*sqrt(d^2))/(sqrt(d^2)*d^2) + 1/16*sqrt(d^2*x^2 - c^2)*b*c^4*x/d^4
 + 1/8*sqrt(d^2*x^2 - c^2)*a*c^2*x/d^2 + 1/8*(d^2*x^2 - c^2)^(3/2)*b*c^2*x/
d^4 + 1/4*(d^2*x^2 - c^2)^(3/2)*a*x/d^2
```

Fricas [A] time = 1.64964, size = 243, normalized size = 1.53

$$\frac{\left(8\,bd^5x^5-2\left(bc^2d^3-6\,ad^5\right)x^3-3\left(bc^4d+2\,ac^2d^3\right)x\right)\sqrt{dx+c}\sqrt{dx-c}+3\left(bc^6+2\,ac^4d^2\right)\log\left(-dx+\sqrt{dx+c}\sqrt{dx-c}\right)}{48\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/48*((8*b*d^5*x^5 - 2*(b*c^2*d^3 - 6*a*d^5)*x^3 - 3*(b*c^4*d + 2*a*c^2*d^3
)*x)*sqrt(d*x + c)*sqrt(d*x - c) + 3*(b*c^6 + 2*a*c^4*d^2)*log(-d*x + sqrt(
d*x + c)*sqrt(d*x - c)))/d^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\left(a+bx^2\right)\sqrt{-c+dx}\sqrt{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)
```

```
[Out] Integral(x**2*(a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x), x)
```

Giac [A] time = 1.31216, size = 311, normalized size = 1.96

$$\frac{6\left(\frac{2\,c^4\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^2}+\left((dx+c)\left(2\,(dx+c)\left(\frac{dx+c}{d^2}-\frac{3\,c}{d^2}\right)+\frac{5\,c^2}{d^2}\right)-\frac{c^3}{d^2}\right)\sqrt{dx+c}\sqrt{dx-c}\right)a+\left(\frac{6\,c^6\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^4}+\left(\left(2\right.\right.\right.}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(6*(2*c^4*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^2 + ((d*x + c)*(2
*(d*x + c)*((d*x + c)/d^2 - 3*c/d^2) + 5*c^2/d^2) - c^3/d^2)*sqrt(d*x + c)*
sqrt(d*x - c))*a + (6*c^6*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^4 + ((
2*((d*x + c)*(4*(d*x + c)*((d*x + c)/d^4 - 5*c/d^4) + 39*c^2/d^4) - 37*c^3/
d^4)*(d*x + c) + 31*c^4/d^4)*(d*x + c) - 3*c^5/d^4)*sqrt(d*x + c)*sqrt(d*x
- c))*b)/d
```

3.345 $\int \sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)\,dx$

Optimal. Leaf size=114

$$\frac{x\sqrt{dx-c}\sqrt{c+dx}\left(4ad^2+bc^2\right)}{8d^2}-\frac{c^2\left(4ad^2+bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4d^3}+\frac{bx(dx-c)^{3/2}(c+dx)^{3/2}}{4d^2}$$

```
[Out] ((b*c^2 + 4*a*d^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(8*d^2) + (b*x*(-c + d*x
)^(3/2)*(c + d*x)^(3/2))/(4*d^2) - (c^2*(b*c^2 + 4*a*d^2)*ArcTanh[Sqrt[-c +
 d*x]/Sqrt[c + d*x]])/(4*d^3)
```

Rubi [A] time = 0.0465434, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {389, 38, 63, 217, 206}

$$\frac{x\sqrt{dx-c}\sqrt{c+dx}\left(4ad^2+bc^2\right)}{8d^2}-\frac{c^2\left(4ad^2+bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4d^3}+\frac{bx(dx-c)^{3/2}(c+dx)^{3/2}}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] ((b*c^2 + 4*a*d^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(8*d^2) + (b*x*(-c + d*x
)^(3/2)*(c + d*x)^(3/2))/(4*d^2) - (c^2*(b*c^2 + 4*a*d^2)*ArcTanh[Sqrt[-c +
 d*x]/Sqrt[c + d*x]])/(4*d^3)
```

Rule 389

```
Int[((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_
.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*x*(a1 + b1*x^(n/2))^(p +
1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*(n*(p + 1) + 1)), x] - Dist[(a1*a2*d -
 b1*b2*c*(n*(p + 1) + 1))/(b1*b2*(n*(p + 1) + 1)), Int[(a1 + b1*x^(n/2))^p*
(a2 + b2*x^(n/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p}, x] && Eq
Q[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)dx = \frac{bx(-c+dx)^{3/2}(c+dx)^{3/2}}{4d^2} - \frac{\left(-bc^2-4ad^2\right)\int \sqrt{-c+dx}\sqrt{c+dx}\,dx}{4d^2}$$
$$= \frac{\left(bc^2+4ad^2\right)x\sqrt{-c+dx}\sqrt{c+dx}}{8d^2} + \frac{bx(-c+dx)^{3/2}(c+dx)^{3/2}}{4d^2} + \frac{\left(c^2\left(-bc^2-4ad^2\right)\right)\int}{8d^2}$$
$$= \frac{\left(bc^2+4ad^2\right)x\sqrt{-c+dx}\sqrt{c+dx}}{8d^2} + \frac{bx(-c+dx)^{3/2}(c+dx)^{3/2}}{4d^2} - \frac{\left(c^2\left(bc^2+4ad^2\right)\right)\mathrm{Su}}{}$$
$$= \frac{\left(bc^2+4ad^2\right)x\sqrt{-c+dx}\sqrt{c+dx}}{8d^2} + \frac{bx(-c+dx)^{3/2}(c+dx)^{3/2}}{4d^2} - \frac{\left(c^2\left(bc^2+4ad^2\right)\right)\mathrm{Su}}{}$$
$$= \frac{\left(bc^2+4ad^2\right)x\sqrt{-c+dx}\sqrt{c+dx}}{8d^2} + \frac{bx(-c+dx)^{3/2}(c+dx)^{3/2}}{4d^2} - \frac{c^2\left(bc^2+4ad^2\right)\tanh}{4d^3}$$

Mathematica [A] time = 0.234537, size = 129, normalized size = 1.13

$$\frac{dx\left(c^2-d^2x^2\right)\left(b\left(c^2-2d^2x^2\right)-4ad^2\right)-2c^{5/2}\sqrt{dx-c}\sqrt{\frac{dx}{c}+1}\left(4ad^2+bc^2\right)\sinh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{2}\sqrt{c}}\right)}{8d^3\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2),x]
```

```
[Out] (d*x*(c^2 - d^2*x^2)*(-4*a*d^2 + b*(c^2 - 2*d^2*x^2)) - 2*c^(5/2)*(b*c^2 +
4*a*d^2)*Sqrt[-c + d*x]*Sqrt[1 + (d*x)/c]*ArcSinh[Sqrt[-c + d*x]/(Sqrt[2]*S
qrt[c])])/(8*d^3*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

Maple [C] time = 0.01, size = 180, normalized size = 1.6

$$-\frac{\mathrm{csgn}\left(d\right)}{8\,d^3}\sqrt{dx-c}\sqrt{dx+c}\left(-2\,\mathrm{csgn}\left(d\right)x^3bd^3\sqrt{d^2x^2-c^2}-4\,\mathrm{csgn}\left(d\right)d^3\sqrt{d^2x^2-c^2}xa+\mathrm{csgn}\left(d\right)d\sqrt{d^2x^2-c^2}xbc^2+4\,\ln\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x)
```

```
[Out] -1/8*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(-2*csgn(d)*x^3*b*d^3*(d^2*x^2-c^2)^(1/2)-
4*csgn(d)*d^3*(d^2*x^2-c^2)^(1/2)*x*a+csgn(d)*d*(d^2*x^2-c^2)^(1/2)*x*b*c^2
+4*ln(((d^2*x^2-c^2)^(1/2)*csgn(d)+d*x)*csgn(d))*a*c^2*d^2+ln(((d^2*x^2-c^2
)^(1/2)*csgn(d)+d*x)*csgn(d))*b*c^4)*csgn(d)/(d^2*x^2-c^2)^(1/2)/d^3
```

Maxima [A] time = 0.942171, size = 205, normalized size = 1.8

$$-\frac{ac^2\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\sqrt{d^2}\right)}{2\,\sqrt{d^2}} - \frac{bc^4\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\sqrt{d^2}\right)}{8\,\sqrt{d^2}d^2} + \frac{1}{2}\sqrt{d^2x^2-c^2}ax + \frac{\sqrt{d^2x^2-c^2}bc^2x}{8\,d^2} + \frac{\left(d^2x^2\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*a*c^2*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*sqrt(d^2))/sqrt(d^2) - 1/8*b
*c^4*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*sqrt(d^2))/(sqrt(d^2)*d^2) + 1/2*s
qrt(d^2*x^2 - c^2)*a*x + 1/8*sqrt(d^2*x^2 - c^2)*b*c^2*x/d^2 + 1/4*(d^2*x^2
 - c^2)^(3/2)*b*x/d^2
```

Fricas [A] time = 1.58721, size = 190, normalized size = 1.67

$$\frac{\left(2\,bd^3x^3-\left(bc^2d-4\,ad^3\right)x\right)\sqrt{dx+c}\sqrt{dx-c}+\left(bc^4+4\,ac^2d^2\right)\log\left(-dx+\sqrt{dx+c}\sqrt{dx-c}\right)}{8\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*((2*b*d^3*x^3 - (b*c^2*d - 4*a*d^3)*x)*sqrt(d*x + c)*sqrt(d*x - c) + (b
*c^4 + 4*a*c^2*d^2)*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c)))/d^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+bx^2\right)\sqrt{-c+dx}\sqrt{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x), x)
```

Giac [A] time = 1.2615, size = 204, normalized size = 1.79

$$\frac{4\left(\sqrt{dx+c}\sqrt{dx-c}dx+2\,c^2\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)\right)a+\left(\frac{2\,c^4\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{d^2}+\left((dx+c)\left(2\,(dx+c)\left(\frac{dx+c}{d^2}-\frac{3\,c}{d^2}\right)\right.\right.\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(4*(sqrt(d*x + c)*sqrt(d*x - c)*d*x + 2*c^2*log(abs(-sqrt(d*x + c) + sq
rt(d*x - c))))*a + (2*c^4*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^2 + ((
d*x + c)*(2*(d*x + c)*((d*x + c)/d^2 - 3*c/d^2) + 5*c^2/d^2) - c^3/d^2)*sqr
t(d*x + c)*sqrt(d*x - c))*b)/d
```

3.346 $\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^2} dx$

Optimal. Leaf size=104

$$\frac{1}{2}x\sqrt{dx-c}\sqrt{c+dx}\left(b-\frac{2ad^2}{c^2}\right)-\frac{\left(bc^2-2ad^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d}+\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{c^2x}$$

```
[Out] ((b - (2*a*d^2)/c^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/2 + (a*(-c + d*x)^(3/2
)*(c + d*x)^(3/2))/(c^2*x) - ((b*c^2 - 2*a*d^2)*ArcTanh[Sqrt[-c + d*x]/Sqrt
[c + d*x]])/d
```

Rubi [A] time = 0.0869517, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {454, 38, 63, 217, 206}

$$\frac{1}{2}x\sqrt{dx-c}\sqrt{c+dx}\left(b-\frac{2ad^2}{c^2}\right)-\frac{\left(bc^2-2ad^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d}+\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{c^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^2,x]
```

```
[Out] ((b - (2*a*d^2)/c^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/2 + (a*(-c + d*x)^(3/2
)*(c + d*x)^(3/2))/(c^2*x) - ((b*c^2 - 2*a*d^2)*ArcTanh[Sqrt[-c + d*x]/Sqrt
[c + d*x]])/d
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)}{x^2}\,dx = \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{c^2x} + \left(b-\frac{2ad^2}{c^2}\right)\int \sqrt{-c+dx}\sqrt{c+dx}\,dx$$

$$= \frac{1}{2}\left(b-\frac{2ad^2}{c^2}\right)x\sqrt{-c+dx}\sqrt{c+dx} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{c^2x} + \frac{1}{2}\left(-bc^2+2ad^2\right)\int$$

$$= \frac{1}{2}\left(b-\frac{2ad^2}{c^2}\right)x\sqrt{-c+dx}\sqrt{c+dx} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{c^2x} + \frac{\left(-bc^2+2ad^2\right)\text{Sul}}{}$$

$$= \frac{1}{2}\left(b-\frac{2ad^2}{c^2}\right)x\sqrt{-c+dx}\sqrt{c+dx} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{c^2x} + \frac{\left(-bc^2+2ad^2\right)\text{Sul}}{}$$

$$= \frac{1}{2}\left(b-\frac{2ad^2}{c^2}\right)x\sqrt{-c+dx}\sqrt{c+dx} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{c^2x} - \frac{\left(bc^2-2ad^2\right)\tanh}{d}$$

Mathematica [A] time = 0.0802886, size = 101, normalized size = 0.97

$$\frac{\sqrt{dx-c}\sqrt{c+dx}\left(cd\left(bx^2-2a\right)\sqrt{1-\frac{d^2x^2}{c^2}}+x\left(bc^2-2ad^2\right)\sin^{-1}\left(\frac{dx}{c}\right)\right)}{2cdx\sqrt{1-\frac{d^2x^2}{c^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^2,x]
```

```
[Out] (Sqrt[-c + d*x]*Sqrt[c + d*x]*(c*d*(-2*a + b*x^2)*Sqrt[1 - (d^2*x^2)/c^2] +
 (b*c^2 - 2*a*d^2)*x*ArcSin[(d*x)/c]))/(2*c*d*x*Sqrt[1 - (d^2*x^2)/c^2])
```

Maple [C] time = 0.014, size = 153, normalized size = 1.5

$$\frac{\text{csgn}(d)}{2\,dx}\sqrt{dx-c}\sqrt{dx+c}\left(\text{csgn}(d)\,x^2bd\sqrt{d^2x^2-c^2}+2\,\ln\left(\left(\sqrt{d^2x^2-c^2}\text{csgn}(d)+dx\right)\text{csgn}(d)\right)xad^2-\ln\left(\left(\sqrt{d^2x^2-\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^2,x)
```

```
[Out] 1/2*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(csgn(d)*x^2*b*d*(d^2*x^2-c^2)^(1/2)+2*ln((
(d^2*x^2-c^2)^(1/2)*csgn(d)+d*x)*csgn(d))*x*a*d^2-ln(((d^2*x^2-c^2)^(1/2)*c
sgn(d)+d*x)*csgn(d))*x*b*c^2-2*csgn(d)*d*(d^2*x^2-c^2)^(1/2)*a)*csgn(d)/(d^
2*x^2-c^2)^(1/2)/x/d
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.57855, size = 181, normalized size = 1.74

$$-\frac{2\,ad^2x-\left(bc^2-2\,ad^2\right)x\log\left(-dx+\sqrt{dx+c}\sqrt{dx-c}\right)-\left(bdx^2-2\,ad\right)\sqrt{dx+c}\sqrt{dx-c}}{2\,dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*d^2*x - (b*c^2 - 2*a*d^2)*x*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c
)) - (b*d*x^2 - 2*a*d)*sqrt(d*x + c)*sqrt(d*x - c))/(d*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a+bx^2\right)\sqrt{-c+dx}\sqrt{c+dx}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2)/x**2,x)
```

```
[Out] Integral((a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x)/x**2, x)
```

Giac [A] time = 1.24287, size = 149, normalized size = 1.43

$$-\frac{\frac{6144\,ac^2d^2}{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2}-2\left((dx+c)b-bc\right)\sqrt{dx+c}\sqrt{dx-c}-\left(bc^2-2\,ad^2\right)\log\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4\right)}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] -1/768*(6144*a*c^2*d^2/((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2) - 2*((d*
x + c)*b - b*c)*sqrt(d*x + c)*sqrt(d*x - c) - (b*c^2 - 2*a*d^2)*log((sqrt(d
*x + c) - sqrt(d*x - c))^4))/d
```

3.347 $\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^4}\,dx$

Optimal. Leaf size=84

$$\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{3c^2x^3}-\frac{b\sqrt{dx-c}\sqrt{c+dx}}{x}+2bd\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)$$

```
[Out] -((b*Sqrt[-c + d*x]*Sqrt[c + d*x])/x) + (a*(-c + d*x)^(3/2)*(c + d*x)^(3/2)
)/(3*c^2*x^3) + 2*b*d*ArcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]]
```

Rubi [A] time = 0.0803436, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {454, 97, 12, 63, 217, 206}

$$\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{3c^2x^3}-\frac{b\sqrt{dx-c}\sqrt{c+dx}}{x}+2bd\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^4,x]
```

```
[Out] -((b*Sqrt[-c + d*x]*Sqrt[c + d*x])/x) + (a*(-c + d*x)^(3/2)*(c + d*x)^(3/2)
)/(3*c^2*x^3) + 2*b*d*ArcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]]
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-c+dx}\sqrt{c+dx}\left(a+bx^2\right)}{x^4}\,dx &= \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{3c^2x^3}+b\int \frac{\sqrt{-c+dx}\sqrt{c+dx}}{x^2}\,dx\\
&= -\frac{b\sqrt{-c+dx}\sqrt{c+dx}}{x}+\frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{3c^2x^3}+b\int \frac{d^2}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx\\
&= -\frac{b\sqrt{-c+dx}\sqrt{c+dx}}{x}+\frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{3c^2x^3}+\left(bd^2\right)\int \frac{1}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx\\
&= -\frac{b\sqrt{-c+dx}\sqrt{c+dx}}{x}+\frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{3c^2x^3}+(2bd)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{2c+x^2}}\,dx,x,\ldots\right.\\
&= -\frac{b\sqrt{-c+dx}\sqrt{c+dx}}{x}+\frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{3c^2x^3}+(2bd)\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\ldots\right.\\
&= -\frac{b\sqrt{-c+dx}\sqrt{c+dx}}{x}+\frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{3c^2x^3}+2bd\tanh^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)
\end{aligned}$$

Mathematica [A] time = 0.075709, size = 105, normalized size = 1.25

$$-\frac{\sqrt{dx-c}\sqrt{c+dx}\left(\sqrt{1-\frac{d^2x^2}{c^2}}\left(a\left(c^2-d^2x^2\right)+3bc^2x^2\right)+3bcdx^3\sin^{-1}\left(\frac{dx}{c}\right)\right)}{3c^2x^3\sqrt{1-\frac{d^2x^2}{c^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^4,x]
```

```
[Out] -(Sqrt[-c + d*x]*Sqrt[c + d*x]*(Sqrt[1 - (d^2*x^2)/c^2]*(3*b*c^2*x^2 + a*(c
^2 - d^2*x^2)) + 3*b*c*d*x^3*ArcSin[(d*x)/c]))/(3*c^2*x^3*Sqrt[1 - (d^2*x^2
)/c^2])
```

Maple [C] time = 0.016, size = 153, normalized size = 1.8

$$\frac{\mathrm{csgn}(d)}{3c^2x^3}\sqrt{dx-c}\sqrt{dx+c}\left(3\ln\left(\left(\sqrt{d^2x^2-c^2}\,\mathrm{csgn}(d)+dx\right)\mathrm{csgn}(d)\right)x^3bc^2d+\mathrm{csgn}(d)\,x^2ad^2\sqrt{d^2x^2-c^2}-3\,\mathrm{csgn}(d)\,x^2b\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^4,x)
```

```
[Out] 1/3*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(3*ln(((d^2*x^2-c^2)^(1/2)*csgn(d)+d*x)*csg
n(d))*x^3*b*c^2*d+csgn(d)*x^2*a*d^2*(d^2*x^2-c^2)^(1/2)-3*csgn(d)*x^2*b*c^2
*(d^2*x^2-c^2)^(1/2)-csgn(d)*a*c^2*(d^2*x^2-c^2)^(1/2))*csgn(d)/(d^2*x^2-c^
2)^(1/2)/c^2/x^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.53373, size = 216, normalized size = 2.57

$$-\frac{3\,bc^2dx^3\log\left(-dx+\sqrt{dx+c}\sqrt{dx-c}\right)+\left(3\,bc^2d-ad^3\right)x^3+\left(ac^2+\left(3\,bc^2-ad^2\right)x^2\right)\sqrt{dx+c}\sqrt{dx-c}}{3\,c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*(3*b*c^2*d*x^3*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c)) + (3*b*c^2*d -
a*d^3)*x^3 + (a*c^2 + (3*b*c^2 - a*d^2)*x^2)*sqrt(d*x + c)*sqrt(d*x - c))/(
c^2*x^3)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: MellinTransformStripError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2)/x**4,x)
```

```
[Out] Exception raised: MellinTransformStripError
```

Giac [B] time = 1.32079, size = 231, normalized size = 2.75

$$-\frac{3\,bd^2\log\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4\right)+\frac{16\left(3\,bc^2d^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^8-3\,ad^4\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^8+24\,bc^4d^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+48\,bc^6d^2-16\,ac^4d^4\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/6*(3*b*d^2*log((sqrt(d*x + c) - sqrt(d*x - c))^4) + 16*(3*b*c^2*d^2*(sqr
t(d*x + c) - sqrt(d*x - c))^8 - 3*a*d^4*(sqrt(d*x + c) - sqrt(d*x - c))^8 +
 24*b*c^4*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^4 + 48*b*c^6*d^2 - 16*a*c^4*d
^4)/((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2)^3)/d
```

3.348 $\int \frac{x^4(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

Optimal. Leaf size=125

$$\frac{x^3\sqrt{cx-1}\sqrt{cx+1}\left(6ac^2+5b\right)}{24c^4}+\frac{x\sqrt{cx-1}\sqrt{cx+1}\left(6ac^2+5b\right)}{16c^6}+\frac{\left(6ac^2+5b\right)\cosh^{-1}(cx)}{16c^7}+\frac{bx^5\sqrt{cx-1}\sqrt{cx+1}}{6c^2}$$

```
[Out] ((5*b + 6*a*c^2)*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*c^6) + ((5*b + 6*a*c^2
)*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(24*c^4) + (b*x^5*Sqrt[-1 + c*x]*Sqrt[1
 + c*x])/(6*c^2) + ((5*b + 6*a*c^2)*ArcCosh[c*x])/(16*c^7)
```

Rubi [A] time = 0.0848769, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {460, 100, 12, 90, 52}

$$\frac{x^3\sqrt{cx-1}\sqrt{cx+1}\left(6ac^2+5b\right)}{24c^4}+\frac{x\sqrt{cx-1}\sqrt{cx+1}\left(6ac^2+5b\right)}{16c^6}+\frac{\left(6ac^2+5b\right)\cosh^{-1}(cx)}{16c^7}+\frac{bx^5\sqrt{cx-1}\sqrt{cx+1}}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] ((5*b + 6*a*c^2)*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*c^6) + ((5*b + 6*a*c^2
)*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(24*c^4) + (b*x^5*Sqrt[-1 + c*x]*Sqrt[1
 + c*x])/(6*c^2) + ((5*b + 6*a*c^2)*ArcCosh[c*x])/(16*c^7)
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
```

```
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4\left(a+bx^2\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx &= \frac{bx^5\sqrt{-1+cx}\sqrt{1+cx}}{6c^2}-\frac{1}{6}\left(-6a-\frac{5b}{c^2}\right)\int \frac{x^4}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx \\
&= \frac{\left(5b+6ac^2\right)x^3\sqrt{-1+cx}\sqrt{1+cx}}{24c^4}+\frac{bx^5\sqrt{-1+cx}\sqrt{1+cx}}{6c^2}+\frac{\left(5b+6ac^2\right)\int \frac{3x^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{24c^4} \\
&= \frac{\left(5b+6ac^2\right)x^3\sqrt{-1+cx}\sqrt{1+cx}}{24c^4}+\frac{bx^5\sqrt{-1+cx}\sqrt{1+cx}}{6c^2}+\frac{\left(5b+6ac^2\right)\int \frac{x^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{8c^4} \\
&= \frac{\left(5b+6ac^2\right)x\sqrt{-1+cx}\sqrt{1+cx}}{16c^6}+\frac{\left(5b+6ac^2\right)x^3\sqrt{-1+cx}\sqrt{1+cx}}{24c^4}+\frac{bx^5\sqrt{-1+cx}\sqrt{1+cx}}{6c^2}+ \\
&= \frac{\left(5b+6ac^2\right)x\sqrt{-1+cx}\sqrt{1+cx}}{16c^6}+\frac{\left(5b+6ac^2\right)x^3\sqrt{-1+cx}\sqrt{1+cx}}{24c^4}+\frac{bx^5\sqrt{-1+cx}\sqrt{1+cx}}{6c^2}+
\end{aligned}$$

Mathematica [A] time = 0.103118, size = 117, normalized size = 0.94

$$\frac{cx\left(c^2x^2-1\right)\left(6ac^2\left(2c^2x^2+3\right)+b\left(8c^4x^4+10c^2x^2+15\right)\right)+3\sqrt{c^2x^2-1}\left(6ac^2+5b\right)\tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{48c^7\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (c*x*(-1 + c^2*x^2)*(6*a*c^2*(3 + 2*c^2*x^2) + b*(15 + 10*c^2*x^2 + 8*c^4*x
^4)) + 3*(5*b + 6*a*c^2)*Sqrt[-1 + c^2*x^2]*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2
]])/(48*c^7*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Maple [C] time = 0.077, size = 191, normalized size = 1.5

$$\frac{\operatorname{csgn}(c)}{48\,c^7}\sqrt{cx-1}\sqrt{cx+1}\left(8\operatorname{csgn}(c)\,x^5bc^5\sqrt{c^2x^2-1}+12\operatorname{csgn}(c)\,x^3ac^5\sqrt{c^2x^2-1}+10\sqrt{c^2x^2-1}\operatorname{csgn}(c)\,c^3x^3b+18\sqrt{c^2x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)
```

```
[Out] 1/48*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(8*csgn(c)*x^5*b*c^5*(c^2*x^2-1)^(1/2)+12*
csgn(c)*x^3*a*c^5*(c^2*x^2-1)^(1/2)+10*(c^2*x^2-1)^(1/2)*csgn(c)*c^3*x^3*b+
18*(c^2*x^2-1)^(1/2)*csgn(c)*c^3*x*a+15*(c^2*x^2-1)^(1/2)*csgn(c)*c*x*b+18*
ln(((c^2*x^2-1)^(1/2)*csgn(c)+c*x)*csgn(c))*a*c^2+15*ln(((c^2*x^2-1)^(1/2)*
```

```
csgn(c)+c*x)*csgn(c))*b)*csgn(c)/c^7/(c^2*x^2-1)^(1/2)
```

Maxima [A] time = 0.961023, size = 231, normalized size = 1.85

$$\frac{\sqrt{c^2x^2-1}bx^5}{6\,c^2}+\frac{\sqrt{c^2x^2-1}ax^3}{4\,c^2}+\frac{5\,\sqrt{c^2x^2-1}bx^3}{24\,c^4}+\frac{3\,\sqrt{c^2x^2-1}ax}{8\,c^4}+\frac{3\,a\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}\sqrt{c^2}\right)}{8\,\sqrt{c^2}c^4}+\frac{5\,\sqrt{c^2x^2-1}b}{16\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(c^2*x^2 - 1)*b*x^5/c^2 + 1/4*sqrt(c^2*x^2 - 1)*a*x^3/c^2 + 5/24*sq
rt(c^2*x^2 - 1)*b*x^3/c^4 + 3/8*sqrt(c^2*x^2 - 1)*a*x/c^4 + 3/8*a*log(2*c^2
*x + 2*sqrt(c^2*x^2 - 1)*sqrt(c^2))/(sqrt(c^2)*c^4) + 5/16*sqrt(c^2*x^2 - 1
)*b*x/c^6 + 5/16*b*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*sqrt(c^2))/(sqrt(c^2)*
c^6)
```

Fricas [A] time = 1.54684, size = 224, normalized size = 1.79

$$\frac{\left(8\,bc^5x^5+2\left(6\,ac^5+5\,bc^3\right)x^3+3\left(6\,ac^3+5\,bc\right)x\right)\sqrt{cx+1}\sqrt{cx-1}-3\left(6\,ac^2+5\,b\right)\log\left(-cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{48\,c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/48*((8*b*c^5*x^5 + 2*(6*a*c^5 + 5*b*c^3)*x^3 + 3*(6*a*c^3 + 5*b*c)*x)*sqr
t(c*x + 1)*sqrt(c*x - 1) - 3*(6*a*c^2 + 5*b)*log(-c*x + sqrt(c*x + 1)*sqrt(
c*x - 1)))/c^7
```

Sympy [C] time = 59.9497, size = 216, normalized size = 1.73

$$\frac{aG_{6,6}^{6,2}\left(\begin{matrix} -\frac{7}{4},-\frac{5}{4} & -\frac{3}{2},-\frac{3}{2},-1,1 \\ -2,-\frac{7}{4},-\frac{3}{2},-\frac{5}{4},-1,0 & \end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^5}-\frac{iaG_{6,6}^{2,6}\left(\begin{matrix} -\frac{5}{2},-\frac{9}{4},-2,-\frac{7}{4},-\frac{3}{2},1 & \\ -\frac{9}{4},-\frac{7}{4} & -\frac{5}{2},-2,-2,0 \end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^5}+\frac{bG_{6,6}^{6,2}\left(\begin{matrix} \\ -3,-\frac{11}{4} \end{matrix}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] a*meijerg(((-7/4, -5/4), (-3/2, -3/2, -1, 1)), ((-2, -7/4, -3/2, -5/4, -1,
0), ()), 1/(c**2*x**2))/(4*pi**(3/2)*c**5) - I*a*meijerg(((-5/2, -9/4, -2,
-7/4, -3/2, 1), ()), ((-9/4, -7/4), (-5/2, -2, -2, 0)), exp_polar(2*I*pi)/(
c**2*x**2))/(4*pi**(3/2)*c**5) + b*meijerg(((-11/4, -9/4), (-5/2, -5/2, -2,
 1)), ((-3, -11/4, -5/2, -9/4, -2, 0), ()), 1/(c**2*x**2))/(4*pi**(3/2)*c**
7) - I*b*meijerg(((-7/2, -13/4, -3, -11/4, -5/2, 1), ()), ((-13/4, -11/4),
(-7/2, -3, -3, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2)*c**7)
```

Giac [A] time = 1.23898, size = 205, normalized size = 1.64

$$-\frac{\left(30\,ac^{38}+33\,bc^{36}-\left(54\,ac^{38}+85\,bc^{36}-2\left(18\,ac^{38}+55\,bc^{36}-\left(6\,ac^{38}+45\,bc^{36}+4\left((cx+1)bc^{36}-5\,bc^{36}\right)(cx+1)\right)(cx -}{34603008\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/34603008*((30*a*c^38 + 33*b*c^36 - (54*a*c^38 + 85*b*c^36 - 2*(18*a*c^38
 + 55*b*c^36 - (6*a*c^38 + 45*b*c^36 + 4*((c*x + 1)*b*c^36 - 5*b*c^36)*(c*x
 + 1))*(c*x + 1))*(c*x + 1))*(c*x + 1))*sqrt(c*x + 1)*sqrt(c*x - 1) + 6*(6*
a*c^38 + 5*b*c^36)*log(abs(-sqrt(c*x + 1) + sqrt(c*x - 1))))/c
```

3.349 $\int \frac{x^3(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

Optimal. Leaf size=103

$$\frac{x^2\sqrt{cx-1}\sqrt{cx+1}\left(5ac^2+4b\right)}{15c^4}+\frac{2\sqrt{cx-1}\sqrt{cx+1}\left(5ac^2+4b\right)}{15c^6}+\frac{bx^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}$$

```
[Out] (2*(4*b + 5*a*c^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(15*c^6) + ((4*b + 5*a*c^2
)*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(15*c^4) + (b*x^4*Sqrt[-1 + c*x]*Sqrt[1
 + c*x])/(5*c^2)
```

Rubi [A] time = 0.0749051, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {460, 100, 12, 74}

$$\frac{x^2\sqrt{cx-1}\sqrt{cx+1}\left(5ac^2+4b\right)}{15c^4}+\frac{2\sqrt{cx-1}\sqrt{cx+1}\left(5ac^2+4b\right)}{15c^6}+\frac{bx^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (2*(4*b + 5*a*c^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(15*c^6) + ((4*b + 5*a*c^2
)*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(15*c^4) + (b*x^4*Sqrt[-1 + c*x]*Sqrt[1
 + c*x])/(5*c^2)
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
```

```
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\left(a+bx^2\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx &= \frac{bx^4\sqrt{-1+cx}\sqrt{1+cx}}{5c^2}-\frac{1}{5}\left(-5a-\frac{4b}{c^2}\right)\int \frac{x^3}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx\\ &= \frac{\left(4b+5ac^2\right)x^2\sqrt{-1+cx}\sqrt{1+cx}}{15c^4}+\frac{bx^4\sqrt{-1+cx}\sqrt{1+cx}}{5c^2}+\frac{\left(4b+5ac^2\right)\int \frac{2x}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{15c^4}\\ &= \frac{\left(4b+5ac^2\right)x^2\sqrt{-1+cx}\sqrt{1+cx}}{15c^4}+\frac{bx^4\sqrt{-1+cx}\sqrt{1+cx}}{5c^2}+\frac{\left(2\left(4b+5ac^2\right)\right)\int \frac{x}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{15c^4}\\ &= \frac{2\left(4b+5ac^2\right)\sqrt{-1+cx}\sqrt{1+cx}}{15c^6}+\frac{\left(4b+5ac^2\right)x^2\sqrt{-1+cx}\sqrt{1+cx}}{15c^4}+\frac{bx^4\sqrt{-1+cx}\sqrt{1+cx}}{5c^2}\end{aligned}$$

Mathematica [A] time = 0.0415032, size = 70, normalized size = 0.68

$$\frac{\left(c^2x^2-1\right)\left(5ac^2\left(c^2x^2+2\right)+b\left(3c^4x^4+4c^2x^2+8\right)\right)}{15c^6\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] ((-1 + c^2*x^2)*(5*a*c^2*(2 + c^2*x^2) + b*(8 + 4*c^2*x^2 + 3*c^4*x^4)))/(1
5*c^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Maple [A] time = 0.003, size = 57, normalized size = 0.6

$$\frac{3\,bx^4c^4+5\,ac^4x^2+4\,bc^2x^2+10\,ac^2+8\,b}{15\,c^6}\sqrt{cx-1}\sqrt{cx+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)
```

```
[Out] 1/15*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(3*b*c^4*x^4+5*a*c^4*x^2+4*b*c^2*x^2+10*a*
c^2+8*b)/c^6
```

Maxima [A] time = 0.946007, size = 128, normalized size = 1.24

$$\frac{\sqrt{c^2x^2-1}bx^4}{5\,c^2}+\frac{\sqrt{c^2x^2-1}ax^2}{3\,c^2}+\frac{4\,\sqrt{c^2x^2-1}bx^2}{15\,c^4}+\frac{2\,\sqrt{c^2x^2-1}a}{3\,c^4}+\frac{8\,\sqrt{c^2x^2-1}b}{15\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(c^2*x^2 - 1)*b*x^4/c^2 + 1/3*sqrt(c^2*x^2 - 1)*a*x^2/c^2 + 4/15*sq
rt(c^2*x^2 - 1)*b*x^2/c^4 + 2/3*sqrt(c^2*x^2 - 1)*a/c^4 + 8/15*sqrt(c^2*x^2
```

```
- 1)*b/c^6
```

Fricas [A] time = 1.54227, size = 128, normalized size = 1.24

$$\frac{\left(3\,bc^4x^4+10\,ac^2+\left(5\,ac^4+4\,bc^2\right)x^2+8\,b\right)\sqrt{cx+1}\sqrt{cx-1}}{15\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*b*c^4*x^4 + 10*a*c^2 + (5*a*c^4 + 4*b*c^2)*x^2 + 8*b)*sqrt(c*x + 1)
*sqrt(c*x - 1)/c^6
```

Sympy [C] time = 45.7106, size = 216, normalized size = 2.1

$$\frac{aG_{6,6}^{6,2}\left(\begin{matrix}-\frac{5}{4},-\frac{3}{4} & -1,-1,-\frac{1}{2},1\\ -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},0 & \end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^4}+\frac{iaG_{6,6}^{2,6}\left(\begin{matrix}-2,-\frac{7}{4},-\frac{3}{2},-\frac{5}{4},-1,1 & \\ -\frac{7}{4},-\frac{5}{4} & -2,-\frac{3}{2},-\frac{3}{2},0\end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^4}+\frac{bG_{6,6}^{6,2}\left(-\frac{5}{2},-\frac{9}{4}\ldots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] a*meijerg(((-5/4, -3/4), (-1, -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -1/2,
0), ()), 1/(c**2*x**2))/(4*pi**(3/2)*c**4) + I*a*meijerg(((-2, -7/4, -3/2,
-5/4, -1, 1), ()), ((-7/4, -5/4), (-2, -3/2, -3/2, 0)), exp_polar(2*I*pi)/(
c**2*x**2))/(4*pi**(3/2)*c**4) + b*meijerg(((-9/4, -7/4), (-2, -2, -3/2, 1)
), ((-5/2, -9/4, -2, -7/4, -3/2, 0), ()), 1/(c**2*x**2))/(4*pi**(3/2)*c**6)
 + I*b*meijerg(((-3, -11/4, -5/2, -9/4, -2, 1), ()), ((-11/4, -9/4), (-3, -
5/2, -5/2, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2)*c**6)
```

Giac [A] time = 1.18833, size = 130, normalized size = 1.26

$$\frac{\left(15\,ac^{27}+15\,bc^{25}-\left(10\,ac^{27}+20\,bc^{25}-\left(5\,ac^{27}+22\,bc^{25}+3\left((cx+1)bc^{25}-4\,bc^{25}\right)(cx+1)\right)(cx+1)\right)(cx+1)\right)\sqrt{cx+\ldots}}{276480\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/276480*(15*a*c^27 + 15*b*c^25 - (10*a*c^27 + 20*b*c^25 - (5*a*c^27 + 22*b
*c^25 + 3*((c*x + 1)*b*c^25 - 4*b*c^25)*(c*x + 1))*(c*x + 1))*(c*x + 1))*sq
rt(c*x + 1)*sqrt(c*x - 1)/c
```

3.350 $\int \frac{x^2(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

Optimal. Leaf size=87

$$\frac{x\sqrt{cx-1}\sqrt{cx+1}\left(4ac^2+3b\right)}{8c^4}+\frac{\left(4ac^2+3b\right)\cosh^{-1}(cx)}{8c^5}+\frac{bx^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}$$

```
[Out] ((3*b + 4*a*c^2)*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(8*c^4) + (b*x^3*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x])/(4*c^2) + ((3*b + 4*a*c^2)*ArcCosh[c*x])/(8*c^5)
```

Rubi [A] time = 0.0694708, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {460, 90, 52}

$$\frac{x\sqrt{cx-1}\sqrt{cx+1}\left(4ac^2+3b\right)}{8c^4}+\frac{\left(4ac^2+3b\right)\cosh^{-1}(cx)}{8c^5}+\frac{bx^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] ((3*b + 4*a*c^2)*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(8*c^4) + (b*x^3*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x])/(4*c^2) + ((3*b + 4*a*c^2)*ArcCosh[c*x])/(8*c^5)
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{x^2(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{bx^3\sqrt{-1+cx}\sqrt{1+cx}}{4c^2} - \frac{1}{4}\left(-4a-\frac{3b}{c^2}\right)\int \frac{x^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx$$
$$= \frac{(3b+4ac^2)\,x\sqrt{-1+cx}\sqrt{1+cx}}{8c^4} + \frac{bx^3\sqrt{-1+cx}\sqrt{1+cx}}{4c^2} + \frac{(3b+4ac^2)\int \frac{1}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{8c^4}$$
$$= \frac{(3b+4ac^2)\,x\sqrt{-1+cx}\sqrt{1+cx}}{8c^4} + \frac{bx^3\sqrt{-1+cx}\sqrt{1+cx}}{4c^2} + \frac{(3b+4ac^2)\cosh^{-1}(cx)}{8c^5}$$

Mathematica [A] time = 0.0664952, size = 98, normalized size = 1.13

$$\frac{cx(c^2x^2-1)\left(4ac^2+b(2c^2x^2+3)\right)+\sqrt{c^2x^2-1}\,(4ac^2+3b)\tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{8c^5\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (c*x*(-1 + c^2*x^2)*(4*a*c^2 + b*(3 + 2*c^2*x^2)) + (3*b + 4*a*c^2)*Sqrt[-1
 + c^2*x^2]*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]])/(8*c^5*Sqrt[-1 + c*x]*Sqrt[1
 + c*x])
```

Maple [C] time = 0.018, size = 147, normalized size = 1.7

$$\frac{\mathrm{csgn}(c)}{8c^5}\sqrt{cx-1}\sqrt{cx+1}\Big(2\sqrt{c^2x^2-1}\mathrm{csgn}(c)c^3x^3b+4\sqrt{c^2x^2-1}\mathrm{csgn}(c)c^3xa+3\sqrt{c^2x^2-1}\mathrm{csgn}(c)cxb+4\ln\Big(\Big(\sqrt{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)
```

```
[Out] 1/8*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(2*(c^2*x^2-1)^(1/2)*csgn(c)*c^3*x^3*b+4*(c
^2*x^2-1)^(1/2)*csgn(c)*c^3*x*a+3*(c^2*x^2-1)^(1/2)*csgn(c)*c*x*b+4*ln(((c^
2*x^2-1)^(1/2)*csgn(c)+c*x)*csgn(c))*a*c^2+3*ln(((c^2*x^2-1)^(1/2)*csgn(c)+
c*x)*csgn(c))*b)*csgn(c)/c^5/(c^2*x^2-1)^(1/2)
```

Maxima [A] time = 0.966629, size = 177, normalized size = 2.03

$$\frac{\sqrt{c^2x^2-1}bx^3}{4c^2}+\frac{\sqrt{c^2x^2-1}ax}{2c^2}+\frac{a\log\left(2c^2x+2\sqrt{c^2x^2-1}\sqrt{c^2}\right)}{2\sqrt{c^2}c^2}+\frac{3\sqrt{c^2x^2-1}bx}{8c^4}+\frac{3b\log\left(2c^2x+2\sqrt{c^2x^2-1}\sqrt{c^2}\right)}{8\sqrt{c^2}c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(c^2*x^2 - 1)*b*x^3/c^2 + 1/2*sqrt(c^2*x^2 - 1)*a*x/c^2 + 1/2*a*log
(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*sqrt(c^2))/(sqrt(c^2)*c^2) + 3/8*sqrt(c^2*x^
2 - 1)*b*x/c^4 + 3/8*b*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*sqrt(c^2))/(sqrt(c
^2)*c^4)
```

Fricas [A] time = 1.5332, size = 180, normalized size = 2.07

$$\frac{\left(2\,bc^{3}x^{3}+\left(4\,ac^{3}+3\,bc\right)x\right)\sqrt{cx+1}\sqrt{cx-1}-\left(4\,ac^{2}+3\,b\right)\log\left(-cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{8\,c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*((2*b*c^3*x^3 + (4*a*c^3 + 3*b*c)*x)*sqrt(c*x + 1)*sqrt(c*x - 1) - (4*a
*c^2 + 3*b)*log(-c*x + sqrt(c*x + 1)*sqrt(c*x - 1)))/c^5
```

Sympy [C] time = 40.417, size = 212, normalized size = 2.44

$$\frac{aG_{6,6}^{6,2}\left(\begin{matrix}-\frac{3}{4},-\frac{1}{4} & -\frac{1}{2},-\frac{1}{2},0,1\\ -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,0 & \end{matrix}\middle|\frac{1}{c^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}c^{3}}-\frac{iaG_{6,6}^{2,6}\left(\begin{matrix}-\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},1 & \\ -\frac{5}{4},-\frac{3}{4} & -\frac{3}{2},-1,-1,0\end{matrix}\middle|\frac{e^{2i\pi}}{c^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}c^{3}}+\frac{bG_{6,6}^{6,2}\left(\begin{matrix}-\frac{7}{4},-\frac{5}{4} & \\ -2,-\frac{7}{4},-\frac{3}{2},- & \end{matrix}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] a*meijerg(((-3/4, -1/4), (-1/2, -1/2, 0, 1)), ((-1, -3/4, -1/2, -1/4, 0, 0)
, ()), 1/(c**2*x**2))/(4*pi**(3/2)*c**3) - I*a*meijerg(((-3/2, -5/4, -1, -3
/4, -1/2, 1), ()), ((-5/4, -3/4), (-3/2, -1, -1, 0)), exp_polar(2*I*pi)/(c*
*2*x**2))/(4*pi**(3/2)*c**3) + b*meijerg(((-7/4, -5/4), (-3/2, -3/2, -1, 1)
), ((-2, -7/4, -3/2, -5/4, -1, 0), ()), 1/(c**2*x**2))/(4*pi**(3/2)*c**5) -
 I*b*meijerg(((-5/2, -9/4, -2, -7/4, -3/2, 1), ()), ((-9/4, -7/4), (-5/2, -
2, -2, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2)*c**5)
```

Giac [A] time = 1.22821, size = 151, normalized size = 1.74

$$-\frac{\left(4\,ac^{18}+5\,bc^{16}-\left(4\,ac^{18}+9\,bc^{16}+2\left((cx+1)bc^{16}-3\,bc^{16}\right)(cx+1)\right)(cx+1)\right)\sqrt{cx+1}\sqrt{cx-1}+2\left(4\,ac^{18}+3\,bc^{16}\right)\log\left(\right.}{114688\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/114688*((4*a*c^18 + 5*b*c^16 - (4*a*c^18 + 9*b*c^16 + 2*((c*x + 1)*b*c^1
6 - 3*b*c^16)*(c*x + 1))*(c*x + 1))*sqrt(c*x + 1)*sqrt(c*x - 1) + 2*(4*a*c^
18 + 3*b*c^16)*log(abs(-sqrt(c*x + 1) + sqrt(c*x - 1))))/c
```

3.351 $\int \frac{x(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

Optimal. Leaf size=65

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(3ac^2+2b\right)}{3c^4}+\frac{bx^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}$$

```
[Out] ((2*b + 3*a*c^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^4) + (b*x^2*Sqrt[-1 + c
*x]*Sqrt[1 + c*x])/(3*c^2)
```

Rubi [A] time = 0.0402388, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {460, 74}

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(3ac^2+2b\right)}{3c^4}+\frac{bx^2\sqrt{cx-1}\sqrt{cx+1}}{3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] ((2*b + 3*a*c^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^4) + (b*x^2*Sqrt[-1 + c
*x]*Sqrt[1 + c*x])/(3*c^2)
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(a+bx^2\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx &= \frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}}{3c^2}-\frac{1}{3}\left(-3a-\frac{2b}{c^2}\right)\int \frac{x}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx\\ &= \frac{\left(2b+3ac^2\right)\sqrt{-1+cx}\sqrt{1+cx}}{3c^4}+\frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}}{3c^2}\end{aligned}$$

Mathematica [A] time = 0.0273294, size = 52, normalized size = 0.8

$$\frac{\left(c^2x^2-1\right)\left(3ac^2+b\left(c^2x^2+2\right)\right)}{3c^4\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] ((-1 + c^2*x^2)*(3*a*c^2 + b*(2 + c^2*x^2)))/(3*c^4*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x])
```

Maple [A] time = 0.004, size = 38, normalized size = 0.6

$$\frac{b x^2 c^2 + 3 a c^2 + 2 b}{3 c^4} \sqrt{c x - 1} \sqrt{c x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)
```

```
[Out] 1/3*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(b*c^2*x^2+3*a*c^2+2*b)/c^4
```

Maxima [A] time = 0.986657, size = 73, normalized size = 1.12

$$\frac{\sqrt{c^2 x^2 - 1} b x^2}{3 c^2} + \frac{\sqrt{c^2 x^2 - 1} a}{c^2} + \frac{2 \sqrt{c^2 x^2 - 1} b}{3 c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(c^2*x^2 - 1)*b*x^2/c^2 + sqrt(c^2*x^2 - 1)*a/c^2 + 2/3*sqrt(c^2*x^
2 - 1)*b/c^4
```

Fricas [A] time = 1.47574, size = 88, normalized size = 1.35

$$\frac{\left(b c^2 x^2 + 3 a c^2 + 2 b\right) \sqrt{c x + 1} \sqrt{c x - 1}}{3 c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b*c^2*x^2 + 3*a*c^2 + 2*b)*sqrt(c*x + 1)*sqrt(c*x - 1)/c^4
```

Sympy [C] time = 27.9257, size = 202, normalized size = 3.11

$$\frac{a G_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4}, \frac{1}{4} & 0, 0, \frac{1}{2}, 1 \\ -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 0 & \end{matrix} \middle| \frac{1}{c^2 x^2}\right)}{4 \pi^{\frac{3}{2}} c^2} + \frac{i a G_{6,6}^{2,6}\left(\begin{matrix} -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 1 & \\ -\frac{3}{4}, -\frac{1}{4} & -1, -\frac{1}{2}, -\frac{1}{2}, 0 \end{matrix} \middle| \frac{e^{2 i \pi}}{c^2 x^2}\right)}{4 \pi^{\frac{3}{2}} c^2} + \frac{b G_{6,6}^{6,2}\left(\begin{matrix} -\frac{5}{4}, -\frac{3}{4} & \\ -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, -\frac{1}{2}, 0 & \end{matrix} \right.}{4 \pi^{\frac{3}{2}} c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] a*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0), ())
, 1/(c**2*x**2))/(4*pi**(3/2)*c**2) + I*a*meijerg(((-1, -3/4, -1/2, -1/4, 0
, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(2*I*pi)/(c**2*x**
2))/(4*pi**(3/2)*c**2) + b*meijerg(((-5/4, -3/4), (-1, -1, -1/2, 1)), ((-3/
2, -5/4, -1, -3/4, -1/2, 0), ()), 1/(c**2*x**2))/(4*pi**(3/2)*c**4) + I*b*m
eijerg(((-2, -7/4, -3/2, -5/4, -1, 1), ()), ((-7/4, -5/4), (-2, -3/2, -3/2,
 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2)*c**4)
```

Giac [A] time = 1.1637, size = 74, normalized size = 1.14

$$\frac{\left(3\,ac^{11}+3\,bc^{9}+\left((cx+1)bc^{9}-2\,bc^{9}\right)(cx+1)\right)\sqrt{cx+1}\sqrt{cx-1}}{1920\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1920*(3*a*c^11 + 3*b*c^9 + ((c*x + 1)*b*c^9 - 2*b*c^9)*(c*x + 1))*sqrt(c*
x + 1)*sqrt(c*x - 1)/c
```

3.352 $\int \frac{a+bx^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

Optimal. Leaf size=47

$$\frac{(2ac^2+b)\cosh^{-1}(cx)}{2c^3}+\frac{bx\sqrt{cx-1}\sqrt{cx+1}}{2c^2}$$

```
[Out] (b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*c^2) + ((b + 2*a*c^2)*ArcCosh[c*x])/(
2*c^3)
```

Rubi [A] time = 0.0195731, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {389, 52}

$$\frac{(2ac^2+b)\cosh^{-1}(cx)}{2c^3}+\frac{bx\sqrt{cx-1}\sqrt{cx+1}}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*c^2) + ((b + 2*a*c^2)*ArcCosh[c*x])/(
2*c^3)
```

Rule 389

```
Int[((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_
.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*x*(a1 + b1*x^(n/2))^(p +
1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*(n*(p + 1) + 1)), x] - Dist[(a1*a2*d -
 b1*b2*c*(n*(p + 1) + 1))/(b1*b2*(n*(p + 1) + 1)), Int[(a1 + b1*x^(n/2))^p*
(a2 + b2*x^(n/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p}, x] && Eq
Q[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx &= \frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{2c^2}-\frac{(-b-2ac^2)\int \frac{1}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{2c^2}\\ &= \frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{2c^2}+\frac{(b+2ac^2)\cosh^{-1}(cx)}{2c^3}\end{aligned}$$

Mathematica [B] time = 0.152255, size = 101, normalized size = 2.15

$$\frac{4(ac^2+b)\tanh^{-1}\left(\sqrt{\frac{cx-1}{cx+1}}\right)+\frac{b\left(cx\sqrt{-(cx-1)^2}\sqrt{cx+1}-2\sqrt{cx-1}\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\right)}{\sqrt{1-cx}}}{2c^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^2)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] ((b*(c*x*Sqrt[-(-1 + c*x)^2]*Sqrt[1 + c*x] - 2*Sqrt[-1 + c*x]*ArcSin[Sqrt[1
 - c*x]/Sqrt[2]]))/Sqrt[1 - c*x] + 4*(b + a*c^2)*ArcTanh[Sqrt[(-1 + c*x)/(1
 + c*x)]])/(2*c^3)
```

Maple [C] time = 0.013, size = 103, normalized size = 2.2

$$\frac{\operatorname{csgn}(c)}{2c^3}\sqrt{cx-1}\sqrt{cx+1}\left(\sqrt{c^2x^2-1}\operatorname{csgn}(c)\,cxb+2\ln\left(\left(\sqrt{c^2x^2-1}\operatorname{csgn}(c)+cx\right)\operatorname{csgn}(c)\right)ac^2+\ln\left(\left(\sqrt{c^2x^2-1}\operatorname{csgn}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)
```

```
[Out] 1/2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*((c^2*x^2-1)^(1/2)*csgn(c)*c*x*b+2*ln(((c^2
*x^2-1)^(1/2)*csgn(c)+c*x)*csgn(c))*a*c^2+ln(((c^2*x^2-1)^(1/2)*csgn(c)+c*x
)*csgn(c))*b)*csgn(c)/c^3/(c^2*x^2-1)^(1/2)
```

Maxima [B] time = 0.977853, size = 120, normalized size = 2.55

$$\frac{a\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}\sqrt{c^2}\right)}{\sqrt{c^2}}+\frac{\sqrt{c^2x^2-1}bx}{2\,c^2}+\frac{b\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}\sqrt{c^2}\right)}{2\,\sqrt{c^2}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] a*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*sqrt(c^2))/sqrt(c^2) + 1/2*sqrt(c^2*x^2
 - 1)*b*x/c^2 + 1/2*b*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*sqrt(c^2))/(sqrt(c^
2)*c^2)
```

Fricas [A] time = 1.45222, size = 136, normalized size = 2.89

$$\frac{\sqrt{cx+1}\sqrt{cx-1}bcx-\left(2\,ac^2+b\right)\log\left(-cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{2\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(c*x + 1)*sqrt(c*x - 1)*b*c*x - (2*a*c^2 + b)*log(-c*x + sqrt(c*x
+ 1)*sqrt(c*x - 1)))/c^3
```

Sympy [C] time = 14.6635, size = 182, normalized size = 3.87

$$\frac{aG_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1 \\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix} \middle| \frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c} - \frac{iaG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0 \end{matrix} \middle| \frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c} + \frac{bG_{6,6}^{6,2}\left(\begin{matrix} -\frac{3}{4},-\frac{1}{4} & -\frac{1}{2},-\frac{1}{2},(\\ -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,0 & \end{matrix}\right.}{4\pi^{\frac{3}{2}}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] a*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), 1
/(c**2*x**2))/(4*pi**(3/2)*c) - I*a*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1),
()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**
(3/2)*c) + b*meijerg(((-3/4, -1/4), (-1/2, -1/2, 0, 1)), ((-1, -3/4, -1/2,
-1/4, 0, 0), ()), 1/(c**2*x**2))/(4*pi**(3/2)*c**3) - I*b*meijerg(((-3/2, -
5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-3/2, -1, -1, 0)), exp_polar(
2*I*pi)/(c**2*x**2))/(4*pi**(3/2)*c**3)
```

Giac [A] time = 1.22789, size = 96, normalized size = 2.04

$$\frac{\left((cx+1)bc^4 - bc^4\right)\sqrt{cx+1}\sqrt{cx-1} - 2\left(2\,ac^6 + bc^4\right)\log\left(\left|-\sqrt{cx+1} + \sqrt{cx-1}\right|\right)}{384\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/384*(((c*x + 1)*b*c^4 - b*c^4)*sqrt(c*x + 1)*sqrt(c*x - 1) - 2*(2*a*c^6 +
 b*c^4)*log(abs(-sqrt(c*x + 1) + sqrt(c*x - 1))))/c
```

3.353 $\int \frac{a+bx^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

Optimal. Leaf size=46

$$a\tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{b\sqrt{cx-1}\sqrt{cx+1}}{c^2}$$

```
[Out] (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c^2 + a*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x
]]
```

Rubi [A] time = 0.0612882, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {460, 92, 205}

$$a\tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{b\sqrt{cx-1}\sqrt{cx+1}}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c^2 + a*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x
]]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx &= \frac{b\sqrt{-1+cx}\sqrt{1+cx}}{c^2}+a\int \frac{1}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx\\
&= \frac{b\sqrt{-1+cx}\sqrt{1+cx}}{c^2}+(ac)\,\text{Subst}\left(\int \frac{1}{c+cx^2}\,dx,x,\sqrt{-1+cx}\sqrt{1+cx}\right)\\
&= \frac{b\sqrt{-1+cx}\sqrt{1+cx}}{c^2}+a\tan^{-1}\left(\sqrt{-1+cx}\sqrt{1+cx}\right)
\end{aligned}$$

Mathematica [A] time = 0.0245972, size = 66, normalized size = 1.43

$$\frac{ac^2\sqrt{c^2x^2-1}\tan^{-1}\left(\sqrt{c^2x^2-1}\right)+b\left(c^2x^2-1\right)}{c^2\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (b*(-1 + c^2*x^2) + a*c^2*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]])/(c
^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Maple [A] time = 0.018, size = 62, normalized size = 1.4

$$\frac{1}{c^2}\left(-\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)ac^2+b\sqrt{c^2x^2-1}\right)\sqrt{cx-1}\sqrt{cx+1}\frac{1}{\sqrt{c^2x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)
```

```
[Out] (-arctan(1/(c^2*x^2-1)^(1/2))*a*c^2+b*(c^2*x^2-1)^(1/2))*(c*x-1)^(1/2)*(c*x
+1)^(1/2)/(c^2*x^2-1)^(1/2)/c^2
```

Maxima [A] time = 1.45888, size = 42, normalized size = 0.91

$$-a\arcsin\left(\frac{1}{\sqrt{c^2}|x|}\right)+\frac{\sqrt{c^2x^2-1}b}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -a*arcsin(1/(sqrt(c^2)*abs(x))) + sqrt(c^2*x^2 - 1)*b/c^2
```

Fricas [A] time = 1.49658, size = 122, normalized size = 2.65

$$\frac{2\,ac^2\arctan\left(-cx+\sqrt{cx+1}\sqrt{cx-1}\right)+\sqrt{cx+1}\sqrt{cx-1}b}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (2*a*c^2*arctan(-c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + sqrt(c*x + 1)*sqrt(c*
x - 1)*b)/c^2
```

Sympy [C] time = 18.0023, size = 162, normalized size = 3.52

$$-\frac{aG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2} \\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0 \end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{iaG_{6,6}^{2,6}\left(\begin{matrix} 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0 \end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{bG_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1 \\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c^2} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] -a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)), 1
/(c**2*x**2))/(4*pi**(3/2)) + I*a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()), (
(1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2))
 + b*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0),
()), 1/(c**2*x**2))/(4*pi**(3/2)*c**2) + I*b*meijerg(((-1, -3/4, -1/2, -1/4
, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(2*I*pi)/(c**2*
x**2))/(4*pi**(3/2)*c**2)
```

Giac [A] time = 1.13846, size = 61, normalized size = 1.33

$$-2\,a\arctan\left(\frac{1}{2}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^2\right)+\frac{\sqrt{cx+1}\sqrt{cx-1}b}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -2*a*arctan(1/2*(sqrt(c*x + 1) - sqrt(c*x - 1))^2) + sqrt(c*x + 1)*sqrt(c*x
 - 1)*b/c^2
```

3.354 $\int \frac{a+bx^2}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

Optimal. Leaf size=33

$$\frac{a\sqrt{cx-1}\sqrt{cx+1}}{x}+\frac{b\cosh^{-1}(cx)}{c}$$

```
[Out] (a*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/x + (b*ArcCosh[c*x])/c
```

Rubi [A] time = 0.0565493, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {454, 52}

$$\frac{a\sqrt{cx-1}\sqrt{cx+1}}{x}+\frac{b\cosh^{-1}(cx)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (a*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/x + (b*ArcCosh[c*x])/c
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,dx &= \frac{a\sqrt{-1+cx}\sqrt{1+cx}}{x}+b\int \frac{1}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx\\ &= \frac{a\sqrt{-1+cx}\sqrt{1+cx}}{x}+\frac{b\cosh^{-1}(cx)}{c}\end{aligned}$$

Mathematica [B] time = 0.0228771, size = 73, normalized size = 2.21

$$\frac{\sqrt{c^2x^2-1}\left(\frac{a\sqrt{c^2x^2-1}}{x}+\frac{b\tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{c}\right)}{\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (Sqrt[-1 + c^2*x^2]*((a*Sqrt[-1 + c^2*x^2])/x + (b*ArcTanh[(c*x)/Sqrt[-1 +
c^2*x^2]])/c))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Maple [C] time = 0.018, size = 77, normalized size = 2.3

$$\frac{\operatorname{csgn}(c)}{cx}\sqrt{cx-1}\sqrt{cx+1}\left(\operatorname{csgn}(c)\,c\sqrt{c^2x^2-1}a+\ln\left(\left(\sqrt{c^2x^2-1}\operatorname{csgn}(c)+cx\right)\operatorname{csgn}(c)\right)xb\right)\frac{1}{\sqrt{c^2x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)
```

```
[Out] (c*x-1)^(1/2)*(c*x+1)^(1/2)*(csgn(c)*c*(c^2*x^2-1)^(1/2)*a+ln(((c^2*x^2-1)^
(1/2)*csgn(c)+c*x)*csgn(c))*x*b)*csgn(c)/(c^2*x^2-1)^(1/2)/c/x
```

Maxima [A] time = 1.47724, size = 68, normalized size = 2.06

$$\frac{b\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}\sqrt{c^2}\right)}{\sqrt{c^2}}+\frac{\sqrt{c^2x^2-1}a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] b*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*sqrt(c^2))/sqrt(c^2) + sqrt(c^2*x^2 - 1
)*a/x
```

Fricas [A] time = 1.56034, size = 131, normalized size = 3.97

$$\frac{ac^2x+\sqrt{cx+1}\sqrt{cx-1}ac-bx\log\left(-cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (a*c^2*x + sqrt(c*x + 1)*sqrt(c*x - 1)*a*c - b*x*log(-c*x + sqrt(c*x + 1)*s
qrt(c*x - 1)))/(c*x)
```

Sympy [C] time = 18.89, size = 148, normalized size = 4.48

$$-\frac{acG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{iacG_{6,6}^{2,6}\left(\begin{matrix}\frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\\frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0\end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{bG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}c}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**2/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] -a*c*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,)),
 1/(c**2*x**2))/(4*pi**(3/2)) - I*a*c*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1),
()), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3
/2)) + b*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0),
 ()), 1/(c**2*x**2))/(4*pi**(3/2)*c) - I*b*meijerg(((-1/2, -1/4, 0, 1/4, 1/
2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(2*I*pi)/(c**2*x**2))/
(4*pi**(3/2)*c)
```

Giac [A] time = 1.65691, size = 78, normalized size = 2.36

$$\frac{\frac{16\,ac^2}{\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^4+4} - b\log\left(\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^4\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(16*a*c^2/((sqrt(c*x + 1) - sqrt(c*x - 1))^4 + 4) - b*log((sqrt(c*x + 1
) - sqrt(c*x - 1))^4))/c
```

3.355 $\int \frac{a+bx^2}{x^3\sqrt{-1+cx}\sqrt{1+cx}} dx$

Optimal. Leaf size=60

$$\frac{1}{2}\left(ac^2+2b\right)\tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{a\sqrt{cx-1}\sqrt{cx+1}}{2x^2}$$

```
[Out] (a*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*x^2) + ((2*b + a*c^2)*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/2
```

Rubi [A] time = 0.0630479, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {454, 92, 205}

$$\frac{1}{2}\left(ac^2+2b\right)\tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{a\sqrt{cx-1}\sqrt{cx+1}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (a*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*x^2) + ((2*b + a*c^2)*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/2
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m + 1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a+bx^2}{x^3\sqrt{-1+cx}\sqrt{1+cx}}\,dx = \frac{a\sqrt{-1+cx}\sqrt{1+cx}}{2x^2} + \frac{1}{2}\left(2b+ac^2\right)\int \frac{1}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx$$
$$= \frac{a\sqrt{-1+cx}\sqrt{1+cx}}{2x^2} + \frac{1}{2}\left(c\left(2b+ac^2\right)\right)\text{Subst}\left(\int \frac{1}{c+cx^2}\,dx, x, \sqrt{-1+cx}\sqrt{1+cx}\right)$$
$$= \frac{a\sqrt{-1+cx}\sqrt{1+cx}}{2x^2} + \frac{1}{2}\left(2b+ac^2\right)\tan^{-1}\left(\sqrt{-1+cx}\sqrt{1+cx}\right)$$

Mathematica [A] time = 0.0454952, size = 77, normalized size = 1.28

$$\frac{x^2\sqrt{c^2x^2-1}\left(ac^2+2b\right)\tan^{-1}\left(\sqrt{c^2x^2-1}\right)+a\left(c^2x^2-1\right)}{2x^2\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (a*(-1 + c^2*x^2) + (2*b + a*c^2)*x^2*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c
^2*x^2]])/(2*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Maple [A] time = 0.019, size = 84, normalized size = 1.4

$$-\frac{1}{2x^2}\sqrt{cx-1}\sqrt{cx+1}\left(\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)x^2ac^2+2\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)x^2b-\sqrt{c^2x^2-1}a\right)\frac{1}{\sqrt{c^2x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^3/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)
```

```
[Out] -1/2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(arctan(1/(c^2*x^2-1)^(1/2))*x^2*a*c^2+2*a
rctan(1/(c^2*x^2-1)^(1/2))*x^2*b-(c^2*x^2-1)^(1/2)*a)/(c^2*x^2-1)^(1/2)/x^2
```

Maxima [A] time = 1.42945, size = 66, normalized size = 1.1

$$-\frac{1}{2}ac^2\arcsin\left(\frac{1}{\sqrt{c^2}|x|}\right)-b\arcsin\left(\frac{1}{\sqrt{c^2}|x|}\right)+\frac{\sqrt{c^2x^2-1}a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^3/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*a*c^2*arcsin(1/(sqrt(c^2)*abs(x))) - b*arcsin(1/(sqrt(c^2)*abs(x))) +
1/2*sqrt(c^2*x^2 - 1)*a/x^2
```

Fricas [A] time = 1.5189, size = 143, normalized size = 2.38

$$\frac{2\left(ac^2+2b\right)x^2\arctan\left(-cx+\sqrt{cx+1}\sqrt{cx-1}\right)+\sqrt{cx+1}\sqrt{cx-1}a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^3/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a*c^2 + 2*b)*x^2*arctan(-c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + sqrt(
c*x + 1)*sqrt(c*x - 1)*a)/x^2
```

Sympy [C] time = 26.1148, size = 141, normalized size = 2.35

$$-\frac{ac^2G_{6,6}^{5,3}\left(\begin{matrix}\frac{7}{4},\frac{9}{4},1 & 2,2,\frac{5}{2}\\ \frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2} & 0\end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{iac^2G_{6,6}^{2,6}\left(\begin{matrix}1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2,1 & \\ \frac{5}{4},\frac{7}{4} & 1,\frac{3}{2},\frac{3}{2},0\end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{bG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2}\\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0\end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{ib\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**3/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] -a*c**2*meijerg(((7/4, 9/4, 1), (2, 2, 5/2)), ((3/2, 7/4, 2, 9/4, 5/2), (0,
)), 1/(c**2*x**2))/(4*pi**(3/2)) + I*a*c**2*meijerg(((1, 5/4, 3/2, 7/4, 2,
1), ()), ((5/4, 7/4), (1, 3/2, 3/2, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*
pi**(3/2)) - b*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/
2), (0,)), 1/(c**2*x**2))/(4*pi**(3/2)) + I*b*meijerg(((0, 1/4, 1/2, 3/4, 1
, 1), ()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(
4*pi**(3/2))
```

Giac [B] time = 1.1904, size = 154, normalized size = 2.57

$$-\frac{\left(ac^3+2\,bc\right)\arctan\left(\frac{1}{2}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^2\right)+\frac{2\left(ac^3\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^6-4\,ac^3\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^2\right)}{\left(\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^4+4\right)^2}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^3/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -((a*c^3 + 2*b*c)*arctan(1/2*(sqrt(c*x + 1) - sqrt(c*x - 1))^2) + 2*(a*c^3*
(sqrt(c*x + 1) - sqrt(c*x - 1))^6 - 4*a*c^3*(sqrt(c*x + 1) - sqrt(c*x - 1))
^2)/((sqrt(c*x + 1) - sqrt(c*x - 1))^4 + 4)^2)/c
```

3.356 $\int \frac{a+bx^2}{x^4\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

Optimal. Leaf size=62

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(2ac^2+3b\right)}{3x}+\frac{a\sqrt{cx-1}\sqrt{cx+1}}{3x^3}$$

```
[Out] (a*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*x^3) + ((3*b + 2*a*c^2)*Sqrt[-1 + c*x]*
Sqrt[1 + c*x])/(3*x)
```

Rubi [A] time = 0.0623655, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {454, 95}

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(2ac^2+3b\right)}{3x}+\frac{a\sqrt{cx-1}\sqrt{cx+1}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (a*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*x^3) + ((3*b + 2*a*c^2)*Sqrt[-1 + c*x]*
Sqrt[1 + c*x])/(3*x)
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x^4\sqrt{-1+cx}\sqrt{1+cx}}\,dx &= \frac{a\sqrt{-1+cx}\sqrt{1+cx}}{3x^3}+\frac{1}{3}\left(3b+2ac^2\right)\int \frac{1}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,dx\\ &= \frac{a\sqrt{-1+cx}\sqrt{1+cx}}{3x^3}+\frac{\left(3b+2ac^2\right)\sqrt{-1+cx}\sqrt{1+cx}}{3x}\end{aligned}$$

Mathematica [A] time = 0.0171105, size = 51, normalized size = 0.82

$$\frac{\left(c^2x^2-1\right)\left(2ac^2x^2+a+3bx^2\right)}{3x^3\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] ((-1 + c^2*x^2)*(a + 3*b*x^2 + 2*a*c^2*x^2))/(3*x^3*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x])
```

Maple [A] time = 0.004, size = 37, normalized size = 0.6

$$\frac{2\,ac^2x^2+3\,bx^2+a}{3\,x^3}\sqrt{cx-1}\sqrt{cx+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)
```

```
[Out] 1/3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*(2*a*c^2*x^2+3*b*x^2+a)/x^3
```

Maxima [A] time = 1.46325, size = 73, normalized size = 1.18

$$\frac{2\,\sqrt{c^2x^2-1}ac^2}{3\,x}+\frac{\sqrt{c^2x^2-1}b}{x}+\frac{\sqrt{c^2x^2-1}a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(c^2*x^2 - 1)*a*c^2/x + sqrt(c^2*x^2 - 1)*b/x + 1/3*sqrt(c^2*x^2 -
1)*a/x^3
```

Fricas [A] time = 1.5221, size = 120, normalized size = 1.94

$$\frac{\left(2\,ac^3+3\,bc\right)x^3+\left(\left(2\,ac^2+3\,b\right)x^2+a\right)\sqrt{cx+1}\sqrt{cx-1}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*((2*a*c^3 + 3*b*c)*x^3 + ((2*a*c^2 + 3*b)*x^2 + a)*sqrt(c*x + 1)*sqrt(c
*x - 1))/x^3
```

Sympy [C] time = 37.4068, size = 146, normalized size = 2.35

$$-\frac{ac^3G_{6,6}^{5,3}\left(\begin{matrix}\frac{9}{4},\frac{11}{4},1 & \frac{5}{2},\frac{5}{2},3\\ 2,\frac{9}{4},\frac{5}{2},\frac{11}{4},3 & 0\end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{iac^3G_{6,6}^{2,6}\left(\begin{matrix}\frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2},1 & \\ \frac{7}{4},\frac{9}{4} & \frac{3}{2},2,2,0\end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{bcG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**4/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] -a*c**3*meijerg(((9/4, 11/4, 1), (5/2, 5/2, 3)), ((2, 9/4, 5/2, 11/4, 3), (
0,)), 1/(c**2*x**2))/(4*pi**(3/2)) - I*a*c**3*meijerg(((3/2, 7/4, 2, 9/4, 5
/2, 1), ()), ((7/4, 9/4), (3/2, 2, 2, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(
4*pi**(3/2)) - b*c*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7
/4, 2), (0,)), 1/(c**2*x**2))/(4*pi**(3/2)) - I*b*c*meijerg(((1/2, 3/4, 1,
5/4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(2*I*pi)/(c**2*x*
*2))/(4*pi**(3/2))
```

Giac [B] time = 1.189, size = 157, normalized size = 2.53

$$\frac{8\left(3\,bc^2\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^8+24\,ac^4\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^4+24\,bc^2\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^4+32\,ac^4+48\,bc^2\right)}{3\left(\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^4+4\right)^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 8/3*(3*b*c^2*(sqrt(c*x + 1) - sqrt(c*x - 1))^8 + 24*a*c^4*(sqrt(c*x + 1) -
sqrt(c*x - 1))^4 + 24*b*c^2*(sqrt(c*x + 1) - sqrt(c*x - 1))^4 + 32*a*c^4 +
48*b*c^2)/(((sqrt(c*x + 1) - sqrt(c*x - 1))^4 + 4)^3*c)
```

3.357 $\int \frac{a+bx^2}{x^5\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

Optimal. Leaf size=99

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(3ac^2+4b\right)}{8x^2}+\frac{1}{8}c^2\left(3ac^2+4b\right)\tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{a\sqrt{cx-1}\sqrt{cx+1}}{4x^4}$$

```
[Out] (a*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*x^4) + ((4*b + 3*a*c^2)*Sqrt[-1 + c*x]*
Sqrt[1 + c*x])/(8*x^2) + (c^2*(4*b + 3*a*c^2)*ArcTan[Sqrt[-1 + c*x]*Sqrt[1
+ c*x]])/8
```

Rubi [A] time = 0.0780453, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {454, 103, 12, 92, 205}

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(3ac^2+4b\right)}{8x^2}+\frac{1}{8}c^2\left(3ac^2+4b\right)\tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{a\sqrt{cx-1}\sqrt{cx+1}}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (a*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*x^4) + ((4*b + 3*a*c^2)*Sqrt[-1 + c*x]*
Sqrt[1 + c*x])/(8*x^2) + (c^2*(4*b + 3*a*c^2)*ArcTan[Sqrt[-1 + c*x]*Sqrt[1
+ c*x]])/8
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
```

```
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x^5\sqrt{-1+cx}\sqrt{1+cx}}\,dx &= \frac{a\sqrt{-1+cx}\sqrt{1+cx}}{4x^4}+\frac{1}{4}\left(4b+3ac^2\right)\int \frac{1}{x^3\sqrt{-1+cx}\sqrt{1+cx}}\,dx\\ &= \frac{a\sqrt{-1+cx}\sqrt{1+cx}}{4x^4}+\frac{\left(4b+3ac^2\right)\sqrt{-1+cx}\sqrt{1+cx}}{8x^2}+\frac{1}{8}\left(4b+3ac^2\right)\int \frac{c^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\\ &= \frac{a\sqrt{-1+cx}\sqrt{1+cx}}{4x^4}+\frac{\left(4b+3ac^2\right)\sqrt{-1+cx}\sqrt{1+cx}}{8x^2}+\frac{1}{8}\left(c^2\left(4b+3ac^2\right)\right)\int \frac{1}{x\sqrt{-1+cx}\sqrt{1+}}\\ &= \frac{a\sqrt{-1+cx}\sqrt{1+cx}}{4x^4}+\frac{\left(4b+3ac^2\right)\sqrt{-1+cx}\sqrt{1+cx}}{8x^2}+\frac{1}{8}\left(c^3\left(4b+3ac^2\right)\right)\mathrm{Subst}\left(\int \frac{1}{c+cx^2}\right.\\ &= \frac{a\sqrt{-1+cx}\sqrt{1+cx}}{4x^4}+\frac{\left(4b+3ac^2\right)\sqrt{-1+cx}\sqrt{1+cx}}{8x^2}+\frac{1}{8}c^2\left(4b+3ac^2\right)\tan^{-1}\left(\sqrt{-1+cx}\sqrt{1}\right.\end{aligned}$$

Mathematica [A] time = 0.09147, size = 102, normalized size = 1.03

$$\frac{\left(c^2x^2-1\right)\left(a\left(3c^2x^2+2\right)+4bx^2\right)-c^2x^4\sqrt{1-c^2x^2}\left(3ac^2+4b\right)\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)}{8x^4\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] ((-1 + c^2*x^2)*(4*b*x^2 + a*(2 + 3*c^2*x^2)) - c^2*(4*b + 3*a*c^2)*x^4*Sqr
t[1 - c^2*x^2]*ArcTanh[Sqrt[1 - c^2*x^2]])/(8*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c
*x])
```

Maple [A] time = 0.02, size = 125, normalized size = 1.3

$$-\frac{1}{8\,x^4}\sqrt{cx-1}\sqrt{cx+1}\left(3\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)x^4ac^4+4\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)x^4bc^2-3\,\sqrt{c^2x^2-1}x^2ac^2-4\,\sqrt{c^2x^2-1}x^2b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^5/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)
```

```
[Out] -1/8*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(3*arctan(1/(c^2*x^2-1)^(1/2))*x^4*a*c^4+4
*arctan(1/(c^2*x^2-1)^(1/2))*x^4*b*c^2-3*(c^2*x^2-1)^(1/2)*x^2*a*c^2-4*(c^2
*x^2-1)^(1/2)*x^2*b-2*(c^2*x^2-1)^(1/2)*a)/(c^2*x^2-1)^(1/2)/x^4
```

Maxima [A] time = 1.41214, size = 120, normalized size = 1.21

$$-\frac{3}{8}ac^4\arcsin\left(\frac{1}{\sqrt{c^2}|x|}\right)-\frac{1}{2}bc^2\arcsin\left(\frac{1}{\sqrt{c^2}|x|}\right)+\frac{3\sqrt{c^2x^2-1}ac^2}{8x^2}+\frac{\sqrt{c^2x^2-1}b}{2x^2}+\frac{\sqrt{c^2x^2-1}a}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^5/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/8*a*c^4*arcsin(1/(sqrt(c^2)*abs(x))) - 1/2*b*c^2*arcsin(1/(sqrt(c^2)*abs
(x))) + 3/8*sqrt(c^2*x^2 - 1)*a*c^2/x^2 + 1/2*sqrt(c^2*x^2 - 1)*b/x^2 + 1/4
*sqrt(c^2*x^2 - 1)*a/x^4
```

Fricas [A] time = 1.52103, size = 186, normalized size = 1.88

$$\frac{2\left(3ac^4+4bc^2\right)x^4\arctan\left(-cx+\sqrt{cx+1}\sqrt{cx-1}\right)+\left(\left(3ac^2+4b\right)x^2+2a\right)\sqrt{cx+1}\sqrt{cx-1}}{8x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^5/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(2*(3*a*c^4 + 4*b*c^2)*x^4*arctan(-c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) +
 ((3*a*c^2 + 4*b)*x^2 + 2*a)*sqrt(c*x + 1)*sqrt(c*x - 1))/x^4
```

Sympy [C] time = 52.0269, size = 148, normalized size = 1.49

$$-\frac{ac^4G_{6,6}^{5,3}\left(\begin{matrix}\frac{11}{4},\frac{13}{4},1 & 3,3,\frac{7}{2}\\ \frac{5}{2},\frac{11}{4},3,\frac{13}{4},\frac{7}{2} & 0\end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{iac^4G_{6,6}^{2,6}\left(\begin{matrix}2,\frac{9}{4},\frac{5}{2},\frac{11}{4},3,1 & \\ \frac{9}{4},\frac{11}{4} & 2,\frac{5}{2},\frac{5}{2},0\end{matrix}\middle|\frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{bc^2G_{6,6}^{5,3}\left(\begin{matrix}\frac{7}{4},\frac{9}{4},1 & 2,2,\frac{5}{2}\\ \frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2} & 0\end{matrix}\middle|\frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**5/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] -a*c**4*meijerg(((11/4, 13/4, 1), (3, 3, 7/2)), ((5/2, 11/4, 3, 13/4, 7/2),
 (0,)), 1/(c**2*x**2))/(4*pi**(3/2)) + I*a*c**4*meijerg(((2, 9/4, 5/2, 11/4
, 3, 1), ()), ((9/4, 11/4), (2, 5/2, 5/2, 0)), exp_polar(2*I*pi)/(c**2*x**2
))/(4*pi**(3/2)) - b*c**2*meijerg(((7/4, 9/4, 1), (2, 2, 5/2)), ((3/2, 7/4,
 2, 9/4, 5/2), (0,)), 1/(c**2*x**2))/(4*pi**(3/2)) + I*b*c**2*meijerg(((1,
5/4, 3/2, 7/4, 2, 1), ()), ((5/4, 7/4), (1, 3/2, 3/2, 0)), exp_polar(2*I*pi
)/(c**2*x**2))/(4*pi**(3/2))
```

Giac [B] time = 1.16859, size = 362, normalized size = 3.66

$$-\frac{\left(3ac^5+4bc^3\right)\arctan\left(\frac{1}{2}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^2\right)+\frac{2\left(3ac^5\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^{14}+4bc^3\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^{14}+44ac^5\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^{10}+16bc^3\cdots\right.}{\cdots}}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^5/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*((3*a*c^5 + 4*b*c^3)*arctan(1/2*(sqrt(c*x + 1) - sqrt(c*x - 1))^2) + 2
*(3*a*c^5*(sqrt(c*x + 1) - sqrt(c*x - 1))^14 + 4*b*c^3*(sqrt(c*x + 1) - sqr
t(c*x - 1))^14 + 44*a*c^5*(sqrt(c*x + 1) - sqrt(c*x - 1))^10 + 16*b*c^3*(sq
rt(c*x + 1) - sqrt(c*x - 1))^10 - 176*a*c^5*(sqrt(c*x + 1) - sqrt(c*x - 1))
^6 - 64*b*c^3*(sqrt(c*x + 1) - sqrt(c*x - 1))^6 - 192*a*c^5*(sqrt(c*x + 1)
- sqrt(c*x - 1))^2 - 256*b*c^3*(sqrt(c*x + 1) - sqrt(c*x - 1))^2)/((sqrt(c*
x + 1) - sqrt(c*x - 1))^4 + 4)^4)/c
```

3.358 $\int \frac{x^4(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

Optimal. Leaf size=164

$$\frac{x^3\sqrt{dx-c}\sqrt{c+dx}\left(6ad^2+5bc^2\right)}{24d^4}+\frac{c^2x\sqrt{dx-c}\sqrt{c+dx}\left(6ad^2+5bc^2\right)}{16d^6}+\frac{c^4\left(6ad^2+5bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{8d^7}+\frac{bx^5\sqrt{dx}}{6}$$

```
[Out] (c^2*(5*b*c^2 + 6*a*d^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(16*d^6) + ((5*b*c
^2 + 6*a*d^2)*x^3*Sqrt[-c + d*x]*Sqrt[c + d*x])/(24*d^4) + (b*x^5*Sqrt[-c +
 d*x]*Sqrt[c + d*x])/(6*d^2) + (c^4*(5*b*c^2 + 6*a*d^2)*ArcTanh[Sqrt[-c + d
*x]/Sqrt[c + d*x]])/(8*d^7)
```

Rubi [A] time = 0.119734, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {460, 100, 12, 90, 63, 217, 206}

$$\frac{x^3\sqrt{dx-c}\sqrt{c+dx}\left(6ad^2+5bc^2\right)}{24d^4}+\frac{c^2x\sqrt{dx-c}\sqrt{c+dx}\left(6ad^2+5bc^2\right)}{16d^6}+\frac{c^4\left(6ad^2+5bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{8d^7}+\frac{bx^5\sqrt{dx}}{6}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (c^2*(5*b*c^2 + 6*a*d^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(16*d^6) + ((5*b*c
^2 + 6*a*d^2)*x^3*Sqrt[-c + d*x]*Sqrt[c + d*x])/(24*d^4) + (b*x^5*Sqrt[-c +
 d*x]*Sqrt[c + d*x])/(6*d^2) + (c^4*(5*b*c^2 + 6*a*d^2)*ArcTanh[Sqrt[-c + d
*x]/Sqrt[c + d*x]])/(8*d^7)
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^4\left(a+bx^2\right)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{bx^5\sqrt{-c+dx}\sqrt{c+dx}}{6d^2} - \frac{1}{6}\left(-6a-\frac{5bc^2}{d^2}\right)\int \frac{x^4}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx$$

$$= \frac{\left(5bc^2+6ad^2\right)x^3\sqrt{-c+dx}\sqrt{c+dx}}{24d^4} + \frac{bx^5\sqrt{-c+dx}\sqrt{c+dx}}{6d^2} + \frac{\left(5bc^2+6ad^2\right)\int \frac{3c^2x^2}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx}{24d^4}$$

$$= \frac{\left(5bc^2+6ad^2\right)x^3\sqrt{-c+dx}\sqrt{c+dx}}{24d^4} + \frac{bx^5\sqrt{-c+dx}\sqrt{c+dx}}{6d^2} + \frac{\left(c^2\left(5bc^2+6ad^2\right)\right)\int \frac{x^2}{\sqrt{-c+dx}\sqrt{c+}}}{8d^4}$$

$$= \frac{c^2\left(5bc^2+6ad^2\right)x\sqrt{-c+dx}\sqrt{c+dx}}{16d^6} + \frac{\left(5bc^2+6ad^2\right)x^3\sqrt{-c+dx}\sqrt{c+dx}}{24d^4} + \frac{bx^5\sqrt{-c+dx}\sqrt{c}}{6d^2}$$

$$= \frac{c^2\left(5bc^2+6ad^2\right)x\sqrt{-c+dx}\sqrt{c+dx}}{16d^6} + \frac{\left(5bc^2+6ad^2\right)x^3\sqrt{-c+dx}\sqrt{c+dx}}{24d^4} + \frac{bx^5\sqrt{-c+dx}\sqrt{c}}{6d^2}$$

$$= \frac{c^2\left(5bc^2+6ad^2\right)x\sqrt{-c+dx}\sqrt{c+dx}}{16d^6} + \frac{\left(5bc^2+6ad^2\right)x^3\sqrt{-c+dx}\sqrt{c+dx}}{24d^4} + \frac{bx^5\sqrt{-c+dx}\sqrt{c}}{6d^2}$$

$$= \frac{c^2\left(5bc^2+6ad^2\right)x\sqrt{-c+dx}\sqrt{c+dx}}{16d^6} + \frac{\left(5bc^2+6ad^2\right)x^3\sqrt{-c+dx}\sqrt{c+dx}}{24d^4} + \frac{bx^5\sqrt{-c+dx}\sqrt{c}}{6d^2}$$

$$= \frac{c^2\left(5bc^2+6ad^2\right)x\sqrt{-c+dx}\sqrt{c+dx}}{16d^6} + \frac{\left(5bc^2+6ad^2\right)x^3\sqrt{-c+dx}\sqrt{c+dx}}{24d^4} + \frac{bx^5\sqrt{-c+dx}\sqrt{c}}{6d^2}$$

Mathematica [A] time = 0.115963, size = 148, normalized size = 0.9

$$\frac{dx\left(d^2x^2-c^2\right)\left(6ad^2\left(3c^2+2d^2x^2\right)+b\left(10c^2d^2x^2+15c^4+8d^4x^4\right)\right)+3c^4\sqrt{d^2x^2-c^2}\left(6ad^2+5bc^2\right)\tanh^{-1}\left(\frac{dx}{\sqrt{d^2x^2-c^2}}\right)}{48d^7\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (d*x*(-c^2 + d^2*x^2)*(6*a*d^2*(3*c^2 + 2*d^2*x^2) + b*(15*c^4 + 10*c^2*d^2
*x^2 + 8*d^4*x^4)) + 3*c^4*(5*b*c^2 + 6*a*d^2)*Sqrt[-c^2 + d^2*x^2]*ArcTanh
[(d*x)/Sqrt[-c^2 + d^2*x^2]])/(48*d^7*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

Maple [C] time = 0.028, size = 240, normalized size = 1.5

$$\frac{\mathrm{csgn}(d)}{48\,d^7}\sqrt{dx-c}\sqrt{dx+c}\left(8\,\mathrm{csgn}(d)\,x^5bd^5\sqrt{d^2x^2-c^2}+12\,\mathrm{csgn}(d)\,x^3ad^5\sqrt{d^2x^2-c^2}+10\,\mathrm{csgn}(d)\,x^3bc^2d^3\sqrt{d^2x^2-c^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] 1/48*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(8*csgn(d)*x^5*b*d^5*(d^2*x^2-c^2)^(1/2)+1
2*csgn(d)*x^3*a*d^5*(d^2*x^2-c^2)^(1/2)+10*csgn(d)*x^3*b*c^2*d^3*(d^2*x^2-c
^2)^(1/2)+18*csgn(d)*d^3*(d^2*x^2-c^2)^(1/2)*x*a*c^2+15*csgn(d)*d*(d^2*x^2-
c^2)^(1/2)*x*b*c^4+18*ln(((d^2*x^2-c^2)^(1/2)*csgn(d)+d*x)*csgn(d))*a*c^4*d
^2+15*ln(((d^2*x^2-c^2)^(1/2)*csgn(d)+d*x)*csgn(d))*b*c^6)*csgn(d)/d^7/(d^2
*x^2-c^2)^(1/2)
```

Maxima [A] time = 0.960679, size = 289, normalized size = 1.76

$$\frac{\sqrt{d^2x^2-c^2}bx^5}{6\,d^2}+\frac{5\,\sqrt{d^2x^2-c^2}bc^2x^3}{24\,d^4}+\frac{\sqrt{d^2x^2-c^2}ax^3}{4\,d^2}+\frac{5\,bc^6\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\sqrt{d^2}\right)}{16\,\sqrt{d^2}d^6}+\frac{3\,ac^4\log\left(2\,d^2x+2\,\sqrt{\right.}}{8\,\sqrt{d^2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(d^2*x^2 - c^2)*b*x^5/d^2 + 5/24*sqrt(d^2*x^2 - c^2)*b*c^2*x^3/d^4
+ 1/4*sqrt(d^2*x^2 - c^2)*a*x^3/d^2 + 5/16*b*c^6*log(2*d^2*x + 2*sqrt(d^2*x
^2 - c^2)*sqrt(d^2))/(sqrt(d^2)*d^6) + 3/8*a*c^4*log(2*d^2*x + 2*sqrt(d^2*x
^2 - c^2)*sqrt(d^2))/(sqrt(d^2)*d^4) + 5/16*sqrt(d^2*x^2 - c^2)*b*c^4*x/d^6
 + 3/8*sqrt(d^2*x^2 - c^2)*a*c^2*x/d^4
```

Fricas [A] time = 1.58638, size = 251, normalized size = 1.53

$$\frac{\left(8\,bd^5x^5+2\left(5\,bc^2d^3+6\,ad^5\right)x^3+3\left(5\,bc^4d+6\,ac^2d^3\right)x\right)\sqrt{dx+c}\sqrt{dx-c}-3\left(5\,bc^6+6\,ac^4d^2\right)\log\left(-dx+\sqrt{dx+c}\sqrt{d\right.}}{48\,d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/48*((8*b*d^5*x^5 + 2*(5*b*c^2*d^3 + 6*a*d^5)*x^3 + 3*(5*b*c^4*d + 6*a*c^2
*d^3)*x)*sqrt(d*x + c)*sqrt(d*x - c) - 3*(5*b*c^6 + 6*a*c^4*d^2)*log(-d*x +
 sqrt(d*x + c)*sqrt(d*x - c)))/d^7
```

Sympy [C] time = 87.1952, size = 240, normalized size = 1.46

$$\frac{ac^4G_{6,6}^{6,2}\left(\begin{matrix} -\frac{7}{4},-\frac{5}{4} & -\frac{3}{2},-\frac{3}{2},-1,1 \\ -2,-\frac{7}{4},-\frac{3}{2},-\frac{5}{4},-1,0 & \end{matrix}\middle| \frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^5}-\frac{iac^4G_{6,6}^{2,6}\left(\begin{matrix} -\frac{5}{2},-\frac{9}{4},-2,-\frac{7}{4},-\frac{3}{2},1 & \\ -\frac{9}{4},-\frac{7}{4} & -\frac{5}{2},-2,-2,0\end{matrix}\middle| \frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^5}+\frac{bc^6G_{6,6}^{6,2}\left(-3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] a*c**4*meijerg(((-7/4, -5/4), (-3/2, -3/2, -1, 1)), ((-2, -7/4, -3/2, -5/4,
 -1, 0), ()), c**2/(d**2*x**2))/(4*pi**(3/2)*d**5) - I*a*c**4*meijerg(((-5/
2, -9/4, -2, -7/4, -3/2, 1), ()), ((-9/4, -7/4), (-5/2, -2, -2, 0)), c**2*e
xp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**5) + b*c**6*meijerg(((-11/4,
-9/4), (-5/2, -5/2, -2, 1)), ((-3, -11/4, -5/2, -9/4, -2, 0), ()), c**2/(d*
*2*x**2))/(4*pi**(3/2)*d**7) - I*b*c**6*meijerg(((-7/2, -13/4, -3, -11/4, -
5/2, 1), ()), ((-13/4, -11/4), (-7/2, -3, -3, 0)), c**2*exp_polar(2*I*pi)/(
d**2*x**2))/(4*pi**(3/2)*d**7)
```

Giac [A] time = 1.27013, size = 246, normalized size = 1.5

$$-\frac{\left(33\,bc^5d^{36}+30\,ac^3d^{38}-\left(85\,bc^4d^{36}+54\,ac^2d^{38}-2\left(55\,bc^3d^{36}+18\,acd^{38}-\left(45\,bc^2d^{36}+6\,ad^{38}+4\left((dx+c)bd^{36}-5\,bcd\right.\right.\right.\right.\right.}{34603008\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/34603008*((33*b*c^5*d^36 + 30*a*c^3*d^38 - (85*b*c^4*d^36 + 54*a*c^2*d^3
8 - 2*(55*b*c^3*d^36 + 18*a*c*d^38 - (45*b*c^2*d^36 + 6*a*d^38 + 4*((d*x +
c)*b*d^36 - 5*b*c*d^36)*(d*x + c))*(d*x + c))*(d*x + c))*(d*x + c))*sqrt(d*
x + c)*sqrt(d*x - c) + 6*(5*b*c^6*d^36 + 6*a*c^4*d^38)*log(abs(-sqrt(d*x +
c) + sqrt(d*x - c))))/d
```

3.359 $\int \frac{x^3(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

Optimal. Leaf size=118

$$\frac{x^2\sqrt{dx-c}\sqrt{c+dx}\,(5ad^2+4bc^2)}{15d^4}+\frac{2c^2\sqrt{dx-c}\sqrt{c+dx}\,(5ad^2+4bc^2)}{15d^6}+\frac{bx^4\sqrt{dx-c}\sqrt{c+dx}}{5d^2}$$

```
[Out] (2*c^2*(4*b*c^2 + 5*a*d^2)*Sqrt[-c + d*x]*Sqrt[c + d*x])/(15*d^6) + ((4*b*c
^2 + 5*a*d^2)*x^2*Sqrt[-c + d*x]*Sqrt[c + d*x])/(15*d^4) + (b*x^4*Sqrt[-c +
 d*x]*Sqrt[c + d*x])/(5*d^2)
```

Rubi [A] time = 0.0864362, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {460, 100, 12, 74}

$$\frac{x^2\sqrt{dx-c}\sqrt{c+dx}\,(5ad^2+4bc^2)}{15d^4}+\frac{2c^2\sqrt{dx-c}\sqrt{c+dx}\,(5ad^2+4bc^2)}{15d^6}+\frac{bx^4\sqrt{dx-c}\sqrt{c+dx}}{5d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (2*c^2*(4*b*c^2 + 5*a*d^2)*Sqrt[-c + d*x]*Sqrt[c + d*x])/(15*d^6) + ((4*b*c
^2 + 5*a*d^2)*x^2*Sqrt[-c + d*x]*Sqrt[c + d*x])/(15*d^4) + (b*x^4*Sqrt[-c +
 d*x]*Sqrt[c + d*x])/(5*d^2)
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
```

```
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\int \frac{x^3\left(a+bx^2\right)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{bx^4\sqrt{-c+dx}\sqrt{c+dx}}{5d^2} - \frac{1}{5}\left(-5a-\frac{4bc^2}{d^2}\right)\int \frac{x^3}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx$$
$$= \frac{\left(4bc^2+5ad^2\right)x^2\sqrt{-c+dx}\sqrt{c+dx}}{15d^4} + \frac{bx^4\sqrt{-c+dx}\sqrt{c+dx}}{5d^2} + \frac{\left(4bc^2+5ad^2\right)\int \frac{2c^2x}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx}{15d^4}$$
$$= \frac{\left(4bc^2+5ad^2\right)x^2\sqrt{-c+dx}\sqrt{c+dx}}{15d^4} + \frac{bx^4\sqrt{-c+dx}\sqrt{c+dx}}{5d^2} + \frac{\left(2c^2\left(4bc^2+5ad^2\right)\right)\int \frac{x}{\sqrt{-c+dx}\sqrt{c}}$$
$$= \frac{2c^2\left(4bc^2+5ad^2\right)\sqrt{-c+dx}\sqrt{c+dx}}{15d^6} + \frac{\left(4bc^2+5ad^2\right)x^2\sqrt{-c+dx}\sqrt{c+dx}}{15d^4} + \frac{bx^4\sqrt{-c+dx}\sqrt{c}}{5d^2}$$

Mathematica [A] time = 0.0544678, size = 87, normalized size = 0.74

$$\frac{\left(d^2x^2-c^2\right)\left(5ad^2\left(2c^2+d^2x^2\right)+b\left(4c^2d^2x^2+8c^4+3d^4x^4\right)\right)}{15d^6\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] ((-c^2 + d^2*x^2)*(5*a*d^2*(2*c^2 + d^2*x^2) + b*(8*c^4 + 4*c^2*d^2*x^2 + 3
*d^4*x^4)))/(15*d^6*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

Maple [A] time = 0.006, size = 68, normalized size = 0.6

$$\frac{3\,bd^4x^4+5\,ad^4x^2+4\,bc^2d^2x^2+10\,ac^2d^2+8\,bc^4}{15\,d^6}\sqrt{dx+c}\sqrt{dx-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] 1/15*(d*x+c)^(1/2)*(3*b*d^4*x^4+5*a*d^4*x^2+4*b*c^2*d^2*x^2+10*a*c^2*d^2+8*
b*c^4)/d^6*(d*x-c)^(1/2)
```

Maxima [A] time = 0.966855, size = 167, normalized size = 1.42

$$\frac{\sqrt{d^2x^2-c^2}bx^4}{5\,d^2} + \frac{4\,\sqrt{d^2x^2-c^2}bc^2x^2}{15\,d^4} + \frac{\sqrt{d^2x^2-c^2}ax^2}{3\,d^2} + \frac{8\,\sqrt{d^2x^2-c^2}bc^4}{15\,d^6} + \frac{2\,\sqrt{d^2x^2-c^2}ac^2}{3\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(d^2*x^2 - c^2)*b*x^4/d^2 + 4/15*sqrt(d^2*x^2 - c^2)*b*c^2*x^2/d^4
+ 1/3*sqrt(d^2*x^2 - c^2)*a*x^2/d^2 + 8/15*sqrt(d^2*x^2 - c^2)*b*c^4/d^6 +
```

```
2/3*sqrt(d^2*x^2 - c^2)*a*c^2/d^4
```

Fricas [A] time = 1.57225, size = 144, normalized size = 1.22

$$\frac{\left(3\,bd^4x^4+8\,bc^4+10\,ac^2d^2+\left(4\,bc^2d^2+5\,ad^4\right)x^2\right)\sqrt{dx+c}\sqrt{dx-c}}{15\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*b*d^4*x^4 + 8*b*c^4 + 10*a*c^2*d^2 + (4*b*c^2*d^2 + 5*a*d^4)*x^2)*s
qrt(d*x + c)*sqrt(d*x - c)/d^6
```

Sympy [C] time = 60.904, size = 240, normalized size = 2.03

$$\frac{ac^3G_{6,6}^{6,2}\left(\begin{matrix}-\frac{5}{4},-\frac{3}{4} & -1,-1,-\frac{1}{2},1\\ -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},0 & \end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^4}+\frac{iac^3G_{6,6}^{2,6}\left(\begin{matrix}-2,-\frac{7}{4},-\frac{3}{2},-\frac{5}{4},-1,1 & \\ -\frac{7}{4},-\frac{5}{4} & -2,-\frac{3}{2},-\frac{3}{2},0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^4}+\frac{bc^5G_{6,6}^{6,2}\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] a*c**3*meijerg(((-5/4, -3/4), (-1, -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -
1/2, 0), ()), c**2/(d**2*x**2))/(4*pi**(3/2)*d**4) + I*a*c**3*meijerg(((-2,
 -7/4, -3/2, -5/4, -1, 1), ()), ((-7/4, -5/4), (-2, -3/2, -3/2, 0)), c**2*e
xp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**4) + b*c**5*meijerg(((-9/4, -
7/4), (-2, -2, -3/2, 1)), ((-5/2, -9/4, -2, -7/4, -3/2, 0), ()), c**2/(d**2
*x**2))/(4*pi**(3/2)*d**6) + I*b*c**5*meijerg(((-3, -11/4, -5/2, -9/4, -2,
1), ()), ((-11/4, -9/4), (-3, -5/2, -5/2, 0)), c**2*exp_polar(2*I*pi)/(d**2
*x**2))/(4*pi**(3/2)*d**6)
```

Giac [A] time = 1.21164, size = 151, normalized size = 1.28

$$\frac{\left(15\,bc^4d^{25}+15\,ac^2d^{27}-\left(20\,bc^3d^{25}+10\,acd^{27}-\left(22\,bc^2d^{25}+5\,ad^{27}+3\left((dx+c)bd^{25}-4\,bcd^{25}\right)(dx+c)\right)(dx+c)\right)(dx}{276480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/276480*(15*b*c^4*d^25 + 15*a*c^2*d^27 - (20*b*c^3*d^25 + 10*a*c*d^27 - (2
2*b*c^2*d^25 + 5*a*d^27 + 3*((d*x + c)*b*d^25 - 4*b*c*d^25)*(d*x + c))*(d*x
 + c))*(d*x + c))*sqrt(d*x + c)*sqrt(d*x - c)/d
```

3.360 $\int \frac{x^2(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

Optimal. Leaf size=118

$$\frac{x\sqrt{dx-c}\sqrt{c+dx}\left(4ad^2+3bc^2\right)}{8d^4}+\frac{c^2\left(4ad^2+3bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4d^5}+\frac{bx^3\sqrt{dx-c}\sqrt{c+dx}}{4d^2}$$

```
[Out] ((3*b*c^2 + 4*a*d^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(8*d^4) + (b*x^3*Sqrt[
-c + d*x]*Sqrt[c + d*x])/(4*d^2) + (c^2*(3*b*c^2 + 4*a*d^2)*ArcTanh[Sqrt[-c
 + d*x]/Sqrt[c + d*x]])/(4*d^5)
```

Rubi [A] time = 0.096774, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {460, 90, 12, 63, 217, 206}

$$\frac{x\sqrt{dx-c}\sqrt{c+dx}\left(4ad^2+3bc^2\right)}{8d^4}+\frac{c^2\left(4ad^2+3bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4d^5}+\frac{bx^3\sqrt{dx-c}\sqrt{c+dx}}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] ((3*b*c^2 + 4*a*d^2)*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(8*d^4) + (b*x^3*Sqrt[
-c + d*x]*Sqrt[c + d*x])/(4*d^2) + (c^2*(3*b*c^2 + 4*a*d^2)*ArcTanh[Sqrt[-c
 + d*x]/Sqrt[c + d*x]])/(4*d^5)
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2\left(a+bx^2\right)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{bx^3\sqrt{-c+dx}\sqrt{c+dx}}{4d^2} - \frac{1}{4}\left(-4a-\frac{3bc^2}{d^2}\right)\int \frac{x^2}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx$$

$$= \frac{\left(3bc^2+4ad^2\right)x\sqrt{-c+dx}\sqrt{c+dx}}{8d^4} + \frac{bx^3\sqrt{-c+dx}\sqrt{c+dx}}{4d^2} + \frac{\left(3bc^2+4ad^2\right)\int \frac{c^2}{\sqrt{-c+dx}\sqrt{c+dx}}}{8d^4}$$

$$= \frac{\left(3bc^2+4ad^2\right)x\sqrt{-c+dx}\sqrt{c+dx}}{8d^4} + \frac{bx^3\sqrt{-c+dx}\sqrt{c+dx}}{4d^2} + \frac{\left(c^2\left(3bc^2+4ad^2\right)\right)\int \frac{1}{\sqrt{-c+dx}\sqrt{}}}{8d^4}$$

$$= \frac{\left(3bc^2+4ad^2\right)x\sqrt{-c+dx}\sqrt{c+dx}}{8d^4} + \frac{bx^3\sqrt{-c+dx}\sqrt{c+dx}}{4d^2} + \frac{\left(c^2\left(3bc^2+4ad^2\right)\right)\text{Subst}\left(\int\right.}{4d^5}$$

$$= \frac{\left(3bc^2+4ad^2\right)x\sqrt{-c+dx}\sqrt{c+dx}}{8d^4} + \frac{bx^3\sqrt{-c+dx}\sqrt{c+dx}}{4d^2} + \frac{\left(c^2\left(3bc^2+4ad^2\right)\right)\text{Subst}\left(\int\right.}{4d^5}$$

$$= \frac{\left(3bc^2+4ad^2\right)x\sqrt{-c+dx}\sqrt{c+dx}}{8d^4} + \frac{bx^3\sqrt{-c+dx}\sqrt{c+dx}}{4d^2} + \frac{c^2\left(3bc^2+4ad^2\right)\tanh^{-1}\left(\frac{\sqrt{-c}}{\sqrt{c}}\right.}{4d^5}$$

Mathematica [A] time = 0.0817847, size = 121, normalized size = 1.03

$$\frac{dx\left(d^2x^2-c^2\right)\left(4ad^2+3bc^2+2bd^2x^2\right)+c^2\sqrt{d^2x^2-c^2}\left(4ad^2+3bc^2\right)\tanh^{-1}\left(\frac{dx}{\sqrt{d^2x^2-c^2}}\right)}{8d^5\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (d*x*(-c^2 + d^2*x^2)*(3*b*c^2 + 4*a*d^2 + 2*b*d^2*x^2) + c^2*(3*b*c^2 + 4*
a*d^2)*Sqrt[-c^2 + d^2*x^2]*ArcTanh[(d*x)/Sqrt[-c^2 + d^2*x^2]])/(8*d^5*Sqr
t[-c + d*x]*Sqrt[c + d*x])
```

Maple [C] time = 0.02, size = 182, normalized size = 1.5

$$\frac{\text{csgn}(d)}{8\,d^5}\sqrt{dx-c}\sqrt{dx+c}\left(2\,\text{csgn}(d)\,x^3bd^3\sqrt{d^2x^2-c^2}+4\,\text{csgn}(d)\,d^3\sqrt{d^2x^2-c^2}xa+3\,\text{csgn}(d)\,d\sqrt{d^2x^2-c^2}xbc^2+4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] 1/8*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(2*csgn(d)*x^3*b*d^3*(d^2*x^2-c^2)^(1/2)+4*
csgn(d)*d^3*(d^2*x^2-c^2)^(1/2)*x*a+3*csgn(d)*d*(d^2*x^2-c^2)^(1/2)*x*b*c^2
+4*ln(((d^2*x^2-c^2)^(1/2)*csgn(d)+d*x)*csgn(d))*a*c^2*d^2+3*ln(((d^2*x^2-c
^2)^(1/2)*csgn(d)+d*x)*csgn(d))*b*c^4)*csgn(d)/d^5/(d^2*x^2-c^2)^(1/2)
```

Maxima [A] time = 0.992695, size = 216, normalized size = 1.83

$$\frac{\sqrt{d^2x^2-c^2}bx^3}{4\,d^2}+\frac{3\,bc^4\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\sqrt{d^2}\right)}{8\,\sqrt{d^2}d^4}+\frac{ac^2\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\sqrt{d^2}\right)}{2\,\sqrt{d^2}d^2}+\frac{3\,\sqrt{d^2x^2-c^2}bc^2x}{8\,d^4}+\frac{\sqrt{d^2x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(d^2*x^2 - c^2)*b*x^3/d^2 + 3/8*b*c^4*log(2*d^2*x + 2*sqrt(d^2*x^2
- c^2)*sqrt(d^2))/(sqrt(d^2)*d^4) + 1/2*a*c^2*log(2*d^2*x + 2*sqrt(d^2*x^2
- c^2)*sqrt(d^2))/(sqrt(d^2)*d^2) + 3/8*sqrt(d^2*x^2 - c^2)*b*c^2*x/d^4 + 1
/2*sqrt(d^2*x^2 - c^2)*a*x/d^2
```

Fricas [A] time = 1.49252, size = 196, normalized size = 1.66

$$\frac{\left(2\,bd^3x^3+\left(3\,bc^2d+4\,ad^3\right)x\right)\sqrt{dx+c}\sqrt{dx-c}-\left(3\,bc^4+4\,ac^2d^2\right)\log\left(-dx+\sqrt{dx+c}\sqrt{dx-c}\right)}{8\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*((2*b*d^3*x^3 + (3*b*c^2*d + 4*a*d^3)*x)*sqrt(d*x + c)*sqrt(d*x - c) -
(3*b*c^4 + 4*a*c^2*d^2)*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c)))/d^5
```

Sympy [C] time = 49.7872, size = 236, normalized size = 2.

$$\frac{ac^2G_{6,6}^{6,2}\left(\begin{matrix}-\frac{3}{4},-\frac{1}{4} & -\frac{1}{2},-\frac{1}{2},0,1\\ -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,0 & \end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^3}-\frac{iac^2G_{6,6}^{2,6}\left(\begin{matrix}-\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},1 & \\ -\frac{5}{4},-\frac{3}{4} & -\frac{3}{2},-1,-1,0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^3}+\frac{bc^4G_{6,6}^{6,2}\left(\begin{matrix} \\ -2,- \end{matrix}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] a*c**2*meijerg(((-3/4, -1/4), (-1/2, -1/2, 0, 1)), ((-1, -3/4, -1/2, -1/4,
0, 0), ()), c**2/(d**2*x**2))/(4*pi**(3/2)*d**3) - I*a*c**2*meijerg(((-3/2,
 -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-3/2, -1, -1, 0)), c**2*exp
_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**3) + b*c**4*meijerg(((-7/4, -5/
4), (-3/2, -3/2, -1, 1)), ((-2, -7/4, -3/2, -5/4, -1, 0), ()), c**2/(d**2*x
**2))/(4*pi**(3/2)*d**5) - I*b*c**4*meijerg(((-5/2, -9/4, -2, -7/4, -3/2, 1
), ()), ((-9/4, -7/4), (-5/2, -2, -2, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**
```

```
2))/(4*pi**(3/2)*d**5)
```

Giac [A] time = 1.21896, size = 176, normalized size = 1.49

$$-\frac{\left(5\,bc^3d^{16}+4\,acd^{18}-\left(9\,bc^2d^{16}+4\,ad^{18}+2\,\left((dx+c)bd^{16}-3\,bcd^{16}\right)(dx+c)\right)(dx+c)\right)\sqrt{dx+c}\sqrt{dx-c}+2\,\left(3\,bc^4d^{16}\right.}{114688\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/114688*((5*b*c^3*d^16 + 4*a*c*d^18 - (9*b*c^2*d^16 + 4*a*d^18 + 2*((d*x
+ c)*b*d^16 - 3*b*c*d^16)*(d*x + c))*(d*x + c))*sqrt(d*x + c)*sqrt(d*x - c)
 + 2*(3*b*c^4*d^16 + 4*a*c^2*d^18)*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))
)/d
```

3.361 $\int \frac{x(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

Optimal. Leaf size=72

$$\frac{\sqrt{dx-c}\sqrt{c+dx}\left(3ad^2+2bc^2\right)}{3d^4}+\frac{bx^2\sqrt{dx-c}\sqrt{c+dx}}{3d^2}$$

```
[Out] ((2*b*c^2 + 3*a*d^2)*Sqrt[-c + d*x]*Sqrt[c + d*x])/(3*d^4) + (b*x^2*Sqrt[-c
 + d*x]*Sqrt[c + d*x])/(3*d^2)
```

Rubi [A] time = 0.0463676, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {460, 74}

$$\frac{\sqrt{dx-c}\sqrt{c+dx}\left(3ad^2+2bc^2\right)}{3d^4}+\frac{bx^2\sqrt{dx-c}\sqrt{c+dx}}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] ((2*b*c^2 + 3*a*d^2)*Sqrt[-c + d*x]*Sqrt[c + d*x])/(3*d^4) + (b*x^2*Sqrt[-c
 + d*x]*Sqrt[c + d*x])/(3*d^2)
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(a+bx^2\right)}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx &= \frac{bx^2\sqrt{-c+dx}\sqrt{c+dx}}{3d^2}-\frac{1}{3}\left(-3a-\frac{2bc^2}{d^2}\right)\int \frac{x}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx\\ &= \frac{\left(2bc^2+3ad^2\right)\sqrt{-c+dx}\sqrt{c+dx}}{3d^4}+\frac{bx^2\sqrt{-c+dx}\sqrt{c+dx}}{3d^2}\end{aligned}$$

Mathematica [A] time = 0.0327275, size = 61, normalized size = 0.85

$$\frac{\left(d^2x^2-c^2\right)\left(3ad^2+2bc^2+bd^2x^2\right)}{3d^4\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^2))/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] ((-c^2 + d^2*x^2)*(2*b*c^2 + 3*a*d^2 + b*d^2*x^2))/(3*d^4*Sqrt[-c + d*x]*Sq
rt[c + d*x])
```

Maple [A] time = 0.004, size = 43, normalized size = 0.6

$$\frac{bd^2x^2 + 3\,ad^2 + 2\,bc^2}{3\,d^4}\sqrt{dx+c}\sqrt{dx-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] 1/3*(d*x+c)^(1/2)*(b*d^2*x^2+3*a*d^2+2*b*c^2)/d^4*(d*x-c)^(1/2)
```

Maxima [A] time = 0.981241, size = 93, normalized size = 1.29

$$\frac{\sqrt{d^2x^2-c^2}bx^2}{3\,d^2} + \frac{2\,\sqrt{d^2x^2-c^2}bc^2}{3\,d^4} + \frac{\sqrt{d^2x^2-c^2}a}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(d^2*x^2 - c^2)*b*x^2/d^2 + 2/3*sqrt(d^2*x^2 - c^2)*b*c^2/d^4 + sqr
t(d^2*x^2 - c^2)*a/d^2
```

Fricas [A] time = 1.4751, size = 93, normalized size = 1.29

$$\frac{\left(bd^2x^2 + 2\,bc^2 + 3\,ad^2\right)\sqrt{dx+c}\sqrt{dx-c}}{3\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b*d^2*x^2 + 2*b*c^2 + 3*a*d^2)*sqrt(d*x + c)*sqrt(d*x - c)/d^4
```

Sympy [C] time = 33.112, size = 223, normalized size = 3.1

$$\frac{acG_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4}, \frac{1}{4} & 0, 0, \frac{1}{2}, 1 \\ -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 0 & \end{matrix} \middle| \frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2} + \frac{iacG_{6,6}^{2,6}\left(\begin{matrix} -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 1 & \\ -\frac{3}{4}, -\frac{1}{4} & -1, -\frac{1}{2}, -\frac{1}{2}, 0 \end{matrix} \middle| \frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2} + \frac{bc^3G_{6,6}^{6,2}\left(\begin{matrix} -\frac{5}{4}, \cdots \\ -\frac{3}{2}, -\frac{5}{4}, -1, \cdots \end{matrix}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] a*c*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0), (
)), c**2/(d**2*x**2))/(4*pi**(3/2)*d**2) + I*a*c*meijerg(((-1, -3/4, -1/2,
-1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), c**2*exp_polar(2*I*p
i)/(d**2*x**2))/(4*pi**(3/2)*d**2) + b*c**3*meijerg(((-5/4, -3/4), (-1, -1,
 -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -1/2, 0), ()), c**2/(d**2*x**2))/(4*pi*
*(3/2)*d**4) + I*b*c**3*meijerg(((-2, -7/4, -3/2, -5/4, -1, 1), ()), ((-7/4
, -5/4), (-2, -3/2, -3/2, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(
3/2)*d**4)
```

Giac [A] time = 1.14379, size = 82, normalized size = 1.14

$$\frac{\left(3\,bc^2d^9 + 3\,ad^{11} + \left((dx+c)bd^9 - 2\,bcd^9\right)(dx+c)\right)\sqrt{dx+c}\sqrt{dx-c}}{1920\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1920*(3*b*c^2*d^9 + 3*a*d^11 + ((d*x + c)*b*d^9 - 2*b*c*d^9)*(d*x + c))*s
qrt(d*x + c)*sqrt(d*x - c)/d
```

3.362 $\int \frac{a+bx^2}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

Optimal. Leaf size=68

$$\frac{\left(2ad^2+bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^3}+\frac{bx\sqrt{dx-c}\sqrt{c+dx}}{2d^2}$$

```
[Out] (b*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(2*d^2) + ((b*c^2 + 2*a*d^2)*ArcTanh[Sqr
t[-c + d*x]/Sqrt[c + d*x]])/d^3
```

Rubi [A] time = 0.0323963, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {389, 63, 217, 206}

$$\frac{\left(2ad^2+bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^3}+\frac{bx\sqrt{dx-c}\sqrt{c+dx}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (b*x*Sqrt[-c + d*x]*Sqrt[c + d*x])/(2*d^2) + ((b*c^2 + 2*a*d^2)*ArcTanh[Sqr
t[-c + d*x]/Sqrt[c + d*x]])/d^3
```

Rule 389

```
Int[((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_
.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*x*(a1 + b1*x^(n/2))^(p +
1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*(n*(p + 1) + 1)), x] - Dist[(a1*a2*d -
 b1*b2*c*(n*(p + 1) + 1))/(b1*b2*(n*(p + 1) + 1)), Int[(a1 + b1*x^(n/2))^p*
(a2 + b2*x^(n/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p}, x] && Eq
Q[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a+bx^2}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{bx\sqrt{-c+dx}\sqrt{c+dx}}{2d^2} - \frac{(-bc^2-2ad^2)\int \frac{1}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx}{2d^2}$$
$$= \frac{bx\sqrt{-c+dx}\sqrt{c+dx}}{2d^2} + \frac{(bc^2+2ad^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{2c+x^2}}\,dx, x, \sqrt{-c+dx}\right)}{d^3}$$
$$= \frac{bx\sqrt{-c+dx}\sqrt{c+dx}}{2d^2} + \frac{(bc^2+2ad^2)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d^3}$$
$$= \frac{bx\sqrt{-c+dx}\sqrt{c+dx}}{2d^2} + \frac{(bc^2+2ad^2)\tanh^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d^3}$$

Mathematica [A] time = 0.261142, size = 119, normalized size = 1.75

$$\frac{4\left(ad^2+bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right) - \frac{2bc^{5/2}\sqrt{\frac{dx}{c}+1}\sinh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{2}\sqrt{c}}\right)}{\sqrt{c+dx}} + bdx\sqrt{dx-c}\sqrt{c+dx}}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (b*d*x*Sqrt[-c + d*x]*Sqrt[c + d*x] - (2*b*c^(5/2)*Sqrt[1 + (d*x)/c]*ArcSin
h[Sqrt[-c + d*x]/(Sqrt[2]*Sqrt[c])])/Sqrt[c + d*x] + 4*(b*c^2 + a*d^2)*ArcT
anh[Sqrt[-c + d*x]/Sqrt[c + d*x]])/(2*d^3)
```

Maple [C] time = 0.016, size = 124, normalized size = 1.8

$$\frac{\text{csgn}(d)}{2d^3}\sqrt{dx-c}\sqrt{dx+c}\left(\text{csgn}(d)\,d\sqrt{d^2x^2-c^2}xb + 2\ln\left(\left(\sqrt{d^2x^2-c^2}\text{csgn}(d)+dx\right)\text{csgn}(d)\right)ad^2 + \ln\left(\left(\sqrt{d^2x^2-c^2}\text{csg}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] 1/2*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(csgn(d)*d*(d^2*x^2-c^2)^(1/2)*x*b+2*ln(((d
^2*x^2-c^2)^(1/2)*csgn(d)+d*x)*csgn(d))*a*d^2+ln(((d^2*x^2-c^2)^(1/2)*csgn(
d)+d*x)*csgn(d))*b*c^2)*csgn(d)/d^3/(d^2*x^2-c^2)^(1/2)
```

Maxima [A] time = 0.976806, size = 140, normalized size = 2.06

$$\frac{a\log\left(2\,d^2x + 2\,\sqrt{d^2x^2-c^2}\sqrt{d^2}\right)}{\sqrt{d^2}} + \frac{bc^2\log\left(2\,d^2x + 2\,\sqrt{d^2x^2-c^2}\sqrt{d^2}\right)}{2\,\sqrt{d^2}d^2} + \frac{\sqrt{d^2x^2-c^2}bx}{2\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] a*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*sqrt(d^2))/sqrt(d^2) + 1/2*b*c^2*log(
2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*sqrt(d^2))/(sqrt(d^2)*d^2) + 1/2*sqrt(d^2*x
```

```
^2 - c^2)*b*x/d^2
```

Fricas [A] time = 1.52222, size = 142, normalized size = 2.09

$$\frac{\sqrt{dx+c}\sqrt{dx-c}bdx - \left(bc^2+2\,ad^2\right)\log\left(-dx+\sqrt{dx+c}\sqrt{dx-c}\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(d*x + c)*sqrt(d*x - c)*b*d*x - (b*c^2 + 2*a*d^2)*log(-d*x + sqrt(
d*x + c)*sqrt(d*x - c)))/d^3
```

Sympy [C] time = 16.4835, size = 199, normalized size = 2.93

$$\frac{aG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d} - \frac{iaG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d} + \frac{bc^2G_{6,6}^{6,2}\left(\begin{matrix}-\frac{3}{4},-\frac{1}{4} & \\ -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,0 & \end{matrix}\right.}{4\pi^{\frac{3}{2}}d^3} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] a*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), c
**2/(d**2*x**2))/(4*pi**(3/2)*d) - I*a*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1
), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))
/(4*pi**(3/2)*d) + b*c**2*meijerg(((-3/4, -1/4), (-1/2, -1/2, 0, 1)), ((-1,
 -3/4, -1/2, -1/4, 0, 0), ()), c**2/(d**2*x**2))/(4*pi**(3/2)*d**3) - I*b*c
**2*meijerg(((-3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-3/2, -1
, -1, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**3)
```

Giac [A] time = 1.20247, size = 107, normalized size = 1.57

$$\frac{\left((dx+c)bd^4 - bcd^4\right)\sqrt{dx+c}\sqrt{dx-c} - 2\left(bc^2d^4+2\,ad^6\right)\log\left(\left|-\sqrt{dx+c}+\sqrt{dx-c}\right|\right)}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/384*(((d*x + c)*b*d^4 - b*c*d^4)*sqrt(d*x + c)*sqrt(d*x - c) - 2*(b*c^2*d
^4 + 2*a*d^6)*log(abs(-sqrt(d*x + c) + sqrt(d*x - c))))/d
```

3.363 $\int \frac{a+bx^2}{x\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

Optimal. Leaf size=56

$$\frac{a\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{c}+\frac{b\sqrt{dx-c}\sqrt{c+dx}}{d^2}$$

```
[Out] (b*Sqrt[-c + d*x]*Sqrt[c + d*x])/d^2 + (a*ArcTan[(Sqrt[-c + d*x]*Sqrt[c + d
*x])/c])/c
```

Rubi [A] time = 0.0687923, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {460, 92, 205}

$$\frac{a\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{c}+\frac{b\sqrt{dx-c}\sqrt{c+dx}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (b*Sqrt[-c + d*x]*Sqrt[c + d*x])/d^2 + (a*ArcTan[(Sqrt[-c + d*x]*Sqrt[c + d
*x])/c])/c
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a+bx^2}{x\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{b\sqrt{-c+dx}\sqrt{c+dx}}{d^2} + a\int \frac{1}{x\sqrt{-c+dx}\sqrt{c+dx}}\,dx$$
$$= \frac{b\sqrt{-c+dx}\sqrt{c+dx}}{d^2} + (ad)\,\mathrm{Subst}\left(\int \frac{1}{c^2d+dx^2}\,dx, x, \sqrt{-c+dx}\sqrt{c+dx}\right)$$
$$= \frac{b\sqrt{-c+dx}\sqrt{c+dx}}{d^2} + \frac{a\tan^{-1}\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)}{c}$$

Mathematica [A] time = 0.0324644, size = 87, normalized size = 1.55

$$\frac{ad^2\sqrt{d^2x^2-c^2}\tan^{-1}\left(\frac{\sqrt{d^2x^2-c^2}}{c}\right) - bc^3 + bcd^2x^2}{cd^2\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (-(b*c^3) + b*c*d^2*x^2 + a*d^2*Sqrt[-c^2 + d^2*x^2]*ArcTan[Sqrt[-c^2 + d^2
*x^2]/c])/(c*d^2*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

Maple [B] time = 0.019, size = 108, normalized size = 1.9

$$\frac{1}{d^2}\left(-\ln\left(-2\,\frac{c^2-\sqrt{-c^2}\sqrt{d^2x^2-c^2}}{x}\right)ad^2 + b\sqrt{-c^2}\sqrt{d^2x^2-c^2}\right)\sqrt{dx-c}\sqrt{dx+c}\frac{1}{\sqrt{-c^2}}\frac{1}{\sqrt{d^2x^2-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x/(d*x-c)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] (-ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*a*d^2+b*(-c^2)^(1/2)*(d^2
*x^2-c^2)^(1/2))*(d*x-c)^(1/2)*(d*x+c)^(1/2)/(d^2*x^2-c^2)^(1/2)/d^2/(-c^2)
^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.5188, size = 135, normalized size = 2.41

$$\frac{2\,ad^2\arctan\left(-\frac{dx-\sqrt{dx+c}\sqrt{dx-c}}{c}\right) + \sqrt{dx+c}\sqrt{dx-c}bc}{cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] (2*a*d^2*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c))/c) + sqrt(d*x + c)*sqr
t(d*x - c)*b*c)/(c*d^2)
```

Sympy [C] time = 21.1566, size = 178, normalized size = 3.18

$$-\frac{aG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2} \\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0 \end{matrix} \middle| \frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c}+\frac{iaG_{6,6}^{2,6}\left(\begin{matrix} 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0 \end{matrix} \middle| \frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c}+\frac{bcG_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1 \\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix} \middle| \frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] -a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)), c
**2/(d**2*x**2))/(4*pi**(3/2)*c) + I*a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), (
)), ((1/4, 3/4), (0, 1/2, 1/2, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(4*
pi**(3/2)*c) + b*c*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0,
1/4, 1/2, 0), ()), c**2/(d**2*x**2))/(4*pi**(3/2)*d**2) + I*b*c*meijerg(((-
1, -3/4, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), c**2*
exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**2)
```

Giac [A] time = 1.16348, size = 74, normalized size = 1.32

$$-\frac{2\,a\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2\,c}\right)}{c}+\frac{\sqrt{dx+c}\sqrt{dx-c}b}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -2*a*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/c)/c + sqrt(d*x + c)*sqrt
(d*x - c)*b/d^2
```

3.364 $\int \frac{a+bx^2}{x^2\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

Optimal. Leaf size=57

$$\frac{a\sqrt{dx-c}\sqrt{c+dx}}{c^2x}+\frac{2b\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d}$$

```
[Out] (a*Sqrt[-c + d*x]*Sqrt[c + d*x])/(c^2*x) + (2*b*ArcTanh[Sqrt[-c + d*x]/Sqrt
[c + d*x]])/d
```

Rubi [A] time = 0.0747374, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {454, 63, 217, 206}

$$\frac{a\sqrt{dx-c}\sqrt{c+dx}}{c^2x}+\frac{2b\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x^2*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (a*Sqrt[-c + d*x]*Sqrt[c + d*x])/(c^2*x) + (2*b*ArcTanh[Sqrt[-c + d*x]/Sqrt
[c + d*x]])/d
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a+bx^2}{x^2\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{a\sqrt{-c+dx}\sqrt{c+dx}}{c^2x} + b\int \frac{1}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx$$
$$= \frac{a\sqrt{-c+dx}\sqrt{c+dx}}{c^2x} + \frac{(2b)\,\text{Subst}\left(\int \frac{1}{\sqrt{2c+x^2}}\,dx, x, \sqrt{-c+dx}\right)}{d}$$
$$= \frac{a\sqrt{-c+dx}\sqrt{c+dx}}{c^2x} + \frac{(2b)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d}$$
$$= \frac{a\sqrt{-c+dx}\sqrt{c+dx}}{c^2x} + \frac{2b\tanh^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d}$$

Mathematica [A] time = 0.0389778, size = 90, normalized size = 1.58

$$\frac{\sqrt{d^2x^2-c^2}\left(\frac{a\sqrt{d^2x^2-c^2}}{c^2x} + \frac{b\tanh^{-1}\left(\frac{dx}{\sqrt{d^2x^2-c^2}}\right)}{d}\right)}{\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x^2*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (Sqrt[-c^2 + d^2*x^2]*((a*Sqrt[-c^2 + d^2*x^2])/(c^2*x) + (b*ArcTanh[(d*x)/
Sqrt[-c^2 + d^2*x^2]])/d))/(Sqrt[-c + d*x]*Sqrt[c + d*x])
```

Maple [C] time = 0.02, size = 97, normalized size = 1.7

$$\frac{\text{csgn}(d)}{c^2dx}\sqrt{dx-c}\sqrt{dx+c}\left(\ln\left(\left(\sqrt{d^2x^2-c^2}\,\text{csgn}(d)+dx\right)\text{csgn}(d)\right)xbc^2+\text{csgn}(d)\,d\sqrt{d^2x^2-c^2}\,a\right)\frac{1}{\sqrt{d^2x^2-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^2/(d*x-c)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] (d*x-c)^(1/2)*(d*x+c)^(1/2)/c^2*(ln(((d^2*x^2-c^2)^(1/2)*csgn(d)+d*x)*csgn(
d))*x*b*c^2+csgn(d)*d*(d^2*x^2-c^2)^(1/2)*a)*csgn(d)/(d^2*x^2-c^2)^(1/2)/d/
x
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^2/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.52417, size = 143, normalized size = 2.51

$$-\frac{bc^2x\log\left(-dx+\sqrt{dx+c}\sqrt{dx-c}\right)-ad^2x-\sqrt{dx+c}\sqrt{dx-c}ad}{c^2dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^2/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -(b*c^2*x*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c)) - a*d^2*x - sqrt(d*x + c)
*sqrt(d*x - c)*a*d)/(c^2*d*x)
```

Sympy [C] time = 21.4911, size = 165, normalized size = 2.89

$$-\frac{adG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^2}-\frac{iadG_{6,6}^{2,6}\left(\begin{matrix}\frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\\frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^2}+\frac{bG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**2/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] -a*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,)),
 c**2/(d**2*x**2))/(4*pi**(3/2)*c**2) - I*a*d*meijerg(((1/2, 3/4, 1, 5/4, 3
/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**
2))/(4*pi**(3/2)*c**2) + b*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4
, 1/2, 3/4, 1, 0), ()), c**2/(d**2*x**2))/(4*pi**(3/2)*d) - I*b*meijerg(((-
1/2, -1/4, 0, 1/4, 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), c**2*exp_p
olar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d)
```

Giac [A] time = 1.18928, size = 89, normalized size = 1.56

$$\frac{\frac{16\,ad^2}{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2}-b\log\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^2/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(16*a*d^2/((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2) - b*log((sqrt(d*x
 + c) - sqrt(d*x - c))^4))/d
```

3.365 $\int \frac{a+bx^2}{x^3\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

Optimal. Leaf size=76

$$\frac{(ad^2+2bc^2)\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{2c^3}+\frac{a\sqrt{dx-c}\sqrt{c+dx}}{2c^2x^2}$$

```
[Out] (a*Sqrt[-c + d*x]*Sqrt[c + d*x])/(2*c^2*x^2) + ((2*b*c^2 + a*d^2)*ArcTan[(S
qrt[-c + d*x]*Sqrt[c + d*x])/c])/(2*c^3)
```

Rubi [A] time = 0.072997, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {454, 92, 205}

$$\frac{(ad^2+2bc^2)\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{2c^3}+\frac{a\sqrt{dx-c}\sqrt{c+dx}}{2c^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x^3*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (a*Sqrt[-c + d*x]*Sqrt[c + d*x])/(2*c^2*x^2) + ((2*b*c^2 + a*d^2)*ArcTan[(S
qrt[-c + d*x]*Sqrt[c + d*x])/c])/(2*c^3)
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a+bx^2}{x^3\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{a\sqrt{-c+dx}\sqrt{c+dx}}{2c^2x^2}+\frac{1}{2}\left(2b+\frac{ad^2}{c^2}\right)\int \frac{1}{x\sqrt{-c+dx}\sqrt{c+dx}}\,dx$$
$$= \frac{a\sqrt{-c+dx}\sqrt{c+dx}}{2c^2x^2}+\frac{1}{2}\left(d\left(2b+\frac{ad^2}{c^2}\right)\right)\text{Subst}\left(\int \frac{1}{c^2d+dx^2}\,dx, x, \sqrt{-c+dx}\sqrt{c+dx}\right)$$
$$= \frac{a\sqrt{-c+dx}\sqrt{c+dx}}{2c^2x^2}+\frac{(2bc^2+ad^2)\tan^{-1}\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)}{2c^3}$$

Mathematica [A] time = 0.0572556, size = 102, normalized size = 1.34

$$\frac{x^2\sqrt{d^2x^2-c^2}\left(ad^2+2bc^2\right)\tan^{-1}\left(\frac{\sqrt{d^2x^2-c^2}}{c}\right)+a\left(cd^2x^2-c^3\right)}{2c^3x^2\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x^3*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (a*(-c^3 + c*d^2*x^2) + (2*b*c^2 + a*d^2)*x^2*Sqrt[-c^2 + d^2*x^2]*ArcTan[S
qrt[-c^2 + d^2*x^2]/c])/(2*c^3*x^2*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

Maple [B] time = 0.022, size = 158, normalized size = 2.1

$$-\frac{1}{2c^2x^2}\sqrt{dx-c}\sqrt{dx+c}\left(\ln\left(-2\frac{c^2-\sqrt{-c^2}\sqrt{d^2x^2-c^2}}{x}\right)x^2ad^2+2\ln\left(-2\frac{c^2-\sqrt{-c^2}\sqrt{d^2x^2-c^2}}{x}\right)x^2bc^2-\sqrt{-c^2}\sqrt{d^2x^2-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^3/(d*x-c)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] -1/2*(d*x-c)^(1/2)*(d*x+c)^(1/2)/c^2*(ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)
^(1/2))/x)*x^2*a*d^2+2*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*x^2*
b*c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2)*a)/(d^2*x^2-c^2)^(1/2)/(-c^2)^(1/2)/
x^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^3/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.51693, size = 165, normalized size = 2.17

$$\frac{2\left(2\,bc^2+ad^2\right)x^2\arctan\left(-\frac{dx-\sqrt{dx+c}\sqrt{dx-c}}{c}\right)+\sqrt{dx+c}\sqrt{dx-c}ac}{2\,c^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^3/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(2*b*c^2 + a*d^2)*x^2*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c))/c)
 + sqrt(d*x + c)*sqrt(d*x - c)*a*c)/(c^3*x^2)
```

Sympy [C] time = 28.438, size = 162, normalized size = 2.13

$$-\frac{ad^2 G_{6,6}^{5,3}\left(\begin{matrix} \frac{7}{4},\frac{9}{4},1 & 2,2,\frac{5}{2} \\ \frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2} & 0 \end{matrix}\middle| \frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^3}+\frac{iad^2 G_{6,6}^{2,6}\left(\begin{matrix} 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2,1 & \\ \frac{5}{4},\frac{7}{4} & 1,\frac{3}{2},\frac{3}{2},0 \end{matrix}\middle| \frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^3}-\frac{bG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2} \\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0 \end{matrix}\middle| \frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c}+\frac{ibG}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**3/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] -a*d**2*meijerg(((7/4, 9/4, 1), (2, 2, 5/2)), ((3/2, 7/4, 2, 9/4, 5/2), (0,
)), c**2/(d**2*x**2))/(4*pi**(3/2)*c**3) + I*a*d**2*meijerg(((1, 5/4, 3/2,
7/4, 2, 1), ()), ((5/4, 7/4), (1, 3/2, 3/2, 0)), c**2*exp_polar(2*I*pi)/(d*
*2*x**2))/(4*pi**(3/2)*c**3) - b*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/
2, 3/4, 1, 5/4, 3/2), (0,)), c**2/(d**2*x**2))/(4*pi**(3/2)*c) + I*b*meijer
g(((0, 1/4, 1/2, 3/4, 1, 1), ()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), c**2*exp_
polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*c)
```

Giac [B] time = 1.24414, size = 190, normalized size = 2.5

$$-\frac{\frac{\left(2\,bc^2d+ad^3\right)\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2\,c}\right)}{c^3}+\frac{2\left(ad^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^6-4\,ac^2d^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2\right)^2c^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^3/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -((2*b*c^2*d + a*d^3)*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/c)/c^3 +
 2*(a*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^6 - 4*a*c^2*d^3*(sqrt(d*x + c) -
sqrt(d*x - c))^2)/(((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2)^2*c^2))/d
```

3.366 $\int \frac{a+bx^2}{x^4\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

Optimal. Leaf size=75

$$\frac{\sqrt{dx-c}\sqrt{c+dx}\left(2ad^2+3bc^2\right)}{3c^4x}+\frac{a\sqrt{dx-c}\sqrt{c+dx}}{3c^2x^3}$$

```
[Out] (a*Sqrt[-c + d*x]*Sqrt[c + d*x])/(3*c^2*x^3) + ((3*b*c^2 + 2*a*d^2)*Sqrt[-c
 + d*x]*Sqrt[c + d*x])/(3*c^4*x)
```

Rubi [A] time = 0.0693193, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {454, 95}

$$\frac{\sqrt{dx-c}\sqrt{c+dx}\left(2ad^2+3bc^2\right)}{3c^4x}+\frac{a\sqrt{dx-c}\sqrt{c+dx}}{3c^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x^4*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (a*Sqrt[-c + d*x]*Sqrt[c + d*x])/(3*c^2*x^3) + ((3*b*c^2 + 2*a*d^2)*Sqrt[-c
 + d*x]*Sqrt[c + d*x])/(3*c^4*x)
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x^4\sqrt{-c+dx}\sqrt{c+dx}}\,dx&=\frac{a\sqrt{-c+dx}\sqrt{c+dx}}{3c^2x^3}+\frac{1}{3}\left(3b+\frac{2ad^2}{c^2}\right)\int \frac{1}{x^2\sqrt{-c+dx}\sqrt{c+dx}}\,dx\\&=\frac{a\sqrt{-c+dx}\sqrt{c+dx}}{3c^2x^3}+\frac{\left(3bc^2+2ad^2\right)\sqrt{-c+dx}\sqrt{c+dx}}{3c^4x}\end{aligned}$$

Mathematica [A] time = 0.0257343, size = 66, normalized size = 0.88

$$-\frac{\left(c^2-d^2x^2\right)\left(a\left(c^2+2d^2x^2\right)+3bc^2x^2\right)}{3c^4x^3\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x^4*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] -((c^2 - d^2*x^2)*(3*b*c^2*x^2 + a*(c^2 + 2*d^2*x^2)))/(3*c^4*x^3*Sqrt[-c +
 d*x]*Sqrt[c + d*x])
```

Maple [A] time = 0.005, size = 49, normalized size = 0.7

$$\frac{2\,a{d}^{2}{x}^{2}+3\,b{c}^{2}{x}^{2}+a{c}^{2}}{3\,{x}^{3}{c}^{4}}\sqrt{dx+c}\sqrt{dx-c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^4/(d*x-c)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] 1/3*(d*x+c)^(1/2)*(2*a*d^2*x^2+3*b*c^2*x^2+a*c^2)/x^3/c^4*(d*x-c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^4/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.48573, size = 144, normalized size = 1.92

$$\frac{{\left(3\,b c^{2} d+2\,a d^{3}\right)}x^{3}+{\left(a c^{2}+{\left(3\,b c^{2}+2\,a d^{2}\right)}x^{2}\right)}\sqrt{dx+c}\sqrt{dx-c}}{3\,c^{4}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^4/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*((3*b*c^2*d + 2*a*d^3)*x^3 + (a*c^2 + (3*b*c^2 + 2*a*d^2)*x^2)*sqrt(d*x
 + c)*sqrt(d*x - c))/(c^4*x^3)
```

Sympy [C] time = 44.3197, size = 170, normalized size = 2.27

$$-\frac{ad^{3}G_{6,6}^{5,3}\left(\begin{matrix}\frac{9}{4},\frac{11}{4},1 & \frac{5}{2},\frac{5}{2},3\\ 2,\frac{9}{4},\frac{5}{2},\frac{11}{4},3 & 0\end{matrix}\middle|\frac{c^{2}}{d^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}c^{4}}-\frac{iad^{3}G_{6,6}^{2,6}\left(\begin{matrix}\frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2},1 & \\ \frac{7}{4},\frac{9}{4} & \frac{3}{2},2,2,0\end{matrix}\middle|\frac{c^{2}e^{2i\pi}}{d^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}c^{4}}-\frac{bdG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{c^{2}}{d^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}c^{2}}-\ldots i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**4/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] -a*d**3*meijerg(((9/4, 11/4, 1), (5/2, 5/2, 3)), ((2, 9/4, 5/2, 11/4, 3), (
0,)), c**2/(d**2*x**2))/(4*pi**(3/2)*c**4) - I*a*d**3*meijerg(((3/2, 7/4, 2
, 9/4, 5/2, 1), ()), ((7/4, 9/4), (3/2, 2, 2, 0)), c**2*exp_polar(2*I*pi)/(
d**2*x**2))/(4*pi**(3/2)*c**4) - b*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2))
, ((1, 5/4, 3/2, 7/4, 2), (0,)), c**2/(d**2*x**2))/(4*pi**(3/2)*c**2) - I*b
*d*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)),
c**2*exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*c**2)
```

Giac [B] time = 1.19709, size = 185, normalized size = 2.47

$$\frac{8\left(3\,bd^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^8+24\,bc^2d^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+24\,ad^4\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+48\,bc^4d^2+32\,ac^2d^4\right)}{3\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2\right)^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^4/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 8/3*(3*b*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^8 + 24*b*c^2*d^2*(sqrt(d*x + c
) - sqrt(d*x - c))^4 + 24*a*d^4*(sqrt(d*x + c) - sqrt(d*x - c))^4 + 48*b*c^
4*d^2 + 32*a*c^2*d^4)/(((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2)^3*d)
```

3.367 $\int \frac{a+bx^2}{x^5\sqrt{-c+dx}\sqrt{c+dx}}\,dx$

Optimal. Leaf size=123

$$\frac{\sqrt{dx-c}\sqrt{c+dx}\left(3ad^2+4bc^2\right)}{8c^4x^2}+\frac{d^2\left(3ad^2+4bc^2\right)\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{8c^5}+\frac{a\sqrt{dx-c}\sqrt{c+dx}}{4c^2x^4}$$

```
[Out] (a*Sqrt[-c + d*x]*Sqrt[c + d*x])/(4*c^2*x^4) + ((4*b*c^2 + 3*a*d^2)*Sqrt[-c
 + d*x]*Sqrt[c + d*x])/(8*c^4*x^2) + (d^2*(4*b*c^2 + 3*a*d^2)*ArcTan[(Sqrt[
-c + d*x]*Sqrt[c + d*x])/c])/(8*c^5)
```

Rubi [A] time = 0.0953812, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {454, 103, 12, 92, 205}

$$\frac{\sqrt{dx-c}\sqrt{c+dx}\left(3ad^2+4bc^2\right)}{8c^4x^2}+\frac{d^2\left(3ad^2+4bc^2\right)\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{8c^5}+\frac{a\sqrt{dx-c}\sqrt{c+dx}}{4c^2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x^5*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] (a*Sqrt[-c + d*x]*Sqrt[c + d*x])/(4*c^2*x^4) + ((4*b*c^2 + 3*a*d^2)*Sqrt[-c
 + d*x]*Sqrt[c + d*x])/(8*c^4*x^2) + (d^2*(4*b*c^2 + 3*a*d^2)*ArcTan[(Sqrt[
-c + d*x]*Sqrt[c + d*x])/c])/(8*c^5)
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a+bx^2}{x^5\sqrt{-c+dx}\sqrt{c+dx}}\,dx = \frac{a\sqrt{-c+dx}\sqrt{c+dx}}{4c^2x^4} + \frac{1}{4}\left(4b+\frac{3ad^2}{c^2}\right)\int \frac{1}{x^3\sqrt{-c+dx}\sqrt{c+dx}}\,dx$$

$$= \frac{a\sqrt{-c+dx}\sqrt{c+dx}}{4c^2x^4} + \frac{\left(4bc^2+3ad^2\right)\sqrt{-c+dx}\sqrt{c+dx}}{8c^4x^2} + \frac{\left(4bc^2+3ad^2\right)\int \frac{d^2}{x\sqrt{-c+dx}\sqrt{c+dx}}}{8c^4}$$

$$= \frac{a\sqrt{-c+dx}\sqrt{c+dx}}{4c^2x^4} + \frac{\left(4bc^2+3ad^2\right)\sqrt{-c+dx}\sqrt{c+dx}}{8c^4x^2} + \frac{\left(d^2\left(4bc^2+3ad^2\right)\right)\int \frac{1}{x\sqrt{-c+dx}\sqrt{}}}{8c^4}$$

$$= \frac{a\sqrt{-c+dx}\sqrt{c+dx}}{4c^2x^4} + \frac{\left(4bc^2+3ad^2\right)\sqrt{-c+dx}\sqrt{c+dx}}{8c^4x^2} + \frac{\left(d^3\left(4bc^2+3ad^2\right)\right)\text{Subst}\left(\int \frac{}{c}\right.}{8}$$

$$= \frac{a\sqrt{-c+dx}\sqrt{c+dx}}{4c^2x^4} + \frac{\left(4bc^2+3ad^2\right)\sqrt{-c+dx}\sqrt{c+dx}}{8c^4x^2} + \frac{d^2\left(4bc^2+3ad^2\right)\tan^{-1}\left(\sqrt{-c+d}\right.}{8c^5}$$

Mathematica [A] time = 0.0936209, size = 144, normalized size = 1.17

$$-\frac{\left(c^2-d^2x^2\right)\left(c^2\sqrt{1-\frac{d^2x^2}{c^2}}\left(2ac^2+3ad^2x^2+4bc^2x^2\right)+d^2x^4\left(3ad^2+4bc^2\right)\tanh^{-1}\left(\sqrt{1-\frac{d^2x^2}{c^2}}\right)\right)}{8c^6x^4\sqrt{dx-c}\sqrt{c+dx}\sqrt{1-\frac{d^2x^2}{c^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x^5*Sqrt[-c + d*x]*Sqrt[c + d*x]),x]
```

```
[Out] -((c^2 - d^2*x^2)*(c^2*(2*a*c^2 + 4*b*c^2*x^2 + 3*a*d^2*x^2)*Sqrt[1 - (d^2*
x^2)/c^2] + d^2*(4*b*c^2 + 3*a*d^2)*x^4*ArcTanh[Sqrt[1 - (d^2*x^2)/c^2]]))/
(8*c^6*x^4*Sqrt[-c + d*x]*Sqrt[c + d*x]*Sqrt[1 - (d^2*x^2)/c^2])
```

Maple [B] time = 0.02, size = 227, normalized size = 1.9

$$-\frac{1}{8c^4x^4}\sqrt{dx-c}\sqrt{dx+c}\left(3\ln\left(-2\frac{c^2-\sqrt{-c^2}\sqrt{d^2x^2-c^2}}{x}\right)x^4ad^4+4\ln\left(-2\frac{c^2-\sqrt{-c^2}\sqrt{d^2x^2-c^2}}{x}\right)x^4bc^2d^2-3\sqrt{-c^2}\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^5/(d*x-c)^(1/2)/(d*x+c)^(1/2),x)
```

```
[Out] -1/8*(d*x-c)^(1/2)*(d*x+c)^(1/2)/c^4*(3*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^
2)^(1/2))/x)*x^4*a*d^4+4*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*x^
4*b*c^2*d^2-3*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2)*x^2*a*d^2-4*(-c^2)^(1/2)*(d^
```

```
2*x^2-c^2)^(1/2)*x^2*b*c^2-2*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2)*a*c^2)/(d^2*x
^2-c^2)^(1/2)/(-c^2)^(1/2)/x^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^5/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.58047, size = 219, normalized size = 1.78

$$\frac{2\left(4\,bc^2d^2+3\,ad^4\right)x^4\arctan\left(-\frac{dx-\sqrt{dx+c}\sqrt{dx-c}}{c}\right)+\left(2\,ac^3+\left(4\,bc^3+3\,acd^2\right)x^2\right)\sqrt{dx+c}\sqrt{dx-c}}{8\,c^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^5/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(2*(4*b*c^2*d^2 + 3*a*d^4)*x^4*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x -
c))/c) + (2*a*c^3 + (4*b*c^3 + 3*a*c*d^2)*x^2)*sqrt(d*x + c)*sqrt(d*x - c))
/(c^5*x^4)
```

Sympy [C] time = 61.5923, size = 172, normalized size = 1.4

$$-\frac{ad^4G_{6,6}^{5,3}\left(\begin{matrix}\frac{11}{4},\frac{13}{4},1 & 3,3,\frac{7}{2}\\ \frac{5}{2},\frac{11}{4},3,\frac{13}{4},\frac{7}{2} & 0\end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^5}+\frac{iad^4G_{6,6}^{2,6}\left(\begin{matrix}2,\frac{9}{4},\frac{5}{2},\frac{11}{4},3,1 & \\ \frac{9}{4},\frac{11}{4} & 2,\frac{5}{2},\frac{5}{2},0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^5}-\frac{bd^2G_{6,6}^{5,3}\left(\begin{matrix}\frac{7}{4},\frac{9}{4},1 & 2,2,\frac{5}{2}\\ \frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2} & 0\end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**5/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] -a*d**4*meijerg(((11/4, 13/4, 1), (3, 3, 7/2)), ((5/2, 11/4, 3, 13/4, 7/2),
 (0,)), c**2/(d**2*x**2))/(4*pi**(3/2)*c**5) + I*a*d**4*meijerg(((2, 9/4, 5
/2, 11/4, 3, 1), ()), ((9/4, 11/4), (2, 5/2, 5/2, 0)), c**2*exp_polar(2*I*p
i)/(d**2*x**2))/(4*pi**(3/2)*c**5) - b*d**2*meijerg(((7/4, 9/4, 1), (2, 2,
5/2)), ((3/2, 7/4, 2, 9/4, 5/2), (0,)), c**2/(d**2*x**2))/(4*pi**(3/2)*c**3
) + I*b*d**2*meijerg(((1, 5/4, 3/2, 7/4, 2, 1), ()), ((5/4, 7/4), (1, 3/2,
3/2, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*c**3)
```

Giac [B] time = 1.20858, size = 439, normalized size = 3.57

$$-\frac{\frac{\left(4\,bc^2d^3+3\,ad^5\right)\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2\,c}\right)}{c^5}+\frac{2\left(4\,bc^2d^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{14}+3\,ad^5\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{14}+16\,bc^4d^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{10}+44\,ac^2d^5\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^{10}\cdots\right.}{\left(\left(\sqrt{dx+c}-\sqrt{\cdots}\right.\right.}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^5/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*((4*b*c^2*d^3 + 3*a*d^5)*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/
c)/c^5 + 2*(4*b*c^2*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^14 + 3*a*d^5*(sqrt(
d*x + c) - sqrt(d*x - c))^14 + 16*b*c^4*d^3*(sqrt(d*x + c) - sqrt(d*x - c))
^10 + 44*a*c^2*d^5*(sqrt(d*x + c) - sqrt(d*x - c))^10 - 64*b*c^6*d^3*(sqrt(
d*x + c) - sqrt(d*x - c))^6 - 176*a*c^4*d^5*(sqrt(d*x + c) - sqrt(d*x - c))
^6 - 256*b*c^8*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^2 - 192*a*c^6*d^5*(sqrt(
d*x + c) - sqrt(d*x - c))^2)/(((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2)^4
*c^4))/d
```

3.368 $\int \frac{x^4(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=161

$$-\frac{x^3\left(4ad^2+5bc^2\right)}{4d^4\sqrt{dx-c}\sqrt{c+dx}}+\frac{3x\sqrt{dx-c}\sqrt{c+dx}\left(4ad^2+5bc^2\right)}{8d^6}+\frac{3c^2\left(4ad^2+5bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4d^7}+\frac{bx^5}{4d^2\sqrt{dx-c}\sqrt{c+dx}}$$

```
[Out] -((5*b*c^2 + 4*a*d^2)*x^3)/(4*d^4*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (b*x^5)/(
4*d^2*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (3*(5*b*c^2 + 4*a*d^2)*x*Sqrt[-c + d*
x]*Sqrt[c + d*x])/(8*d^6) + (3*c^2*(5*b*c^2 + 4*a*d^2)*ArcTanh[Sqrt[-c + d*
x]/Sqrt[c + d*x]])/(4*d^7)
```

Rubi [A] time = 0.122936, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {460, 98, 21, 90, 12, 63, 217, 206}

$$-\frac{x^3\left(4ad^2+5bc^2\right)}{4d^4\sqrt{dx-c}\sqrt{c+dx}}+\frac{3x\sqrt{dx-c}\sqrt{c+dx}\left(4ad^2+5bc^2\right)}{8d^6}+\frac{3c^2\left(4ad^2+5bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4d^7}+\frac{bx^5}{4d^2\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -((5*b*c^2 + 4*a*d^2)*x^3)/(4*d^4*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (b*x^5)/(
4*d^2*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (3*(5*b*c^2 + 4*a*d^2)*x*Sqrt[-c + d*
x]*Sqrt[c + d*x])/(8*d^6) + (3*c^2*(5*b*c^2 + 4*a*d^2)*ArcTanh[Sqrt[-c + d*
x]/Sqrt[c + d*x]])/(4*d^7)
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
```

```
a + b*x])
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^4 \left(a+bx^2\right)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{bx^5}{4d^2\sqrt{-c+dx}\sqrt{c+dx}} - \frac{1}{4}\left(-4a-\frac{5bc^2}{d^2}\right)\int \frac{x^4}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$$

$$= -\frac{\left(5bc^2+4ad^2\right)x^3}{4d^4\sqrt{-c+dx}\sqrt{c+dx}} + \frac{bx^5}{4d^2\sqrt{-c+dx}\sqrt{c+dx}} - \frac{\left(4a+\frac{5bc^2}{d^2}\right)\int \frac{x^2\left(-3c^2-3cdx\right)}{\sqrt{-c+dx}(c+dx)^{3/2}}\,dx}{4cd^2}$$

$$= -\frac{\left(5bc^2+4ad^2\right)x^3}{4d^4\sqrt{-c+dx}\sqrt{c+dx}} + \frac{bx^5}{4d^2\sqrt{-c+dx}\sqrt{c+dx}} + \frac{\left(3\left(5bc^2+4ad^2\right)\right)\int \frac{x^2}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx}{4d^4}$$

$$= -\frac{\left(5bc^2+4ad^2\right)x^3}{4d^4\sqrt{-c+dx}\sqrt{c+dx}} + \frac{bx^5}{4d^2\sqrt{-c+dx}\sqrt{c+dx}} + \frac{3\left(5bc^2+4ad^2\right)x\sqrt{-c+dx}\sqrt{c+dx}}{8d^6} +$$

$$= -\frac{\left(5bc^2+4ad^2\right)x^3}{4d^4\sqrt{-c+dx}\sqrt{c+dx}} + \frac{bx^5}{4d^2\sqrt{-c+dx}\sqrt{c+dx}} + \frac{3\left(5bc^2+4ad^2\right)x\sqrt{-c+dx}\sqrt{c+dx}}{8d^6} +$$

$$= -\frac{\left(5bc^2+4ad^2\right)x^3}{4d^4\sqrt{-c+dx}\sqrt{c+dx}} + \frac{bx^5}{4d^2\sqrt{-c+dx}\sqrt{c+dx}} + \frac{3\left(5bc^2+4ad^2\right)x\sqrt{-c+dx}\sqrt{c+dx}}{8d^6} +$$

$$= -\frac{\left(5bc^2+4ad^2\right)x^3}{4d^4\sqrt{-c+dx}\sqrt{c+dx}} + \frac{bx^5}{4d^2\sqrt{-c+dx}\sqrt{c+dx}} + \frac{3\left(5bc^2+4ad^2\right)x\sqrt{-c+dx}\sqrt{c+dx}}{8d^6} +$$

$$= -\frac{\left(5bc^2+4ad^2\right)x^3}{4d^4\sqrt{-c+dx}\sqrt{c+dx}} + \frac{bx^5}{4d^2\sqrt{-c+dx}\sqrt{c+dx}} + \frac{3\left(5bc^2+4ad^2\right)x\sqrt{-c+dx}\sqrt{c+dx}}{8d^6} +$$

Mathematica [A] time = 0.141607, size = 119, normalized size = 0.74

$$\frac{3c^3\sqrt{1-\frac{d^2x^2}{c^2}}\left(4ad^2+5bc^2\right)\sin^{-1}\left(\frac{dx}{c}\right)+4ad^3x\left(d^2x^2-3c^2\right)+bdx\left(5c^2d^2x^2-15c^4+2d^4x^4\right)}{8d^7\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (4*a*d^3*x*(-3*c^2 + d^2*x^2) + b*d*x*(-15*c^4 + 5*c^2*d^2*x^2 + 2*d^4*x^4)
 + 3*c^3*(5*b*c^2 + 4*a*d^2)*Sqrt[1 - (d^2*x^2)/c^2]*ArcSin[(d*x)/c])/(8*d^
7*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

Maple [C] time = 0.03, size = 316, normalized size = 2.

$$\frac{\mathrm{csgn}(d)}{8\,d^7}\left(2\,\mathrm{csgn}(d)\,x^5bd^5\sqrt{d^2x^2-c^2}+4\,\mathrm{csgn}(d)\,x^3ad^5\sqrt{d^2x^2-c^2}+5\,\mathrm{csgn}(d)\,x^3bc^2d^3\sqrt{d^2x^2-c^2}+12\,\ln\left(\left(\sqrt{d^2x^2-c^2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)
```

```
[Out] 1/8*(2*csgn(d)*x^5*b*d^5*(d^2*x^2-c^2)^(1/2)+4*csgn(d)*x^3*a*d^5*(d^2*x^2-c
^2)^(1/2)+5*csgn(d)*x^3*b*c^2*d^3*(d^2*x^2-c^2)^(1/2)+12*ln(((d^2*x^2-c^2)^
(1/2)*csgn(d)+d*x)*csgn(d))*x^2*a*c^2*d^4+15*ln(((d^2*x^2-c^2)^(1/2)*csgn(d
)+d*x)*csgn(d))*x^2*b*c^4*d^2-12*csgn(d)*d^3*(d^2*x^2-c^2)^(1/2)*x*a*c^2-15
*csgn(d)*d*(d^2*x^2-c^2)^(1/2)*x*b*c^4-12*ln(((d^2*x^2-c^2)^(1/2)*csgn(d)+d
*x)*csgn(d))*a*c^4*d^2-15*ln(((d^2*x^2-c^2)^(1/2)*csgn(d)+d*x)*csgn(d))*b*c
```

```
^6)*csgn(d)/(d^2*x^2-c^2)^(1/2)/d^7/(d*x+c)^(1/2)/(d*x-c)^(1/2)
```

Maxima [A] time = 0.951779, size = 289, normalized size = 1.8

$$\frac{bx^5}{4\sqrt{d^2x^2-c^2}d^2}+\frac{5\,bc^2x^3}{8\sqrt{d^2x^2-c^2}d^4}+\frac{ax^3}{2\sqrt{d^2x^2-c^2}d^2}-\frac{15\,bc^4x}{8\sqrt{d^2x^2-c^2}d^6}-\frac{3\,ac^2x}{2\sqrt{d^2x^2-c^2}d^4}+\frac{15\,bc^4\log\left(2\,d^2x+2\sqrt{d^2x\right.}}{8\sqrt{d^2}d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*b*x^5/(sqrt(d^2*x^2 - c^2)*d^2) + 5/8*b*c^2*x^3/(sqrt(d^2*x^2 - c^2)*d^
4) + 1/2*a*x^3/(sqrt(d^2*x^2 - c^2)*d^2) - 15/8*b*c^4*x/(sqrt(d^2*x^2 - c^2
)*d^6) - 3/2*a*c^2*x/(sqrt(d^2*x^2 - c^2)*d^4) + 15/8*b*c^4*log(2*d^2*x + 2
*sqrt(d^2*x^2 - c^2)*sqrt(d^2))/(sqrt(d^2)*d^6) + 3/2*a*c^2*log(2*d^2*x + 2
*sqrt(d^2*x^2 - c^2)*sqrt(d^2))/(sqrt(d^2)*d^4)
```

Fricas [A] time = 1.51211, size = 390, normalized size = 2.42

$$\frac{8\,bc^6+8\,ac^4d^2-8\left(bc^4d^2+ac^2d^4\right)x^2+\left(2\,bd^5x^5+\left(5\,bc^2d^3+4\,ad^5\right)x^3-3\left(5\,bc^4d+4\,ac^2d^3\right)x\right)\sqrt{dx+c}\sqrt{dx-c}+3\,(5}{8\left(d^9x^2-c^2d^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(8*b*c^6 + 8*a*c^4*d^2 - 8*(b*c^4*d^2 + a*c^2*d^4)*x^2 + (2*b*d^5*x^5 +
 (5*b*c^2*d^3 + 4*a*d^5)*x^3 - 3*(5*b*c^4*d + 4*a*c^2*d^3)*x)*sqrt(d*x + c)
*sqrt(d*x - c) + 3*(5*b*c^6 + 4*a*c^4*d^2 - (5*b*c^4*d^2 + 4*a*c^2*d^4)*x^2
)*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c)))/(d^9*x^2 - c^2*d^7)
```

Sympy [C] time = 99.7226, size = 233, normalized size = 1.45

$$a\left(\frac{c^2G_{6,6}^{6,2}\left(\begin{matrix}-\frac{5}{4},-\frac{3}{4} & -\frac{3}{2},-\frac{1}{2},0,1\\ -\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},0,0 & \end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}d^5}+\frac{ic^2G_{6,6}^{2,6}\left(\begin{matrix}-\frac{5}{2},-2,-\frac{7}{4},-\frac{3}{2},-\frac{5}{4},1 & \\ -\frac{7}{4},-\frac{5}{4} & -\frac{5}{2},-2,-1,0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}d^5}\right)+b\left(\frac{c^4G_{6,6}^{6,2}\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] a*(c**2*meijerg(((-5/4, -3/4), (-3/2, -1/2, 0, 1)), ((-5/4, -1, -3/4, -1/2,
 0, 0), ()), c**2/(d**2*x**2))/(2*pi**(3/2)*d**5) + I*c**2*meijerg(((-5/2,
-2, -7/4, -3/2, -5/4, 1), ()), ((-7/4, -5/4), (-5/2, -2, -1, 0)), c**2*exp_
polar(2*I*pi)/(d**2*x**2))/(2*pi**(3/2)*d**5)) + b*(c**4*meijerg(((-9/4, -7
/4), (-5/2, -3/2, -1, 1)), ((-9/4, -2, -7/4, -3/2, -1, 0), ()), c**2/(d**2*
x**2))/(2*pi**(3/2)*d**7) + I*c**4*meijerg(((-7/2, -3, -11/4, -5/2, -9/4, 1
), ()), ((-11/4, -9/4), (-7/2, -3, -2, 0)), c**2*exp_polar(2*I*pi)/(d**2*x*
```

```
*2))/(2*pi**(3/2)*d**7))
```

Giac [A] time = 1.32636, size = 290, normalized size = 1.8

$$-\frac{1}{688128}\left(5\,bc^3d^{35}+4\,acd^{37}\right)\log\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2\right)-\frac{\left(\left(\left(2\left(5\,bd^{35}-\frac{(dx+c)bd^{35}}{c}\right)(dx+c)-\frac{25\,bc^2d^{35}+4\,ad^{37}}{c}\right)(dx+c)+\right.\right.}{2064384\,\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/688128*(5*b*c^3*d^35 + 4*a*c*d^37)*log((sqrt(d*x + c) - sqrt(d*x - c))^2
) - 1/2064384*(((2*(5*b*d^35 - (d*x + c)*b*d^35/c)*(d*x + c) - (25*b*c^2*d^
35 + 4*a*d^37)/c)*(d*x + c) + (35*b*c^3*d^35 + 12*a*c*d^37)/c)*(d*x + c) -
2*(7*b*c^4*d^35 + 2*a*c^2*d^37)/c)*sqrt(d*x + c)/sqrt(d*x - c) - 2*(b*c^5 +
 a*c^3*d^2)/(((sqrt(d*x + c) - sqrt(d*x - c))^2 + 2*c)*d^7)
```

3.369 $\int \frac{x^3(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}} dx$

Optimal. Leaf size=115

$$-\frac{x^2(3ad^2+4bc^2)}{3d^4\sqrt{dx-c}\sqrt{c+dx}}+\frac{2\sqrt{dx-c}\sqrt{c+dx}\,(3ad^2+4bc^2)}{3d^6}+\frac{bx^4}{3d^2\sqrt{dx-c}\sqrt{c+dx}}$$

```
[Out] -((4*b*c^2 + 3*a*d^2)*x^2)/(3*d^4*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (b*x^4)/(
3*d^2*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (2*(4*b*c^2 + 3*a*d^2)*Sqrt[-c + d*x]
*Sqrt[c + d*x])/(3*d^6)
```

Rubi [A] time = 0.0943516, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {460, 98, 21, 74}

$$-\frac{x^2(3ad^2+4bc^2)}{3d^4\sqrt{dx-c}\sqrt{c+dx}}+\frac{2\sqrt{dx-c}\sqrt{c+dx}\,(3ad^2+4bc^2)}{3d^6}+\frac{bx^4}{3d^2\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -((4*b*c^2 + 3*a*d^2)*x^2)/(3*d^4*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (b*x^4)/(
3*d^2*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (2*(4*b*c^2 + 3*a*d^2)*Sqrt[-c + d*x]
*Sqrt[c + d*x])/(3*d^6)
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3\left(a+bx^2\right)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx &= \frac{bx^4}{3d^2\sqrt{-c+dx}\sqrt{c+dx}}-\frac{1}{3}\left(-3a-\frac{4bc^2}{d^2}\right)\int \frac{x^3}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx\\ &= -\frac{\left(4bc^2+3ad^2\right)x^2}{3d^4\sqrt{-c+dx}\sqrt{c+dx}}+\frac{bx^4}{3d^2\sqrt{-c+dx}\sqrt{c+dx}}-\frac{\left(3a+\frac{4bc^2}{d^2}\right)\int \frac{x\left(-2c^2-2cdx\right)}{\sqrt{-c+dx}(c+dx)^{3/2}}\,dx}{3cd^2}\\ &= -\frac{\left(4bc^2+3ad^2\right)x^2}{3d^4\sqrt{-c+dx}\sqrt{c+dx}}+\frac{bx^4}{3d^2\sqrt{-c+dx}\sqrt{c+dx}}+\frac{\left(2\left(3a+\frac{4bc^2}{d^2}\right)\right)\int \frac{x}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx}{3d^2}\\ &= -\frac{\left(4bc^2+3ad^2\right)x^2}{3d^4\sqrt{-c+dx}\sqrt{c+dx}}+\frac{bx^4}{3d^2\sqrt{-c+dx}\sqrt{c+dx}}+\frac{2\left(4bc^2+3ad^2\right)\sqrt{-c+dx}\sqrt{c+dx}}{3d^6}\end{aligned}$$

Mathematica [A] time = 0.0500077, size = 72, normalized size = 0.63

$$\frac{-6ac^2d^2+3ad^4x^2+4bc^2d^2x^2-8bc^4+bd^4x^4}{3d^6\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (-8*b*c^4 - 6*a*c^2*d^2 + 4*b*c^2*d^2*x^2 + 3*a*d^4*x^2 + b*d^4*x^4)/(3*d^6
*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

Maple [A] time = 0.005, size = 68, normalized size = 0.6

$$-\frac{-bd^4x^4-3\,ad^4x^2-4\,bc^2d^2x^2+6\,ac^2d^2+8\,bc^4}{3\,d^6}\frac{1}{\sqrt{dx+c}}\frac{1}{\sqrt{dx-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)
```

```
[Out] -1/3*(-b*d^4*x^4-3*a*d^4*x^2-4*b*c^2*d^2*x^2+6*a*c^2*d^2+8*b*c^4)/(d*x+c)^(
1/2)/d^6/(d*x-c)^(1/2)
```

Maxima [A] time = 0.95142, size = 166, normalized size = 1.44

$$\frac{bx^4}{3\sqrt{d^2x^2-c^2}d^2}+\frac{4\,bc^2x^2}{3\sqrt{d^2x^2-c^2}d^4}+\frac{ax^2}{\sqrt{d^2x^2-c^2}d^2}-\frac{8\,bc^4}{3\sqrt{d^2x^2-c^2}d^6}-\frac{2\,ac^2}{\sqrt{d^2x^2-c^2}d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*b*x^4/(sqrt(d^2*x^2 - c^2)*d^2) + 4/3*b*c^2*x^2/(sqrt(d^2*x^2 - c^2)*d^
4) + a*x^2/(sqrt(d^2*x^2 - c^2)*d^2) - 8/3*b*c^4/(sqrt(d^2*x^2 - c^2)*d^6)
- 2*a*c^2/(sqrt(d^2*x^2 - c^2)*d^4)
```

Fricas [A] time = 1.51864, size = 161, normalized size = 1.4

$$\frac{\left(bd^4x^4 - 8\,bc^4 - 6\,ac^2d^2 + \left(4\,bc^2d^2 + 3\,ad^4\right)x^2\right)\sqrt{dx+c}\sqrt{dx-c}}{3\left(d^8x^2 - c^2d^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(b*d^4*x^4 - 8*b*c^4 - 6*a*c^2*d^2 + (4*b*c^2*d^2 + 3*a*d^4)*x^2)*sqrt(
d*x + c)*sqrt(d*x - c)/(d^8*x^2 - c^2*d^6)
```

Sympy [C] time = 79.9069, size = 226, normalized size = 1.97

$$a\left(\frac{cG_{6,6}^{6,2}\left(\begin{matrix} -\frac{3}{4}, -\frac{1}{4} & -1, 0, \frac{1}{2}, 1 \\ -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{2}, 0 & \end{matrix} \middle| \frac{c^2}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}d^4} - \frac{icG_{6,6}^{2,6}\left(\begin{matrix} -2, -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, 1 & \\ -\frac{5}{4}, -\frac{3}{4} & -2, -\frac{3}{2}, -\frac{1}{2}, 0 \end{matrix} \middle| \frac{c^2e^{2i\pi}}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}d^4}\right) + b\left(\frac{c^3G_{6,6}^{6,2}\left(-\frac{7}{4}, -\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] a*(c*meijerg(((-3/4, -1/4), (-1, 0, 1/2, 1)), ((-3/4, -1/2, -1/4, 0, 1/2, 0
), ()), c**2/(d**2*x**2))/(2*pi**(3/2)*d**4) - I*c*meijerg(((-2, -3/2, -5/4
, -1, -3/4, 1), ()), ((-5/4, -3/4), (-2, -3/2, -1/2, 0)), c**2*exp_polar(2*
I*pi)/(d**2*x**2))/(2*pi**(3/2)*d**4)) + b*(c**3*meijerg(((-7/4, -5/4), (-2
, -1, -1/2, 1)), ((-7/4, -3/2, -5/4, -1, -1/2, 0), ()), c**2/(d**2*x**2))/(
2*pi**(3/2)*d**6) - I*c**3*meijerg(((-3, -5/2, -9/4, -2, -7/4, 1), ()), ((-
9/4, -7/4), (-3, -5/2, -3/2, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(2*pi
**(3/2)*d**6))
```

Giac [A] time = 1.22954, size = 194, normalized size = 1.69

$$-\frac{\left(2\left(\left(4\,bd^{24} - \frac{(dx+c)bd^{24}}{c}\right)(dx+c) - \frac{10\,bc^2d^{24}+3\,ad^{26}}{c}\right)(dx+c) + \frac{3\left(9\,bc^3d^{24}+5\,acd^{26}\right)}{c}\right)\sqrt{dx+c}}{23040\sqrt{dx-c}} + \frac{2\left(bc^4 + ac^2d^2\right)}{\left(\left(\sqrt{dx+c} - \sqrt{dx-c}\right)^2 + 2\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/23040*(2*((4*b*d^24 - (d*x + c)*b*d^24/c)*(d*x + c) - (10*b*c^2*d^24 + 3
*a*d^26)/c)*(d*x + c) + 3*(9*b*c^3*d^24 + 5*a*c*d^26)/c)*sqrt(d*x + c)/sqrt
(d*x - c) + 2*(b*c^4 + a*c^2*d^2)/(((sqrt(d*x + c) - sqrt(d*x - c))^2 + 2*c
)*d^6)
```

3.370 $\int \frac{x^2(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=152

$$-\frac{\sqrt{dx-c}\left(2ad^2+3bc^2\right)}{2d^5\sqrt{c+dx}}-\frac{c\left(2ad^2+3bc^2\right)}{2d^5\sqrt{dx-c}\sqrt{c+dx}}+\frac{\left(2ad^2+3bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^5}+\frac{bx^3}{2d^2\sqrt{dx-c}\sqrt{c+dx}}$$

```
[Out] -(c*(3*b*c^2 + 2*a*d^2))/(2*d^5*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (b*x^3)/(2*
d^2*Sqrt[-c + d*x]*Sqrt[c + d*x]) - ((3*b*c^2 + 2*a*d^2)*Sqrt[-c + d*x])/(2
*d^5*Sqrt[c + d*x]) + ((3*b*c^2 + 2*a*d^2)*ArcTanh[Sqrt[-c + d*x]/Sqrt[c +
d*x]])/d^5
```

Rubi [A] time = 0.11468, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {460, 89, 12, 78, 63, 217, 206}

$$-\frac{\sqrt{dx-c}\left(2ad^2+3bc^2\right)}{2d^5\sqrt{c+dx}}-\frac{c\left(2ad^2+3bc^2\right)}{2d^5\sqrt{dx-c}\sqrt{c+dx}}+\frac{\left(2ad^2+3bc^2\right)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^5}+\frac{bx^3}{2d^2\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -(c*(3*b*c^2 + 2*a*d^2))/(2*d^5*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (b*x^3)/(2*
d^2*Sqrt[-c + d*x]*Sqrt[c + d*x]) - ((3*b*c^2 + 2*a*d^2)*Sqrt[-c + d*x])/(2
*d^5*Sqrt[c + d*x]) + ((3*b*c^2 + 2*a*d^2)*ArcTanh[Sqrt[-c + d*x]/Sqrt[c +
d*x]])/d^5
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\left(a+bx^2\right)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx &= \frac{bx^3}{2d^2\sqrt{-c+dx}\sqrt{c+dx}}-\frac{1}{2}\left(-2a-\frac{3bc^2}{d^2}\right)\int\frac{x^2}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx\\
&=-\frac{c\left(3bc^2+2ad^2\right)}{2d^5\sqrt{-c+dx}\sqrt{c+dx}}+\frac{bx^3}{2d^2\sqrt{-c+dx}\sqrt{c+dx}}-\frac{\left(-2a-\frac{3bc^2}{d^2}\right)\int\frac{cd^2x}{\sqrt{-c+dx}(c+dx)^{3/2}}\,dx}{2cd^3}\\
&=-\frac{c\left(3bc^2+2ad^2\right)}{2d^5\sqrt{-c+dx}\sqrt{c+dx}}+\frac{bx^3}{2d^2\sqrt{-c+dx}\sqrt{c+dx}}+\frac{\left(3bc^2+2ad^2\right)\int\frac{x}{\sqrt{-c+dx}(c+dx)^{3/2}}\,dx}{2d^3}\\
&=-\frac{c\left(3bc^2+2ad^2\right)}{2d^5\sqrt{-c+dx}\sqrt{c+dx}}+\frac{bx^3}{2d^2\sqrt{-c+dx}\sqrt{c+dx}}-\frac{\left(3bc^2+2ad^2\right)\sqrt{-c+dx}}{2d^5\sqrt{c+dx}}+\frac{\left(3bc^2+\right.}{}\\
&=-\frac{c\left(3bc^2+2ad^2\right)}{2d^5\sqrt{-c+dx}\sqrt{c+dx}}+\frac{bx^3}{2d^2\sqrt{-c+dx}\sqrt{c+dx}}-\frac{\left(3bc^2+2ad^2\right)\sqrt{-c+dx}}{2d^5\sqrt{c+dx}}+\frac{\left(3bc^2+\right.}{}\\
&=-\frac{c\left(3bc^2+2ad^2\right)}{2d^5\sqrt{-c+dx}\sqrt{c+dx}}+\frac{bx^3}{2d^2\sqrt{-c+dx}\sqrt{c+dx}}-\frac{\left(3bc^2+2ad^2\right)\sqrt{-c+dx}}{2d^5\sqrt{c+dx}}+\frac{\left(3bc^2+\right.}{}\\
&=-\frac{c\left(3bc^2+2ad^2\right)}{2d^5\sqrt{-c+dx}\sqrt{c+dx}}+\frac{bx^3}{2d^2\sqrt{-c+dx}\sqrt{c+dx}}-\frac{\left(3bc^2+2ad^2\right)\sqrt{-c+dx}}{2d^5\sqrt{c+dx}}+\frac{\left(3bc^2+\right.}{}
\end{aligned}$$

Mathematica [A] time = 0.107926, size = 90, normalized size = 0.59

$$\frac{c\sqrt{1-\frac{d^2x^2}{c^2}}\left(2ad^2+3bc^2\right)\sin^{-1}\left(\frac{dx}{c}\right)-2ad^3x-3bc^2dx+bd^3x^3}{2d^5\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (-3*b*c^2*d*x - 2*a*d^3*x + b*d^3*x^3 + c*(3*b*c^2 + 2*a*d^2)*Sqrt[1 - (d^2
*x^2)/c^2]*ArcSin[(d*x)/c])/(2*d^5*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

Maple [C] time = 0.022, size = 254, normalized size = 1.7

$$\frac{\operatorname{csgn}(d)}{2\,d^5}\left(\operatorname{csgn}(d)\,x^3bd^3\sqrt{d^2x^2-c^2}+2\,\ln\left(\left(\sqrt{d^2x^2-c^2}\operatorname{csgn}(d)+dx\right)\operatorname{csgn}(d)\right)x^2ad^4+3\,\ln\left(\left(\sqrt{d^2x^2-c^2}\operatorname{csgn}(d)+dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)
```

```
[Out] 1/2*(csgn(d)*x^3*b*d^3*(d^2*x^2-c^2)^(1/2)+2*ln(((d^2*x^2-c^2)^(1/2)*csgn(d
)+d*x)*csgn(d))*x^2*a*d^4+3*ln(((d^2*x^2-c^2)^(1/2)*csgn(d)+d*x)*csgn(d))*x
^2*b*c^2*d^2-2*csgn(d)*d^3*(d^2*x^2-c^2)^(1/2)*x*a-3*csgn(d)*d*(d^2*x^2-c^2
)^(1/2)*x*b*c^2-2*ln(((d^2*x^2-c^2)^(1/2)*csgn(d)+d*x)*csgn(d))*a*c^2*d^2-3
*ln(((d^2*x^2-c^2)^(1/2)*csgn(d)+d*x)*csgn(d))*b*c^4)*csgn(d)/(d^2*x^2-c^2)
^(1/2)/d^5/(d*x+c)^(1/2)/(d*x-c)^(1/2)
```

Maxima [A] time = 0.955846, size = 211, normalized size = 1.39

$$\frac{bx^3}{2\sqrt{d^2x^2-c^2}d^2}-\frac{3\,bc^2x}{2\sqrt{d^2x^2-c^2}d^4}-\frac{ax}{\sqrt{d^2x^2-c^2}d^2}+\frac{3\,bc^2\log\left(2\,d^2x+2\sqrt{d^2x^2-c^2}\sqrt{d^2}\right)}{2\sqrt{d^2}d^4}+\frac{a\log\left(2\,d^2x+2\sqrt{d^2x^2-c^2}\right.}{\sqrt{d^2}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*x^3/(sqrt(d^2*x^2 - c^2)*d^2) - 3/2*b*c^2*x/(sqrt(d^2*x^2 - c^2)*d^4)
 - a*x/(sqrt(d^2*x^2 - c^2)*d^2) + 3/2*b*c^2*log(2*d^2*x + 2*sqrt(d^2*x^2 -
 c^2)*sqrt(d^2))/(sqrt(d^2)*d^4) + a*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*sq
rt(d^2))/(sqrt(d^2)*d^2)
```

Fricas [A] time = 1.54815, size = 325, normalized size = 2.14

$$\frac{2\,bc^4+2\,ac^2d^2-2\left(bc^2d^2+ad^4\right)x^2+\left(bd^3x^3-\left(3\,bc^2d+2\,ad^3\right)x\right)\sqrt{dx+c}\sqrt{dx-c}+\left(3\,bc^4+2\,ac^2d^2-\left(3\,bc^2d^2+2\,ad^4\right)x\right.}{2\left(d^7x^2-c^2d^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*c^4 + 2*a*c^2*d^2 - 2*(b*c^2*d^2 + a*d^4)*x^2 + (b*d^3*x^3 - (3*b*
c^2*d + 2*a*d^3)*x)*sqrt(d*x + c)*sqrt(d*x - c) + (3*b*c^4 + 2*a*c^2*d^2 -
(3*b*c^2*d^2 + 2*a*d^4)*x^2)*log(-d*x + sqrt(d*x + c)*sqrt(d*x - c)))/(d^7*
x^2 - c^2*d^5)
```

Sympy [C] time = 66.7475, size = 212, normalized size = 1.39

$$a\left(\frac{G_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4}, \frac{1}{4} & -\frac{1}{2}, \frac{1}{2}, 1, 1 \\ -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 1, 0 & \end{matrix} \middle| \frac{c^{2}}{d^{2}x^{2}}\right)}{2\pi^{\frac{3}{2}}d^{3}} + \frac{iG_{6,6}^{2,6}\left(\begin{matrix} -\frac{3}{2}, -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 1 & \\ -\frac{3}{4}, -\frac{1}{4} & -\frac{3}{2}, -1, 0, 0 \end{matrix} \middle| \frac{c^{2}e^{2i\pi}}{d^{2}x^{2}}\right)}{2\pi^{\frac{3}{2}}d^{3}}\right) + b\left(\frac{c^{2}G_{6,6}^{6,2}\left(\begin{matrix} -\frac{5}{4}, - \\ -\frac{5}{4}, -1, -\frac{3}{4}, \end{matrix}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] a*(meijerg(((-1/4, 1/4), (-1/2, 1/2, 1, 1)), ((-1/4, 0, 1/4, 1/2, 1, 0), ()
), c**2/(d**2*x**2))/(2*pi**(3/2)*d**3) + I*meijerg(((-3/2, -1, -3/4, -1/2,
 -1/4, 1), ()), ((-3/4, -1/4), (-3/2, -1, 0, 0)), c**2*exp_polar(2*I*pi)/(d
**2*x**2))/(2*pi**(3/2)*d**3)) + b*(c**2*meijerg(((-5/4, -3/4), (-3/2, -1/2
, 0, 1)), ((-5/4, -1, -3/4, -1/2, 0, 0), ()), c**2/(d**2*x**2))/(2*pi**(3/2
)*d**5) + I*c**2*meijerg(((-5/2, -2, -7/4, -3/2, -5/4, 1), ()), ((-7/4, -5/
4), (-5/2, -2, -1, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(2*pi**(3/2)*d*
*5))
```

Giac [A] time = 1.28983, size = 208, normalized size = 1.37

$$-\frac{\left(\left(3\,bd^{15} - \frac{(dx+c)bd^{15}}{c}\right)(dx+c) - \frac{bc^{2}d^{15} - ad^{17}}{c}\right)\sqrt{dx+c}}{768\sqrt{dx-c}} - \frac{\left(3\,bc^{2}d^{15} + 2\,ad^{17}\right)\log\left(\left(\sqrt{dx+c} - \sqrt{dx-c}\right)^{2}\right)}{768\,c} - \frac{2\,(b}{\left(\left(\sqrt{dx+c} - \right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/768*((3*b*d^15 - (d*x + c)*b*d^15/c)*(d*x + c) - (b*c^2*d^15 - a*d^17)/c
)*sqrt(d*x + c)/sqrt(d*x - c) - 1/768*(3*b*c^2*d^15 + 2*a*d^17)*log((sqrt(d
*x + c) - sqrt(d*x - c))^2)/c - 2*(b*c^3 + a*c*d^2)/(((sqrt(d*x + c) - sqrt
(d*x - c))^2 + 2*c)*d^5)
```

3.371 $\int \frac{x(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=76

$$\frac{\sqrt{dx-c}\sqrt{c+dx}\left(ad^2+2bc^2\right)}{c^2d^4}-\frac{x^2\left(\frac{a}{c^2}+\frac{b}{d^2}\right)}{\sqrt{dx-c}\sqrt{c+dx}}$$

```
[Out] -(((a/c^2 + b/d^2)*x^2)/(Sqrt[-c + d*x]*Sqrt[c + d*x])) + ((2*b*c^2 + a*d^2
)*Sqrt[-c + d*x]*Sqrt[c + d*x])/(c^2*d^4)
```

Rubi [A] time = 0.0535191, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {458, 74}

$$\frac{\sqrt{dx-c}\sqrt{c+dx}\left(ad^2+2bc^2\right)}{c^2d^4}-\frac{x^2\left(\frac{a}{c^2}+\frac{b}{d^2}\right)}{\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -(((a/c^2 + b/d^2)*x^2)/(Sqrt[-c + d*x]*Sqrt[c + d*x])) + ((2*b*c^2 + a*d^2
)*Sqrt[-c + d*x]*Sqrt[c + d*x])/(c^2*d^4)
```

Rule 458

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b1*b2*
c - a1*a2*d)*(e*x)^(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p +
 1))/(a1*a2*b1*b2*e*n*(p + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n
*(p + 1) + 1))/(a1*a2*b1*b2*n*(p + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^(p +
1)*(a2 + b2*x^(n/2))^(p + 1), x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m,
n}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && LtQ[p, -1] && (( !Inte
gerQ[p + 1/2] && NeQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p +
 1/2, 0] && LeQ[-1, m, -(n*(p + 1))]))
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(a+bx^2\right)}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx &= -\frac{\left(\frac{a}{c^2}+\frac{b}{d^2}\right)x^2}{\sqrt{-c+dx}\sqrt{c+dx}}-\left(-\frac{a}{c^2}-\frac{2b}{d^2}\right)\int \frac{x}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx\\ &= -\frac{\left(\frac{a}{c^2}+\frac{b}{d^2}\right)x^2}{\sqrt{-c+dx}\sqrt{c+dx}}+\frac{\left(\frac{a}{c^2}+\frac{2b}{d^2}\right)\sqrt{-c+dx}\sqrt{c+dx}}{d^2}\end{aligned}$$

Mathematica [A] time = 0.0283267, size = 45, normalized size = 0.59

$$\frac{-ad^2 - 2bc^2 + bd^2x^2}{d^4\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (-2*b*c^2 - a*d^2 + b*d^2*x^2)/(d^4*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

Maple [A] time = 0.004, size = 43, normalized size = 0.6

$$-\frac{-bd^2x^2 + ad^2 + 2bc^2}{d^4}\frac{1}{\sqrt{dx+c}}\frac{1}{\sqrt{dx-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)
```

```
[Out] -(-b*d^2*x^2+a*d^2+2*b*c^2)/(d*x+c)^(1/2)/d^4/(d*x-c)^(1/2)
```

Maxima [A] time = 0.966589, size = 93, normalized size = 1.22

$$\frac{bx^2}{\sqrt{d^2x^2 - c^2}d^2} - \frac{2bc^2}{\sqrt{d^2x^2 - c^2}d^4} - \frac{a}{\sqrt{d^2x^2 - c^2}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] b*x^2/(sqrt(d^2*x^2 - c^2)*d^2) - 2*b*c^2/(sqrt(d^2*x^2 - c^2)*d^4) - a/(sq
rt(d^2*x^2 - c^2)*d^2)
```

Fricas [A] time = 1.50678, size = 107, normalized size = 1.41

$$\frac{\left(bd^2x^2 - 2bc^2 - ad^2\right)\sqrt{dx+c}\sqrt{dx-c}}{d^6x^2 - c^2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] (b*d^2*x^2 - 2*b*c^2 - a*d^2)*sqrt(d*x + c)*sqrt(d*x - c)/(d^6*x^2 - c^2*d^
4)
```

Sympy [C] time = 52.8629, size = 201, normalized size = 2.64

$$a\left(-\frac{G_{6,6}^{5,3}\left(\begin{matrix}\frac{1}{4},\frac{3}{4},1 & 0,1,\frac{3}{2}\\ \frac{1}{4},\frac{1}{2},\frac{3}{4},1,\frac{3}{2} & 0\end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}cd^2}-\frac{iG_{6,6}^{2,6}\left(\begin{matrix}-1,-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},1 & \\ -\frac{1}{4},\frac{1}{4} & -1,-\frac{1}{2},\frac{1}{2},0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}cd^2}\right)+b\left(\frac{cG_{6,6}^{6,2}\left(\begin{matrix}-\frac{3}{4},-\frac{1}{4} & -1,0\\ -\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,\frac{1}{2},0 & \end{matrix}\right.}{2\pi^{\frac{3}{2}}d^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] a*(-meijerg(((1/4, 3/4, 1), (0, 1, 3/2)), ((1/4, 1/2, 3/4, 1, 3/2), (0,)),
c**2/(d**2*x**2))/(2*pi**(3/2)*c*d**2) - I*meijerg(((-1, -1/2, -1/4, 0, 1/4
, 1), ()), ((-1/4, 1/4), (-1, -1/2, 1/2, 0)), c**2*exp_polar(2*I*pi)/(d**2*
x**2))/(2*pi**(3/2)*c*d**2)) + b*(c*meijerg(((-3/4, -1/4), (-1, 0, 1/2, 1))
, ((-3/4, -1/2, -1/4, 0, 1/2, 0), ()), c**2/(d**2*x**2))/(2*pi**(3/2)*d**4)
 - I*c*meijerg(((-2, -3/2, -5/4, -1, -3/4, 1), ()), ((-5/4, -3/4), (-2, -3/
2, -1/2, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(2*pi**(3/2)*d**4))
```

Giac [A] time = 1.25029, size = 127, normalized size = 1.67

$$\frac{\left(2\,(dx+c)bd^8-\frac{5\,bc^2d^8+ad^{10}}{c}\right)\sqrt{dx+c}}{32\,\sqrt{dx-c}}+\frac{2\left(bc^2+ad^2\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+2\,c\right)d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 1/32*(2*(d*x + c)*b*d^8 - (5*b*c^2*d^8 + a*d^10)/c)*sqrt(d*x + c)/sqrt(d*x
- c) + 2*(b*c^2 + a*d^2)/(((sqrt(d*x + c) - sqrt(d*x - c))^2 + 2*c)*d^4)
```

3.372 $\int \frac{a+bx^2}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=63

$$\frac{2b\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^3}-\frac{x\left(\frac{a}{c^2}+\frac{b}{d^2}\right)}{\sqrt{dx-c}\sqrt{c+dx}}$$

```
[Out] -(((a/c^2 + b/d^2)*x)/(Sqrt[-c + d*x]*Sqrt[c + d*x])) + (2*b*ArcTanh[Sqrt[-
c + d*x]/Sqrt[c + d*x]])/d^3
```

Rubi [A] time = 0.0316689, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {386, 63, 217, 206}

$$\frac{2b\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^3}-\frac{x\left(\frac{a}{c^2}+\frac{b}{d^2}\right)}{\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -(((a/c^2 + b/d^2)*x)/(Sqrt[-c + d*x]*Sqrt[c + d*x])) + (2*b*ArcTanh[Sqrt[-
c + d*x]/Sqrt[c + d*x]])/d^3
```

Rule 386

```
Int[((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_
.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b1*b2*c - a1*a2*d)*x*(a1
+ b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*b1*b2*n*(p + 1)), x
] - Dist[(a1*a2*d - b1*b2*c*(n*(p + 1) + 1))/(a1*a2*b1*b2*n*(p + 1)), Int[(
a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1), x], x] /; FreeQ[{a1, b1
, a2, b2, c, d, n}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (LtQ[p
, -1] || ILtQ[1/n + p, 0])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{a+bx^2}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = -\frac{\left(\frac{a}{c^2}+\frac{b}{d^2}\right)x}{\sqrt{-c+dx}\sqrt{c+dx}} + \frac{b\int \frac{1}{\sqrt{-c+dx}\sqrt{c+dx}}\,dx}{d^2}$$
$$= -\frac{\left(\frac{a}{c^2}+\frac{b}{d^2}\right)x}{\sqrt{-c+dx}\sqrt{c+dx}} + \frac{(2b)\,\text{Subst}\left(\int \frac{1}{\sqrt{2c+x^2}}\,dx, x, \sqrt{-c+dx}\right)}{d^3}$$
$$= -\frac{\left(\frac{a}{c^2}+\frac{b}{d^2}\right)x}{\sqrt{-c+dx}\sqrt{c+dx}} + \frac{(2b)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d^3}$$
$$= -\frac{\left(\frac{a}{c^2}+\frac{b}{d^2}\right)x}{\sqrt{-c+dx}\sqrt{c+dx}} + \frac{2b\tanh^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d^3}$$

Mathematica [A] time = 0.167137, size = 86, normalized size = 1.37

$$\frac{2bc^{5/2}\sqrt{\frac{dx}{c}+1}\sinh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{2}\sqrt{c}}\right)-\frac{dx\left(ad^2+bc^2\right)}{\sqrt{dx-c}}}{c^2d^3\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (-((d*(b*c^2 + a*d^2)*x)/Sqrt[-c + d*x]) + 2*b*c^(5/2)*Sqrt[1 + (d*x)/c]*Ar
cSinh[Sqrt[-c + d*x]/(Sqrt[2]*Sqrt[c])])/(c^2*d^3*Sqrt[c + d*x])
```

Maple [C] time = 0.018, size = 160, normalized size = 2.5

$$\frac{\text{csgn}(d)}{c^2d^3}\left(\ln\left(\left(\text{csgn}(d)\sqrt{(dx-c)(dx+c)}+dx\right)\text{csgn}(d)\right)x^2bc^2d^2-\text{csgn}(d)\,d^3\sqrt{d^2x^2-c^2}xa-\text{csgn}(d)\,d\sqrt{d^2x^2-c^2}xbc^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)
```

```
[Out] (ln((csgn(d)*((d*x-c)*(d*x+c))^(1/2)+d*x)*csgn(d))*x^2*b*c^2*d^2-csgn(d)*d^
3*(d^2*x^2-c^2)^(1/2)*x*a-csgn(d)*d*(d^2*x^2-c^2)^(1/2)*x*b*c^2-ln((csgn(d)
*((d*x-c)*(d*x+c))^(1/2)+d*x)*csgn(d))*b*c^4)*csgn(d)/(d^2*x^2-c^2)^(1/2)/c
^2/d^3/(d*x+c)^(1/2)/(d*x-c)^(1/2)
```

Maxima [A] time = 0.990491, size = 115, normalized size = 1.83

$$-\frac{ax}{\sqrt{d^2x^2-c^2}c^2}-\frac{bx}{\sqrt{d^2x^2-c^2}d^2}+\frac{b\log\left(2\,d^2x+2\,\sqrt{d^2x^2-c^2}\sqrt{d^2}\right)}{\sqrt{d^2}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -a*x/(sqrt(d^2*x^2 - c^2)*c^2) - b*x/(sqrt(d^2*x^2 - c^2)*d^2) + b*log(2*d^
2*x + 2*sqrt(d^2*x^2 - c^2)*sqrt(d^2))/(sqrt(d^2)*d^2)
```

Fricas [B] time = 1.51159, size = 252, normalized size = 4.

$$\frac{bc^4 + ac^2d^2 - \left(bc^2d + ad^3\right)\sqrt{dx + c}\sqrt{dx - c}x - \left(bc^2d^2 + ad^4\right)x^2 - \left(bc^2d^2x^2 - bc^4\right)\log\left(-dx + \sqrt{dx + c}\sqrt{dx - c}\right)}{c^2d^5x^2 - c^4d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] (b*c^4 + a*c^2*d^2 - (b*c^2*d + a*d^3)*sqrt(d*x + c)*sqrt(d*x - c)*x - (b*c
^2*d^2 + a*d^4)*x^2 - (b*c^2*d^2*x^2 - b*c^4)*log(-d*x + sqrt(d*x + c)*sqrt
(d*x - c)))/(c^2*d^5*x^2 - c^4*d^3)
```

Sympy [C] time = 39.6289, size = 182, normalized size = 2.89

$$a\left(-\frac{G_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & \frac{1}{2},\frac{3}{2},2\\ \frac{3}{4},1,\frac{5}{4},\frac{3}{2},2 & 0\end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}c^2d}+\frac{iG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1 & \\ \frac{1}{4},\frac{3}{4} & -\frac{1}{2},0,1,0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}c^2d}\right)+b\left(\frac{G_{6,6}^{6,2}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4} & -\frac{1}{2},\frac{1}{2},1,1\\ -\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1,0 & \end{matrix}\middle|\ \right.}{2\pi^{\frac{3}{2}}d^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] a*(-meijerg(((3/4, 5/4, 1), (1/2, 3/2, 2)), ((3/4, 1, 5/4, 3/2, 2), (0,)),
c**2/(d**2*x**2))/(2*pi**(3/2)*c**2*d) + I*meijerg(((-1/2, 0, 1/4, 1/2, 3/4
, 1), ()), ((1/4, 3/4), (-1/2, 0, 1, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2
))/(2*pi**(3/2)*c**2*d)) + b*(meijerg(((-1/4, 1/4), (-1/2, 1/2, 1, 1)), ((-
1/4, 0, 1/4, 1/2, 1, 0), ()), c**2/(d**2*x**2))/(2*pi**(3/2)*d**3) + I*meij
erg(((-3/2, -1, -3/4, -1/2, -1/4, 1), ()), ((-3/4, -1/4), (-3/2, -1, 0, 0))
, c**2*exp_polar(2*I*pi)/(d**2*x**2))/(2*pi**(3/2)*d**3))
```

Giac [B] time = 1.25857, size = 153, normalized size = 2.43

$$-\frac{b\log\left(\left(\sqrt{dx + c} - \sqrt{dx - c}\right)^2\right)}{d^3} - \frac{2\left(bc^2 + ad^2\right)}{\left(\left(\sqrt{dx + c} - \sqrt{dx - c}\right)^2 + 2\,c\right)cd^3} - \frac{\left(bc^2d^3 + ad^5\right)\sqrt{dx + c}}{2\,\sqrt{dx - c}c^2d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -b*log((sqrt(d*x + c) - sqrt(d*x - c))^2)/d^3 - 2*(b*c^2 + a*d^2)/(((sqrt(d
*x + c) - sqrt(d*x - c))^2 + 2*c)*c*d^3) - 1/2*(b*c^2*d^3 + a*d^5)*sqrt(d*x
 + c)/(sqrt(d*x - c)*c^2*d^6)
```

3.373 $\int \frac{a+bx^2}{x(-c+dx)^{3/2}(c+dx)^{3/2}} \, dx$

Optimal. Leaf size=65

$$-\frac{\frac{a}{c^2}+\frac{b}{d^2}}{\sqrt{dx-c}\sqrt{c+dx}}-\frac{a\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{c^3}$$

```
[Out] -((a/c^2 + b/d^2)/(Sqrt[-c + d*x]*Sqrt[c + d*x])) - (a*ArcTan[(Sqrt[-c + d*
x]*Sqrt[c + d*x])/c])/c^3
```

Rubi [A] time = 0.0805375, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {458, 92, 205}

$$-\frac{\frac{a}{c^2}+\frac{b}{d^2}}{\sqrt{dx-c}\sqrt{c+dx}}-\frac{a\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -((a/c^2 + b/d^2)/(Sqrt[-c + d*x]*Sqrt[c + d*x])) - (a*ArcTan[(Sqrt[-c + d*
x]*Sqrt[c + d*x])/c])/c^3
```

Rule 458

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b1*b2*
c - a1*a2*d)*(e*x)^(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p +
 1))/(a1*a2*b1*b2*e*n*(p + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n
*(p + 1) + 1))/(a1*a2*b1*b2*n*(p + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^(p +
1)*(a2 + b2*x^(n/2))^(p + 1), x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m,
n}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && LtQ[p, -1] && (( !Inte
gerQ[p + 1/2] && NeQ[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p +
 1/2, 0] && LeQ[-1, m, -(n*(p + 1))]))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a+bx^2}{x(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = -\frac{\frac{a}{c^2}+\frac{b}{d^2}}{\sqrt{-c+dx}\sqrt{c+dx}} - \frac{a\int \frac{1}{x\sqrt{-c+dx}\sqrt{c+dx}}\,dx}{c^2}$$
$$= -\frac{\frac{a}{c^2}+\frac{b}{d^2}}{\sqrt{-c+dx}\sqrt{c+dx}} - \frac{(ad)\,\text{Subst}\left(\int \frac{1}{c^2d+dx^2}\,dx, x, \sqrt{-c+dx}\sqrt{c+dx}\right)}{c^2}$$
$$= -\frac{\frac{a}{c^2}+\frac{b}{d^2}}{\sqrt{-c+dx}\sqrt{c+dx}} - \frac{a\tan^{-1}\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)}{c^3}$$

Mathematica [A] time = 0.0388531, size = 84, normalized size = 1.29

$$-\frac{ad^2\sqrt{d^2x^2-c^2}\tan^{-1}\left(\frac{\sqrt{d^2x^2-c^2}}{c}\right)+acd^2+bc^3}{c^3d^2\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -((b*c^3 + a*c*d^2 + a*d^2*Sqrt[-c^2 + d^2*x^2]*ArcTan[Sqrt[-c^2 + d^2*x^2]
/c])/(c^3*d^2*Sqrt[-c + d*x]*Sqrt[c + d*x]))
```

Maple [B] time = 0.021, size = 188, normalized size = 2.9

$$\frac{1}{c^2d^2}\left(\ln\left(-2\frac{c^2-\sqrt{-c^2}\sqrt{d^2x^2-c^2}}{x}\right)x^2ad^4-\ln\left(-2\frac{c^2-\sqrt{-c^2}\sqrt{d^2x^2-c^2}}{x}\right)ac^2d^2-\sqrt{-c^2}\sqrt{d^2x^2-c^2}ad^2-bc^2\sqrt{-c^2}\sqrt{d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)
```

```
[Out] 1/c^2*(ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*x^2*a*d^4-ln(-2*(c^2
-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*a*c^2*d^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^
(1/2)*a*d^2-b*c^2*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/(-c^2)^(1/2)/(d^2*x^2-c
^2)^(1/2)/d^2/(d*x+c)^(1/2)/(d*x-c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.53798, size = 201, normalized size = 3.09

$$-\frac{(bc^3+acd^2)\sqrt{dx+c}\sqrt{dx-c}+2\left(ad^4x^2-ac^2d^2\right)\arctan\left(-\frac{dx-\sqrt{dx+c}\sqrt{dx-c}}{c}\right)}{c^3d^4x^2-c^5d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -((b*c^3 + a*c*d^2)*sqrt(d*x + c)*sqrt(d*x - c) + 2*(a*d^4*x^2 - a*c^2*d^2)
*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c))/c))/(c^3*d^4*x^2 - c^5*d^2)
```

Sympy [C] time = 51.5601, size = 172, normalized size = 2.65

$$a\left(-\frac{G_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & 1,2,\frac{5}{2}\\ \frac{5}{4},\frac{3}{2},\frac{7}{4},2,\frac{5}{2} & 0\end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}c^3}-\frac{iG_{6,6}^{2,6}\left(\begin{matrix}0,\frac{1}{2},\frac{3}{4},1,\frac{5}{4},1 & \\ \frac{3}{4},\frac{5}{4} & 0,\frac{1}{2},\frac{3}{2},0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}c^3}\right)+b\left(-\frac{G_{6,6}^{5,3}\left(\begin{matrix}\frac{1}{4},\frac{3}{4},1 & 0,1,\frac{3}{2}\\ \frac{1}{4},\frac{1}{2},\frac{3}{4},1,\frac{3}{2} & 0\end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}cd^2}-\frac{iG_6^2}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] a*(-meijerg(((5/4, 7/4, 1), (1, 2, 5/2)), ((5/4, 3/2, 7/4, 2, 5/2), (0,)),
c**2/(d**2*x**2))/(2*pi**(3/2)*c**3) - I*meijerg(((0, 1/2, 3/4, 1, 5/4, 1),
 ()), ((3/4, 5/4), (0, 1/2, 3/2, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(
2*pi**(3/2)*c**3)) + b*(-meijerg(((1/4, 3/4, 1), (0, 1, 3/2)), ((1/4, 1/2,
3/4, 1, 3/2), (0,)), c**2/(d**2*x**2))/(2*pi**(3/2)*c*d**2) - I*meijerg(((-
1, -1/2, -1/4, 0, 1/4, 1), ()), ((-1/4, 1/4), (-1, -1/2, 1/2, 0)), c**2*exp
_polar(2*I*pi)/(d**2*x**2))/(2*pi**(3/2)*c*d**2))
```

Giac [B] time = 1.31751, size = 155, normalized size = 2.38

$$\frac{2\,a\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2\,c}\right)}{c^3}-\frac{\left(bc^2+ad^2\right)\sqrt{dx+c}}{2\,\sqrt{dx-c}c^3d^2}+\frac{2\left(bc^2+ad^2\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+2\,c\right)c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 2*a*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/c)/c^3 - 1/2*(b*c^2 + a*d^
2)*sqrt(d*x + c)/(sqrt(d*x - c)*c^3*d^2) + 2*(b*c^2 + a*d^2)/(((sqrt(d*x +
c) - sqrt(d*x - c))^2 + 2*c)*c^2*d^2)
```

3.374 $\int \frac{a+bx^2}{x^2(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=67

$$\frac{a}{c^2x\sqrt{dx-c}\sqrt{c+dx}}-\frac{x\left(2ad^2+bc^2\right)}{c^4\sqrt{dx-c}\sqrt{c+dx}}$$

```
[Out] a/(c^2*x*Sqrt[-c + d*x]*Sqrt[c + d*x]) - ((b*c^2 + 2*a*d^2)*x)/(c^4*Sqrt[-c
 + d*x]*Sqrt[c + d*x])
```

Rubi [A] time = 0.076683, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {454, 39}

$$\frac{a}{c^2x\sqrt{dx-c}\sqrt{c+dx}}-\frac{x\left(2ad^2+bc^2\right)}{c^4\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x^2*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] a/(c^2*x*Sqrt[-c + d*x]*Sqrt[c + d*x]) - ((b*c^2 + 2*a*d^2)*x)/(c^4*Sqrt[-c
 + d*x]*Sqrt[c + d*x])
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2}{x^2(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx &= \frac{a}{c^2x\sqrt{-c+dx}\sqrt{c+dx}}+\left(b+\frac{2ad^2}{c^2}\right)\int \frac{1}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx\\ &= \frac{a}{c^2x\sqrt{-c+dx}\sqrt{c+dx}}-\frac{\left(bc^2+2ad^2\right)x}{c^4\sqrt{-c+dx}\sqrt{c+dx}}\end{aligned}$$

Mathematica [A] time = 0.0241339, size = 51, normalized size = 0.76

$$\frac{a\left(c^2-2d^2x^2\right)-bc^2x^2}{c^4x\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x^2*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (-(b*c^2*x^2) + a*(c^2 - 2*d^2*x^2))/(c^4*x*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

Maple [A] time = 0.004, size = 48, normalized size = 0.7

$$\frac{-2\,ad^2x^2 - bc^2x^2 + ac^2}{xc^4}\frac{1}{\sqrt{dx+c}}\frac{1}{\sqrt{dx-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^2/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)
```

```
[Out] (-2*a*d^2*x^2-b*c^2*x^2+a*c^2)/(d*x+c)^(1/2)/x/c^4/(d*x-c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^2/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.49661, size = 197, normalized size = 2.94

$$-\frac{\left(bc^2d^2 + 2\,ad^4\right)x^3 - \left(ac^2d - \left(bc^2d + 2\,ad^3\right)x^2\right)\sqrt{dx+c}\sqrt{dx-c} - \left(bc^4 + 2\,ac^2d^2\right)x}{c^4d^3x^3 - c^6dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^2/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -((b*c^2*d^2 + 2*a*d^4)*x^3 - (a*c^2*d - (b*c^2*d + 2*a*d^3)*x^2)*sqrt(d*x
+ c)*sqrt(d*x - c) - (b*c^4 + 2*a*c^2*d^2)*x)/(c^4*d^3*x^3 - c^6*d*x)
```

Sympy [C] time = 58.0321, size = 165, normalized size = 2.46

$$a\left(-\frac{dG_{6,6}^{5,3}\left(\begin{matrix} \frac{7}{4}, \frac{9}{4}, 1 & \frac{3}{2}, \frac{5}{2}, 3 \\ \frac{7}{4}, 2, \frac{9}{4}, \frac{5}{2}, 3 & 0 \end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}c^4} + \frac{idG_{6,6}^{2,6}\left(\begin{matrix} \frac{1}{2}, 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 1 & \\ \frac{5}{4}, \frac{7}{4} & \frac{1}{2}, 1, 2, 0 \end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}c^4}\right) + b\left(-\frac{G_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4}, \frac{5}{4}, 1 & \frac{1}{2}, \frac{3}{2}, 2 \\ \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2}, 2 & 0 \end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}c^2d} + \cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**2/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] a*(-d*meijerg(((7/4, 9/4, 1), (3/2, 5/2, 3)), ((7/4, 2, 9/4, 5/2, 3), (0,))
, c**2/(d**2*x**2))/(2*pi**(3/2)*c**4) + I*d*meijerg(((1/2, 1, 5/4, 3/2, 7/
4, 1), ()), ((5/4, 7/4), (1/2, 1, 2, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2
))/(2*pi**(3/2)*c**4)) + b*(-meijerg(((3/4, 5/4, 1), (1/2, 3/2, 2)), ((3/4,
 1, 5/4, 3/2, 2), (0,)), c**2/(d**2*x**2))/(2*pi**(3/2)*c**2*d) + I*meijerg
(((-1/2, 0, 1/4, 1/2, 3/4, 1), ()), ((1/4, 3/4), (-1/2, 0, 1, 0)), c**2*exp
_polar(2*I*pi)/(d**2*x**2))/(2*pi**(3/2)*c**2*d))
```

Giac [B] time = 1.36761, size = 296, normalized size = 4.42

$$-\frac{\left(bc^2+ad^2\right)\sqrt{dx+c}}{2\sqrt{dx-c}c^4d}-\frac{2\left(bc^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+ad^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,acd^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+4\,bc\right.}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^6+2\,c\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4+4\,c^2\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+8\,c^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^2/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(b*c^2 + a*d^2)*sqrt(d*x + c)/(sqrt(d*x - c)*c^4*d) - 2*(b*c^2*(sqrt(d
*x + c) - sqrt(d*x - c))^4 + a*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*a*
c*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^2 + 4*b*c^4 + 12*a*c^2*d^2)/(((sqrt(d
*x + c) - sqrt(d*x - c))^6 + 2*c*(sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2*
(sqrt(d*x + c) - sqrt(d*x - c))^2 + 8*c^3)*c^3*d)
```

3.375 $\int \frac{a+bx^2}{x^3(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=117

$$-\frac{3ad^2+2bc^2}{2c^4\sqrt{dx-c}\sqrt{c+dx}}-\frac{(3ad^2+2bc^2)\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{2c^5}+\frac{a}{2c^2x^2\sqrt{dx-c}\sqrt{c+dx}}$$

```
[Out] -(2*b*c^2 + 3*a*d^2)/(2*c^4*Sqrt[-c + d*x]*Sqrt[c + d*x]) + a/(2*c^2*x^2*Sq
rt[-c + d*x]*Sqrt[c + d*x]) - ((2*b*c^2 + 3*a*d^2)*ArcTan[(Sqrt[-c + d*x]*S
qrt[c + d*x])/c])/(2*c^5)
```

Rubi [A] time = 0.0975361, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {454, 104, 21, 92, 205}

$$-\frac{3ad^2+2bc^2}{2c^4\sqrt{dx-c}\sqrt{c+dx}}-\frac{(3ad^2+2bc^2)\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{2c^5}+\frac{a}{2c^2x^2\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x^3*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] -(2*b*c^2 + 3*a*d^2)/(2*c^4*Sqrt[-c + d*x]*Sqrt[c + d*x]) + a/(2*c^2*x^2*Sq
rt[-c + d*x]*Sqrt[c + d*x]) - ((2*b*c^2 + 3*a*d^2)*ArcTan[(Sqrt[-c + d*x]*S
qrt[c + d*x])/c])/(2*c^5)
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a+bx^2}{x^3(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{a}{2c^2x^2\sqrt{-c+dx}\sqrt{c+dx}} + \frac{1}{2}\left(2b+\frac{3ad^2}{c^2}\right)\int \frac{1}{x(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$$

$$= -\frac{2bc^2+3ad^2}{2c^4\sqrt{-c+dx}\sqrt{c+dx}} + \frac{a}{2c^2x^2\sqrt{-c+dx}\sqrt{c+dx}} + \frac{\left(-2b-\frac{3ad^2}{c^2}\right)\int \frac{cd+d^2x}{x\sqrt{-c+dx}(c+dx)^{3/2}}\,d}{2c^2d}$$

$$= -\frac{2bc^2+3ad^2}{2c^4\sqrt{-c+dx}\sqrt{c+dx}} + \frac{a}{2c^2x^2\sqrt{-c+dx}\sqrt{c+dx}} - \frac{\left(2bc^2+3ad^2\right)\int \frac{1}{x\sqrt{-c+dx}\sqrt{c+dx}}\,d}{2c^4}$$

$$= -\frac{2bc^2+3ad^2}{2c^4\sqrt{-c+dx}\sqrt{c+dx}} + \frac{a}{2c^2x^2\sqrt{-c+dx}\sqrt{c+dx}} - \frac{\left(d\left(2bc^2+3ad^2\right)\right)\operatorname{Subst}\left(\int \frac{}{c^2d}\right.}{2c^4}$$

$$= -\frac{2bc^2+3ad^2}{2c^4\sqrt{-c+dx}\sqrt{c+dx}} + \frac{a}{2c^2x^2\sqrt{-c+dx}\sqrt{c+dx}} - \frac{\left(2bc^2+3ad^2\right)\tan^{-1}\left(\frac{\sqrt{-c+dx}\sqrt{c+}}{c}\right.}{2c^5}$$

Mathematica [C] time = 0.0282738, size = 75, normalized size = 0.64

$$\frac{ac^2-x^2\left(3ad^2+2bc^2\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1-\frac{d^2x^2}{c^2}\right)}{2c^4x^2\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x^3*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (a*c^2 - (2*b*c^2 + 3*a*d^2)*x^2*Hypergeometric2F1[-1/2, 1, 1/2, 1 - (d^2*x
^2)/c^2])/(2*c^4*x^2*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

Maple [B] time = 0.024, size = 315, normalized size = 2.7

$$\frac{1}{2\,c^4x^2}\left(3\,\ln\left(-2\,\frac{c^2-\sqrt{-c^2}\sqrt{d^2x^2-c^2}}{x}\right)x^4ad^4+2\,\ln\left(-2\,\frac{c^2-\sqrt{-c^2}\sqrt{d^2x^2-c^2}}{x}\right)x^4bc^2d^2-3\,\ln\left(-2\,\frac{c^2-\sqrt{-c^2}\sqrt{d^2x^2-}}{x}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^3/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)
```

```
[Out] 1/2/c^4*(3*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*x^4*a*d^4+2*ln(-
2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*x^4*b*c^2*d^2-3*ln(-2*(c^2-(-c^
2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*x^2*a*c^2*d^2-2*ln(-2*(c^2-(-c^2)^(1/2)*(d
```

```
^2*x^2-c^2)^(1/2))/x)*x^2*b*c^4-3*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2)*x^2*a*d^
2-2*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2)*x^2*b*c^2+(-c^2)^(1/2)*(d^2*x^2-c^2)^(
1/2)*a*c^2)/x^2/(-c^2)^(1/2)/(d^2*x^2-c^2)^(1/2)/(d*x+c)^(1/2)/(d*x-c)^(1/2
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^3/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.59939, size = 278, normalized size = 2.38

$$\frac{\left(ac^3-\left(2\,bc^3+3\,acd^2\right)x^2\right)\sqrt{dx+c}\sqrt{dx-c}-2\left(\left(2\,bc^2d^2+3\,ad^4\right)x^4-\left(2\,bc^4+3\,ac^2d^2\right)x^2\right)\arctan\left(-\frac{dx-\sqrt{dx+c}\sqrt{dx-c}}{c}\right)}{2\left(c^5d^2x^4-c^7x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^3/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((a*c^3 - (2*b*c^3 + 3*a*c*d^2)*x^2)*sqrt(d*x + c)*sqrt(d*x - c) - 2*((
2*b*c^2*d^2 + 3*a*d^4)*x^4 - (2*b*c^4 + 3*a*c^2*d^2)*x^2)*arctan(-(d*x - sq
rt(d*x + c)*sqrt(d*x - c))/c))/(c^5*d^2*x^4 - c^7*x^2)
```

Sympy [C] time = 79.3979, size = 165, normalized size = 1.41

$$a\left(-\frac{d^2G_{6,6}^{5,3}\left(\begin{matrix}\frac{9}{4},\frac{11}{4},1 & 2,3,\frac{7}{2}\\ \frac{9}{4},\frac{5}{2},\frac{11}{4},3,\frac{7}{2} & 0\end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}c^5}-\frac{id^2G_{6,6}^{2,6}\left(\begin{matrix}1,\frac{3}{2},\frac{7}{4},2,\frac{9}{4},1 & \\ \frac{7}{4},\frac{9}{4} & 1,\frac{3}{2},\frac{5}{2},0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}c^5}\right)+b\left(-\frac{G_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & 1,2,\frac{5}{2}\\ \frac{5}{4},\frac{3}{2},\frac{7}{4},2,\frac{5}{2} & 0\end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**3/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] a*(-d**2*meijerg(((9/4, 11/4, 1), (2, 3, 7/2)), ((9/4, 5/2, 11/4, 3, 7/2),
(0,)), c**2/(d**2*x**2))/(2*pi**(3/2)*c**5) - I*d**2*meijerg(((1, 3/2, 7/4,
 2, 9/4, 1), ()), ((7/4, 9/4), (1, 3/2, 5/2, 0)), c**2*exp_polar(2*I*pi)/(d
**2*x**2))/(2*pi**(3/2)*c**5)) + b*(-meijerg(((5/4, 7/4, 1), (1, 2, 5/2)),
((5/4, 3/2, 7/4, 2, 5/2), (0,)), c**2/(d**2*x**2))/(2*pi**(3/2)*c**3) - I*m
eijerg(((0, 1/2, 3/4, 1, 5/4, 1), ()), ((3/4, 5/4), (0, 1/2, 3/2, 0)), c**2
*exp_polar(2*I*pi)/(d**2*x**2))/(2*pi**(3/2)*c**3))
```

Giac [B] time = 1.38762, size = 285, normalized size = 2.44

$$\frac{\left(2\,bc^2+3\,ad^2\right)\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2\,c}\right)}{c^5}-\frac{\left(bc^2+ad^2\right)\sqrt{dx+c}}{2\,\sqrt{dx-c}c^5}+\frac{2\left(bc^2+ad^2\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+2\,c\right)c^4}+\frac{2\left(ad^2\left(\sqrt{dx+c}-\right.\right.}{\left(\left(\sqrt{\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^3/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] (2*b*c^2 + 3*a*d^2)*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/c)/c^5 - 1
/2*(b*c^2 + a*d^2)*sqrt(d*x + c)/(sqrt(d*x - c)*c^5) + 2*(b*c^2 + a*d^2)/((
(sqrt(d*x + c) - sqrt(d*x - c))^2 + 2*c)*c^4) + 2*(a*d^2*(sqrt(d*x + c) - s
qrt(d*x - c))^6 - 4*a*c^2*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^2)/(((sqrt(d*
x + c) - sqrt(d*x - c))^4 + 4*c^2)^2*c^4)
```

3.376 $\int \frac{a+bx^2}{x^4(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=119

$$-\frac{2d^2x\left(4ad^2+3bc^2\right)}{3c^6\sqrt{dx-c}\sqrt{c+dx}}+\frac{4ad^2+3bc^2}{3c^4x\sqrt{dx-c}\sqrt{c+dx}}+\frac{a}{3c^2x^3\sqrt{dx-c}\sqrt{c+dx}}$$

```
[Out] a/(3*c^2*x^3*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (3*b*c^2 + 4*a*d^2)/(3*c^4*x*S
qrt[-c + d*x]*Sqrt[c + d*x]) - (2*d^2*(3*b*c^2 + 4*a*d^2)*x)/(3*c^6*Sqrt[-c
 + d*x]*Sqrt[c + d*x])
```

Rubi [A] time = 0.0956436, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {454, 103, 12, 39}

$$-\frac{2d^2x\left(4ad^2+3bc^2\right)}{3c^6\sqrt{dx-c}\sqrt{c+dx}}+\frac{4ad^2+3bc^2}{3c^4x\sqrt{dx-c}\sqrt{c+dx}}+\frac{a}{3c^2x^3\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x^4*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] a/(3*c^2*x^3*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (3*b*c^2 + 4*a*d^2)/(3*c^4*x*S
qrt[-c + d*x]*Sqrt[c + d*x]) - (2*d^2*(3*b*c^2 + 4*a*d^2)*x)/(3*c^6*Sqrt[-c
 + d*x]*Sqrt[c + d*x])
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
```

```
Q[b*c + a*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+bx^2}{x^4(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx &= \frac{a}{3c^2x^3\sqrt{-c+dx}\sqrt{c+dx}} + \frac{1}{3}\left(3b+\frac{4ad^2}{c^2}\right)\int \frac{1}{x^2(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx \\
&= \frac{a}{3c^2x^3\sqrt{-c+dx}\sqrt{c+dx}} + \frac{3bc^2+4ad^2}{3c^4x\sqrt{-c+dx}\sqrt{c+dx}} + \frac{\left(3b+\frac{4ad^2}{c^2}\right)\int \frac{2d^2}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx}{3c^2} \\
&= \frac{a}{3c^2x^3\sqrt{-c+dx}\sqrt{c+dx}} + \frac{3bc^2+4ad^2}{3c^4x\sqrt{-c+dx}\sqrt{c+dx}} + \frac{\left(2d^2\left(3b+\frac{4ad^2}{c^2}\right)\right)\int \frac{1}{(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx}{3c^2} \\
&= \frac{a}{3c^2x^3\sqrt{-c+dx}\sqrt{c+dx}} + \frac{3bc^2+4ad^2}{3c^4x\sqrt{-c+dx}\sqrt{c+dx}} - \frac{2d^2\left(3bc^2+4ad^2\right)x}{3c^6\sqrt{-c+dx}\sqrt{c+dx}}
\end{aligned}
$$

Mathematica [A] time = 0.02844, size = 77, normalized size = 0.65

$$\frac{a\left(4c^2d^2x^2+c^4-8d^4x^4\right)+3bc^2x^2\left(c^2-2d^2x^2\right)}{3c^6x^3\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x^4*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (3*b*c^2*x^2*(c^2 - 2*d^2*x^2) + a*(c^4 + 4*c^2*d^2*x^2 - 8*d^4*x^4))/(3*c^
6*x^3*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

Maple [A] time = 0.006, size = 73, normalized size = 0.6

$$\frac{-8\,ad^4x^4-6\,bc^2d^2x^4+4\,ac^2d^2x^2+3\,bc^4x^2+ac^4}{3\,x^3c^6}\frac{1}{\sqrt{dx+c}}\frac{1}{\sqrt{dx-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^4/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)
```

```
[Out] 1/3*(-8*a*d^4*x^4-6*b*c^2*d^2*x^4+4*a*c^2*d^2*x^2+3*b*c^4*x^2+a*c^4)/(d*x+c
)^(1/2)/x^3/c^6/(d*x-c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^4/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.5375, size = 265, normalized size = 2.23

$$-\frac{2\left(3\,bc^2d^3+4\,ad^5\right)x^5-2\left(3\,bc^4d+4\,ac^2d^3\right)x^3-\left(ac^4-2\left(3\,bc^2d^2+4\,ad^4\right)x^4+\left(3\,bc^4+4\,ac^2d^2\right)x^2\right)\sqrt{dx+c}\sqrt{dx-c}}{3\left(c^6d^2x^5-c^8x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^4/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*(3*b*c^2*d^3 + 4*a*d^5)*x^5 - 2*(3*b*c^4*d + 4*a*c^2*d^3)*x^3 - (a*
c^4 - 2*(3*b*c^2*d^2 + 4*a*d^4)*x^4 + (3*b*c^4 + 4*a*c^2*d^2)*x^2)*sqrt(d*x
 + c)*sqrt(d*x - c))/(c^6*d^2*x^5 - c^8*x^3)
```

Sympy [C] time = 129.492, size = 165, normalized size = 1.39

$$a\left(-\frac{d^3G_{6,6}^{5,3}\left(\begin{matrix}\frac{11}{4},\frac{13}{4},1 & \frac{5}{2},\frac{7}{2},4\\ \frac{11}{4},3,\frac{13}{4},\frac{7}{2},4 & 0\end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}c^6}+\frac{id^3G_{6,6}^{2,6}\left(\begin{matrix}\frac{3}{2},2,\frac{9}{4},\frac{5}{2},\frac{11}{4},1 & \\ \frac{9}{4},\frac{11}{4} & \frac{3}{2},2,3,0\end{matrix}\middle|\frac{c^2e^{2i\pi}}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}c^6}\right)+b\left(-\frac{dG_{6,6}^{5,3}\left(\begin{matrix}\frac{7}{4},\frac{9}{4},1 & \frac{3}{2},\frac{5}{2},3\\ \frac{7}{4},2,\frac{9}{4},\frac{5}{2},3 & 0\end{matrix}\middle|\frac{c^2}{d^2x^2}\right)}{2\pi^{\frac{3}{2}}c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**4/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] a*(-d**3*meijerg(((11/4, 13/4, 1), (5/2, 7/2, 4)), ((11/4, 3, 13/4, 7/2, 4)
, (0,)), c**2/(d**2*x**2))/(2*pi**(3/2)*c**6) + I*d**3*meijerg(((3/2, 2, 9/
4, 5/2, 11/4, 1), ()), ((9/4, 11/4), (3/2, 2, 3, 0)), c**2*exp_polar(2*I*pi
)/(d**2*x**2))/(2*pi**(3/2)*c**6)) + b*(-d*meijerg(((7/4, 9/4, 1), (3/2, 5/
2, 3)), ((7/4, 2, 9/4, 5/2, 3), (0,)), c**2/(d**2*x**2))/(2*pi**(3/2)*c**4)
 + I*d*meijerg(((1/2, 1, 5/4, 3/2, 7/4, 1), ()), ((5/4, 7/4), (1/2, 1, 2, 0
)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(2*pi**(3/2)*c**4))
```

Giac [B] time = 1.50363, size = 327, normalized size = 2.75

$$-\frac{\left(bc^2d+ad^3\right)\sqrt{dx+c}}{2\sqrt{dx-c}c^6}-\frac{2\left(bc^2d+ad^3\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+2c\right)c^5}-\frac{8\left(3\,bc^2d\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^8+3\,ad^3\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^8\right.}{3\left(\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^4/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(b*c^2*d + a*d^3)*sqrt(d*x + c)/(sqrt(d*x - c)*c^6) - 2*(b*c^2*d + a*d
^3)/(((sqrt(d*x + c) - sqrt(d*x - c))^2 + 2*c)*c^5) - 8/3*(3*b*c^2*d*(sqrt(
d*x + c) - sqrt(d*x - c))^8 + 3*a*d^3*(sqrt(d*x + c) - sqrt(d*x - c))^8 + 2
4*b*c^4*d*(sqrt(d*x + c) - sqrt(d*x - c))^4 + 48*a*c^2*d^3*(sqrt(d*x + c) -
 sqrt(d*x - c))^4 + 48*b*c^6*d + 80*a*c^4*d^3)/(((sqrt(d*x + c) - sqrt(d*x
- c))^4 + 4*c^2)^3*c^4)
```

3.377 $\int \frac{a+bx^2}{x^5(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=166

$$\frac{5ad^2+4bc^2}{8c^4x^2\sqrt{dx-c}\sqrt{c+dx}}-\frac{3d^2\left(5ad^2+4bc^2\right)}{8c^6\sqrt{dx-c}\sqrt{c+dx}}-\frac{3d^2\left(5ad^2+4bc^2\right)\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{8c^7}+\frac{a}{4c^2x^4\sqrt{dx-c}\sqrt{c+dx}}$$

```
[Out] (-3*d^2*(4*b*c^2 + 5*a*d^2))/(8*c^6*Sqrt[-c + d*x]*Sqrt[c + d*x]) + a/(4*c^
2*x^4*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (4*b*c^2 + 5*a*d^2)/(8*c^4*x^2*Sqrt[-
c + d*x]*Sqrt[c + d*x]) - (3*d^2*(4*b*c^2 + 5*a*d^2)*ArcTan[(Sqrt[-c + d*x]
*Sqrt[c + d*x])/c])/(8*c^7)
```

Rubi [A] time = 0.120213, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {454, 103, 12, 104, 21, 92, 205}

$$\frac{5ad^2+4bc^2}{8c^4x^2\sqrt{dx-c}\sqrt{c+dx}}-\frac{3d^2\left(5ad^2+4bc^2\right)}{8c^6\sqrt{dx-c}\sqrt{c+dx}}-\frac{3d^2\left(5ad^2+4bc^2\right)\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{8c^7}+\frac{a}{4c^2x^4\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)/(x^5*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```


```
[Out] (-3*d^2*(4*b*c^2 + 5*a*d^2))/(8*c^6*Sqrt[-c + d*x]*Sqrt[c + d*x]) + a/(4*c^
2*x^4*Sqrt[-c + d*x]*Sqrt[c + d*x]) + (4*b*c^2 + 5*a*d^2)/(8*c^4*x^2*Sqrt[-
c + d*x]*Sqrt[c + d*x]) - (3*d^2*(4*b*c^2 + 5*a*d^2)*ArcTan[(Sqrt[-c + d*x]
*Sqrt[c + d*x])/c])/(8*c^7)
```


Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{a+bx^2}{x^5(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx = \frac{a}{4c^2x^4\sqrt{-c+dx}\sqrt{c+dx}} + \frac{1}{4}\left(4b+\frac{5ad^2}{c^2}\right)\int \frac{1}{x^3(-c+dx)^{3/2}(c+dx)^{3/2}}\,dx$$

$$= \frac{a}{4c^2x^4\sqrt{-c+dx}\sqrt{c+dx}} + \frac{4bc^2+5ad^2}{8c^4x^2\sqrt{-c+dx}\sqrt{c+dx}} + \frac{\left(4bc^2+5ad^2\right)\int \frac{3d^2}{x(-c+dx)^{3/2}(c+dx)^{3/2}}}{8c^4}$$

$$= \frac{a}{4c^2x^4\sqrt{-c+dx}\sqrt{c+dx}} + \frac{4bc^2+5ad^2}{8c^4x^2\sqrt{-c+dx}\sqrt{c+dx}} + \frac{\left(3d^2\left(4bc^2+5ad^2\right)\right)\int \frac{1}{x(-c+dx)^{3/2}(c}}{8c^4}$$

$$= -\frac{3d^2\left(4bc^2+5ad^2\right)}{8c^6\sqrt{-c+dx}\sqrt{c+dx}} + \frac{a}{4c^2x^4\sqrt{-c+dx}\sqrt{c+dx}} + \frac{4bc^2+5ad^2}{8c^4x^2\sqrt{-c+dx}\sqrt{c+dx}} - \frac{(3d\,(4b}{}$$

$$= -\frac{3d^2\left(4bc^2+5ad^2\right)}{8c^6\sqrt{-c+dx}\sqrt{c+dx}} + \frac{a}{4c^2x^4\sqrt{-c+dx}\sqrt{c+dx}} + \frac{4bc^2+5ad^2}{8c^4x^2\sqrt{-c+dx}\sqrt{c+dx}} - \frac{(3d^2\,(4}{}$$

$$= -\frac{3d^2\left(4bc^2+5ad^2\right)}{8c^6\sqrt{-c+dx}\sqrt{c+dx}} + \frac{a}{4c^2x^4\sqrt{-c+dx}\sqrt{c+dx}} + \frac{4bc^2+5ad^2}{8c^4x^2\sqrt{-c+dx}\sqrt{c+dx}} - \frac{(3d^3\,(4}{}$$

$$= -\frac{3d^2\left(4bc^2+5ad^2\right)}{8c^6\sqrt{-c+dx}\sqrt{c+dx}} + \frac{a}{4c^2x^4\sqrt{-c+dx}\sqrt{c+dx}} + \frac{4bc^2+5ad^2}{8c^4x^2\sqrt{-c+dx}\sqrt{c+dx}} - \frac{3d^2\,(4b}{}$$

Mathematica [C] time = 0.0299248, size = 78, normalized size = 0.47

$$\frac{ac^4 - d^2x^4\left(5ad^2+4bc^2\right)\,{}_2F_1\left(-\frac{1}{2},2;\frac{1}{2};1-\frac{d^2x^2}{c^2}\right)}{4c^6x^4\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x^5*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

```
[Out] (a*c^4 - d^2*(4*b*c^2 + 5*a*d^2)*x^4*Hypergeometric2F1[-1/2, 2, 1/2, 1 - (d
^2*x^2)/c^2])/(4*c^6*x^4*Sqrt[-c + d*x]*Sqrt[c + d*x])
```

Maple [B] time = 0.026, size = 387, normalized size = 2.3

$$\frac{1}{8c^6x^4}\left(15\ln\left(-2\frac{c^2-\sqrt{-c^2}\sqrt{d^2x^2-c^2}}{x}\right)x^6ad^6+12\ln\left(-2\frac{c^2-\sqrt{-c^2}\sqrt{d^2x^2-c^2}}{x}\right)x^6bc^2d^4-15\ln\left(-2\frac{c^2-\sqrt{-c^2}\sqrt{d^2\ldots}}{x}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/x^5/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)
```

```
[Out] 1/8/c^6*(15*ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*x^6*a*d^6+12*ln
(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*x^6*b*c^2*d^4-15*ln(-2*(c^2-(
-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*x^4*a*c^2*d^4-12*ln(-2*(c^2-(-c^2)^(1/2
)*(d^2*x^2-c^2)^(1/2))/x)*x^4*b*c^4*d^2-15*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2)
*x^4*a*d^4-12*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2)*x^4*b*c^2*d^2+5*x^2*a*c^2*d^
2*(d^2*x^2-c^2)^(1/2)*(-c^2)^(1/2)+4*x^2*b*c^4*(d^2*x^2-c^2)^(1/2)*(-c^2)^(
1/2)+2*a*c^4*(d^2*x^2-c^2)^(1/2)*(-c^2)^(1/2))/x^4/(-c^2)^(1/2)/(d^2*x^2-c^
2)^(1/2)/(d*x+c)^(1/2)/(d*x-c)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^5/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.62221, size = 335, normalized size = 2.02

$$\frac{\left(2\,ac^5-3\left(4\,bc^3d^2+5\,acd^4\right)x^4+\left(4\,bc^5+5\,ac^3d^2\right)x^2\right)\sqrt{dx+c}\sqrt{dx-c}-6\left(\left(4\,bc^2d^4+5\,ad^6\right)x^6-\left(4\,bc^4d^2+5\,ac^2d^4\right)x^4\ldots\right.}{8\left(c^7d^2x^6-c^9x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^5/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*((2*a*c^5 - 3*(4*b*c^3*d^2 + 5*a*c*d^4)*x^4 + (4*b*c^5 + 5*a*c^3*d^2)*x
^2)*sqrt(d*x + c)*sqrt(d*x - c) - 6*((4*b*c^2*d^4 + 5*a*d^6)*x^6 - (4*b*c^4
*d^2 + 5*a*c^2*d^4)*x^4)*arctan(-(d*x - sqrt(d*x + c)*sqrt(d*x - c))/c))/(c
^7*d^2*x^6 - c^9*x^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**5/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.61696, size = 543, normalized size = 3.27

$$\frac{3\left(4\,bc^2d^2+5\,ad^4\right)\arctan\left(\frac{\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2}{2\,c}\right)}{4\,c^7}-\frac{\left(bc^2d^2+ad^4\right)\sqrt{dx+c}}{2\,\sqrt{dx-c}c^7}+\frac{2\left(bc^2d^2+ad^4\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^2+2\,c\right)c^6}+\frac{4\,bc^2d^2\left(\sqrt{dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^5/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] 3/4*(4*b*c^2*d^2 + 5*a*d^4)*arctan(1/2*(sqrt(d*x + c) - sqrt(d*x - c))^2/c)
/c^7 - 1/2*(b*c^2*d^2 + a*d^4)*sqrt(d*x + c)/(sqrt(d*x - c)*c^7) + 2*(b*c^2
*d^2 + a*d^4)/(((sqrt(d*x + c) - sqrt(d*x - c))^2 + 2*c)*c^6) + 1/2*(4*b*c^
2*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^14 + 7*a*d^4*(sqrt(d*x + c) - sqrt(d*
x - c))^14 + 16*b*c^4*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^10 + 60*a*c^2*d^4
*(sqrt(d*x + c) - sqrt(d*x - c))^10 - 64*b*c^6*d^2*(sqrt(d*x + c) - sqrt(d*
x - c))^6 - 240*a*c^4*d^4*(sqrt(d*x + c) - sqrt(d*x - c))^6 - 256*b*c^8*d^2
*(sqrt(d*x + c) - sqrt(d*x - c))^2 - 448*a*c^6*d^4*(sqrt(d*x + c) - sqrt(d*
x - c))^2)/(((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2)^4*c^6)
```

3.378 $\int \frac{1+c^2x^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx$

Optimal. Leaf size=40

$$\sqrt{cx-1}\sqrt{cx+1}+\tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right)$$

```
[Out] Sqrt[-1 + c*x]*Sqrt[1 + c*x] + ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]]
```

Rubi [A] time = 0.0544183, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {460, 92, 205}

$$\sqrt{cx-1}\sqrt{cx+1}+\tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + c^2*x^2)/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] Sqrt[-1 + c*x]*Sqrt[1 + c*x] + ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+c^2x^2}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx &= \sqrt{-1+cx}\sqrt{1+cx}+\int \frac{1}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx\\
&= \sqrt{-1+cx}\sqrt{1+cx}+c\,\text{Subst}\left(\int \frac{1}{c+cx^2}\,dx,x,\sqrt{-1+cx}\sqrt{1+cx}\right)\\
&= \sqrt{-1+cx}\sqrt{1+cx}+\tan^{-1}\left(\sqrt{-1+cx}\sqrt{1+cx}\right)
\end{aligned}$$

Mathematica [A] time = 0.0239749, size = 56, normalized size = 1.4

$$\frac{c^2x^2+\sqrt{c^2x^2-1}\tan^{-1}\left(\sqrt{c^2x^2-1}\right)-1}{\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + c^2*x^2)/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]),x]
```

```
[Out] (-1 + c^2*x^2 + Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]])/(Sqrt[-1 + c
*x]*Sqrt[1 + c*x])
```

Maple [A] time = 0.071, size = 53, normalized size = 1.3

$$\left(\sqrt{c^2x^2-1}-\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)\right)\sqrt{cx-1}\sqrt{cx+1}\frac{1}{\sqrt{c^2x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^2*x^2+1)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)
```

```
[Out] ((c^2*x^2-1)^(1/2)-arctan(1/(c^2*x^2-1)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)
/(c^2*x^2-1)^(1/2)
```

Maxima [A] time = 1.65547, size = 34, normalized size = 0.85

$$\sqrt{c^2x^2-1}-\arcsin\left(\frac{1}{\sqrt{c^2}|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(c^2*x^2 - 1) - arcsin(1/(sqrt(c^2)*abs(x)))
```

Fricas [A] time = 1.55084, size = 103, normalized size = 2.58

$$\sqrt{cx+1}\sqrt{cx-1}+2\arctan\left(-cx+\sqrt{cx+1}\sqrt{cx-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x + 1)*sqrt(c*x - 1) + 2*arctan(-c*x + sqrt(c*x + 1)*sqrt(c*x - 1))
```

Sympy [C] time = 16.1366, size = 148, normalized size = 3.7

$$\frac{G_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1 \\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix} \middle| \frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{G_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2} \\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0 \end{matrix} \middle| \frac{1}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{iG_{6,6}^{2,6}\left(\begin{matrix} -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0 \end{matrix} \middle| \frac{e^{2i\pi}}{c^2x^2}\right)}{4\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**2*x**2+1)/x/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0), ()),
1/(c**2*x**2))/(4*pi**(3/2)) - meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2,
 3/4, 1, 5/4, 3/2), (0,)), 1/(c**2*x**2))/(4*pi**(3/2)) + I*meijerg(((-1, -
3/4, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar
(2*I*pi)/(c**2*x**2))/(4*pi**(3/2)) + I*meijerg(((0, 1/4, 1/2, 3/4, 1, 1),
()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**
(3/2))
```

Giac [A] time = 1.13011, size = 54, normalized size = 1.35

$$\sqrt{cx+1}\sqrt{cx-1} - 2\arctan\left(\frac{1}{2}\left(\sqrt{cx+1} - \sqrt{cx-1}\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^2*x^2+1)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(c*x + 1)*sqrt(c*x - 1) - 2*arctan(1/2*(sqrt(c*x + 1) - sqrt(c*x - 1))^
2)
```

3.379 $\int \frac{x^{-\frac{2b^2c+a^2d}{b^2c+a^2d}}\left(c+dx^2\right)}{\sqrt{-a+bx}\sqrt{a+bx}}\,dx$

Optimal. Leaf size=53

$$\sqrt{bx-a}\sqrt{a+bx}\left(\frac{c}{a^2}+\frac{d}{b^2}\right)x^{-\frac{b^2c}{a^2d+b^2c}}$$

```
[Out] ((c/a^2 + d/b^2)*Sqrt[-a + b*x]*Sqrt[a + b*x])/x^((b^2*c)/(b^2*c + a^2*d))
```

Rubi [A] time = 0.0907204, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 57, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.018, Rules used = {450}

$$\sqrt{bx-a}\sqrt{a+bx}\left(\frac{c}{a^2}+\frac{d}{b^2}\right)x^{-\frac{b^2c}{a^2d+b^2c}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2)/(x^((2*b^2*c + a^2*d)/(b^2*c + a^2*d))*Sqrt[-a + b*x]*Sqrt[
a + b*x]),x]
```

```
[Out] ((c/a^2 + d/b^2)*Sqrt[-a + b*x]*Sqrt[a + b*x])/x^((b^2*c)/(b^2*c + a^2*d))
```

Rule 450

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2, n/2] &
& EqQ[a2*b1 + a1*b2, 0] && EqQ[a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1
), 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^{-\frac{2b^2c+a^2d}{b^2c+a^2d}}\left(c+dx^2\right)}{\sqrt{-a+bx}\sqrt{a+bx}}\,dx=\left(\frac{c}{a^2}+\frac{d}{b^2}\right)x^{-\frac{b^2c}{b^2c+a^2d}}\sqrt{-a+bx}\sqrt{a+bx}$$

Mathematica [C] time = 0.258403, size = 244, normalized size = 4.6

$$\frac{\sqrt{1-\frac{b^2x^2}{a^2}}\left(a^2d+b^2c\right)x^{-\frac{b^2c}{a^2d+b^2c}}\left(b^2dx^2\,{}_2F_1\left(\frac{1}{2},\frac{2da^2+b^2c}{2da^2+2b^2c};\frac{4da^2+3b^2c}{2da^2+2b^2c};\frac{b^2x^2}{a^2}\right)-\left(2a^2d+b^2c\right)\,{}_2F_1\left(\frac{1}{2},-\frac{b^2c}{2\left(da^2+b^2c\right)};\frac{2da^2+b^2c}{2da^2+2b^2c};\frac{b^2x^2}{a^2}\right)\right)}{b^2\sqrt{bx-a}\sqrt{a+bx}\left(2a^2d+b^2c\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2)/(x^((2*b^2*c + a^2*d)/(b^2*c + a^2*d))*Sqrt[-a + b*x]
*Sqrt[a + b*x]),x]
```

```
[Out] ((b^2*c + a^2*d)*Sqrt[1 - (b^2*x^2)/a^2]*(-((b^2*c + 2*a^2*d)*Hypergeometri
c2F1[1/2, -(b^2*c)/(2*(b^2*c + a^2*d)), (b^2*c + 2*a^2*d)/(2*b^2*c + 2*a^2*
```

```
d), (b^2*x^2)/a^2]) + b^2*d*x^2*Hypergeometric2F1[1/2, (b^2*c + 2*a^2*d)/(2
*b^2*c + 2*a^2*d), (3*b^2*c + 4*a^2*d)/(2*b^2*c + 2*a^2*d), (b^2*x^2)/a^2])
)/(b^2*(b^2*c + 2*a^2*d)*x^((b^2*c)/(b^2*c + a^2*d))*Sqrt[-a + b*x]*Sqrt[a
+ b*x])
```

Maple [A] time = 0.008, size = 66, normalized size = 1.3

$$\frac{x\left(a^2d+b^2c\right)}{b^2a^2}\sqrt{bx+a}\sqrt{bx-a}\left(x^{\frac{a^2d+2b^2c}{a^2d+b^2c}}\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)/(x^((a^2*d+2*b^2*c)/(a^2*d+b^2*c)))/(b*x-a)^(1/2)/(b*x+a)^(1/
2),x)
```

```
[Out] x*(a^2*d+b^2*c)*(b*x+a)^(1/2)/b^2/a^2/(x^((a^2*d+2*b^2*c)/(a^2*d+b^2*c)))*(
b*x-a)^(1/2)
```

Maxima [A] time = 1.61435, size = 107, normalized size = 2.02

$$\frac{\left(b^2c+a^2d\right)\sqrt{bx+a}\sqrt{bx-a}\,xe^{\left(-\frac{2b^2c\log(x)}{b^2c+a^2d}-\frac{a^2d\log(x)}{b^2c+a^2d}\right)}}{a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(x^((a^2*d+2*b^2*c)/(a^2*d+b^2*c)))/(b*x-a)^(1/2)/(b*x+
a)^(1/2),x, algorithm="maxima")
```

```
[Out] (b^2*c + a^2*d)*sqrt(b*x + a)*sqrt(b*x - a)*x*e^(-2*b^2*c*log(x)/(b^2*c + a
^2*d) - a^2*d*log(x)/(b^2*c + a^2*d))/(a^2*b^2)
```

Fricas [A] time = 1.79243, size = 128, normalized size = 2.42

$$\frac{\left(b^2c+a^2d\right)\sqrt{bx+a}\sqrt{bx-a}\,x}{a^2b^2x^{\frac{2b^2c+a^2d}{b^2c+a^2d}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(x^((a^2*d+2*b^2*c)/(a^2*d+b^2*c)))/(b*x-a)^(1/2)/(b*x+
a)^(1/2),x, algorithm="fricas")
```

```
[Out] (b^2*c + a^2*d)*sqrt(b*x + a)*sqrt(b*x - a)*x/(a^2*b^2*x^((2*b^2*c + a^2*d)
/(b^2*c + a^2*d)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(x**((a**2*d+2*b**2*c)/(a**2*d+b**2*c)))/(b*x-a)**(1/2
)/(b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{dx^2 + c}{\sqrt{bx + a}\sqrt{bx - a}x^{\frac{2b^2c + a^2d}{b^2c + a^2d}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)/(x^((a^2*d+2*b^2*c)/(a^2*d+b^2*c)))/(b*x-a)^(1/2)/(b*x+
a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)/(sqrt(b*x + a)*sqrt(b*x - a)*x^((2*b^2*c + a^2*d)/(b^
2*c + a^2*d))), x)
```

3.380 $\int \frac{1}{\sqrt{-1-\sqrt{x}}\sqrt{-1+\sqrt{x}}\sqrt{1+x}}\,dx$

Optimal. Leaf size=36

$$\frac{\sqrt{1-x}\sin^{-1}(x)}{\sqrt{-\sqrt{x}-1}\sqrt{\sqrt{x}-1}}$$

```
[Out] (Sqrt[1 - x]*ArcSin[x])/(Sqrt[-1 - Sqrt[x]]*Sqrt[-1 + Sqrt[x]])
```

Rubi [A] time = 0.0203589, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {519, 41, 216}

$$\frac{\sqrt{1-x}\sin^{-1}(x)}{\sqrt{-\sqrt{x}-1}\sqrt{\sqrt{x}-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 - Sqrt[x]]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + x]),x]
```

```
[Out] (Sqrt[1 - x]*ArcSin[x])/(Sqrt[-1 - Sqrt[x]]*Sqrt[-1 + Sqrt[x]])
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{-1-\sqrt{x}}\sqrt{-1+\sqrt{x}}\sqrt{1+x}}\,dx = \frac{\sqrt{1-x}\int \frac{1}{\sqrt{1-x}\sqrt{1+x}}\,dx}{\sqrt{-1-\sqrt{x}}\sqrt{-1+\sqrt{x}}}$$
$$= \frac{\sqrt{1-x}\int \frac{1}{\sqrt{1-x^2}}\,dx}{\sqrt{-1-\sqrt{x}}\sqrt{-1+\sqrt{x}}}$$
$$= \frac{\sqrt{1-x}\sin^{-1}(x)}{\sqrt{-1-\sqrt{x}}\sqrt{-1+\sqrt{x}}}$$

Mathematica [A] time = 0.0163369, size = 36, normalized size = 1.

$$\frac{\sqrt{1-x}\sin^{-1}(x)}{\sqrt{-\sqrt{x}-1}\sqrt{\sqrt{x}-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 - Sqrt[x]]*Sqrt[-1 + Sqrt[x]]*Sqrt[1 + x]),x]
```

```
[Out] (Sqrt[1 - x]*ArcSin[x])/(Sqrt[-1 - Sqrt[x]]*Sqrt[-1 + Sqrt[x]])
```

Maple [F] time = 0.601, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{1+x}}\frac{1}{\sqrt{-1-\sqrt{x}}}\frac{1}{\sqrt{-1+\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)^(1/2)/(-1-x^(1/2))^(1/2)/(-1+x^(1/2))^(1/2),x)
```

```
[Out] int(1/(1+x)^(1/2)/(-1-x^(1/2))^(1/2)/(-1+x^(1/2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x+1}\sqrt{\sqrt{x}-1}\sqrt{-\sqrt{x}-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/2)/(-1-x^(1/2))^(1/2)/(-1+x^(1/2))^(1/2),x, algorithm=
"maxima")
```

```
[Out] integrate(1/(sqrt(x + 1)*sqrt(sqrt(x) - 1)*sqrt(-sqrt(x) - 1)), x)
```

Fricas [C] time = 1.70318, size = 194, normalized size = 5.39

$$-i \log\left(\frac{\sqrt{x+1}\sqrt{\sqrt{x}-1}\sqrt{-\sqrt{x}-1}+i\,x-1}{x}\right)+i \log\left(\frac{\sqrt{x+1}\sqrt{\sqrt{x}-1}\sqrt{-\sqrt{x}-1}-i\,x-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/2)/(-1-x^(1/2))^(1/2)/(-1+x^(1/2))^(1/2),x, algorithm=
"fricas")
```

```
[Out] -I*log((sqrt(x + 1)*sqrt(sqrt(x) - 1)*sqrt(-sqrt(x) - 1) + I*x - 1)/x) + I*
log((sqrt(x + 1)*sqrt(sqrt(x) - 1)*sqrt(-sqrt(x) - 1) - I*x - 1)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\sqrt{x}-1}\sqrt{\sqrt{x}-1}\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)**(1/2)/(-1-x**(1/2))**(1/2)/(-1+x**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-sqrt(x) - 1)*sqrt(sqrt(x) - 1)*sqrt(x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x+1}\sqrt{\sqrt{x}-1}\sqrt{-\sqrt{x}-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)^(1/2)/(-1-x^(1/2))^(1/2)/(-1+x^(1/2))^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate(1/(sqrt(x + 1)*sqrt(sqrt(x) - 1)*sqrt(-sqrt(x) - 1)), x)
```

3.381 $\int \frac{1}{\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}\sqrt{a^2+b^2x}}\,dx$

Optimal. Leaf size=75

$$-\frac{2\sqrt{a^2-b^2x}\tan^{-1}\left(\frac{\sqrt{a^2-b^2x}}{\sqrt{a^2+b^2x}}\right)}{b^2\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}}$$

```
[Out] (-2*Sqrt[a^2 - b^2*x]*ArcTan[Sqrt[a^2 - b^2*x]/Sqrt[a^2 + b^2*x]])/(b^2*Sqr
t[a - b*Sqrt[x]]*Sqrt[a + b*Sqrt[x]])
```

Rubi [A] time = 0.0502034, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {519, 63, 217, 203}

$$-\frac{2\sqrt{a^2-b^2x}\tan^{-1}\left(\frac{\sqrt{a^2-b^2x}}{\sqrt{a^2+b^2x}}\right)}{b^2\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a - b*Sqrt[x]]*Sqrt[a + b*Sqrt[x]]*Sqrt[a^2 + b^2*x]),x]
```

```
[Out] (-2*Sqrt[a^2 - b^2*x]*ArcTan[Sqrt[a^2 - b^2*x]/Sqrt[a^2 + b^2*x]])/(b^2*Sqr
t[a - b*Sqrt[x]]*Sqrt[a + b*Sqrt[x]])
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}\sqrt{a^2+b^2x}}\,dx = \frac{\sqrt{a^2-b^2x}\int \frac{1}{\sqrt{a^2-b^2x}\sqrt{a^2+b^2x}}\,dx}{\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}}$$
$$= -\frac{\left(2\sqrt{a^2-b^2x}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{2a^2-x^2}}\,dx, x, \sqrt{a^2-b^2x}\right)}{b^2\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}}$$
$$= -\frac{\left(2\sqrt{a^2-b^2x}\right)\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sqrt{a^2-b^2x}}{\sqrt{a^2+b^2x}}\right)}{b^2\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}}$$
$$= -\frac{2\sqrt{a^2-b^2x}\tan^{-1}\left(\frac{\sqrt{a^2-b^2x}}{\sqrt{a^2+b^2x}}\right)}{b^2\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}}$$

Mathematica [A] time = 0.0332435, size = 75, normalized size = 1.

$$-\frac{2\sqrt{a^2-b^2x}\tan^{-1}\left(\frac{\sqrt{a^2-b^2x}}{\sqrt{a^2+b^2x}}\right)}{b^2\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a - b*Sqrt[x]]*Sqrt[a + b*Sqrt[x]]*Sqrt[a^2 + b^2*x]),x]
```

```
[Out] (-2*Sqrt[a^2 - b^2*x]*ArcTan[Sqrt[a^2 - b^2*x]/Sqrt[a^2 + b^2*x]])/(b^2*Sqr
t[a - b*Sqrt[x]]*Sqrt[a + b*Sqrt[x]])
```

Maple [F] time = 0.663, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b^2x+a^2}}\frac{1}{\sqrt{a-b\sqrt{x}}}\frac{1}{\sqrt{a+b\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x+a^2)^(1/2)/(a-b*x^(1/2))^(1/2)/(a+b*x^(1/2))^(1/2),x)
```

```
[Out] int(1/(b^2*x+a^2)^(1/2)/(a-b*x^(1/2))^(1/2)/(a+b*x^(1/2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b^2x+a^2}\sqrt{b\sqrt{x}+a}\sqrt{-b\sqrt{x}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x+a^2)^(1/2)/(a-b*x^(1/2))^(1/2)/(a+b*x^(1/2))^(1/2),x, al
gorithm="maxima")
```

```
[Out] integrate(1/(sqrt(b^2*x + a^2)*sqrt(b*sqrt(x) + a)*sqrt(-b*sqrt(x) + a)), x
)
```

Fricas [A] time = 1.8122, size = 124, normalized size = 1.65

$$-\frac{2\arctan\left(-\frac{a^2-\sqrt{b^2x+a^2}\sqrt{b\sqrt{x}+a}\sqrt{-b\sqrt{x}+a}}{b^2x}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x+a^2)^(1/2)/(a-b*x^(1/2))^(1/2)/(a+b*x^(1/2))^(1/2),x, al
gorithm="fricas")
```

```
[Out] -2*arctan(-(a^2 - sqrt(b^2*x + a^2)*sqrt(b*sqrt(x) + a)*sqrt(-b*sqrt(x) + a
))/(b^2*x))/b^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a-b\sqrt{x}}\sqrt{a+b\sqrt{x}}\sqrt{a^2+b^2x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x+a**2)**(1/2)/(a-b*x**(1/2))**(1/2)/(a+b*x**(1/2))**(1/2
),x)
```

```
[Out] Integral(1/(sqrt(a - b*sqrt(x))*sqrt(a + b*sqrt(x))*sqrt(a**2 + b**2*x)), x
)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x+a^2)^(1/2)/(a-b*x^(1/2))^(1/2)/(a+b*x^(1/2))^(1/2),x, al
gorithm="giac")
```

```
[Out] Timed out
```

3.382 $\int (a - bx^n)^p (a + bx^n)^p \left(c + dx^{2n}\right)^q dx$

Optimal. Leaf size=113

$$x(a - bx^n)^p (a + bx^n)^p \left(1 - \frac{b^2x^{2n}}{a^2}\right)^{-p} \left(c + dx^{2n}\right)^q \left(\frac{dx^{2n}}{c} + 1\right)^{-q} F_1\left(\frac{1}{2n}; -p, -q; \frac{1}{2}\left(2 + \frac{1}{n}\right); \frac{b^2x^{2n}}{a^2}, -\frac{dx^{2n}}{c}\right)$$

```
[Out] (x*(a - b*x^n)^p*(a + b*x^n)^p*(c + d*x^(2*n))^q*AppellF1[1/(2*n), -p, -q,
(2 + n^(-1))/2, (b^2*x^(2*n))/a^2, -((d*x^(2*n))/c)])/((1 - (b^2*x^(2*n))/a
^2)^p*(1 + (d*x^(2*n))/c)^q)
```

Rubi [A] time = 0.0904555, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {519, 430, 429}

$$x(a - bx^n)^p (a + bx^n)^p \left(1 - \frac{b^2x^{2n}}{a^2}\right)^{-p} \left(c + dx^{2n}\right)^q \left(\frac{dx^{2n}}{c} + 1\right)^{-q} F_1\left(\frac{1}{2n}; -p, -q; \frac{1}{2}\left(2 + \frac{1}{n}\right); \frac{b^2x^{2n}}{a^2}, -\frac{dx^{2n}}{c}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^n)^p*(a + b*x^n)^p*(c + d*x^(2*n))^q,x]
```

```
[Out] (x*(a - b*x^n)^p*(a + b*x^n)^p*(c + d*x^(2*n))^q*AppellF1[1/(2*n), -p, -q,
(2 + n^(-1))/2, (b^2*x^(2*n))/a^2, -((d*x^(2*n))/c)])/((1 - (b^2*x^(2*n))/a
^2)^p*(1 + (d*x^(2*n))/c)^q)
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int (a-bx^n)^p (a+bx^n)^p \left(c+dx^{2n}\right)^q dx = \left((a-bx^n)^p (a+bx^n)^p \left(a^2-b^2x^{2n}\right)^{-p}\right) \int \left(a^2-b^2x^{2n}\right)^p \left(c+dx^{2n}\right)^q dx$$

$$= \left((a-bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p}\right) \int \left(1-\frac{b^2x^{2n}}{a^2}\right)^p \left(c+dx^{2n}\right)^q dx$$

$$= \left((a-bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} \left(c+dx^{2n}\right)^q \left(1+\frac{dx^{2n}}{c}\right)^{-q}\right) \int \left(1-\frac{b^2x^{2n}}{a^2}\right)^p \Big($$

$$= x(a-bx^n)^p (a+bx^n)^p \left(1-\frac{b^2x^{2n}}{a^2}\right)^{-p} \left(c+dx^{2n}\right)^q \left(1+\frac{dx^{2n}}{c}\right)^{-q} F_1\left(\frac{1}{2n};-p,-q;\frac{1}{2}\right.$$

Mathematica [F] time = 0.238882, size = 0, normalized size = 0.

$$\int (a-bx^n)^p (a+bx^n)^p \left(c+dx^{2n}\right)^q dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a - b*x^n)^p*(a + b*x^n)^p*(c + d*x^(2*n))^q,x]
```

```
[Out] Integrate[(a - b*x^n)^p*(a + b*x^n)^p*(c + d*x^(2*n))^q, x]
```

Maple [F] time = 1.492, size = 0, normalized size = 0.

$$\int (a-bx^n)^p (a+bx^n)^p \left(c+dx^{2n}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-b*x^n)^p*(a+b*x^n)^p*(c+d*x^(2*n))^q,x)
```

```
[Out] int((a-b*x^n)^p*(a+b*x^n)^p*(c+d*x^(2*n))^q,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(dx^{2n}+c\right)^q (bx^n+a)^p (-bx^n+a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^n)^p*(a+b*x^n)^p*(c+d*x^(2*n))^q,x, algorithm="maxima")
```

```
[Out] integrate((d*x^(2*n) + c)^q*(b*x^n + a)^p*(-b*x^n + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(dx^{2n}+c\right)^q (bx^n+a)^p (-bx^n+a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^n)^p*(a+b*x^n)^p*(c+d*x^(2*n))^q,x, algorithm="fricas")
```

```
[Out] integral((d*x^(2*n) + c)^q*(b*x^n + a)^p*(-b*x^n + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x**n)**p*(a+b*x**n)**p*(c+d*x**(2*n))**q,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(dx^{2\,n}+c\right)^{q}(bx^{n}+a)^{p}(-bx^{n}+a)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^n)^p*(a+b*x^n)^p*(c+d*x^(2*n))^q,x, algorithm="giac")
```

```
[Out] integrate((d*x^(2*n) + c)^q*(b*x^n + a)^p*(-b*x^n + a)^p, x)
```

3.383 $\int (a-bx^n)^p (a+bx^n)^p \left(a^2+b^2x^{2n}\right)^p \, dx$

Optimal. Leaf size=87

$$x(a-bx^n)^p (a+bx^n)^p \left(a^2+b^2x^{2n}\right)^p \left(1-\frac{b^4x^{4n}}{a^4}\right)^{-p} {}_2F_1\left(\frac{1}{4n},-p;\frac{1}{4}\left(4+\frac{1}{n}\right);\frac{b^4x^{4n}}{a^4}\right)$$

```
[Out] (x*(a - b*x^n)^p*(a + b*x^n)^p*(a^2 + b^2*x^(2*n))^p*Hypergeometric2F1[1/(4
*n), -p, (4 + n^(-1))/4, (b^4*x^(4*n))/a^4])/(1 - (b^4*x^(4*n))/a^4)^p
```

Rubi [A] time = 0.0587613, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {519, 253, 246, 245}

$$x(a-bx^n)^p (a+bx^n)^p \left(a^2+b^2x^{2n}\right)^p \left(1-\frac{b^4x^{4n}}{a^4}\right)^{-p} {}_2F_1\left(\frac{1}{4n},-p;\frac{1}{4}\left(4+\frac{1}{n}\right);\frac{b^4x^{4n}}{a^4}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^n)^p*(a + b*x^n)^p*(a^2 + b^2*x^(2*n))^p,x]
```

```
[Out] (x*(a - b*x^n)^p*(a + b*x^n)^p*(a^2 + b^2*x^(2*n))^p*Hypergeometric2F1[1/(4
*n), -p, (4 + n^(-1))/4, (b^4*x^(4*n))/a^4])/(1 - (b^4*x^(4*n))/a^4)^p
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 253

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Dist[((a1 + b1*x^n)^FracPart[p]*(a2 + b2*x^n)^FracPart[p])/(a1*a2
+ b1*b2*x^(2*n))^FracPart[p], Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Free
Q[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] && !IntegerQ[p]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int (a-bx^n)^p (a+bx^n)^p \left(a^2+b^2x^{2n}\right)^p dx = \left((a-bx^n)^p (a+bx^n)^p \left(a^2-b^2x^{2n}\right)^{-p}\right) \int \left(a^2-b^2x^{2n}\right)^p \left(a^2+b^2x^{2n}\right)^p dx$$
$$= \left((a-bx^n)^p (a+bx^n)^p \left(a^2+b^2x^{2n}\right)^p \left(a^4-b^4x^{4n}\right)^{-p}\right) \int \left(a^4-b^4x^{4n}\right)^p dx$$
$$= \left((a-bx^n)^p (a+bx^n)^p \left(a^2+b^2x^{2n}\right)^p \left(1-\frac{b^4x^{4n}}{a^4}\right)^{-p}\right) \int \left(1-\frac{b^4x^{4n}}{a^4}\right)^p dx$$
$$= x(a-bx^n)^p (a+bx^n)^p \left(a^2+b^2x^{2n}\right)^p \left(1-\frac{b^4x^{4n}}{a^4}\right)^{-p} {}_2F_1\left(\frac{1}{4n},-p;\frac{1}{4}\left(4+\frac{1}{n}\right)\right.$$

Mathematica [A] time = 0.0419371, size = 87, normalized size = 1.

$$x(a-bx^n)^p (a+bx^n)^p \left(a^2+b^2x^{2n}\right)^p \left(1-\frac{b^4x^{4n}}{a^4}\right)^{-p} {}_2F_1\left(\frac{1}{4n},-p;1+\frac{1}{4n};\frac{b^4x^{4n}}{a^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^n)^p*(a + b*x^n)^p*(a^2 + b^2*x^(2*n))^p,x]
```

```
[Out] (x*(a - b*x^n)^p*(a + b*x^n)^p*(a^2 + b^2*x^(2*n))^p*Hypergeometric2F1[1/(4
*n), -p, 1 + 1/(4*n), (b^4*x^(4*n))/a^4])/(1 - (b^4*x^(4*n))/a^4)^p
```

Maple [F] time = 1.464, size = 0, normalized size = 0.

$$\int (a-bx^n)^p (a+bx^n)^p \left(a^2+b^2x^{2n}\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-b*x^n)^p*(a+b*x^n)^p*(a^2+b^2*x^(2*n))^p,x)
```

```
[Out] int((a-b*x^n)^p*(a+b*x^n)^p*(a^2+b^2*x^(2*n))^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2x^{2n}+a^2\right)^p (bx^n+a)^p (-bx^n+a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^n)^p*(a+b*x^n)^p*(a^2+b^2*x^(2*n))^p,x, algorithm="maxima"
)
```

```
[Out] integrate((b^2*x^(2*n) + a^2)^p*(b*x^n + a)^p*(-b*x^n + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2x^{2n}+a^2\right)^p (bx^n+a)^p (-bx^n+a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^n)^p*(a+b*x^n)^p*(a^2+b^2*x^(2*n))^p,x, algorithm="fricas"
)
```

```
[Out] integral((b^2*x^(2*n) + a^2)^p*(b*x^n + a)^p*(-b*x^n + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x**n)**p*(a+b*x**n)**p*(a**2+b**2*x**(2*n))**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b^2 x^{2\,n} + a^2\right)^p (bx^n + a)^p (-bx^n + a)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^n)^p*(a+b*x^n)^p*(a^2+b^2*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] integrate((b^2*x^(2*n) + a^2)^p*(b*x^n + a)^p*(-b*x^n + a)^p, x)
```

3.384 $\int \frac{(c+dx^{2n})^p}{(a-bx^n)(a+bx^n)}\,dx$

Optimal. Leaf size=76

$$\frac{x\left(c+dx^{2n}\right)^p\left(\frac{dx^{2n}}{c}+1\right)^{-p}F_1\left(\frac{1}{2n};1,-p;\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2},-\frac{dx^{2n}}{c}\right)}{a^2}$$

```
[Out] (x*(c + d*x^(2*n))^p*AppellF1[1/(2*n), 1, -p, (2 + n^(-1))/2, (b^2*x^(2*n))
/a^2, -((d*x^(2*n))/c)])/(a^2*(1 + (d*x^(2*n))/c)^p)
```

Rubi [A] time = 0.056669, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {517, 430, 429}

$$\frac{x\left(c+dx^{2n}\right)^p\left(\frac{dx^{2n}}{c}+1\right)^{-p}F_1\left(\frac{1}{2n};1,-p;\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2},-\frac{dx^{2n}}{c}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^(2*n))^p/((a - b*x^n)*(a + b*x^n)),x]
```

```
[Out] (x*(c + d*x^(2*n))^p*AppellF1[1/(2*n), 1, -p, (2 + n^(-1))/2, (b^2*x^(2*n))
/a^2, -((d*x^(2*n))/c)])/(a^2*(1 + (d*x^(2*n))/c)^p)
```

Rule 517

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \frac{\left(c+dx^{2n}\right)^p}{(a-bx^n)(a+bx^n)}\,dx = \int \frac{\left(c+dx^{2n}\right)^p}{a^2-b^2x^{2n}}\,dx$$
$$=\left(\left(c+dx^{2n}\right)^p\left(1+\frac{dx^{2n}}{c}\right)^{-p}\right)\int \frac{\left(1+\frac{dx^{2n}}{c}\right)^p}{a^2-b^2x^{2n}}\,dx$$
$$=\frac{x\left(c+dx^{2n}\right)^p\left(1+\frac{dx^{2n}}{c}\right)^{-p} F_1\left(\frac{1}{2n};1,-p;\frac{1}{2}\left(2+\frac{1}{n}\right);\frac{b^2x^{2n}}{a^2},-\frac{dx^{2n}}{c}\right)}{a^2}$$

Mathematica [B] time = 0.289537, size = 258, normalized size = 3.39

$$\frac{a^2c(2n+1)x\left(c+dx^{2n}\right)^p F_1\left(\frac{1}{2n};-p,1;1+\frac{1}{2n};-\frac{dx^{2n}}{c},\frac{b^2x^{2n}}{a^2}\right)}{\left(a^2-b^2x^{2n}\right)\left(2a^2dnpx^{2n}F_1\left(1+\frac{1}{2n};1-p,1;2+\frac{1}{2n};-\frac{dx^{2n}}{c},\frac{b^2x^{2n}}{a^2}\right)+2b^2cnx^{2n}F_1\left(1+\frac{1}{2n};-p,2;2+\frac{1}{2n};-\frac{dx^{2n}}{c},\frac{b^2x^{2n}}{a^2}\right)+a^2c(2\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x^(2*n))^p/((a - b*x^n)*(a + b*x^n)),x]
```

```
[Out] (a^2*c*(1 + 2*n)*x*(c + d*x^(2*n))^p*AppellF1[1/(2*n), -p, 1, 1 + 1/(2*n),
-((d*x^(2*n))/c), (b^2*x^(2*n))/a^2])/((a^2 - b^2*x^(2*n))*(2*a^2*d*n*p*x^(
2*n)*AppellF1[1 + 1/(2*n), 1 - p, 1, 2 + 1/(2*n), -((d*x^(2*n))/c), (b^2*x^
(2*n))/a^2] + 2*b^2*c*n*x^(2*n)*AppellF1[1 + 1/(2*n), -p, 2, 2 + 1/(2*n), -
((d*x^(2*n))/c), (b^2*x^(2*n))/a^2] + a^2*c*(1 + 2*n)*AppellF1[1/(2*n), -p,
 1, 1 + 1/(2*n), -((d*x^(2*n))/c), (b^2*x^(2*n))/a^2]))
```

Maple [F] time = 1.048, size = 0, normalized size = 0.

$$\int \frac{\left(c+dx^{2n}\right)^p}{(a-bx^n)(a+bx^n)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d*x^(2*n))^p/(a-b*x^n)/(a+b*x^n),x)
```

```
[Out] int((c+d*x^(2*n))^p/(a-b*x^n)/(a+b*x^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(dx^{2n}+c\right)^p}{(bx^n+a)(bx^n-a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(2*n))^p/(a-b*x^n)/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] -integrate((d*x^(2*n) + c)^p/((b*x^n + a)*(b*x^n - a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(dx^{2n}+c\right)^{p}}{b^{2}x^{2n}-a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(2*n))^p/(a-b*x^n)/(a+b*x^n),x, algorithm="fricas")
```

```
[Out] integral(-(d*x^(2*n) + c)^p/(b^2*x^(2*n) - a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**(2*n))**p/(a-b*x**n)/(a+b*x**n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(dx^{2n}+c\right)^{p}}{(bx^{n}+a)(bx^{n}-a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^(2*n))^p/(a-b*x^n)/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate(-(d*x^(2*n) + c)^p/((b*x^n + a)*(b*x^n - a)), x)
```

3.385 $\int \left(a-bx^{n/2}\right)^p \left(a+bx^{n/2}\right)^p \left(\frac{a^2d(1+p)}{b^2\left(1+\frac{-1-2n-np}{n}\right)}+dx^n\right)^{\frac{-1-2n-np}{n}} dx$

Optimal. Leaf size=96

$$-\frac{b^2x(np+n+1)\left(a-bx^{n/2}\right)^{p+1}\left(a+bx^{n/2}\right)^{p+1}\left(dx^n-\frac{a^2dn(p+1)}{b^2(np+n+1)}\right)^{-\frac{np+n+1}{n}}}{a^4dn(p+1)}$$

```
[Out] -((b^2*(1 + n + n*p)*x*(a - b*x^(n/2))^(1 + p)*(a + b*x^(n/2))^(1 + p))/(a^
4*d*n*(1 + p)*(-((a^2*d*n*(1 + p))/(b^2*(1 + n + n*p))) + d*x^n)^((1 + n +
n*p)/n)))
```

Rubi [A] time = 0.12239, antiderivative size = 104, normalized size of antiderivative = 1.08, number of steps used = 2, number of rules used = 2, integrand size = 76, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {519, 381}

$$-\frac{b^2x(np+n+1)\left(a^2-b^2x^n\right)\left(a-bx^{n/2}\right)^p\left(a+bx^{n/2}\right)^p\left(dx^n-\frac{a^2dn(p+1)}{b^2(np+n+1)}\right)^{-\frac{np+n+1}{n}}}{a^4dn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^(n/2))^p*(a + b*x^(n/2))^p*((a^2*d*(1 + p))/(b^2*(1 + (-1 - 2*
n - n*p)/n)) + d*x^n)^((-1 - 2*n - n*p)/n),x]
```

```
[Out] -((b^2*(1 + n + n*p)*x*(a - b*x^(n/2))^p*(a + b*x^(n/2))^p*(a^2 - b^2*x^n))
/(a^4*d*n*(1 + p)*(-((a^2*d*n*(1 + p))/(b^2*(1 + n + n*p))) + d*x^n)^((1 +
n + n*p)/n)))
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 381

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c), x] /; FreeQ[{a,
b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] &&
EqQ[a*d*(p + 1) + b*c*(q + 1), 0]
```

Rubi steps

$$\int \left(a-bx^{n/2}\right)^p \left(a+bx^{n/2}\right)^p \left(\frac{a^2d(1+p)}{b^2\left(1+\frac{-1-2n-np}{n}\right)}+dx^n\right)^{\frac{-1-2n-np}{n}} dx = \left(\left(a-bx^{n/2}\right)^p \left(a+bx^{n/2}\right)^p \left(a^2-b^2x^n\right)^{-p}\right) \int \left(a^2-b\right.$$

$$= -\frac{b^2(1+n+np)x\left(a-bx^{n/2}\right)^p \left(a+bx^{n/2}\right)^p \left(a^2-b^2\right.}{a^4dn(1+p)}$$

Mathematica [A] time = 0.275633, size = 103, normalized size = 1.07

$$-\frac{b^2x(np+n+1)\left(a^2-b^2x^n\right)\left(a-bx^{n/2}\right)^p\left(a+bx^{n/2}\right)^p\left(d\left(x^n-\frac{a^2n(p+1)}{b^2(np+n+1)}\right)\right)^{-\frac{np+n+1}{n}}}{a^4dn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^(n/2))^p*(a + b*x^(n/2))^p*((a^2*d*(1 + p))/(b^2*(1 + (-
1 - 2*n - n*p)/n)) + d*x^n)^((-1 - 2*n - n*p)/n),x]
```

```
[Out] -((b^2*(1 + n + n*p)*x*(a - b*x^(n/2))^p*(a + b*x^(n/2))^p*(a^2 - b^2*x^n))
/(a^4*d*n*(1 + p)*(d*(-((a^2*n*(1 + p))/(b^2*(1 + n + n*p))) + x^n))^((1 +
n + n*p)/n)))
```

Maple [F] time = 2.152, size = 0, normalized size = 0.

$$\int \left(a-bx^{\frac{n}{2}}\right)^p \left(a+bx^{\frac{n}{2}}\right)^p \left(\frac{a^2d\left(1+p\right)}{b^2}\left(1+\frac{-np-2n-1}{n}\right)^{-1}+dx^n\right)^{\frac{-np-2n-1}{n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-b*x^(1/2*n))^p*(a+b*x^(1/2*n))^p*(a^2*d*(1+p)/b^2/(1+(-n*p-2*n-1)/n)
+d*x^n)^((-n*p-2*n-1)/n),x)
```

```
[Out] int((a-b*x^(1/2*n))^p*(a+b*x^(1/2*n))^p*(a^2*d*(1+p)/b^2/(1+(-n*p-2*n-1)/n)
+d*x^n)^((-n*p-2*n-1)/n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^{\frac{1}{2}n}+a\right)^p\left(-bx^{\frac{1}{2}n}+a\right)^p}{\left(dx^n-\frac{a^2d\left(p+1\right)}{b^2\left(\frac{np+2n+1}{n}-1\right)}\right)^{\frac{np+2n+1}{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^(1/2*n))^p*(a+b*x^(1/2*n))^p*(a^2*d*(1+p)/b^2/(1+(-n*p-2*n
-1)/n)+d*x^n)^((-n*p-2*n-1)/n),x, algorithm="maxima")
```

```
[Out] integrate((b*x^(1/2*n) + a)^p*(-b*x^(1/2*n) + a)^p/(d*x^n - a^2*d*(p + 1)/(
b^2*((n*p + 2*n + 1)/n - 1)))^((n*p + 2*n + 1)/n), x)
```

Fricas [A] time = 1.90123, size = 369, normalized size = 3.84

$$\frac{\left(\left(b^4np+b^4n+b^4\right)xx^{2n}-\left(2a^2b^2np+2a^2b^2n+a^2b^2\right)xx^n+\left(a^4np+a^4n\right)x\right)\left(bx^{\frac{1}{2}n}+a\right)^p\left(-bx^{\frac{1}{2}n}+a\right)^p}{\left(a^4np+a^4n\right)\left(-\frac{a^2dnp+a^2dn-\left(b^2dnp+b^2dn+b^2d\right)x^n}{b^2np+b^2n+b^2}\right)^{\frac{np+2n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^(1/2*n))^p*(a+b*x^(1/2*n))^p*(a^2*d*(1+p)/b^2/(1+(-n*p-2*n
-1)/n)+d*x^n)^((-n*p-2*n-1)/n),x, algorithm="fricas")
```

```
[Out] ((b^4*n*p + b^4*n + b^4)*x*x^(2*n) - (2*a^2*b^2*n*p + 2*a^2*b^2*n + a^2*b^2
)*x*x^n + (a^4*n*p + a^4*n)*x)*(b*x^(1/2*n) + a)^p*(-b*x^(1/2*n) + a)^p/((a
^4*n*p + a^4*n)*(-(a^2*d*n*p + a^2*d*n - (b^2*d*n*p + b^2*d*n + b^2*d)*x^n)
/(b^2*n*p + b^2*n + b^2))^((n*p + 2*n + 1)/n))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x**(1/2*n))**p*(a+b*x**(1/2*n))**p*(a**2*d*(1+p)/b**2/(1+(-n
*p-2*n-1)/n)+d*x**n)**((-n*p-2*n-1)/n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(bx^{\frac{1}{2}n}+a\right)^p\left(-bx^{\frac{1}{2}n}+a\right)^p}{\left(dx^n-\frac{a^2d(p+1)}{b^2\left(\frac{np+2n+1}{n}-1\right)}\right)^{\frac{np+2n+1}{n}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-b*x^(1/2*n))^p*(a+b*x^(1/2*n))^p*(a^2*d*(1+p)/b^2/(1+(-n*p-2*n
-1)/n)+d*x^n)^((-n*p-2*n-1)/n),x, algorithm="giac")
```

```
[Out] integrate((b*x^(1/2*n) + a)^p*(-b*x^(1/2*n) + a)^p/(d*x^n - a^2*d*(p + 1)/(
b^2*((n*p + 2*n + 1)/n - 1)))^((n*p + 2*n + 1)/n), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal, debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=", ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
```

```
                      #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```